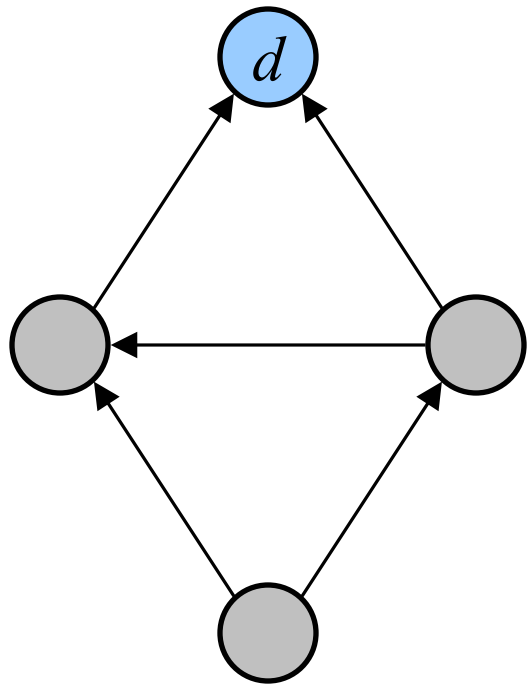
# The complexity of a restricted variant of the stable paths problem

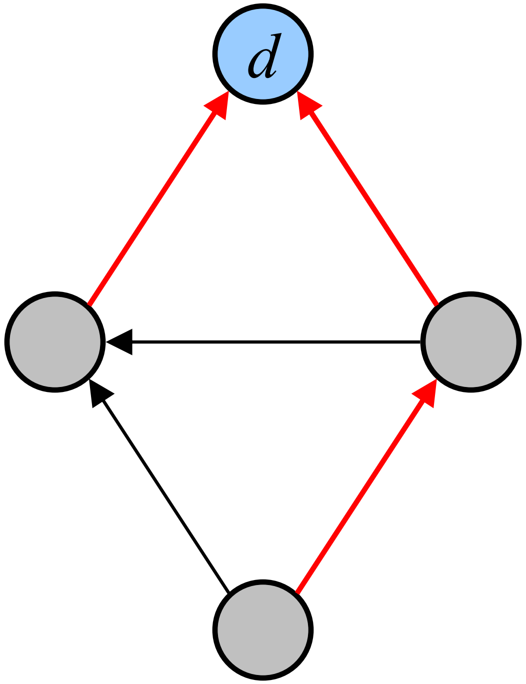
Andrei Lapets

Based on joint work with Kevin Donnelly and Assaf Kfoury

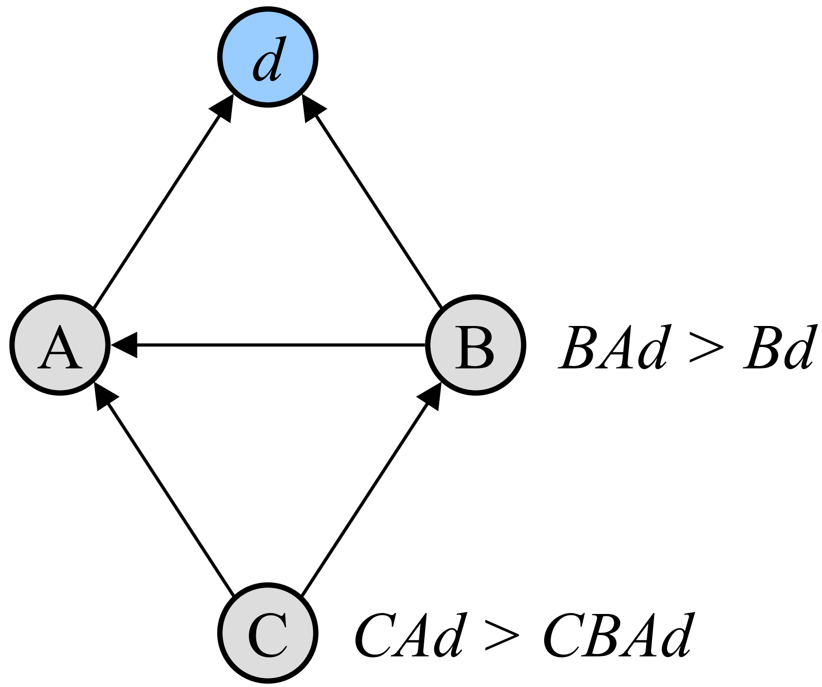
April 23, 2010



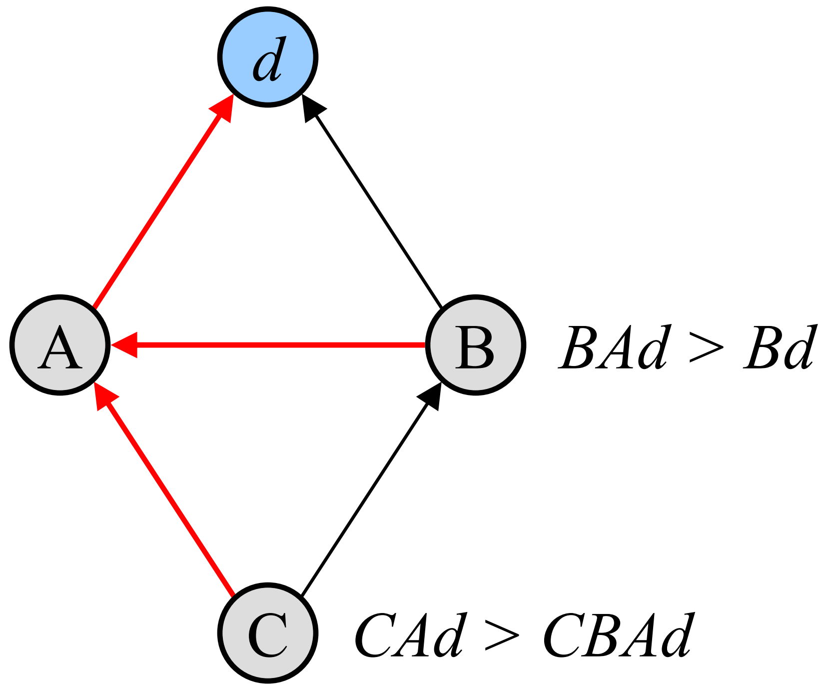

no restrictions



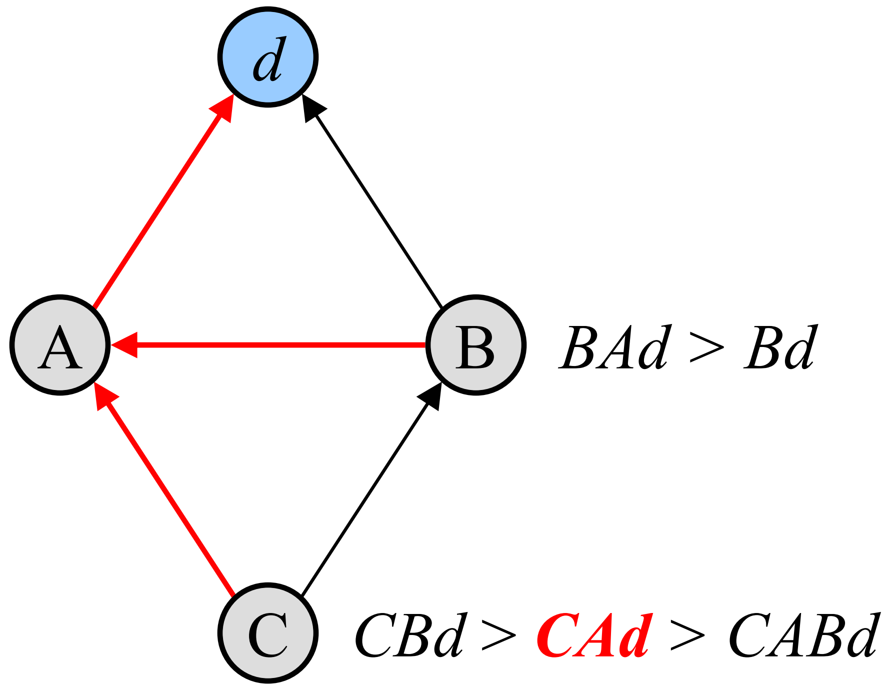

no restrictions




path rankings



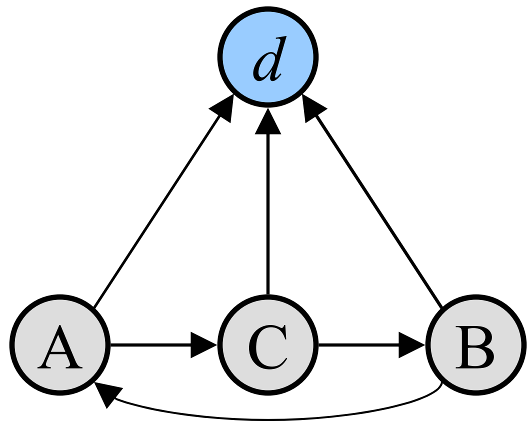

path rankings




path rankings
prefix preservation

# “unsolvable” instance: no stable solutions

*ACd > Ad > ACBd*

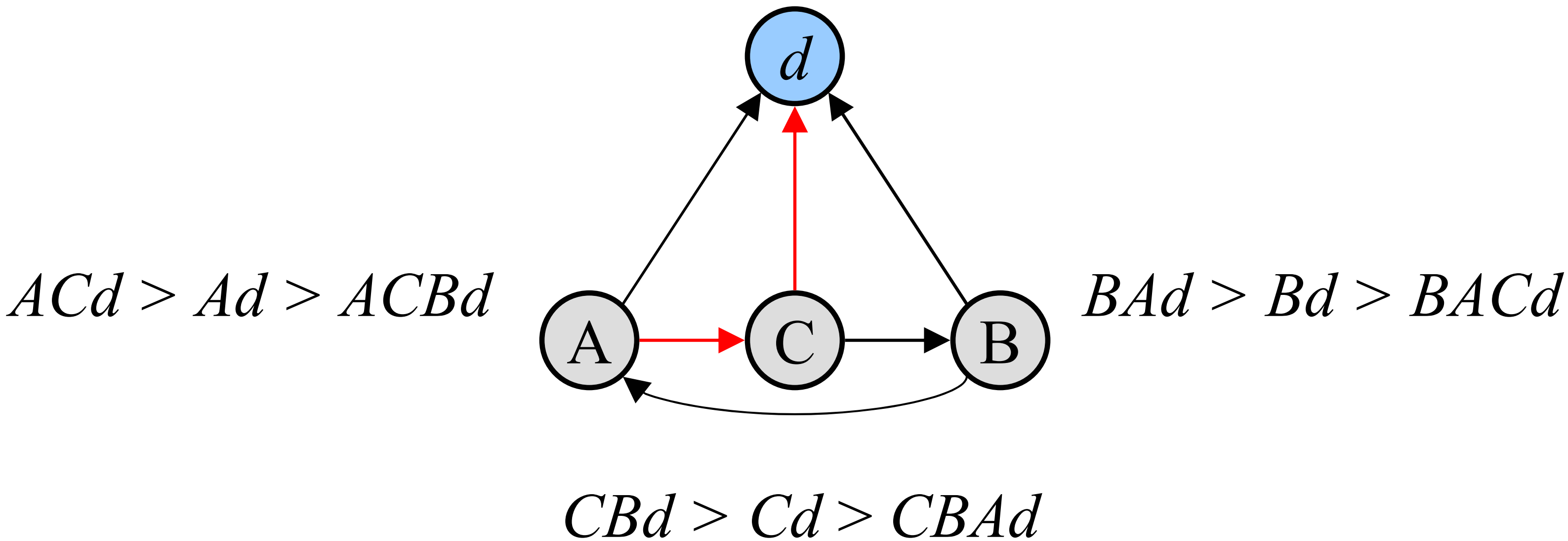



*BAd > Bd > BACd*

*CBd > Cd > CBAd*

path rankings
prefix preservation

# “unsolvable” instance: no stable solutions




path rankings
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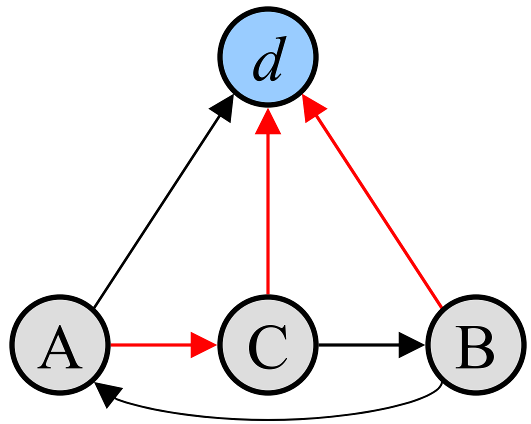


*ACd* > *Ad* > *ACBd*

*BAd* > *Bd* > *BACd*

*CBd* > *Cd* > *CBAd*

path rankings
prefix preservation

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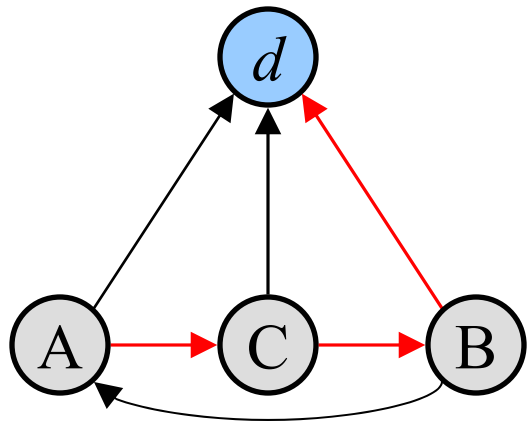


*ACd > Ad > ACBd*

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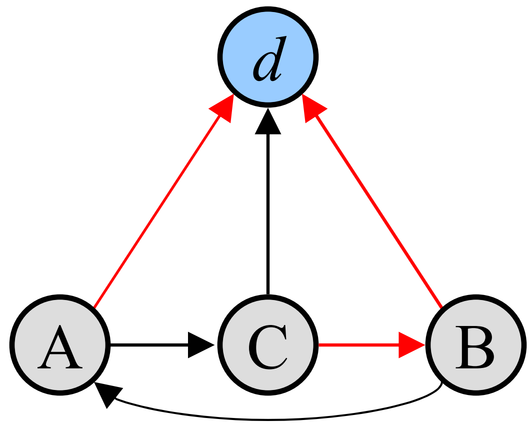
path rankings
prefix preservation

# “unsolvable” instance: no stable solutions




*ACd > Ad > ACBd*

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path rankings
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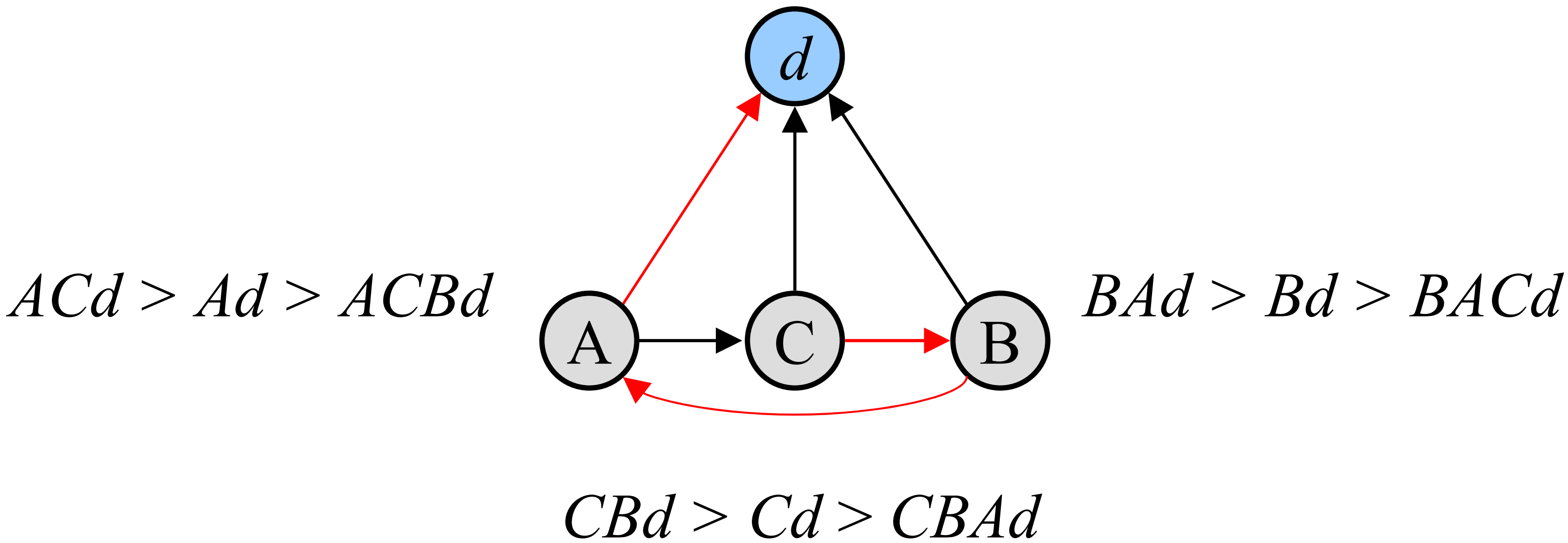


path rankings
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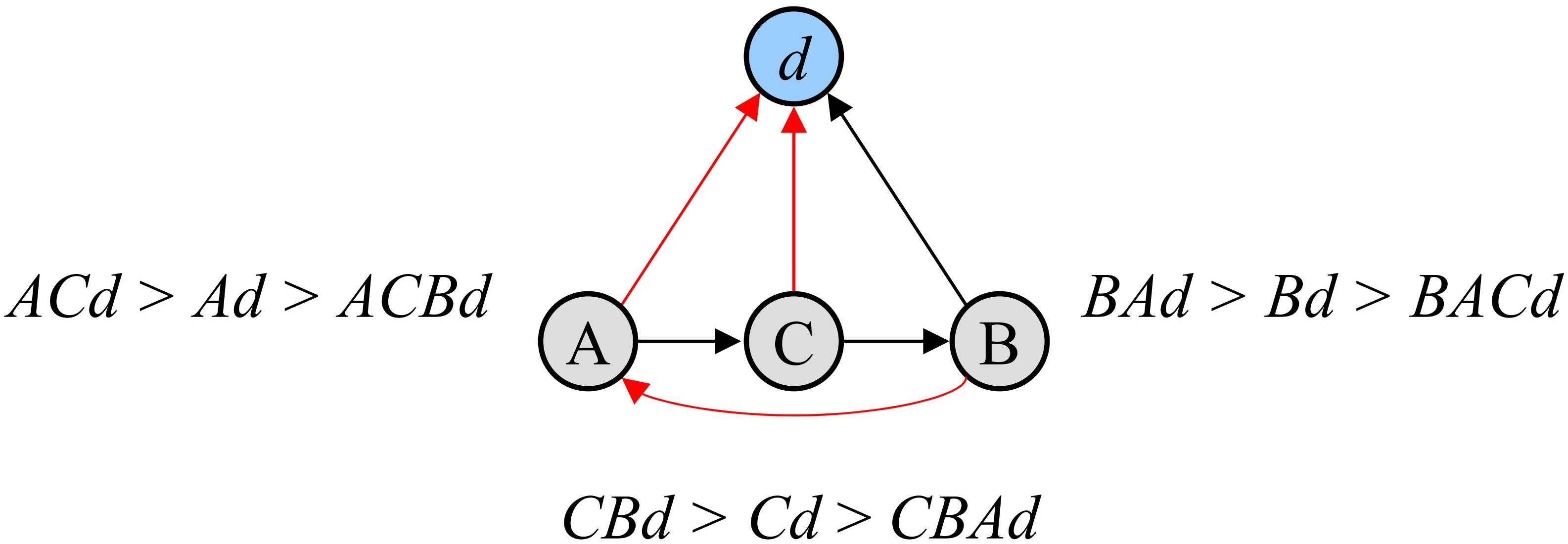
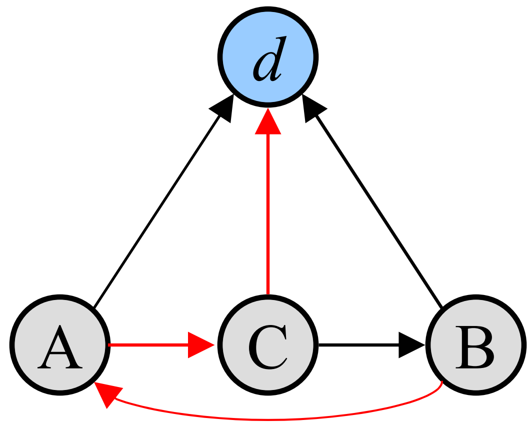
# “unsolvable” instance: no stable solutions




path rankings
prefix preservation

# “unsolvable” instance: no stable solutions




*ACd > Ad > ACBd*

*BAd > Bd > BACd*

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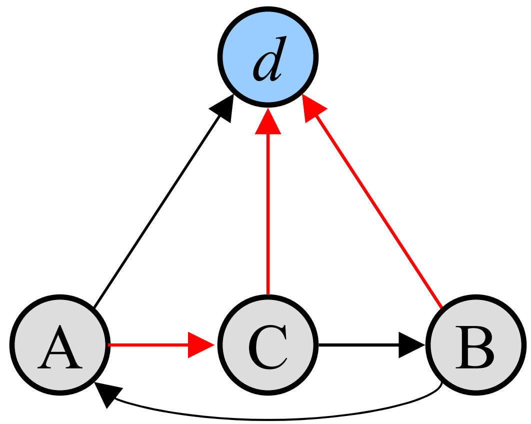
path rankings
prefix preservation

