

# 代码库

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## 1 数学

### 1.1 快速求逆元

使用条件:  $x \in [0, mod)$  并且  $x$  与  $mod$  互质

```
LL inv(LL a, LL p) {
    LL d, x, y;
    exgcd(a, p, d, x, y);
    return d == 1 ? (x + p) % p : -1;
}
```

### 1.2 扩展欧几里德算法

返回结果:  $ax + by = gcd(a, b)$

```
LL exgcd(LL a, LL b, LL &x, LL &y) {
    if(!b) return x = 1, y = 0, a;
    else {
        LL d = exgcd(b, a % b, x, y);
        LL t = x; x = y;
        y = t - a / b * y;
        return d;
    }
}
```

### 1.3 中国剩余定理

返回结果:

$$x \equiv r_i(mod\ p_i) \quad (0 \leq i < n)$$

```
LL china(int n, int *a, int *m) {
    LL M = 1, d, x = 0, y;
    for(int i = 0; i < n; i++)
        M *= m[i];
    for(int i = 0; i < n; i++) {
        LL w = M / m[i];
        d = exgcd(m[i], w, d, y);

```

```
        y = (y % M + M) % M;
        x = (x + y * w % M * a[i]) % M;
    }
    while(x < 0) x += M;
    return x;
}
```

### 1.4 中国剩余定理 2

```
//merge Ax=B and ax=b to A'x=B'
void merge(LL &A, LL &B, LL a, LL b){
    LL x, y;
    sol(A, -a, b - B, x, y);
    A = lcm(A, a);
    B = (a * y + b) % A;
    B = (B + A) % A;
}
```

### 1.5 卢卡斯定理

```
LL Lucas(LL n, LL m, LL p) {
    LL ans = 1;
    while(n && m) {
        LL a = n % p, b = m % p;
        if(a < b) return 0;
        ans = (ans * C(a, b, p)) % p;
        n /= p; m /= p;
    }
    return ans % p;
}
```

### 1.6 小步大步

返回结果:  $a^x = b \pmod{p}$       使用条件:  $p$  为质数

```
LL BSGS(LL a, LL b, LL p) {
    LL m = sqrt(p) + .5, v = inv(pw(a, m, p), p), e = 1;
    map<LL, LL> hash; hash[1] = 0;

```

```

for(int i = 1; i < m; i++) e = e * a % p, hash[e] = i;
for(int i = 0; i <= m; i++)
    if(hash.count(b)) return i * m + hash[b];
    else b = b * v % p;
return -1;
}

```

## 1.7 Miller Rabin 素数测试

```

const int BASE[12] = {2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37};
bool check(long long n, int base) {
    long long n2 = n - 1, res; int s = 0;
    while(n2 % 2 == 0) n2 >>= 1, s++;
    res = pw(base, n2, n);
    if((res == 1) || (res == n - 1)) return 1;
    while(s--) {
        res = mul(res, res, n);
        if(res == n - 1) return 1;
    }
    return 0; // n is not a strong pseudo prime
}
bool isprime(const long long &n) {
    if(n == 2) return true;
    if(n < 2 || n % 2 == 0) return false;
    for(int i = 0; i < 12 && BASE[i] < n; i++)
        if(!check(n, BASE[i])) return false;
    return true;
}

```

## 1.8 Pollard Rho 大数分解

```

LL prho(LL n, LL c) {
    LL i = 1, k = 2, x = rand() % (n - 1) + 1, y = x;
    while(1) {
        i++; x = (x * x % n + c) % n;
        LL d = __gcd((y - x + n) % n, n);
        if(d > 1 && d < n) return d;
    }
}

```

```

    if(y == x) return n;
    if(i == k) y = x, k <= 1;
}
}
void factor(LL n, vector<LL>&fat) {
    if(n == 1) return;
    if(isprime(n)) {fat.push_back(n); return;}
    LL p = n;
    while(p >= n) p = prho(p, rand() % (n - 1) + 1);
    factor(p, fat); factor(n / p, fat);
}

```

## 1.9 快速数论变换 (zky)

返回结果:  $c_i = \sum_{0 \leq j \leq i} a_j \cdot b_{i-j} \pmod{} (0 \leq i < n)$

```

/*{(mod,G)}={(81788929,7),(101711873,3),(167772161,3),
    (377487361,7),(998244353,3),(1224736769,3),
    (1300234241,3),(1484783617,5)}*/
int mo = 998244353, G = 3;
void NTT(int a[], int n, int f) {
    for(register int i = 0; i < n; i++)
        if(i < rev[i])
            swap(a[i], a[rev[i]]);
    for (register int i = 2; i <= n; i <= 1) {
        static int exp[maxn];
        exp[0] = 1;
        exp[1] = pw(G, (mo - 1) / i);
        if(f == -1) exp[1] = pw(exp[1], mo - 2);
        for(register int k = 2; k < (i >> 1); k++)
            exp[k] = 1LL * exp[k - 1] * exp[1] % mo;
        for(register int j = 0; j < n; j += i) {
            for(register int k = 0; k < (i >> 1); k++) {
                register int &pA = a[j + k], &pB = a[j + k + (i >> 1)];
                register int A = pA, B = 1LL * pB * exp[k] % mo;
                pA = (A + B) % mo;
                pB = (A - B + mo) % mo;
            }
        }
    }
}

```

```

    }
}
if(f == -1) {
    int rv = pw(n, mo - 2) % mo;
    for(int i = 0; i < n; i++)
        a[i] = 1LL * a[i] * rv % mo;
}
}

void mul(int m, int a[], int b[], int c[]) {
    int n = 1, len = 0;
    while(n < m)n <= 1, len++;
    for (int i = 1; i < n; i++)
        rev[i] = (rev[i >> 1] >> 1) | ((i & 1) << (len - 1));
    NTT(a, n, 1);
    NTT(b, n, 1);
    for(int i = 0; i < n; i++)
        c[i] = 1LL * a[i] * b[i] % mo;
    NTT(c, n, -1);
}

```

### 1.10 原根

```

vector<LL>fct;
bool check(LL x, LL g) {
    for(int i = 0; i < fct.size(); i++)
        if(pw(g, (x - 1) / fct[i], x) == 1)
            return 0;
    return 1;
}

LL findrt(LL x) {
    LL tmp = x - 1;
    for(int i = 2; i * i <= tmp; i++) {
        if(tmp % i == 0) {
            fct.push_back(i);
            while(tmp % i == 0)tmp /= i;
        }
    }
    if(tmp > 1) fct.push_back(tmp);
}

```

```

// x is 1,2,4,p^n,2p^n
// x has phi(phi(x)) primitive roots
for(int i = 2; i < int(1e9); i++)
    if(check(x, i)) return i;
return -1;
}

```

### 1.11 线性递推

//已知  $a_0, a_1, \dots, a_{m-1} \setminus \setminus$   
 $a_n = c_0 * a_{n-m} + \dots + c_{m-1} * a_{n-1} \setminus \setminus$   
 求  $a_n = v_0 * a_0 + v_1 * a_1 + \dots + v_{m-1} * a_{m-1} \setminus \setminus$

```

void linear_recurrence(long long n, int m, int a[], int c[], int p) {
    long long v[M] = {1 % p}, u[M << 1], msk = !n;
    for(long long i(n); i > 1; i >= 1) {
        msk <= 1;
    }
    for(long long x(0); msk; msk >= 1, x <= 1) {
        fill_n(u, m << 1, 0);
        int b(!(n & msk));
        x |= b;
        if(x < m) {
            u[x] = 1 % p;
        } else {
            for(int i(0); i < m; i++) {
                for(int j(0), t(i + b); j < m; j++, t++) {
                    u[t] = (u[t] + v[i] * v[j]) % p;
                }
            }
        }
        for(int i((m << 1) - 1); i >= m; i--) {
            for(int j(0), t(i - m); j < m; j++, t++) {
                u[t] = (u[t] + c[j] * u[i]) % p;
            }
        }
    }
    copy(u, u + m, v);
}

```

```
//a[n] = v[0] * a[0] + v[1] * a[1] + ... + v[m - 1] * a[m - 1].
for(int i(m); i < 2 * m; i++) {
    a[i] = 0;
    for(int j(0); j < m; j++) {
        a[i] = (a[i] + (long long)c[j] * a[i + j - m]) % p;
    }
}
for(int j(0); j < m; j++) {
    b[j] = 0;
    for(int i(0); i < m; i++) {
        b[j] = (b[j] + v[i] * a[i + j]) % p;
    }
}
for(int j(0); j < m; j++) {
    a[j] = b[j];
}
}
```

### 1.12 直线下整点个数

返回结果:  $\sum_{0 \leq i < n} \lfloor \frac{a+b \cdot i}{m} \rfloor$       使用条件:  $n, m > 0, a, b \geq 0$       时间复杂度:  $\mathcal{O}(n \log n)$

```
LL solve(LL n, LL a, LL b, LL m) {
    if(b == 0)
        return n * (a / m);
    if(a >= m || b >= m)
        return n * (a / m) + (n - 1) * n / 2 * (b / m) + solve(n, a % m, b % m, m);
    return solve((a + b * n) / m, (a + b * n) % m, m, b);
}
```

### 1.13 1e9+7 FFT

```
// double 精度对  $10^9 + 7$  取模最多可以做到  $2^{20}$ 
const int MOD = 1000003;
const double PI = acos(-1);
typedef complex<double> Complex;
const int N = 65536, L = 15, MASK = (1 << L) - 1;
```

```
Complex w[N];
void FFTInit() {
    for (int i = 0; i < N; ++i)
        w[i] = Complex(cos(2 * i * PI / N), sin(2 * i * PI / N));
}
void FFT(Complex p[], int n) {
    for (int i = 1, j = 0; i < n - 1; ++i) {
        for (int s = n; j ^= s >= 1, ~j & s;);
        if (i < j) swap(p[i], p[j]);
    }
    for (int d = 0; (1 << d) < n; ++d) {
        int m = 1 << d, m2 = m * 2, rm = n >> (d + 1);
        for (int i = 0; i < n; i += m2) {
            for (int j = 0; j < m; ++j) {
                Complex &p1 = p[i + j + m], &p2 = p[i + j];
                Complex t = w[rm * j] * p1;
                p1 = p2 - t, p2 = p2 + t;
            }
        }
    }
    Complex A[N], B[N], C[N], D[N];
    void mul(int a[N], int b[N]) {
        for (int i = 0; i < N; ++i) {
            A[i] = Complex(a[i] >> L, a[i] & MASK);
            B[i] = Complex(b[i] >> L, b[i] & MASK);
        }
        FFT(A, N), FFT(B, N);
        for (int i = 0; i < N; ++i) {
            int j = (N - i) % N;
            Complex da = (A[i] - conj(A[j])) * Complex(0, -0.5),
                    db = (A[i] + conj(A[j])) * Complex(0.5, 0),
                    dc = (B[i] - conj(B[j])) * Complex(0, -0.5),
                    dd = (B[i] + conj(B[j])) * Complex(0.5, 0);
            C[j] = da * dd + da * dc * Complex(0, 1);
            D[j] = db * dd + db * dc * Complex(0, 1);
        }
        FFT(C, N), FFT(D, N);
        for (int i = 0; i < N; ++i) {
            long long da = (long long)(C[i].imag() / N + 0.5) % MOD,
```

```

        db = (long long)(C[i].real() / N + 0.5) % MOD,
        dc = (long long)(D[i].imag() / N + 0.5) % MOD,
        dd = (long long)(D[i].real() / N + 0.5) % MOD;
        a[i] = ((dd << (L * 2)) + ((db + dc) << L) + da) % MOD;
    }
}

```

### 1.14 自适应辛普森

```

double area(const double &left, const double &right) {
    double mid = (left + right) / 2;
    return (right - left) * (calc(left) + 4 * calc(mid) + calc(right)) / 6;
}

```

```

double simpson(const double &left, const double &right,
               const double &eps, const double &area_sum) {
    double mid = (left + right) / 2;
    double area_left = area(left, mid);
    double area_right = area(mid, right);
    double area_total = area_left + area_right;
    if (std::abs(area_total - area_sum) < 15 * eps) {
        return area_total + (area_total - area_sum) / 15;
    }
    return simpson(left, mid, eps / 2, area_left)
        + simpson(mid, right, eps / 2, area_right);
}

```

```

double simpson(const double &left, const double &right, const double &eps) {
    return simpson(left, right, eps, area(left, right));
}

```

### 1.15 多项式求根

```

const double eps=1e-12;
double a[10][10];
typedef vector<double> vd;
int sgn(double x) { return x < -eps ? -1 : x > eps; }

```

```

double mypow(double x,int num){
    double ans=1.0;
    for(int i=1;i<=num;++i)ans*=x;
    return ans;
}

double f(int n,double x){
    double ans=0;
    for(int i=n;i>=0;--i)ans+=a[n][i]*mypow(x,i);
    return ans;
}

double getRoot(int n,double l,double r){
    if(sgn(f(n,l))==0)return l;
    if(sgn(f(n,r))==0)return r;
    double temp;
    if(sgn(f(n,l))>0)temp=-1;else temp=1;
    double m;
    for(int i=1;i<=10000;++i){
        m=(l+r)/2;
        double mid=f(n,m);
        if(sgn(mid)==0){
            return m;
        }
        if(mid*temp<0)l=m;else r=m;
    }
    return (l+r)/2;
}

vd did(int n){
    vd ret;
    if(n==1){
        ret.push_back(-1e10);
        ret.push_back(-a[n][0]/a[n][1]);
        ret.push_back(1e10);
        return ret;
    }
    vd mid=did(n-1);
    ret.push_back(-1e10);
    for(int i=0;i+1<mid.size();++i){
        int t1=sgn(f(n,mid[i])),t2=sgn(f(n,mid[i+1]));

```



```

    if(t1*t2>0)continue;
    ret.push_back(getRoot(n,mid[i],mid[i+1]));
}
ret.push_back(1e10);
return ret;
}

int main(){
    int n; scanf("%d",&n);
    for(int i=n;i>=0;--i){
        scanf("%lf",&a[n][i]);
    }
    for(int i=n-1;i>=0;--i)
        for(int j=0;j<=i;++j)a[i][j]=a[i+1][j+1]*(j+1);
    vd ans=did(n);
    sort(ans.begin(),ans.end());
    for(int i=1;i+1<ans.size();++i)printf("%.10f\n",ans[i]);
    return 0;
}

```

## 1.16 魔幻多项式

### 多项式求逆

原理：令  $G(x) = x * A - 1$ （其中  $A$  是一个多项式系数），根据牛顿迭代法有：

$$\begin{aligned}
 F_{t+1}(x) &\equiv F_t(x) - \frac{F_t(x) * A(x) - 1}{A(x)} \\
 &\equiv 2F_t(x) - F_t(x)^2 * A(x) \pmod{x^{2t}}
 \end{aligned}$$

注意事项：

1.  $F(x)$  的常数项系数必然不为 0，否则没有逆元；
2. 复杂度是  $O(n \log n)$  但是常数比较大（ $10^5$  大概需要 0.3 秒左右）；
3. 传入的两个数组必须不同，但传入的次数界没有必要是 2 的次幂；

```

void getInv(int *a, int *b, int n) {
    static int tmp[MAXN];

```

```

b[0] = fpm(a[0], MOD - 2, MOD);
for (int c = 2, M = 1; c < (n << 1); c <= 1) {
    for (; M <= 3 * (c - 1); M <= 1);
    meminit(b, c, M);
    meminit(tmp, c, M);
    memcpy(tmp, a, 0, c);
    DFT(tmp, M, 0);
    DFT(b, M, 0);
    for (int i = 0; i < M; i++) {
        b[i] = 1ll * b[i] * (2ll - 1ll * tmp[i] * b[i] % MOD + MOD) % MOD;
    }
    DFT(b, M, 1);
    meminit(b, c, M);
}
}

```

## 2 数据结构

### 2.1 lct

```

struct LCT
{
    int fa[N], c[N][2], rev[N], sz[N];

    void update(int o) {
        sz[o] = sz[c[o][0]] + sz[c[o][1]] + 1;
    }

    void pushdown(int o) {
        if(rev[o]) {
            rev[o] = 0;
            rev[c[o][0]] ^= 1;
            rev[c[o][1]] ^= 1;
            swap(c[o][0], c[o][1]);
        }
    }

    bool ch(int o) {
        return o == c[fa[o]][1];
    }
}

```

```

}
bool isroot(int o) {
    return c[fa[o]][0] != o && c[fa[o]][1] != o;
}
void setc(int x, int y, bool d) {
    if(x) fa[x] = y;
    if(y) c[y][d] = x;
}
void rotate(int x) {
    if(isroot(x)) return;
    int p = fa[x], d = ch(x);
    if(isroot(p)) fa[x] = fa[p];
    else setc(x, fa(p), ch(p));
    setc(c[x][d^1], p, d);
    setc(p, x, d^1);
    update(p);
    update(x);
}
void splay(int x) {
    static int q[N], top;
    int y = q[top = 1] = x;
    while(!isroot(y)) q[++top] = y = fa[y];
    while(top) pushdown(q[top--]);
    while(!isroot(x)) {
        if(!isroot(fa[x]))
            rotate(ch(fa[x]) == ch(x) ? fa[x] : x);
        rotate(x);
    }
}
void access(int x) {
    for(int y = 0; x; y = x, x = fa[x])
        splay(x), c[x][1] = y, update(x);
}
void makeroot(int x) {
    access(x), splay(x), rev(x) ^= 1;
}
void link(int x, int y) {
    makeroot(x), fa[x] = y, splay(x);
}

```

```

}
void cut(int x, int y) {
    makeroot(x);
    access(y);
    splay(y);
    c[y][0] = fa[x] = 0;
}
};

```

## 2.2 k-d 树

```

struct Point{
    int data[MAXK], id;
}p[MAXN];

struct KdNode{
    int l, r;
    Point p, dmin, dmax;
    KdNode() {}
    KdNode(const Point &rhs) : l(0), r(0), p(rhs), dmin(rhs), dmax(rhs) {}
    inline void merge(const KdNode &rhs) {
        for (register int i = 0; i < k; i++) {
            dmin.data[i] = std::min(dmin.data[i], rhs.dmin.data[i]);
            dmax.data[i] = std::max(dmax.data[i], rhs.dmax.data[i]);
        }
    }
    inline long long getMinDist(const Point &rhs)const {
        register long long ret = 0;
        for (register int i = 0; i < k; i++) {
            if (dmin.data[i] <= rhs.data[i] && rhs.data[i] <= dmax.data[i]) continue;
            ret += std::min(1ll * (dmin.data[i] - rhs.data[i]) * (dmin.data[i] -
↪ rhs.data[i]),
                1ll * (dmax.data[i] - rhs.data[i]) * (dmax.data[i] - rhs.data[i]));
        }
        return ret;
    }
    long long getMaxDist(const Point &rhs) {
        long long ret = 0;
    }
}

```

```

    for (register int i = 0; i < k; i++) {
        int tmp = std::max(std::abs(dmin.data[i] - rhs.data[i]),
            std::abs(dmax.data[i] - rhs.data[i]));
        ret += 1ll * tmp * tmp;
    }
    return ret;
}
}tree[MAXN * 4];

struct Result{
    long long dist;
    Point d;
    Result() {}
    Result(const long long &dist, const Point &d) : dist(dist), d(d) {}
    bool operator >(const Result &rhs)const {
        return dist > rhs.dist || (dist == rhs.dist && d.id < rhs.d.id);
    }
    bool operator <(const Result &rhs)const {
        return dist < rhs.dist || (dist == rhs.dist && d.id > rhs.d.id);
    }
};

inline long long sqrdist(const Point &a, const Point &b) {
    register long long ret = 0;
    for (register int i = 0; i < k; i++) {
        ret += 1ll * (a.data[i] - b.data[i]) * (a.data[i] - b.data[i]);
    }
    return ret;
}

inline int alloc() {
    size++;
    tree[size].l = tree[size].r = 0;
    return size;
}

void build(const int &depth, int &rt, const int &l, const int &r) {
    if (l > r) return;

```

```

    register int middle = l + r >> 1;
    std::nth_element(p + l, p + middle, p + r + 1,
        [=](const Point &a, const Point &b){return a.data[depth] < b.data[depth];});
    tree[rt = alloc()] = KdNode(p[middle]);
    if (l == r) return;
    build((depth + 1) % k, tree[rt].l, l, middle - 1);
    build((depth + 1) % k, tree[rt].r, middle + 1, r);
    if (tree[rt].l) tree[rt].merge(tree[tree[rt].l]);
    if (tree[rt].r) tree[rt].merge(tree[tree[rt].r]);
}

```

```
std::priority_queue<Result, std::vector<Result>, std::greater<Result> > heap;
```

```

void getMinKth(const int &depth, const int &rt, const int &m, const Point &d) {
    ↪ // 求 K 近点
    Result tmp = Result(sqrdist(tree[rt].p, d), tree[rt].p);
    if ((int)heap.size() < m) {
        heap.push(tmp);
    } else if (tmp < heap.top()) {
        heap.pop();
        heap.push(tmp);
    }
    int x = tree[rt].l, y = tree[rt].r;
    if (x != 0 && y != 0 && sqrdist(d, tree[x].p) > sqrdist(d, tree[y].p)) std::swap(x,
    ↪ y);
    if (x != 0 && ((int)heap.size() < m || tree[x].getMinDist(d) < heap.top().dist)) {
        getMinKth((depth + 1) % k, x, m, d);
    }
    if (y != 0 && ((int)heap.size() < m || tree[y].getMinDist(d) < heap.top().dist)) {
        getMinKth((depth + 1) % k, y, m, d);
    }
}

```

```

void getMaxKth(const int &depth, const int &rt, const int &m, const Point &d) {
    ↪ // 求 K 远点
    Result tmp = Result(sqrdist(tree[rt].p, d), tree[rt].p);
    if ((int)heap.size() < m) {
        heap.push(tmp);
    }
}

```

```

} else if (tmp > heap.top()) {
    heap.pop();
    heap.push(tmp);
}
int x = tree[rt].l, y = tree[rt].r;
if (x != 0 && y != 0 && sqrdist(d, tree[x].p) < sqrdist(d, tree[y].p)) std::swap(x,
↪ y);
if (x != 0 && ((int)heap.size() < m || tree[x].getMaxDist(d) >= heap.top().dist)) {
↪ // 这里的 >= 是因为在距离相等的时候需要按照 id 排序
    getMaxKth((depth + 1) % k, x, m, d);
}
if (y != 0 && ((int)heap.size() < m || tree[y].getMaxDist(d) >= heap.top().dist)) {
    getMaxKth((depth + 1) % k, y, m, d);
}
}
}

```

## 2.3 树上莫队

```
#include <bits/stdc++.h>
```

```
using namespace std;
```

```
const int N = 40005;
const int M = 100005;
const int LOGN = 17;
```

```
int n, m;
int w[N];
vector<int> g[N];
int bid[N << 1];
```

```
struct Query
{
    int l, r, extra, i;
    friend bool operator < (const Query &a, const Query &b)
    {
        if(bid[a.l] != bid[b.l])
            return bid[a.l] < bid[b.l];
    }
};

```

```
return a.r < b.r;
}
} q[M];
```

```
void input()
{
    vector<int> vs;
    scanf("%d%d", &n, &m);
    for(int i = 1; i <= n; i++)
    {
        scanf("%d", &w[i]);
        vs.push_back(w[i]);
    }
    sort(vs.begin(), vs.end());
    vs.resize(unique(vs.begin(), vs.end()) - vs.begin());
    for(int i = 1; i <= n; i++)
        w[i] = lower_bound(vs.begin(), vs.end(), w[i]) - vs.begin() + 1;
    for(int i = 2; i <= n; i++)
    {
        int a, b;
        scanf("%d%d", &a, &b);
        g[a].push_back(b);
        g[b].push_back(a);
    }
    for(int i = 1; i <= m; i++)
    {
        scanf("%d%d", &q[i].l, &q[i].r);
        q[i].i = i;
    }
}

```

```
int dfs_clock;
int st[N], ed[N];
int fa[N][LOGN], dep[N];
int col[N << 1];
int id[N << 1];
```

```
void dfs(int x, int p)
```

```

{
    col[st[x] = ++dfs_clock] = w[x];
    id[st[x]] = x;
    fa[x][0] = p; dep[x] = dep[p] + 1;
    for(int i = 0; fa[x][i]; i++)
        fa[x][i + 1] = fa[fa[x][i]][i];
    for(auto y: g[x])
        if(y != p)
            dfs(y, x);
    col[ed[x] = ++dfs_clock] = w[x];
    id[ed[x]] = x;
}

int lca(int x, int y)
{
    if(dep[x] < dep[y]) swap(x, y);
    for(int i = LOGN - 1; i >= 0; i--)
        if(dep[fa[x][i]] >= dep[y])
            x = fa[x][i];
    if(x == y) return x;
    for(int i = LOGN - 1; i >= 0; i--)
        if(fa[x][i] != fa[y][i])
            x = fa[x][i], y = fa[y][i];
    return fa[x][0];
}

void prepare()
{
    dfs_clock = 0;
    dfs(1, 0);
    int BS = (int)sqrt(dfs_clock + 0.5);
    for(int i = 1; i <= dfs_clock; i++)
        bid[i] = (i + BS - 1) / BS;
    for(int i = 1; i <= m; i++)
    {
        int a = q[i].l;
        int b = q[i].r;
        int c = lca(a, b);

```

```

        if(st[a] > st[b]) swap(a, b);
        if(c == a)
        {
            q[i].l = st[a];
            q[i].r = st[b];
            q[i].extra = 0;
        }
        else
        {
            q[i].l = ed[a];
            q[i].r = st[b];
            q[i].extra = c;
        }
    }
    sort(q + 1, q + m + 1);
}

int curans;
int ans[M];
int cnt[N];
bool state[N];

void rev(int x)
{
    int &c = cnt[col[x]];
    curans -= !!c;
    c += (state[id[x]] ^ 1) ? 1 : -1;
    curans += !!c;
}

void solve()
{
    prepare();
    curans = 0;
    memset(cnt, 0, sizeof(cnt));
    memset(state, 0, sizeof(state));

    int l = 1, r = 0;

```

```

for(int i = 1; i <= m; i++)
{
    while(l < q[i].l) rev(l++);
    while(l > q[i].l) rev(--l);
    while(r < q[i].r) rev(++r);
    while(r > q[i].r) rev(r--);
    if(q[i].extra) rev(st[q[i].extra]);
    ans[q[i].i] = curans;
    if(q[i].extra) rev(st[q[i].extra]);
}
for(int i = 1; i <= m; i++)
    printf("%d\n", ans[i]);
}

int main()
{
    input();
    solve();
    return 0;
}

```

## 2.4 树状数组 kth

```

int find(int k){
    int cnt=0,ans=0;
    for(int i=22;i>=0;i--){
        ans+=(1<<i);
        if(ans>n || cnt+d[ans]>=k)ans-=(1<<i);
        else cnt+=d[ans];
    }
    return ans+1;
}

```

## 2.5 虚树

```

int a[maxn*2],sta[maxn*2];
int top=0,k;

```

```

void build(){
    top=0;
    sort(a,a+k,bydfn);
    k=unique(a,a+k)-a;
    sta[top++]=1;_n=k;
    for(int i=0;i<k;i++){
        int LCA=lca(a[i],sta[top-1]);
        while(dep[LCA]<dep[sta[top-1]]){
            if(dep[LCA]>=dep[sta[top-2]]){
                add_edge(LCA,sta[top-1]);
                if(sta[top-1]!=LCA)sta[top++]=LCA;
                break;
            }add_edge(sta[top-2],sta[top-1]);top--;
        }if(sta[top-1]!=a[i])sta[top++]=a[i];
    }
    while(top>1)
        add_edge(sta[top-2],sta[top-1]),top--;
    for(int i=0;i<k;i++)inr[a[i]]=1;
}

```

## 3 图论

### 3.1 点双连通分量 (lyx)

```
#define SZ(x) ((int)x.size())
```

```

const int N = 400005; // N 开 2 倍点数，因为新树会加入最多 n 个新点
const int M = 200005;

```

```
vector<int> g[N];
```

```
int bccno[N], bcc_cnt;
```

```
vector<int> bcc[N];
```

```
bool iscut[N];
```

```

struct Edge {
    int u, v;
}

```

```

} stk[M << 2];
int top; // 注意栈大小为边数 4 倍
int dfn[N], low[N], dfs_clock;

void dfs(int x, int fa)
{
    low[x] = dfn[x] = ++dfs_clock;
    int child = 0;
    for(int i = 0; i < SZ(g[x]); i++) {
        int y = g[x][i];
        if(!dfn[y]) {
            child++;
            stk[++top] = (Edge){x, y};
            dfs(y, x);
            low[x] = min(low[x], low[y]);
            if(low[y] >= dfn[x]) {
                iscut[x] = true;
                bcc[++bcc_cnt].clear();
                for(;;) {
                    Edge e = stk[top--];
                    if(bccno[e.u] != bcc_cnt) { bcc[bcc_cnt].push_back(e.u); bccno[e.u] =
↪ bcc_cnt; }
                    if(bccno[e.v] != bcc_cnt) { bcc[bcc_cnt].push_back(e.v); bccno[e.v] =
↪ bcc_cnt; }
                    if(e.u == x && e.v == y) break;
                }
            }
        }
        else if(y != fa && dfn[y] < dfn[x]) {
            stk[++top] = (Edge){x, y};
            low[x] = min(low[x], dfn[y]);
        }
    }
    if(fa == 0 && child == 1) iscut[x] = false;
}

void find_bcc() // 求点双联通分量, 需要时手动 1 到 n 清空, 1-based
{
    memset(dfn, 0, sizeof(dfn));

```

```

    memset(iscut, 0, sizeof(iscut));
    memset(bccno, 0, sizeof(bccno));
    dfs_clock = bcc_cnt = 0;
    for(int i = 1; i <= n; i++)
        if(!dfn[i])
            dfs(i, 0);
}

vector<int> G[N];

void prepare() { // 建出缩点后的树
    for(int i = 1; i <= n + bcc_cnt; i++)
        G[i].clear();
    for(int i = 1; i <= bcc_cnt; i++) {
        int x = i + n;
        for(int j = 0; j < SZ(bcc[i]); j++) {
            int y = bcc[i][j];
            G[x].push_back(y);
            G[y].push_back(x);
        }
    }
}

```

### 3.2 Hopcroft-Karp 求最大匹配

```

int matchx[N], matchy[N], level[N];

bool dfs(int x) {
    for (int i = 0; i < (int)edge[x].size(); ++i) {
        int y = edge[x][i];
        int w = matchy[y];
        if (w == -1 || level[x] + 1 == level[w] && dfs(w)) {
            matchx[x] = y;
            matchy[y] = x;
            return true;
        }
    }
    level[x] = -1;
}

