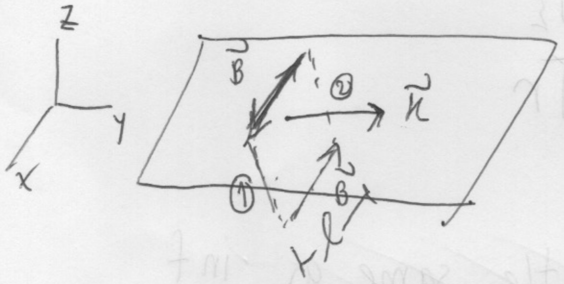
~~(a)~~ HW 1.

1. (a) $\vec{B}$ should be parallel to the plane but perpendicular to $\vec{n}$ (due to symmetry)



$\vec{n} \longrightarrow \hat{y}$

$\hat{B} \longrightarrow \hat{x}$

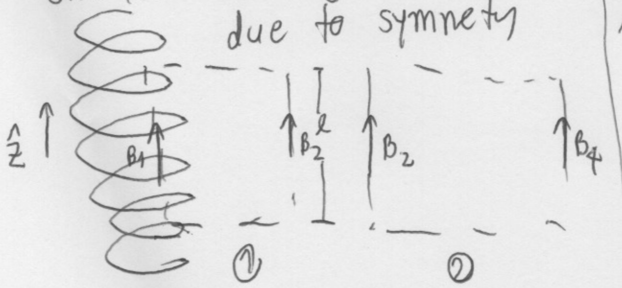
$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I$$

$$Bl + \underbrace{0}_{} + Bl + \underbrace{0}_{} = \mu_0 n l$$

$\vec{B} \perp d\vec{l}$ at ② & ①

$$\vec{B} = \frac{\mu_0 n \cancel{l}}{2\cancel{l}}\hat{x} = \frac{\mu_0 n}{2}\hat{x}$$

2. $\vec{B}$ should be along $\hat{z}$ due to symmetry



loop 2

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I$$

$$B_2 l - B_4 l = 0$$ ← no I

$$B_2 = B_4$$

but since $B_4 = 0$ at ∞ (far away)

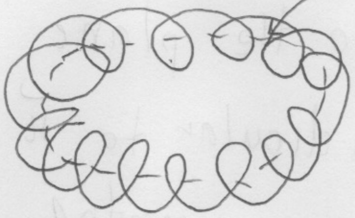
$$B_2 = 0$$

loop ①

$$B_1 l - \cancel{B_2}^{B_2 = 0} l = \mu_0 n I l$$

$$\vec{B}_1 = \mu_0 n I \hat{z}$$

(C)



$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I$$

$$B 2\pi r = \mu_0 N I$$

$$B = \frac{\mu_0 N I}{2\pi r}$$

~~interestingly, $\vec{B}$ is the same as inf~~

outside

$$B 2\pi r = 0$$

I poning up is the same as pointing down

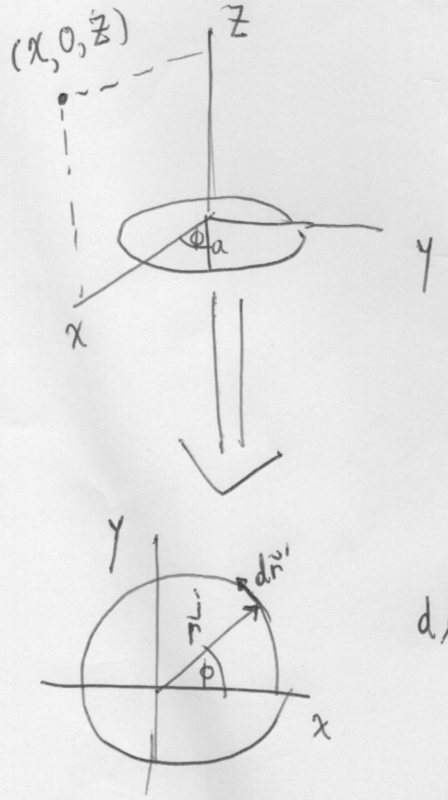
$$B = 0$$

HW 2

(b) $\vec{A} = \frac{\mu_0 I}{4\pi} \int \frac{d\vec{l}}{\mathscr{r}}$

$A_x = \frac{\mu_0 I}{4\pi} \int_0^{2\pi} \frac{-a\sin\phi \, d\phi}{\mathscr{r}}$ $\qquad A_y = \frac{\mu_0 I}{4\pi} \int_0^{2\pi} \frac{a\cos\phi}{\mathscr{r}} \, d\phi$

$\mathscr{r} = \sqrt{x^2 + a^2 + z^2 - 2az\cos\phi}$



$\vec{r} = (x, 0, z)$

$\vec{r}' = (a\cos\phi, a\sin\phi, 0)$

$\vec{\mathscr{r}} = (x - a\cos\phi, -a\sin\phi, z)$

$|\vec{\mathscr{r}}| = \sqrt{(x - a\cos\phi)^2 + a^2\sin^2\phi + z^2}$

$= \sqrt{x^2 - 2xa\cos\phi + a^2 + z^2}$

$d\vec{l}' = d\vec{r}' = (-a\sin\phi \, d\phi, a\cos\phi \, d\phi, 0)$

(C) $A_x = -\frac{\mu_0 I a}{4\pi} \int_0^{2\pi} \frac{\sin\phi \, d\phi}{\sqrt{x^2 + a^2 + z^2 - 2az\cos\phi}}$

