Introduction to Magnetism

E&M

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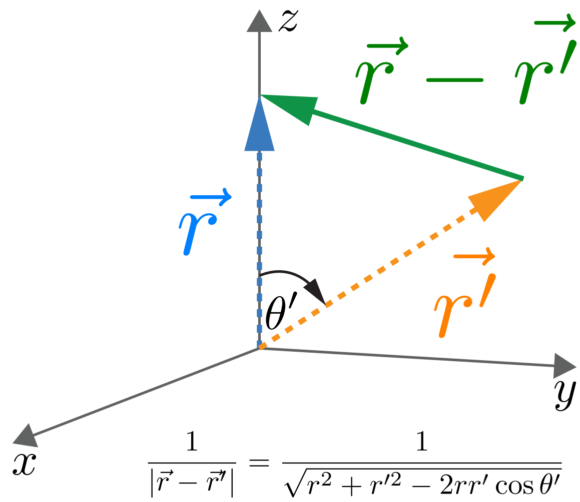

Figure 1.1

1 Multipole Expansion

If the Biot-Savart or vector potential method is too difficult, then it might be a good idea to approximate the problem. One way is to expand the vector potential out with the help of the expansion of the $\mathscr{r}$ term:

$$\frac{1}{|\vec{r} - \vec{r'}|} = \frac{1}{\mathscr{r}} = \frac{1}{\sqrt{(r^2 + r'^2 - 2rr'\cos\theta')}}.$$

where the second equality assumes that we align $\vec{r}$ along the z axis and we define the angle between them as θ' (Fig 1.1). If $r > r'$ then,

$$\frac{1}{\mathscr{r}} = \frac{1}{r\sqrt{1 + (\frac{r'}{r})^2 - \frac{r'}{r}\cos\theta'}} = \frac{1}{r}\sum_{n=0}^{\infty}(\frac{r'}{r})^n P_n(\cos\theta'),$$

This follow from the definition of Legender polynomial $P_n(x)$ where $\frac{1}{\sqrt{1-2xu+u^2}} = \sum_{n=0}^{\infty} P_n(x)u^n$. In our case, $u = \frac{r'}{r}$ and $x = \cos\theta'$. On the other hand, if $r' > r$, you have to switch from r to r'. This ends up with the expression

$$\frac{1}{\mathscr{r}} = \frac{1}{r\sqrt{1 + (\frac{r_<}{r_>})^2 - \frac{r_<}{r_>}\cos\theta'}} = \frac{1}{r}\sum_{n=0}^{\infty}(\frac{r_<}{r_>})^n P_n(\cos\theta'),$$

with $r_<$ is the smaller r between r and r'.
With the expansion, the vector potential becomes

$$\begin{aligned}\vec{A}(\vec{r}) =& \frac{\mu_0 I}{4\pi}\oint \frac{d\vec{l'}}{\mathscr{r}} \\ =& \frac{\mu_0 I}{4\pi}\oint d\vec{l'} + \frac{\mu_0 I}{4\pi}\oint \frac{r'\cos(\theta')d\vec{l'}}{r^2} + ... \\ =& 0 + \frac{\mu_0}{4\pi}\frac{\vec{m}\times\hat{r}}{r^2}\end{aligned} \tag{1.1}$$

The first term is zero since the integral goes over the current loop. The second term is called a dipole term. Here we define the magnetic dipole moment as current times area. The direction of the dipole is from the right-hand rule.

Definition 1.1 (Magnetic Dipole Moment)**.**

$$\vec{m} = I \int d\vec{a}$$

By taking the curl, we can calculate magnetic induction

Theorem 1.1 (Magnetic Dipole Field)**.**

$$\vec{B} = \frac{\mu_0}{4\pi r^3}[3(\vec{m} \cdot \hat{r})\hat{r} - \vec{m}]$$

The missing mathematical steps from this section can be found in homework.

Example 1.1. Find magnetic dipole moment of a rotating sphere.

Theorem 1.2 (Torque on Magnetic Dipole)**.**

$$\vec{N} = \vec{m} \times \vec{B}$$

Example 1.2. Drive the torque equation.

Theorem 1.3 (Force on Magnetic Dipoles)**.**

$$\vec{F} = \vec{\nabla}(\vec{m} \cdot \vec{B})$$

Theorem 1.4 (Potential Energy of Magnetic Dipole)**.**

$$U = -\vec{m} \cdot \vec{B}$$

2 Magnetization

Magnetization is defined as magnetic dipole moment per unit volumn.

Definition 2.1.

$$\vec{m} = \int \vec{M} d\tau'$$

3 Fields due to Magnetization

Example 3.1 (Vector Potential Due to Magnetization)**.** Show that magnetization is equivalent to current as the following

$$\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi} \int \frac{\vec{J}_b(\vec{r'})}{\mathscr{r}} d\tau' + \frac{\mu_0}{4\pi} \oint \frac{\vec{K}_b(\vec{r'})}{\mathscr{r}} da',$$

where $\vec{J}_b = \nabla \times \vec{M}$ and $\vec{K}_b = \vec{M} \times \hat{n}$.

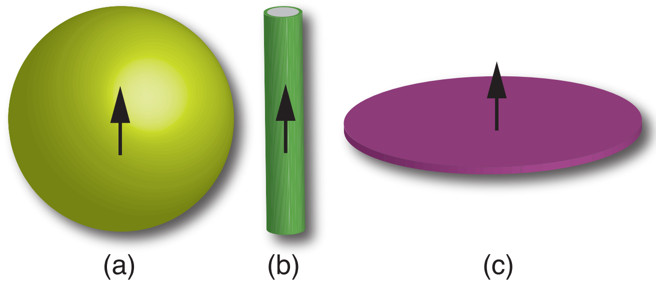

Figure 3.1

After calculating $\vec{A}$, we could take the curl to calulate $\vec{B} = \vec{\nabla} \times \vec{A}$. But if the symmetry permits, we could use the Amperian tricks.

Example 3.2. What is the magnetic induction of a uniformly magnetize sphere (Fig. 3.1a)?

Example 3.3. What is the magnetic induction of an infinitely long cylinder with a uniform magnetization $\vec{M}$ along the axis (Fig. 3.1b)?

Example 3.4. What is the magnetic induction of a very short cylinder but with infinitely radius? The magnetization $\vec{M}$ perpendicular to the plane. This is a good represention of a magnetic thin film with perpendicular magnetization as found in computer hard drives. (Fig. 3.1c)

4 Magnetic Field

- So far we haven't intruduced you to the magnetic field $\vec{H}$.
- From Ampere's Law, we have two sources of currents

$$\vec{\nabla} \times \vec{B} = \mu_0 J = \mu_0(\vec{J}_b + \vec{J}_f)$$

  or (with theorm 3.1)

$$\vec{\nabla} \times \vec{B} = \mu_0(\nabla \times \vec{M}) + \mu_0 \vec{J}_f.$$

  This allow us to write

$$\vec{\nabla} \times \vec{H} = \vec{J}_f, \tag{4.1}$$

  after introducing a new term $\vec{H}$.

- Notice that the source of $\vec{H}$ is the applied current (e.g. from a power supply). But the source of $\vec{B}$ are from both applied current and bound current (e.g due to magnetization).

Definition 4.1 (Magnetic Field $\vec{H}$)**.**

$$\vec{B} = \mu_0(\vec{H} + \vec{M})$$

- The equation always true in all materials.
- In vacuum, $\vec{B} = \mu_0 \vec{H}$. Therefore, it does not matter much to call $\vec{B}$ “magnetic field".

Example 4.1 (Magnetic Field due to Magnetization)**.** What are magnetic field inside the sphere (Ex.3.2), infinitely long cylinder (Ex. 3.3), and a thin film (Ex. 3.4)?

5 Magnetic Charge

- Magnetic monopole does not exist and we know it from $\vec{\nabla} \cdot \vec{B} = 0$. No point source of $\vec{B}$.
- But there is a source for $\vec{H}$! By taking the divergence of the definition

$$\vec{\nabla} \cdot \vec{H} = -\vec{\nabla} \cdot \vec{M} = \rho_m.$$

6 Maxwell's Equation inside Materials

$$\begin{aligned}
\nabla \cdot \vec{D} &= \rho_f & \qquad &(\qquad\qquad\qquad\qquad\qquad\qquad) \\
\nabla \times \vec{E} &= -\frac{\partial B}{\partial t} & \qquad &(\qquad\qquad\qquad\qquad\qquad\qquad) \\
\nabla \cdot \vec{B} &= 0 & \qquad &(\qquad\qquad\qquad\qquad\qquad\qquad) \\
\nabla \times \vec{H} &= \vec{J}_f + \frac{\partial \vec{D}}{\partial t} & \qquad &(\qquad\qquad\qquad\qquad\qquad\qquad)
\end{aligned}$$

7 Demagnetized Field and Stray Field

- The magnetic field inside a magnetized body tends to point opposite to the magnetization; therefore the field inside the magnetic material is called demagnetized field $\vec{H}_d$
- For ellipsoidal shapes (including sphere), the demagnetized field is uniform. This is one of the most important findings in magnetism since it simplifies countless of calculations.

