

ELECTROSTATICS

PHY 104

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1 Electromagnetism

- Four fundamental forces: _____, _____, _____, _____.
- Previously electromagnetism were thought of two separated phenomena: elektron, "amber", and magnetis lithos, "magnesian stone".
- The two were unified by the experimental works of Faraday, Oested and Ampere, and theoretical work by Maxwell. (very simplified history)
- Heart and soul of classical E&M: _____'s Equation

$$\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\epsilon_0} \quad (\quad)$$

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \quad (\quad)$$

$$\vec{\nabla} \cdot \vec{B} = 0 \quad (\quad)$$

$$\vec{\nabla} \times \vec{B} = \mu_0(\vec{J} + \epsilon_0 \frac{\partial \vec{E}}{\partial t}) \quad (\quad)$$

\vec{E} is electric field and \vec{B} is (related to) magnetic fields. ρ is charge density, \vec{J} is current density.

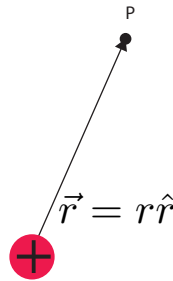
- Guide to read these equations:
 1. Don't worry about the unknown math operations.
 2. Don't worry about the vector notations ($\vec{\nabla}$ is just like $\frac{d}{dx}$)
 3. Two equations for electro (\vec{E} 's) and two equations for magnetic (\vec{B} 's).
 4. The "sources of the fields are on the right hand sides.
 5. The behaviors of the fields around the sources are determined by the left hand sides.
 6. \cdot reads "to diverge out from a point"
 7. \times reads "to curl around"

2 Charge and Coulomb Law

- Unlike charges attract, like ones repel.
- Charge is quantized. The elementary charge is that of electron. We could only found the charges of Ne , where $N = 1, 2, 3, \dots$
- Charge is conserved, so far no one has disproved it. <http://physics.aps.org/synopsis-for/10.1103/PhysRevLett.115.231802>
- Charge has a unit of _____, which is very large.
- Electrostatics = charges do not move

- Charges interact via the force between two point charges (q_1 and q_2) calculated from a Coulomb's Law

$$\vec{F} = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2} \hat{r}$$



- Four features:
 - it's a vector
 - it's an inverse square law
 - linear in charge q_2 (results in the superposition principle of force)
 - symmetry of force on A from B and on B from A.
- The inverse square nature of Coulomb's law has been a subject of test since 1769. The more recent measurement in 1983 show that the deviation factor q (i.e. $\vec{F} \propto \frac{1}{r^{2+q}}$) is in the order of 10^{-17} . The deviation is ascribed to the rest mass of photon.
- From special relativity:

$$E = \sqrt{p^2 c^2 + (mc^2)^2}.$$

When momentum p is zero (a particle at rest), the rest mass energy is _____

Example 2.1. (a) Calculate a Coulomb force between two 1-C charges at 1 meter apart. (b) Calculate a gravitational force between two 1-kg masses at 1 meter apart.

3 What is Field?

- Feynman: A "field" is any physical quantity which takes on different values at different points in space.
- A necessary concept to account for forces when bodies are far apart (action at distant).
- Electromagnetic Field, Gravitational Field, Quantum Field Theory

- Scalar Field (weather forecast, altitude map), Vector Field, Tensor Field
- One way to generate a vector field is to take a gradient. For example, thermal gradient, slope map of the mountains, weather forecasting of wind direction.
- Gradient is a directional slope:

$$\vec{\nabla}\phi(x, y, z) = \frac{\partial}{\partial x}\phi\hat{i} + \frac{\partial}{\partial y}\phi\hat{j} + \frac{\partial}{\partial z}\phi\hat{k}.$$

4 Electric Field

- Another equivalent picture of electric force on charge q is

$$\vec{F} = q\vec{E}$$

- An equivalent approach to Coulomb's Law: q_1 create an electric field of $\vec{E}_1 = \frac{1}{4\pi\epsilon_0} \frac{q_1}{r^2} \hat{r}$ and q_2 interacts with the field \vec{E}_1 in the form of force $\vec{F} = q_2\vec{E}_1$.

- An electric field due to point charge

$$\vec{E}_{point} = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} \hat{r}$$

- A more practical E-field: light, CCD cameras (record intensity - E-field square), a parallel plate capacitor, a cathode-ray TV screen.

Theorem 4.1 (Approximation of $(1+x)^n$).

