PHY104/2015 Mock Pre-Midterm Examination

Fundamental constants:

Elementary charge:	$e = 1.60 \times 10^{-19}$ C
Coulomb constant:	$k = 9.0 \times 10^{9}$ N m^2 C^{-2}
Electron mass:	$m_e = 9.11 \times 10^{-31}$ kg
Proton mass:	$m_p = 1.67 \times 10^{-27}$ kg
Permittivity of free space:	$\epsilon_0 = 8.85 \times 10^{-12}$ F m^{-1}
Gravitational acceleration at Earth's surface:	$g = 10$ m s^{-2}

Remember

Relationship between electric field E and electric potential $V(x)$:

$$E = -\frac{dV}{dx}.$$

Multiply the above equation by charge q:

$$qE = -\frac{d(qV)}{dx}.$$

Because electric force $F = qE$ and potential energy $U = qV$, we have the relationship between force and potential energy:

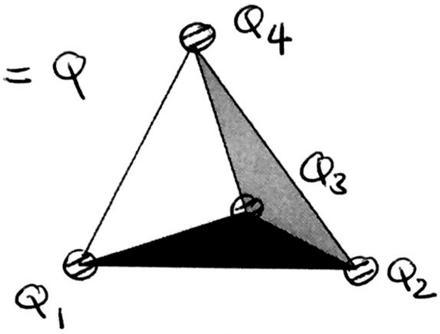
$$F = -\frac{dU}{dx}.$$

Part A

1. A charge $q = -1.5\ \mu\text{C}$ is situated at each vertex of a tetrahedron. All edges of the tetrahedron are equal in length which is 20 cm. Determine the electric potential energy of the system.

[3]

$Q_1 = Q_2 = Q_3 = Q_4 = Q$



Energy between a pair of charges $U = \frac{kQq}{r}$

In this question, there are 6 pairs in total

So $U = \frac{kQ_1Q_2}{a} + \frac{kQ_1Q_3}{a} + \frac{kQ_1Q_4}{a} + \frac{kQ_2Q_3}{a} + \frac{kQ_2Q_4}{a}$

$+ \frac{kQ_3Q_4}{a} = \frac{6kQ^2}{a} = \frac{6 \times 9 \times 10^9 \times (-1.5 \times 10^{-6})^2}{0.2}$

$\therefore U = \underline{0.61\ \text{J.}}$

2. A proton is accelerated from rest by an electric field $E = 1{,}000\ \text{NC}^{-1}$. Calculate the speed of the proton after 1.0 μs.

[3]

$\longrightarrow E = 1{,}000\ \text{N/C}$, $q = 1.6 \times 10^{-19}\ \text{C}$

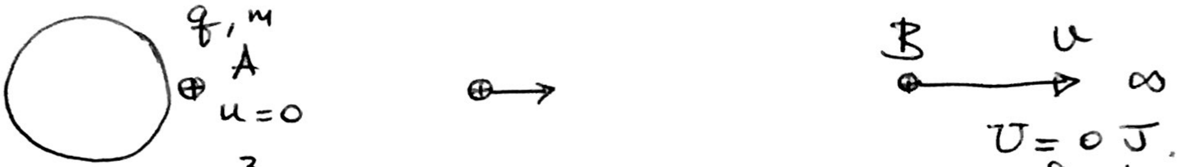
$\oplus \rightarrow F = qE$ Newton's 2nd law $qE = ma$

$a = \frac{qE}{m}$

$\therefore v = \cancel{u}^{\,0} + at \Rightarrow v = \frac{qEt}{m}$

$v = \frac{1.6 \times 10^{-19} \times 1000 \times 10^{-6}}{1.67 \times 10^{-27}} = \underline{9.6 \times 10^4\ \text{m/s}}$

3. A conducting sphere of radius $R = 0.25$ m carries a charge $Q = 2.4 \times 10^{-3}$ C. A proton is released from the surface of the sphere. Find the speed of the proton when it reaches infinity. [3]



$Q = 2.4 \times 10^{-3}$ C

$v = 1.3 \times 10^{8}$ m/s #

Using conservation of energy $\quad E_A = E_B$

$$\cancel{K_A}^{0} + U_A = K_B + \cancel{U_B}^{0}$$

$$\frac{kQq}{R} = \frac{1}{2}mv^2$$

$$v^2 = \frac{2kQq}{mR} \Rightarrow v = \left[\frac{2 \times 9 \times 10^{9} \times 2.4 \times 10^{-3} \times 1.6 \times 10^{-19}}{1.67 \times 10^{-27} \times 0.25}\right]^{\frac{1}{2}}$$