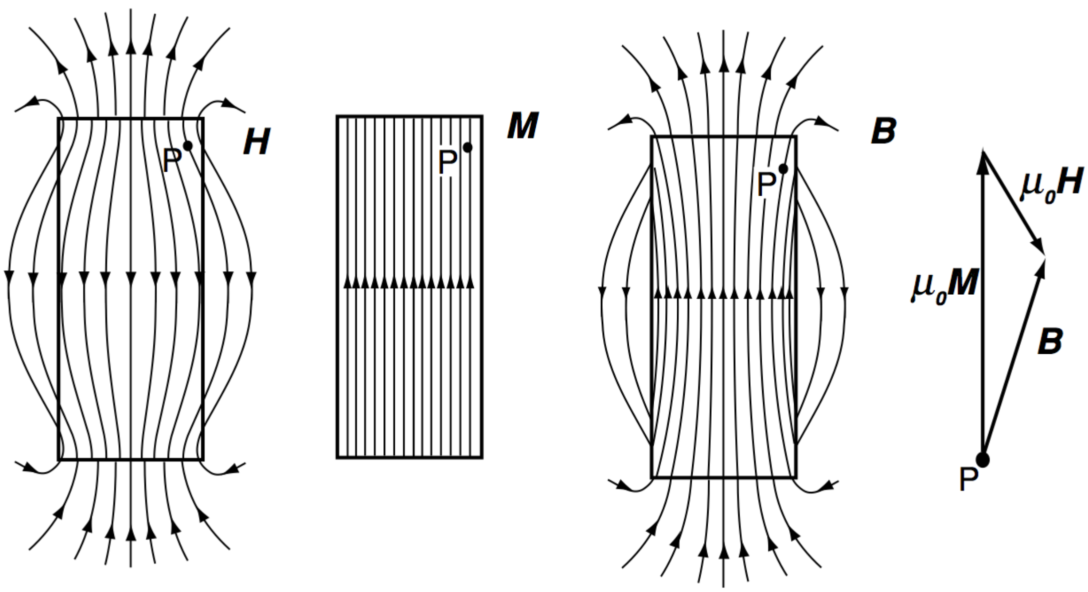

Figure 5.1: from J. M. D. Coey, Magnetism and Magnetic Materials

Reference J. A. Osborn, "Demagnetizing Factors of the General Ellipsoid" Phys. Rev. 67, 351 (1945)

- Therefore, we define a demagnetization factor ($\mathcal{N}$).

Definition 7.1 (Demagnetization Factor of Ellipsoids)**.**

$$\vec{H}_d = -\mathcal{N}\vec{M}$$

- $\mathcal{N}$ of a sphere is _____.
- $\mathcal{N}$ of a ring magnet is _____.
- $\mathcal{N}$ of a thin film with perpendicular magnetization is _____.
- Imagine that there is a bar magnet under the external magnetic field $\vec{H}'$. Total field inside the bar magnet is

$$\vec{H} = \vec{H}_d + \vec{H}' = -\mathcal{N}\vec{M} + \vec{H}'$$

.

- Similarly, the field outside the bar magnet is

$$\vec{H} = \vec{H}_s + \vec{H}'$$

$\vec{H}_s$ is called stray field generated by the magnetization.

8 Magnetic Susceptibility

Definition 8.1 (Magnetic Susceptibility χ)**.**

$$\vec{M} = \chi\vec{H}$$

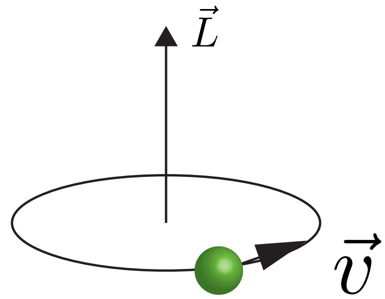

Figure 9.1

- For a very simple material, the appearance of magnetic field would create magnetization. When the field is gone, magnetization disappears.
- This simple magnetic susceptibility formula is only good for linear, isotropic and homogeneous materials
- This equation fails in non-linear materials such as a piece of iron that shows hysteresis loop.
- This equation fail in anisotropic materials since the relationship becomes a tensor.
- Susceptibility is related to permeability.

Definition 8.2 (Magnetic Permeability μ)**.**

$$\vec{B} = \mu \vec{H}$$

- This means that $\mu = \mu_0(1 + \chi)$ for linear, isotropic and homogeneous material.
- The last two topics might confuse you as the formula (definition 8.1) is similar to the definition of the demagnetization field (definition 7.1). The susceptibility tells about the origin of magnetization by applying $\vec{H}$ and only for linear, isotropic and homogeneous materials. But the definition of demagnetization field is valid for any case i.e. magnetizations produce fields opposite to itself inside their own body.