# “unsolvable” instance: no stable solutions

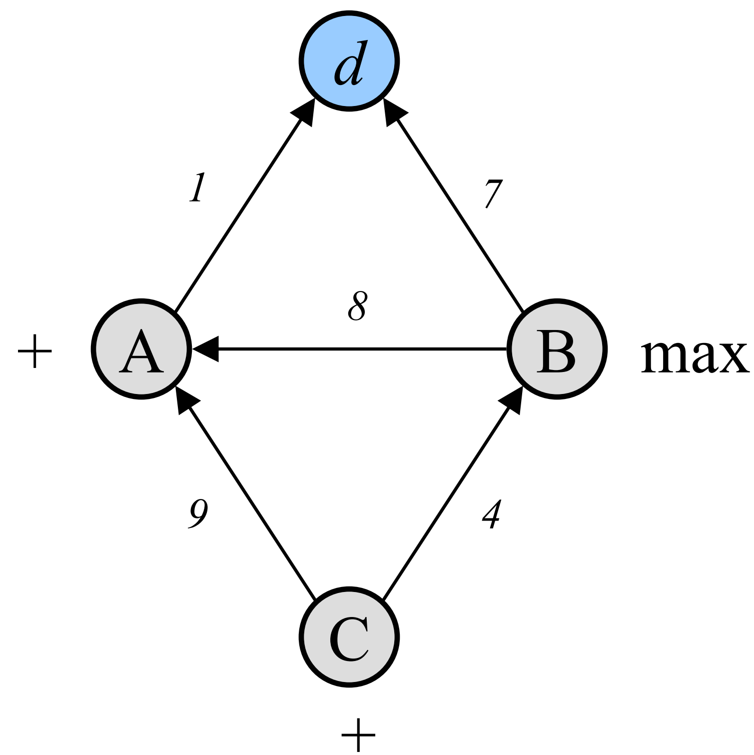



*ACd* > *Ad* > *ACBd*

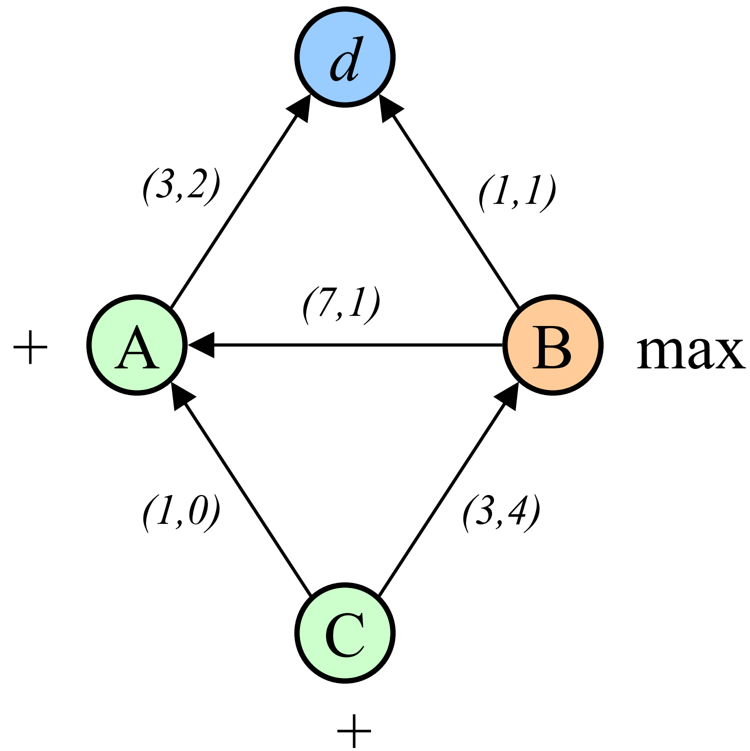
*BAd* > *Bd* > *BACd*

*CBd* > *Cd* > *CBAd*

path rankings
prefix preservation



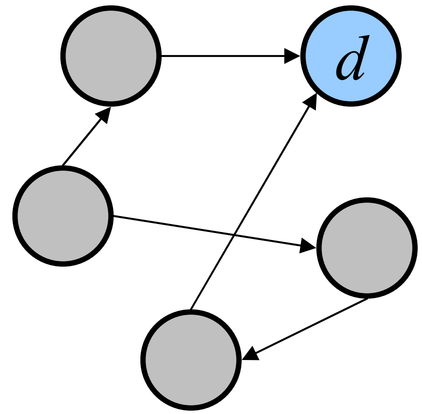

path rankings using aggregates
prefix preservation



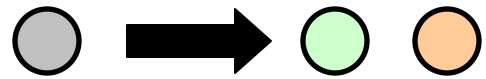

path rankings using aggregates
prefix preservation

# $\overline{f}$-SPP problem instance

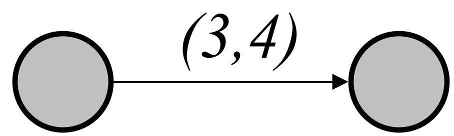
$$(V, E, d)$$



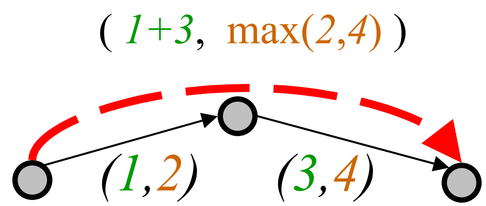

$$\pi : V \to n$$



$$w : E \to \mathbb{R}^n$$




$$\overline{f} = \langle f_1, \ldots, f_n \rangle$$




$$\langle +, \max \rangle$$

Given an $\overline{f}$-SPP instance

$$(V, E, d),\ \pi,\ w,\ \langle f_1, \ldots, f_n \rangle,$$

does there exist a stable solution?

Given an $\overline{f}$-SPP instance

$$(V, E, d),\ w,\ \langle f_1, \ldots, f_n \rangle,$$

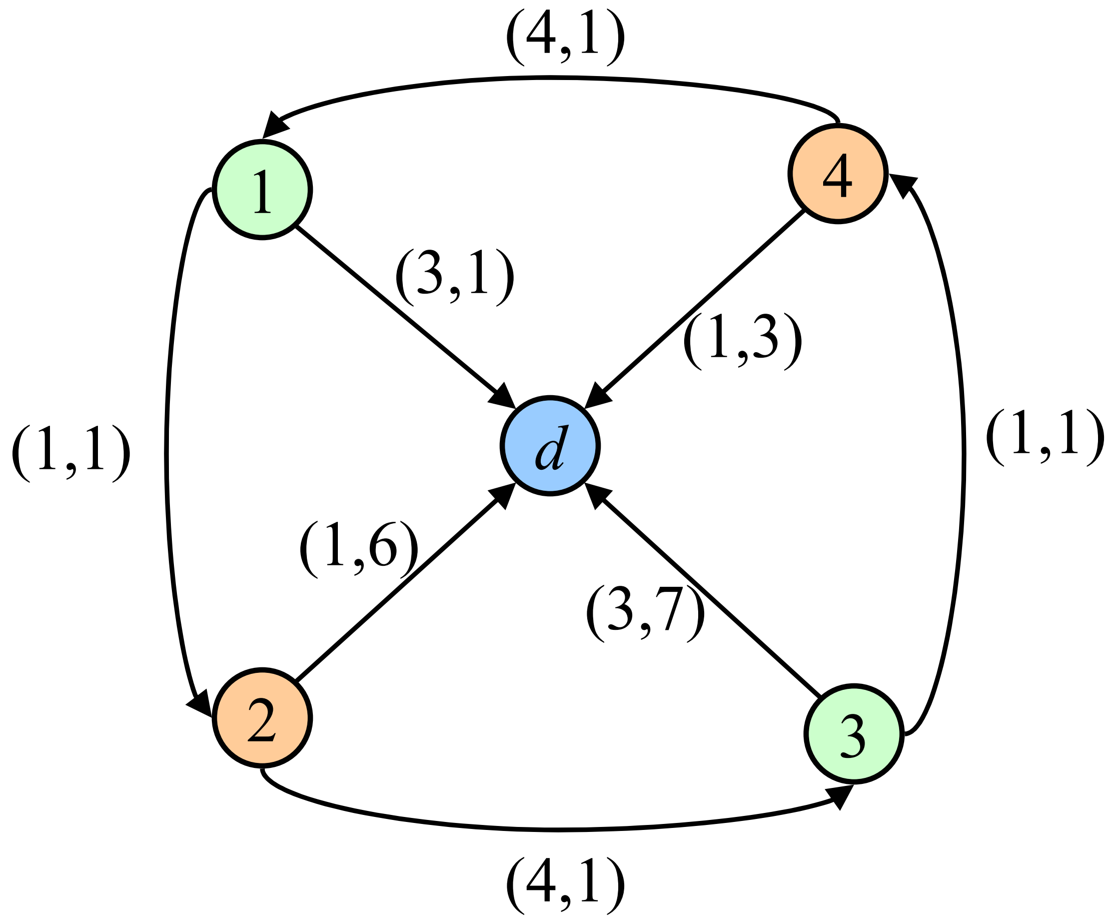
does there exist a solution that is stable under all possible $\pi$?

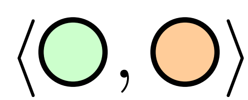
Given an $\overline{f}$-SPP instance

$$(V, E, d),\ \pi,\ w,\ \langle f_1, \ldots, f_n \rangle,$$

does there exist a stable solution?

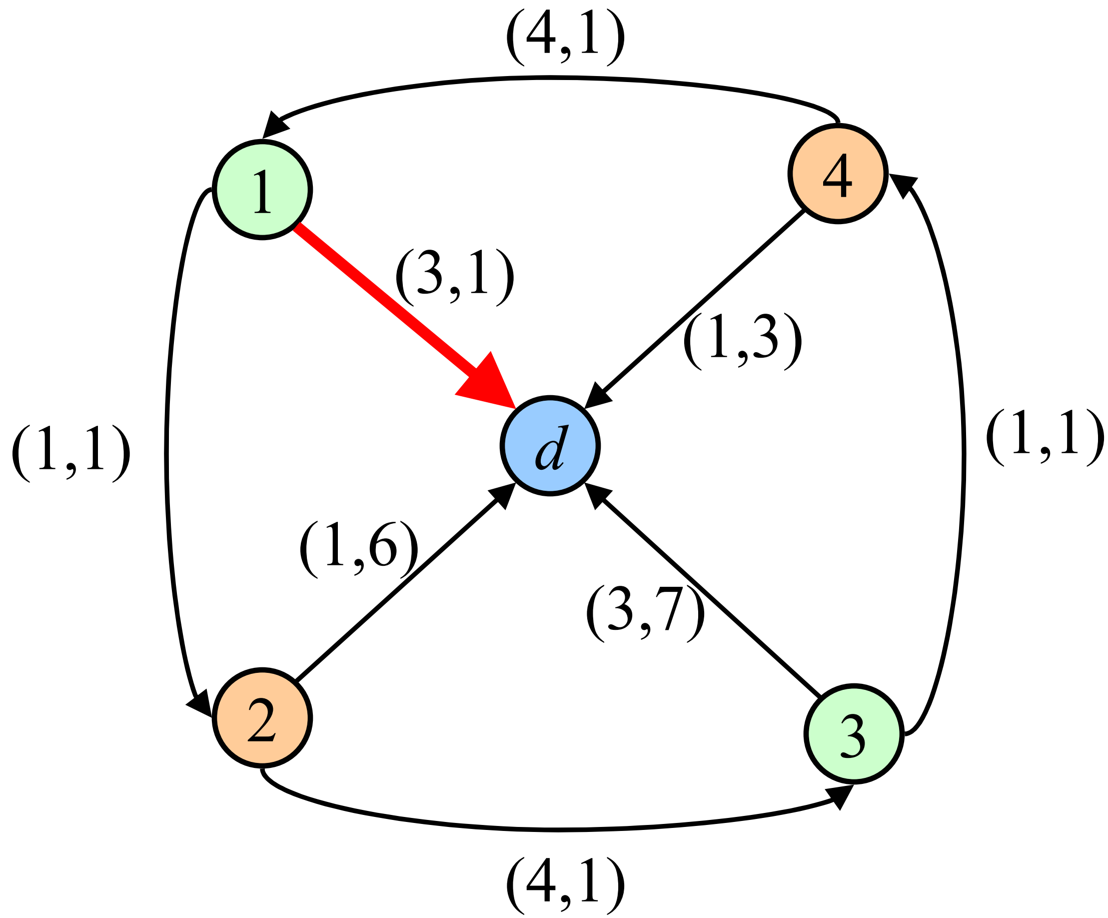
# $\langle \max, \max \rangle$-SPP unsolvable instance



$\overline{f} = \langle \max, \max \rangle$

# $\langle \max, \max \rangle$-SPP unsolvable instance




$\langle$ ◯ , ◯ $\rangle$

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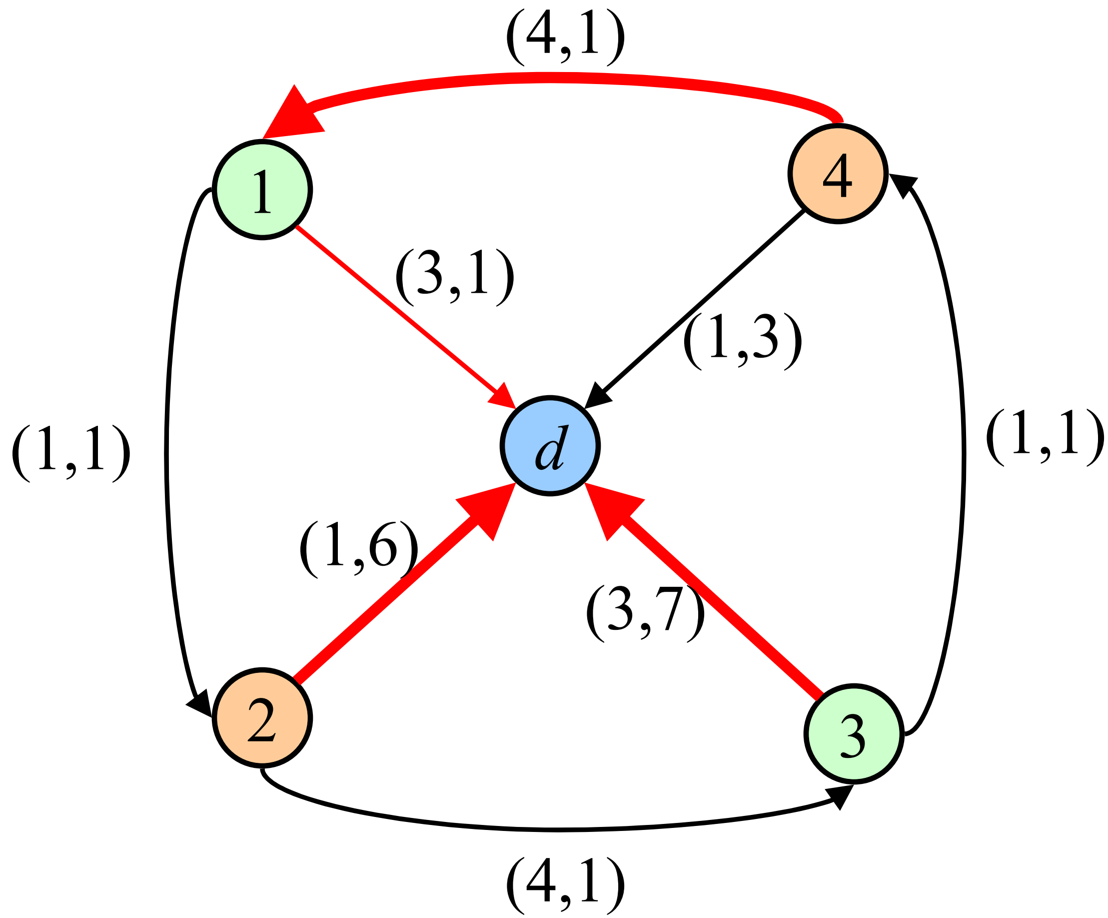
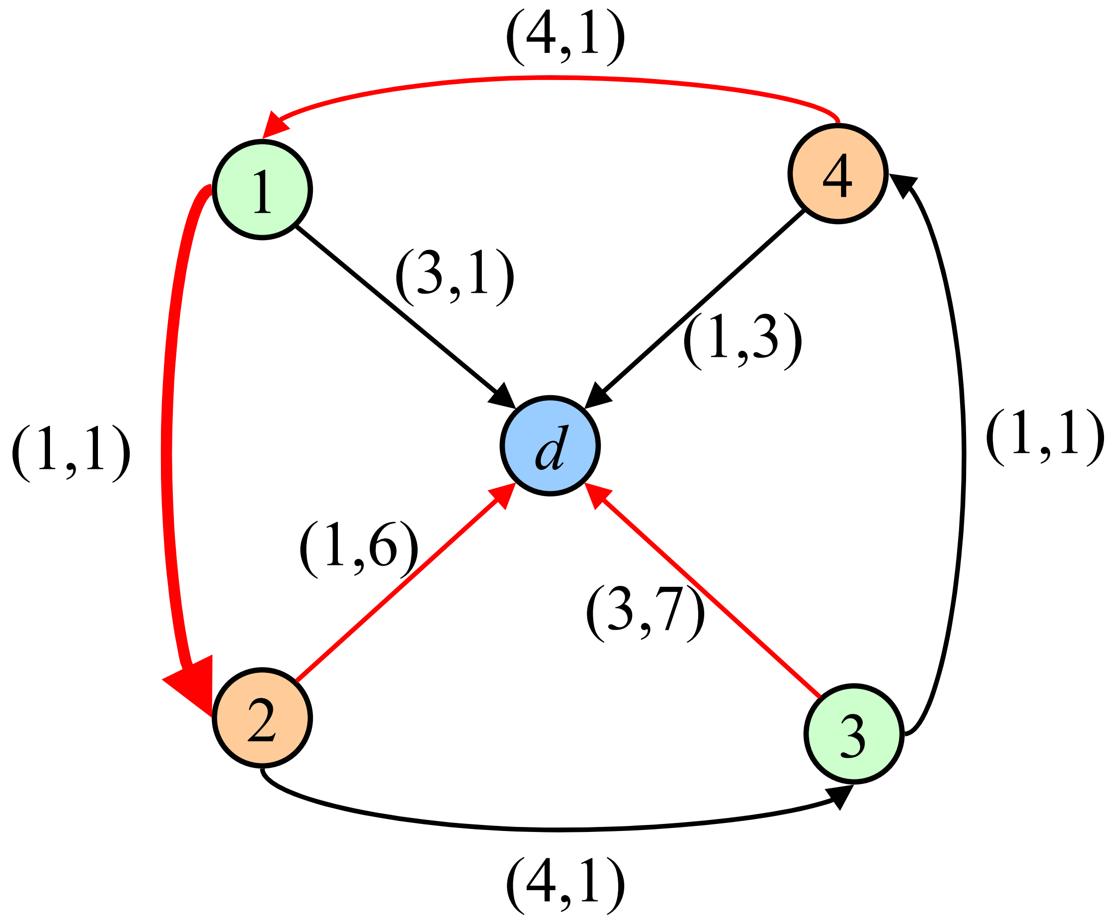
# $\langle \max, \max \rangle$-SPP unsolvable instance