4. An electron of charge q and mass m is travelling from rest (at $x = 0$) along x-axis under potential $V = kx^{4/3}$, where k is a constant. Find the formula for the magnitude of the acceleration of the electron at distance x. [3]

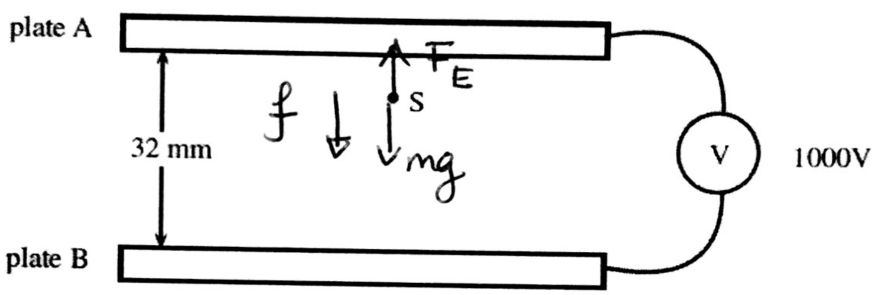
From $\quad F = -\frac{dU}{dx}$

we have $\quad F = -\frac{d}{dx}\left(qkx^{4/3}\right)$

$$= -\frac{4}{3}kqx^{\frac{1}{3}}$$

Using $F = ma$; $\quad a = -\frac{4kqx^{\frac{1}{3}}}{3m}$ *

5. Two parallel plates, separated by distance $d = 32$ mm, have potential difference $V = 1000$ V. A negatively charged particle S with charge of magnitude $q = 1.6 \times 10^{-10}$ C and mass $m = 1.0 \times 10^{-7}$ kg is found to be moving <u>upwards</u> at <u>constant speed</u> $v = 1.25 \times 10^{-3}$ ms^{-1}. The drag force on the particle is given by kv. Calculate the value of k.



[4]

$$F_E = mg + f \quad \text{(Newton's law).}$$

$$qE = mg + kv \quad (f = kv)$$

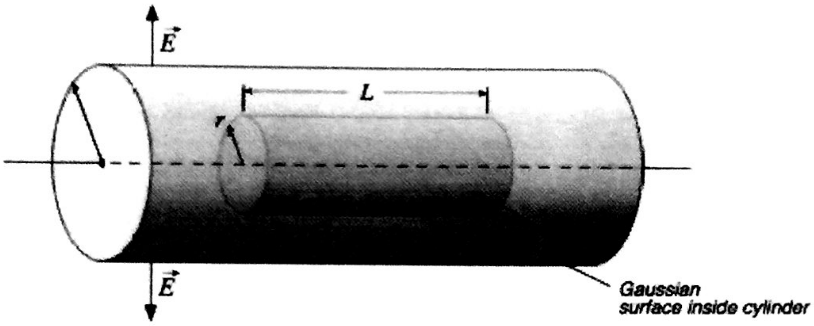
$$q\frac{V}{d} = mg + kv \quad \left(E = \frac{V}{d}\right)$$

$$k = \frac{q\frac{V}{d} - mg}{v}$$

$$k = \frac{1.6\times10^{-10} \times \frac{1000}{32\times10^{-3}} - 10^{-7} \times 10}{1.25\times10^{-3}}$$

$$\underline{k = 3.2\times10^{-3} \ \frac{\text{N}\cdot\text{s}}{\text{m}}}$$

6. An insulating long cylinder has positive charge with charge density (charge per unit volume) $\rho = 6.4 \times 10^{-4}\ \mathrm{Cm^{-3}}$ uniformly distributed all over the volume. Use Gauss' law to calculate the electric field at distance $r = 0.15$ m from the cylinder's axis <u>inside</u> the cylinder.



From Gauss' law : Electric flux $= \dfrac{Q}{\varepsilon_0}$. [4]

$$E\,(\text{Surface area}) = \frac{\rho\,(\text{Volume})}{\varepsilon_0}.$$

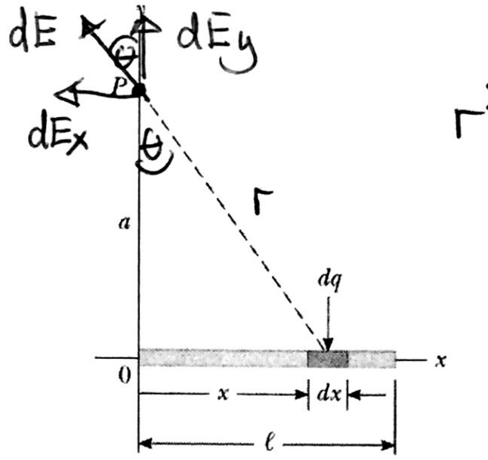
$$E\,(2\pi \cancel{r} \cancel{\ell}) = \rho\,\frac{\pi r^{\cancel{2}} \cancel{\ell}}{\varepsilon_0}$$

