3 Homework

Homework 1. A square copper loop of length L is placed in a uniform field $\vec{B}$ and allowed to fall under gravity.
(a) (10 points) Show that the velocity of the loop as a function of time is in the form of

$$\vec{v}(t) = \frac{g}{\lambda} - \frac{g}{\lambda}e^{-\lambda t}\hat{k}.$$

What is λ in terms of copper density and conductivity.
Hint: Find an induced EMF then calculate the current around the loop due to magnetic induction. Assume that the copper loop has a resistance R. Use the formula $R = \frac{L}{\sigma A}$ to related R to the conductivity (σ), cross sectional area (A) and length (L). Find the magnitude of force (ILB) and use Newton's law.

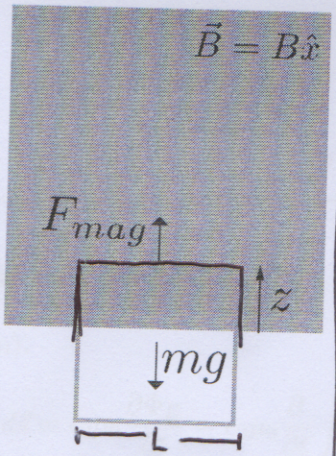

Figure 3.1

$$\frac{1}{\lambda} = \frac{mR}{B^2L^2}$$

$$= \frac{\rho(4\cancel{L}\cancel{A})\cancel{\times}4\cancel{L}}{B^2\cancel{L^2}\ \sigma\ \cancel{A}}$$

$$= \frac{16\rho}{B^2\sigma}, \quad \lambda = \frac{B^2\sigma}{16\rho}$$

$$\Phi_B = BLz$$

$$\frac{d\Phi_B}{dt} = BL\dot{z} = \mathcal{E} = IR$$

$$I = \frac{BL\dot{z}}{R}$$

$$F_B = \frac{B^2L^2\dot{z}}{R}$$

$$\Sigma F = ma$$

$$mg - \frac{B^2L^2\dot{z}}{R} = m\ddot{z}$$

$$g - \frac{B^2L^2\dot{z}}{mR} = \ddot{z}$$

$$\frac{d\dot{z}}{g - \frac{B^2L^2\dot{z}}{mR}} = dt$$

$$-\left(\frac{mR}{B^2L^2}\right)\ln\left(\frac{g - \frac{B^2L^2\dot{z}}{mR}}{g}\right) = t$$

$$\ln\left(\frac{g - \frac{B^2L^2\dot{z}}{mR}}{g}\right) = -\frac{B^2L^2t}{mR}$$

$$1 - \frac{B^2L^2\dot{z}}{gmR} = e^{-\frac{B^2L^2t}{mR}}$$

$$\dot{z} = \frac{mgR}{B^2L^2}\left(1 - e^{-\frac{B^2L^2t}{mR}}\right)$$