The Rest of Maxwell's Equaitions

PHY 104

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1 Faraday's Law of Induction

1.1 Electromotive Force

- Electromotive force $\mathscr{E}$ is related to a force on a charge that goes around the loop. But it is a force per unit charge, $\vec{f} = \frac{\vec{F}}{q}$.
- And it's not a force but line integral of force around the loop.

$$\mathscr{E} = \oint \vec{f} \cdot d\vec{l}$$

- $\vec{f}$ can be anything e.g magnetic ($\vec{f} = \vec{v} \times \vec{B}$) or electric ($\vec{f} = \vec{E}$).

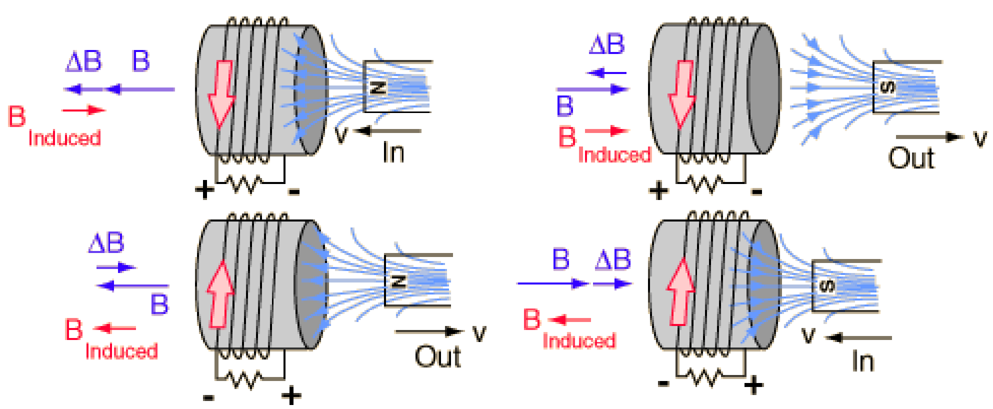
Example 1.1. What is the electromotive force of a a sqare current loop when pulling across the area with magnetic field to the empty space at a constant velocity $\vec{v}$.

Theorem 1.1 (Flux Rule)**.**

$$\mathscr{E} = -\frac{\partial \Phi_B}{\partial t}$$

1.2 Lenz's Law

- An induced current has a direction such that the magnetic field due to the current opposes the change in magnetic flux that induces the current.



1.3 Faraday's Law

- A changing magnetic field produces an electric field.
- A similar law to flux rules holds i.e. $\mathscr{E} = -\frac{\partial \Phi_B}{\partial t}$
- EMF due to electric field is $\oint \vec{E} \cdot d\vec{l}$; therefore

Theorem 1.2 (Faraday's Law in the integral form)**.**

$$\oint \vec{E} \cdot d\vec{l} = -\frac{\partial \Phi_B}{\partial t} = -\frac{\partial}{\partial t} \int \vec{B} \cdot d\vec{a}$$

1.4 A Deceptive Parallel

We are now entering another subject of "electrodynamics." The electric and magnetic field is now allowed to be changed over time. The results from the previous chapters might be not be applied. For example, the definition of potential difference,

$$\Delta V = \int_i^f \vec{E} \cdot d\vec{s}$$

is not valid in non-static case.

Example 1.2. A circular loop is located in an circular area containing an increasing magnetic field of $B = B_0 t$. (a) Find an induced electric field at radius r from the center (b) Find $\oint \vec{E} \cdot d\vec{s}$ around the circular loop.

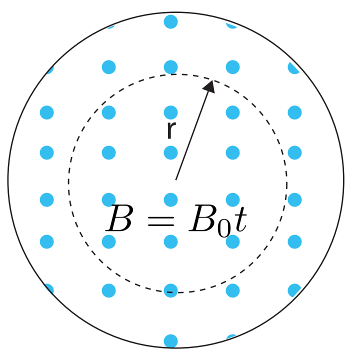


Figure 1.1

1.5 Inductance

Definition 1.1.

$$L = \frac{\phi_B}{I}$$

Example 1.3 (Solenoid Inductance)**.** Find the inductance of a solenoid.

2 Maxwell's Law of Induction

Theorem 2.1 (Maxwell's Law of Induction)**.**

$$\oint \vec{B} \cdot d\vec{s} = \epsilon_0 \mu_0 \frac{\partial \Phi_E}{\partial t} = \epsilon_0 \mu_0 \frac{\partial}{\partial t} \int \vec{E} \cdot d\vec{a}$$

2.1 Displacement Current and Maxwell's Equation

- Ampere's law (which is only good for magnetostatics),

  $$\oint \vec{B} \cdot d\vec{s} = \mu_0 I_{enc},$$

  allows us to define the the Maxwell's displacement current as

  $$I_d = \epsilon_0 \frac{\partial \Phi_E}{\partial t}.$$

  And we can introduce Ampere-Maxwell's Law

  $$\oint \vec{B} \cdot d\vec{s} = \mu_0 I_{enc} + \mu_0 I_d$$

- The last addition completes Maxwell'ls Equations:

$$\oint \vec{E} \cdot d\vec{a} = \frac{q_i n}{\epsilon_0} \tag{2.1}$$

$$\oint \vec{B} \cdot d\vec{a} = 0 \tag{2.2}$$

$$\oint \vec{E} \cdot d\vec{s} = -\frac{\partial \Phi_B}{\partial t} \tag{2.3}$$

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 I_{enc} + \mu_0 I_d \tag{2.4}$$

$$\tag{2.5}$$

3 Homework

Homework 1. A square copper loop of length L is placed in a uniform field $\vec{B}$ and allowed to fall under gravity.
(a) (10 points) Show that the velocity of the loop as a function of time is in the form of

$$\vec{v}(t) = \frac{g}{\lambda} - \frac{g}{\lambda}e^{-\lambda t}\hat{k}.$$

What is λ in terms of copper density and conductivity.
Hint: Find an induced EMF then calculate the current around the loop due to magnetic induction. Assume that the copper loop has a resistance R. Use the formula $R = \frac{L}{\sigma A}$ to related R to the conductivity (σ), cross sectional area (A) and length (L). Find the magnitude of force (ILB) and use Newton's law.

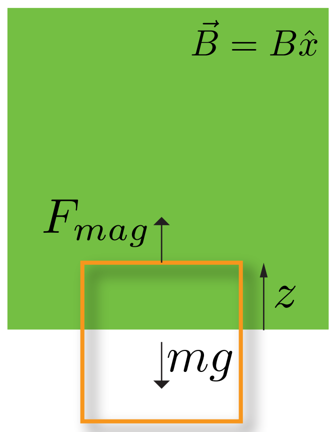


Figure 3.1