$$E = \frac{\rho r}{2\varepsilon_0}$$

$$E = \frac{6.4\times10^{-4}\times 0.15}{2\times 8.85\times10^{-12}} = 5.4\times10^{6}\ \mathrm{N/C}.$$

Part B

7. A line of charge Q and length ℓ is lying along x-axis. One end of the line is at the origin.



$r^2 = a^2 + x^2$

$dq = \lambda dx$

$\lambda = \frac{Q}{\ell}$

Point P is at a distance a above the origin. Let E_x be horizontal component of electric field and E_y the vertical component of the field at point P.

a) Show that

$$E_x = \frac{Q}{4\pi\varepsilon_0\ell}\left(\frac{1}{a} - \frac{1}{\sqrt{a^2+\ell^2}}\right) \quad \text{and} \quad E_y = \frac{Q}{4\pi\varepsilon_0 a}\frac{1}{\sqrt{a^2+\ell^2}}.$$

Hint: $\int \frac{x}{(x^2+a^2)^{3/2}}dx = \frac{-1}{\sqrt{x^2+a^2}} + C, \quad \int \frac{1}{(x^2+a^2)^{3/2}}dx = \frac{x}{a^2\sqrt{x^2+a^2}} + C.$

[7]

From $E = \frac{kQ}{r^2}$ we have $dE = \frac{k\,dq}{r^2}$

$$dE_x = dE \sin\theta$$

$$= \frac{k\lambda dx}{r^2} \cdot \frac{x}{r}$$

$$= k\lambda \frac{x\,dx}{(x^2+a^2)^{3/2}}$$

$$E_x = k\lambda \int_0^\ell \frac{x\,dx}{(x^2+a^2)^{3/2}}$$

$$dE_y = dE\cos\theta$$

$$= \frac{k\lambda dx\, a}{(x^2+a^2)^{3/2}}$$

$$E_y = k\lambda a \int_0^\ell \frac{dx}{\sqrt{(x^2+a^2)}^{3/2}}$$

$$= \frac{k\lambda a\, x}{a^2\sqrt{x^2+a^2}}\Bigg|_0^\ell = \frac{Q}{4\pi\varepsilon_0 a}\frac{1}{\sqrt{a^2+\ell^2}} \;\ast$$

$$E_x = -k\lambda\left[\frac{1}{(x^2+a^2)^{1/2}}\right]_0^\ell = \frac{Q}{4\pi\varepsilon_0\ell}\left(\frac{1}{a} - \frac{1}{\sqrt{a^2+\ell^2}}\right) \ast$$

b) Given that $Q = 1.6 \times 10^{-10}$ C, $\ell = 1.0$ m and $a = 1.0$ m, calculate the magnitude of the resultant force on an electron sitting at point P. [3]

$$E_x = \frac{1.6\times10^{-10}\times9\times10^{9}}{1}\left(\frac{1}{1}-\frac{1}{\sqrt{1+1}}\right)$$

$$= 0.4218 \ \text{N/C}$$

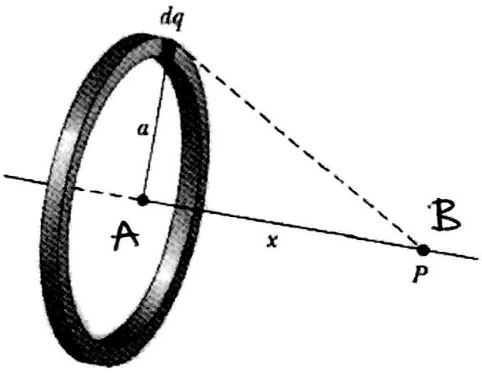
$$E_y = \frac{1.6\times10^{-10}\times9\times10^{9}}{1}\ \frac{1}{\sqrt{2}} = 1.0182 \ \text{N/C}$$

$$E = \sqrt{E_x^2+E_y^2} = 1.102 \ \text{N/C}$$

$$\therefore \quad F = qE = 1.6\times10^{-19}\times1.102$$

$$F = 1.8\times10^{-19} \ \text{N}.$$

8. The diagram shows a thin ring of radius a with positive charge uniformly distributed. The linear charge density (charge per unit length) is given by λ. Point P is at distance x from the center of the ring.



