PHY104/2015 Mock Pre-Midterm Examination

Fundamental constants:

Elementary charge:	$e = 1.60 \times 10^{-19}$ C
Coulomb constant:	$k = 9.0 \times 10^{9}$ N m^2 C^{-2}
Electron mass:	$m_e = 9.11 \times 10^{-31}$ kg
Proton mass:	$m_p = 1.67 \times 10^{-27}$ kg
Permittivity of free space:	$\epsilon_0 = 8.85 \times 10^{-12}$ F m^{-1}
Gravitational acceleration at Earth's surface:	$g = 10$ m s^{-2}

Remember

Relationship between electric field E and electric potential $V(x)$:

$$E = -\frac{dV}{dx}.$$

Multiply the above equation by charge q:

$$qE = -\frac{d(qV)}{dx}.$$

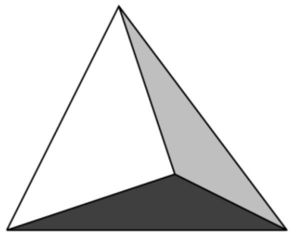
Because electric force $F = qE$ and potential energy $U = qV$, we have the relationship between force and potential energy:

$$F = -\frac{dU}{dx}.$$

Part A

1. A charge $q = -1.5\ \mu\text{C}$ is situated at each vertex of a tetrahedron. All edges of the tetrahedron are equal in length which is 20 cm. Determine the electric potential energy of the system.

[3]

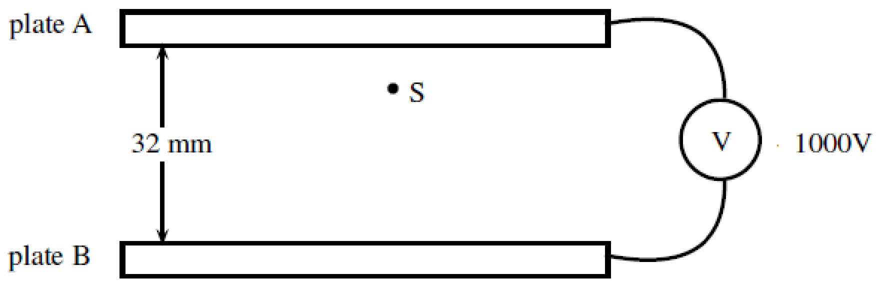


2. A proton is accelerated from rest by an electric field $E = 1{,}000\ \text{NC}^{-1}$. Calculate the speed of the proton after 1.0 μs. [3]

3. A conducting sphere of radius $R = 0.25$ m carries a charge $Q = 2.4 \times 10^{-3}$ C. A proton is released from the surface of the sphere. Find the speed of the proton when it reaches infinity. [3]

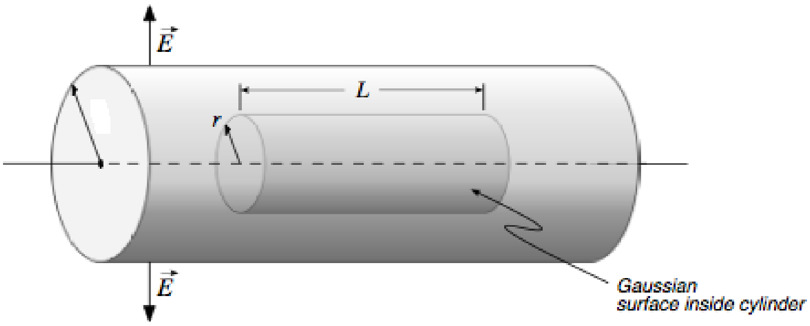
4. An electron of charge q and mass m is travelling from rest (at $x = 0$) along x-axis under potential $V = kx^{4/3}$, where k is a constant. Find the formula for the magnitude of the acceleration of the electron at distance x. [3]

5. Two parallel plates, separated by distance $d = 32$ mm, have potential difference $V = 1000$ V. A negatively charged particle S with charge of magnitude $q = 1.6 \times 10^{-10}$ C and mass $m = 1.0 \times 10^{-7}$ kg is found to be moving <u>upwards</u> at <u>constant speed</u> $v = 1.25 \times 10^{-3}$ ms^{-1}. The drag force on the particle is given by kv. Calculate the value of k.



