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A new formulation and solution for the nurse scheduling problem: A case study in Egypt

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Abstract Nurse Scheduling Problem (NSP) is the assignment of a number of nurses to a number of shifts in order to satisfy hospital's demand. The objectives of NSP are the minimization of the overall hospital cost, and the maximization of nurses' preferences while taking into consideration the governmental rules and hospital standards. In this article, a proposed mathematical model for the NSP is presented, which is based on the idea of multi-commodity network flow model. The proposed model was verified using hypothetical instances as well as benchmark instances, then, it is applied to a real case study in an Egyptian hospital. The results demonstrate the advantage of using the proposed model in generating schedule required to solve the problem. Furthermore, it proves the superiority of the obtained schedule to those generated manually by the supervisor head nurse as it improves the level of nurses' satisfaction by creating fair schedule system take care about nurses' preferences as well as decreases the overall overtime cost by 36%.

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1. Introduction

The ability to have an excellent staff on duty at the right time is an important factor for organizations to satisfy their customers' requirements. So that, staff scheduling problem is of importance for many organizations in industry as well as in services. Generally, scheduling problems have received attention over the past years as they based on the small-action-big-effect concept that maximizes the profitability of different

organizations through exploitation of workers and equipment in the best possible way. In the last twenty years, more than 300 articles discussed various problems related to the scheduling problem [1].

Nurses' main responsibility is to take care of the patient in order to improve patient satisfaction. However, nurses nowadays spend a lot of time in recording health care related documents through electronic medical records to improve care coordination. As a result, in USA, more than 50% of full-time nurses work an average of seven hours of overtime each week. This situation may solve the NSP in hospitals in short term, but it has a lot of consequences in the long term for example, the nurses who work 40 h in a week or 12 h in a shift are more likely to have job dissatisfaction. This in return negatively affects the hospital profitability [2].

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Egypt is facing the shortage number in nurses compared to the number of patients. According to undersecretary of the faculty of Nursing in Cairo University, nowadays one nurse services 20 patients although in nursing standers one nurse services 8–10 patients.

Nurse Scheduling Problem (NSP) aims to assign a number of nurses to a number of shifts in order to satisfy hospital's demand. During latest 50 years, NSP became important problem in field of operation research and artificial intelligence. The automatic generation of high quality schedules can lead to improvements in: hospital resource efficiency, staff and patient safety, staff and patient satisfaction, and administrative workload.

The main target of NSP is to assign an optimum number of different skilled nurses for each shift, while minimizing the hospital's cost, and maximizing the nurses' preferences [3]. There are several factors affecting the NSP: the governmental regulations, labor laws, hospital policy, and the status of nurses.

In NSP, there are three main terms:

- **Planning horizon:** Time span covered by a particular plan
- **Shift:** Each day is divided into time slots called shifts. The number of nurses required in each shift is pre-specified.
- **Off day:** A day when a nurse has no shifts to work.

The output of the NSP is as shown in Fig. 1. The planning horizon is K days, each day is divided into a number of shifts (e.g. morning, evening, and night), each shift has a pre-specified number of nurses should be satisfied while considering the maximum number of shifts that a nurse can work.

Due to governmental rules and hospital policy, a set of constraints must be satisfied as follows:

1. Each nurse can have at most two shifts a day.
2. Each nurse must have at least a pre-specified number of night shifts in the week.
3. Each nurse must have at least a pre-specified number of evening shifts in the week.
4. Each nurse should work between a minimum and a maximum number of shifts during the planning horizon.
5. If a nurse has a shift in a specific day, then, he/she should be off for next two consecutive shifts. However, if a nurse has two consecutive shifts in a specific day, he/she should be off for next three consecutive shifts.

The rest of the paper is organized as follows; review of recent literature is presented in the next section. In the third section, the proposed mathematical model is presented. Finally, the results of implementing the proposed model in a real case study are discussed.

2. Literature review

In addition, and in order to elaborate the progress in the NSP, the articles related to the problem were presented in their chronological appearance. At the end of the literature, we focused on our contribution which is based on proposing a new model that captures most of the aspects related to the NSP based on the idea of MCF.

2.1. Exact techniques

There are several articles which use exact methods to solve the NSP. Yilmaz et al. [4], developed a mathematical model that minimize nurses' total idle time during a week planning horizon. The model was verified using a numerical example and then LINGO8.0 software is used to ensure the global optimum solution.

Alkhabbaz et al. [5] studied a real case study in Kuwaiti health care units. They developed a mixed integer model that minimize outsourcing nurses, and considered nurses' preferences. EL-Rifai et al. [3], proposed a stochastic mixed integer programming model that used sample average approximation approach to create suitable shift schedule for the emergency department in the University Hospital Center of Lille France. Then, they evaluated the solutions using a discrete-event simulation model.

Legrain et al. [6] studied the scheduling process for two types of nursing teams, regular teams and float teams. Float teams are used to cover the shortages in the hospital. They proposed two different models that are flexible and easy enough to be implemented on spreadsheets. The first model that minimized the total cost, while maximizing the nurses' preferences. The second one was a local search-based model. They illustrated the advantages of their proposed models over commercial software packages by comparing the models results with optimization software.

Bateni et al. [7] proposed a mathematical model that maximized the nurses' preferences and minimized the total outsource nurses to cover the demands of each day. Their proposed model considered hospital's policies and governmental regulations.

Hamed Jafari et al. [8] developed a mathematical model that maximize nurses' preferences. An optimal solution was generated by their proposed model, and then they used simulated annealing approach to solve the problem in a reasonable time. The experiments showed that the computation time of the mathematical model was 6128.70 s, and the computation time of the simulated annealing approach was 502.48 s.

Liang et al. [9] developed two multi-objective optimization models, the first one was functional care delivery model, and the other one was primary care delivery model. Those models aimed to minimize the total patient waiting time and find optimal appointment times to minimize the total nurse overtime. They applied the models in one of largest oncology clinics. The results showed that the proposed models were flexible, competitive and capable of reaching an optimal solution.

Zakria et al. [10] proposed stochastic optimization model and sample average approximation method to obtain an optimal case-mix plan for a surgery department with considering nurses capacity constraints. Their model was solved under three capacity constraints: nurses, beds, and operating room time.

Ohki et al. [11] proposed an effective mutation operator that guarantee the consistency of the nurse scheduling generation. They used a new penalty function which evaluates the difference between the original schedule and the optimum schedule. Edmund et al. [12] proposed a multi-objective model that use integer programming and variable neighborhood search heuristic. Their model has two steps. In the first step,

	Day 1			Day 2			...	Day 7			Number of shifts
Shifts	M	E	N	M	E	N	...	M	E	N	
Nurse 1							...				6
Nurse 2							...				7
Nurse 3							...				7
Nurse 4							...				7
.....
Nurse J							...				6
Number of nurses assigned	15	18	13	13	18	13	...	13	14	13	314

Fig. 1 Typical nurse schedule.

an integer programming is used to generate a feasible solution. In the second steps, a variable neighborhood search heuristic is used to enhance the feasible solution. They applied their approach on real case study arising in a Dutch hospital.

2.2. Approximant techniques

A several articles use approximant techniques to solve the NSP. Zhang et al. [13] proposed hybrid swarm-based optimization algorithm that combine both genetic algorithms and a variable neighborhood search heuristic to solve the NSP. The solution approach has three steps, in the first step, the problem is divided into several sub-problems and a genetic algorithm is used to solve those sub-problems. In the second step, a hybrid genetic algorithm is used to develop better feasible timetable. In the third step, the feasible solution is used as initial solution for the variable neighborhood search heuristic to generate a final timetable. The results showed the effectiveness and flexibility of the generated schedules.

Bilgin et al. [14] proposed one general high-level hyper-heuristic approach that can deal with the NSP. Constantino et al. [15] proposed two phase heuristic algorithm generated work schedules for a set of nurses to maximize the satisfaction of nurses' preferences and minimize the violation of soft constraints.

Akbari et al. [16] developed a simulated annealing approach to solve NSP. They considered fatigue as a factor that affects workers during their work. Burke et al. [17] presented a branch and price algorithm and an ejection chain method to solve the NSP. Tassopoulos et al. [18] proposed a two-phase variable neighborhood search heuristic to solve the NSP. They applied the proposed heuristic to benchmark instances and the results were remarkable.

Rahimian et al. [19] presented a novel hybrid algorithm that combine the strengths of integer programming and variable neighborhood search heuristic to design a method for solving the NSP. Their model has four stages which are used to generate and improve the solutions.

In this work, a new formulation is proposed based on the idea of Multi-Commodity Network Flow (MCF). The MCF problem is a network flow problem with multiple commodities between different sources and sink nodes. The idea of MCF has been implemented in a wide range of applications such

as: energy management [20], airlines scheduling [21], routing plan for the hazard materials in industrial facilities [22], maintenance planning problem [23], traveling salesman problem [24], and [25], and inventory management [26].

In the MCF problem, several commodities with known quantities to be transported from a number of source nodes to a number of sink nodes with the objective of transporting these commodities with minimum cost while fulfilling the demand requirements of the sink nodes. NSP could be viewed as a MCF problem. The source nodes are the nurses available for work, the sink nodes are the shifts in different days in the planning horizon, while the demand of the sink nodes is the number of nurses required in each shift in the planning horizon. The commodities here are: the head nurse, the senior nurse, and the training nurse.

3. The proposed mathematical model

In this section, a proposed model that captures most of the aspects related to the NSP is presented. Most of hospitals consider their planning horizon as a week. Each day in the week is divided into three types of shifts (i.e. morning, evening, and night shifts). Beside morning shifts, a nurse should work a specified number of evening and night shifts per week, usually two afternoon shifts, and one night shift. Each shift is eight hours, and a nurse can't work more than two consecutive shifts.

The proposed mathematical model is based on the idea of the multi-commodity-network flow model. Fig. 2 shows the network representation of the proposed model. The figure illustrates how the idea of the MCF model could be applied in the context of NSP. In the network, there are a specified number of nurses (I), and a specified number of shifts (J). For each nurse (i), the arcs represent all possible assignments of that nurse to all the shifts. The nurses represent source nodes, while the shifts represent sink nodes. The demand of each of the sink nodes is equal to the number of nurses required for each of the sink nodes, while the capacity of the source nodes is equal to one which is the nurse that may be assigned for one of the sink nodes. The assignment of a nurse to a certain shift is represented by an arc between a source node and a sink node. The model indices, parameters, decision variables, objectives, and constraints are described below.

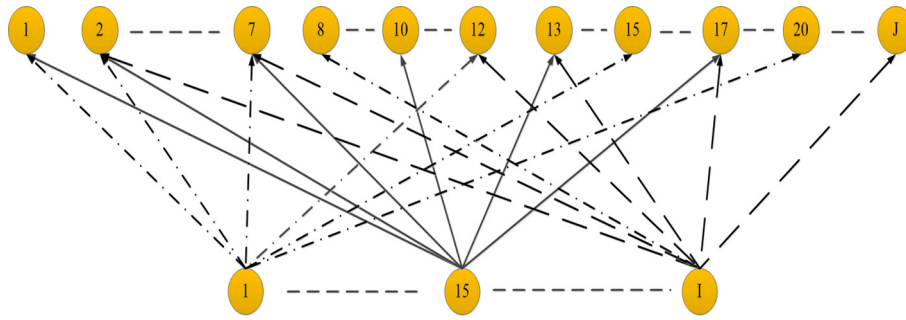


Fig. 2 Network representation of the proposed model.

Indices and parameters:

c_1 : Cost per shift of a nurse

c_2 : Overtime cost per shift of a nurse

c_3 : Cost per shift of a head nurse

I : Set of nurses $\{1, 2, \dots, H, H + 1, H + 2, \dots, I\}$

Note: The set nurses are divided into two categories, from 1 to H which is the number of head nurses, and from $H + 1$ to I which is the number of nurses.

J : Set of shifts $\{1, 2, 3, \dots, J\}$ during the planning horizon

Note: the working day is divided into three types of shifts: morning, afternoon, and night shifts. If the planning horizon is a week for example, then 21 shifts are available during the planning horizon. Morning shifts take the values of $\{1, 4, 7, \dots, J-2\}$, afternoon shifts take the values of $\{2, 5, 8, \dots, J-1\}$, while the night shifts take the values of $\{3, 6, 9, \dots, J\}$

U_i : Maximum number of shifts that nurse (i) can work during the planning horizon.