```

```

    return false;
}

int solve() {
    std::fill(matchx, matchx + n, -1);
    std::fill(matchy, matchy + m, -1);
    for (int answer = 0; ; ) {
        std::vector<int> queue;
        for (int i = 0; i < n; ++i) {
            if (matchx[i] == -1) {
                level[i] = 0;
                queue.push_back(i);
            } else {
                level[i] = -1;
            }
        }
        for (int head = 0; head < (int)queue.size(); ++head) {
            int x = queue[head];
            for (int i = 0; i < (int)edge[x].size(); ++i) {
                int y = edge[x][i];
                int w = matchy[y];
                if (w != -1 && level[w] < 0) {
                    level[w] = level[x] + 1;
                    queue.push_back(w);
                }
            }
        }
        int delta = 0;
        for (int i = 0; i < n; ++i) {
            if (matchx[i] == -1 && dfs(i)) {
                delta++;
            }
        }
        if (delta == 0) {
            return answer;
        } else {
            answer += delta;
        }
    }
}

```

```

    }
}

```

### 3.3 KM 带权匹配

**注意事项：**最小权完美匹配，复杂度为  $\mathcal{O}(|V|^3)$ 。

```

int DFS(int x){
    visx[x] = 1;
    for (int y = 1; y <= ny; y++){
        if (visy[y]) continue;
        int t = lx[x] + ly[y] - w[x][y];
        if (t == 0) {
            visy[y] = 1;
            if (link[y] == -1 || DFS(link[y])){
                link[y] = x;
                return 1;
            }
        }
        else slack[y] = min(slack[y], t);
    }
    return 0;
}

int KM(){
    int i, j;
    memset(link, -1, sizeof(link));
    memset(ly, 0, sizeof(ly));
    for (i = 1; i <= nx; i++){
        for (j = 1, lx[i] = -inf; j <= ny; j++){
            lx[i] = max(lx[i], w[i][j]);
        }
        for (int x = 1; x <= nx; x++){
            for (i = 1; i <= ny; i++) slack[i] = inf;
            while (true) {
                memset(visx, 0, sizeof(visx));
                memset(visy, 0, sizeof(visy));
                if (DFS(x)) break;
                int d = inf;
                for (i = 1; i <= ny; i++)

```



```

        if (!visy[i] && d > slack[i]) d = slack[i];
    for (i = 1; i <= nx; i++)
        if (visx[i]) lx[i] -= d;
    for (i = 1; i <= ny; i++)
        if (visy[i]) ly[i] += d;
        else slack[i] -= d;
    }
}
int res = 0;
for (i = 1; i <= ny; i++)
    if (link[i] > -1) res += w[link[i]][i];
return res;
}

```

### 3.4 2-SAT 问题

```

int stamp, comps, top;
int dfn[N], low[N], comp[N], stack[N];

void add(int x, int a, int y, int b) {
    edge[x << 1 | a].push_back(y << 1 | b);
}

void tarjan(int x) {
    dfn[x] = low[x] = ++stamp;
    stack[top++] = x;
    for (int i = 0; i < (int)edge[x].size(); ++i) {
        int y = edge[x][i];
        if (!dfn[y]) {
            tarjan(y);
            low[x] = std::min(low[x], low[y]);
        } else if (!comp[y]) {
            low[x] = std::min(low[x], dfn[y]);
        }
    }
}

if (low[x] == dfn[x]) {
    comps++;
    do {

```

```

        int y = stack[--top];
        comp[y] = comps;
    } while (stack[top] != x);
}

bool solve() {
    int counter = n + n + 1;
    stamp = top = comps = 0;
    std::fill(dfn, dfn + counter, 0);
    std::fill(comp, comp + counter, 0);
    for (int i = 0; i < counter; ++i) {
        if (!dfn[i]) {
            tarjan(i);
        }
    }
    for (int i = 0; i < n; ++i) {
        if (comp[i << 1] == comp[i << 1 | 1]) {
            return false;
        }
        answer[i] = (comp[i << 1 | 1] < comp[i << 1]);
    }
    return true;
}

```

### 3.5 有根树的同构

```

const unsigned long long MAGIC = 4423;

unsigned long long magic[N];
std::pair<unsigned long long, int> hash[N];

void solve(int root) {
    magic[0] = 1;
    for (int i = 1; i <= n; ++i) {
        magic[i] = magic[i - 1] * MAGIC;
    }
    std::vector<int> queue;

```

```

queue.push_back(root);
for (int head = 0; head < (int)queue.size(); ++head) {
    int x = queue[head];
    for (int i = 0; i < (int)son[x].size(); ++i) {
        int y = son[x][i];
        queue.push_back(y);
    }
}
for (int index = n - 1; index >= 0; --index) {
    int x = queue[index];
    hash[x] = std::make_pair(0, 0);

    std::vector<std::pair<unsigned long long, int> > value;
    for (int i = 0; i < (int)son[x].size(); ++i) {
        int y = son[x][i];
        value.push_back(hash[y]);
    }
    std::sort(value.begin(), value.end());

    hash[x].first = hash[x].first * magic[1] + 37;
    hash[x].second++;
    for (int i = 0; i < (int)value.size(); ++i) {
        hash[x].first = hash[x].first * magic[value[i].second] + value[i].first;
        hash[x].second += value[i].second;
    }
    hash[x].first = hash[x].first * magic[1] + 41;
    hash[x].second++;
}
}

```

### 3.6 Dominator Tree

```

#include <bits/stdc++.h>
using namespace std;

const int MAXN = 50101;
const int MAXM = 110101;

```

```

class Edge
{public:
    int size;
    int begin[MAXN], dest[MAXM], next[MAXM];
    void clear(int n)
    {
        size = 0;
        fill(begin, begin + n, -1);
    }
    Edge(int n = MAXN)
    {
        clear(n);
    }
    void add_edge(int u, int v)
    {
        dest[size] = v;
        next[size] = begin[u];
        begin[u] = size++;
    }
};

class dominator
{public:
    int dfn[MAXN], sdom[MAXN], idom[MAXN], id[MAXN], f[MAXN], fa[MAXN], smin[MAXN],
    ↪ stamp;

    void predfs(int x, const Edge &succ)
    {
        id[dfn[x] = stamp++] = x;
        for(int i = succ.begin[x]; ~i; i = succ.next[i])
        {
            int y = succ.dest[i];
            if(dfn[y] < 0)
            {
                f[y] = x;
                predfs(y, succ);
            }
        }
    }
}

```

```

}
int getfa(int x)
{
    if(fa[x] == x)
        return x;
    int ret = getfa(fa[x]);
    if(dfn[sdom[smin[fa[x]]]] < dfn[sdom[smin[x]]])
        smin[x] = smin[fa[x]];
    return fa[x] = ret;
}
void solve(int s, int n, const Edge &succ)
{
    fill(dfn, dfn + n, -1);
    fill(idom, idom + n, -1);
    static Edge pred, tmp;
    pred.clear(n);
    for(int i = 0; i < n; ++i)
        for(int j = succ.begin[i]; ~j; j = succ.next[j])
            pred.add_edge(succ.dest[j], i);
    stamp = 0;
    tmp.clear(n);
    predfs(s, succ);
    for(int i = 0; i < stamp; ++i)
        fa[id[i]] = smin[id[i]] = id[i];
    for(int o = stamp - 1; o >= 0; --o)
    {
        int x = id[o];
        if(o)
        {
            sdom[x] = f[x];
            for(int i = pred.begin[x]; ~i; i = pred.next[i])
            {
                int p = pred.dest[i];
                if(dfn[p] < 0)
                    continue;
                if(dfn[p] > dfn[x])
                {
                    getfa(p);

```

```

                p = sdom[smin[p]];
            }
            if(dfn[sdom[x]] > dfn[p])
                sdom[x] = p;
        }
        tmp.add_edge(sdom[x], x);
    }
    while(~tmp.begin[x])
    {
        int y = tmp.dest[tmp.begin[x]];
        tmp.begin[x] = tmp.next[tmp.begin[x]];
        getfa(y);
        if(x != sdom[smin[y]])
            idom[y] = smin[y];
        else
            idom[y] = x;
    }
    for(int i = succ.begin[x]; ~i; i = succ.next[i])
        if(f[succ.dest[i]] == x)
            fa[succ.dest[i]] = x;
    }
    idom[s] = s;
    for(int i = 1; i < stamp; ++i)
    {
        int x = id[i];
        if(idom[x] != sdom[x])
            idom[x] = idom[idom[x]];
    }
    };

    int ans[MAXN];

    Edge e;
    dominator dom1;

    int dfs(int x)
    {

```

```

if(dom1.idom[x] <= 0)
    return 0;
if(ans[x] > 0)
    return ans[x];
if(dom1.idom[x] == x)
    return ans[x] = x;
return ans[x] = x + dfs(dom1.idom[x]);
}

int main()
{
    int n, m;
    while(scanf("%d%d", &n, &m) == 2)
    {
        e.clear(n + 1);
        fill(ans, ans + n + 1, 0);
        for(int i = 0; i < m; ++i)
        {
            int u, v;
            scanf("%d%d", &u, &v);
            e.add_edge(u, v);
        }
        dom1.solve(n, n + 1, e);
        for(int i = 1; i <= n; ++i)
            printf("%d%c", dfs(i), " \n"[i == n]);
    }
    return 0;
}

```

### 3.7 哈密尔顿回路 (ORE 性质的图)

```

int left[N], right[N], next[N], last[N];

void cover(int x) {
    left[right[x]] = left[x];
    right[left[x]] = right[x];
}

```

```

int adjacent(int x) {
    for (int i = right[0]; i <= n; i = right[i]) {
        if (graph[x][i]) {
            return i;
        }
    }
    return 0;
}

std::vector<int> solve() {
    for (int i = 1; i <= n; ++i) {
        left[i] = i - 1;
        right[i] = i + 1;
    }
    int head, tail;
    for (int i = 2; i <= n; ++i) {
        if (graph[1][i]) {
            head = 1;
            tail = i;
            cover(head);
            cover(tail);
            next[head] = tail;
            break;
        }
    }
    while (true) {
        int x;
        while (x = adjacent(head)) {
            next[x] = head;
            head = x;
            cover(head);
        }
        while (x = adjacent(tail)) {
            next[tail] = x;
            tail = x;
            cover(tail);
        }
        if (!graph[head][tail]) {

```

```

    for (int i = head, j; i != tail; i = next[i]) {
        if (graph[head][next[i]] && graph[tail][i]) {
            for (j = head; j != i; j = next[j]) {
                last[next[j]] = j;
            }
            j = next[head];
            next[head] = next[i];
            next[tail] = i;
            tail = j;
            for (j = i; j != head; j = last[j]) {
                next[j] = last[j];
            }
            break;
        }
    }
}
next[tail] = head;
if (right[0] > n) {
    break;
}
for (int i = head; i != tail; i = next[i]) {
    if (adjacent(i)) {
        head = next[i];
        tail = i;
        next[tail] = 0;
        break;
    }
}
}
std::vector<int> answer;
for (int i = head; ; i = next[i]) {
    if (i == 1) {
        answer.push_back(i);
        for (int j = next[i]; j != i; j = next[j]) {
            answer.push_back(j);
        }
        answer.push_back(i);
        break;
    }
}

```

```

    }
    if (i == tail) {
        break;
    }
}
return answer;
}

```

### 3.8 无向图最小割

```

int node[N], dist[N];
bool visit[N];

int solve(int n) {
    int answer = INT_MAX;
    for (int i = 0; i < n; ++i) {
        node[i] = i;
    }
    while (n > 1) {
        int max = 1;
        for (int i = 0; i < n; ++i) {
            dist[node[i]] = graph[node[0]][node[i]];
            if (dist[node[i]] > dist[node[max]]) {
                max = i;
            }
        }
        int prev = 0;
        memset(visit, 0, sizeof(visit));
        visit[node[0]] = true;
        for (int i = 1; i < n; ++i) {
            if (i == n - 1) {
                answer = std::min(answer, dist[node[max]]);
                for (int k = 0; k < n; ++k) {
                    graph[node[k]][node[prev]] =
                        (graph[node[prev]][node[k]] += graph[node[k]][node[max]]);
                }
                node[max] = node[--n];
            }
        }
    }
}

```

```

    visit[node[max]] = true;
    prev = max;
    max = -1;
    for (int j = 1; j < n; ++j) {
        if (!visit[node[j]]) {
            dist[node[j]] += graph[node[prev]][node[j]];
            if (max == -1 || dist[node[max]] < dist[node[j]]) {
                max = j;
            }
        }
    }
}
return answer;
}

```

### 3.9 带花树

```

int match[N], belong[N], next[N], mark[N], visit[N];
std::vector<int> queue;

```

```

int find(int x) {
    if (belong[x] != x) {
        belong[x] = find(belong[x]);
    }
    return belong[x];
}

```

```

void merge(int x, int y) {
    x = find(x);
    y = find(y);
    if (x != y) {
        belong[x] = y;
    }
}

```

```

int lca(int x, int y) {
    static int stamp = 0;

```

```

    stamp++;
    while (true) {
        if (x != -1) {
            x = find(x);
            if (visit[x] == stamp) {
                return x;
            }
            visit[x] = stamp;
            if (match[x] != -1) {
                x = next[match[x]];
            } else {
                x = -1;
            }
        }
        std::swap(x, y);
    }
}

```

```

void group(int a, int p) {
    while (a != p) {
        int b = match[a], c = next[b];
        if (find(c) != p) {
            next[c] = b;
        }
        if (mark[b] == 2) {
            mark[b] = 1;
            queue.push_back(b);
        }
        if (mark[c] == 2) {
            mark[c] = 1;
            queue.push_back(c);
        }
        merge(a, b);
        merge(b, c);
        a = c;
    }
}

```

```

void augment(int source) {
    queue.clear();
    for (int i = 0; i < n; ++i) {
        next[i] = visit[i] = -1;
        belong[i] = i;
        mark[i] = 0;
    }
    mark[source] = 1;
    queue.push_back(source);
    for (int head = 0; head < (int)queue.size() && match[source] == -1; ++head) {
        int x = queue[head];
        for (int i = 0; i < (int)edge[x].size(); ++i) {
            int y = edge[x][i];
            if (match[x] == y || find(x) == find(y) || mark[y] == 2) {
                continue;
            }
            if (mark[y] == 1) {
                int r = lca(x, y);
                if (find(x) != r) {
                    next[x] = y;
                }
                if (find(y) != r) {
                    next[y] = x;
                }
                group(x, r);
                group(y, r);
            } else if (match[y] == -1) {
                next[y] = x;
                for (int u = y; u != -1; ) {
                    int v = next[u];
                    int mv = match[v];
                    match[v] = u;
                    match[u] = v;
                    u = mv;
                }
                break;
            } else {
                next[y] = x;

```

```

                mark[y] = 2;
                mark[match[y]] = 1;
                queue.push_back(match[y]);
            }
        }
    }
}

int solve() {
    std::fill(match, match + n, -1);
    for (int i = 0; i < n; ++i) {
        if (match[i] == -1) {
            augment(i);
        }
    }
    int answer = 0;
    for (int i = 0; i < n; ++i) {
        answer += (match[i] != -1);
    }
    return answer;
}

```

## 4 字符串

### 4.1 KMP 算法

```

void getnex(char *s, int *nex){
    int n = strlen(s + 1);
    for(int j = 0, i = 2; i <= n; i++){
        while(j && s[j + 1] != s[i])j = nex[j];
        if(s[i] == s[j + 1]) j++;
        nex[i] = j;
    }
}