[4]

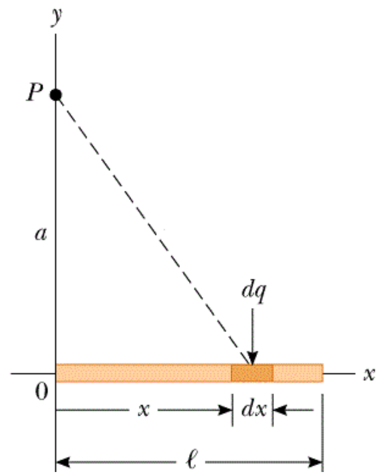
6. An insulating long cylinder has positive charge with charge density (charge per unit volume) $\rho = 6.4 \times 10^{-4}\ \mathrm{Cm}^{-3}$ uniformly distributed all over the volume. Use Gauss' law to calculate the electric field at distance $r = 0.15$ m from the cylinder's axis <u>inside</u> the cylinder.



[4]

Part B

7. A line of charge Q and length ℓ is lying along x-axis. One end of the line is at the origin.



Point P is at a distance a above the origin. Let E_x be horizontal component of electric field and E_y the vertical component of the field at point P.

a) Show that

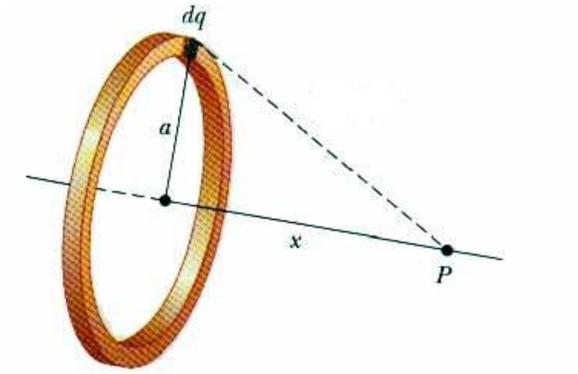
$$E_x = \frac{Q}{4\pi\varepsilon_0 \ell}\left(\frac{1}{a} - \frac{1}{\sqrt{a^2+\ell^2}}\right) \quad \text{and} \quad E_y = \frac{Q}{4\pi\varepsilon_0 a}\frac{1}{\sqrt{a^2+\ell^2}}.$$

Hint: $\displaystyle\int \frac{x}{(x^2+a^2)^{3/2}}\,dx = \frac{-1}{\sqrt{x^2+a^2}} + C, \quad \int \frac{1}{(x^2+a^2)^{3/2}}\,dx = \frac{x}{a^2\sqrt{x^2+a^2}} + C.$

[7]

b) Given that $Q = 1.6 \times 10^{-10}$ C, $\ell = 1.0$ m and $a = 1.0$ m, calculate the magnitude of the resultant force on an electron sitting at point P. [3]

8. The diagram shows a thin ring of radius a with positive charge uniformly distributed. The linear charge density (charge per unit length) is given by λ. Point P is at distance x from the center of the ring.



a) Show that the potential at point P is given by

$$V(x) = \frac{\lambda a}{2\pi\varepsilon_0\sqrt{x^2 + a^2}}.$$

[3]

b) Use the result in a) to find electric field at point P. [3]

c) Given that $\lambda = 10^{-12}$ Cm^{-1}, $a = 0.5$ m and $x = 1.0$ m. An electron is released at point P. Calculate the speed of an electron when it passes the center of the ring.

[4]

9. Figure 1 shows an electric dipole which consists of two opposite charges equal in magnitude q, separated by a distance a. A vector drawn from negative charge to positive charge is called <u>electric dipole</u> p whose magnitude is defined as

$$p = qa.$$

Figure 2 shows an electric dipole making an angle θ with a uniform electric field E.