9 Origins of Magnetization

9.1 Magnetization and Angular Momentum

- Magnetization carries angular momentum.
- Einstein-Dehass Experiment
- **Angular Momentum due to Atomic Orbital:** When an electron circles in a loop of radius R (Fig. 9.1). The magnetic moment is

$$m = I\pi R^2 = -\frac{e}{T}\pi R^2 = -\frac{ev}{2\pi R}\pi R^2 = -\frac{emvR}{2m} = -\frac{eL}{2m} = \gamma L$$

  where $\gamma = -\frac{e}{2m}$ or a gyromagnetic ratio.

- There is also contribution from spin angular momentum so $\vec{L}$ becomes $\vec{J} = \vec{L} + \vec{S}$.

Theorem 9.1 (Magnetic Moment as Angular Momentum)**.**

$$\vec{m} = \gamma \vec{J}$$

- If an external field is applied along z direction, the z projection of angular momentum will be a good quantum number. $J_z = M_J \hbar$. So the magnetic moment should be in the unit of Bohr Magnetron $\mu_B = \gamma\hbar = \frac{-e\hbar}{2m}$.

Theorem 9.2 (Projection of Magnetic Moment along z Direction)**.**

$$m_z = \mu_B M_J$$

9.2 Paramagnetism

- Origin is due to quantum mechanics.
- Assume that each atoms process a built in magnetic moment (localized approximation).
- From quantum mechanics, an atom with angular momentum $\vec{J}$ carries magnetic moment of $\vec{m} = -\frac{g\mu_B \vec{J}}{\hbar}$. When the applied magnetic field is along the z axis ($\vec{H} = H\hat{z}$), the degeneracy is lifted of and the energy states are split into $2J+1$ levels where the energy of each state is governed by a quantum number M_J running from $-J, -J+1, -J+1, ..., J$.

  The energy of each level is $\varepsilon_i = U_i = -\vec{m}\cdot\vec{B} = g\mu_B\mu_0 H M_J$.

  Magnetic moment of each level is $(m_z)_i = -g\mu_B M_J$ (Fig 9.2).

- The averaged of magnetic moment along z direction is

$$\langle m_z \rangle = \frac{\sum\limits_{M_J=-J}^{J} m_i \exp\frac{-\varepsilon_i}{k_B T}}{\sum\limits_{M_J=-J}^{J} \exp\frac{-\varepsilon_i}{k_B T}}. \tag{9.1}$$

  The sum could be done analytically and the result is

Theorem 9.3 (Paramagnetic Magnetization)**.**

$$M = n\langle m_z\rangle = M_0 B_J(x),$$

where $B_J(x) = \frac{2J+1}{2J}\coth\frac{2J+1}{2J}x - \frac{1}{2J}\coth\frac{x}{2J}$ is a Brillouin function. $M_0 = nm_0 = gn\mu_B J$ is defined as the maximum possible magnetic moment level and x is the ratio between the maximum energy $g\mu_B\mu_0 H' J$ and the thermal energy $k_B T$ i.e. $x = \frac{g\mu_B\mu_0 H' J}{k_B T}$. n is number of atom per unit volume.

The detail is very interesting but I will leave it for your homework.

- Magnetization is magnetic moment per volume. So magnetization is simply $M = n\langle m_z\rangle$, where n is number of atoms per unit volume.
- Notice that $B_J(x)$ is linear when small x. This suggests that by taking the limit of small x and take the slope we could measure susceptibility χ (see definition 8.1).

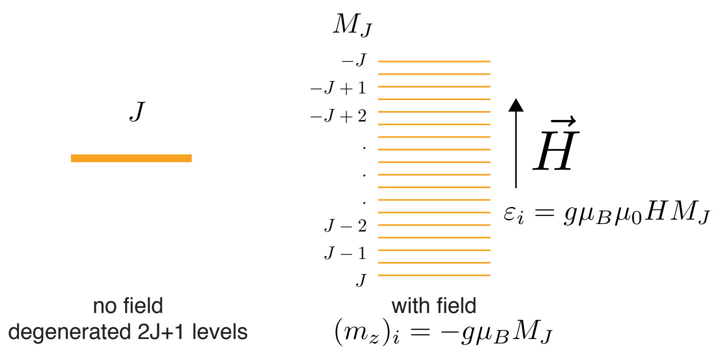

Figure 9.2

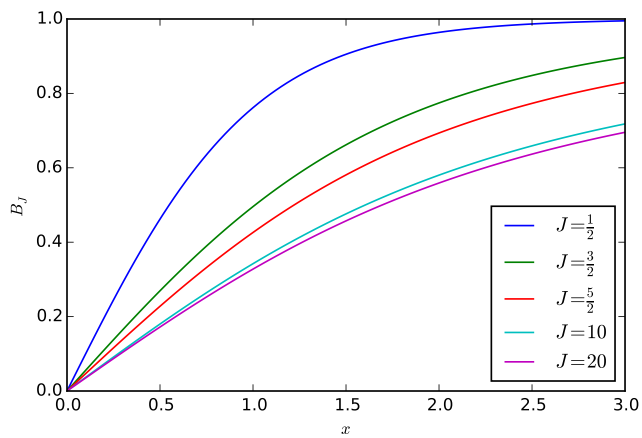

Figure 9.3

- To do this analytically, we expand $B_J(x)$ for small x:

$$B_J(x) \approx \frac{J+1}{3J}x$$

.

