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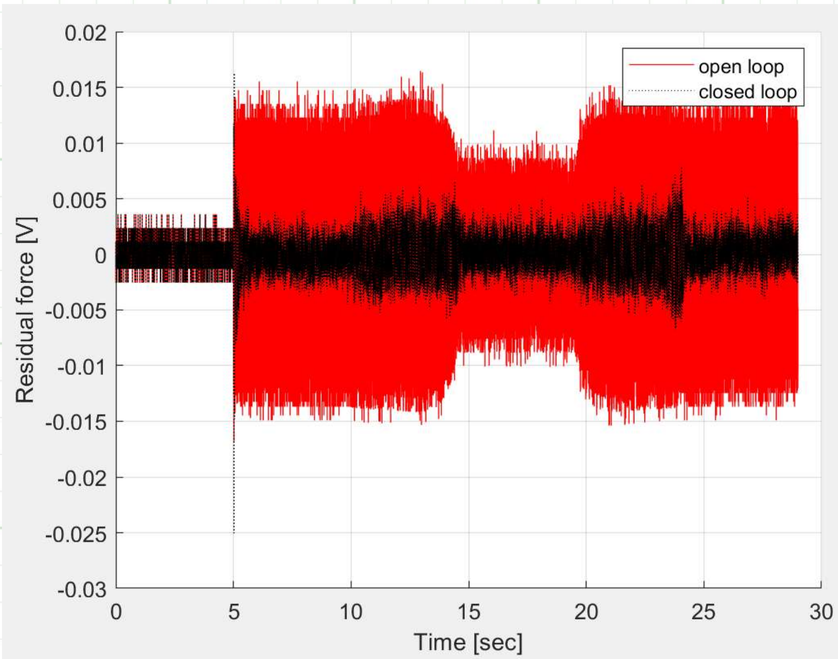
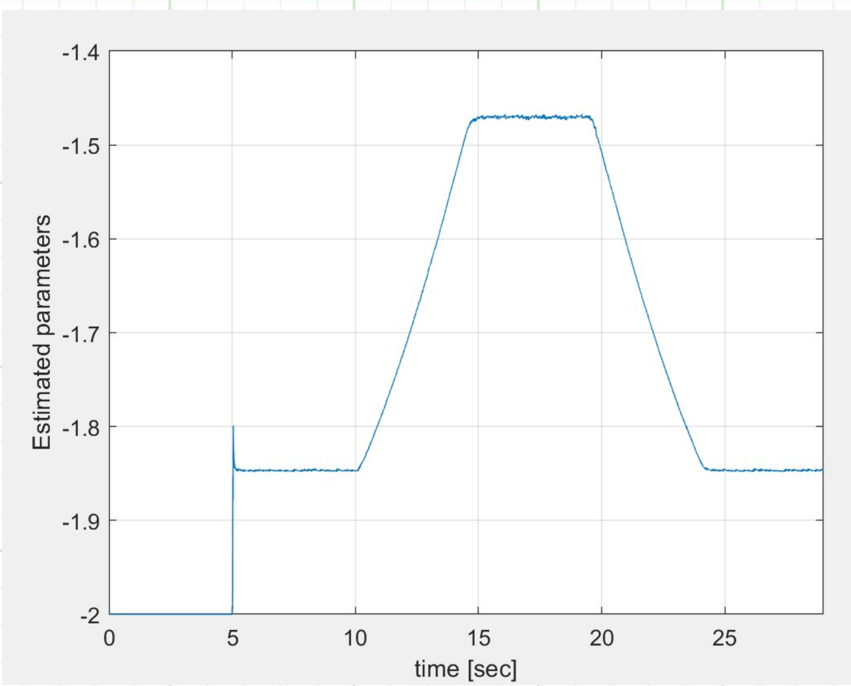
(a) The default code will run adaptive control against disturbances with a sudden change of frequency. Configure the code and test the performance of disturbance rejection against a chirp disturbance, namely, a disturbance that linearly changes its frequency between different set points. Plot the disturbance rejection result and convergence of the parameter.





