Example 5.3 Find electric potential as a function of distance r for the center of a sphere with uniform charge density ρ and radius a. Defining the potential of $V = 0$ at the **infinity** (Typo in the lecture note!).

Solution From Example 4.1 using Gauss's law, Electric field at distance r away from the center of the sphere is

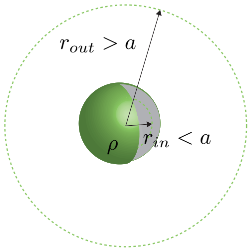

Figure 1:

$$E(r) = \begin{cases} \frac{\rho r}{3\epsilon_0} & r < a \\ \frac{\rho a^3}{3\epsilon_0 r^2} & r > a. \end{cases}$$

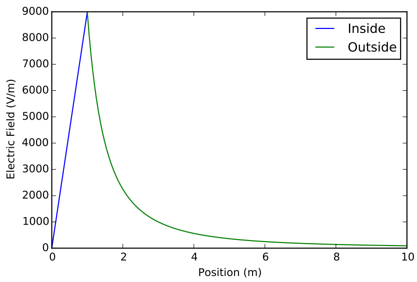

Figure 2: Electric Field due to a uniform sphere (total charge = 1 μC and a = 1 m)

Then use the definition of the electric potential difference,

$$V_f - V_i = -\int_i^f \vec{E} \cdot d\vec{s}.$$

First, we want to calculate potential outside the sphere ($r > a$). We set $V_i = 0$ when $i \to \infty$.

$$V_{out} = -\int_\infty^r \frac{\rho a^3}{3\epsilon_0 r^2} dr = \frac{\rho a^3}{3\epsilon_0 r}.$$

Next, the integral for potential inside ($r < a$) the sphere has to be executed in two steps:

$$V_{in} = -\int_\infty^a \frac{\rho a^3}{3\epsilon_0 r^2} dr - \int_a^r \frac{\rho r}{3\epsilon_0} dr = \frac{\rho a^3}{3\epsilon_0 a} - \frac{\rho}{6\epsilon_0}(r^2 - a^2).$$

Check that when you take the gradient of the potential, its negative should be the electric field.

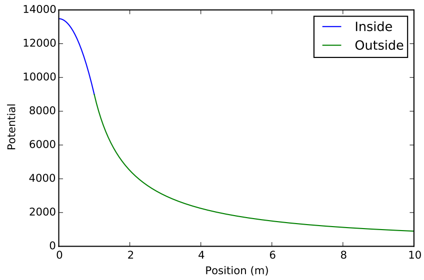

Figure 3: Potential of a uniform sphere (total charge = 1 μC and $a = 1$ m)