Theorem 9.4 (Paramagnetic Susceptibility)**.**

$$\chi = \frac{\mu_0 g^2 \mu_B^2 n J(J+1)}{3k_B T}$$

- The inverse temperature dependent of χ is called Curie law of $\chi = \frac{C}{T}$ where C is Curie constant and

$$C = \frac{\mu_0 g^2 \mu_B^2 n J(J+1)}{3k_B} \tag{9.2}$$

9.3 Ferromagnetism: Spontaneous Magnetization

- There is such a thing like spontaneous magnetization M_s (i.e. M when field is zero).

- M_s can be accounted by Weiss model or a molecular field theory where ferromagnetism is paramagnetism with a 'molecular field'. This field occurs hen the material is magnetized and produced a field of $n_w\vec{M}$.
- This simple model is very powerful when we consider when calculating spontaneous magnetization M_s as a function of temperature.
- Similar to the case of paramagnets, we calculate the magnetization from $M = M_0 B_J(x)$. The x now becomes $x = \frac{\mu_0 g \mu_B J(H' + n_w M)}{k_B T}$.
- Notice that M appears on both sides of the equation so in general we can only use numerical approaches.
- If we want to calculate spontaneous magnetization $M = M_s = M_0 B_J(x_0)$ where $x = x_0 = \frac{\mu_0 g \mu_B J n_w M}{k_B T}$.
- The numerical solving can be less scary with the help of two observations

  (1) We can related x_0 to the Curie constant (C) from Eqn. 9.2. $M_s = \frac{M_0 x_0 (J+1) T}{3 J n_w C}$

  (2) From dimension analysis $n_w C$ has a unit of temperature, so it is convenient to display the temperature is a reduced unit T', where $T' = \frac{T}{T_C}$ with $T_C = n_w C$

  With (1) and (2), we need to find x_0 where $\frac{x_0(J+1)T'}{3J} = B_J(x_0)$. And then take B_J for the spontaneous magnetization for a given T'.

Example 9.1. Derive Curie-Weiss Law.

$$\chi = \frac{C}{T - T_c}.$$

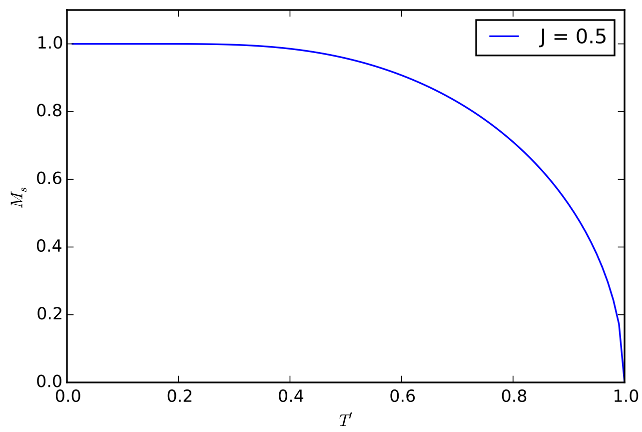

Figure 9.4

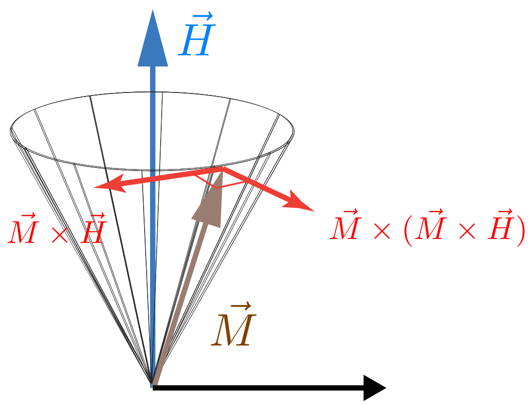

Figure 10.1

10 Magnetization Dynamics and Landau-Lifshitz-Gilbert Equation

When magnetic moment from an atom is interacted with magnetic field the equation of motion is

$$\vec{N} = \frac{d\vec{J}}{dt} = \vec{m} \times \vec{B}.$$

See Theorem 1.2. But next we know that the angular momentum $\vec{J}$ is related to the magnetic moment (Theorm 9.1); therefore

$$\frac{d\vec{M}}{dt} = \gamma \vec{M} \times \vec{B} \tag{10.1}$$

Here we also make use of 'macro-spin approximation' by assume that magnetic moment is uniformly distributed through out the volume τ ($\vec{M} = \frac{\vec{m}}{\tau}$). This equation is called Larmor Formula and γ is related to Larmor frequency. The result is that magnetization would precess around the field $\vec{B}$ (Fig 10.1 and Numerical Homework 2).

