2 Short Problems

Exam 1 (Lecture 2). (2 points) Explain why the electric potential is the same every where inside a conductor.
Hint: What is zero inside any conductor?

$E = 0$ inside a conductor

so $\Delta V = -\int_i^f \vec{E} \cdot d\vec{s} = 0$

Exam 2 (Mock). (2 points) A proton of mass m is accelerated from rest by an electric field E. Calculate the speed of the proton after time t.

$F = eE = ma$

$a = \frac{eE}{m} \longrightarrow$ constant

$v = \frac{eEt}{m}$

Exam 3 (Lecture 3). (4 points) (a) (2 points) Show that the capacitance of two infinite plates is

$$C = \frac{\epsilon_0 A}{d}$$

since

$E = \frac{\sigma}{\varepsilon_0}$

$V = \frac{\sigma d}{\varepsilon_0} = \frac{qd}{\varepsilon_0 A}$ $\nearrow$ $C = \frac{q}{V} = \frac{\varepsilon_0 A}{d}$

(b) (1 point each up to 3 points) Suggest methods to increase the capacitance above.

1. large A
2. small d
3. insert a dielectric to increase $\varepsilon_0 \rightarrow \kappa \varepsilon_0$

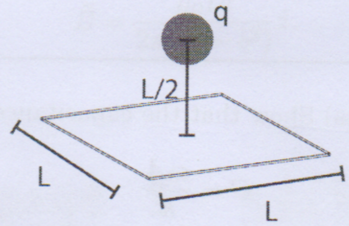
Exam 4 (Mock). (2 points) An electron of charge q and mass m is travelling from rest (at x = 0) along x-axis under potential $V = kx^{4/3}$, where k is a constant. Find the formula for the magnitude of the acceleration of the electron at distance x.

$$E = -\frac{dV}{dx} = -k\frac{4}{3}x^{\frac{1}{3}}$$

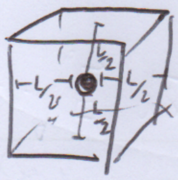
$$F = qE = -qk\frac{4}{3}x^{\frac{1}{3}} = ma$$

$$|a| = \frac{4qk}{3m}x^{\frac{1}{3}}$$

Exam 5 (HW 2). (5 points) (a) (3 points) Find the electric field flux that goes through the square surface of length L due to charge q located at $L/2$ above from the center.



due to symmetry, ~~this~~ we could construct a cube Gaussian by putting the charge at the center



Total flux through 6 faces $= \frac{q}{\varepsilon_0}$

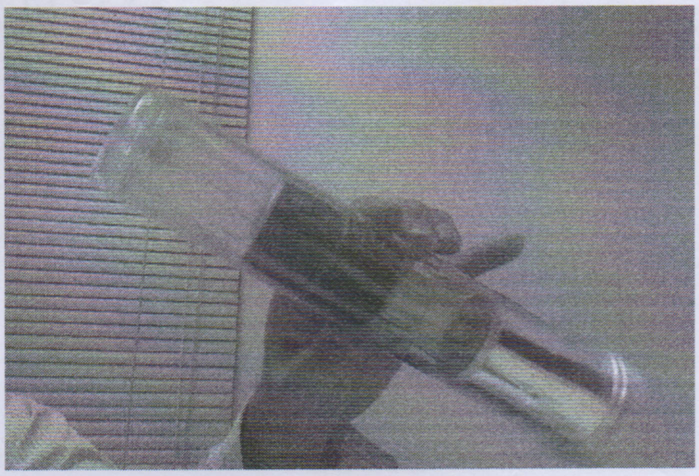
" ——————— " 1 face $= \frac{q}{6\varepsilon_0}$

(b) (2 points) From previous problem, what is the smallest possible flux that we could have physically.

smallest possible q is $e = 1.6\times10^{-19}$ C

smallest flux $= \frac{e}{\varepsilon_0}$

Exam 6 (Week 5). (3 points) Explain how does this thing work?

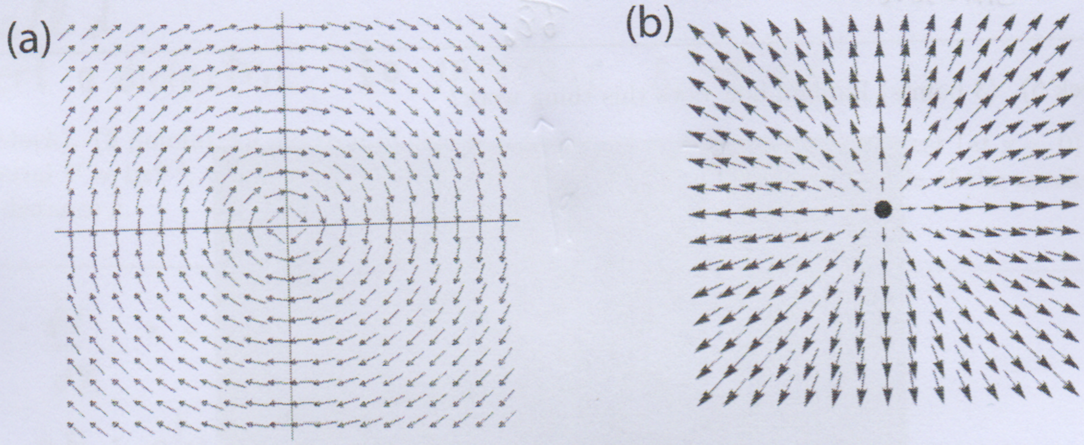


~~By~~ changing magnetic flux indues E field which drives current.