$\langle$ ◯ , ◯ $\rangle$

$\overline{f} = \langle \max, \max \rangle$

# ⟨max, max⟩-SPP unsolvable instance

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$\overline{f} = \langle \max, \max \rangle$

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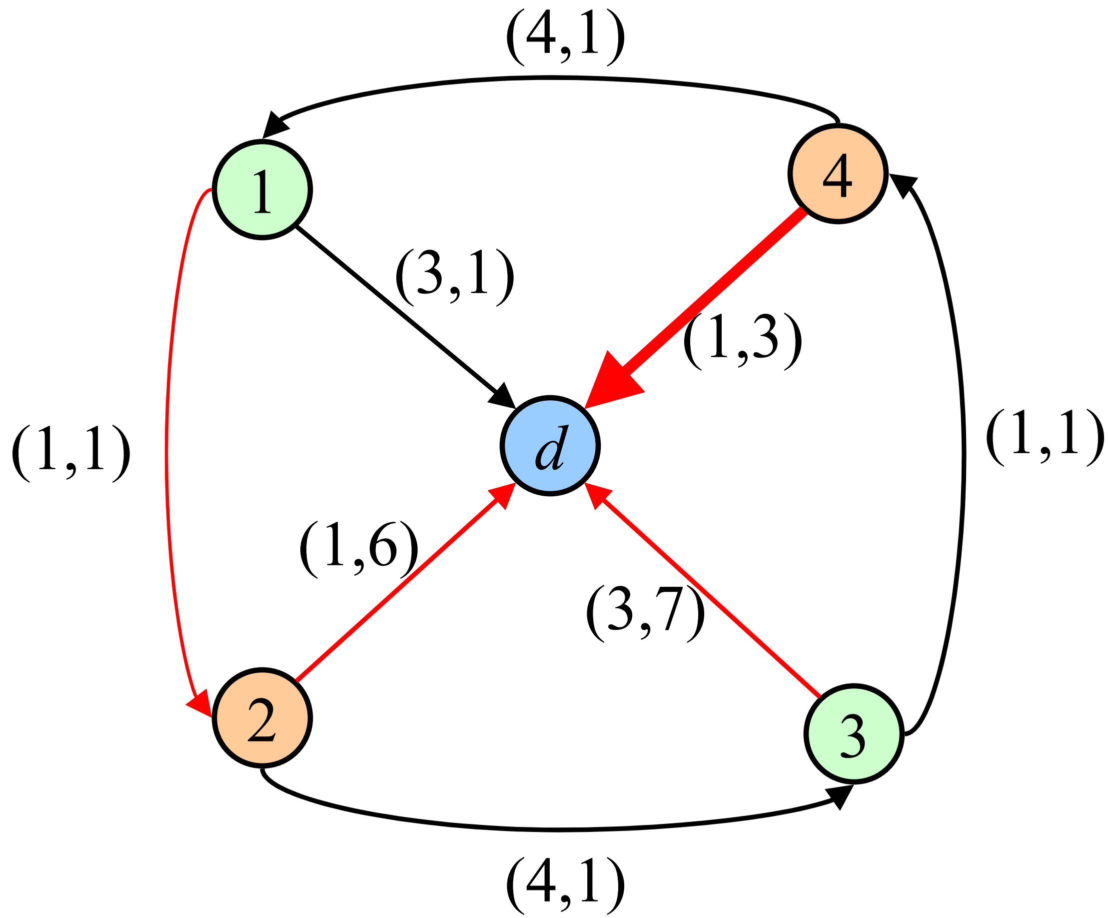


$\langle$ ◯, ◯ $\rangle$

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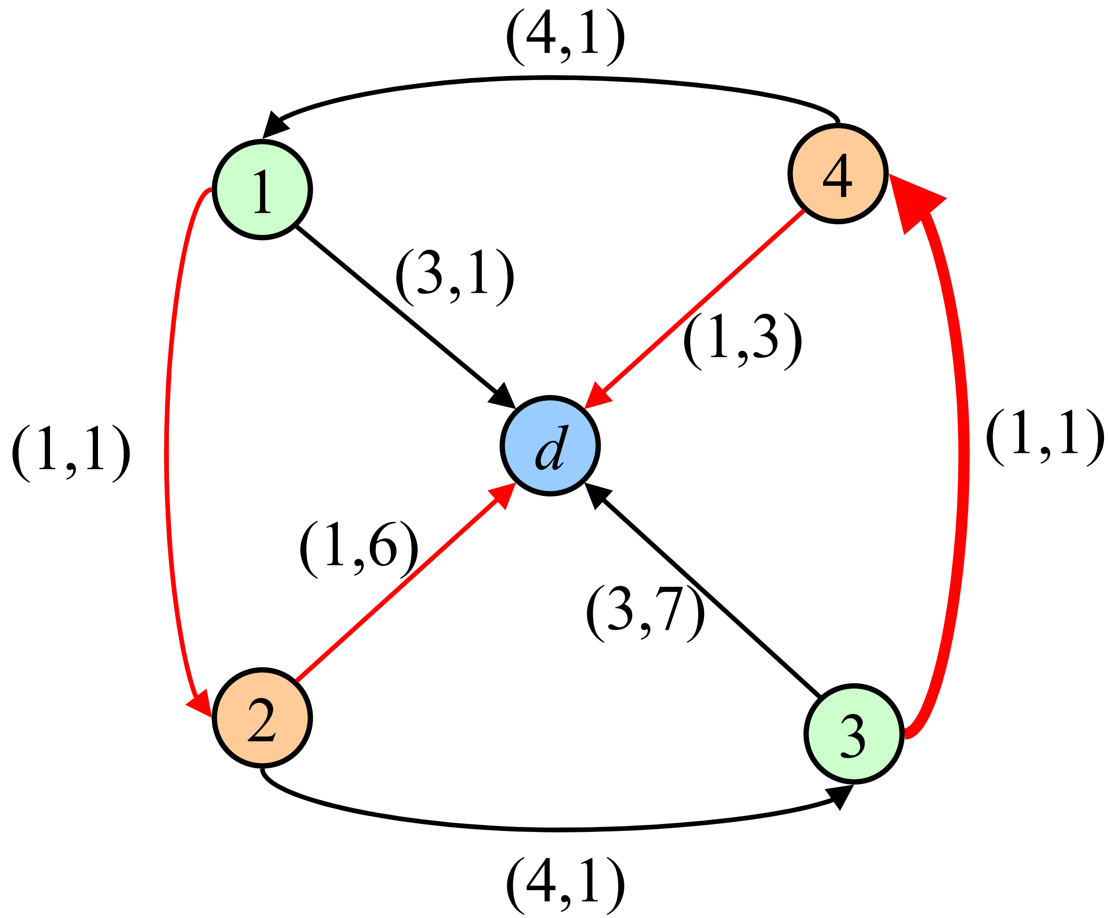


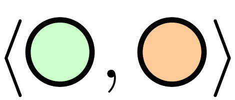

$\overline{f} = \langle \max, \max \rangle$

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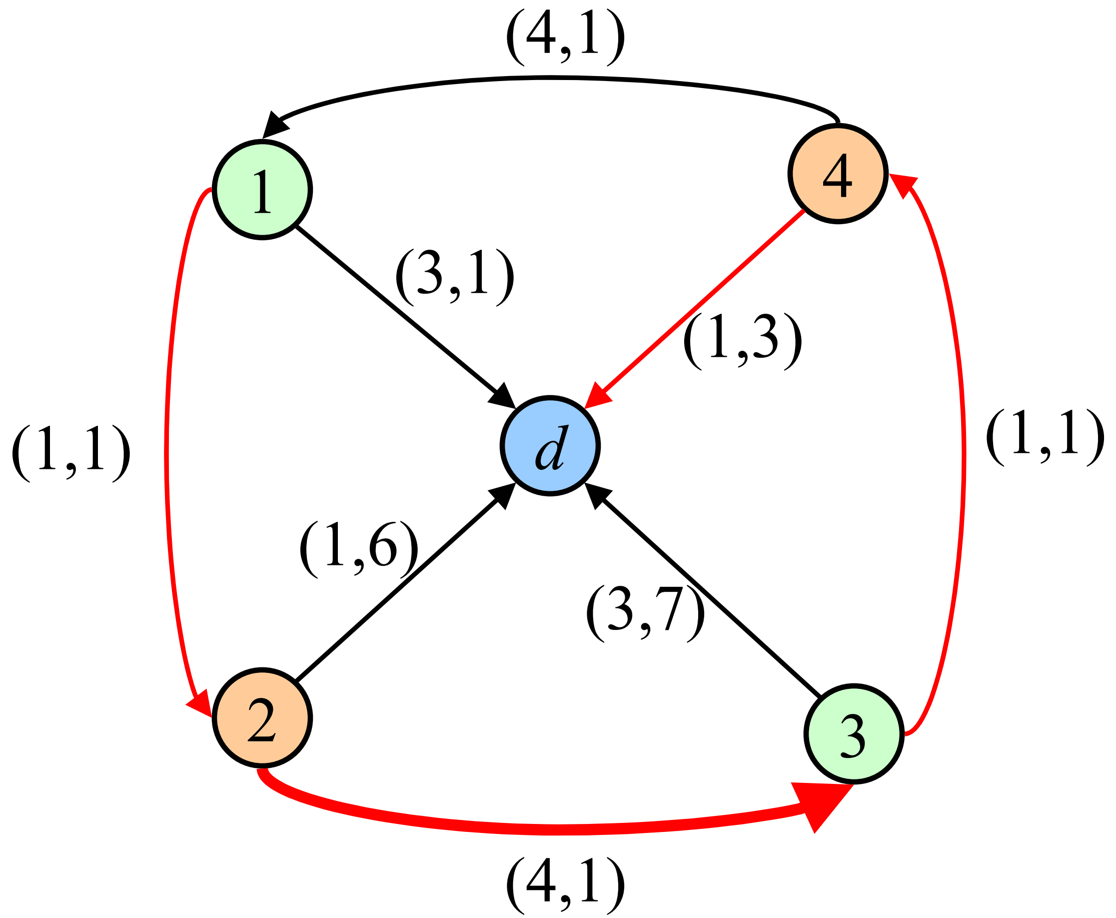


$\langle$ ◯ , ◯ $\rangle$

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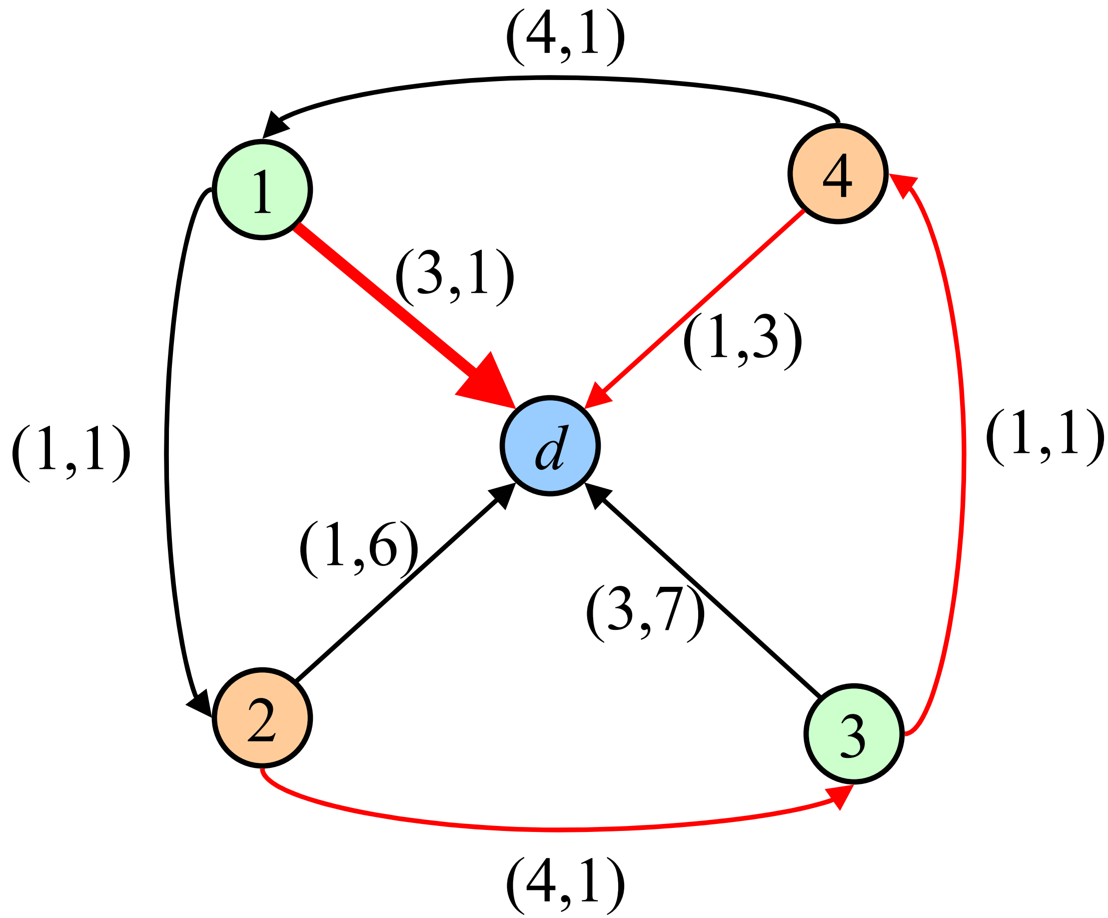


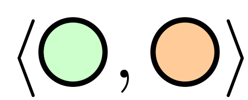

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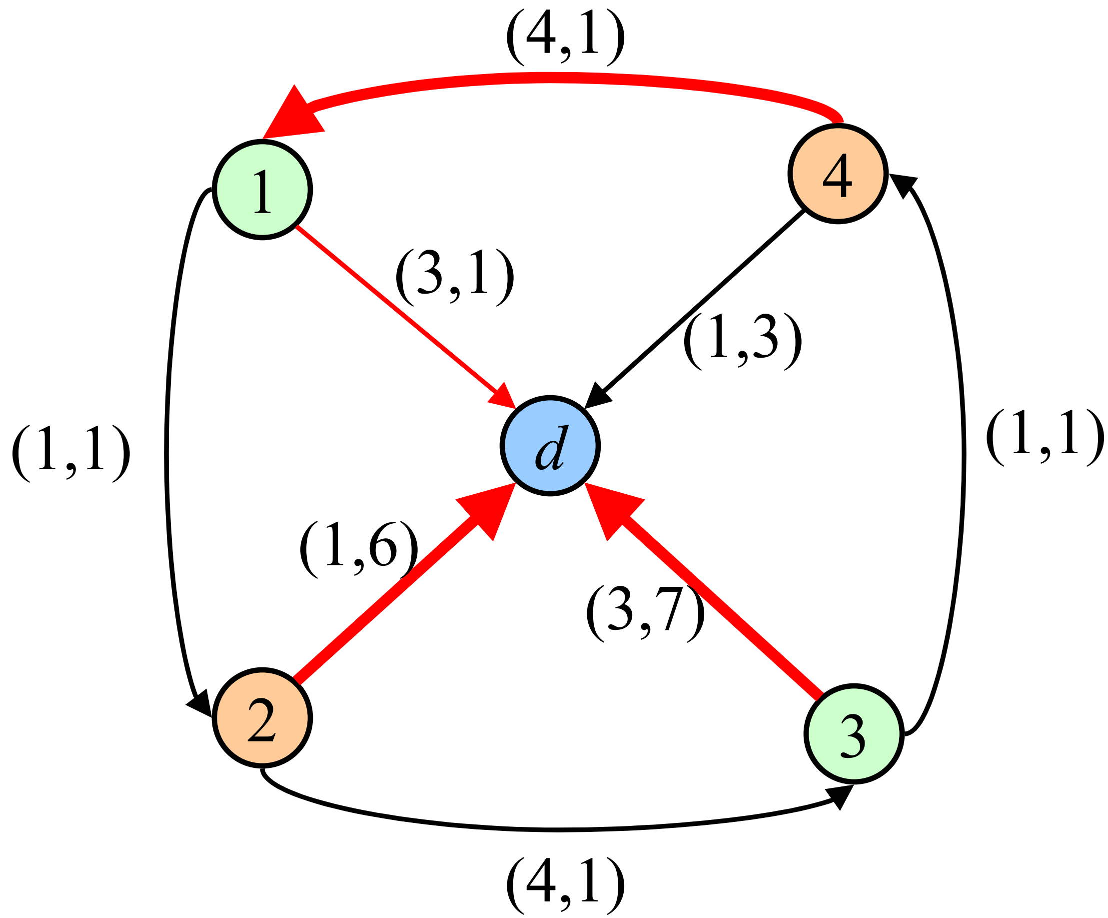