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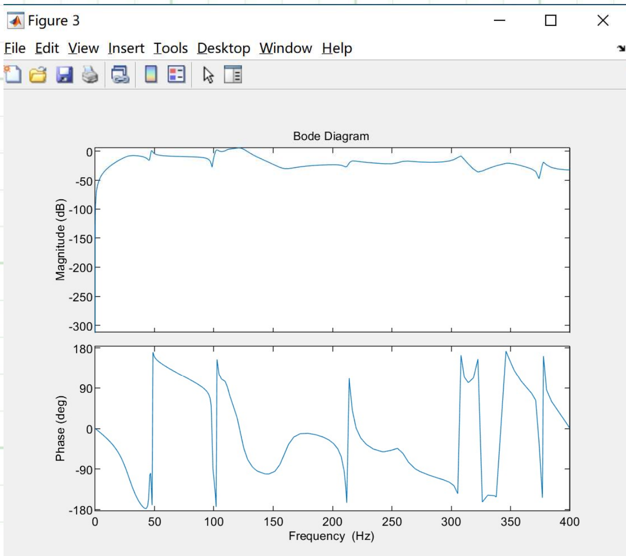
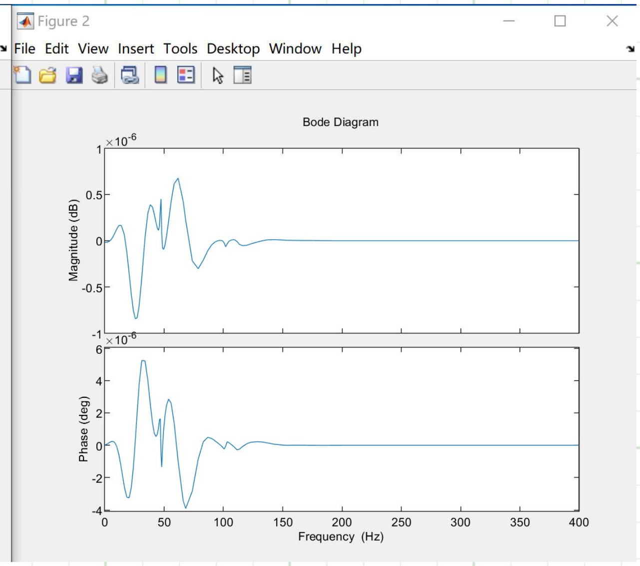
(b) Plot the bode plots of the transfer function from the control input to the plant ouput y, and the transfer function of the baseline controller.





(c) Identify the sampling frequency T_s from the codebase.

$F_s = 800\,Hz \quad T_s = 1/F_s \quad = 1.25\times10^{-3}\ s$

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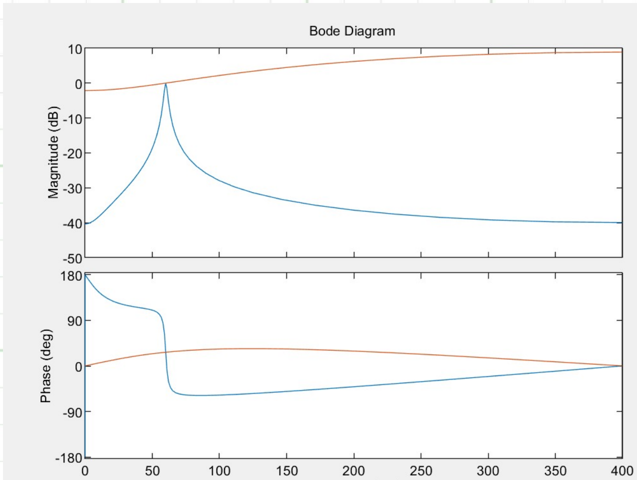
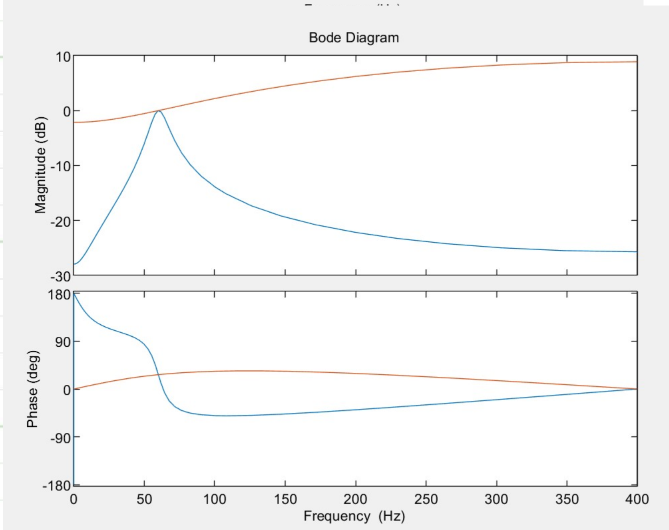
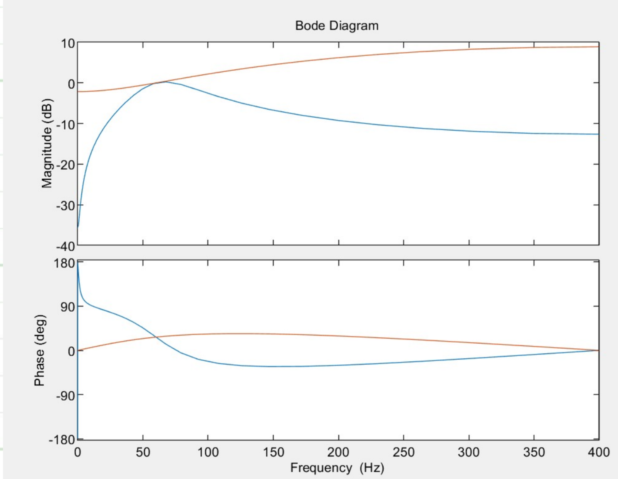
(d) When the plant has a one-step delay, the transfer function of the Q filter follows Eq. (7) of Chen 2015. In this Eq. (7), let $\alpha = 0.99, 0.95, 0.8$ respectively, and let the disturbance frequency be $\omega_0 = 2\pi \times 60 T_s$ rad/s (60 Hz).