a) Show that the potential at point P is given by

$$V(x) = \frac{\lambda a}{2\varepsilon_0\sqrt{x^2+a^2}}.$$

[3]

Using $dV = \frac{k\,dq}{r} = \frac{k\,dq}{\sqrt{x^2+a^2}}$

$$\therefore V = \frac{k}{\sqrt{x^2+a^2}}\int_0^Q dq = \frac{kQ}{\sqrt{x^2+a^2}}.$$

Find Q from $Q = \lambda 2\pi a \Rightarrow V = \frac{1}{\cancel{4\pi}_{2}\varepsilon_0}\,\frac{\cancel{Q}\ \lambda\cancel{2\pi}a}{\sqrt{x^2+a^2}}$

b) Use the result in a) to find electric field at point P.

[3]

$$V = \frac{\lambda a}{2\varepsilon_0\sqrt{x^2+a^2}} \quad *$$

$$E = -\frac{dV}{dx} = \frac{-\lambda a}{2\varepsilon_0}\frac{d}{dx}(a^2+x^2)^{-\frac{1}{2}}$$

$$= \frac{\lambda a}{4\varepsilon_0}\cdot\frac{2x}{(x^2+a^2)^{3/2}} = \frac{\lambda a\, x}{2\varepsilon_0(x^2+a^2)^{3/2}}$$

c) **Given that $\lambda = 10^{-12}$ Cm^{-1}, $a = 0.5$ m and $x = 1.0$ m. An electron is released at point P. Calculate the speed of an electron when it passes the center of the ring.**

[4]

Use conservation of energy

$$E_A = E_B$$

$$K_A + U_A = \cancel{K_B}^{0} + U_B$$

$$q = +1.6\times10^{-19}.$$

$$\frac{1}{2}mv^2 - qV_A = -qV_B$$

$$\frac{1}{2}mv^2 = q(V_A - V_B)$$

$$\frac{1}{2}\times 9.11\times10^{-31}\times v^2 = 1.6\times10^{-19}\times \frac{10^{-12}\times 0.5}{2\times 8.85\times10^{-12}}\left[\frac{1}{0.5} - \frac{1}{\sqrt{0.5^2+1^2}}\right].$$

$$v = 1.05\times10^{5}\ \text{m/s}$$

9. Figure 1 shows an electric dipole which consists of two opposite charges equal in magnitude q, separated by a distance a. A vector drawn from negative charge to positive charge is called electric dipole p whose magnitude is defined as

$$p = qa.$$

Figure 2 shows an electric dipole making an angle θ with a uniform electric field E.

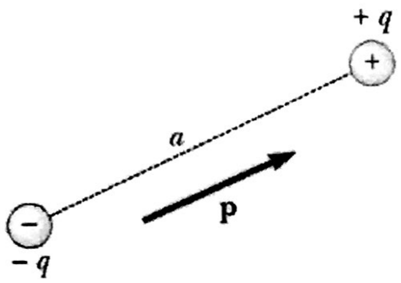


Figure 1 Electric dipole

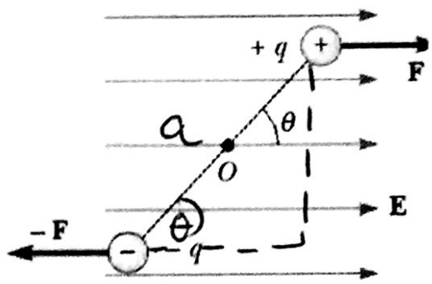


Figure 2 Electric dipole in uniform electric field

a) Consider Fig. 2, show that the torque on the dipole is given by

$$\tau = pE\sin\theta.$$

[3]

$$\tau = F r_{\perp} = Fa\sin\theta$$

$$\tau = qEa\sin\theta.$$

Using $p = qa$

$$\tau = pE\sin\theta$$

In the presence of electric field as in Fig. 2, the dipole rotates. The work done by electric force is equal to the change in potential energy according to

$$\Delta U = \int_{\theta_1}^{\theta_2} \tau \, d\theta.$$

b) By using the result in a), show that, for any angle θ, the potential energy is given by

$$U = -pE\cos\theta.$$

[3]

$$\Delta U = \int pE\sin\theta \, d\theta$$

$$= -pE\cos\theta + C$$

People around the world agree with this

To find C, we use $U = 0$ at $\theta = \frac{\pi}{2}$.

$$0 = -pE\cancel{\cos\frac{\pi}{2}}^{\,0} + C$$

So they define it the way we use it here.

$$C = 0$$

$$\therefore \quad U = -pE\cos\theta \; *$$

Figure 3 shows a conducting spherical star with positive charge Q. A dumbbell-shaped satellite modeled as an electric dipole p is at distance x far away from the star.