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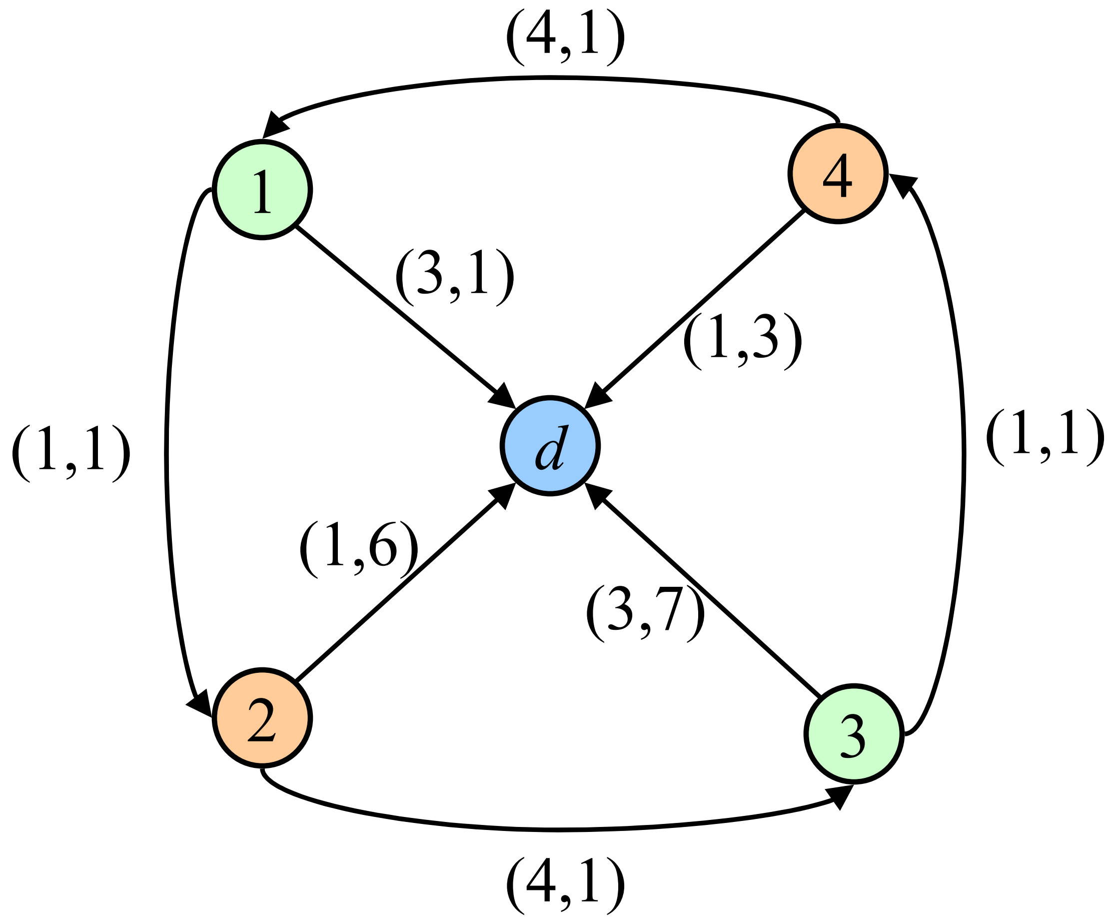
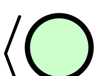
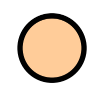



$\langle$  ,  $\rangle$

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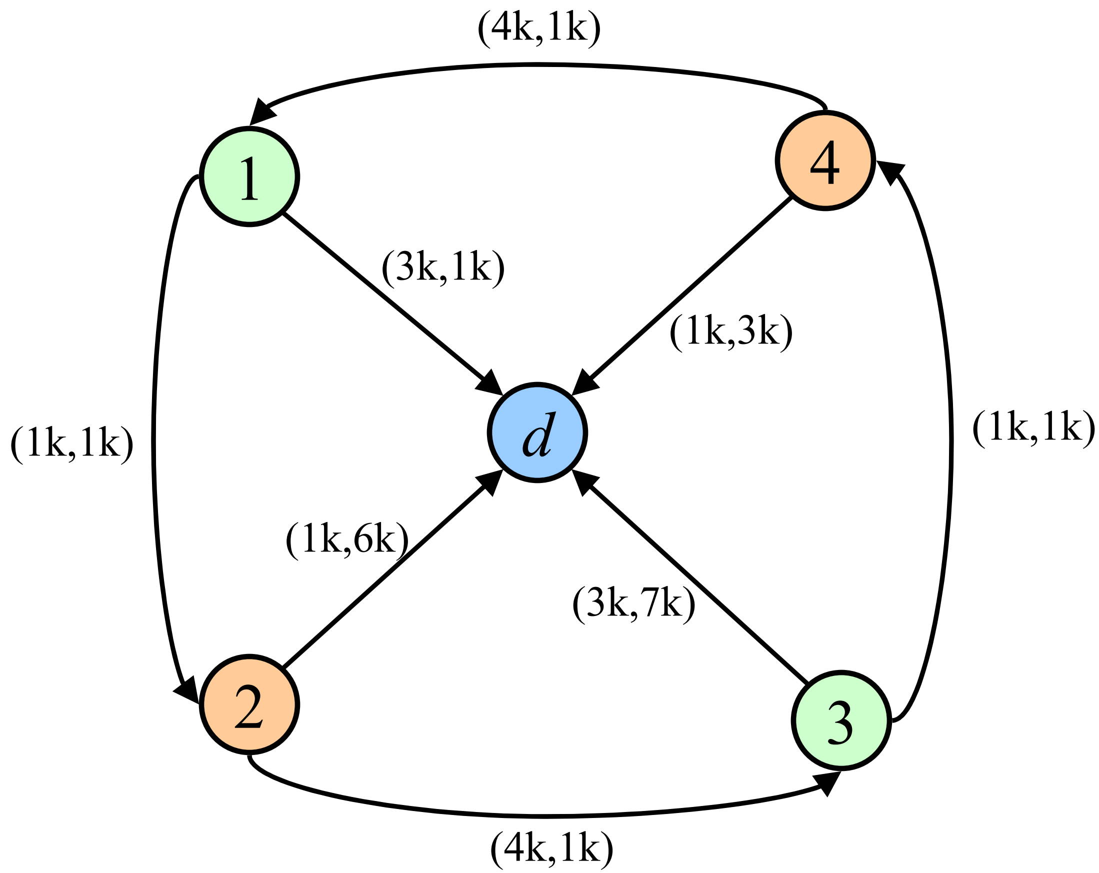


$\langle$ ●, ● $\rangle$

$\overline{f} = \langle \max, \max \rangle$

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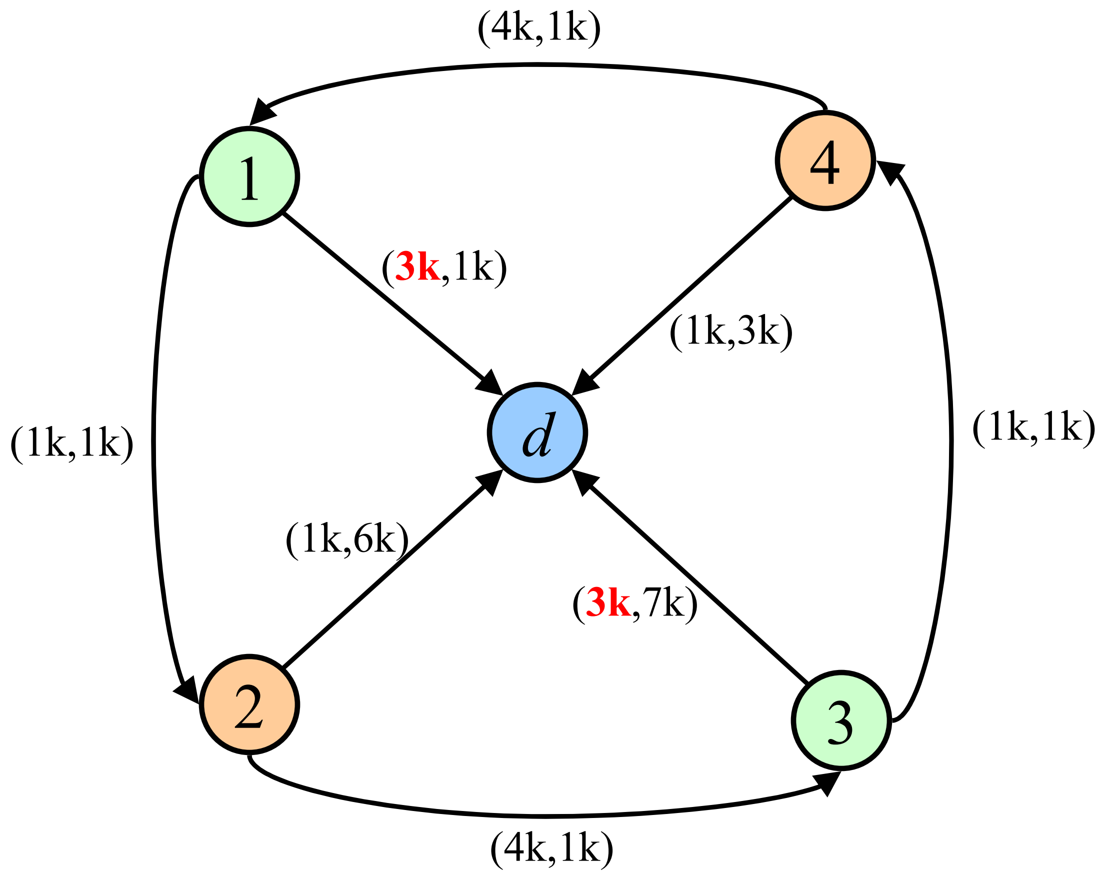
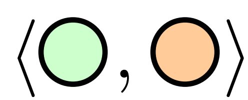
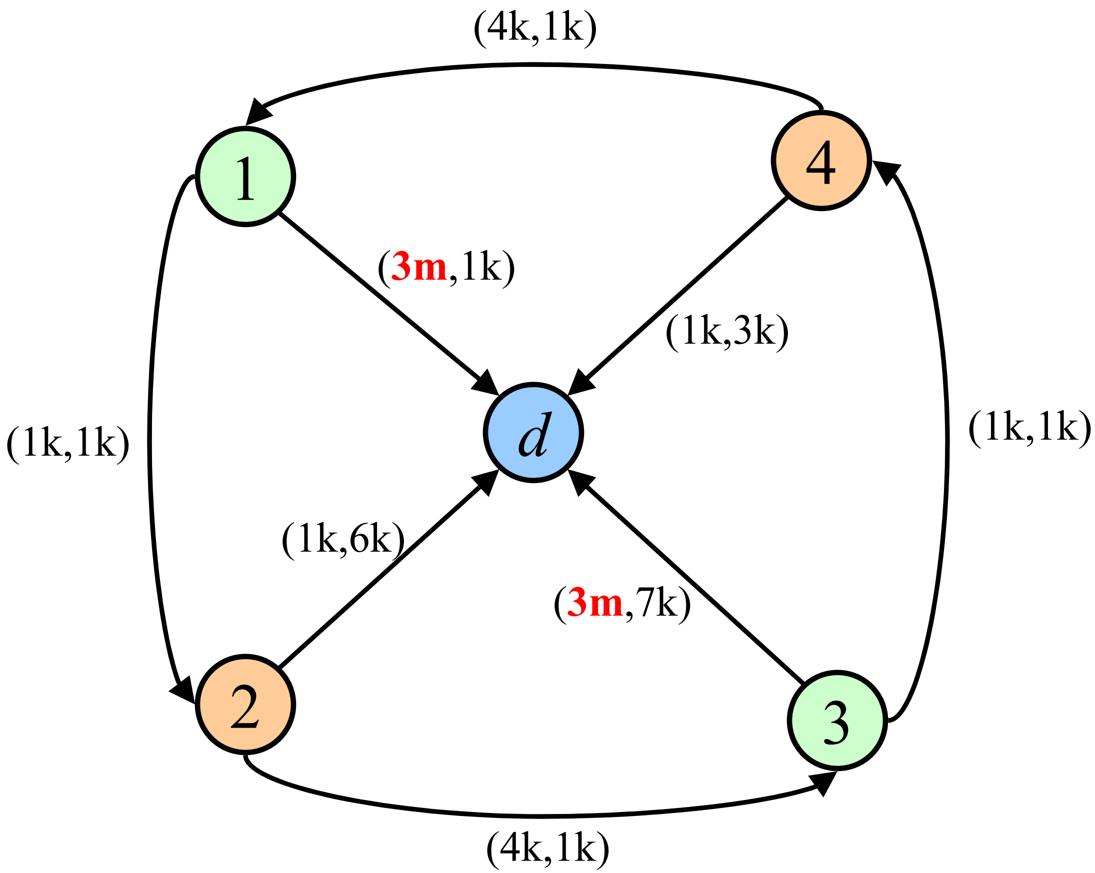


$$\overline{f} = \langle \max, \max \rangle$$

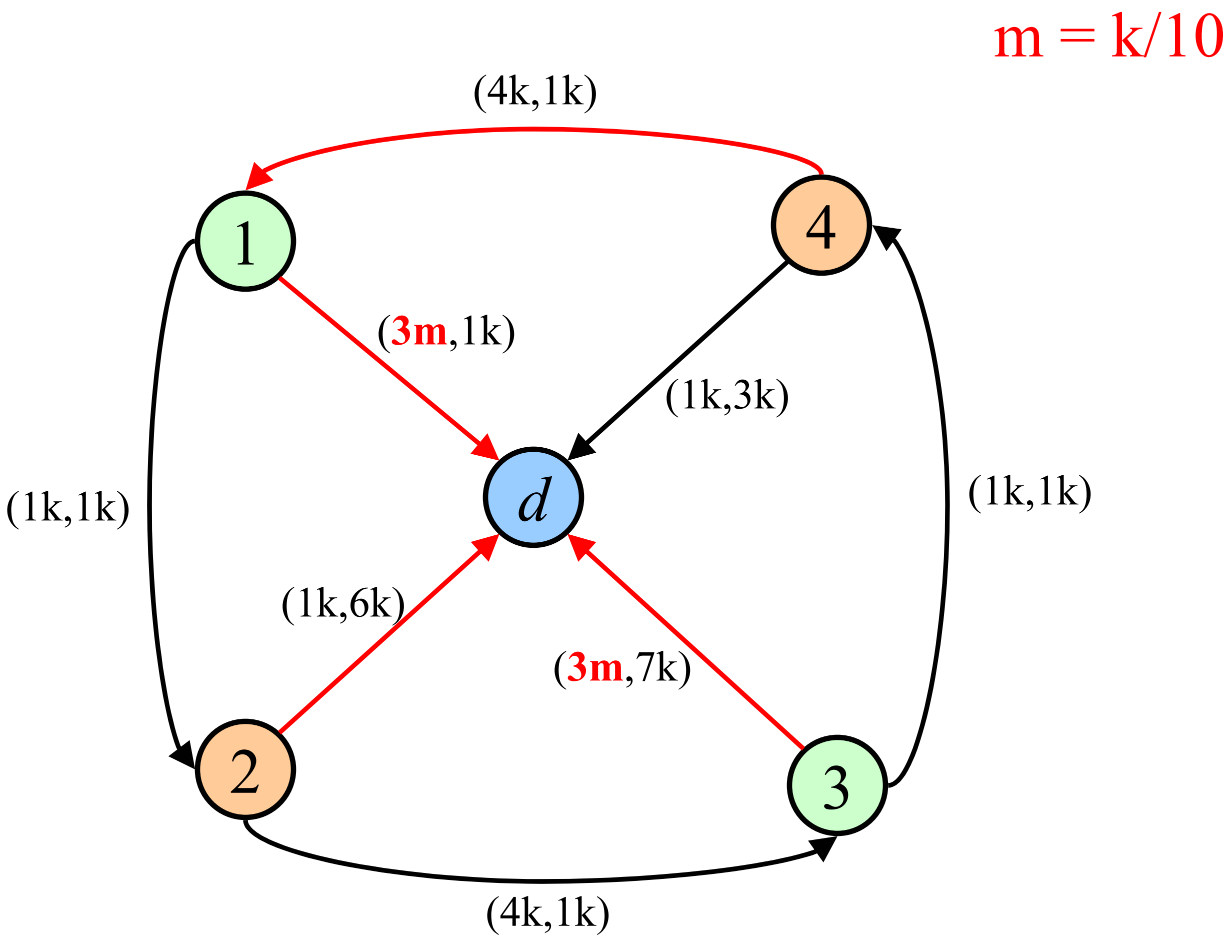
# $\langle \max, \max \rangle$-SPP *solvable* instance

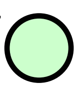
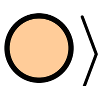
m = k/10 or m = k*10