i. Plot the bode plots of these different Q filter designs. Compare the results with the frequency response of the FIR Q solution $Q(z^{-1}) = 2\cos\omega_0 - z^{-1}$. Provide your insights.



When alpha value is large, it provide a very narrow band pass filter. When alpha is 0.8, the band pass is wider.
On the other hand the FIR will amplify high frequency noise





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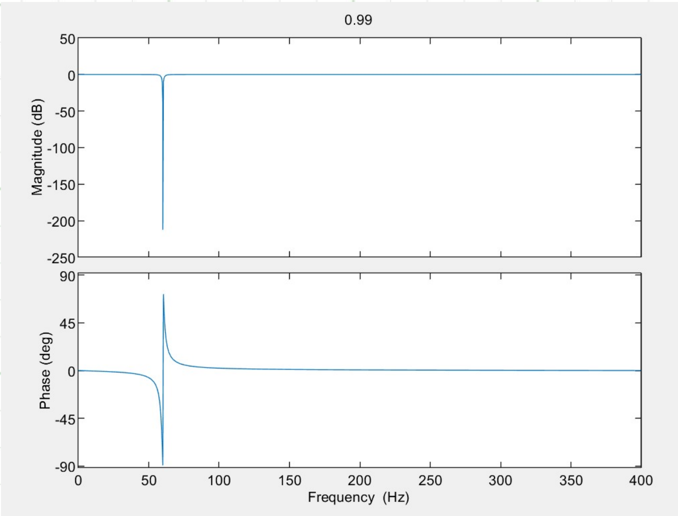
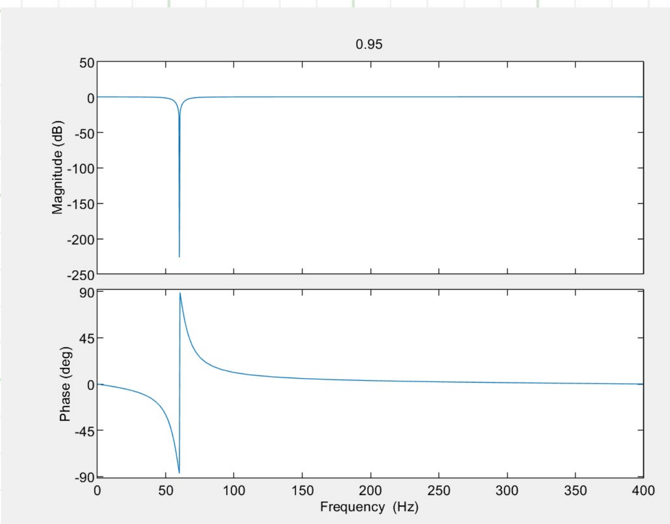
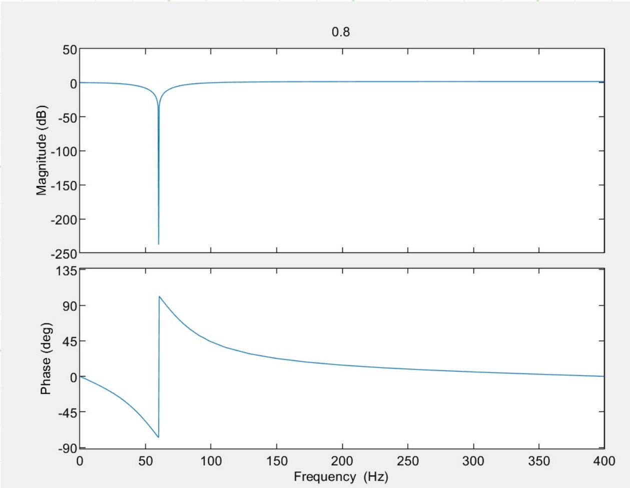
ii. Plot the frequency responses of $1 - z^{-1}Q\left(z^{-1}\right)$ and observe the magnitude at the disturbance frequency. Explain why such a filter will help reject disturbances at a particular frequency.