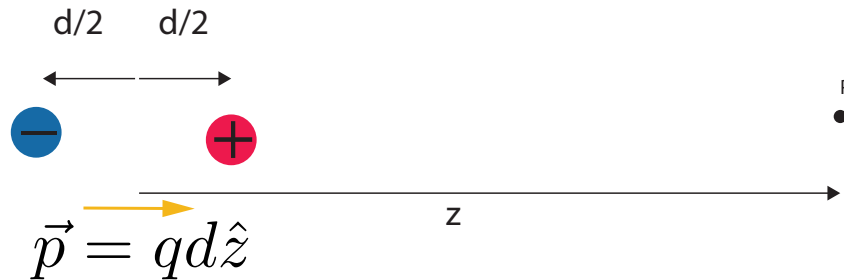
$$(1+x)^n \approx 1+nx,$$

if x is much smaller than 1.

Example 4.1. Show that electric field of a dipole (two point charges $+e$ and $-e$ separated by distance d) at distance z along the direction of the dipole is

$$\vec{E}_{dipole} = \frac{1}{4\pi\epsilon_0} \frac{2p}{z^3} \hat{k},$$

where $p = qd$ is called the dipole moment.

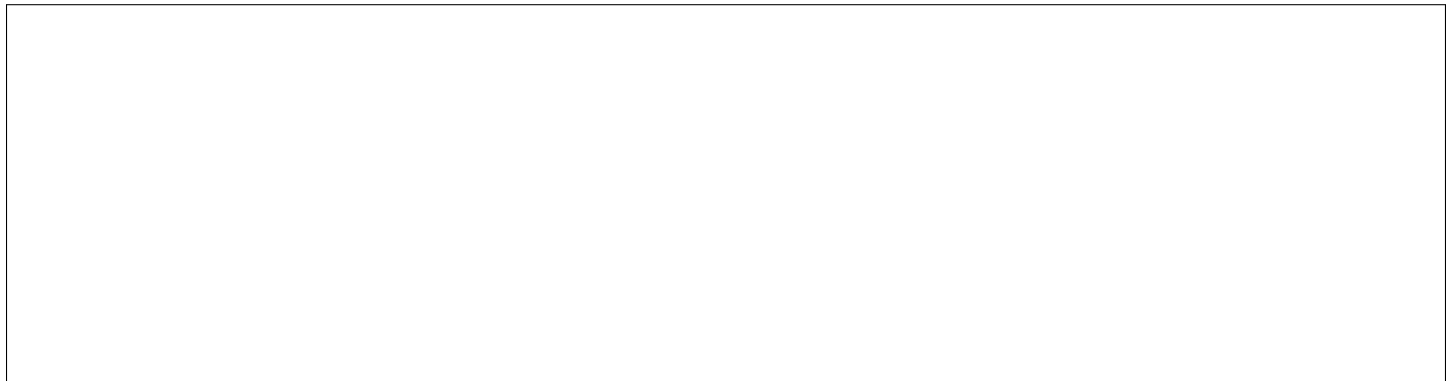
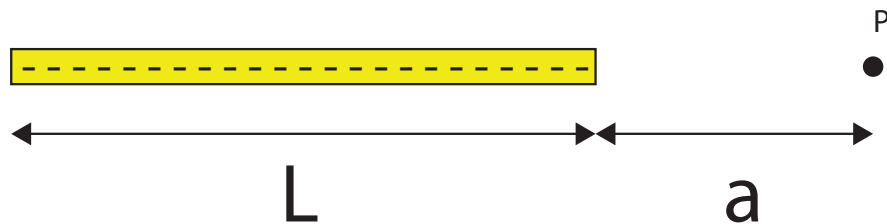


4.1 Electric Field Calculation from Continuous Charge Distributions

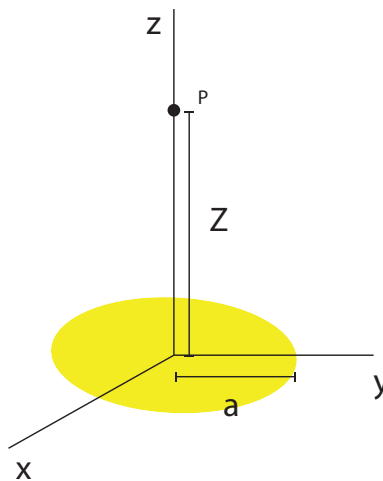
- The physics is over and it's time for calculus (boring, difficult, irrelevant etc). Important skill: you need to know if you stuck with math or with physics.