N_i : Number of shifts that nurse (i) should work during the planning horizon.

M_j : Number of nurses required for shift (j).

A : Minimum number of afternoon shifts that a nurse should work.

B : Minimum number of night shifts that a nurse should work.

F : Minimum number of shifts of type (j) that a head nurse should work.

$$f_i = \begin{cases} 1 & \text{if nurse } (i) \text{ is female} \\ 0 & \text{otherwise} \end{cases}$$

Decision variables:

$$x_{ij} = \begin{cases} 1 & \text{if nurse } (i) \text{ is assigned to shift } (j) \\ 0 & \text{otherwise} \end{cases}$$

Objective function:

$$\text{Min} \sum_{i=H+1}^I \sum_{j=1}^J (c_1 * x_{ij}) + c_2 \left[\left(\sum_{i=H+1}^I \sum_{j=1}^J x_{ij} \right) - N_i \right] + \sum_{i=1}^H \sum_{j=1}^J (c_3 * x_{ij})$$

Constraints:

$$\sum_{i=1}^I x_{ij} \geq M_j \quad \forall j \in J \quad (1)$$

$$\sum_{j=1}^J x_{ij} \geq N_i \quad \forall i \in I \quad (2)$$

$$\sum_{j=1}^J x_{ij} \leq U_i \quad \forall i \in I \quad (3)$$

$$\sum_{j=0}^{(\frac{J}{3})-1} x_{i(3j+2)} \geq A \quad \forall i \in I \setminus \{H+1, H+2, \dots, I\} \quad (4)$$

$$\sum_{j=1}^{J/3} x_{i(3j)} \geq B \quad \forall i \in I \setminus \{H+1, H+2, \dots, I\} \quad (5)$$

$$\sum_{j=0}^{(\frac{J}{3})-1} x_{i(1+3j)} \geq F \quad \forall i \in I \setminus \{1, 2, \dots, H\} \quad (6)$$

$$\sum_{j=0}^{(\frac{J}{3})-1} (x_{i(2+3j)} + x_{i(3+3j)}) = 0 \quad \forall i \in I \setminus \{H+1, \dots, I\} \quad (7)$$

$$\sum_{i=1}^I f_i * x_{ij} \geq 1 \quad \forall j \in J \quad (8)$$

$$x_{ij} + x_{i(j+2)} \leq 1 \quad \begin{matrix} \forall i \in I \setminus \{1, 2, \dots, H\} \\ \forall j \in J \setminus \{1, 2, 3, \dots, J-2\} \end{matrix} \quad (9)$$

$$\sum_j^{j+4} x_{ij} \leq 2 \quad \begin{matrix} \forall i \in I \setminus \{1, 2, \dots, H\} \\ \forall j \in J \setminus \{1, 2, 3, \dots, J-4\} \end{matrix} \quad (10)$$

The objective function of the proposed model is to minimize the overall cost. The overall cost is divided into three parts. The first one is the cost of normal shifts of nurses; the second part is the overtime cost of nurses, while the third part is the cost of head nurses. A head nurse can only work in the morning shifts. Generally, if a nurse works more than the minimum number of shifts that he/she can work; then, the extra shifts are considered as overtime shifts.

Table 1 Results of model verification.

Instance	Number of nurses	Planning horizon (weeks)	Shift/day	Required number of shifts in the planning horizon	Cost of regular shifts
1	8	2	1	71	71
2	14	2	2	108	108
3	20	2	3	154	154
4	10	4	2	180	180
5	16	4	2	288	288
6	18	4	3	297	297
7	20	4	3	315	315
8	30	4	4	440	440
9	36	4	4	410	410
10	40	4	5	693	693
11	50	4	6	811	811
12	32	6	4	692	735
13	45	6	6	941	1081
14	20	8	3	671	677
15	32	8	4	1086	1086
16	22	12	3	1116	1132
17	40	12	5	1857	1924

Table 2 Compare between two different instances.