```

## 4.2 扩展 KMP 算法

```
//nex[i] 表示 s 和其后缀 s[i, n] 的 lcp 的长度
void getNext(char s[], int n, int nex[])
{
    nex[1] = n;
    int &t = nex[2] = 0;
    for(; t + 2 <= n && s[1 + t] == s[2 + t]; t++);
    int pos = 2;
    for(int i = 3; i <= n; i++){
        if(i + nex[i - pos + 1] < pos + nex[pos])
            nex[i] = nex[i - pos + 1];
        else{
            int j = max(0, nex[pos] + pos - i);
            for(; i + j <= n && s[i + j] == s[j + 1]; j++);
            nex[i] = j; pos = i;
        }
    }
}

//extend[i] 表示 s2 和 s1 后缀 s1[i, n] 的 lcp 的长度
void getextend(char s1[], char s2[], int extend[])
{
    int n = strlen(s1 + 1), m = strlen(s2 + 1);
    getNext(s2, m, next);
    int &t = extend[1] = 0, pos = 1;
    for(; t < n && t < m && s1[1 + t] == s2[1 + t]; t++);
    for(int i = 2; i <= n; i++){
        if(i + nex[i - pos + 1] < pos + extend[pos])
            extend[i] = nex[i - pos + 1];
        else{
            int j = max(0, extend[pos] + pos - i);
            for(; i + j <= n && j < m && s1[i + j] == s2[j + 1]; j++);
            extend[i] = j; pos = i;
        }
    }
}
```

## 4.3 AC 自动机

```
const int C = 26, L = 1e5 + 5, N = 5e5 + 10;
int n, root, cnt, fail[N], son[N][26], num[N];
char s[L];
inline int newNode(){
    cnt++; fail[cnt] = num[cnt] = 0;
    memset(son[cnt], 0, sizeof(son[cnt]));
    return cnt;
}
void insert(char *s){
    int n = strlen(s + 1), now = 1;
    for(int i = 1; i <= n; i++){
        int c = s[i] - 'a';
        if(!son[now][c]) son[now][c] = newNode();
        now = son[now][c];
    }
    num[now]++;
}
void getfail(){
    static queue<int> Q;
    fail[root] = 0;
    Q.push(root);
    while(!Q.empty()){
        int now = Q.front();
        Q.pop();
        for(int i = 0; i < C; i++){
            if(son[now][i]){
                Q.push(son[now][i]);
                int p = fail[now];
                while(!son[p][i]) p = fail[p];
                fail[son[now][i]] = son[p][i];
            }
            else son[now][i] = son[fail[now]][i];
        }
    }
}
int main(){
    cnt = 0; root = newNode();
```



```

scanf("%d", &n);
for(int i = 0; i < C; i++) son[0][i] = 1;
for(int i = 1; i <= n; i++){
    scanf("%s", s + 1);
    insert(s);
}
getfail();
return 0;
}

```

## 4.4 后缀自动机

### 4.4.1 广义后缀自动机（多串）

**注意事项：**空间是插入字符串总长度的 2 倍并注意字符集大小。

```

const int N = 251010, C = 26;
int tot, las, root;
struct Node
{
    int son[C], len, par;
    void clear(){
        memset(son, 0, sizeof(son));
        par = len = 0;
    }
}node[N << 1];
inline int newNode(){return node[++tot].clear(), tot;}
void extend(int c)
{
    int p = las;
    if (node[p].son[c]) {
        int q = node[p].son[c];
        if (node[p].len + 1 == node[q].len) las = q;
        else{
            int nq = newNode();
            las = nq; node[nq] = node[q];
            node[nq].len = node[p].len + 1; node[q].par = nq;
            for (; p && node[p].son[c] == q; p = node[p].par)

```

```

        node[p].son[c] = nq;
    }
}
else{ // Naive Suffix Automaton
    int np = newNode();
    las = np; node[np].len = node[p].len + 1;
    for (; p && !node[p].son[c]; p = node[p].par)
        node[p].son[c] = np;
    if (!p) node[np].par = root;
    else{
        int q = node[p].son[c];
        if (node[p].len + 1 == node[q].len)
            node[np].par = q;
        else{
            int nq = newNode();
            node[nq] = node[q];
            node[nq].len = node[p].len + 1;
            node[q].par = node[np].par = nq;
            for (; p && node[p].son[c] == q; p = node[p].par)
                node[p].son[c] = nq;
        }
    }
}
}
void add(char *s)
{
    int len = strlen(s + 1); las = root;
    for(int i = 1; i <= len; i++) extend(s[i] - 'a');
}

```

### 4.4.2 sam-ypm

#### sam-nsustr

```

//SAM 利用后缀树进行计算，由儿子向 parent 更新
#include <bits/stdc++.h>
using namespace std;
typedef long long LL;

```

```

typedef pair<int, int> pii;
const int inf = 1e9;
const int N = 251010, C = 26;
int tot, las, root;
struct Node
{
    int son[C], len, par, count;
    void clear(){
        memset(son, 0, sizeof(son));
        par = count = len = 0;
    }
}node[N << 1];
inline int newNode(){return node[++tot].clear(), tot;}
void extend(int c)//传入转化为数字之后的字符, 从 0 开始
{
    int p = las, np = newNode(); las = np;
    node[np].len = node[p].len + 1;
    for(;p && !node[p].son[c]; p = node[p].par)
        node[p].son[c] = np;
    if(p == 0) node[np].par = root;
    else{
        int q = node[p].son[c];
        if(node[p].len + 1 == node[q].len)
            node[np].par = q;
        else{
            int nq = newNode();
            node[nq] = node[q];
            node[nq].len = node[p].len + 1;
            node[q].par = node[np].par = nq;
            for(;p && node[p].son[c] == q; p = node[p].par)
                node[p].son[c] = nq;
        }
    }
}
int main(){
    static char s[N];
    while(scanf("%s", s + 1) == 1){
        tot = 0;

```

```

        root = las = newNode();
        int n = strlen(s + 1);
        for(int i = 1; i <= n; i++) extend(s[i] - 'a');
        static int cnt[N], order[N << 1];
        memset(cnt, 0, sizeof(*cnt) * (n + 5));
        for(int i = 1; i <= tot; i++) cnt[node[i].len]++;
        for(int i = 1; i <= n; i++) cnt[i] += cnt[i - 1];
        for(int i = tot; i; i--) order[ cnt[node[i].len]-- ] = i;
        static int dp[N]; memset(dp, 0, sizeof(dp));
        //dp[i] 为长度为 i 的子串中出现次数最多的串的出现次数
        for(int now = root, i = 1; i <= n; i++){
            now = node[now].son[s[i] - 'a'];
            node[now].count++;
        }
        for(int i = tot; i; i--){
            Node &now = node[order[i]];
            dp[now.len] = max(dp[now.len], now.count);
            node[now.par].count += now.count;
        }
        for(int i = n - 1; i; i--) dp[i] = max(dp[i], dp[i + 1]);
        for(int i = 1; i <= n; i++) printf("%d\n", dp[i]);
    }
}

```

### sam-lcs

```

#include <bits/stdc++.h>
using namespace std;
typedef long long LL;
typedef pair<int, int> pii;
const int inf = 1e9;
const int N = 101010, C = 26;

int tot, las, root;
struct Node{
    int son[C], len, par, count;
    void clear(){
        memset(son, 0, sizeof(son));

```

```

    par = count = len = 0;
}
}node[N << 1];
inline int newNode(){return node[++tot].clear(), tot;}
void extend(int c)//传入转化为数字之后的字符, 从 0 开始
{
    int p = las, np = newNode(); las = np;
    node[np].len = node[p].len + 1;
    for(;p && !node[p].son[c]; p = node[p].par)
        node[p].son[c] = np;
    if(p == 0) node[np].par = root;
    else{
        int q = node[p].son[c];
        if(node[p].len + 1 == node[q].len)
            node[np].par = q;
        else{
            int nq = newNode(); node[nq] = node[q];
            node[nq].len = node[p].len + 1;
            node[q].par = node[np].par = nq;
            for(;p && node[p].son[c] == q; p = node[p].par)
                node[p].son[c] = nq;
        }
    }
}
}
int main(){
    static char s[N];
    scanf("%s", s + 1);
    tot = 0; root = las = newNode();
    int n = strlen(s + 1);
    for(int i = 1; i <= n; i++){
        extend(s[i] - 'a');
    }
    static int cnt[N], order[N << 1];
    memset(cnt, 0, sizeof(*cnt) * (n + 5));
    for(int i = 1; i <= tot; i++) cnt[node[i].len]++;
    for(int i = 1; i <= n; i++) cnt[i] += cnt[i - 1];
    for(int i = tot; i; i--) order[ cnt[node[i].len]-- ] = i;
    static int ANS[N << 1], dp[N << 1];
    memset(dp, 0, sizeof(*dp) * (tot + 5));

```

```

    for(int i = 1; i <= tot; i++) ANS[i] = node[i].len;
    while(scanf("%s", s + 1) == 1){
        n = strlen(s + 1);
        for(int now = root, len = 0, i = 1; i <= n; i++){
            int c = s[i] - 'a';
            while(now != root && !node[now].son[c])
                now = node[now].par;
            if(node[now].son[c]){
                len = min(len, node[now].len) + 1;
                now = node[now].son[c];
            }
            else len = 0;
            dp[now] = max(dp[now], len);
        }
        for(int i = tot; i; i--){
            int now = order[i];
            dp[node[now].par] = max(dp[node[now].par], dp[now]);
            ANS[now] = min(ANS[now], dp[now]);
            dp[now] = 0;
        }
    }
    int ans = 0;
    for(int i = 1; i <= tot; i++) ans = max(ans, ANS[i]);
    printf("%d\n", ans);
}

```

## 4.5 后缀数组

注意事项:  $\mathcal{O}(n \log n)$  倍增构造。

```

#define ws wws
const int MAXN = 201010;
int wa[MAXN], wb[MAXN], wv[MAXN], ws[MAXN];
int sa[MAXN], rk[MAXN], height[MAXN];
char s[MAXN];
inline bool cmp(int *r, int a, int b, int l)
{return r[a] == r[b] && r[a + l] == r[b + l];}
void SA(char *r, int *sa, int n, int m){

```

```

int *x = wa, *y = wb;
for(int i = 1; i <= m; i++)ws[i] = 0;
for(int i = 1; i <= n; i++)ws[x[i]] = r[i]++;
for(int i = 1; i <= m; i++)ws[i] += ws[i - 1];
for(int i = n; i > 0; i--)sa[ws[x[i]]--] = i;
for(int j = 1, p = 0; p < n; j <= 1, m = p){
    p = 0;
    for(int i = n - j + 1; i <= n; i++)y[++p] = i;
    for(int i = 1; i <= n; i++)if(sa[i] > j) y[++p] = sa[i] - j;
    for(int i = 1; i <= n; i++)wv[i] = x[y[i]];
    for(int i = 1; i <= m; i++)ws[i] = 0;
    for(int i = 1; i <= n; i++)ws[wv[i]]++;
    for(int i = 1; i <= m; i++)ws[i] += ws[i - 1];
    for(int i = n; i > 0; i--)sa[ws[wv[i]]--] = y[i];
    swap(x, y); x[sa[1]] = p = 1;
    for(int i = 2; i <= n; i++)
        x[sa[i]] = cmp(y, sa[i - 1], sa[i], j) ? p : ++p;
}
}
void getheight(char *r, int *sa, int *rk, int *h, int n){
    for(int i = 1; i <= n; i++) rk[sa[i]] = i;
    for(int i = 1, p = 0; i <= n; i++, p ? p-- : 0){
        int j = sa[rk[i] - 1];
        while(r[i + p] == r[j + p]) p++;
        h[rk[i]] = p;
    }
}

```

## 4.6 Manacher

注意事项：1-based 算法，请注意下标。

```

void manacher(char *st){
    static char s[N << 1];
    static int p[N << 1];
    int n = strlen(st + 1);
    s[0] = '$'; s[1] = '#';
    for(int i = 1; i <= n; i++)

```

```

        s[i << 1] = st[i], s[(i << 1) + 1] = '#';
    s[(n = n * 2 + 1) + 1] = 0;
    int pos, mx = 0, res = 0;
    for(int i = 1; i <= n; i++){
        p[i] = (mx > i) ? min(p[pos * 2 - i], mx - i) : 1;
        while(s[i + p[i]] == s[i - p[i]]) p[i]++;
        if(p[i] + i - 1 > mx) mx = p[i] + i - 1, pos = i;
    }
}

```

## 4.7 循环串的最小表示

注意事项：0-Based 算法，请注意下标。

```

int getmin(char *s, int n){// 0-base
    int i = 0, j = 1, k = 0;
    while(i < n && j < n && k < n){
        int x = i + k; if(x >= n) x -= n;
        int y = j + k; if(y >= n) y -= n;
        if(s[x] == s[y]) k++;
        else{
            if(s[x] > s[y]) i += k + 1;
            else j += k + 1;
            if(i == j) j++;
            k = 0;
        }
    }
    return min(i, j);
}

```

## 5 计算几何

### 5.1 二维几何

// 求圆与直线的交点

```

bool isCL(Circle a, Line l, P &p1, P &p2) {
    D x = (l.s - a.o) % l.d,

```

```

    y = l.d.sqrLen(),
    d = x * x - y * ((l.s - a.o).sqrLen() - a.r * a.r);
    if (sign(d) < 0) return false;
    P p = l.s - x / y * l.d, delta = sqrt(max((D)0., d)) / y * l.d;
    p1 = p + delta, p2 = p - delta;
    return true;
}
// 求圆与圆的交面积
D areaCC(const Circle &c1, const Circle &c2) {
    D d = (c1.o - c2.o).len();
    if (sign(d - (c1.r + c2.r)) >= 0) {
        return 0;
    }
    if (sign(d - abs(c1.r - c2.r)) <= 0) {
        D r = min(c1.r, c2.r);
        return r * r * pi;
    }
    D x = (d * d + c1.r * c1.r - c2.r * c2.r) / (2 * d),
        t1 = acos(min(1., max(-1., x / c1.r))), t2 = acos(min(1., max(-1., (d - x) /
↪ c2.r)));
    return c1.r * c1.r * t1 + c2.r * c2.r * t2 - d * c1.r * sin(t1);
}
// 求圆与圆的交点, 注意调用前要先判定重圆
bool isCC(Circle a, Circle b, P &p1, P &p2) {
    D s1 = (a.o - b.o).len();
    if (sign(s1 - a.r - b.r) > 0 || sign(s1 - abs(a.r - b.r)) < 0) return false;
    D s2 = (a.r * a.r - b.r * b.r) / s1;
    D aa = (s1 + s2) * 0.5, bb = (s1 - s2) * 0.5;
    P o = aa / (aa + bb) * (b.o - a.o) + a.o;
    P delta = sqrt(max(0., a.r * a.r - aa * aa)) * (b.o - a.o).zoom(1).rev();
    p1 = o + delta, p2 = o - delta;
    return true;
}
// 求点到圆的切点, 按关于点的顺时针方向返回两个点, rev 必须是 (-y, x)
bool tanCP(const Circle &c, const P &p0, P &p1, P &p2) {
    D x = (p0 - c.o).sqrLen(), d = x - c.r * c.r;
    if (d < eps) return false; // 点在圆上认为没有切点
    P p = c.r * c.r / x * (p0 - c.o);

```

```

    P delta = (-c.r * sqrt(d) / x * (p0 - c.o)).rev();
    p1 = c.o + p + delta;
    p2 = c.o + p - delta;
    return true;
}
// 求圆到圆的外共切线, 按关于 c1.o 的顺时针方向返回两条线, rev 必须是 (-y, x)
vector<Line> extanCC(const Circle &c1, const Circle &c2) {
    vector<Line> ret;
    if (sign(c1.r - c2.r) == 0) {
        P dir = c2.o - c1.o;
        dir = (c1.r / dir.len() * dir).rev();
        ret.push_back(Line(c1.o + dir, c2.o - c1.o));
        ret.push_back(Line(c1.o - dir, c2.o - c1.o));
    } else {
        P p = 1. / (c1.r - c2.r) * (-c2.r * c1.o + c1.r * c2.o);
        P p1, p2, q1, q2;
        if (tanCP(c1, p, p1, p2) && tanCP(c2, p, q1, q2)) {
            if (c1.r < c2.r) swap(p1, p2), swap(q1, q2);
            ret.push_back(Line(p1, q1 - p1));
            ret.push_back(Line(p2, q2 - p2));
        }
    }
    return ret;
}
// 求圆到圆的内共切线, 按关于 c1.o 的顺时针方向返回两条线, rev 必须是 (-y, x)
vector<Line> intanCC(const Circle &c1, const Circle &c2) {
    vector<Line> ret;
    P p = 1. / (c1.r + c2.r) * (c2.r * c1.o + c1.r * c2.o);
    P p1, p2, q1, q2;
    if (tanCP(c1, p, p1, p2) && tanCP(c2, p, q1, q2)) { // 两圆相切认为没有切线
        ret.push_back(Line(p1, q1 - p1));
        ret.push_back(Line(p2, q2 - p2));
    }
    return ret;
}
bool contain(vector<P> poly, P p) { // 判断点 p 是否被多边形包含, 包括落在边界上
    int ret = 0, n = poly.size();
    for(int i = 0; i < n; ++i) {

```

```

    P u = poly[i], v = poly[(i + 1) % n];
    if (onSeg(p, u, v)) return true; // 在边界上
    if (sign(u.y - v.y) <= 0) swap(u, v);
    if (sign(p.y - u.y) > 0 || sign(p.y - v.y) <= 0) continue;
    ret += sign((v - p) * (u - p)) > 0;
}
return ret & 1;
}

vector<P> convexCut(const vector<P>&ps, Line l) {
    ↪ // 用半平面 (s,d) 的逆时针方向去切凸多边形
    vector<P> qs;
    int n = ps.size();
    for (int i = 0; i < n; ++i) {
        Point p1 = ps[i], p2 = ps[(i + 1) % n];
        int d1 = sign(l.d * (p1 - l.s)), d2 = sign(l.d * (p2 - l.s));
        if (d1 >= 0) qs.push_back(p1);
        if (d1 * d2 < 0) qs.push_back(isLL(Line(p1, p2 - p1), l));
    }
    return qs;
}

```

## 5.2 凸包

```

inline bool turn_left(const Point &a, const Point &b, const Point &c) {
    return sgn(det(b - a, c - a)) >= 0;
}

```

```

void convex_hull(vector<Data> p, vector<Data> &res) {
    int n = (int)p.size(), cnt = 0;
    sort(p.begin(), p.end(), [&](const Data &a, const Data &b) {
        if(fabs(a.p.x - b.p.x) < eps) return a.p.y > b.p.y;
        return a.p.x < b.p.x; });
    res.clear();
    for(int i = 0; i < n; i++) {
        while(cnt > 1 && turn_left(res[cnt - 2].p, p[i].p, res[cnt - 1].p)) {
            cnt--;
            res.pop_back();
        }
    }
}

```

```

        res.push_back(p[i]);
        ++cnt;
    }
    int fixed = cnt;
    for(int i = n - 2; i >= 0; i--) {
        while(cnt > fixed && turn_left(res[cnt - 2].p, p[i].p, res[cnt - 1].p)) {
            --cnt;
            res.pop_back();
        }
        res.push_back(p[i]);
        ++cnt;
    }
}

```

## 5.3 阿波罗尼茨圆

硬币问题：易知两两相切的圆半径为  $r_1, r_2, r_3$ ，求与他们都相切的圆的半径  $r_4$

分母取负号，答案再取绝对值，为外切圆半径

分母取正号为内切圆半径

$$// r_4^{\pm} = \frac{r_1 r_2 r_3}{r_1 r_2 + r_1 r_3 + r_2 r_3 \pm 2\sqrt{r_1 r_2 r_3 (r_1 + r_2 + r_3)}}$$

## 5.4 最小覆盖球

// 注意，无法处理小于四点的退化情况

```

struct P;
P a[33];
P intersect(const Plane &a, const Plane &b, const Plane &c) {
    P c1(a.nor.x, b.nor.x, c.nor.x), c2(a.nor.y, b.nor.y, c.nor.y), c3(a.nor.z, b.nor.z,
    ↪ c.nor.z), c4(a.m, b.m, c.m);
    return 1 / ((c1 * c2) % c3) * Point((c4 * c2) % c3, (c1 * c4) % c3, (c1 * c2) % c4);
}
bool in(const P &a, const Circle &b) {
    return sign((a - b.o).len() - b.r) <= 0;
}
vector<P> vec;
Circle calc() {
    if (vec.empty()) {

```

```

    return Circle(Point(0, 0, 0), 0);
} else if(1 == (int)vec.size()) {
    return Circle(vec[0], 0);
} else if(2 == (int)vec.size()) {
    return Circle(0.5 * (vec[0] + vec[1]), 0.5 * (vec[0] - vec[1]).len());
} else if(3 == (int)vec.size()) {
    double r((vec[0] - vec[1]).len() * (vec[1] - vec[2]).len() * (vec[2] -
↪ vec[0]).len() / 2 /
        fabs(((vec[0] - vec[2]) * (vec[1] - vec[2])).len()));
    return Circle(intersect(Plane(vec[1] - vec[0], 0.5 * (vec[1] + vec[0])),
        Plane(vec[2] - vec[1], 0.5 * (vec[2] + vec[1])),
        Plane((vec[1] - vec[0]) * (vec[2] - vec[0]), vec[0])), r);
} else {
    P o(intersect(Plane(vec[1] - vec[0], 0.5 * (vec[1] + vec[0])),
        Plane(vec[2] - vec[0], 0.5 * (vec[2] + vec[0])),
        Plane(vec[3] - vec[0], 0.5 * (vec[3] + vec[0]))));
    return Circle(o, (o - vec[0]).len());
}
}

Circle miniBall(int n) {
    Circle res(calc());
    for(int i(0); i < n; i++) {
        if(!in(a[i], res)) {
            vec.push_back(a[i]);
            res = miniBall(i);
            vec.pop_back();
            if (i) { Point tmp(a[i]); memmove(a + 1, a, sizeof(Point) * i); a[0] = tmp; }
        }
    }
    return res;
}

int main() {
    for(int i(0); i < n; i++) a[i].scan();
    sort(a, a + n);
    n = unique(a, a + n) - a;
    vec.clear();
    random_shuffle(a, a + n);
    printf("%.10f\n", miniBall(n).r);
}

```

```

}

```

## 5.5 三角形与圆交

// 反三角函数要在  $[-1, 1]$  中, sqrt 要与 0 取 max 别忘了取正负

// 改成周长请用注释, res1 为直线长度, res2 为弧线长度

// 多边形与圆求交时, 相切精度比较差

```

D areaCT(P pa, P pb, D r) { //, D & res1, D & res2) {
    if (pa.len() < pb.len()) swap(pa, pb);
    if (sign(pb.len()) == 0) return 0;
↪ // if (sign(pb.len()) == 0) { res1 += min(r, pa.len()); return; }
    D a = pb.len(), b = pa.len(), c = (pb - pa).len();
    D sinB = fabs(pb * (pb - pa)), cosB = pb % (pb - pa), area = fabs(pa * pb);
    D S, B = atan2(sinB, cosB), C = atan2(area, pa % pb);
    sinB /= a * c; cosB /= a * c;
    if (a > r) {
        S = C / 2 * r * r; D h = area / c; //res2 += -1 * sgn * C * r; D h = area / c;
        if (h < r && B < pi / 2) {
            //res2 -= -1 * sgn * 2 * acos(max((D)-1., min((D)1., h / r))) * r;
            //res1 += 2 * sqrt(max((D)0., r * r - h * h));
            S -= (acos(max((D)-1., min((D)1., h / r))) * r * r - h * sqrt(max((D)0., r
↪ * r - h * h)));
        }
    } else if (b > r) {
        D theta = pi - B - asin(max((D)-1., min((D)1., sinB / r * a)));
        S = a * r * sin(theta) / 2 + (C - theta) / 2 * r * r;
        //res2 += -1 * sgn * (C - theta) * r;
        //res1 += sqrt(max((D)0., r * r + a * a - 2 * r * a * cos(theta)));
    } else S = area / 2; //res1 += (pb - pa).len();
    return S;
}

```

## 5.6 圆并

```

struct Event {
    P p; D ang; int delta;
}

```

```

    Event (P p = Point(0, 0), D ang = 0, int delta = 0) : p(p), ang(ang), delta(delta)
    ↪ {}
};

bool operator < (const Event &a, const Event &b) { return a.ang < b.ang; }

void addEvent(const Circle &a, const Circle &b, vector<Event> &evt, int &cnt) {
    D d2 = (a.o - b.o).sqrLen(), dRatio = ((a.r - b.r) * (a.r + b.r) / d2 + 1) / 2,
    pRatio = sqrt(max((D)0., -(d2 - sqrt(a.r - b.r)) * (d2 - sqrt(a.r + b.r)) / (d2 * d2
    ↪ * 4)));
    P d = b.o - a.o, p = d.rot(pi / 2),
    q0 = a.o + d * dRatio + p * pRatio,
    q1 = a.o + d * dRatio - p * pRatio;
    D ang0 = (q0 - a.o).ang(), ang1 = (q1 - a.o).ang();
    evt.emplace_back(q1, ang1, 1); evt.emplace_back(q0, ang0, -1);
    cnt += ang1 > ang0;
}

bool issame(const Circle &a, const Circle &b) { return sign((a.o - b.o).len()) == 0 &&
    ↪ sign(a.r - b.r) == 0; }

bool overlap(const Circle &a, const Circle &b) { return sign(a.r - b.r - (a.o -
    ↪ b.o).len()) >= 0; }

bool intersect(const Circle &a, const Circle &b) { return sign((a.o - b.o).len() - a.r
    ↪ - b.r) < 0; }

int C;
Circle c[N];
double area[N];

void solve() { // 返回覆盖至少 k 次的面积
    memset(area, 0, sizeof(D) * (C + 1));
    for (int i = 0; i < C; ++i) {
        int cnt = 1;
        vector<Event> evt;
        for (int j = 0; j < i; ++j) if (issame(c[i], c[j])) ++cnt;
        for (int j = 0; j < C; ++j)
            if (j != i && !issame(c[i], c[j]) && overlap(c[j], c[i]))
                ++cnt;
        for (int j = 0; j < C; ++j)
            if (j != i && !overlap(c[j], c[i]) && !overlap(c[i], c[j]) && intersect(c[i],
            ↪ c[j]))
                addEvent(c[i], c[j], evt, cnt);
        if (evt.empty()) area[cnt] += PI * c[i].r * c[i].r;
    }
}

```

```

    else {
        sort(evt.begin(), evt.end());
        evt.push_back(evt.front());
        for (int j = 0; j + 1 < (int)evt.size(); ++j) {
            cnt += evt[j].delta;
            area[cnt] += det(evt[j].p, evt[j + 1].p) / 2;
            D ang = evt[j + 1].ang - evt[j].ang;
            if (ang < 0) ang += PI * 2;
            area[cnt] += ang * c[i].r * c[i].r / 2 - sin(ang) * c[i].r * c[i].r / 2;
        }
    }
}
}
}
}

```

## 5.7 整数半平面交

```

typedef __int128 J; // 坐标 |1e9| 就要用 int128 来判断

struct Line {
    bool include(P a) const { return (a - s) * d >= 0; } // 严格去掉 =
    bool include(Line a, Line b) const {
        J l1(a.d * b.d);
        if (!l1) return true;
        J x(l1 * (a.s.x - s.x)), y(l1 * (a.s.y - s.y));
        J l2((b.s - a.s) * b.d);
        x += l2 * a.d.x; y += l2 * a.d.y;
        J res(x * d.y - y * d.x);
        return l1 > 0 ? res >= 0 : res <= 0; // 严格去掉 =
    }
};

bool HPI(vector<Line> v) { // 返回 v 中每个射线的右侧的交是否非空
    sort(v.begin(), v.end()); // 按方向排极角序
    { // 同方向取最严格的一个
        vector<Line> t; int n(v.size());
        for (int i(0), j; i < n; i = j) {
            LL mx(-9e18); int mxi;
            for (j = i; j < n && v[i].d * v[j].d == 0; j++) {
                LL tmp(v[j].s * v[i].d);
                if (tmp > mx)
                    mx = tmp, mxi = j;
            }
            t.push_back(v[mxi]);
        }
    }
}

```



```

    }
    swap(v, t);
}
deque<Line> res;
bool emp(false);
for(auto i : v) {
    if(res.size() == 1) {
        if(res[0].d * i.d == 0 && !i.include(res[0].s)) {
            res.pop_back();
            emp = true;
        }
    } else if(res.size() >= 2) {
        while(res.size() >= 2u && !i.include(res.back(), res[res.size() - 2])) {
            if(i.d * res[res.size() - 2].d == 0 || !res.back().include(i, res[res.size() - 2])) {
                emp = true;
                break;
            }
            res.pop_back();
        }
        while(res.size() >= 2u && !i.include(res[0], res[1])) res.pop_front();
    }
    if(emp) break;
    res.push_back(i);
}
while (res.size() > 2u && !res[0].include(res.back(), res[res.size() - 2]))
    res.pop_back();
return !emp; // emp: 是否为空, res 按顺序即为半平面交
}

```

## 5.8 三角形

```

P ferma(const P& a, const P& b, const P& c) {
    D ab((b - a).len()), bc((b - c).len()), ca((c - a).len());
    D cosa((b - a) % (c - a) / ab / ca);
    D cosb((a - b) % (c - b) / ab / bc);
    D cosc((b - c) % (a - c) / ca / bc);
    P mid; D sq3(sqrt(3) / 2);

```

```

    if(sign((b - a) * (c - a)) < 0) swap(b, c);
    if(sign(cosa + 0.5) < 0) mid = a;
    else if(sign(cosb + 0.5) < 0) mid = b;
    else if(sign(cosc + 0.5) < 0) mid = c;
    else mid = intersection(Line(a, c + (b - c).rot(sq3) - a), Line(c, b + (a -
    ↪ b).rot(sq3) - c));
    return mid;
    // mid 为三角形 abc 费马点, 要求 abc 非退化
    length = (mid - a).len() + (mid - b).len() + (mid - c).len();
    // 以下求法仅在三角形三个角均小于 120 度时, 可以求出 ans 为费马点到 abc 三点距离和
    length = (a - c - (b - c).rot(sq3)).len();
}
P inCenter(const P & A, const P & B, const P & C) { // 内心
    D a = (B - C).len(), b = (C - A).len(), c = (A - B).len(),
    s = abs((B - A) * (C - A)),
    r = s / (a + b + c); // 内接圆半径
    return 1. / (a + b + c) * (A * a + B * b + C * c);
    ↪ // 偏心则将对应点前两个加号改为减号
}
P circumCenter(const P & a, const P & b, const P & c) { // 外心
    P bb = b - a, cc = c - a;
    // 半径为 a * b * c / 4 / S, a, b, c 为边长, S 为面积
    D db = bb.sqrLen(), dc = cc.sqrLen(), d = 2 * (bb * cc);
    return a - 1. / d * P(bb.y * dc - cc.y * db, cc.x * db - bb.x * dc);
}
P othroCenter(const P & a, const P & b, const P & c) { // 垂心
    P ba = b - a, ca = c - a, bc = b - c;
    D Y = ba.y * ca.y * bc.y,
    A = ca.x * ba.y - ba.x * ca.y,
    x0 = (Y + ca.x * ba.y * b.x - ba.x * ca.y * c.x) / A,
    y0 = -ba.x * (x0 - c.x) / ba.y + ca.y;
    return P(x0, y0);
}