Exam 7 (Week 1-5). (6 points) Choose between (a) and (b) or both.
What is (are) the possible field distribution(s) of

1. Electrostatic field due to a point charge **(a, b, both)**
2. Magnetic field due to a current carrying wire **(a, b, both)**
3. Induced electric field due to the change in magnetic flux (Faraday's law) **(a, b, both)**
4. Induced magneitc field due to the change in electric flux (Maxwell-Ampere's law) **(a, b, both)**
5. Any electric field **(a, b, both)**
6. Any magnetic field **(a, b, both)**

- (b) can't be magnetic field ($\vec{B}$) since there is no ~~[illegible]~~ monopole (answered 2, 4, 6)
- point charges produce (b) (answered 1).
- Faraday's Law $\oint \vec{E}\cdot d\vec{s} = -\frac{d\vec{B}}{dt}$ is mathematically similar to Ampere's law $\oint \vec{B}\cdot d\vec{s} = \mu_0 I$ (answered 3) so $\vec{E}$ should run around the loop



3 Long Problems

Exam 8 (Github Week 2). (8 points) Given a sphere with uniform charge density ρ and radius a: (a) (4 points) Show that the electric field as a function of distance r for the center is

$$E(r) = \begin{cases} \frac{\rho r}{3\epsilon_0} & r < a \\ \frac{\rho a^3}{3\epsilon_0 r^2} & r > a. \end{cases}$$

(b) (4 points) Find electric potential (V). Defining the potential of $V = 0$ at the infinity.

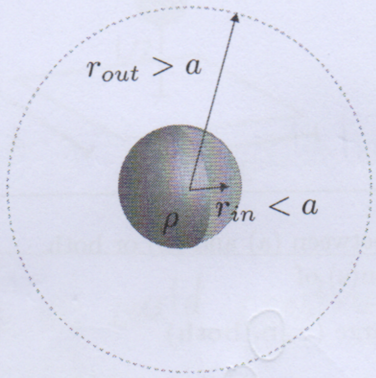


Figure 3.1

(a) Gauss's Law

(b) $\Delta V = -\int \vec{E} \cdot d\vec{s}$

} see solution in our Github Repo (week 2)

Exam 9 (Github Week 2 and Mock). (8 points) (a) (4 points) Show that the electric field at point P due to a ring of uniformly distributed charge density with total positive charge Q and radius R is

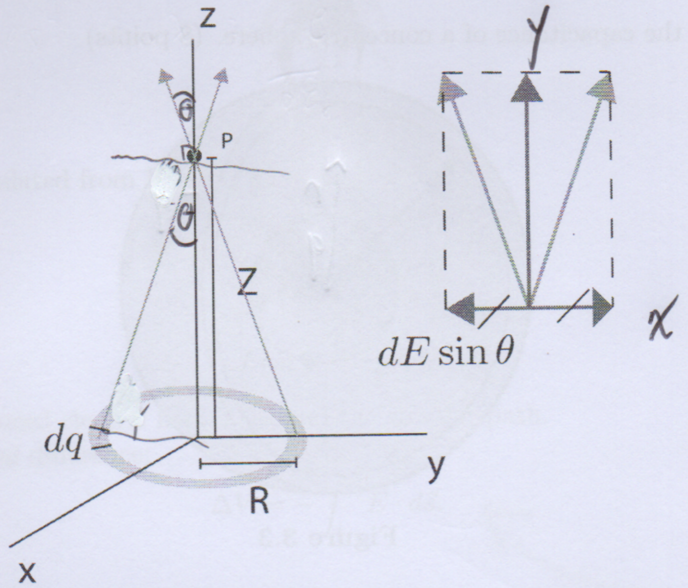


Figure 3.2

$$E = \frac{1}{4\pi\epsilon_0}\frac{Qz}{(R^2+Z^2)^{3/2}}$$

(b)(4 points) A negative charge $-q$ and mass m is placed near the plane of the ring (when Z is much smaller than R). It will undergoes a simple harmonic motion. Find the oscillation frequency.