Nurses No.		Planning horizon				Required number of shifts	Optimum solution
From	To	Min. Required shifts/ nurse	Max. Required shifts/ nurse	Min. nurses shifts	Max. nurses shifts		
<i>Instance No. 3</i>							
1	15	7	9	120	160	154	154
16	20	3	5				
<i>Instance No. 12</i>							
1	27	25	27	735	799	692	735
28	32	12	14				

Constraint (1) ensures that the required number of nurses in each shift is satisfied. The sets of constraints (2) and (3) ensure that each nurse has a minimum and a maximum number of working shifts. A nurse has to work a number of shifts within these limits.

Constraint (4) ensures that each nurse must work at least a minimum number of afternoon shifts. Constraint (5) ensures that each nurse must work at least a minimum number of night shifts. The set of constraints (6) and (7) ensures that the head nurses work only in the morning shifts, and they are not allowed to work in either afternoon shift or night shift. Constraint (8) ensures that each shift has at least one female nurse.

Constraint (9) and (10) are provided to achieve the hospitals and governmental regulations. Constraint (9) ensures that a nurse will have at least two successive shifts off if he/she works for one shift. Constraint (10) ensures that a nurse will have at least three successive shifts off if he/she works for two consecutive shifts.

For any given instance, the number of constraints (c) and the number of variables (v) can be calculated using formulas (a), and (b) respectively. The main advantage of the proposed model is its simplicity while considering all the important aspects related to the NSP.

$$c = 2J + 6I - 8H + 2HJ \quad (a)$$

$$v = IJ \quad (b)$$

In order to verify the model, a number of instances¹ were used to check the performance of the proposed mathematical model. The cost of both regular time shifts and overtime shifts is set to be unity in order to facilitate the determination of total number of shifts.

The instances were solved using Gurobi 7.0.1 optimization software which use the branch and bound algorithm. All the experiments were performed on Intel (R) Core (TM) i7 – 6500 U CPU @ 2.50 GHz and 8.00 GB installed memory. The characteristics of the instances and their results are shown in Table 1.

The proposed mathematical model was generate the optimum solution of those instance in negligible computation time (less than 0.05 s) the following table compare between two different instances:

In instance 3, the optimum solution generated by the proposed model equal the required number of shifts in the planning horizon because of the required number of shifts is greater than the minimum number of shifts that nurses should work in the planning horizon. However, the optimum solution

¹ These instances are available on: <http://www.cs.nott.ac.uk/~psztzc/NRP/index.html>.

Table 3 Comparison between the results of the first scenario and the manual schedule.

	Manual Schedule						First scenario			Manual schedule & First scenario											
Days	Saturday			Sunday			Monday			Tuesday			Wednesday			Thursday			Friday		
Nurse	M	A	N	M	A	N	M	A	N	M	A	N	M	A	N	M	A	N	M	A	N
1																					
2																					
3																					
4																					
5																					
6																					
7																					
8																					
9																					
10																					
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generated in instance 12 equal the minimum number of shifts that nurses should work in the planning horizon because of the minimum number of shifts that nurses should work is greater than the required number of shifts in the planning horizon (see Table 2).

4. Implementing the proposed model to a real case study in Egypt

The proposed model has been implemented in one of the private hospitals in Alexandria, Egypt. The hospital aims at present high quality of medical services in different fields,

Table 4 Comparison between the results of the second scenario and the manual schedule.

	Manual Schedule							Second scenario							Manual & Second scenario						
Days	Saturday			Sunday			Monday			Tuesday			Wednesday			Thursday			Friday		
Nurse	M	A	N	M	A	N	M	A	N	M	A	N	M	A	N	M	A	N	M	A	N
Critical care unit																					
1																					
7																					
8																					
9																					
10																					
11																					
12																					
13																					
14																					
15																					
16																					
17																					
Intensive care unit for newborns																					
2																					
18																					
19																					
20																					
21																					
22																					
Emergency Unit																					
3																					
23																					
24																					
25																					
26																					
Post Critical Care unit																					
4																					
27																					
28																					
29																					
30																					
31																					
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Outpatients Clinics Unit																					
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especially in emergency cases. The hospital has six departments (Critical care unit, Intensive care unit for newborns, Emergency Unit, Post Critical Care unit, Outpatients Clinics Unit, and Internal Unit). The hospital has six head nurses and 40 regular nurses. There are 17 male nurses, and 29 female nurses.