$\overline{f} = \langle \max, \max \rangle$

# $\langle \max, \max \rangle$-SPP *solvable* instance




$\langle$  ,  $\rangle$

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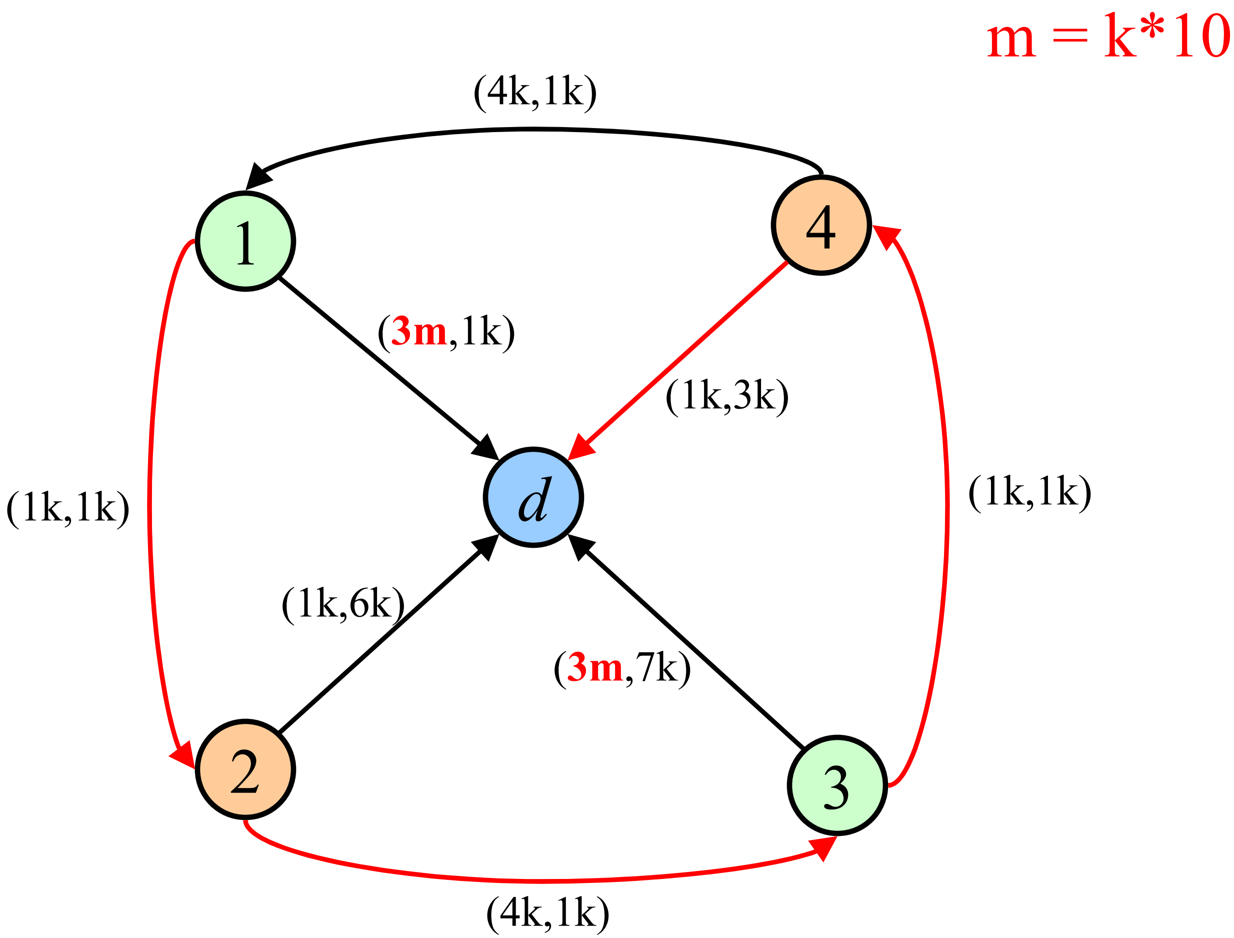



$$\langle \text{green}, \text{orange} \rangle$$

$$\overline{f} = \langle \max, \max \rangle$$

Given an $\overline{f}$-SPP instance

$$(V, E, d),\ \pi,\ w,\ \langle f_1, \ldots, f_n \rangle,$$

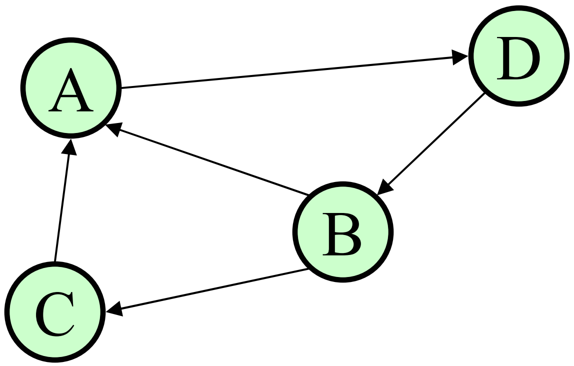
does there exist a stable solution?

instance of the Hamiltonian circuit problem

$$\Downarrow$$

instance of $\langle \max, \max \rangle$-SPP

s.t. a circuit exists iff a stable solution exists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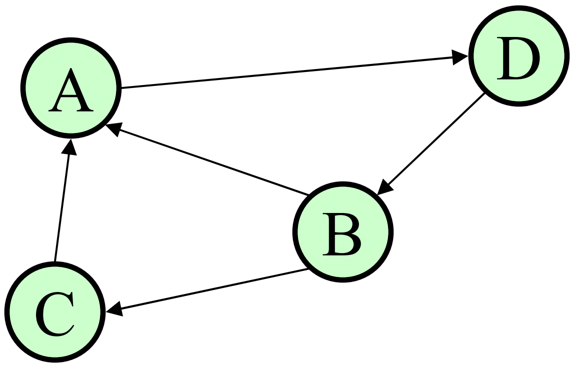
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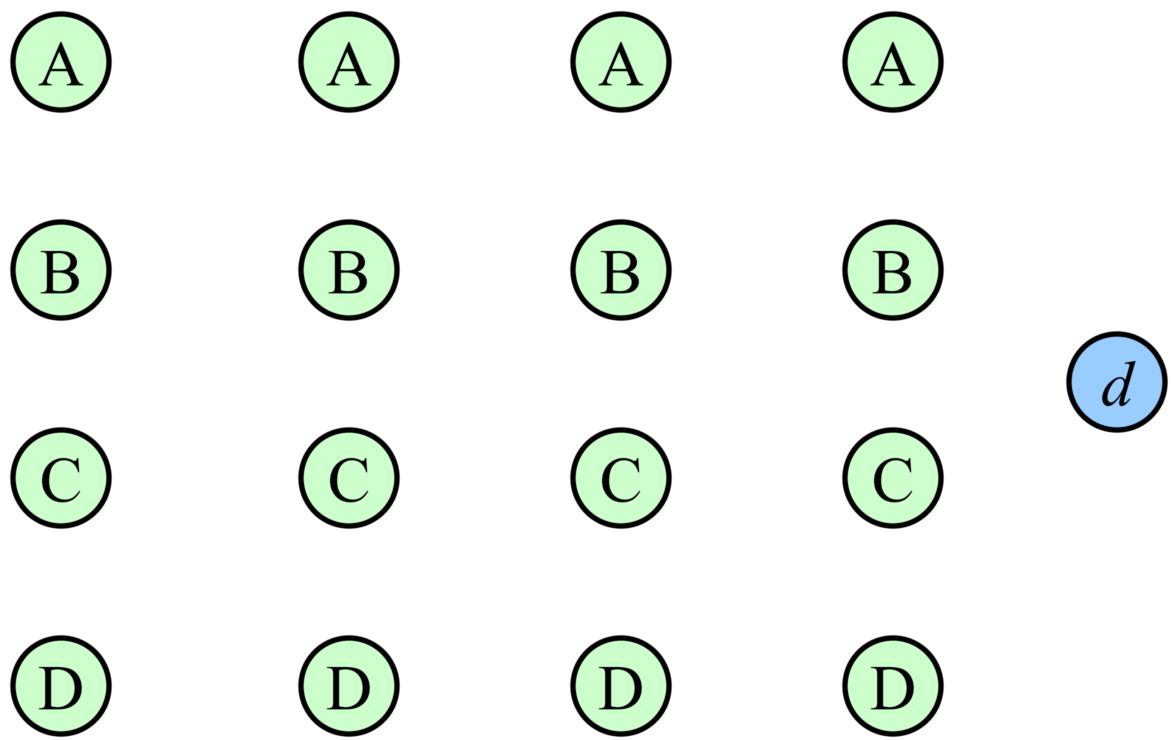
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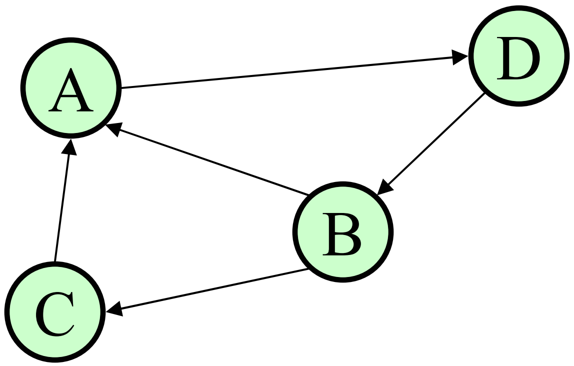
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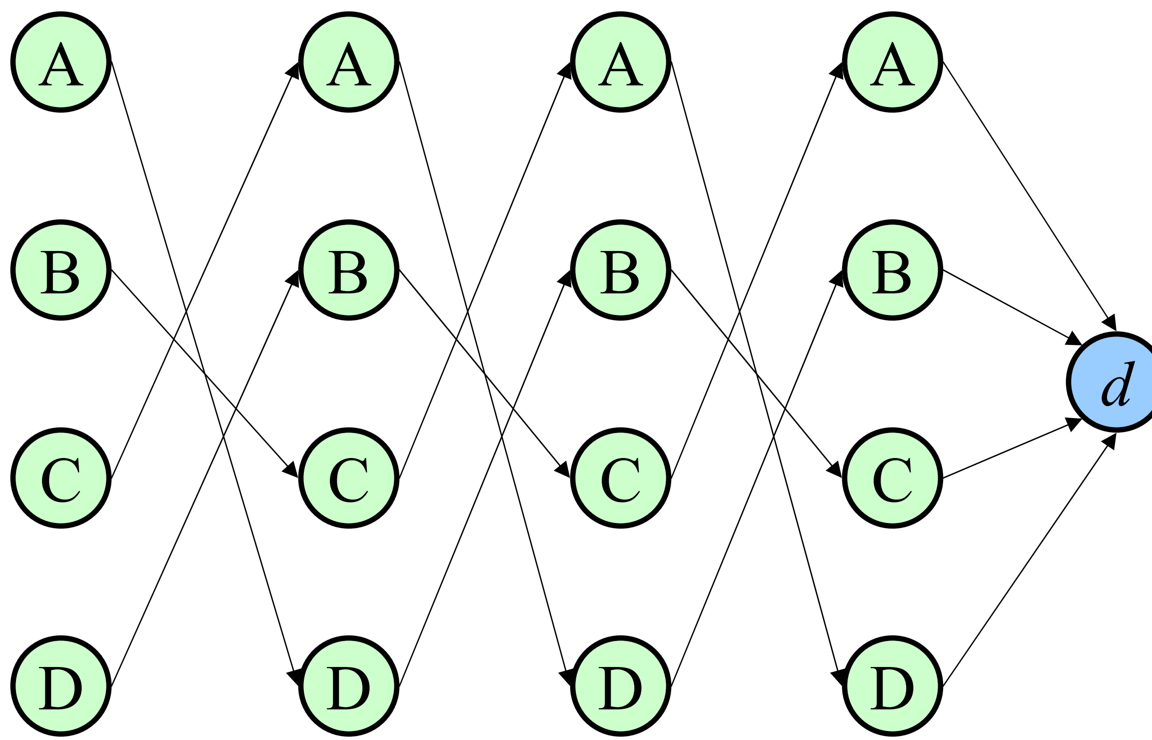
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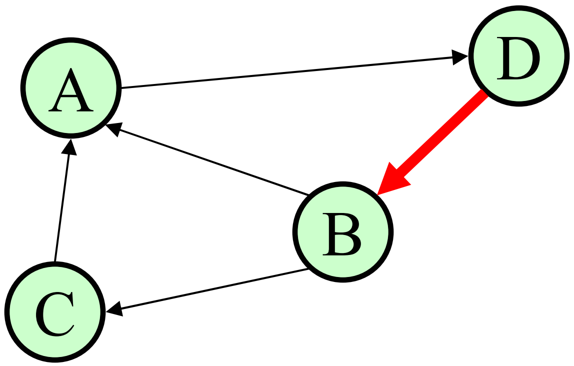

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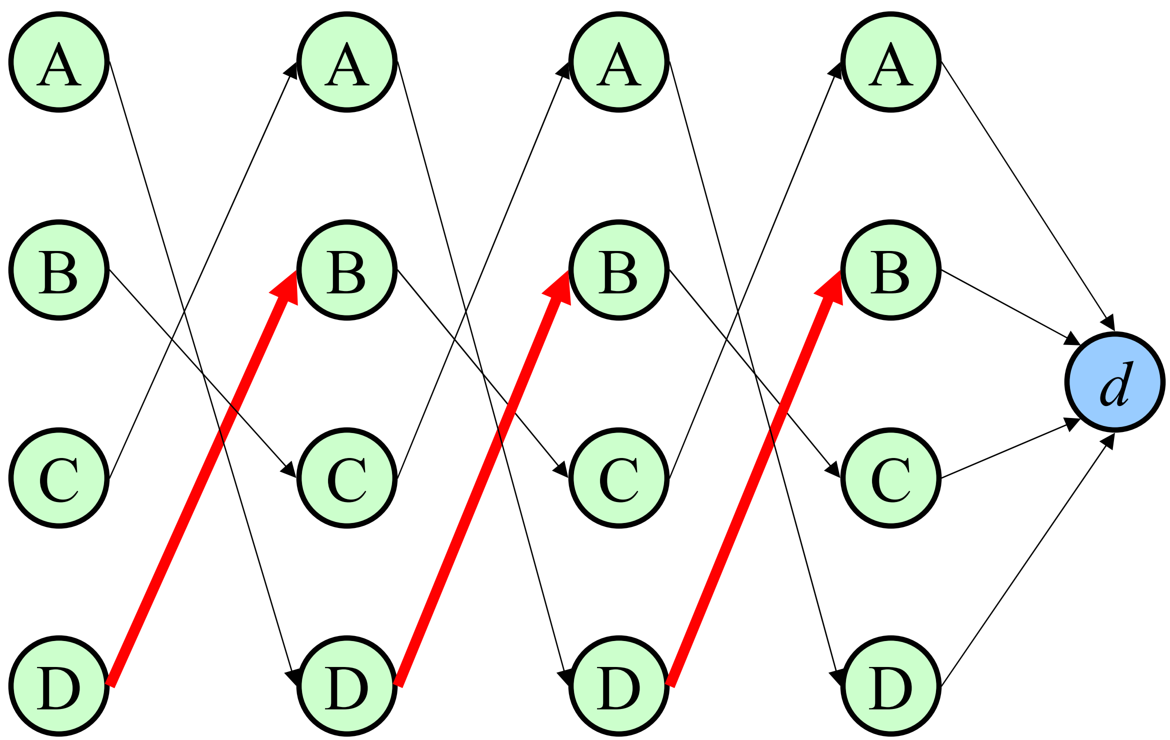

