Electrostatics

PHY 104

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1 Electromagnetism

- Four fundamental forces: __________, __________, __________, __________.
- Previously electromagnetism were thought of two separated phenomena: elektron, "amber", and magnetis lithos, "magnesian stone".
- The two were unified by the experimental works of Faraday, Oested and Ampere, and theoretical work by Maxwell. (very simplified history)
- Heart and soul of classical E&M: __________'s Equation

$$
\begin{aligned}
\vec{\nabla} \cdot \vec{E} &= \frac{\rho}{\varepsilon_0} &\qquad& (\qquad\qquad\qquad\qquad\qquad\qquad) \\
\vec{\nabla} \times \vec{E} &= \frac{\partial \vec{B}}{\partial t} &\qquad& (\qquad\qquad\qquad\qquad\qquad\qquad) \\
\vec{\nabla} \cdot \vec{B} &= 0 &\qquad& (\qquad\qquad\qquad\qquad\qquad\qquad) \\
\vec{\nabla} \times \vec{B} &= \mu_0 (\vec{J} + \varepsilon_0 \frac{\partial \vec{E}}{\partial t}) &\qquad& (\qquad\qquad\qquad\qquad\qquad\qquad)
\end{aligned}
$$

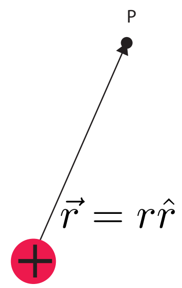
  $\vec{E}$ is electric field and $\vec{B}$ is (related to) magnetic fields. ρ is charge density, $\vec{J}$ is current density.
- Guide to read these equations:
  1. Don't worry about the unknown math operations.
  2. Don't worry about the vector notations ($\vec{\nabla}$ is just like $\frac{d}{dx}$)
  3. Two equations for electro ($\vec{E}$'s) and two equations for magnetic ($\vec{B}$'s).
  4. The "sources of the fields are on the right hand sides.
  5. The behaviors of the fields around the sources are determined by the left hand sides.
  6. $\cdot$ reads "to diverge out from a point"
  7. $\times$ reads "to curl around"

2 Charge and Coulomb Law

- Unlike charges attract, like ones repel.
- Charge is quantized. The elementary charge is that of electron. We could only found the charges of Ne, where N = 1,2,3..
- Charge is conserved, so far no one has disproved it. http://physics.aps.org/synopsis-for/10.1103/PhysRevLett.115.231802
- Charge has a unit of __________, which is very large.
- Electrostatics = charges do not move

- Charges interact via the force between two point charges (q_1 and q_2) calculated from a Coulomb's Law

$$\vec{F} = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r^2}\hat{r}$$



- Four features:
  1. it's a vector
  2. it's an inverse square law
  3. linear in charge q_2 (results in the superposition principle of force)
  4. symmetry of force on A from B and on B from A.

- The inverse square nature of Coulomb's law has be a subject of test since 1769. The more recent measurement in 1983 show that the deviation factor q (i.e. $\vec{F} \propto \frac{1}{r^{2+q}}$) is in the order of 10^{-17}. The deviation is ascribed to the rest mass of photon.

- From special relativity:

$$E = \sqrt{p^2c^2 + (mc^2)^2}.$$

  When momentum p is zero (a particle at rest), the rest mass energy is __________

Example 2.1. (a) Calculate a Coulomb force between two 1-C charges at 1 meter apart. (b) Calculate a gravitational force between two 1-kg masses at 1 meter apart.

3 What is Field?

- Feynman: A "field" is any physical quantity which takes on different values at different points in space.
- A necessary concept to account for forces when bodies are far apart (action at distant).
- Electromagnetic Field, Gravitational Field, Quantum Field Theory

- Scalar Field (weather forecast, altitude map), Vector Field, Tensor Field
- One way to generate a vector field is to take a gradient. For example, thermal gradient, slope map of the mountains, weather forecasting of wind direction.
- Gradient is a directional slope:
$$\vec{\nabla}\phi(x,y,z) = \frac{\partial}{\partial x}\phi\hat{i} + \frac{\partial}{\partial y}\phi\hat{j} + \frac{\partial}{\partial z}\phi\hat{k}.$$

4 Electric Field

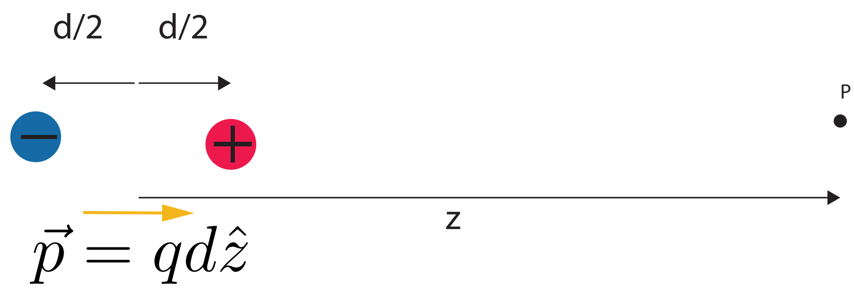
- Another equivalent picture of electric force on charge q is
$$\vec{F} = q\vec{E}$$
- An equivalent approach to Coulomb's Law: q_1 create an electric field of $\vec{E_1} = \frac{1}{4\pi\epsilon_0}\frac{q_1}{r^2}\hat{r}$ and q_2 interacts with the field $\vec{E_1}$ in the form of force $\vec{F} = q_2\vec{E_1}$.
- An electric field due to point charge
$$\vec{E}_{point} = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\hat{r}$$
- A more practical E-field: light, CCD cameras (record intensity - E-field square), a parallel plate capacitor, a cathord-ray TV screen.

Theorem 4.1 (Approximation of $(1+x)^n$)**.**
$$(1+x)^n \approx 1 + nx,$$
if x is much smaller than 1.

Example 4.1. Show that electric field of a dipole (two point charges +e and -e seperated by distance d) at distance z along the direction of the dipole is

$$\vec{E}_{dipole} = \frac{1}{4\pi\epsilon_0}\frac{2p}{z^3}\hat{k},$$

where $p = qd$ is called the dipole moment.

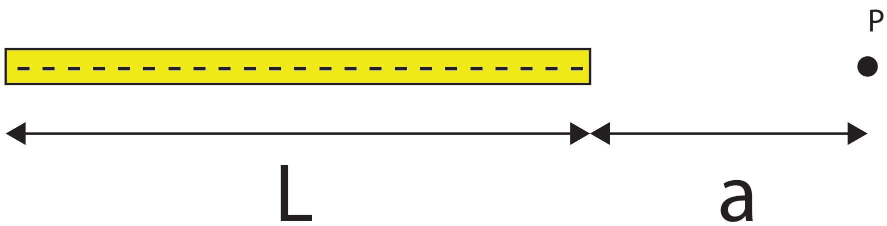


4.1 Electric Field Calculation from Continuous Charge Distributions

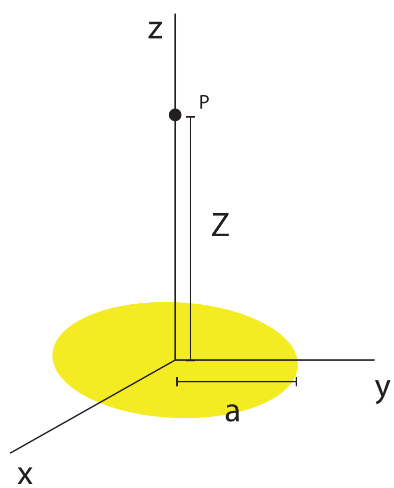
- The physics is over and it's time for calculus (boring, difficult, irrelevant etc). Important skill: you need to know if you stuck with math or with physics.

- Strategy: chop off the shapes into point charges, calculate electric field from each of them, and do the sum (vectorially)
- There are only a handful of geometries that can be solved by integration within a finite about of time. Others are possible but the math will be too complicated.
  1. 1D: Line Charge
  2. 2D: Ring, Disk, Rectangle

Example 4.2. Calculate an electric field from a rod of charge with linear density of λ.



Example 4.3. Calculate an electric field from a disk with areal density of σ.



Example 4.4. Calculate an electric field from an infinite plate with areal density of σ.

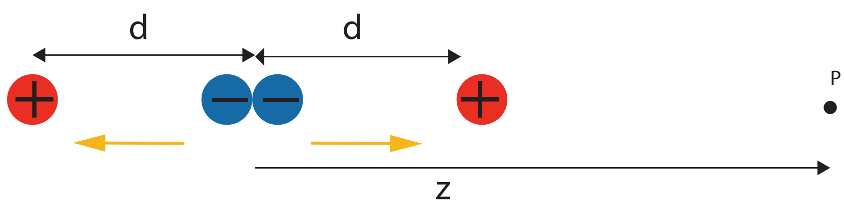
5 Dipole in a uniform electric field

- torque
$$\vec{\tau} = \vec{p} \times \vec{E}$$
- potential energy
$$U = \vec{p} \cdot \vec{E}$$
- To be derived in the homework.