As in most of hospitals in Egypt, the Supervisor Head Nurse (SHN) creates suitable schedule manually. This process is not only limited to find optimal solution, but also it is very time consuming and cumbersome. The following are the regulations of the hospital:

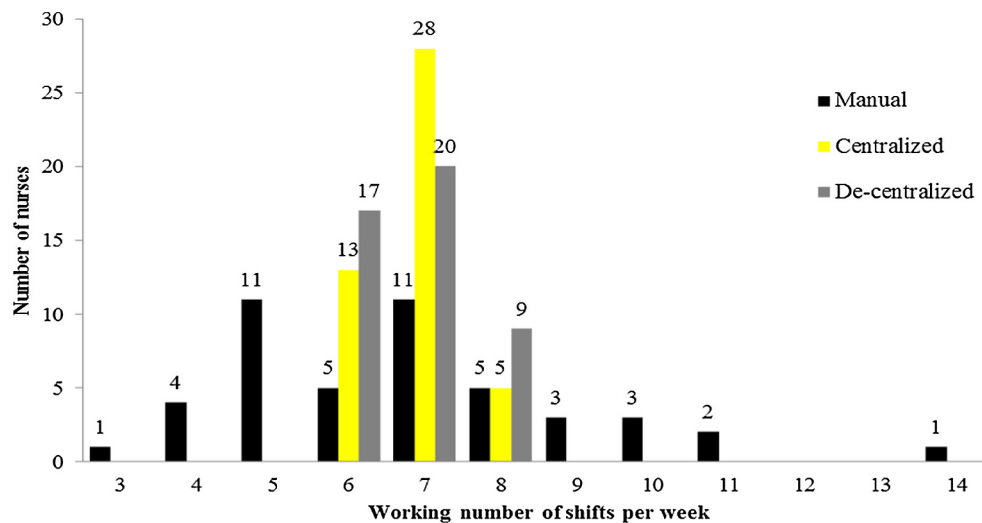


Fig. 3 Numbers of nurses vs. working number of shifts per week at different scenarios.

Table 5 Improvements of both scenarios comparing with current status of the hospital.

	Manual Schedule	First scenario schedule	Second scenario schedule
Normal working hours	1996	2166	2139
Overtime working hours	475	305	332
Overtime Cost Reduction	—	36%	31%
Minimum number of working hours a nurse has worked	24	42	42
Maximum number of working hours a nurse has worked	119	65	65

1. The planning horizon is one week, i.e., 7 days.
2. Each day has three types of shifts: morning shift (7 h) from 8:00 AM to 3:00 PM, afternoon shift (7 h) from 3:00 PM to 10:00 PM, and night shift (10 h) from 10:00 PM to 8:00 AM.
3. Saturday is the first day of each week.
4. The hospital administration is aiming to limit the nurses to work between six and eight shifts a week.
5. The head nurses work only in the morning shifts.

Before implementing the proposed model in the hospital, it is important to show the disadvantages of the manually generated schedule. The manually generated schedule has the following disadvantages:

1. It needs about four hours to be generated.
2. It can't cope with the sudden absence of nurses.
3. Overtime cost per week represents about 50% of the total cost.
4. About 30% of the nurses work less than six shifts per week
5. About 20% of the nurses work more than eight shifts per week.
6. More than 80% of the nurses represents one of the following situations:

7. They work two consecutive shifts without having three successive shifts-off after that
8. They work one shift without having two successive shifts off.
9. More than 42% of the nurses work more than 56 h in the week, which is not acceptable regarding governmental regulations.

The proposed model has been implemented in the hospital considering two scenarios. The first one considers the centralized approach, within which any nurse can work in any department. The other scenario considers the de-centralized approach, within which each department has its own nurses, and they can't be assigned outside that department.

4.1. Implementing the proposed model based on the centralized approach of the hospital case

The following points are concluded the results of applied proposed model on the centralized approach:

1. The time needed to generate the weekly schedule is negligible (i.e. the computation time is less than a second).
2. The overtime cost per week is reduced by 36%.
3. The nurses' preferences have been improved as follow:
 - Each nurse takes at least one day off.
 - Each nurse takes at least two successive shifts off working one shift.
 - If a nurse is assigned to two consecutive shifts, then he/she takes at least three consecutive shifts off.
4. The assignment of nurses to the shifts has been improved as follow:
 - 28% of the nurses work 6 shifts
 - 61% of the nurses work 7 shifts
 - 11% of the nurses work 8 shifts.

