Current and Magnetic Field

PHY 104

Chan La-o-vorakiat • (2015/2) • King Mongkut's University of Technology Thonburi

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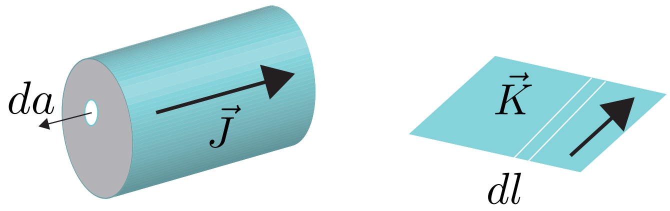


Figure 1.1

1 Charge in Motion and Current

- Charge creates electric field. Charge in motion (current) creates magnetic field.
- Definition of current
$$i = \frac{dq}{dt}$$
- Current comes in various shapes (Fig. ??): **current density**
$$I = \int \vec{J} \cdot d\vec{a}$$
- The conversion is done by dimension analysis.
$$q\vec{v} \propto \vec{I}dl \propto \vec{K}da \propto \vec{J}dV \tag{1.1}$$
- Magnetostatic → current does not change with time

2 Magnetic Field

Consequences of the last two points of Table 1:

1. Magnetic field cannot diverge from a point but electric field can do.
2. Electric field cannot form a loop, but magnetic field can do.

3 Lorentz Force Law

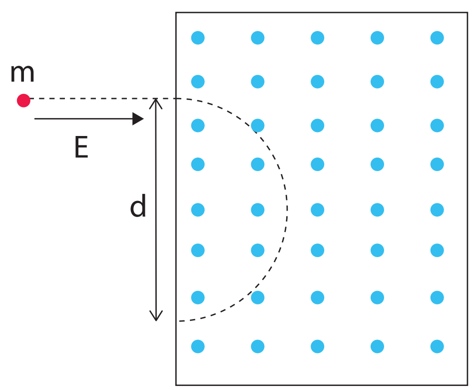
Theorem 3.1 (Force on a Moving Charge q due to Magnetic Field)**.**

$$\vec{F} = q(\vec{v} \times \vec{B}))$$

Example 3.1 (Mass Spectrometer)**.** An ion (of mass m and charge e) is accelerated by the electric field due to potential difference V. Then the fast ion enters a chamber with uniform electric field $\vec{B}$ perpendicular to the path of ion. The magnetic field causes the ion to move in a semicircle orbit. Find the distance of hitting the detector plate.

Table 1: Comparing Electrostatic E-field and Magnetostatic B-field

	$\vec{E}$	$\vec{B}$
name	electric field	magnetic field
unit	V/m	Teala or Gauss
interaction with charge	$\vec{F} = q\vec{E}$	$\vec{F} = q(\vec{v} \times \vec{B})$
source	static charge	moving charge (static current)
charge	yes	has not found yet
flux	$\int \vec{E} \cdot d\vec{a}$	$\int \vec{B} \cdot d\vec{a}$
Line Integral $\int_i^f \vec{E}(\vec{B}) \cdot d\vec{s}$	Potential $V_i - V_f$	no special meaning
Surface Integral around a closed (Gaussian) surface $\oint \vec{E}(\vec{B}) \cdot d\vec{a}$	$\frac{q_{in}}{\epsilon_0}$	0
Line Integral around a closed (Amperian) loop $\oint \vec{E}(\vec{B}) \cdot d\vec{s}$	0	$\mu_0 I_{enc}$
moments	dipole moment $\vec{p} = q\vec{d}$	magnetic moment $\vec{\mu} = I\vec{a}$
interaction with moment	$\vec{\tau} = \vec{p} \times \vec{E}$	$\vec{\tau} = \vec{\mu} \times \vec{B}$
potential energy	$-\vec{p} \cdot \vec{E}$	$-\vec{\mu} \cdot \vec{B}$



Example 3.2 (Hall's Effect)**.** A strip of semiconductor containg a free carrier of charge q and velocity v is located in magnetic field $\vec{B}$. The field deflects the charge and creates a voltage drop along the side of the semiconductor V. The buildup of charge at the sides of the conductors creates electric field that balance this magnetic force.