But this is not what we expect in reality. When you apply field, $\vec{M}$ would eventually align in parallel to $\vec{H}$. There is a need of a component of torque that pull $\vec{M}$ to the direction of $\vec{H}$. This missing torque is parallel to $\vec{M} \times (\vec{M} \times \vec{B})$

$$\frac{d\vec{M}}{dt} = \gamma \vec{M} \times \vec{B} + \lambda(\vec{M} \times (\vec{M} \times \vec{H})) \tag{10.2}$$

Note that γ and λ are negative numbers. The right hand rule of precession would apply and the $\vec{M} \times (\vec{M} \times \vec{B})$ will pull magnetic moment to become parallel to the field (Fig 10.1 and Numerical Homework 2).

This formulation is the same as famous Landau-Lifshitz-Gilbert equation. However the field B is replcted by an effective field H_{eff} which is tricky to calculate. It is the combination of the applied field $\vec{H}'$, demagnetizing field $\vec{H}_d$, anisotropy and other quantum mechanical effects.

11 Homeworks

Homework 1 (Magnetic Dipole Potential)**. (10 points)** Derive the missing steps for the field due to magnetic moment especially the dipole field. First show that

$$I \oint r' \cos\theta' d\vec{l'} = \vec{m} \times \vec{r}$$

(this step is missing from the second to third line of Eq. 1.1) by following these steps.
(a) (4 points) Derive the following vector identity.

$$\int \vec{\nabla} T \times d\vec{a} = -\oint T d\vec{l},$$

where $d\vec{a}$ is an area element and $d\vec{a}$ is a line element

Hints:

1. Use Stroke Theorem: $\int \vec{\nabla} \times \vec{v} \cdot d\vec{a} = \oint \vec{v} \cdot d\vec{l}$. And set $\vec{v} = T\vec{c}$ where $\vec{c}$ is a constant vector.
2. Use the vector identity to expand $\vec{\nabla} \times (T\vec{c})$.
3. Use another identity involving $\vec{A} \cdot (\vec{B} \times \vec{C})$

(b) (4 points) Derive another vector identity

$$\oint (\vec{c} \cdot \vec{r}) d\vec{l} = \vec{a} \times \vec{c},$$

where $\vec{c}$ is a constant by setting $T = \vec{c} \cdot \vec{r}$ in the identity from (a). $\vec{a}$ is a the area vector where $|\vec{a}|$ is equal to total area and the direction perpendicular to the area.

Hints:

1. Expand $\vec{\nabla}(\vec{c} \cdot \vec{r})$. Only one term is non zero.
2. Use definition of area vector $\int d\vec{a} = \vec{a}$

(c) (1 points) Since the cosine term is related to the dot product, show that

$$I \oint r' \cos\theta' d\vec{l'} = I \oint (\hat{r} \cdot \vec{r'}) d\vec{l'}$$

.
(d) (1 points) And since $\hat{r}$ is a constant vector in our integration over the source (prime coordinate), set $\vec{c} = I\hat{r}$ in (b) and write the final expression in terms of magnetic moment $\vec{m}$.

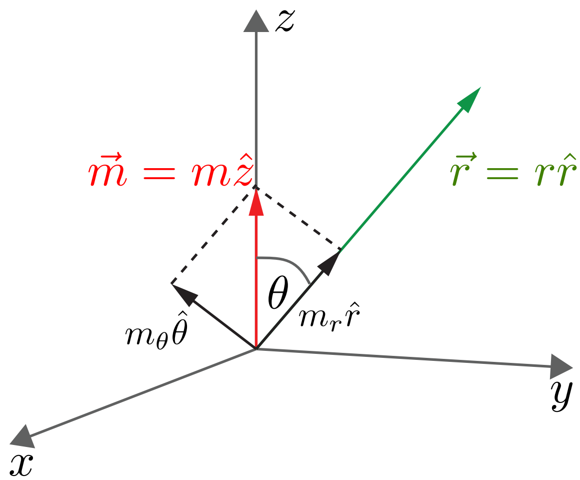

Figure 11.1

Homework 2 (Magnetic Dipole Field)**. (10 points)**
(a) (3 points) By setting the magnetic dipole $\vec{m} = m\hat{z}$ along the z direction (Fig. 11.1), show from Eq.(1.1) that in spherical coordinate

$$\vec{A} = \frac{\mu_0}{4\pi}\frac{m\sin\theta}{r^2}\hat{\phi}$$

(b) (3 points) And from $\vec{B} = \vec{\nabla} \times \vec{A}$, derive the magnetic induction

$$\vec{B} = \frac{\mu_0 m}{4\pi r^3}(2\cos\theta\hat{r} + \sin\theta\hat{\theta})$$