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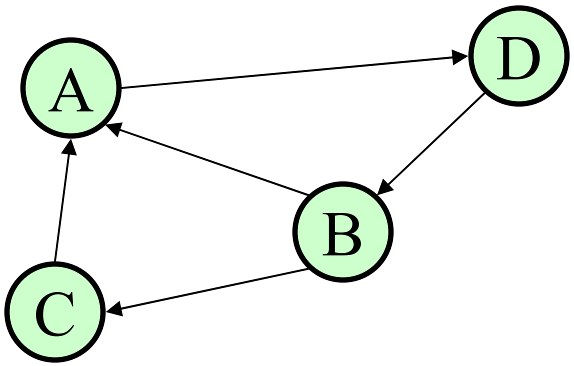
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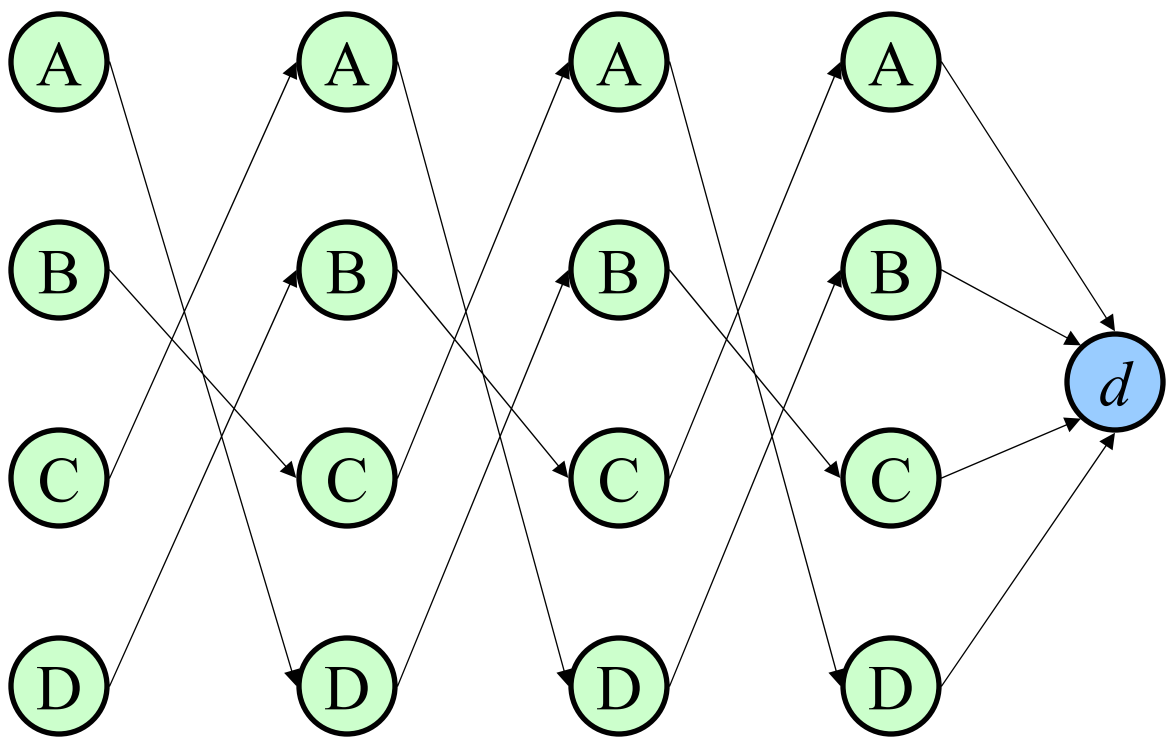
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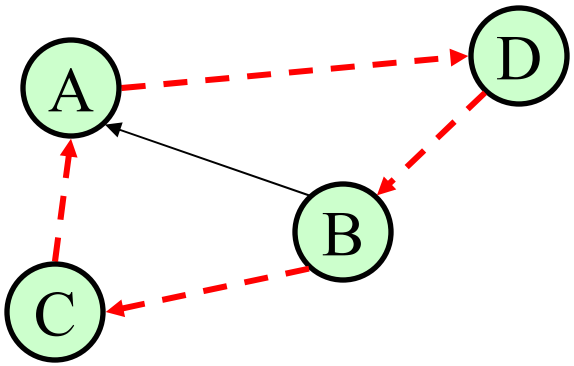
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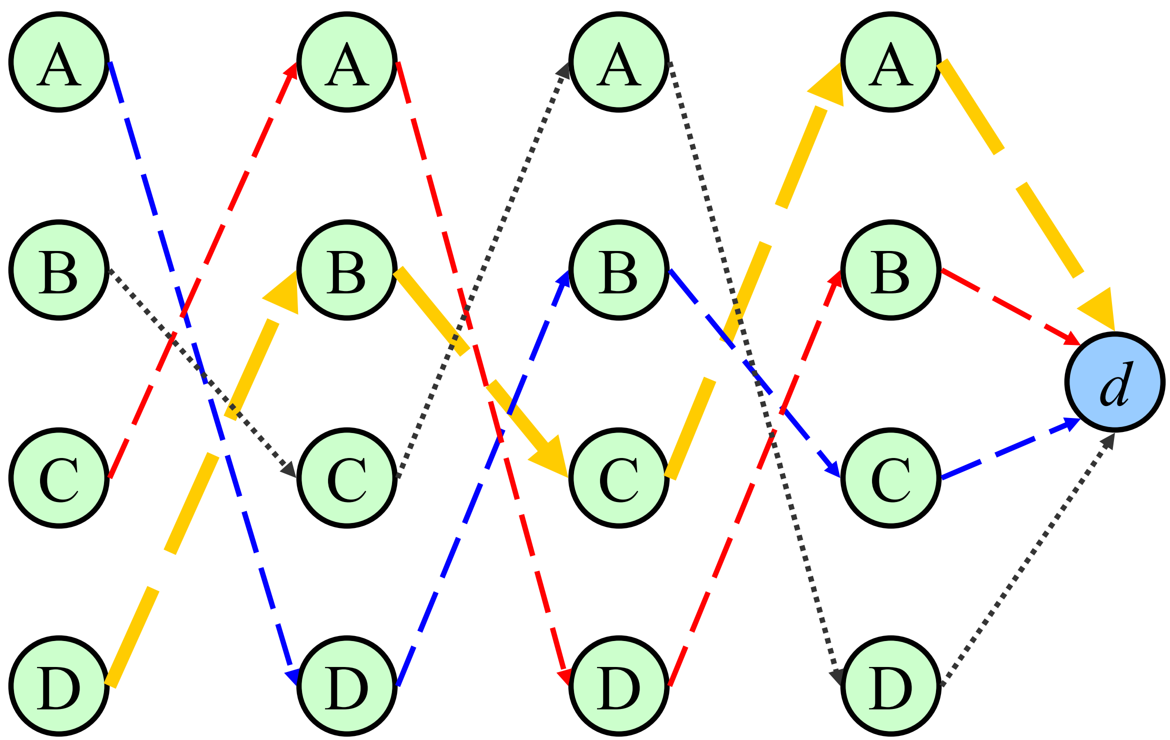
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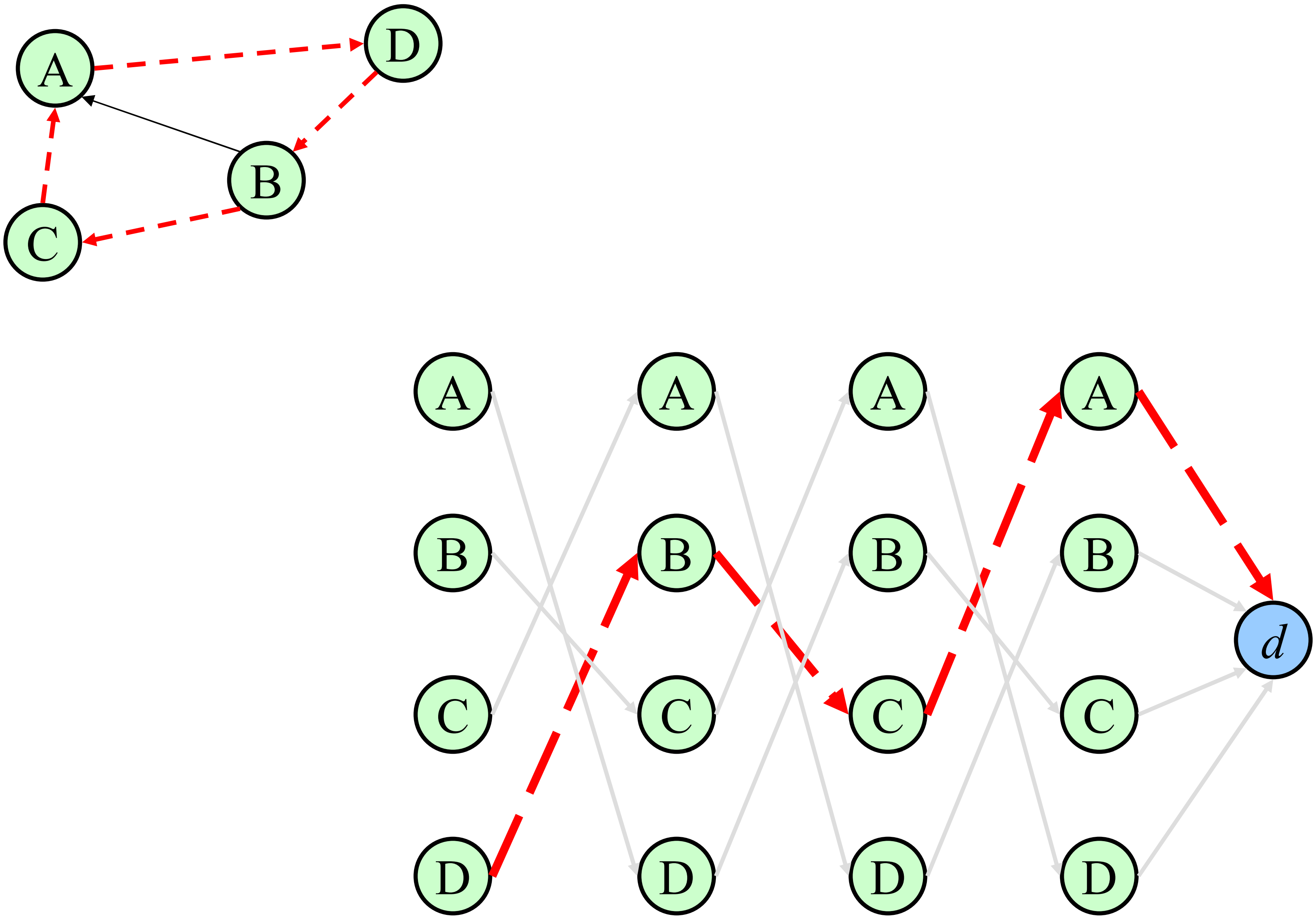
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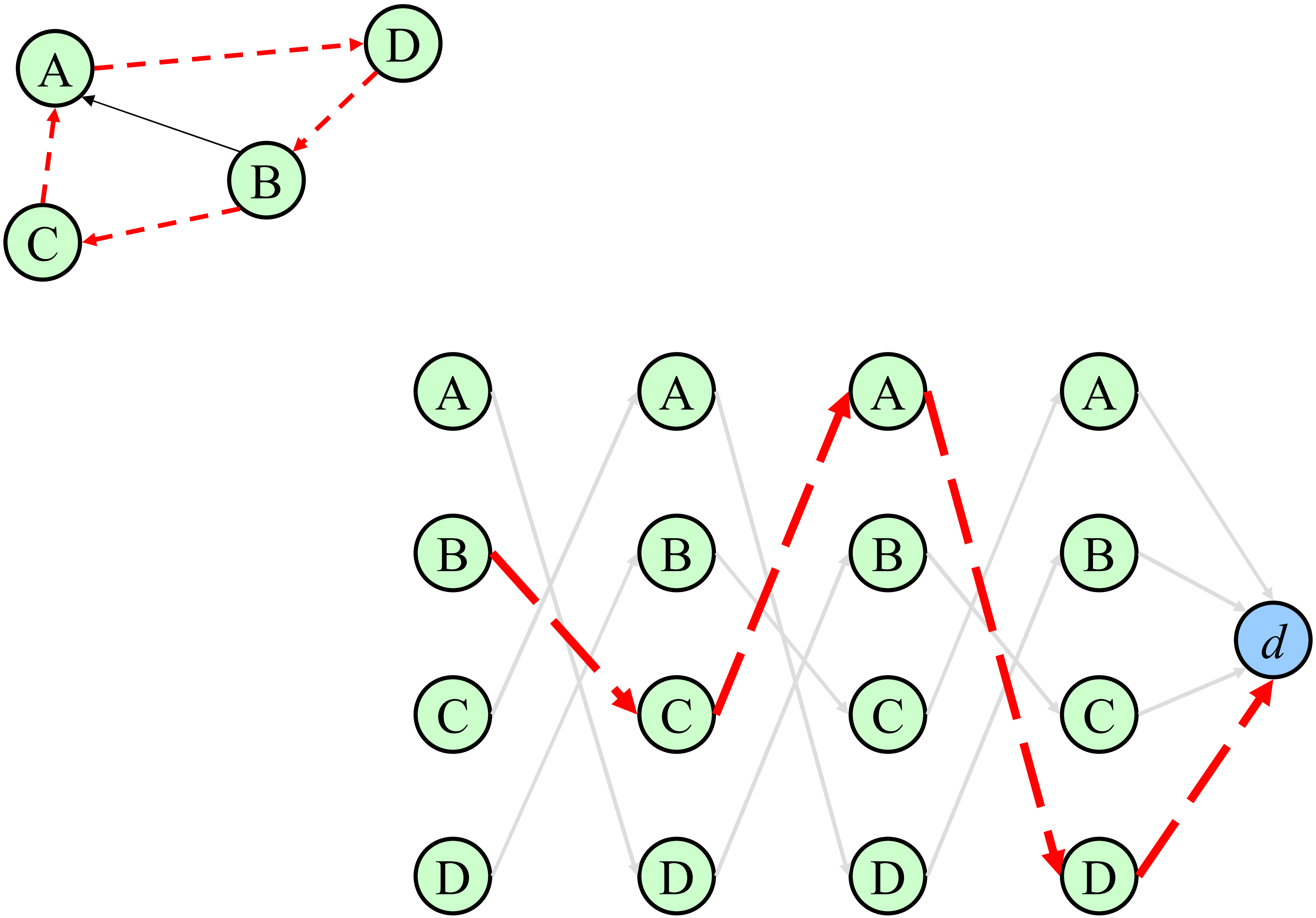
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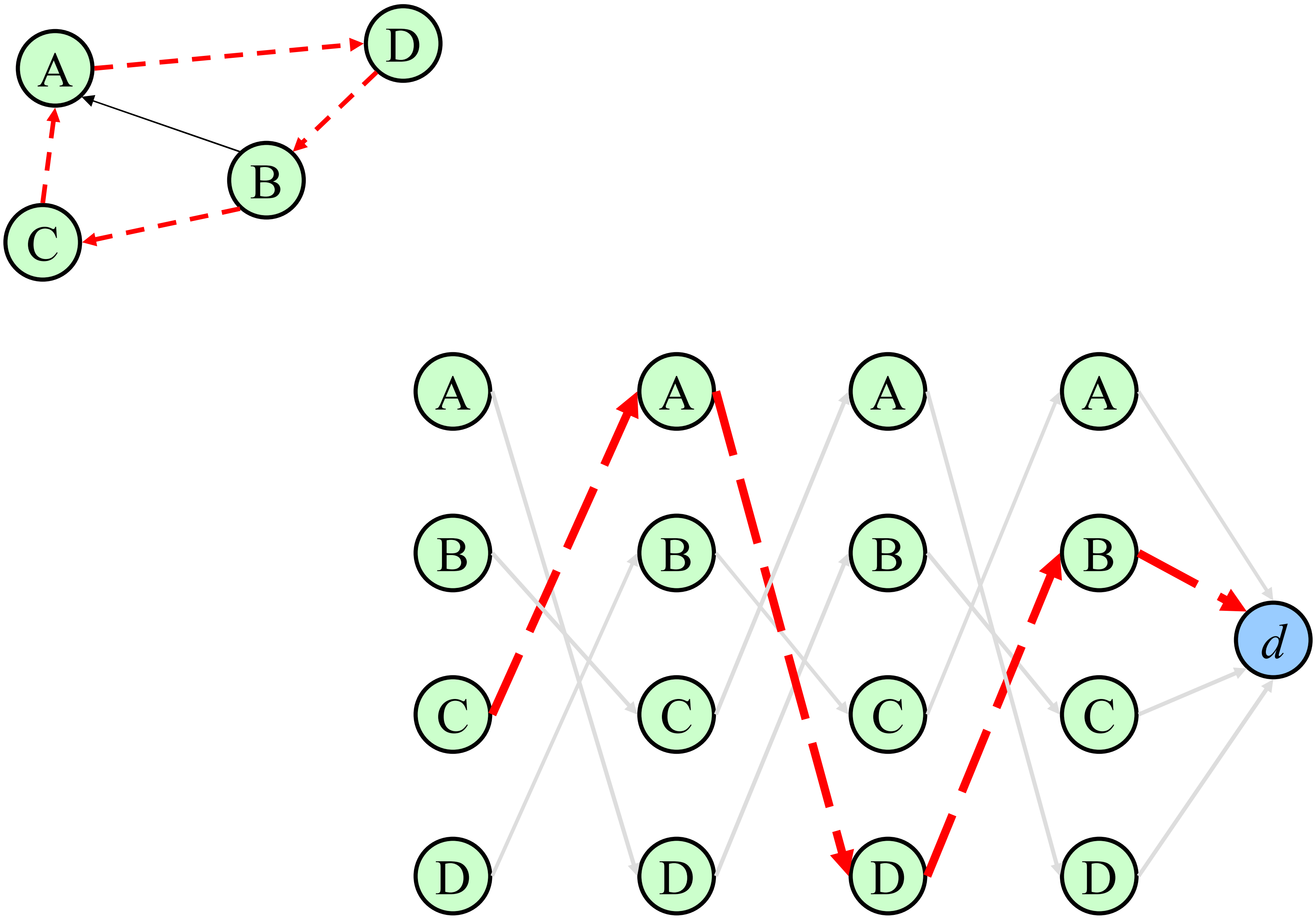
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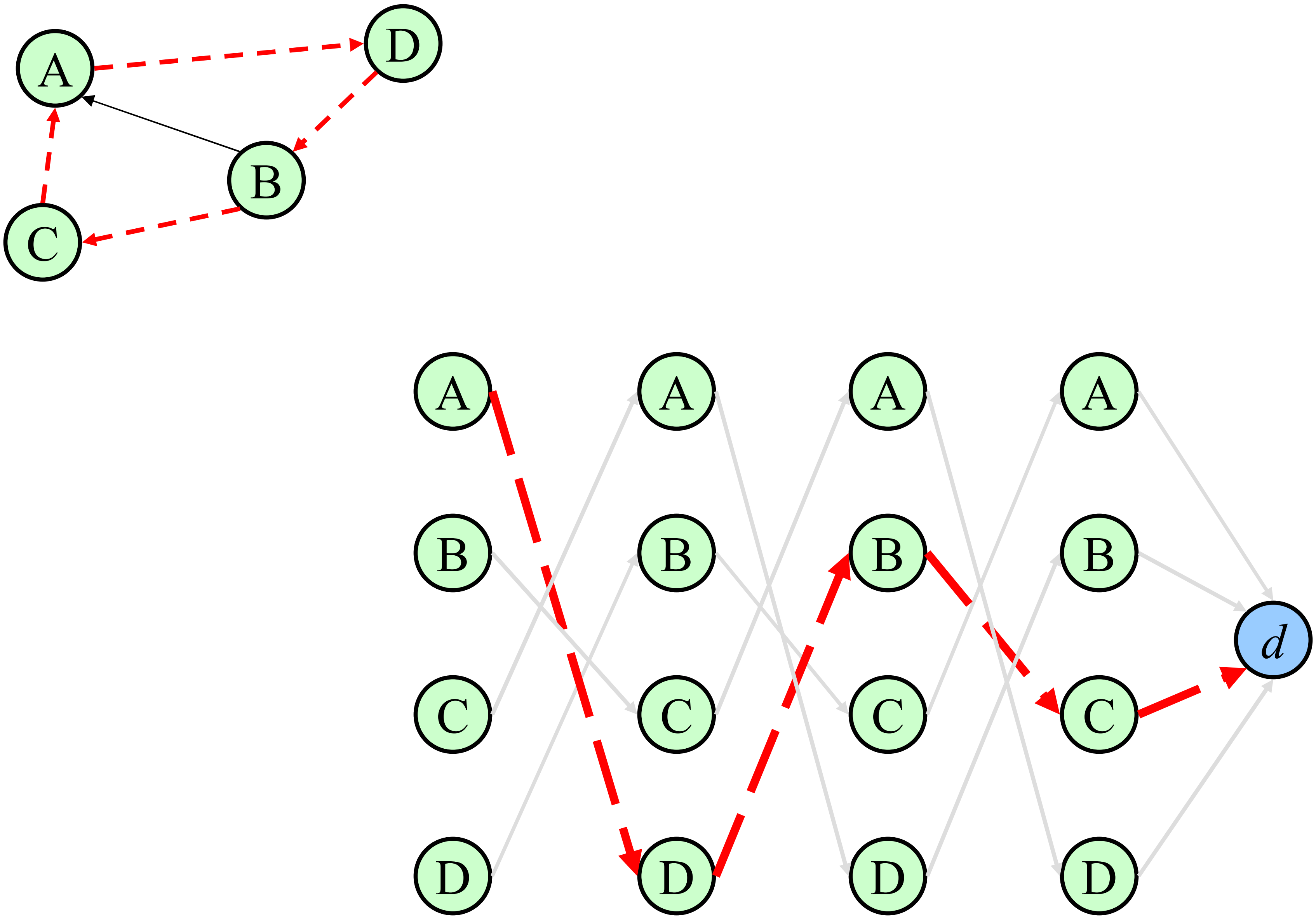
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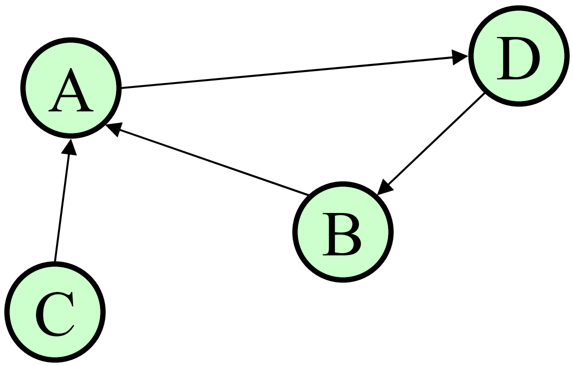
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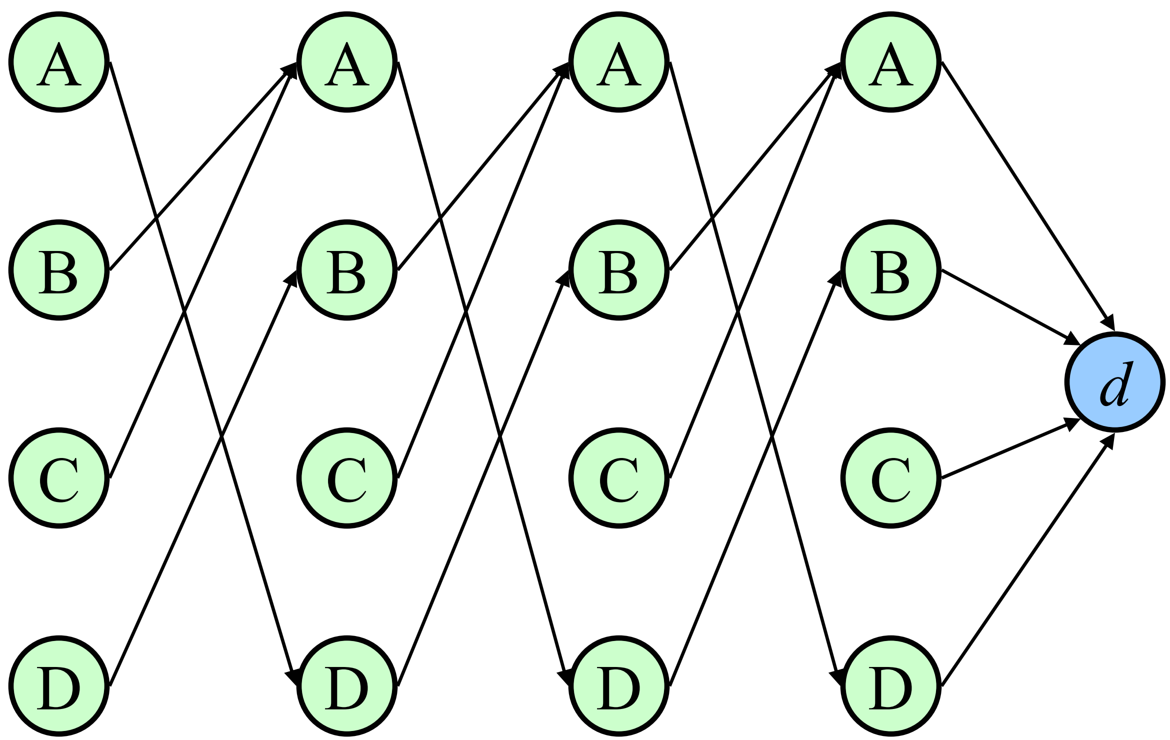
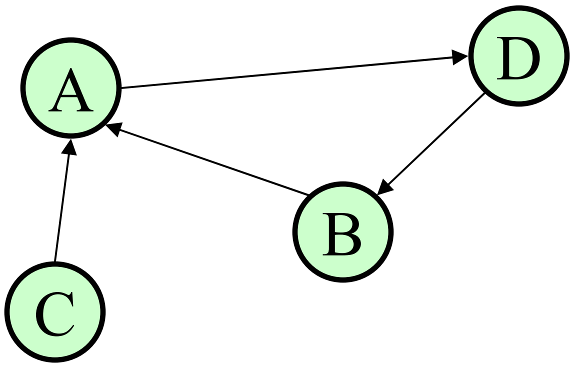
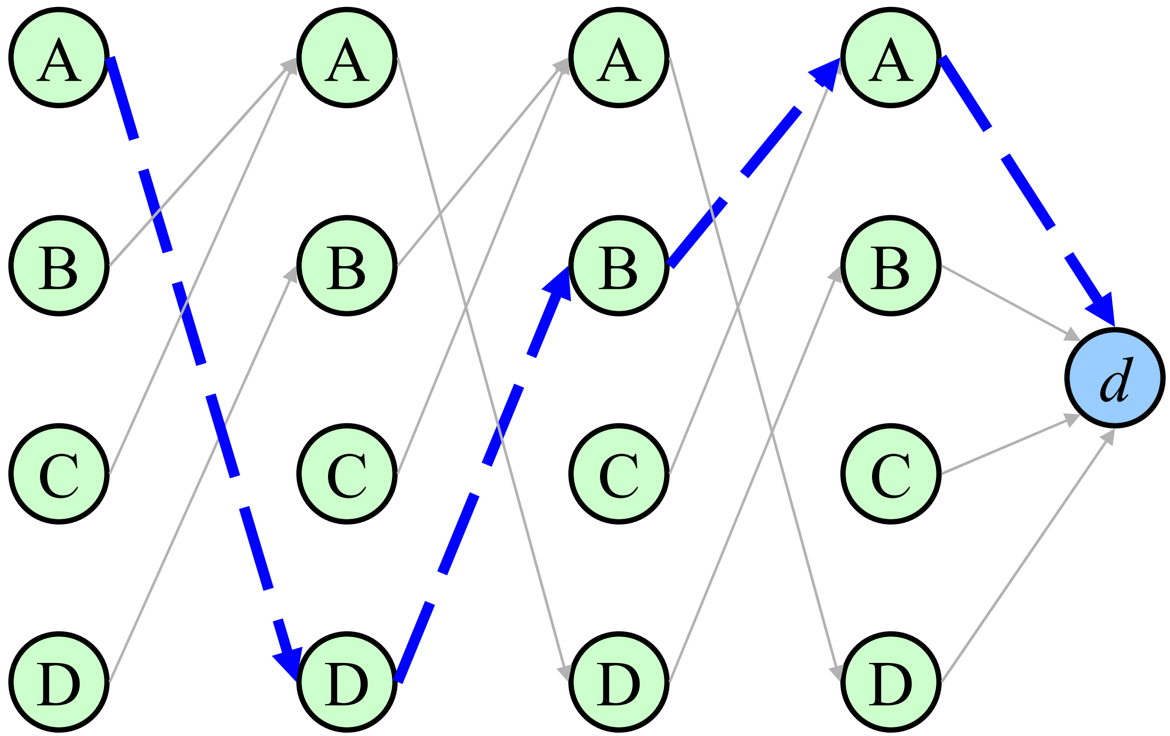
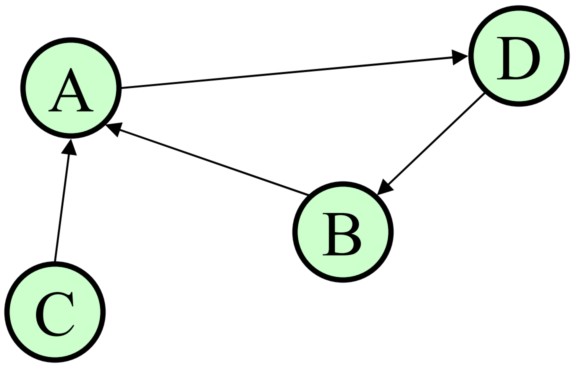
B
C