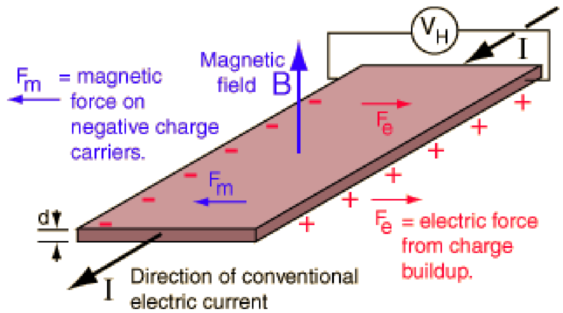


Figure 3.1: http://hyperphysics.phy-astr.gsu.edu/hbase/magnetic/hall.html

Lemma 3.1 (Force on a Current-Carrying Wire due to Magnetic Field)**.**

$$d\vec{F} = I(d\vec{s} \times \vec{B}))$$

4 Biot-Savart's Law

Theorem 4.1 (Biot-Savart Law for a Line Current)**.**

$$\vec{B} = \frac{\mu_0 I}{4\pi} \int \frac{d\vec{s} \times \hat{r}}{r^2} = \frac{\mu_0 I}{4\pi} \int \frac{d\vec{s} \times \vec{r}}{r^3},$$

where $\vec{r}$ is a vector from the current element to point P where we want to calculate the field. For other types of current sources (eqn. 1.1).

- Biot-Savart law is all you need to calculate magnetic field (similar to Coulomb's Law). But it usually leads to a very messy mathematics due to the cross product and an inverse cube dependent. Similar to electrostatics,

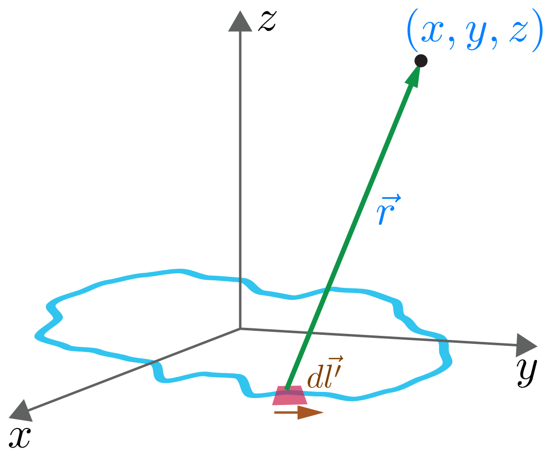


Figure 4.1: Current loop produces magnetic induction $\vec{B}$ via Biot Savart law.

there are two more approaches: one is Gauss's Law like method (Ampere's Law), the other is potential method (to much for 104).

Example 4.1. Find magnetic field from a long straight section of current.

Example 4.2. Find magnetic induction from a current loop

5 Ampere's Law

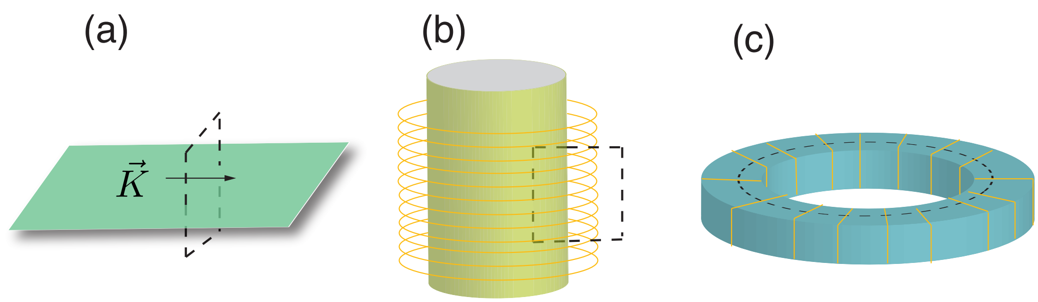
Theorem 5.1 (Ampere's Law in integral Form)**.**

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{enc}$$

- Ampere's law is useful when you know partially about the answer but it's only limited to very few cases with very high symmetry. To list them all

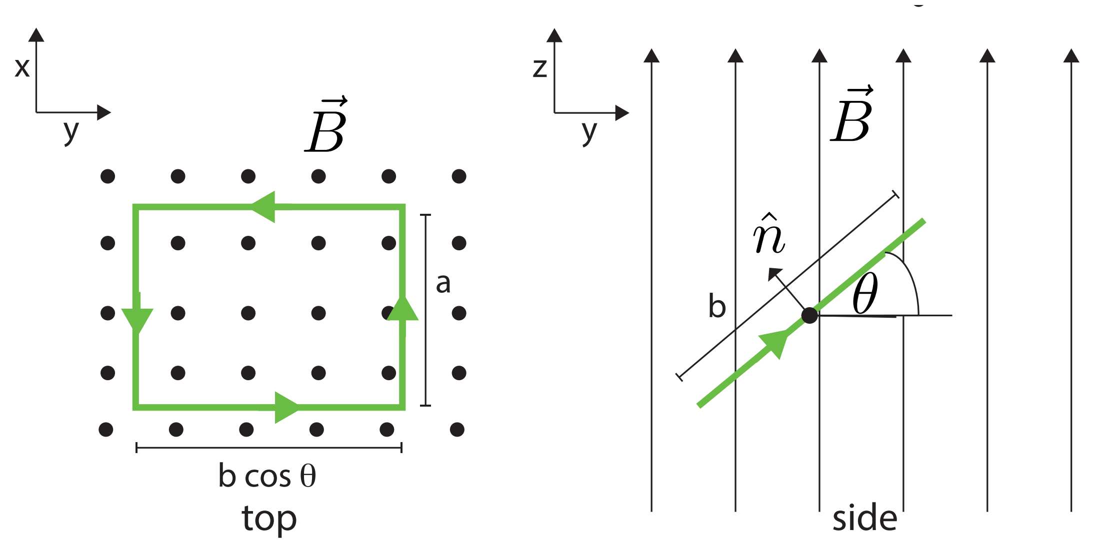
  1. infinite wires
  2. infinite planes
  3. solenoids
  4. toroid

  But still it is very handy when you want to recall some formula quickly.



6 Homework

Homework 1 (Magnetic Moment in a Uniform Field). (9 points) A rectangle current loop of current I is located in a uniform magnetic field as shown.



(a) (3 points) What is the net force the loop according to the figure above.

(b) (3 points) Show that the net torque on the above loop with respect to the center point c is

$$\tau = -IabB\sin\theta$$

(c) (3 points) Show that the torque vector is

$$\vec{\tau} = \vec{\mu} \times \vec{B},$$

where $\vec{\mu} = abI\hat{n}$, where $\hat{n}$ is a normal vector from the plane of the current loop.

Homework 2. (10 points) A closed circular loop of radius a with current I is in a uniform magnetic field $\vec{B}$.Show that the total force is zero when (a) (3 points) the current loop is aligned perpendicular to $\vec{B}$. (show by symmetry only without doing the integral)(b) (7 points) with the loop at the angel θ to the direction of $\vec{B}$.