(a) $E_y = E\cos\theta = \frac{1}{4\pi\varepsilon_0}\int \frac{dq}{(Z^2+R^2)}\frac{Z}{(Z^2+R^2)^{\frac{1}{2}}}$

$E_y = \frac{1}{4\pi\varepsilon_0}\frac{Z}{(Z^2+R^2)^{3/2}}Q$

(b) for $Z \ll R$

$E_y \approx \frac{1}{4\pi\varepsilon_0}\frac{QZ}{R^3}$

$F = -qE = ma = m\ddot{z}$

$\ddot{z} + \frac{qQZ}{4\pi\varepsilon mR^3} = 0$

$\omega^2 = \frac{qQ}{4\pi\varepsilon_0 mR^3} \longrightarrow f = \frac{1}{2\pi}\sqrt{\frac{qQ}{4\pi\varepsilon_0 mR^3}}$

Exam 10 (Week 3). Find the capacitance of a concentric sphere. (8 points)

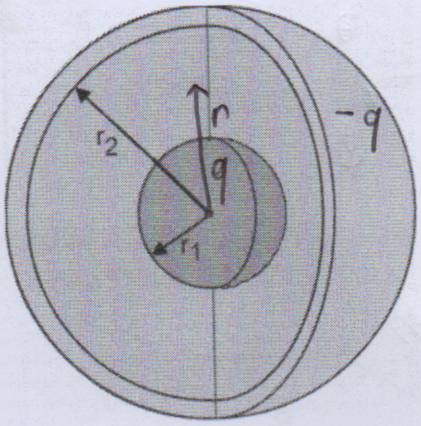


Figure 3.3

Hint: Recipe to calculate C

1. Identify the two plates
2. Put charges of $+q$ and $-q$ onto two plates (does not matter in what order)
3. Calculate E field between two plates
4. Calculate potential difference
5. Take the ratio of q and V

from Gauss's Law

$$\oint \vec{E} \cdot d\vec{A} = \frac{q}{\varepsilon_0}$$

$$E 4\pi r^2 = \frac{q}{\varepsilon_0}$$

$$E = \frac{q}{4\pi\varepsilon_0 r^2}$$

calculate ΔV between two plates

$$\Delta V = -\int_{r_1}^{r_2} \frac{q}{4\pi\varepsilon_0 r^2} dr = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{r_2} - \frac{1}{r_1}\right)$$

$$C = \frac{q}{\Delta V} = \frac{4\pi\varepsilon_0}{\left(\frac{1}{r_2} - \frac{1}{r_1}\right)}$$

Exam 11 (Week 5). (8 points)
A circular wire is located in an circular area containing an increasing magnetic field of $B = B_0 t$.

The induced electric field calculated from Faraday's law is

$$\oint \vec{E} \cdot d\vec{l} = -\frac{\partial \Phi_B}{\partial t} = -\frac{\partial}{\partial t} \int \vec{B} \cdot d\vec{a}$$

or

$$E_{ind} = -\frac{B_0 r}{2},$$

where the direction of the induced electric field is around the circular path.
From the definition of potential difference:

$$\Delta V = -\int_i^f \vec{E} \cdot d\vec{s}.$$

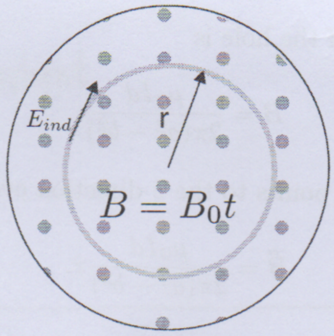


Figure 3.4

it could be seen that the potential difference of the ring is dependent of the number of rounds around the loop (N):

$$\Delta V = -\int_i^f \vec{E}_{ind} \cdot d\vec{s} = \frac{B_0 r}{r} 2\pi r N = N\pi r^2 B_0.$$

This means that we could produce any high voltage depending on the number of turns around the loop (e.g. we could produce the ΔV of 1000 Volts for a circular loop of radius r of 1 m by making 318 turns around the path for a B_0 of 1 T/s). There is no limitation of N.

Question: Can the above principle be applied to generate electricity? If not, why? What is wrong?

~~Potential~~ The concept of electric potential

ΔV

$$\Delta V = -\int \vec{E} \cdot d\vec{s}$$

doesn't apply in this case which is not 'static' (i.e. B is a function of time)

Exam 12 (Homework 4 Rerun). (10 points) A long cylinder conductor of radius a containing a long cylindrical hole of radius b. The axes of the cylinder and hole are parallel and are distance d apart; a current I is uniformly distributed over the cross section.

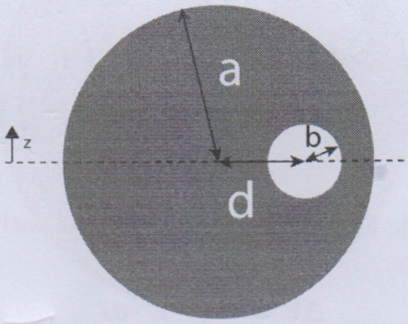


Figure 3.5

Show that (a) magnetic field at the center of the hole is

$$B = \frac{\mu_0 I d}{2\pi(a^2 - b^2)}.$$

(b) magnetic field in the hole is unform and points to the $\hat{z}$ direction everywhere in the hole i.e.

$$\vec{B} = \frac{\mu_0 I d}{2\pi(a^2 - b^2)}\hat{z}.$$

See HW 4 page 3. solution in our Github page.