A
B
C
D
A
B
C
D
A
B
C
D
A
B
C
D
d

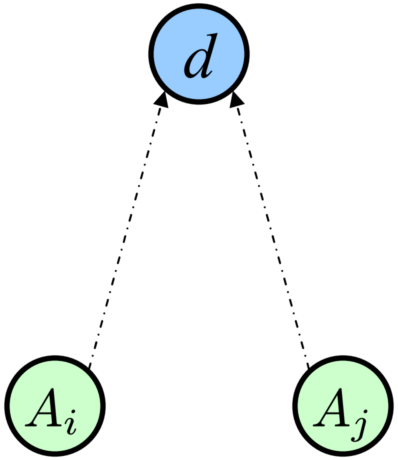

A
D
B
C


A
B
C
D
A
B
C
D
A
B
C
D
A
B
C
D
d

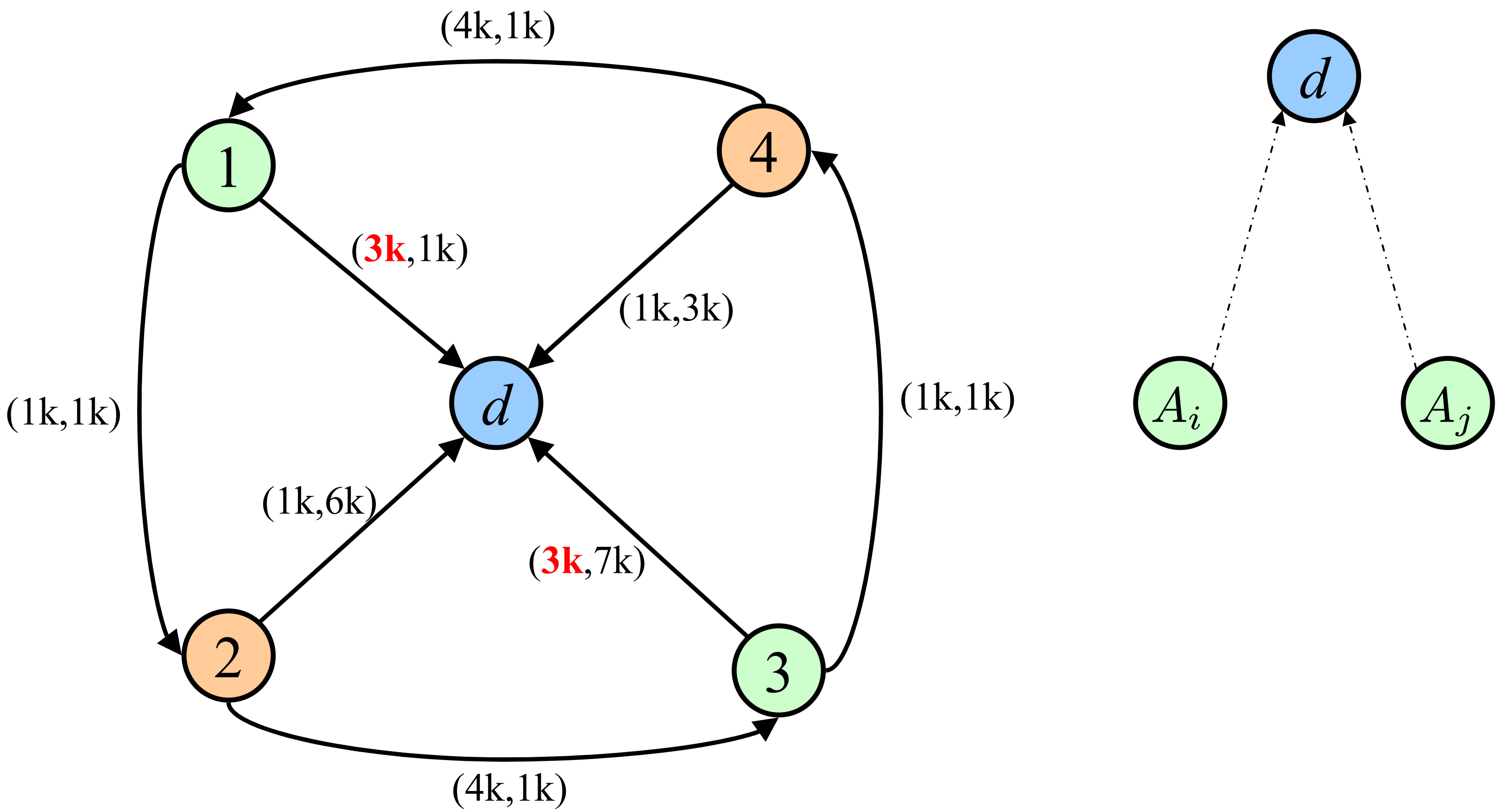

A
D
B
C

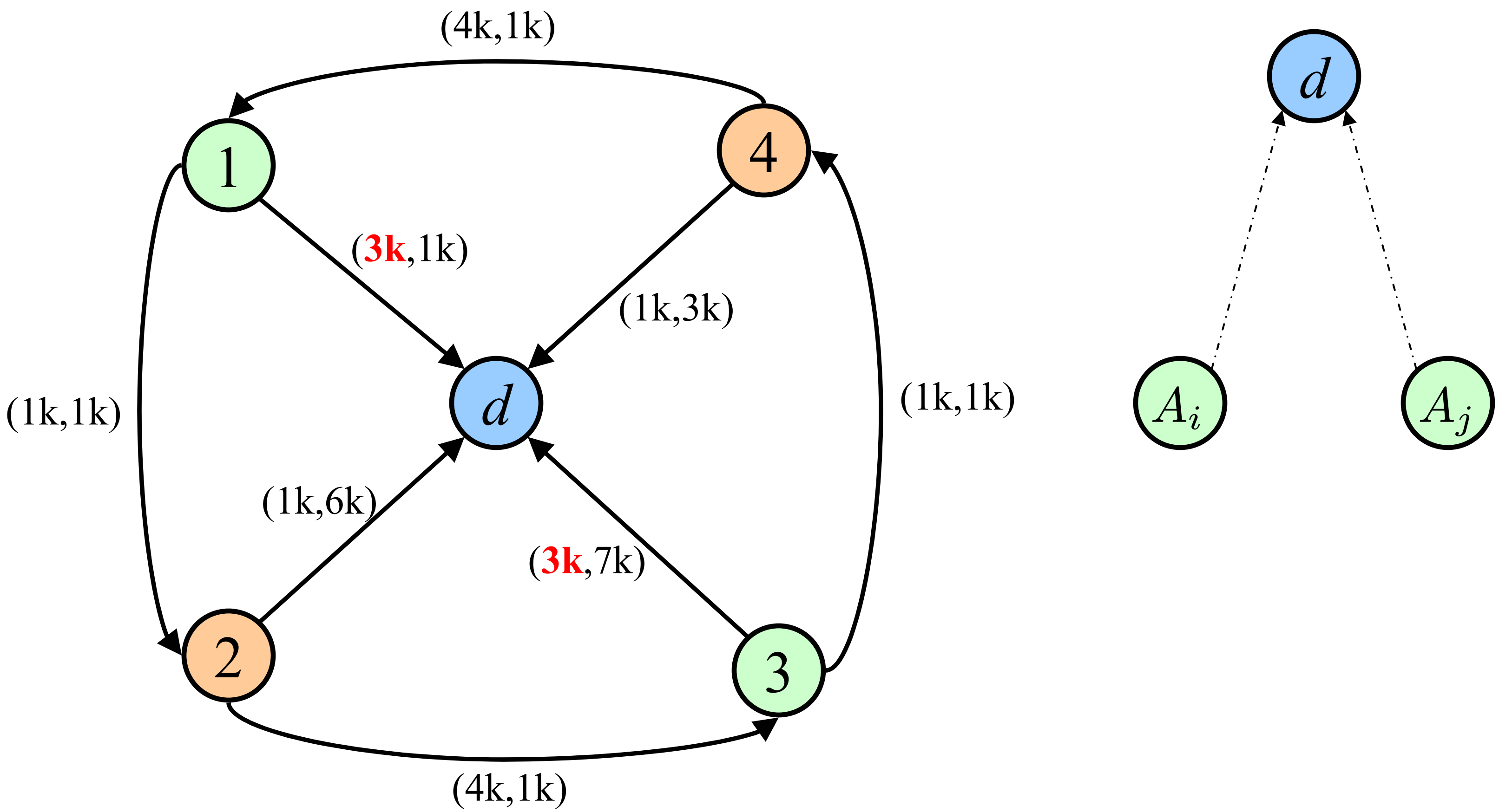

A
A

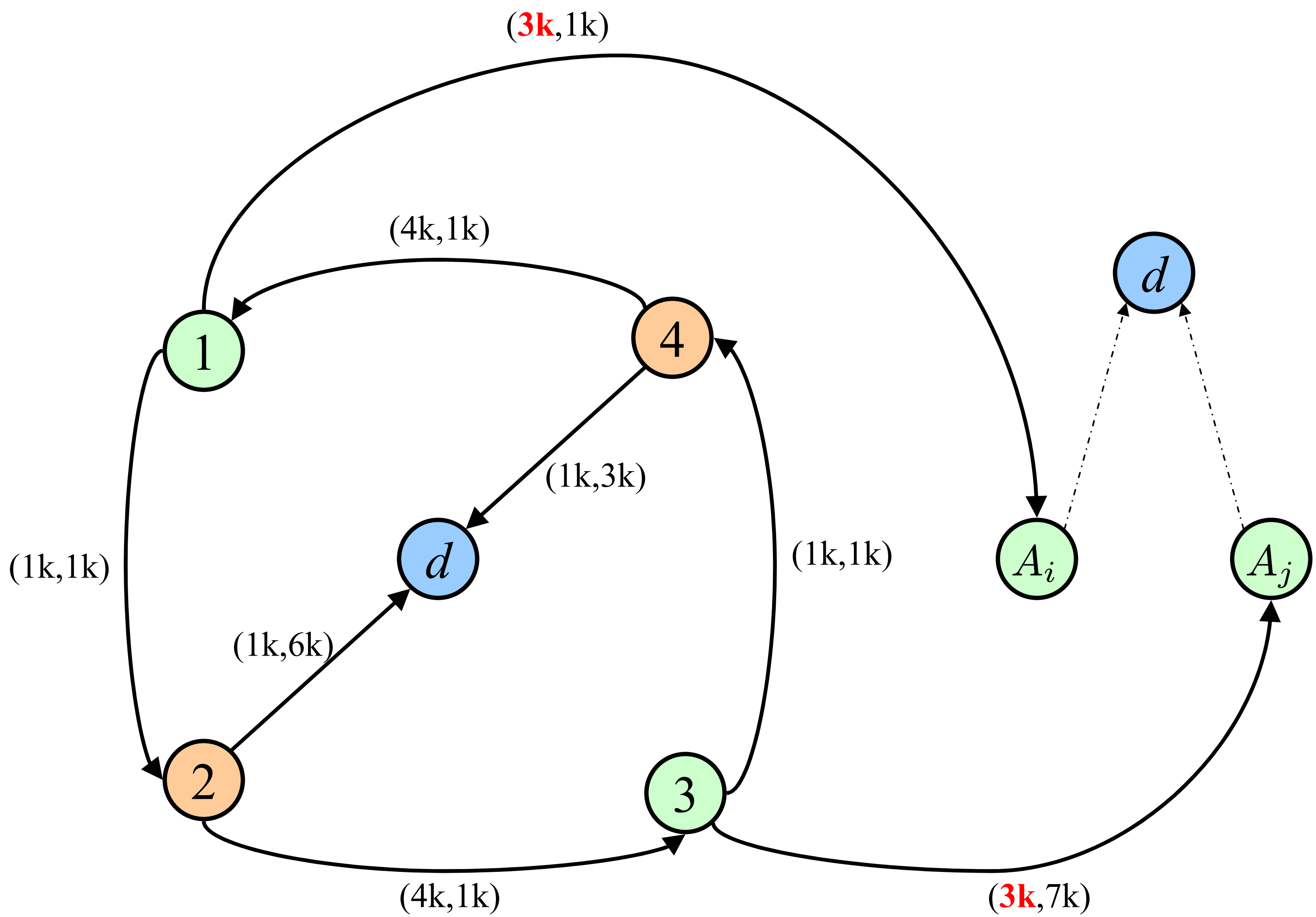

$A_i$
$A_j$



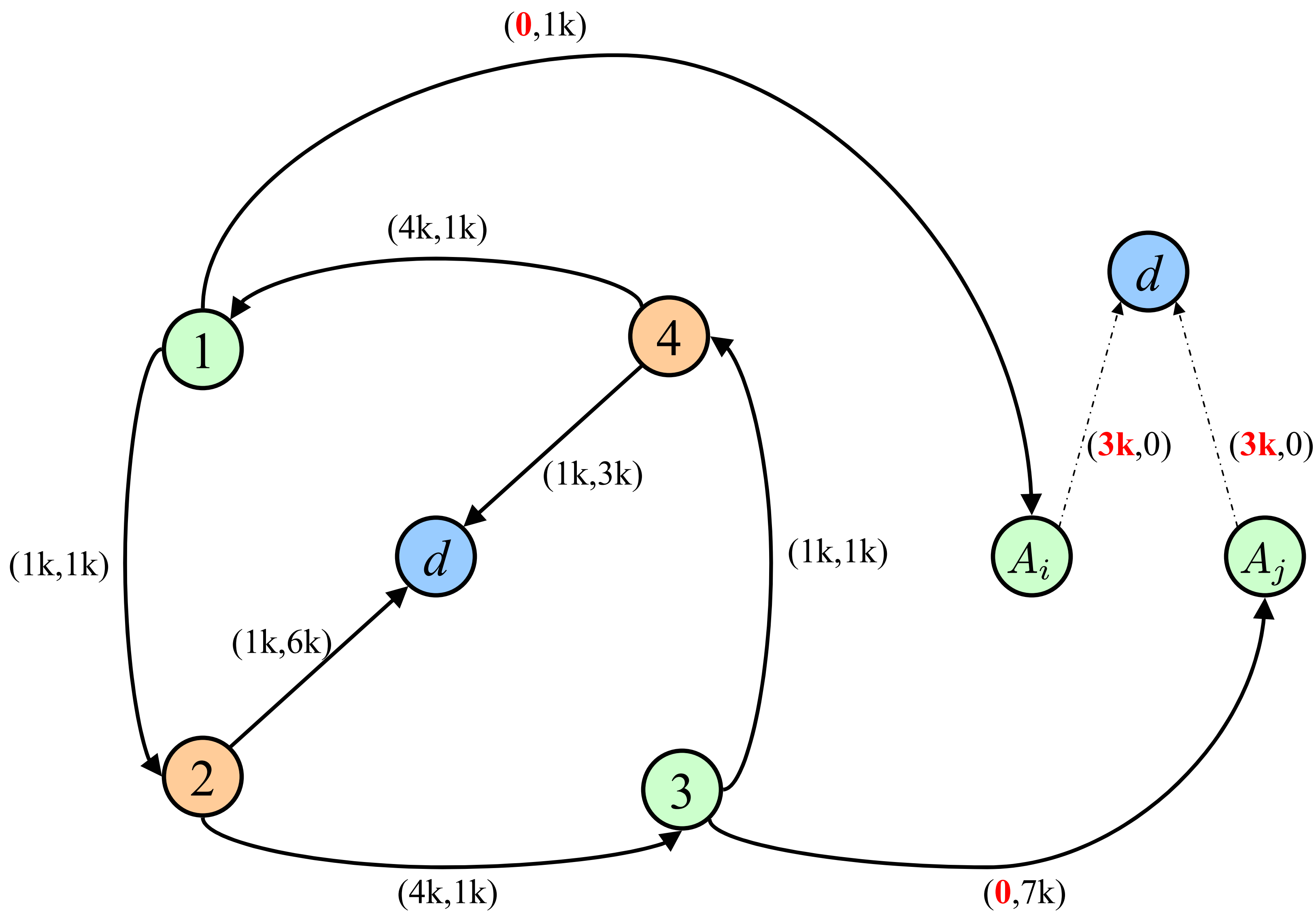
$d$
$A_i$
$A_j$


(4k,1k)
1
4
(3k,1k)
(1k,3k)
(1k,1k)
d
(1k,1k)
(1k,6k)
(3k,7k)
2
3
(4k,1k)
d
$A_i$
$A_j$