(c)(4 points) Show that $\vec{B}$ can be written in a coordinate free form as in Eq. (1.4)

$$\vec{B} = \frac{\mu_0}{4\pi r^3}[3(\vec{m}\cdot\hat{r})\hat{r} - \vec{m}]$$

Hint Use Fig. 11.1 to find the component along $\vec{r}$ and $\vec{\theta}$ of $\vec{m}$

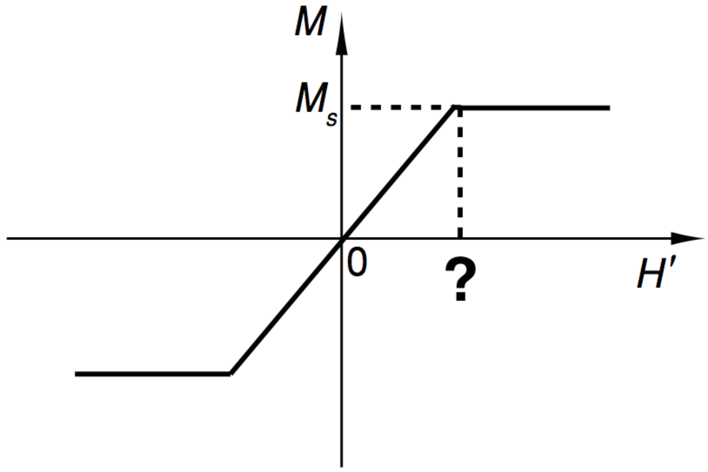

Figure 11.2

Homework 3. (10 points) From the definition of χ (definition 8.1), the magnetic field $\vec{H}$ is the total field inside materials, which is the summation of the applied field $\vec{H}'$ and demagnetized field $\vec{H}_d{}'$. It would be more intuitive to redefine the susceptibility as the function of applied field $\vec{H}'$ instead or

$$\vec{M} = \chi' \vec{H}'.$$

This new parameter χ' is called 'external susceptibility'.
(a) (2 points) Show that the external susceptibility can be written in terms of the internal susceptibility as $\chi' = \frac{\chi}{1+\chi\mathcal{N}}$.

(b) (1 points) In case of a soft ferromagnetic ferromagnetic sphere with high permeability and no hysteresis, we could approximate that χ is very large. This is because only a small amount of magnetic field would induces a very large magnetization. Find χ' of this ferromagnetic sphere.

(c) (3 points)This definition of magnetic susceptibility assumes a linearity in materials. which is valid only in a small range of magnetic field. We could not produce any large magnetization as we like as $\vec{M}$ would saturate at some point called spontaneous magnetization, $\vec{M}_s$. The plot of magnetization and the applied field is then shown in Fig. 11.2. What is the required field to saturate magnetization.

(d)(2 points) Show that the total field inside the sphere $\vec{H}$ is zero during the magnetizing region.

(e)(2 points) Plot $\vec{M}$ vs $\vec{H}$

Homework 4. (10 points) There is a very smart trick to find such a complex summation for the derivation of Brillouin function (Eqn. 9.1).
(a) (3 points) Show that

$$\langle m_z \rangle = g\mu_B \frac{\partial}{\partial y}[\ln \sum_{M_J=-J}^{J} \exp(-M_J y)],$$

where $y = \frac{\mu_0 g \mu_B H}{k_B T}$. Do this by doing the differentiation on the right hand side and compare the result with Eqn. 9.1
(b) (3 points) Recall your high-school math. Do the sum and show that

$$\sum_{M_J=-J}^{J} \exp(-M_J y) = \frac{\sinh\left(\frac{(2J+1)y}{2}\right)}{\sinh(y/2)}$$

(c) (4 points) Finish off your work by showing that

$$\langle m_z \rangle = g\mu_B J\{\frac{2J+1}{2J}\coth\frac{2J+1}{2J}x - \frac{1}{2J}\coth\frac{x}{2J}\}$$

with $x = Jy = \frac{\mu_0 g \mu_B J H}{k_B T}$.