Table 3 shows the comparison between the schedule generated by the proposed model based on the first scenario and the manual schedule generated by the SHN.

4.2. Applying the proposed model to the de-centralized case of the hospital

The following points are concluded the results of applied proposed model on the de-centralized approach:

1. The time needed to generate the weekly schedule is negligible (i.e. computation time is less than a second).
2. The overtime cost per week is reduced by 30%.
3. The nurses' preferences have been as follow:
 - Each nurse takes at least one day off.
 - Each nurse takes at least two successive shifts off working one shift.
 - If a nurse is assigned to two consecutive shifts, then he/she takes at least three consecutive shifts off.
4. The assignment of nurses to the shifts has been improved as follow:
 - 37% of the nurses work 6 shifts
 - 43% of the nurses work 7 shifts
 - 20% of the nurses work 8 shifts.

Table 4 shows the comparison between the schedule generated by the proposed model based on the second scenario and the manual schedule generated by the SHN.

Fig. 3 shows a comparison between the results of the proposed model based on the first scenario, the results of the proposed model based on the second scenario, and the manual schedule generated by the SHN. The comparison shows the number of nurses those worked a specific number of shifts per week. Before implementing the proposed model, the distribution of shifts to nurses was unfair. Some nurses worked less than 4 shifts a week, and on the other hand, more than 9 nurses worked more than 8 shifts a week. After applying the proposed model, almost all the nurses have the same working load per week.

Table 5 shows a comparison between the manual schedule generated by the SHN, the results of the proposed model based on the first scenario, and the results of the proposed model based on the second scenario. The comparison shows the normal working hours, overtime working hours, minimum and maximum numbers of working hours a nurse has worked per planning horizon. Before implementing the proposed model, the distribution of shifts to nurses was unfair and there is lack of utilization of the normal working hours.

5. Conclusions

The NSP is very important problem for hospitals' administration. NSP considers the minimization of the overall hospital cost while considering the hospital's regulations. A new formulation which is based on the idea of MCF model was presented to solve the problem. To the best of our knowledge, this is the first time to attempt the idea of MCF in NSP. The proposed model captures the realistic constraints related to the problem such as governmental constraints, hospital's policy, and global standard in this area. The model was verified using test instances, and then validated by implementing the proposed model in a real case study in Egypt. The results demonstrate the advantage of using the proposed model in generating

timetables schedule required to solve the problem. Furthermore, it proves the superiority of the obtained timetables schedule to those generated manually by the SHN as it improves the level of nurses' satisfaction by creating fear schedule system take care about nurses' preferences as well as decreases the overall overtime cost by 36%. The proposed methodology can be applied to other resource allocation problems with a large number of constraints.