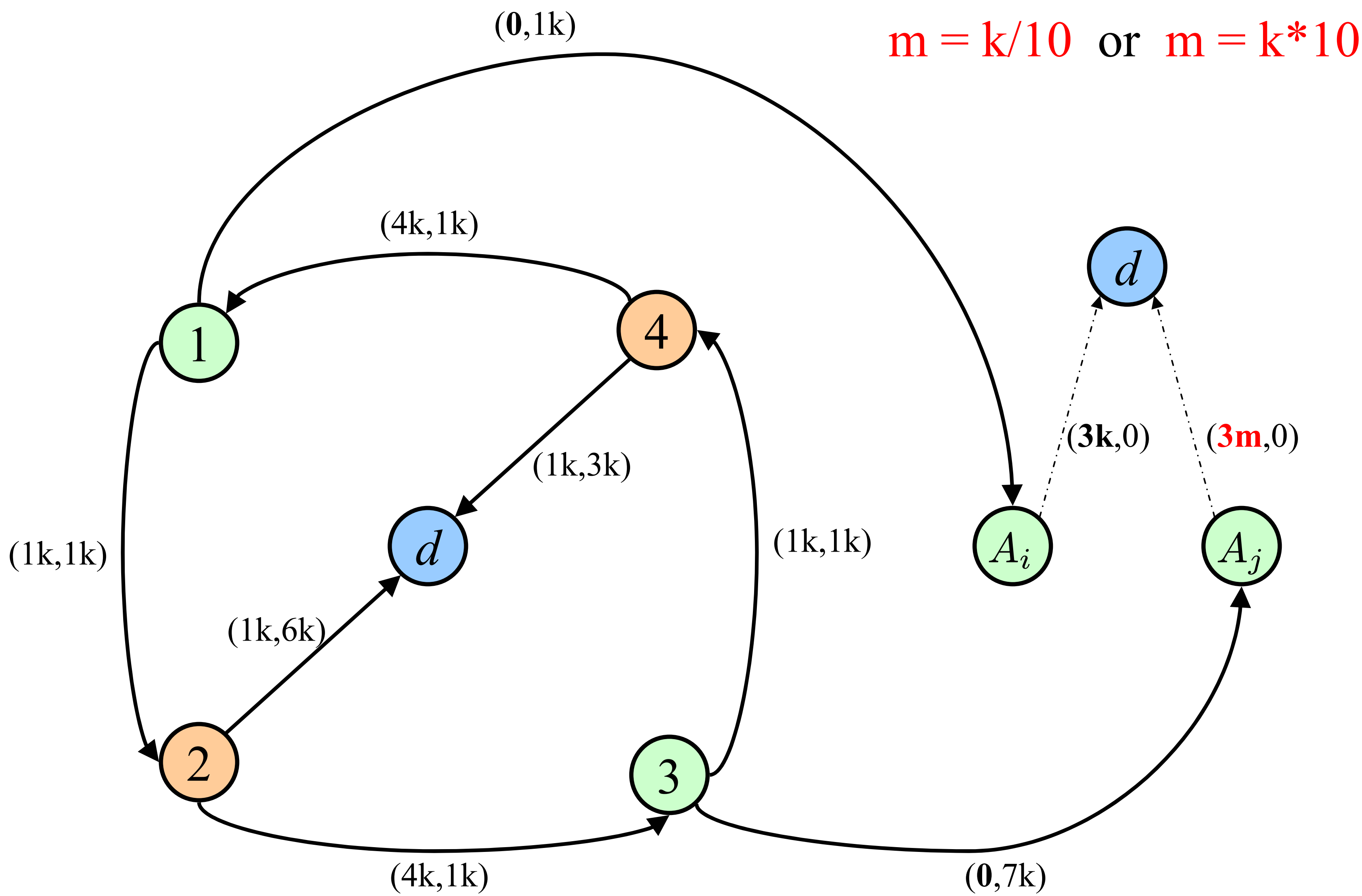

(4k,1k)
1
4
(3k,1k)
(1k,3k)
(1k,1k)
d
(1k,1k)
(1k,6k)
(3k,7k)
2
3
(4k,1k)
d
$A_i$
$A_j$



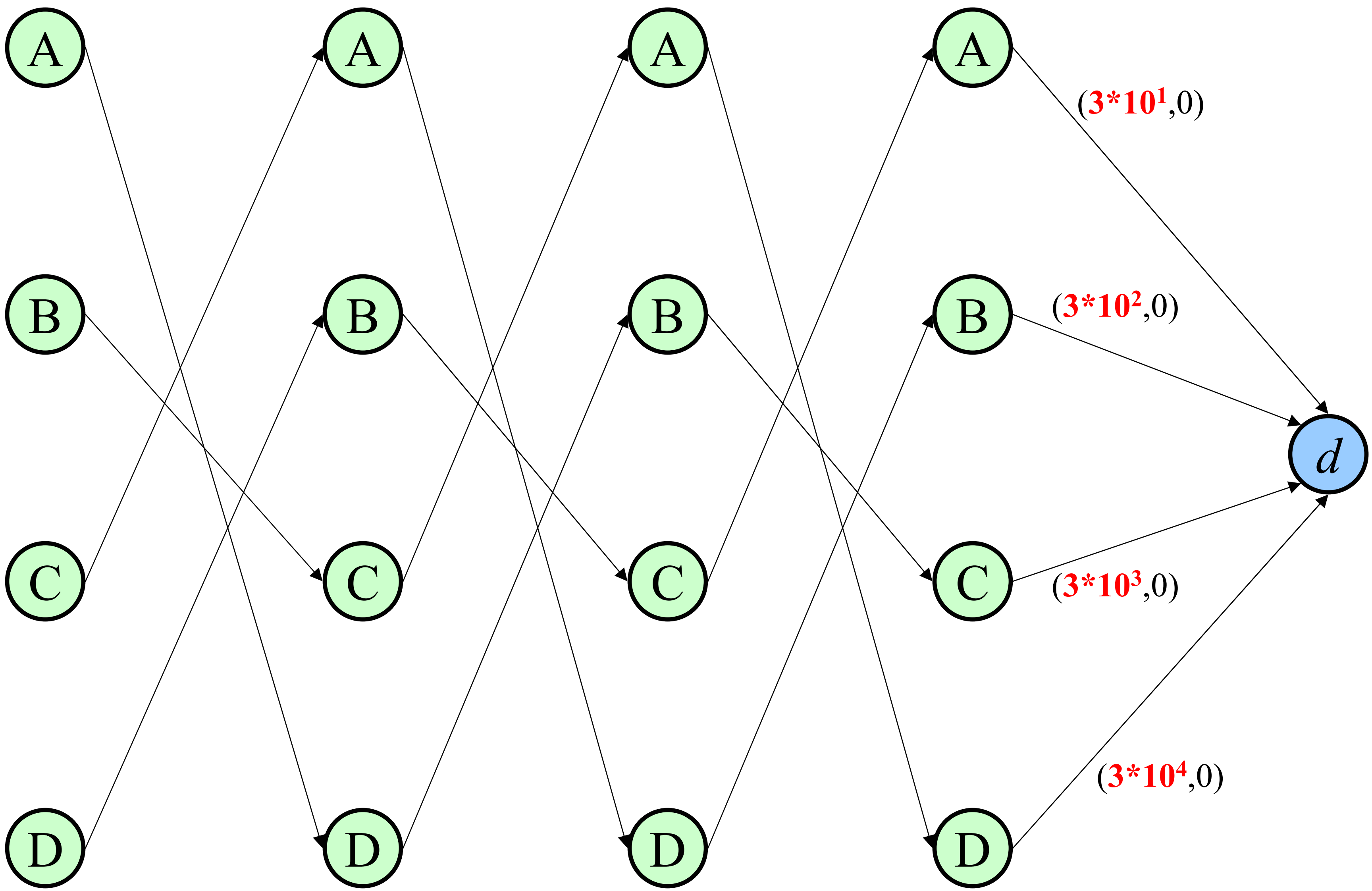
(3k,1k)
(4k,1k)
1
4
d
(1k,3k)
(1k,1k)
(1k,1k)
d
A_i
A_j
(1k,6k)
2
3
(4k,1k)
(3k,7k)

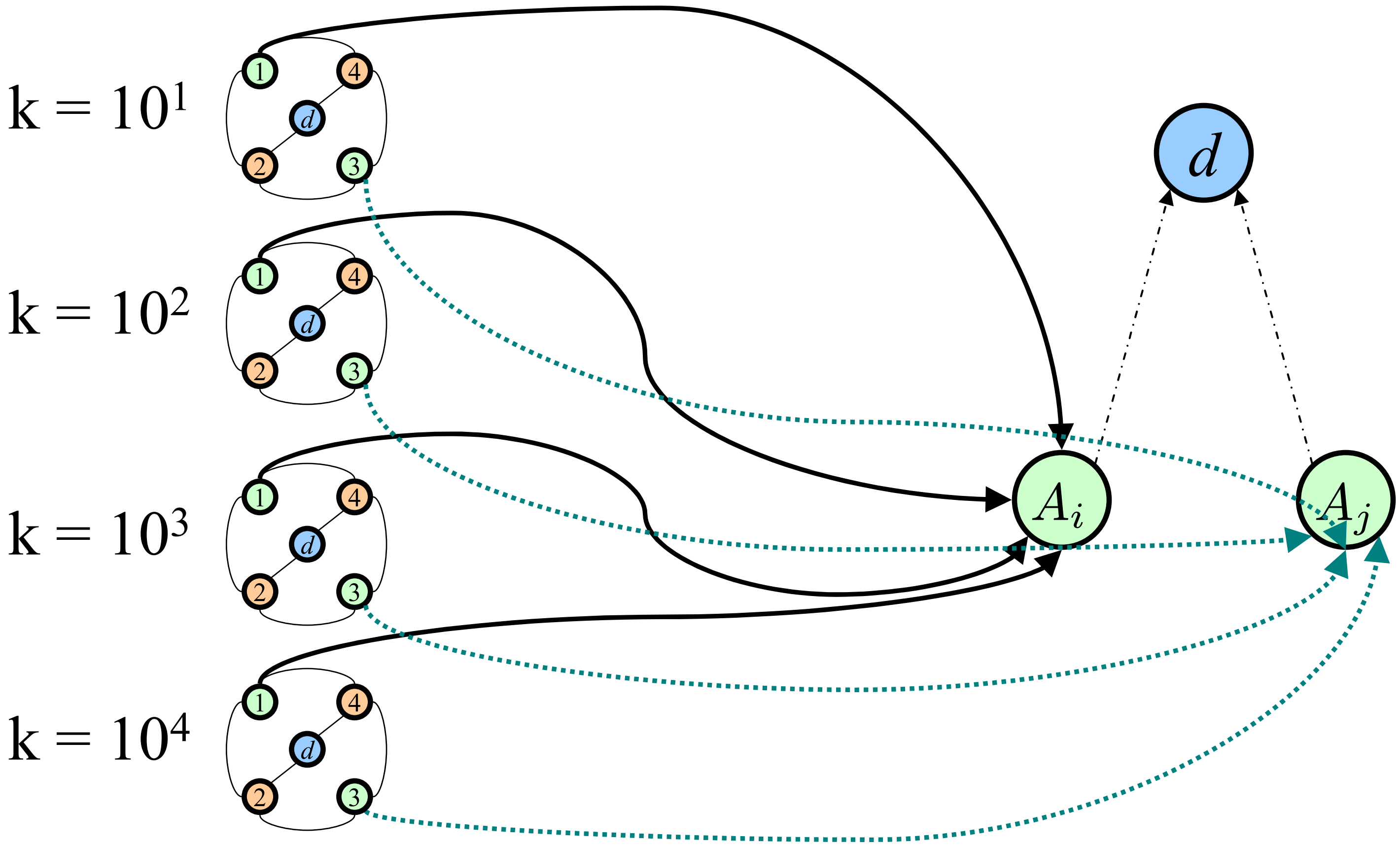


(0,1k)
(4k,1k)
1
4
d
(1k,3k)
(3k,0)
(3k,0)
$A_i$
$A_j$
(1k,1k)
(1k,1k)
(1k,6k)
2
3
(4k,1k)
(0,7k)

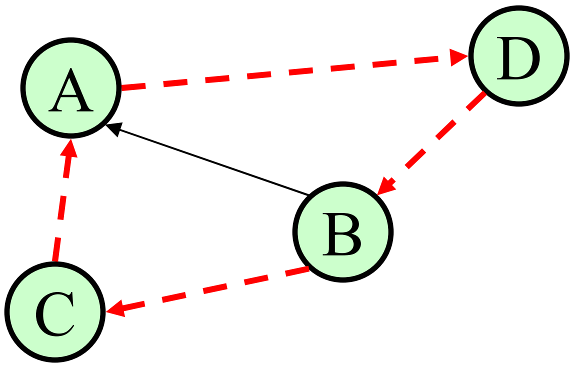


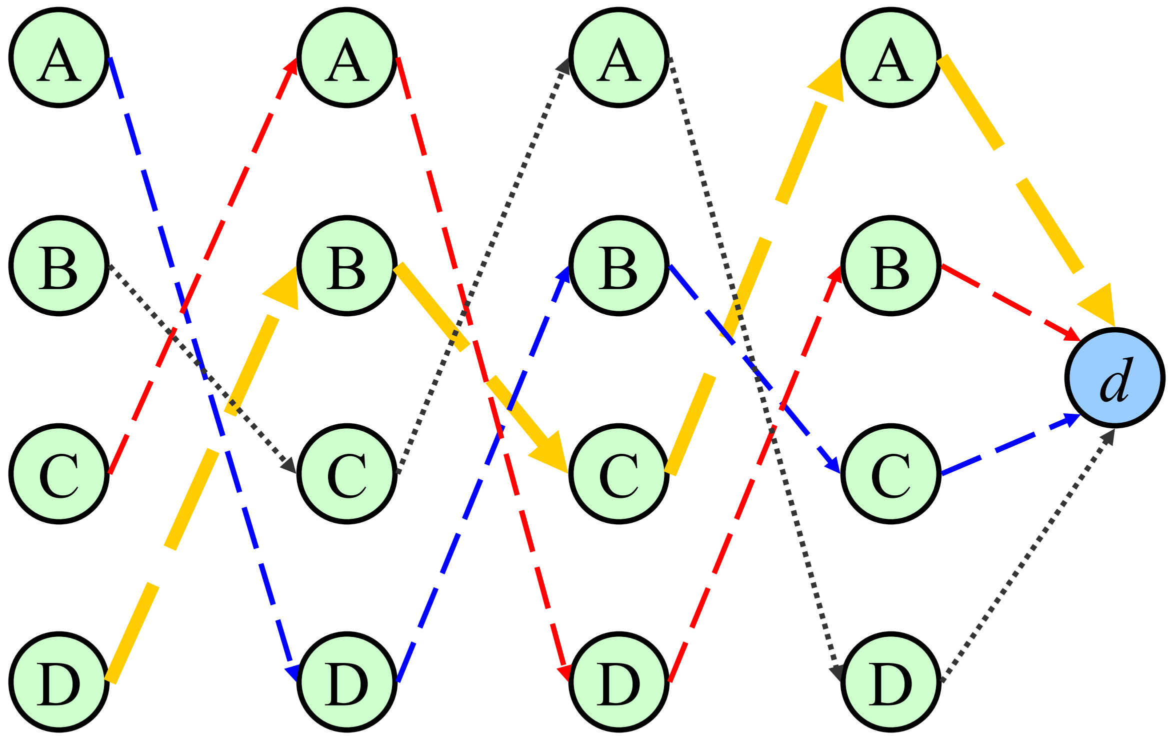
m = k/10 or m = k*10
(0,1k)
(4k,1k)
(1k,3k)
(1k,1k)
(1k,1k)
(1k,6k)
(4k,1k)
(0,7k)
(3k,0)
(3m,0)
1
2
3
4
d
d
$A_i$
$A_j$


A
B
C
D
A
B
C
D
A
B
C
D
A
B
C
D
d
(3*10¹,0)
(3*10²,0)
(3*10³,0)
(3*10⁴,0)



k = 10^1
k = 10^2
k = 10^3
k = 10^4
1
2
3
4
d
$A_i$
$A_j$


A
D
B
C


A
B
C
D
A
B
C
D
A
B
C
D
A
B
C
D
d

stable solution

- no subgraph is unstable
- no path from any $A_i$ crossing another $A_j$
- Hamiltonian circuit must exist

no stable solutions

- some subgraph is unstable
- path through an $A_i$ always crosses an $A_j$
- no Hamiltonian circuit exists

Given an $\overline{f}$-SPP instance

$$(V, E, d),\ w,\ \langle f_1, \ldots, f_n \rangle,$$

does there exist a solution that is stable under all possible $\pi$?