6 Homework

Homework 1 (Electric Quadrupole)**.** (10 points) Quadrupole consists of two dipoles with dipoles that are equal in magnitude but opposite direction. Show that the value of electric field on the axis if the quadrupole for a point P at distance z from the center is given by (assuming $z \gg d$):

$$\vec{E}_{quad} = \frac{3Q}{4\pi\epsilon_0 z^4}\hat{k}$$



Hint: You can solve this problem in two ways

1. Follow the steps in Ex 4.1 but instead, summing electric field contribution from 4 points charges:

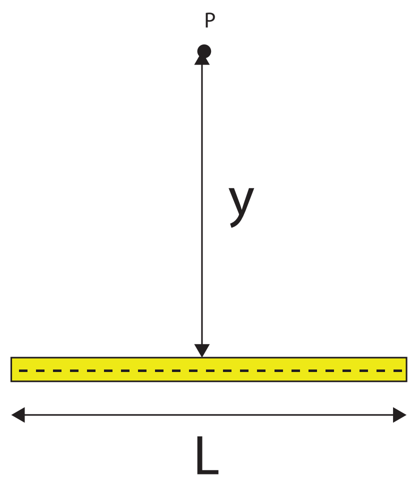
$$\vec{E}_{quad} = \vec{E}_{point}(z-d) - 2\vec{E}_{point}(z) + \vec{E}_{point}(z+d).$$

2. Use the final results from Ex 4.1 and do the summation from two dipoles.

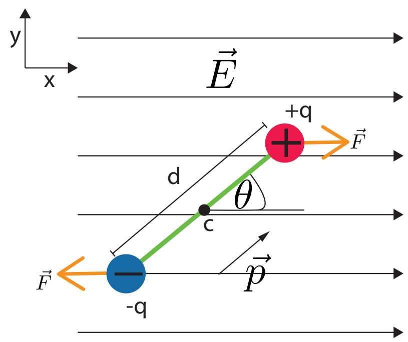
$$\vec{E}_{quad} = -\vec{E}_{dipole}(z-d/2) + \vec{E}_{dipole}(z+d/2)$$

Homework 2 (Electric Field from a rod – revisited). (10 points) A thin nonconducting rod of finite lenght L has a charge q spread uniformly along it. Show that the magnitude of electric field at point p is

$$E = \frac{q}{2\pi\epsilon_0 y}\frac{1}{\sqrt{L^2+4y^2}^{1/2}}$$



Homework 3. Torque and Energy of a Dipole in an Electric Field (30 points)



(a) (3 points) What is the net force due to electric field on a dipole according to the figure above.
(b) (7 points) Show that the net torque on the above dipole with respect to the center point c is

$$\tau = -qdE\sin\theta = -pE\sin\theta$$

Hint: Minus sign suggests that torque gives rise to the rotation in -θ directon (clockwise)
(c) (3 points) Explain why the direction of torque is along the negative z axis (pointing into the page) and therefore

$$\vec{\tau} = -pE\sin\theta\hat{k} = \vec{p}\times\vec{E}$$

(d) (7 points) Show that the potential energy of an electric dipole is $U = -\vec{p}\cdot\vec{E}$ by defining the angle of $\theta = \frac{\pi}{2}$ as a reference point.
Hints:

1. work done by the field to rotate this dipole form angle ($\theta = \frac{\pi}{2}$) to angle θ is $W = \int_{\pi/2}^{\theta}\tau d\theta$
2. potential energy energy is the negative of the work done by the field $U = -W$.
3. $\vec{x}\cdot\vec{y} = xy\cos\theta$

(e) (3 points) Find the angle θ where the energy is minimized. This is the equilibrium position.
(f) (7 points) Find the frequency of oscillation of this electric dipole and with rotational inertia ($I = \frac{md^2}{2}$), for small amplitude of oscillation about its equilibrium position in a uniform electric field E. Assuming that the charges have mass of m [Answer: $f = \frac{1}{2\pi}\sqrt{\frac{2qE}{md}}$]