When alpha is at 0.99, it reject a narrow band of noise.
When alpha is at 0.8, it reject a wider band of noise with stronger magnitude.





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i. Verify that the Q design in Eq. (7) satisfies

$$1 - z^{-1}Q\left(z^{-1}\right) = \frac{A\left(z^{-1}\right)}{A\left(\alpha z^{-1}\right)}$$

for the case of $n = 1$ in Eq. (9).

the damped IIR design provides a bandpass filter characteristics

FIR: high pass, amplifying the noise

$$Q(z^{-1}) = \frac{-2(\alpha-1)\cos w_0 + (\alpha^2-1)z^{-1}}{1-2\alpha\cos w_0 z^{-1} + \alpha^2 z^{-2}} \quad (7)$$

$$-z^{-1}Q(z^{-1}) = \frac{-z^{-1}\cdot(-2)(\alpha-1)\cos w_0 - (\alpha^2-1)z^{-2}}{1-2\alpha\cos w_0 z^{-1} + \alpha^2 z^{-2}}$$

$$1-z^{-1}Q(z^{-1}) = 1 - \frac{-z^{-1}\cdot(-2)(\alpha-1)\cos w_0 - (\alpha^2-1)z^{-2}}{1-2\alpha\cos w_0 z^{-1} + \alpha^2 z^{-2}}$$

$$= \frac{1-2\alpha\cos w_0 z^{-1} + \alpha^2 z^{-2}}{1-2\alpha\cos w_0 z^{-1} + \alpha^2 z^{-2}} - \frac{-z^{-1}\cdot(-2)(\alpha-1)\cos w_0 - (\alpha^2-1)z^{-2}}{1-2\alpha\cos w_0 z^{-1} + \alpha^2 z^{-2}}$$

$$= \frac{1-2\cos w_0 z^{-1} + z^{-2}}{-2\alpha\cos w_0 z^{-1} + \alpha^2 z^{-2}} = \frac{A(z^{-1})}{A(\alpha z^{-1})}$$

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ii. Chen 2013 and Chen 2015 provide a RLS PAA and a parallel PAA with a fixed compensator. In Chen 2015, the RLS PAA is defined from Eq. (15) to Eq. (24). The parallel PAA is provided from Eq. (25) to Eq. (36). Complete the following:

A. For $n = 1$ and 2, write down the specific forms of θ and the regressor vectors for the RLS PAA and the parallel PAA with a fixed compensator. Your results should be explicit, e.g.: $\psi_1(k-1) = ...$, $\psi_2(k-1) = ...$

B. In the deterministic case when a set of disturbance frequencies ω_i's are given, write down the formulas to calculate θ. Hint: check Eq. (37).

