Derivation of Vector Potential Due to Magnetization

October 17, 2015

Since the vector potential due to a magnetic moment $\vec{m}$ is

$$d\vec{A} = \frac{\mu_0}{4\pi}\frac{\vec{m} \times \hat{\mathscr{r}}}{\mathscr{r}^2}.$$

Therefore total vector potential becomes

$$\vec{A} = \frac{\mu_0}{4\pi}\int \frac{\vec{M} \times \hat{\mathscr{r}}}{\mathscr{r}^2} d\tau'$$

Rewrite the equation with the help of $\frac{\hat{\mathscr{r}}}{\mathscr{r}^2} = -\vec{\nabla}\frac{1}{\mathscr{r}} = \vec{\nabla}'\frac{1}{\mathscr{r}}$.
With this

$$\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi}\int \vec{M}(\vec{r'}) \times \vec{\nabla}'\frac{1}{\mathscr{r}} d\tau'.$$

By use of the vector identity

$$\vec{\nabla} \times (f\vec{A}) = f(\vec{\nabla} \times \vec{A}) - \vec{A} \times (\vec{\nabla} f),$$

we could rearrange the terms such that

$$\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi}\int \frac{\nabla \times \vec{M}(\vec{r'})}{\mathscr{r}} d\tau' + \frac{\mu_0}{4\pi}\oint \frac{\vec{M}(\vec{r'}) \times d\vec{a'}}{\mathscr{r}}.$$

By comparing the formular to the expression for the vector potential due to volumn current and surface current, we could see that having magnetization is equivalent to having current sources $\vec{K_b}$ and $\vec{J_b}$: