Gauss's Law and Electric Potential

PHY 104

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1 Recap

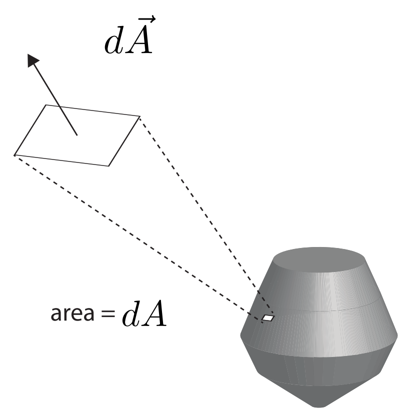
	E field produced	force due to E-field	Potential Energy in E-field*	Torque*
point charge	$\frac{1}{4\pi\epsilon_0}\frac{q}{r^2}$	$q\vec{r}\times\vec{E}$	q V (This lecture)	
dipole	$\propto \frac{p}{r^3}$	HW1	$-\vec{p}\cdot\vec{E}$	$\vec{p}\times\vec{E}$

* Here we assume that the field is uniform to simplify the derivation.

Name	Unit
Charge q or e	C
Dipole Moment p	$C \cdot m$
Quadrupole Moment Q	$C \cdot m^2$
Linear Charge Density λ	C/m
Surface Charge Density σ	C/m^2
Volume Charge Density ρ	C/m^3

2 Closed Surface and Area Vector

- We will have a closer look of a closed surface (a shell of 3d object) called Gaussian surface
- The object can be described by small area vectors $d\vec{A}$ where the direction of the vector is **perpendicular** to the surface and point **outward**.



3 Flux

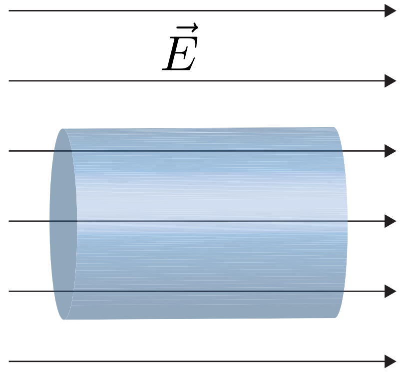
- Flux describes the flow of a physical property in space
- water flow analogy

- Electric flux: defined as electric field component perpendicular to surface times area.
- But soon we will consider the total electric flux around the surface.

Definition 3.1 (Total Electric Flux around a Closed Surface)**.**

$$\Phi = \oint \vec{E} \cdot d\vec{A}$$

Example 3.1. Calculate (a) electric flux through each surface of a cylinder surface inside a uniform field (b) calculated the sum.



4 Gauss's Law

- The total of the electric flux out of a closed surface is equal to the charge enclosed divided by the permittivity.

Theorem 4.1. Gauss's Law in the integral form

$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{in}}{\epsilon_0},$$

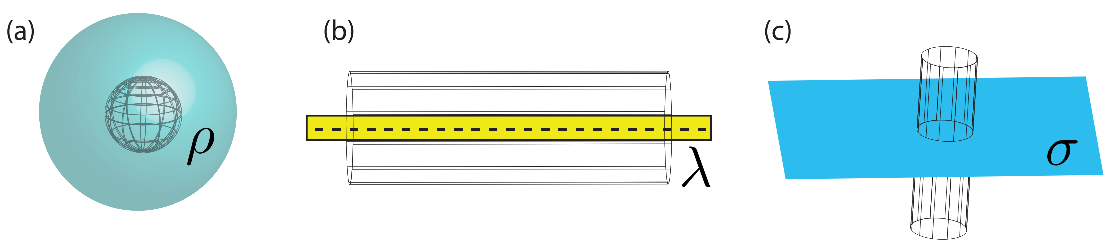
q_{in} is total charge inside the gaussian surface.

- Gauss's law works because electric field decays as an inverse square law and we live in a 3D universe (Homework 2).
- Pro: effortless to calculate electric field from certain geometries Con: Only a few symmetrical are useful.
- water flow analogy

- Alternatively:

Theorem 4.2. Gauss's Law in the differental form

$$\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\epsilon}$$



Example 4.1. Use Gauss's Law to calculate electric field inside and outside of a sphere of radius R with uniformly distributed charge density of ρ

Example 4.2. Use Gauss's Law to calculate electric field of around an infinitely long bar with uniformly distributed charge density of λ

Example 4.3. Use Gauss's Law to calculate electric field of a plane with uniformly distributed charge areal density of σ

5 Electric Potential

- scalar field $\xrightarrow{gradient}$ vector field
- altitude map $\xrightarrow{gradient}$ slope map
- pressure map $\xrightarrow{gradient}$ wind direction map
- electric potential V $\xrightarrow{gradient}$ (minus) electric field

  Example 5.1. Find electric potential of a uniform electric field $\vec{E} = E\hat{z}$

- Define potential V at point f and i as

  Definition 5.1.

$$V_f - V_i = -\int_i^f \vec{E} \cdot d\vec{s}$$

- 1D: $V = -\int_{x_i}^{x_f} E_x dx$
- Important properties:
  1. The value of V does not matter. Only the difference in V matters (potential difference).

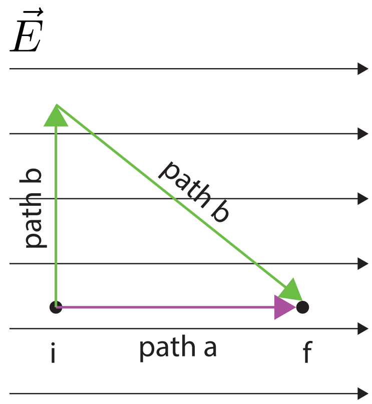
     Theorem 5.1.

$$V = -\int_i^f \vec{E} \cdot d\vec{s},$$

     if choosing the initial point to be zero potential.

2. The path to do the integral does not matter (take them as facts, hard to prove now in 104).

Example 5.2. Find potential of point at f by doing the integral in two paths as shown.



- Physical Meaning of potential $\rightarrow$ potential energy per unit charge.

  derivation: Work by electric field $\vec{E}$ on charge q when draging a charge from point i to f is

$$W = \int_i^f \vec{F} \cdot ds = q \int_i^f \vec{E} \cdot ds.$$

  Potential energy is negative of this work (since we have to move against this electric force to increase the potential energy)

$$U = -W = -q \int_i^f \vec{E} \cdot ds$$

  Potential energy per unit charge q ($\frac{U}{q}$) is therefore electric potential.

- Potential of a Point Charge: by inserting electric field $E = \frac{1}{4\pi\epsilon} \frac{q}{r^2} \hat{r}$ and set potential to be zero at infinity.

  Theorem 5.2 (Electric Potential Due to Point Charge q)**.**

$$V_{point} = \frac{1}{4\pi\epsilon_0} \frac{q}{r}$$

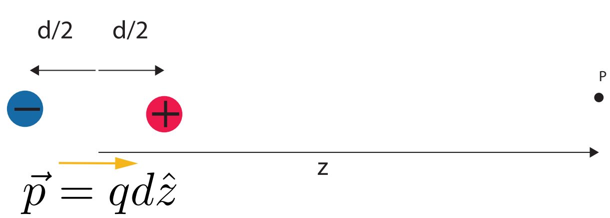
Example 5.3. Find electric potential as a function of distance r for the center of a sphere with uniform charge density ρ. Defining the potential of $V = 0$ at the center.

- Since adding scalar is simpler than adding vector, calculating E-field from a gradient of electric potential might be a simpler approach.

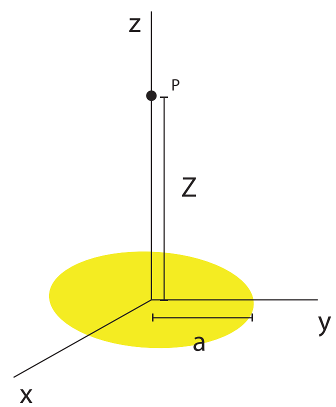
5.1 Calculating Field from Potential

- 3D: $\vec{E} = -\vec{\nabla} V$
- 1D: $E_x = -\frac{\partial V}{\partial x}, E_y = -\frac{\partial V}{\partial y}, E_z = -\frac{\partial V}{\partial z}$

Example 5.4. Calculate an electric field from dipole



Example 5.5. Calculate an electric field from a disk with areal density of σ.



6 E-Field Calculations

Summary: Methods to Calculate Electric field

1.
2.
3.

7 Electric field and Potential of Conductors

- Conductor → (negative)charges can move freely.
- Insulator → charges cannot move so they can be fixed at some points

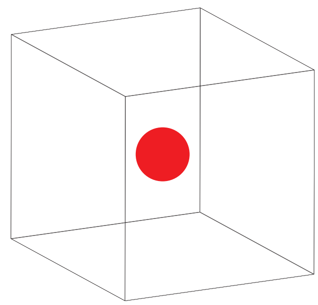
Theorem 7.1. Electric field inside a conductor is zero.

Theorem 7.2. If an excess charge is placed on an isolated conductor, that amount of charge will move entirely to the surface of the conductor. None of the excess charge will be found within the body of the conductor. This applies only in electrostatic.

Theorem 7.3. Electric potential is the same at all points of the conductor - whether on the surface or inside.

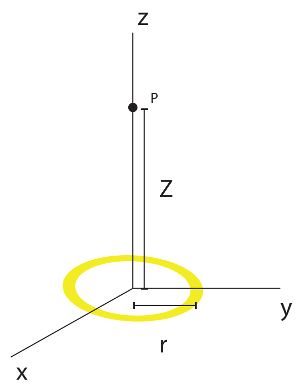
8 Homework

Homework 1 (Charge inside a Cube)**.** A cube of length L has a point charge q at the center. Find (a) the flux that goes through each surface (b) an average electric field through the surface .



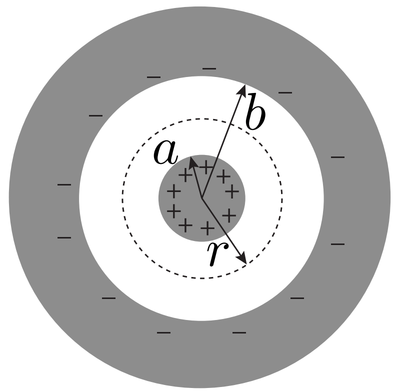
Homework 2 (Gauss Law vs Coulomb Law). Show that the electric potential at a point on the central axis of a thin ring of charge of radius r and a distance Z from the center is

$$V = \frac{1}{4\pi\epsilon_0}\frac{q}{\sqrt{Z^2 + r^2}.}$$



And calculate electric field (all direction x,y,z) by performing the derivation.

Homework 3 (Potential Difference of Coaxial Cylinders)**.** A coaxial cylinder of radiuii a and b of length L. L is much larger than b so that we could approximate that it is infinitely long. Each plate contains a charge of magnitude q. Find the potential difference between two plates in the following steps:



(a) Show that electric field between the cylinders is

$$E = \frac{q}{2\pi\epsilon_0 Lr}$$

(similar to Ex. 4.2).
Note that the total charge outside and the thickness of the shell do not affect the field E.
(b) Show that electric field outside the cylinders is zero.
(c) Show that the potential difference between b and a is

$$V = \frac{q}{2\pi\epsilon_0 L} \ln\frac{a}{b}$$