```

## 5.9 经纬度求球面最短距离

```

double sphereDis(double lon1, double lat1, double lon2, double lat2, double R) {
    return R * acos(cos(lat1) * cos(lat2) * cos(lon1 - lon2) + sin(lat1) * sin(lat2));
}

```

```
}
```

### 5.10 长方体表面两点最短距离

```
int r;
void turn(int i, int j, int x, int y, int z, int x0, int y0, int L, int W, int H) {
    if (z==0) { int R = x*x+y*y; if (R<r) r=R;
    } else {
        if(i>=0 && i< 2) turn(i+1, j, x0+L+z, y, x0+L-x, x0+L, y0, H, W, L);
        if(j>=0 && j< 2) turn(i, j+1, x, y0+W+z, y0+W-y, x0, y0+W, L, H, W);
        if(i<=0 && i>-2) turn(i-1, j, x0-z, y, x-x0, x0-H, y0, H, W, L);
        if(j<=0 && j>-2) turn(i, j-1, x, y0-z, y-y0, x0, y0-H, L, H, W);
    }
}
int main(){
    int L, H, W, x1, y1, z1, x2, y2, z2;
    cin >> L >> W >> H >> x1 >> y1 >> z1 >> x2 >> y2 >> z2;
    if (z1!=0 && z1!=H) if (y1==0 || y1==W)
        swap(y1,z1), std::swap(y2,z2), std::swap(W,H);
    else swap(x1,z1), std::swap(x2,z2), std::swap(L,H);
    if (z1==H) z1=0, z2=H-z2;
    r=0x3fffffff;
    turn(0,0,x2-x1,y2-y1,z2,-x1,-y1,L,W,H);
    cout<<r<<endl;
}
```

### 5.11 点到凸包切线

```
P lb(P x, vector<P> & v, int le, int ri, int sg) {
    if (le > ri) le = ri;
    int s(le), t(ri);
    while (le != ri) {
        int mid((le + ri) / 2);
        if (sign((v[mid] - x) * (v[mid + 1] - v[mid])) == sg)
            le = mid + 1; else ri = mid;
    }
    return x - v[le]; // le 即为下标, 按需返回
}
```

```
}
```

// v[0] 为顺时针上凸壳, v[1] 为顺时针下凸壳, 均允许起始两个点横坐标相同  
// 返回值为真代表严格在凸包外, 顺时针旋转在 d1 方向先碰到凸包

```
bool getTan(P x, vector<P> * v, P & d1, P & d2) {
    if (x.x < v[0][0].x) {
        d1 = lb(x, v[0], 0, sz(v[0]) - 1, 1);
        d2 = lb(x, v[1], 0, sz(v[1]) - 1, -1);
        return true;
    } else if (x.x > v[0].back().x) {
        d1 = lb(x, v[1], 0, sz(v[1]) - 1, 1);
        d2 = lb(x, v[0], 0, sz(v[0]) - 1, -1);
        return true;
    } else {
        for(int d(0); d < 2; d++) {
            int id(lower_bound(v[d].begin(), v[d].end(), x,
                [&](const P & a, const P & b) {
                    return d == 0 ? a < b : b < a;
                }) - v[d].begin());
            if (id && (id == sz(v[d]) || (v[d][id - 1] - x) * (v[d][id] - x) > 0)) {
                d1 = lb(x, v[d], id, sz(v[d]) - 1, 1);
                d2 = lb(x, v[d], 0, id, -1);
                return true;
            }
        }
    }
    return false;
}
```

### 5.12 直线与凸包的交点

// a 是顺时针凸包, i1 为 x 最小的点, j1 为 x 最大的点 需保证 j1 > i1  
// n 是凸包上的点数, a 需复制多份或写循环数组类

```
int lowerBound(int le, int ri, const P & dir) {
    while (le < ri) {
        int mid((le + ri) / 2);
        if (sign((a[mid + 1] - a[mid]) * dir) <= 0) {
            le = mid + 1;
        } else ri = mid;
    }
}
```

```

    }
    return le;
}

int boundLower(int le, int ri, const P & s, const P & t) {
    while (le < ri) {
        int mid((le + ri + 1) / 2);
        if (sign((a[mid] - s) * (t - s)) <= 0) {
            le = mid;
        } else ri = mid - 1;
    }
    return le;
}

void calc(P s, P t) {
    if(t < s) swap(t, s);
    int i3(lowerBound(i1, j1, t - s)); // 和上凸包的切点
    int j3(lowerBound(j1, i1 + n, s - t)); // 和下凸包的切点
    int i4(boundLower(i3, j3, s, t));
    ↪ // 如果有交则是右侧的交点, 与 a[i4]~a[i4+1] 相交 要判断是否有交的话 就手动 check 一下
    int j4(boundLower(j3, i3 + n, t, s)); // 如果有交左侧的交点, 与 a[j4]~a[j4+1] 相交
    // 返回的下标不一定在 [0 ~ n-1] 内
}

```

### 5.13 平面最近点对

// Create: 2017-10-22 20:15:34

#include <bits/stdc++.h>

using namespace std;

const int N = 100005;

```

struct Data {
    double x, y;
};

```

```

double sqr(double x) {
    return x * x;
}

```

```

}

double dis(Data a, Data b) {
    return sqrt(sqr(a.x - b.x) + sqr(a.y - b.y));
}

```

```

int n;
Data p[N], q[N];

```

```

double solve(int l, int r) {
    if(l == r) return 1e18;
    if(l + 1 == r) return dis(p[l], p[r]);
    int m = (l + r) / 2;
    double d = min(solve(l, m), solve(m + 1, r));
    int qt = 0;
    for(int i = l; i <= r; i++) {
        if(fabs(p[m].x - p[i].x) <= d) {
            q[++qt] = p[i];
        }
    }
}

```

```

sort(q + 1, q + qt + 1, [&](const Data &a, const Data &b) {
    return a.y < b.y; });
for(int i = 1; i <= qt; i++) {
    for(int j = i + 1; j <= qt; j++) {
        if(q[j].y - q[i].y >= d) break;
        d = min(d, dis(q[i], q[j]));
    }
}
return d;
}

```

```

int main()
{
    while(scanf("%d", &n) == 1 && n) {
        for(int i = 1; i <= n; i++) {
            scanf("%lf%lf", &p[i].x, &p[i].y);
        }
        sort(p + 1, p + n + 1, [&](const Data &a, const Data &b) {
            return a.x < b.x || (a.x == b.x && a.y < b.y); });
    }
}

```

```

double ans = solve(1, n);
printf("%.2f\n", ans / 2);
}
return 0;
}

```

## 6 其他

### 6.1 斯坦纳树

```
priority_queue<pair<int, int> > Q;
```

```

// m is key point
// n is all point

```

```

for (int s = 0; s < (1 << m); s++){
    for (int i = 1; i <= n; i++){
        for (int s0 = (s&(s-1)); s0 ; s0=(s&(s0-1))){
            f[s][i] = min(f[s][i], f[s0][i] + f[s - s0][i]);
        }
    }
    for (int i = 1; i <= n; i++) vis[i] = 0;
    while (!Q.empty()) Q.pop();
    for (int i = 1; i <= n; i++){
        Q.push(mp(-f[s][i], i));
    }
    while (!Q.empty()){
        while (!Q.empty() && Q.top().first != -f[s][Q.top().second]) Q.pop();
        if (Q.empty()) break;
        int Cur = Q.top().second; Q.pop();
        for (int p = g[Cur]; p; p = nxt[p]){
            int y = adj[p];
            if ( f[s][y] > f[s][Cur] + 1){
                f[s][y] = f[s][Cur] + 1;
                Q.push(mp(-f[s][y], y));
            }
        }
    }
}

```

```

}
}

```

### 6.2 无敌的读入优化

```

namespace Reader {
    const int L = (1 << 20) + 5;
    char buffer[L], *S, *T;
    __inline bool getchar(char &ch) {
        if (S == T) {
            T = (S = buffer) + fread(buffer, 1, L, stdin);
            if (S == T) {
                ch = EOF;
                return false;
            }
        }
        ch = *S ++;
        return true;
    }
    __inline bool getint(int &x) {
        char ch;
        for (; getchar(ch) && (ch < '0' || ch > '9'); );
        if (ch == EOF) return false;
        x = ch - '0';
        for (; getchar(ch), ch >= '0' && ch <= '9'; )
            x = x * 10 + ch - '0';
        return true;
    }
}
Reader::getint(x);
Reader::getint(y);

```

### 6.3 最小树形图

```
const int maxn=1100;
```

```
int n,m , g[maxn][maxn] , used[maxn] , pass[maxn] , eg[maxn] , more , queue[maxn];
```

```

void combine (int id , int &sum ) {
    int tot = 0 , from , i , j , k ;
    for ( ; id!=0 && !pass[ id ] ; id=eg[id] ) {
        queue[tot++]=id ; pass[id]=1;
    }
    for ( from=0; from<tot && queue[from]!=id ; from++);
    if ( from==tot ) return ;
    more = 1 ;
    for ( i=from ; i<tot ; i++) {
        sum+=g[eg[queue[i]]][queue[i]] ;
        if ( i!=from ) {
            used[queue[i]]=1;
            for ( j = 1 ; j <= n ; j++) if ( !used[j] )
                if ( g[queue[i]][j]<g[id][j] ) g[id][j]=g[queue[i]][j] ;
        }
    }
    for ( i=1; i<=n ; i++) if ( !used[i] && i!=id ) {
        for ( j=from ; j<tot ; j++){
            k=queue[j];
            if ( g[i][id]>g[i][k]-g[eg[k]][k] ) g[i][id]=g[i][k]-g[eg[k]][k];
        }
    }
}

int mdst( int root ) { // return the total length of MDST
    int i , j , k , sum = 0 ;
    memset ( used , 0 , sizeof ( used ) ) ;
    for ( more =1; more ; ) {
        more = 0 ;
        memset ( eg,0,sizeof(eg)) ;
        for ( i=1 ; i <= n ; i ++ ) if ( !used[i] && i!=root ) {
            for ( j=1 , k=0 ; j <= n ; j ++ ) if ( !used[j] && i!=j )
                if ( k==0 || g[j][i] < g[k][i] ) k=j ;
            eg[i] = k ;
        }
        memset(pass,0,sizeof(pass));
    }
}

```

```

        for ( i=1; i<=n ; i++) if ( !used[i] && !pass[i] && i!= root ) combine ( i , sum )
    }
    for ( i =1; i<=n ; i ++ ) if ( !used[i] && i!= root ) sum+=g[eg[i]][i];
    return sum ;
}

```

## 6.4 DLX

```

int n,m,K;
struct DLX{
    int L[maxn],R[maxn],U[maxn],D[maxn];
    int sz,col[maxn],row[maxn],s[maxn],H[maxn];
    bool vis[233];
    int ans[maxn],cnt;
    void init(int m){
        for(int i=0;i<=m;i++){
            L[i]=i-1;R[i]=i+1;
            U[i]=D[i]=i;s[i]=0;
        }
        memset(H,-1,sizeof H);
        L[0]=m;R[m]=0;sz=m+1;
    }
    void Link(int r,int c){
        U[sz]=c;D[sz]=D[c];U[D[c]]=sz;D[c]=sz;
        if(H[r]<0)H[r]=L[sz]=R[sz]=sz;
        else{
            L[sz]=H[r];R[sz]=R[H[r]];
            L[R[H[r]]]=sz;R[H[r]]=sz;
        }
        s[c]++;col[sz]=c;row[sz]=r;sz++;
    }
    void remove(int c){
        for(int i=D[c];i!=c;i=D[i])
            L[R[i]]=L[i],R[L[i]]=R[i];
    }
    void resume(int c){
        for(int i=U[c];i!=c;i=U[i])

```

```

    L[R[i]]=R[L[i]]=i;
}
int A(){
    int res=0;
    memset(vis,0,sizeof vis);
    for(int i=R[0];i;i=R[i])if(!vis[i]){
        vis[i]=1;res++;
        for(int j=D[i];j!=i;j=D[j])
            for(int k=R[j];k!=j;k=R[k])
                vis[col[k]]=1;
    }
    return res;
}
void dfs(int d,int &ans){
    if(R[0]==0){ans=min(ans,d);return;}
    if(d+A()>=ans)return;
    int tmp=23333,c;
    for(int i=R[0];i;i=R[i])
        if(tmp>s[i])tmp=s[i],c=i;
    for(int i=D[c];i!=c;i=D[i]){
        remove(i);
        for(int j=R[i];j!=i;j=R[j])remove(j);
        dfs(d+1,ans);
        for(int j=L[i];j!=i;j=L[j])resume(j);
        resume(i);
    }
}
void del(int c){//exactly cover
    L[R[c]]=L[c];R[L[c]]=R[c];
    for(int i=D[c];i!=c;i=D[i])
        for(int j=R[i];j!=i;j=R[j])
            U[D[j]]=U[j],D[U[j]]=D[j],--s[col[j]];
}
void add(int c){ //exactly cover
    R[L[c]]=L[R[c]]=c;
    for(int i=U[c];i!=c;i=U[i])
        for(int j=L[i];j!=i;j=L[j])
            ++s[col[U[D[j]]=D[U[j]]=j]];
}

```

```

}
bool dfs2(int k){//exactly cover
    if(!R[0]){
        cnt=k;return 1;
    }
    int c=R[0];
    for(int i=R[0];i;i=R[i])
        if(s[c]>s[i])c=i;
    del(c);
    for(int i=D[c];i!=c;i=D[i]){
        for(int j=R[i];j!=i;j=R[j])
            del(col[j]);
        ans[k]=row[i];if(dfs2(k+1))return true;
        for(int j=L[i];j!=i;j=L[j])
            add(col[j]);
    }
    add(c);
    return 0;
}
}dlx;
int main(){
    dlx.init(n);
    for(int i=1;i<=m;i++)
        for(int j=1;j<=n;j++)
            if(dis(station[i],city[j])<mid-eps)
                dlx.Link(i,j);
    dlx.dfs(0,ans);
}

```

## 6.5 插头 DP

```

int n,m,l;
struct L{
    int d[11];
    int& operator[](int x){return d[x];}
    int mc(int x){
        int an=1;
        if(d[x]==1){

```

```

        for(x++;x<l;x++){
            an=an+(d[x]==1?-1);
            if(!an)return x;
        }
    }else{
        for(x--;x>=0;x--){
            an=an+(d[x]==2?-1);
            if(!an)return x;
        }
    }
}
int h(){int an=0;for(int i=l-1;i>=0;i--)an=an*3+d[i];return an;}
L s(int x,int y){
    L S=*this;
    S[x]=y;return S;
}
L operator>>(int _){
    L S=*this;
    for(int i=l-1;i>=1;i--)S[i]=S[i-1];
    S[0]=0;return S;
}
};
struct Int{
    int len;
    int a[40];
    Int(){len=1;memset(a,0,sizeof a);}
    Int operator+=(const Int &o){
        int l=max(len,o.len);
        for(int i=0;i<l;i++)
            a[i]=a[i]+o.a[i];
        for(int i=0;i<l;i++)
            a[i+1]+=a[i]/10,a[i]%=10;
        if(a[l])l++;len=l;
        return *this;
    }
}
void print(){
    for(int i=len-1;i>=0;i--)
        printf("%d",a[i]);