References

- [1] J. Van Den Bergh, J. Beliën, P. De Bruecker, E. Demeulemeester, L. De Boeck, Personnel scheduling: A literature review, *Eur. J. Oper. Res.* 226 (2013) 367–385.
- [2] W. Stimpfel, M. Sloane, H. Aiken, The longer the shifts for hospital nurses, the higher the levels of burnout and patient dissatisfaction, *Health Affairs* 31 (11) (2011) 2501–2509.
- [3] O. EL-Rifai, T. Garaix, V. Augusto, X. Xie, A stochastic optimization model for shift scheduling in emergency departments, *Health Care Manage. Sci.* 18 (2015) 289–302.
- [4] E. Yilmaz, A mathematical programming model for scheduling of nurses' labor shifts, *J. Med. Syst.* 36 (2012) 491–496.
- [5] R. M'Hallah, A. Alkhabbaz, Scheduling of nurses: A case study of a Kuwaiti health care unit, *Oper. Res. Heal. Care* 2 (2013) 1–19.
- [6] A. Legrain, H. Bouarab, N. Lahrichi, The nurse scheduling problem in real-life, *J. Med. Syst.* 39 (2014) 1–11.
- [7] H. Jafari, S. Bateni, P. Daneshvar, S. Bateni, Fuzzy mathematical modeling approach for the nurse scheduling problem: A case study, *Int. J. Fuzzy Syst.* (2015).
- [8] H. Jafari, N. Salmasi, Maximizing the nurses' preferences in nurse scheduling problem: Mathematical modeling and a meta-heuristic algorithm, *J. Ind. Eng. Int.* 11 (2015) 439–458.
- [9] B. Liang, A. Turkan, Acuity-based nurse assignment and patient scheduling in oncology clinics, *Health Care Manage. Sci.* 19 (2016) 207–226.
- [10] Y. Zakaria, A. Eltawil, N. Harraz, The operating room case-mix problem under uncertainty and nurses capacity constraints, *Health Care Manage. Sci.* 19 (2016) 383–394.
- [11] M. Ohki, S.Y. Uneme, H. Kawano, Effective mutation operator and parallel processing for nurse scheduling, *Stud. Comput. Intell.* 299 (2010) 229–242.
- [12] E.K. Burke, J. Li, R. Qu, A hybrid model of integer programming and variable neighbourhood search for highly-constrained nurse rostering problems, *Eur. J. Oper. Res.* 203 (2010) 484–493.
- [13] Z. Zhang, Z. Hao, H. Huang, Hybrid swarm-based optimization algorithm of GA & VNS for nurse scheduling problem, in: *Lect. Notes Comput. Sci. (Including Subser. Lect. Notes Artif. Intell. Lect. Notes Bioinformatics)*, vol. 7030, LNCS, 2011, pp. 375–382.
- [14] B. Bilgin, P. Demeester, M. Misir, W. Vancroonenburg, G. Vanden Berghe, One hyper-heuristic approach to two timetabling problems in health care, *J. Heuristics* 18 (2012) 401–434.
- [15] A. Constantino, D. Landa-Silva, E. Luiz de Melo, C. De Mendonça, D. Rizzato, D. Romão, A heuristic algorithm based on multi-assignment procedures for nurse scheduling, *Ann. Oper. Res.* (2013) 1–19.
- [16] M. Akbari, M. Zandieh, B. Dorri, Scheduling part-time and mixed-skilled workers to maximize employee satisfaction, *Int. J. Adv. Manuf. Technol.* 64 (2013) 1017–1027.
- [17] E. Burke, T. Curtois, New approaches to nurse rostering benchmark instances, *Eur. J. Oper. Res.* 237 (2014) 71–81.
- [18] I.X. Tassopoulos, I.P. Solos, G.N. Beligiannis, Computers & operations research A two-phase adaptive variable neighborhood approach for nurse rostering, *Comput. Oper. Res.* 60 (2015) 150–169.

- [19] E. Rahimian, K. Akartunali, J. Levine, A hybrid Integer Programming and variable neighborhood search algorithm to solve nurse rostering problems, *Eur. J. Oper. Res.* 258 (2017) 411–423.
- [20] R.S. Adhikari, N. Aste, M. Manfren, Energy procedia multi-commodity network flow models for dynamic energy management – smart grid applications, *Energy Proc.* 14 (2012) 1374–1379.
- [21] B. Maharjan, T.I. Matis, Computers & industrial engineering multi-commodity flow network model of the flight gate assignment problem, *Comput. Ind. Eng.* 63 (2012) 1135–1144.
- [22] Y. Jiang, X. Zhang, Y. Rong, Z. Zhang, A multimodal location and routing model for hazardous materials transportation based on multi-commodity flow model, *Proc. - Soc. Behav. Sci.* 138 (2014) 791–799.
- [23] M. Mesquita, M. Moz, A. Paias, M. Pato, Discrete Optimization A decompose-and-fix heuristic based on multi-commodity flow models for driver rostering with days-off pattern, *Eur. J. Oper. Res.* 245 (2015) 423–437.
- [24] A.N. Letchford, J. Salazar-gonzález, Stronger multi-commodity flow formulations of the (capacitated) sequential ordering problem, *Eur. J. Oper. Res.* 251 (2016) 74–84.
- [25] N. Boland, T. Kalinowski, F. Rigterink, New multi-commodity flow formulations for the pooling problem, *J. Glob. Optim.* 66 (2016) 669–710.
- [26] F. Schultmann, Freight transportation planning considering carbon emissions and in-transit holding costs: a capacitated multi-commodity network flow model, *EURO J. Transport. Logist.* (2016) 123–160.