(5) a) $\sum_{x=0}^{N-1} \exp(-2\pi i k x/N) \equiv S$

$\alpha \equiv \exp\left(-\frac{2\pi i k}{N}\right)$

$S = \sum_{x=0}^{N-1} \alpha^x$

$\alpha S = \sum_{x=0}^{N-1} \alpha^{x+1}$

$\alpha S - S = \alpha^N - 1$

$(\alpha - 1) S = \alpha^N - 1$

$S = \frac{\alpha^N - 1}{\alpha - 1} = \frac{1 - \alpha^N}{1 - \alpha}$

$\alpha^N = \exp(-2\pi i k)$

$\sum_{x=0}^{N-1} \exp(-2\pi i k x/N) = \frac{1 - \exp(-2\pi i k)}{1 - \exp(-2\pi i k/N)}$

(5) b) for $k \to 0$ $\exp(\beta k) \approx 1 + \beta k$

$\lim_{k \to 0} S \longrightarrow \frac{1 - 1 + 2\pi i k}{1 - 1 + 2\pi i k/N} = \frac{1}{\frac{1}{N}} = N$

if $k \% N \neq 0$:

[illegible]

$\exp(-2\pi i k) = 1$

$\exp(-2\pi i k/N) \neq 1 \equiv \gamma$

$\frac{1-1}{1-\gamma} = \frac{0}{\underset{\neq 0}{1-\gamma}} = 0$

5 c)

$$\sum_{x=0}^{N-1} \exp(-2\pi i k x/N) = \frac{1-\exp(-2\pi i k)}{1-\exp(-2\pi i k/N)}$$

$$F(k) = \sum_x \sin\left(\frac{2\pi k' x}{N}\right) e^{-\frac{2\pi i k x}{N}}$$

$$\sin\left(\frac{2\pi k' x}{N}\right) = \frac{1}{2i}\left(e^{\frac{2\pi i k' x}{N}} - e^{\frac{-2\pi i k' x}{N}}\right)$$

$$F(k) = \sum_x \frac{1}{2i}\left(e^{\frac{-2\pi i(k-k')x}{N}} - e^{\frac{-2\pi i(k+k')x}{N}}\right) =$$

$$= \frac{1}{2i}\left[\frac{1-e^{-2\pi i(k-k')}}{1-e^{-2\pi i(k-k')/N}} - \frac{1-e^{-2\pi i(k+k')}}{1-e^{-2\pi i(k+k')/N}}\right]$$