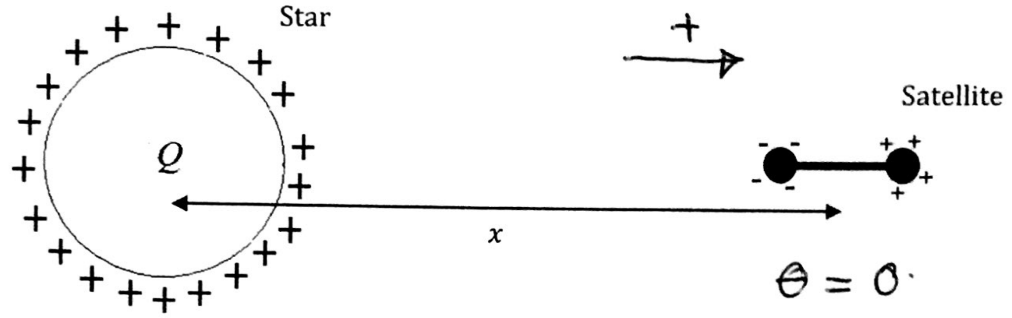


Figure 3 A dipolar satellite far away from a positively charged star

c) There is a force acting on the satellite. Is this force repulsive from or attractive towards the star? [1]

Attractive
(-ve is closer to the star than +ve).

d) From the result in part b), show that the force acting on the satellite

$$F = \frac{-pQ}{2\pi\varepsilon_0 x^3}.$$

[3]

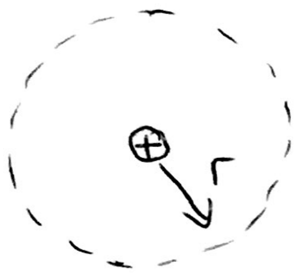
$$F = -\frac{dU}{dx} = -\frac{d}{dx}(-pE) = \frac{d}{dx}(pE)$$

$$E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{x^2}; \text{ Thus } F = \frac{pQ}{4\pi\varepsilon_0}\frac{d}{dx}x^{-2}$$

$$\therefore F = (-)\frac{pQ}{2\pi\varepsilon_0 x^3}$$

Because force is attractive and points to the left.

10. Use Gauss' law to find the electric field $E(r)$ as a function of distance r from a point charge Q. [3]

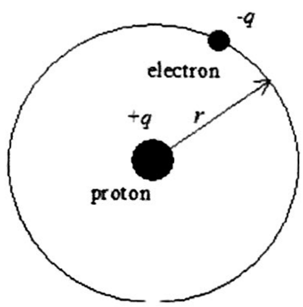


a splere

$$E\,4\pi r^2 = \frac{Q}{\varepsilon_0}$$

$$E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}$$

In a model of hydrogen atom, an electron (charge $-q$) is orbiting around a fixed proton (charge q) in circle with radius $r = 5.3 \times 10^{-11}$ m.



a) Show that the speed of electron is about 2.19×10^6 ms^{-1}. [3]

$$F = ma$$

$$\frac{kq^2}{r^{\cancel{2}}} = m\frac{v^2}{\cancel{r}}$$

$$v^2 = \frac{kq^2}{mr} = \frac{9\times10^9 \times (1.6\times10^{-19})^2}{9.11\times10^{-31} \times 5.3\times10^{-11}}$$

$$v = 2.19\times10^6 \text{ m/s.}$$

b) Calculate kinetic energy, electric potential energy, and total energy of the atom in unit of electronvolt (eV). Note that 1 eV = 1.6×10^{-19} J. [4]

$$K = \frac{1}{2} m v^2 = \frac{1}{2} \times 9.11 \times 10^{-31} \times (2.19 \times 10^{6})^2$$

$$= 2.17 \times 10^{-18} \text{ J}$$

$$= \frac{2.17 \times 10^{-18}}{1.6 \times 10^{-19}} = 13.58 \text{ eV}$$

$$U = -\frac{kq^2}{r} = \frac{-9 \times 10^{9} \times (1.6 \times 10^{-19})^2}{5.3 \times 10^{-11}}$$

$$= -4.35 \times 10^{-18} \text{ J}$$

$$= -27.17 \text{ eV}$$

$$\text{Total} = K + U = -13.6 \text{ eV}.$$