$= +\frac{\mu_0 I a}{4\pi} \int_0^{2\pi} \frac{d\cos\phi}{\sqrt{x^2 + a^2 + z^2 - 2za\cos\phi}}$

$= \frac{\mu_0 I a}{4\pi} \frac{1}{(-2za)} (2) \left(x^2 + a^2 + z^2 - 2za\cos\phi \right) \Big|_{\cos\phi = 0}^{\cos\phi = 2\pi}$

$= 0$

(d)

$$A_y = \frac{\mu_0 I}{4\pi} \int_0^{2\pi} \frac{a\cos\phi \, d\phi}{\sqrt{x^2+a^2+z^2-2\cancel{z}a\cos\phi}}$$

set $\phi = \pi + 2\beta$

$$A_y = -\frac{\mu_0 I}{4\pi} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{a\cos(2\beta)(2)\, d\beta}{\sqrt{x^2+a^2+z^2+2\cancel{z}a\cos(2\beta)}}$$

$$= -\frac{\cancel{4}\mu_0 I}{\cancel{4}\pi} \int_0^{\frac{\pi}{2}} \frac{\cancel{2}a\cos 2\beta \, d\beta}{\sqrt{x^2+a^2+z^2+2\cancel{z}a\cos 2\beta}}$$

use $\cos 2\beta = 1 - 2\sin^2\beta$

$$= -\frac{\mu_0 I a}{\pi} \int_0^{\frac{\pi}{2}} \frac{(1-2\sin^2\beta)\, d\beta}{\sqrt{x^2+a^2+z^2+2\cancel{z}a(1-2\sin^2\beta)}}$$

$\boxed{\text{make a substitution } \; k^2 = \frac{4ax}{(a+x)^2+z^2}}$

$$= -\frac{\mu_0 I a k}{\pi} \int_0^{\frac{\pi}{2}} \frac{(1-2\sin^2\beta)\, d\beta}{\sqrt{4xa}\,(\sqrt{1-k^2\sin^2\beta}}$$

$$= -\frac{\mu_0 I k}{2\pi}\sqrt{\frac{a}{x}} \left[\underbrace{\int_0^{\frac{\pi}{2}} \frac{d\beta}{\sqrt{1-k^2\sin^2\beta}}}_{\tilde{K}(k)} - \underbrace{\int_0^{\frac{\pi}{2}} \frac{2\sin^2\beta \, d\beta}{\sqrt{1-k^2\sin^2\beta}}}_{(*)} \right.$$

(*) is related to $\frac{\partial \tilde{E}}{\partial k}$

$$\tilde{E}(k) = \int_0^{\frac{\pi}{2}} \sqrt{1-k^2\sin^2\beta}\, d\beta$$

$$\frac{\partial \tilde{E}}{\partial k} = \int_0^{\frac{\pi}{2}} \frac{1}{\cancel{2}\sqrt{1-k^2\sin^2\beta}} (\cancel{2}k)(-\sin^2\beta)\, d\beta = -k\int_0^{\frac{\pi}{2}} \frac{\sin^2\beta \, d\beta}{\sqrt{1-k^2\sin^2\beta}}$$

$$A_y = -\frac{\mu_0 I k}{2\pi}\sqrt{\frac{a}{x}}\left[\tilde{K}(k) + \frac{2}{k}\frac{\partial \tilde{E}}{\partial k}\right]$$

$$= -\frac{\mu_0 I k}{2\pi}\sqrt{\frac{a}{x}}\left[\tilde{K} + \frac{2}{k}\left(\frac{\tilde{E}}{k} - \frac{\tilde{K}}{k}\right)\right]$$

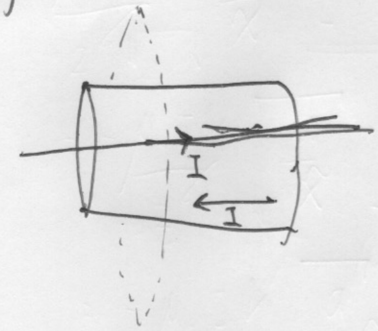
$$= -\frac{\mu_0 I k}{2\pi}\sqrt{\frac{a}{x}}\left[\tilde{K}\left(1 - \frac{2}{k^2}\right) + \frac{2\tilde{E}}{k^2}\right]$$

$$= -\frac{\mu_0 I}{\pi k}\sqrt{\frac{a}{x}}\left[\tilde{K}\left(\frac{k^2}{2} - 1\right) + \hat{E}\right]$$

$$= \frac{\mu_0 I}{\pi k}\sqrt{\frac{a}{x}}\left[\tilde{K}(k)\left(1 - \frac{k^2}{2}\right) - \tilde{E}(k)\right]$$

HW 3.

(a)



outside

$$B\,2\pi r = 0 = \mu_0 (I - I)$$

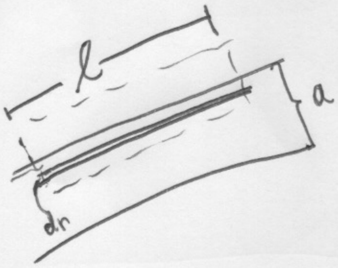
$$B = 0$$

inside

$$B\,2\pi r = \mu_0 I_0 \cos\omega t$$

$$\vec{B} = \frac{\mu_0 I_0}{2\pi r} \cos\omega t \,\hat{\phi}$$

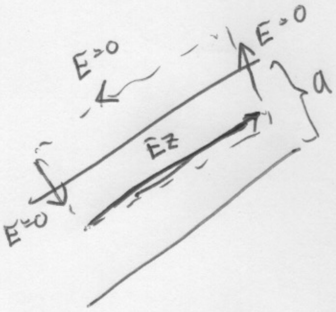
(b) $\Phi_B = \int \vec{B} \cdot d\vec{A}$



$$= \int_r^a \frac{\mu_0 I_0}{2\pi r} \cos\omega t \; l \; dr \quad \Big\} \begin{matrix}\text{outside} \\ B = 0\end{matrix}$$

$$= \frac{\mu_0 I_0}{2\pi} \cos\omega t \; l \ln\left(\frac{a}{r}\right)$$

(c) $\oint \vec{E} \cdot dl = -\frac{\partial \Phi_B}{\partial t}$



$$E_z \cancel{l} = +\frac{\mu_0 I_0}{2\pi} (\omega)(\sin\omega t) \cancel{l} \ln\frac{a}{r}$$

$$E = \frac{\mu_0 I_0 \omega}{2\pi} \sin\omega t \ln\frac{a}{r}\,\hat{z}$$

(d) displacment current density

$$\vec{J}_D = \varepsilon_0 \cancel{\mu_0} \frac{\partial \vec{E}}{\partial t}$$

$$= \frac{\mu_0 \varepsilon_0 I_0}{2\pi} \omega^2 \cos\omega t \ln\frac{a}{r}\,\hat{z}$$

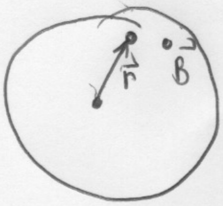
(e) $I_D = \int \vec{J}_D \cdot d\vec{A} = \frac{\mu_0 \varepsilon_0 I_0 \omega^2}{2\pi} \cos\omega t \int_0^a (2\pi)\, r \ln\frac{a}{r}\, dr$

HW 4.

(4) (a)

$$\vec{F}_{mag} = q(\vec{V}\times\vec{B})$$

$$\vec{\tau} = \vec{r}\times\vec{F}_{mag} = q\left[\vec{r}\times(\vec{V}\times\vec{B})\right]$$



$$= q\left[\vec{V}\underbrace{(\vec{r}\cdot\vec{B})}_{0} - \vec{B}(\vec{r}\cdot\vec{V})\right]$$

field into the page

$$\vec{\tau} = -q\vec{B}(\vec{r}\cdot\vec{V})$$

(b) $$\vec{\tau} = \frac{d\vec{L}}{dt} = -q\vec{B}\left(\vec{r}\cdot\frac{d\vec{\ell}}{dt}\right)$$

put $d\vec{\ell} = dr\,\hat{r} + d\phi\,\hat{\phi}$

$$\frac{d\vec{L}}{dt} = \vec{\tau} = -q\vec{B}\left(r\frac{dr}{dt}\right)$$

$$\frac{dL}{dt} = -qBr\frac{dr}{dt} \Rightarrow L - L_0 = -\int_0^R qBr\frac{dr}{\cancel{dt}}\cancel{dt}$$

$$L - L_0 = \frac{-q\Phi_B}{2\pi} \quad \text{where } \Phi_B = 2\pi\int_0^R Br\,dr$$

(c) when $\Phi_B = 0$, then $L = L_0$ (i.e. conserve angular momentum)

If the particle comes out from the center,

$$\vec{L}_0 = 0 \ (\text{since } \vec{r} = 0)$$

then it must exist with $L = 0$ and only possibility is when it travels radially ($\cancel{\vec{L} = \vec{r}\times\vec{p}} = 0$)

~~must be 0~~

$$(\vec{L} = \vec{r}\times\underbrace{\vec{p}}_{m\vec{V}} = 0)$$

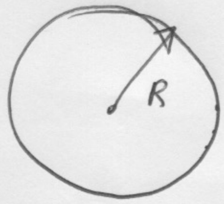
$\vec{V}$ must // to $\vec{r}$

(d)

again $\vec{L}_0 = 0$ since it starts from center

$$\vec{L} = \frac{q\Phi}{2\pi}$$

$$L = m\omega \cancel{R^2} = \frac{qB\cancel{\pi}\cancel{R^2}}{2\cancel{\pi}}$$



$$\omega = \frac{qB}{2m}$$