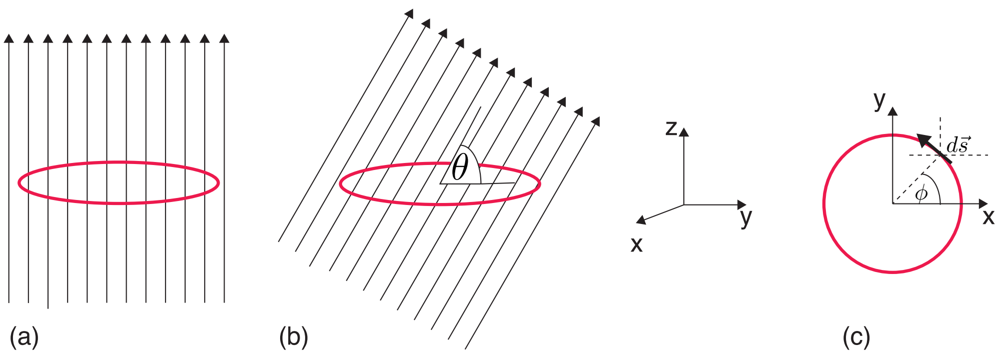


Figure 6.1

Hint: For the (b) part. First show that

$$d\vec{s} = -a\sin\phi d\phi\hat{x} + a\cos\phi d\phi\hat{y},$$

(See Fig 6.1c) and assume that $\vec{B}$ is int the zy plane:

$$\vec{B} = B\cos\theta\hat{y} + B\sin\theta\hat{z}.$$

Note: Total force on magnetic dipole in a uniform magnetic field is zero. Analogously, total force on electric dipole in a uniform electric field is zero (HW1.3).

Homework 3. (15 points) A long cylinder conductor of radius a containing a long cylindrical hole of radius b. The axes of the cylinder and hole are parallel and are distance d apart; a current i is uniformly distributed over the cross section.

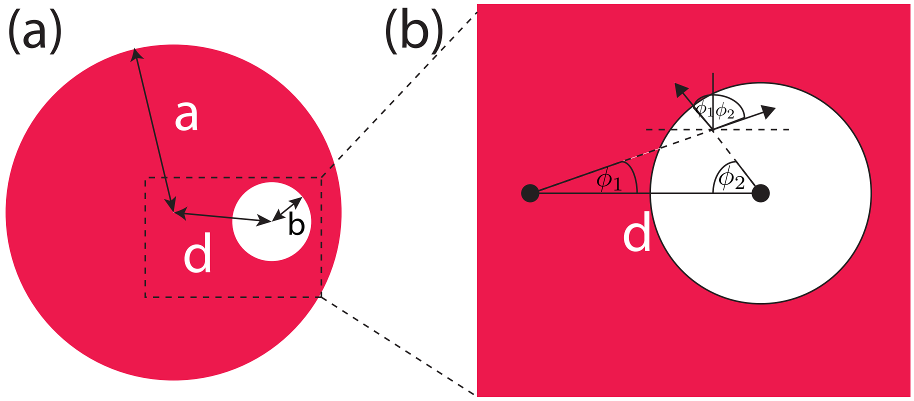


Figure 6.2

Show that (a) (5 points) magnetic field at the center of the hole is

$$B = \frac{\mu_0 I d}{2\pi(a^2 - b^2)}.$$

(b) (10 points) Magnetic field in the hole is

$$\vec{B} = \frac{\mu_0 I d}{2\pi(a^2 - b^2)}\hat{z},$$

everywhere in the hole.

Homework 4. (20 points) Consider two infinite charge surfaces with opposite charge density of σ and $-\sigma$. The surfaces are separated by distance D and they move at the same direction with the same velocity v (Fig 6.3). Find
(a) (5 points) electric force on a charge dq of the top plate due to the bottom plate
(b) (5 points)magnetic force on a charge dq of the top plate due to the bottom plater
(c) (5 points) Find the velocity that two forces are equal (d) (5 points) Use Wolfram Alpha to find the velocity, what is the input command?
Hint:
1. Use the conversion formula $\kappa\ da = dq\ v$
2. ϵ_0 is epsilon_0 and μ_0 is mu_0

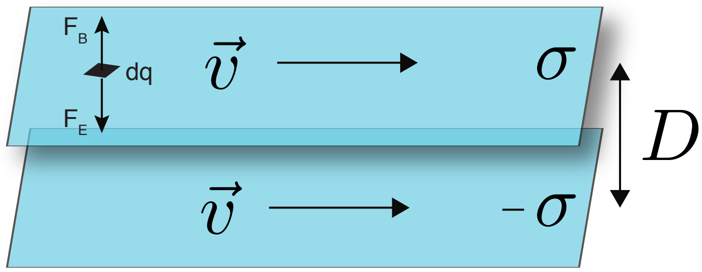


Figure 6.3