for n=1 RLS: $\theta = [a_1]^T$

$$\psi(k-1) = [\psi_1(k-1)]^T$$

$$\psi_1(k-1) = w(k-1) - \alpha v(k-1)$$

parallel: $\theta_c = [c_1]^T$ is a estimate of θ

$$\psi_1(k-1) = \alpha e(k-1)$$

n=2

RLS: $\theta = [a_1, a_2]^T$

$$\psi(k-1) = [\psi_1(k-1), \psi_2(k-1)]^T$$

$$\psi_1(k-1) = w(k-1) - \alpha v(k-1)$$

$$\psi_2(k-1) = w(k-2) - \alpha^2 v(k-2)$$

Parallel $\theta_c = [c_1, c_2]^T$

$$\psi(k-1) = [\psi_1(k-1), \psi_2(k-1)]^T$$

$\psi_1(k-1) = \alpha^1 e(k-1)$, $\psi_2(k-1) = \alpha^2 e(k-2)$

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$$(1-e^{jw_1}z^{-1})(1-e^{-jw_1}z^{-1})\cdot(1-e^{jw_2}z^{-1})(1-e^{-jw_1}z^{-1})$$

$$\underbrace{\left[1-e^{-jw_1}z^{-1}-e^{jw_1}z^{-1}+z^{-2}\right]}_{1-(e^{-jw_1}+e^{jw_1})z^{-1}+z^{-2}}\cdot\left(1-e^{-jw_2}z^{-1}-e^{jw_2}z^{-1}+z^{-2}\right)$$

$$1-2\cos(w)z^{-1}+z^{-2}=1+\underset{\substack{\downarrow\\ -2\cos(w)}}{a_1}z^{-1}+z^{-2}$$

$$\underbrace{(1-2\cos(w_1)z^{-1}+z^{-2})\cdot(1-2\cos(w_2)z^{-1}+z^{-2})}=1+a_1z^{-1}+a_2z^{-2}+a_1z^{-3}+z^{-4}$$

$$=1-2\cos(w_2)z^{-1}+z^{-2}-2\cos(w_1)z^{-1}+4\cos(w_1)\cdot\cos(w_2)z^{-2}-2\cos(w_1)z^{-3}$$
$$+z^{-2}-2\cos(w_2)z^{-3}+z^{-4}$$

$$=1+\underbrace{(-2\cos(w_2)-2\cos(w_1))}_{a_1}z^{-1}+((-2\cos(w_1))\cdot(-2\cos(w_2))+2)z^{-2}$$
$$\underbrace{(-2\cos(w_1)-2\cos(w_2))}_{a_1}z^{-3}$$
$$+z^{-4}$$

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(f) The PAA is implemented in a Matlab Embedded function. Open the simulink file "simulator_1bd_submit.mdl". Double click the magenta disturbance observer block. Then double click the orange PAA block titled "dist. ID". Observe the different inputs and outputs of the Embedded MATLAB Function block titled "PAA" in the center of the block. Explain which lines are implementing

i. the $\hat{\theta}(k)$ update equation for the RLS PAA

```
% parameter estimation update
delta_theta = (F*phi*v/(1+phi_F_phi))/theta;
theta = theta * (1+delta_theta);
```

$$\hat{\theta}(k) = \hat{\theta}(k-1) + \frac{F(k-1)\,\psi(k-1)\,e^{o}(k)}{1+\psi(k-1)^{T}F(k-1)\,\psi(k-1)} \tag{22}$$

ii. the adaptation gain update equation for the RLS PAA

```
F = 1/lambda2 * ( F - F*(phi*phi')*F / (lambda2+ph
```

$$F(k) = \frac{1}{\lambda(k)}\left[F(k-1) - \frac{F(k-1)\,\psi(k-1)\,\psi(k-1)^{T}F(k-1)}{\lambda(k)+\psi(k-1)^{T}F(k-1)\,\psi(k-1)}\right]. \tag{24}$$

iii. the update of the regressor vector (ψ or ϕ) for the RLS PAA

```
phi = -x(1)+alpha2*e_post(1);
```

$$\psi_n(k-1) = w(k-n) - \alpha^{n} v(k-n).$$

Do the same for the parallel PAA with a fixed compensator. Do the codes match with your analysis in 2e?

i. the $\hat{\theta}(k)$ update equation for the RLS PAA

```
delta_theta = (F*phi*e/(lambda+phi_F_phi));
% parameter estimation update
theta = theta + delta_theta;
```

ii. the adaptation gain update equation for the RLS PAA

```
% adaptation gain update
F = 1/lambda * ( F - F*(phi*phi')*F /
(lambda+phi_F_phi) );
```

iii. the update of the regressor vector (ψ or ϕ) for the RLS PAA

```
phi_F_phi = phi'*F*phi;
```