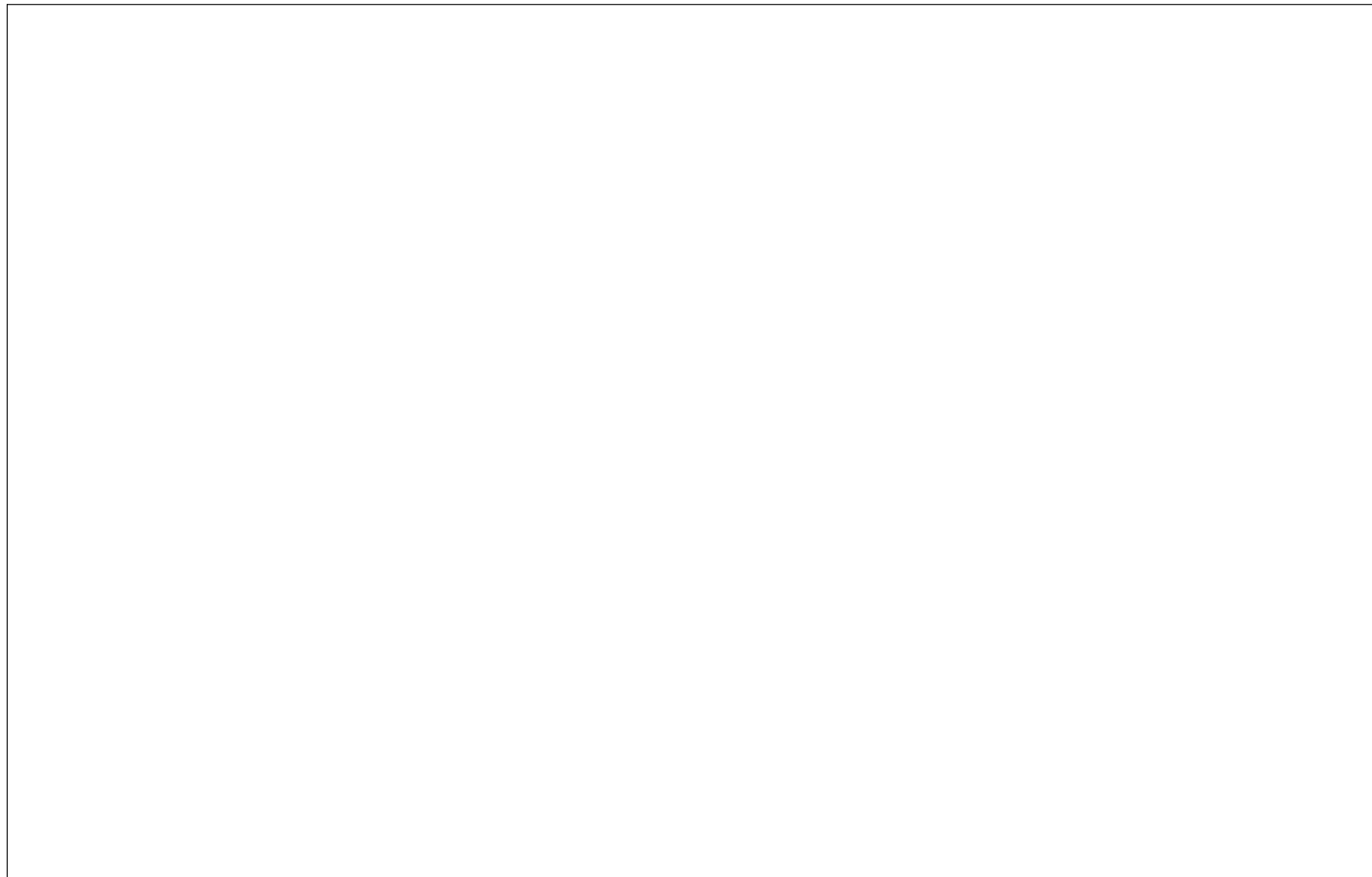
- Strategy: chop off the shapes into point charges, calculate electric field from each of them, and do the sum (vectorially)
- There are only a handful of geometries that can be solved by integration within a finite amount of time. Others are possible but the math will be too complicated.
 1. 1D: Line Charge
 2. 2D: Ring, Disk, Rectangle

Example 4.2. Calculate an electric field from a rod of charge with linear density of λ .



Example 4.3. Calculate an electric field from a disk with areal density of σ .





Example 4.4. Calculate an electric field from an infinite plate with areal density of σ .



