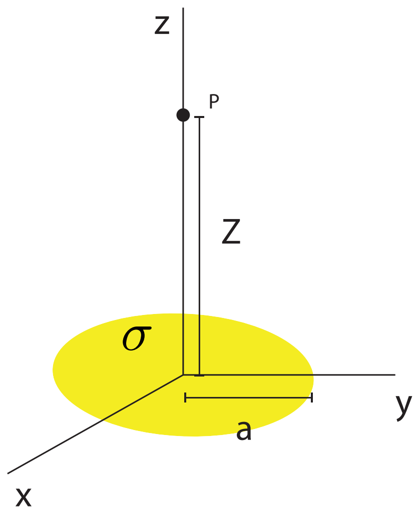

Calculate an electric field from a disk with uniform areal density of σ.

Solution: We have to split the problem into two steps.

Part 1: calculate an electric field from a ring of charge Q and radius r. And pretend that charge density is λ. We chop off the right into a small element of dl (Fig. 1a). Since the charge is uniform:

$$\lambda = \frac{Q}{2\pi r} = \frac{dq}{dl}.$$

Electric field from a point charge dq is

$$dE = \frac{kdq}{r^2 + Z^2}$$

But from symmetry we know that only the z component survive the integral,

$$dE_z = dE\cos\theta = \frac{kdq}{r^2 + Z^2}\frac{Z}{\sqrt{r^2 + Z^2}}.$$

Then we could do the integral,

$$E_z = \int \frac{k\lambda dl}{r^2 + Z^2}\frac{Z}{\sqrt{r^2 + Z^2}} = \frac{k\lambda Z}{(r^2 + Z^2)^{3/2}}\oint dl.$$

$\oint$ is the integral around the circle which is $2\pi r$.

$$E_z = \frac{k2\pi r\lambda Z}{(r^2 + Z^2)^{3/2}} = \frac{kQZ}{(r^2 + Z^2)^{3/2}}.$$

Part 2 (Fig. 2b): The disk could be chopped off into (onion) rings of radius r and thickness dr. Each carriers charge dq, which could be related to the density of

$$\sigma = \frac{dq}{2\pi r dr}$$

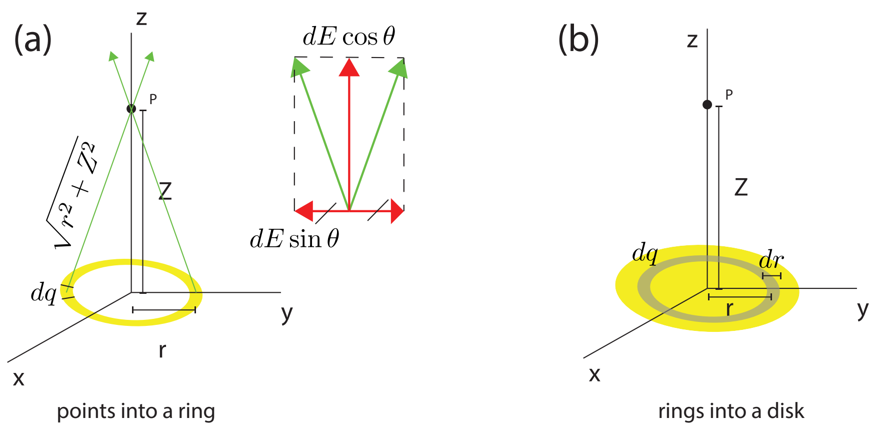

Figure 1:

The field due to a ring in the previous section is transformed to an element:

$$E_z \rightarrow dE_z$$

$$Q \rightarrow dq.$$

The electric field due to one ring becomes

$$dE_z = \frac{kdqZ}{(r^2+Z^2)^{3/2}} = \frac{k\sigma 2\pi r dr Z}{(r^2+Z^2)^{3/2}}.$$

Performing an integral

$$E_z = \int dE_z = k\sigma 2\pi Z \int_0^a \frac{rdr}{(r^2+Z^2)^{3/2}}$$

Making a substitution $u = r^2 + Z^2$ and $du = 2rdr$.

$$E_z = \frac{k\sigma 2\pi Z}{2} \int_{Z^2}^{a^2+Z^2} \frac{du}{u^{3/2}}$$

$$E_z = -k\sigma 2\pi Z[\frac{1}{\sqrt{a^2+Z^2}} - \frac{1}{Z}]$$

extra: If the radius is large, the first term can be omitted

$$E_z = \frac{k\sigma 2\pi Z}{Z} = \frac{\sigma}{2\epsilon_0},$$

which is an electric field from an infinite plane plane.