```

```

        puts("");
    }
};
struct hashtab{
    int sz;
    int tab[177147];
    Int w[177147];
    L s[177147];
    hashtab(){memset(tab,-1,sizeof tab);}
    void cl(){
        for(int i=0;i<sz;i++)tab[s[i].h()]=-1;
        sz=0;
    }
    Int& operator[](L S){
        int h=S.h();
        if(tab[h]==-1)tab[h]=sz,s[sz]=S,w[sz]=Int(),sz++;
        return w[tab[h]];
    }
}f[2];
bool check(L S){
    int cn1=0,cn2=0;
    for(int i=0;i<l;i++){
        cn1+=S[i]==1;
        cn2+=S[i]==2;
    }return cn1==1&&cn2==1;
}
int main(){
    Int One;One.a[0]=1;
    scanf("%d%d",&n,&m);if(n<m)swap(n,m);l=m+1;
    if(n==1||m==1){puts("1");return 0;}
    int cur=0;f[cur].cl();
    for(int i=1;i<=n;i++){
        for(int j=1;j<=m;j++){
            if(i==1&&j==1){
                f[cur][L().s(0,1).s(1,2)]+=One;
                continue;
            }
            cur^=1;f[cur].cl();

```

```

for(int k=0;k<f[!cur].sz;k++){
    L S=f[!cur].s[k];Int w=f[!cur][S];
    int d1=S[j-1],d2=S[j];
    if(d1==0&&d2==0){
        if(i!=n&&j!=m)f[cur][S.s(j-1,1).s(j,2)]+=w;
    }else
    if(d1==0||d2==0){
        if(i!=n)f[cur][S.s(j-1,d1|d2).s(j,0)]+=w;
        if(j!=m)f[cur][S.s(j-1,0).s(j,d1|d2)]+=w;
    }else
    if(d1==1&&d2==2){
        if(i==n&&j==m&&check(S))
            (w+=w).print();
    }else
    if(d1==2&&d2==1){
        f[cur][S.s(j-1,0).s(j,0)]+=w;
    }else
    if((d1==1&&d2==1)||((d1==2&&d2==2))){
        int m1=S.mc(j),m2=S.mc(j-1);
        f[cur][S.s(j-1,0).s(j,0).s(m1,1).s(m2,2)]+=w;
    }
}
}
cur^=1;f[cur].cl();
for(int k=0;k<f[!cur].sz;k++){
    L S=f[!cur].s[k];Int w=f[!cur][S];
    f[cur][S>>1]=w;
}
}
return 0;
}

```

## 6.6 某年某月某日是星期几

```

int solve(int year, int month, int day) {
    int answer;
    if (month == 1 || month == 2) {
        month += 12;

```

```

        year--;
    }
    if ((year < 1752) || (year == 1752 && month < 9) ||
        (year == 1752 && month == 9 && day < 3)) {
        answer = (day + 2 * month + 3 * (month + 1) / 5 + year + year / 4 + 5) % 7;
    } else {
        answer = (day + 2 * month + 3 * (month + 1) / 5 + year + year / 4
            - year / 100 + year / 400) % 7;
    }
    return answer;
}

```

## 6.7 枚举大小为 $k$ 的子集

使用条件:  $k > 0$

```

void solve(int n, int k) {
    for (int comb = (1 << k) - 1; comb < (1 << n); ) {
        // ...
        int x = comb & -comb, y = comb + x;
        comb = (((comb & ~y) / x) >> 1) | y;
    }
}

```

## 6.8 环状最长公共子串

```
int n, a[N << 1], b[N << 1];
```

```

bool has(int i, int j) {
    return a[(i - 1) % n] == b[(j - 1) % n];
}

```

```
const int DELTA[3][2] = {{0, -1}, {-1, -1}, {-1, 0}};
```

```
int from[N][N];
```

```

int solve() {
    memset(from, 0, sizeof(from));
}

```



```

int ret = 0;
for (int i = 1; i <= 2 * n; ++i) {
    from[i][0] = 2;
    int left = 0, up = 0;
    for (int j = 1; j <= n; ++j) {
        int upleft = up + 1 + !!from[i - 1][j];
        if (!has(i, j)) {
            upleft = INT_MIN;
        }
        int max = std::max(left, std::max(upleft, up));
        if (left == max) {
            from[i][j] = 0;
        } else if (upleft == max) {
            from[i][j] = 1;
        } else {
            from[i][j] = 2;
        }
        left = max;
    }
}
if (i >= n) {
    int count = 0;
    for (int x = i, y = n; y; ) {
        int t = from[x][y];
        count += t == 1;
        x += DELTA[t][0];
        y += DELTA[t][1];
    }
    ret = std::max(ret, count);
    int x = i - n + 1;
    from[x][0] = 0;
    int y = 0;
    while (y <= n && from[x][y] == 0) {
        y++;
    }
    for (; x <= i; ++x) {
        from[x][y] = 0;
        if (x == i) {
            break;
        }
    }
}

```

```

}
for (; y <= n; ++y) {
    if (from[x + 1][y] == 2) {
        break;
    }
    if (y + 1 <= n && from[x + 1][y + 1] == 1) {
        y++;
        break;
    }
}
}
}
}
return ret;
}

```

## 6.9 LLMOD

```

LL multiplyMod(LL a, LL b, LL P) { // `需要保证 a 和 b 非负`
    LL t = (a * b - LL((long double)a / P * b + 1e-3) * P) % P;
    return t < 0 : t + P : t;
}

```

## 6.10 STL 内存清空

```

template <typename T>
__inline void clear(T& container) {
    container.clear(); // 或者删除了一堆元素
    T(container).swap(container);
}

```

## 6.11 开栈

```

register char *_sp __asm__("rsp");
int main() {
    const int size = 400 << 20; // 400MB
    static char *sys, *mine(new char[size] + size - 4096);
}

```

```
sys = _sp; _sp = mine; _main(); _sp = sys;
}
```

6.12 32-bit/64-bit 随机素数

32-bit	64-bit
73550053	1249292846855685773
148898719	1701750434419805569
189560747	3605499878424114901
459874703	5648316673387803781
1202316001	6125342570814357977
1431183547	6215155308775851301
1438011109	6294606778040623451
1538762023	6347330550446020547
1557944263	7429632924303725207
1981315913	8524720079480389849

7 vimrc

```
colo morning
set ru nu cin ts=4 sts=4 sw=4 hls is ar acd bs=2 mouse=a ls=2 fdm=syntax fdl=100
set makeprg=g++\ %:r.cpp\ -o\ %:r\ -g\ -std=c++11\ -Wall\ -Wextra\ -Wconversion

nmap <C-A> ggVG
vmap <C-C> "+y
noremap <C-V> "+P

map <F3> :vnew %:r.in<cr>
map <F4> :!gedit %<cr>
map <F5> :!time ./%:r<cr>
map <F8> :!time ./%:r < %:r.in<cr>
map <F9> :make<cr>
map <C-F9> :!g++ %:r.cpp -o %:r -g -O2 -std=c++11<cr>
map <F10> :!gdb ./%:r<cr>
```

8 常用结论

8.1 上下界网络流

$B(u, v)$  表示边  $(u, v)$  流量的下界,  $C(u, v)$  表示边  $(u, v)$  流量的上界,  $F(u, v)$  表示边  $(u, v)$  的流量。设  $G(u, v) = F(u, v) - B(u, v)$ , 显然有

$$0 \leq G(u, v) \leq C(u, v) - B(u, v)$$

无源汇的上下界可行流

建立超级源点  $S^*$  和超级汇点  $T^*$ , 对于原图每条边  $(u, v)$  在新网络中连如下三条边:  $S^* \rightarrow v$ , 容量为  $B(u, v)$ ;  $u \rightarrow T^*$ , 容量为  $B(u, v)$ ;  $u \rightarrow v$ , 容量为  $C(u, v) - B(u, v)$ 。最后求新网络的最大流, 判断从超级源点  $S^*$  出发的边是否都满流即可, 边  $(u, v)$  的最终解中的实际流量为  $G(u, v) + B(u, v)$ 。

有源汇的上下界可行流

从汇点  $T$  到源点  $S$  连一条上界为  $\infty$ , 下界为  $0$  的边。按照无源汇的上下界可行流一样做即可, 流量即为  $T \rightarrow S$  边上的流量。

有源汇的上下界最大流

- 1. 在有源汇的上下界可行流中, 从汇点  $T$  到源点  $S$  的边改为连一条上界为  $\infty$ , 下届为  $x$  的边。 $x$  满足二分性质, 找到最大的  $x$  使得新网络存在无源汇的上下界可行流即为原图的最大流。
- 2. 从汇点  $T$  到源点  $S$  连一条上界为  $\infty$ , 下界为  $0$  的边, 变成无源汇的网络。按照无源汇的上下界可行流的方法, 建立超级源点  $S^*$  和超级汇点  $T^*$ , 求一遍  $S^* \rightarrow T^*$  的最大流, 再将从汇点  $T$  到源点  $S$  的这条边拆掉, 求一次  $S \rightarrow T$  的最大流即可。

有源汇的上下界最小流

- 1. 在有源汇的上下界可行流中, 从汇点  $T$  到源点  $S$  的边改为连一条上界为  $x$ , 下界为  $0$  的边。 $x$  满足二分性质, 找到最小的  $x$  使得新网络存在无源汇的上下界

可行流即为原图的最小流。

- 按照无源汇的上下界可行流的方法，建立超级源点  $S^*$  与超级汇点  $T^*$ ，求一遍  $S^* \rightarrow T^*$  的最大流，但是注意这一次不加上汇点  $T$  到源点  $S$  的这条边，即不使之改为无源汇的网络去求解。求完后，再加上那条汇点  $T$  到源点  $S$  上界  $\infty$  的边。因为这条边下界为 0，所以  $S^*, T^*$  无影响，再直接求一次  $S^* \rightarrow T^*$  的最大流。若超级源点  $S^*$  出发的边全部满流，则  $T \rightarrow S$  边上的流量即为原图的最小流，否则无解。

## 8.2 上下界费用流

来源：BZOJ 3876 设汇  $t$ ，源  $s$ ，超级源  $S$ ，超级汇  $T$ ，本质是每条边的下界为 1，上界为 MAX，跑一遍有源汇的上下界最小费用最小流。（因为上界无穷大，所以只要满足所有下界的最小费用最小流）

- 对每个点  $x$ ：从  $x$  到  $t$  连一条费用为 0，流量为 MAX 的边，表示可以任意停止当前的剧情（接下来的剧情从更优的路径去走，画个样例就知道了）
- 对于每一条边权为  $z$  的边  $x \rightarrow y$ ：
  - 从  $S$  到  $y$  连一条流量为 1，费用为  $z$  的边，代表这条边至少要被走一次。
  - 从  $x$  到  $y$  连一条流量为 MAX，费用为  $z$  的边，代表这条边除了至少走的一次之外还可以随便走。
  - 从  $x$  到  $T$  连一条流量为 1，费用为 0 的边。（注意是每一条  $x \rightarrow y$  的边都连，或者你可以记下  $x$  的出边数  $Kx$ ，连一次流量为  $Kx$ ，费用为 0 的边）。

建完图后从  $S$  到  $T$  跑一遍费用流，即可。（当前跑出来的就是满足上下界的最小费用最小流了）

## 8.3 弦图相关

- 团数  $\leq$  色数，弦图团数 = 色数
- 设  $next(v)$  表示  $N(v)$  中最前的点。令  $w^*$  表示所有满足  $A \in B$  的  $w$  中最后的一个点，判断  $v \cup N(v)$  是否为极大团，只需判断是否存在一个  $w$ ，满足  $Next(w) = v$  且  $|N(v)| + 1 \leq |N(w)|$  即可。

- 最小染色：完美消除序列从后往前依次给每个点染色，给每个点染上可以染的最小的颜色
- 最大独立集：完美消除序列从前往后能选就选
- 弦图最大独立集数 = 最小团覆盖数，最小团覆盖：设最大独立集为  $\{p_1, p_2, \dots, p_t\}$ ，则  $\{p_1 \cup N(p_1), \dots, p_t \cup N(p_t)\}$  为最小团覆盖

## 8.4 Bernoulli 数

- 初始化： $B_0(n) = 1$

- 递推公式：

$$B_m(n) = n^m - \sum_{k=0}^{m-1} \binom{m}{k} \frac{B_k(n)}{m-k+1}$$

- 应用：

$$\sum_{k=1}^n k^m = \frac{1}{m+1} \sum_{k=0}^m \binom{m+1}{k} n^{m+1-k}$$

## 9 常见错误

- 数组或者变量类型开错，例如将 `double` 开成 `int`；
- 函数忘记返回返回值；
- 初始化数组没有初始化完全；
- 对空间限制判断不足导致 MLE；
- 对于重边未注意，
- 对于 0、1base 未弄清楚，用混
- map 的赋值问题 (`dis[] = find(dis[])`)
- 输出格式

## 10 测试列表

1. 检测评测机是否开 O2;
2. 检测 `__int128` 以及 `__float128` 是否能够使用;
3. 检测是否能够使用 C++11;
4. 检测是否能够使用 `Ext Lib`;
5. 检测程序运行所能使用的内存大小;
6. 检测程序运行所能使用的栈大小;
7. 检测是否有代码长度限制;
8. 检测是否能够正常返 `Runtime Error` (assertion、return 1、空指针);
9. 查清楚厕所方位和打印机方位;

## 11 Java

### 11.1 Java Hints

```
import java.util.*;
import java.math.*;
import java.io.*;

public class Main{
    static class Task{
        void solve(int testId, InputReader cin, PrintWriter cout) {
            // Write down the code you want
        }
    };

    public static void main(String args[]) {
        InputStream inputStream = System.in;
        OutputStream outputStream = System.out;
        InputReader in = new InputReader(inputStream);
        PrintWriter out = new PrintWriter(outputStream);
        TaskA solver = new TaskA();
```

```
        solver.solve(1, in, out);
        out.close();
    }

    static class InputReader {
        public BufferedReader reader;
        public StringTokenizer tokenizer;

        public InputReader(InputStream stream) {
            reader = new BufferedReader(new InputStreamReader(stream), 32768);
            tokenizer = null;
        }

        public String next() {
            while (tokenizer == null || !tokenizer.hasMoreTokens()) {
                try {
                    tokenizer = new StringTokenizer(reader.readLine());
                } catch (IOException e) {
                    throw new RuntimeException(e);
                }
            }
            return tokenizer.nextToken();
        }

        public int nextInt() {
            return Integer.parseInt(next());
        }
    };

    // Arrays
    int a[];
    .fill(a[, int fromIndex, int toIndex],val); | .sort(a[, int fromIndex, int toIndex])
    // String
    String s;
    .charAt(int i); | compareTo(String) | compareToIgnoreCase () | contains(String) |
    length () | substring(int l, int len)
    // BigInteger
```

```
.abs() | .add() | bitLength() | subtract() | divide() | remainder() |
↳ divideAndRemainder() | modPow(b, c) |
pow(int) | multiply() | compareTo() |
gcd() | intValue() | longValue() | isProbablePrime(int c) (1 - 1/2^c) |
nextProbablePrime() | shiftLeft(int) | valueOf()
// BigDecimal
.ROUND_CEILING | ROUND_DOWN_FLOOR | ROUND_HALF_DOWN | ROUND_HALF_EVEN | ROUND_HALF_UP
↳ | ROUND_UP
.divide(BigDecimal b, int scale, int round_mode) | doubleValue() |
↳ movePointLeft(int) | pow(int) |
setScale(int scale, int round_mode) | stripTrailingZeros()
BigDecimal.setScale()方法用于格式化小数点
setScale(1)表示保留一位小数, 默认用四舍五入方式
setScale(1,BigDecimal.ROUND_DOWN)直接删除多余的小数位, 如 2.35会变成 2.3
setScale(1,BigDecimal.ROUND_UP)进位处理, 2.35变成 2.4
setScale(1,BigDecimal.ROUND_HALF_UP)四舍五入, 2.35变成 2.4
setScale(1,BigDecimal.ROUND_HALF_DOWN)四舍五入, 2.35变成 2.3, 如果是 5 则向下舍
setScale(1,BigDecimal.ROUND_CEILING)接近正无穷大的舍入
setScale(1,BigDecimal.ROUND_FLOOR)接近负无穷大的舍入, 数字>0和 ROUND_UP 作用一样, 数字<0和 ROUND_DOWN 作用一样
setScale(1,BigDecimal.ROUND_HALF_EVEN)向最接近的数字舍入, 如果与两个相邻数字的距离相等, 则向相邻的偶数舍入, 如 2.5 向 2 舍入, 3.5 向 4 舍入
// StringBuilder
StringBuilder sb = new StringBuilder();
sb.append(elem) | out.println(sb)
// TODO Java STL 的使用方法以及上面这些方法的检验
```