5 Dipole in a uniform electric field

- torque

$$\vec{\tau} = \vec{p} \times \vec{E}$$

- potential energy

$$U = \vec{p} \cdot \vec{E}$$

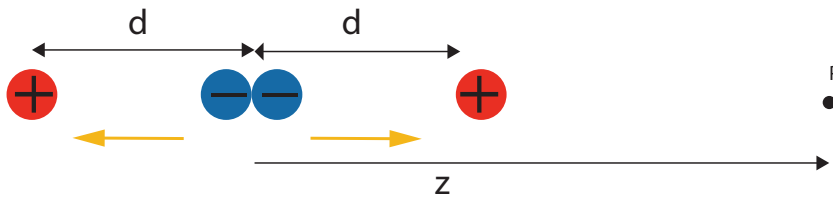
- To be derived in the homework.

6 Homework

Homework 1 (Electric Quadrupole). (10 points) Quadrupole consists of two dipoles with dipoles that are equal in magnitude but opposite direction. Show that the value of electric field on the axis if the quadrupole for a point P at distance z from the center is given by (assuming $z \gg d$):

$$\vec{E}_{quad} = \frac{3Q}{4\pi\epsilon_0 z^4} \hat{k},$$

where $Q = 2qd^2$.



Hint: You can solve this problem in two ways

1. Follow the steps in Ex 4.1 but instead, summing electric field contribution from 4 point charges:

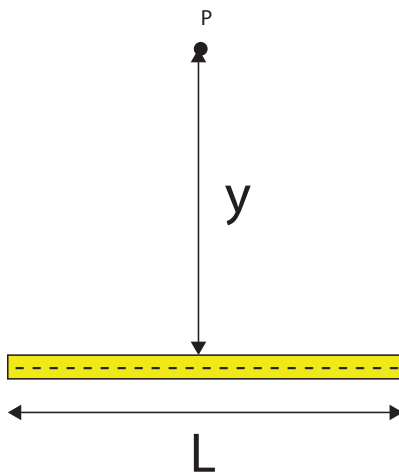
$$\vec{E}_{quad} = \vec{E}_{point}(z - d) - 2\vec{E}_{point}(z) + \vec{E}_{point}(z + d).$$

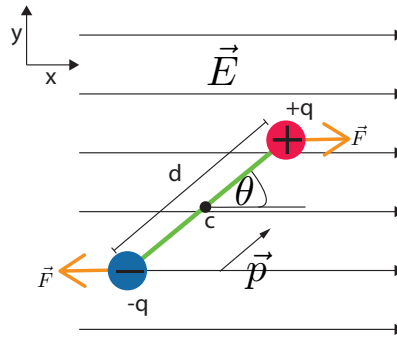
2. Use the final results from Ex 4.1 and do the summation from two dipoles.

$$\vec{E}_{quad} = \vec{E}_{dipole}(z - d/2) - \vec{E}_{dipole}(z + d/2)$$

Homework 2 (Electric Field from a rod – revisited). (10 points) A thin nonconducting rod of finite length L has a charge q spread uniformly along it. Show that the magnitude of electric field at point p is

$$E = \frac{q}{2\pi\epsilon_0 y} \frac{1}{\sqrt{L^2 + 4y^2}^{1/2}}$$



Homework 3. Torque and Energy of a Dipole in an Electric Field (30 points)

- (a) (3 points) What is the net force due to electric field on a dipole according to the figure above.
 (b) (7 points) Show that the net torque on the above dipole with respect to the center point c is

$$\tau = -qdE \sin \theta = -pE \sin \theta$$

Hint: Minus sign suggests that torque gives rise to the rotation in $-\theta$ direction (clockwise)

- (c) (3 points) Explain why the direction of torque is along the negative z axis (pointing into the page) and therefore

$$\vec{\tau} = -pE \sin \theta \hat{k} = \vec{p} \times \vec{E}$$

- (d) (7 points) Show that the potential energy of an electric dipole is $U = -\vec{p} \cdot \vec{E}$ by defining the angle of $\theta = \frac{\pi}{2}$ as a reference point.

Hints:

1. work done by the field to rotate this dipole from angle $(\theta = \frac{\pi}{2})$ to angle θ is $W = \int_{\pi/2}^{\theta} \tau d\theta$
2. potential energy is the negative of the work done by the field $U = -W$.
3. $\vec{x} \cdot \vec{y} = xy \cos \theta$

- (e) (3 points) Find the angle θ where the energy is minimized. This is the equilibrium position.
 (f) (7 points) Find the frequency of oscillation of this electric dipole and with rotational inertia ($I = \frac{md^2}{2}$), for small amplitude of oscillation about its equilibrium position in a uniform electric field E . Assuming that the charges have mass of m [Answer: $f = \frac{1}{2\pi} \sqrt{\frac{2qE}{md^2}}$]