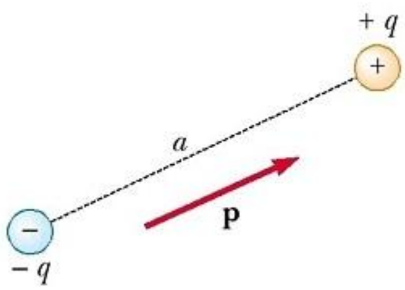


Figure 1 Electric dipole

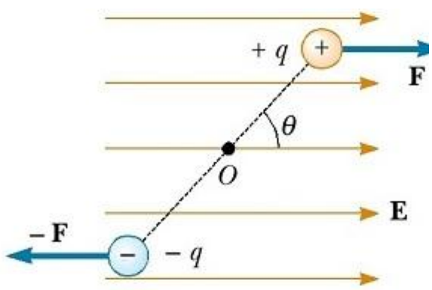


Figure 2 Electric dipole in uniform electric field

a) <u>Consider Fig. 2</u>, show that the torque on the dipole is given by

$$\tau = pE\sin\theta.$$

[3]

In the presence of electric field as in Fig. 2, the dipole rotates. The work done by electric force is equal to the change in potential energy according to

$$\Delta U = \int_{\theta_1}^{\theta_2} \tau \, d\theta.$$

b) By using the result in a), show that, for any angle θ, the potential energy is given by

$$U = -pE \cos\theta.$$

[3]

Figure 3 shows a conducting spherical star with positive charge Q. A dumbbell-shaped satellite modeled as an electric dipole p is at distance x far away from the star.

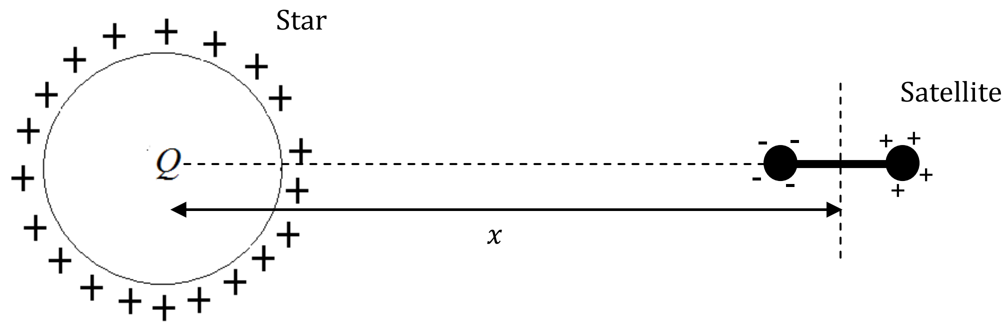


Figure 3 A dipolar satellite far away from a positively charged star

c) There is a force acting on the satellite. Is this force repulsive from or attractive towards the star? [1]

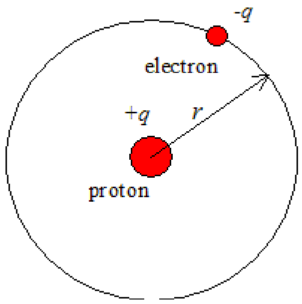
d) From the result in part b), show that the force acting on the satellite

$$F = \frac{pQ}{2\pi\varepsilon_0 x^3}.$$

[3]

10. Use Gauss' law to find the electric field $E(r)$ as a function of distance r from a point charge Q. [3]

In a model of hydrogen atom, an electron (charge $-q$) is orbiting around a fixed proton (charge q) in circle with radius $r = 5.3 \times 10^{-11}$ m.



a) Show that the speed of electron is about 2.19×10^{6} ms^{-1}. [3]

b) Calculate kinetic energy, electric potential energy, and total energy of the atom in unit of electronvolt (eV). Note that $1\ \mathrm{eV} = 1.6 \times 10^{-19}\ \mathrm{J}$. [4]