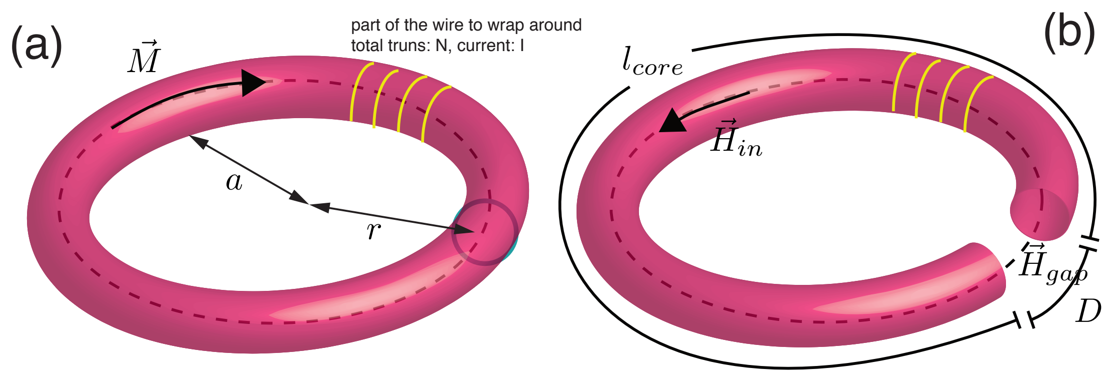

Figure 11.3

Homework 5 (Electromagnet)**. (15 points)** We finish our discussion with an application of this chapter – an electromagnet. This is in contrast the first numerical homework where we have considered the Helmholtz coil. Helmholtz coil is an 'air-core' magnet, but now we are interested in a magnetic-core magnet (Fig 11.3b).
Here we start with a circular ring of uniformly magnetized object along the $\hat{\phi}$ direction (with permeability of μ). In addition, there should be an insulated write to wrap around the ring magnet (Fig 11.3a) with current I and N turning of wire (not drawn completely. I'm lazy ☹).

(a) **(3 points)** Analyze the bound current due to the ring magnetization (Fig 11.3a). What is the equivalent current distribution? Why?

1. Solenoid
2. Infinite Plane
3. Toroid
4. Cylindrical Current

(b) **(2 points)** Show that the magnetic field outside the magnet is zero (i.e. the stray field is zero). So, this ring magnet configuration as in Fig 11.3a is pretty much useless.

(c) **(3 points)** Given that the inner radius of the ring magnetic is a, show that the flux density inside the magnet at the position r is $\vec{B}_{in} = \frac{\mu_0 NI}{2\pi r} + \frac{\mu_0 Ma}{r}\hat{\phi}$. And when the cross-section of the ring is small, $a \approx r$. Show that the magnetic field inside the ring is $\vec{H}_{in} = \frac{NI}{2\pi r}\hat{\phi}$.

Notice that there is no contribution from the magnetization of the object. The field is only due to the free current.

(d) **(7 points)** But what if we remove a small section of material (length D) out from the ring (Fig 11.3b). Show that the field between the gap is approximately $\vec{H} = \frac{NI}{\frac{l_{core}}{\mu}+D}\hat{\phi}$. Assume that our magnetic material is linear. l_{core} is the total length of the magnet. $\vec{H}$ is the amount of the stray field that useful for magnetizing your samples.

Hint

1. Use the integral form of Ampere's Law $\vec{H}$ (Eq. 4.1) and divide the integral into two sections: the loop within the ring with field H_{in} of the length l_{core} and at the small section H_{gap} of the length D.

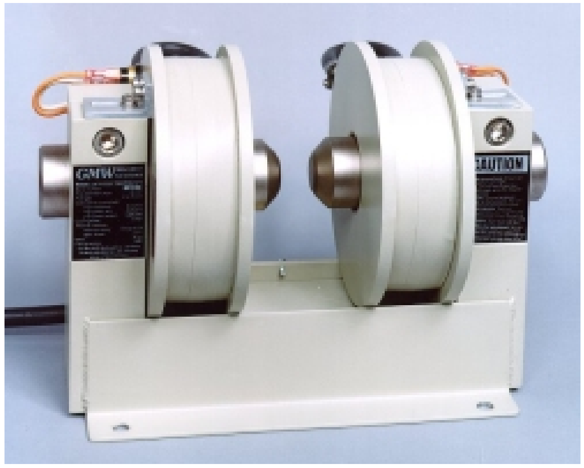

Figure 11.4: from GMW

2. $B = \mu H$.
3. $\nabla \cdot \vec{B} = 0$ implies that the magnetic flux going in and out into the yoke must be the same. Write down the relationship between H_{in} and H_{gap}.

Although our result is pretty simple. But it suggests four very important design criteria for electromagnets.

- First, l_{core} must be small to have large field. The path of the magnetic flux should be as short as possible.
- Second, we expect material with high susceptibility to give larger field.
- Third, the gap D should be as small as possible. But this must be compromised by your working space (e.g. the sample sizes).
- Fourth, the magnetic field is not uniform. It points in the $\hat{\phi}$ direction. So, if you need to have a more uniform field the design of the core should be such that the poles are facing one another (Fig. 11.4).