## 12 数学

### 12.1 常用数学公式

#### 12.1.1 求和公式

- $\sum_{k=1}^n (2k-1)^2 = \frac{n(4n^2-1)}{3}$
- $\sum_{k=1}^n k^3 = \left[\frac{n(n+1)}{2}\right]^2$
- $\sum_{k=1}^n (2k-1)^3 = n^2(2n^2-1)$
- $\sum_{k=1}^n k^4 = \frac{n(n+1)(2n+1)(3n^2+3n-1)}{30}$

- $\sum_{k=1}^n k^5 = \frac{n^2(n+1)^2(2n^2+2n-1)}{12}$
- $\sum_{k=1}^n k(k+1) = \frac{n(n+1)(n+2)}{3}$
- $\sum_{k=1}^n k(k+1)(k+2) = \frac{n(n+1)(n+2)(n+3)}{4}$
- $\sum_{k=1}^n k(k+1)(k+2)(k+3) = \frac{n(n+1)(n+2)(n+3)(n+4)}{5}$

#### 12.1.2 斐波那契数列

- $fib_0 = 0, fib_1 = 1, fib_n = fib_{n-1} + fib_{n-2}$
- $fib_{n+2} \cdot fib_n - fib_{n+1}^2 = (-1)^{n+1}$
- $fib_{-n} = (-1)^{n-1} fib_n$
- $fib_{n+k} = fib_k \cdot fib_{n+1} + fib_{k-1} \cdot fib_n$
- $gcd(fib_m, fib_n) = fib_{gcd(m,n)}$
- $fib_n | fib_m \Leftrightarrow n | m$

#### 12.1.3 错排公式

- $D_n = (n-1)(D_{n-2} - D_{n-1})$
- $D_n = n! \cdot \left(1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \dots + \frac{(-1)^n}{n!}\right)$

#### 12.1.4 莫比乌斯函数

$$\mu(n) = \begin{cases} 1 & \text{若 } n = 1 \\ (-1)^k & \text{若 } n \text{ 无平方数因子, 且 } n = p_1 p_2 \dots p_k \\ 0 & \text{若 } n \text{ 有大于1的平方数因数} \end{cases}$$

$$\sum_{d|n} \mu(d) = \begin{cases} 1 & \text{若 } n = 1 \\ 0 & \text{其他情况} \end{cases}$$

$$g(n) = \sum_{d|n} f(d) \Leftrightarrow f(n) = \sum_{d|n} \mu(d)g\left(\frac{n}{d}\right)$$

$$g(x) = \sum_{n=1}^{[x]} f\left(\frac{x}{n}\right) \Leftrightarrow f(x) = \sum_{n=1}^{[x]} \mu(n)g\left(\frac{x}{n}\right)$$

### 12.1.5 伯恩赛德引理

设  $G$  是一个有限群, 作用在集合  $X$  上. 对每个  $g$  属于  $G$ , 令  $X^g$  表示  $X$  中在  $g$  作用下的不动元素, 轨道数 (记作  $|X/G|$ ) 由如下公式给出:

$$|X/G| = \frac{1}{|G|} \sum_{g \in G} |X^g|.$$

### 12.1.6 五边形数定理

设  $p(n)$  是  $n$  的拆分数, 有

$$p(n) = \sum_{k \in \mathbb{Z} \setminus \{0\}} (-1)^{k-1} p\left(n - \frac{k(3k-1)}{2}\right)$$

### 12.1.7 树的计数

1. 有根树计数:  $n+1$  个结点的有根树的个数为

$$a_{n+1} = \frac{\sum_{j=1}^n j \cdot a_j \cdot S_{n,j}}{n}$$

其中,

$$S_{n,j} = \sum_{i=1}^{n/j} a_{n+1-ij} = S_{n-j,j} + a_{n+1-j}$$

2. 无根树计数: 当  $n$  为奇数时,  $n$  个结点的无根树的个数为

$$a_n - \sum_{i=1}^{n/2} a_i a_{n-i}$$

当  $n$  为偶数时,  $n$  个结点的无根树的个数为

$$a_n - \sum_{i=1}^{n/2} a_i a_{n-i} + \frac{1}{2} a_{\frac{n}{2}} (a_{\frac{n}{2}} + 1)$$

3.  $n$  个结点的完全图的生成树个数为

$$n^{n-2}$$

4. 矩阵 - 树定理: 图  $G$  由  $n$  个结点构成, 设  $\mathbf{A}[G]$  为图  $G$  的邻接矩阵、 $\mathbf{D}[G]$  为图  $G$  的度数矩阵, 则图  $G$  的不同生成树的个数为  $\mathbf{C}[G] = \mathbf{D}[G] - \mathbf{A}[G]$  的任意一个  $n-1$  阶主子式的行列式值。

### 12.1.8 欧拉公式

平面图的顶点个数、边数和面的个数有如下关系:

$$V - E + F = C + 1$$

其中,  $V$  是顶点的数目,  $E$  是边的数目,  $F$  是面的数目,  $C$  是组成图形的连通部分的数目。当图是单连通图的时候, 公式简化为:

$$V - E + F = 2$$

### 12.1.9 皮克定理

给定顶点坐标均是整点 (或正方形格点) 的简单多边形, 其面积  $A$  和内部格点数目  $i$ 、边上格点数目  $b$  的关系:

$$A = i + \frac{b}{2} - 1$$

### 12.1.10 牛顿恒等式

设

$$\prod_{i=1}^n (x - x_i) = a_n + a_{n-1}x + \cdots + a_1x^{n-1} + a_0x^n$$

$$p_k = \sum_{i=1}^n x_i^k$$

则

$$a_0 p_k + a_1 p_{k-1} + \cdots + a_{k-1} p_1 + k a_k = 0$$

特别地, 对于

$$|\mathbf{A} - \lambda \mathbf{E}| = (-1)^n (a_n + a_{n-1} \lambda + \cdots + a_1 \lambda^{n-1} + a_0 \lambda^n)$$

有

$$p_k = \text{Tr}(\mathbf{A}^k)$$

## 12.2 平面几何公式

### 12.2.1 三角形

1. 半周长

$$p = \frac{a + b + c}{2}$$

2. 面积

$$S = \frac{a \cdot H_a}{2} = \frac{ab \cdot \sin C}{2} = \sqrt{p(p-a)(p-b)(p-c)}$$

3. 中线

$$M_a = \frac{\sqrt{2(b^2 + c^2) - a^2}}{2} = \frac{\sqrt{b^2 + c^2 + 2bc \cdot \cos A}}{2}$$

4. 角平分线

$$T_a = \frac{\sqrt{bc \cdot [(b+c)^2 - a^2]}}{b+c} = \frac{2bc}{b+c} \cos \frac{A}{2}$$

5. 高线

$$H_a = b \sin C = c \sin B = \sqrt{b^2 - \left(\frac{a^2 + b^2 - c^2}{2a}\right)^2}$$

6. 内切圆半径

$$\begin{aligned} r &= \frac{S}{p} = \frac{\arcsin \frac{B}{2} \cdot \sin \frac{C}{2}}{\sin \frac{B+C}{2}} = 4R \cdot \sin \frac{A}{2} \sin \frac{B}{2} \sin \frac{C}{2} \\ &= \sqrt{\frac{(p-a)(p-b)(p-c)}{p}} = p \cdot \tan \frac{A}{2} \tan \frac{B}{2} \tan \frac{C}{2} \end{aligned}$$

7. 外接圆半径

$$R = \frac{abc}{4S} = \frac{a}{2\sin A} = \frac{b}{2\sin B} = \frac{c}{2\sin C}$$

### 12.2.2 四边形

$D_1, D_2$  为对角线,  $M$  为对角线中点连线,  $A$  为对角线夹角,  $p$  为半周长

$$1. a^2 + b^2 + c^2 + d^2 = D_1^2 + D_2^2 + 4M^2$$

$$2. S = \frac{1}{2} D_1 D_2 \sin A$$

3. 对于圆内接四边形

$$ac + bd = D_1 D_2$$

4. 对于圆内接四边形

$$S = \sqrt{(p-a)(p-b)(p-c)(p-d)}$$

### 12.2.3 正 $n$ 边形

$R$  为外接圆半径,  $r$  为内切圆半径

1. 中心角

$$A = \frac{2\pi}{n}$$

2. 内角

$$C = \frac{n-2}{n} \pi$$

3. 边长

$$a = 2\sqrt{R^2 - r^2} = 2R \cdot \sin \frac{A}{2} = 2r \cdot \tan \frac{A}{2}$$

## 4. 面积

$$S = \frac{nar}{2} = nr^2 \cdot \tan \frac{A}{2} = \frac{nR^2}{2} \cdot \sin A = \frac{na^2}{4 \cdot \tan \frac{A}{2}}$$

**12.2.4 圆**

## 1. 弧长

$$l = rA$$

## 2. 弦长

$$a = 2\sqrt{2hr - h^2} = 2r \cdot \sin \frac{A}{2}$$

## 3. 弓形高

$$h = r - \sqrt{r^2 - \frac{a^2}{4}} = r(1 - \cos \frac{A}{2}) = \frac{1}{2} \cdot \arctan \frac{A}{4}$$

## 4. 扇形面积

$$S_1 = \frac{rl}{2} = \frac{r^2 A}{2}$$

## 5. 弓形面积

$$S_2 = \frac{rl - a(r - h)}{2} = \frac{r^2}{2}(A - \sin A)$$

**12.2.5 棱柱**

## 1. 体积

$$V = Ah$$

$A$  为底面积,  $h$  为高

## 2. 侧面积

$$S = lp$$

$l$  为棱长,  $p$  为直截面周长

## 3. 全面积

$$T = S + 2A$$

**12.2.6 棱锥**

## 1. 体积

$$V = Ah$$

$A$  为底面积,  $h$  为高

## 2. 正棱锥侧面积

$$S = lp$$

$l$  为棱长,  $p$  为直截面周长

## 3. 正棱锥全面积

$$T = S + 2A$$

**12.2.7 棱台**

## 1. 体积

$$V = (A_1 + A_2 + \sqrt{A_1 A_2}) \cdot \frac{h}{3}$$

$A_1, A_2$  为上下底面积,  $h$  为高

## 2. 正棱台侧面积

$$S = \frac{p_1 + p_2}{2} l$$

$p_1, p_2$  为上下底面周长,  $l$  为斜高

## 3. 正棱台全面积

$$T = S + A_1 + A_2$$

**12.2.8 圆柱**

## 1. 侧面积

$$S = 2\pi rh$$

## 2. 全面积

$$T = 2\pi r(h + r)$$



3. 体积

$$V = \pi r^2 h$$

12.2.9 圆锥

1. 母线

$$l = \sqrt{h^2 + r^2}$$

2. 侧面积

$$S = \pi r l$$

3. 全面积

$$T = \pi r(l + r)$$

4. 体积

$$V = \frac{\pi}{3} r^2 h$$

12.2.10 圆台

1. 母线

$$l = \sqrt{h^2 + (r_1 - r_2)^2}$$

2. 侧面积

$$S = \pi(r_1 + r_2)l$$

3. 全面积

$$T = \pi r_1(l + r_1) + \pi r_2(l + r_2)$$

4. 体积

$$V = \frac{\pi}{3}(r_1^2 + r_2^2 + r_1 r_2)h$$

12.2.11 球

1. 全面积

$$T = 4\pi r^2$$

2. 体积

$$V = \frac{4}{3}\pi r^3$$

12.2.12 球台

1. 侧面积

$$S = 2\pi r h$$

2. 全面积

$$T = \pi(2rh + r_1^2 + r_2^2)$$

3. 体积

$$V = \frac{\pi h[3(r_1^2 + r_2^2) + h^2]}{6}$$

12.2.13 球扇形

1. 全面积

$$T = \pi r(2h + r_0)$$

$h$  为球冠高,  $r_0$  为球冠底面半径

2. 体积

$$V = \frac{2}{3}\pi r^2 h$$

12.3 积分表

$$\int \frac{1}{1+x^2} dx = \tan^{-1} x$$

$$\int \frac{1}{a^2+x^2} dx = \frac{1}{a} \tan^{-1} \frac{x}{a}$$

$$\int \frac{x}{a^2+x^2} dx = \frac{1}{2} \ln |a^2 + x^2|$$

$$\int \frac{x^2}{a^2+x^2} dx = x - a \tan^{-1} \frac{x}{a}$$

$$\int \sqrt{x^2 \pm a^2} dx = \frac{1}{2} x \sqrt{x^2 \pm a^2} \pm \frac{1}{2} a^2 \ln |x + \sqrt{x^2 \pm a^2}|$$

$$\int \sqrt{a^2 - x^2} dx = \frac{1}{2} x \sqrt{a^2 - x^2} + \frac{1}{2} a^2 \tan^{-1} \frac{x}{\sqrt{a^2 - x^2}}$$

$$\int \frac{x^2}{\sqrt{x^2 \pm a^2}} dx = \frac{1}{2} x \sqrt{x^2 \pm a^2} \mp \frac{1}{2} a^2 \ln |x + \sqrt{x^2 \pm a^2}|$$

$$\int \frac{1}{\sqrt{x^2 \pm a^2}} dx = \ln |x + \sqrt{x^2 \pm a^2}|$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1} \frac{x}{a}$$

$$\int \frac{x}{\sqrt{x^2 \pm a^2}} dx = \sqrt{x^2 \pm a^2}$$

$$\int \frac{x}{\sqrt{a^2 - x^2}} dx = -\sqrt{a^2 - x^2}$$

$$\int \sqrt{ax^2 + bx + c} dx = \frac{b+2ax}{4a} \sqrt{ax^2 + bx + c} + \frac{4ac-b^2}{8a^{3/2}} \ln \left| 2ax + b + 2\sqrt{a(ax^2 + bx + c)} \right|$$

$$\int x^n e^{ax} dx = \frac{x^n e^{ax}}{a} - \frac{n}{a} \int x^{n-1} e^{ax} dx$$

$$\int \sin^2 ax dx = \frac{x}{2} - \frac{1}{4a} \sin 2ax$$

$$\int \sin^3 ax dx = -\frac{3 \cos ax}{4a} + \frac{\cos 3ax}{12a}$$

$$\int \cos^2 ax dx = \frac{x}{2} + \frac{\sin 2ax}{4a}$$

$$\int \cos^3 ax dx = \frac{3 \sin ax}{4a} + \frac{\sin 3ax}{12a}$$

$$\int \tan ax dx = -\frac{1}{a} \ln \cos ax$$

$$\int \tan^2 ax dx = -x + \frac{1}{a} \tan ax$$

$$\int x \cos ax dx = \frac{1}{a^2} \cos ax + \frac{x}{a} \sin ax$$

$$\int x^2 \cos ax dx = \frac{2x \cos ax}{a^2} + \frac{a^2 x^2 - 2}{a^3} \sin ax$$

$$\int x \sin ax dx = -\frac{x \cos ax}{a} + \frac{\sin ax}{a^2}$$

$$\int x^2 \sin ax dx = \frac{2 - a^2 x^2}{a^3} \cos ax + \frac{2x \sin ax}{a^2}$$