

Summary

This 3D printed surface is the geometrical representation of the van der Waals Equation of State from Classical Thermodynamics. It shows the coordinates of all the equilibrium points accessible to a dense gas made of interacting molecules with a finite minimal volume, in terms of its three average thermodynamic coordinates: Temperature, Pressure and Volume.

Overview and Background

The surface was generated by the following equation:

$$P' = 8 T' / (3 V' - 1) - 3 / (V'^2)$$

with P' , T' and V' the non-dimensional Pressure, Temperature and Volume of the gas respectively, with

$$P' = P / P_c$$

$$V' = V / V_c$$

$$T' = T / T_c$$

with P_c , V_c and T_c the critical pressure, volume and temperature, respectively. In order for the van der Waals equation to be applicable, these critical coordinate must comply with the following relationship:

$$P_c = a / (27 b^2)$$

$$T_c = 8 a / (27 b R)$$

$$V_c = 3 n b$$

Here, a and b are the classical van der Waals parameters, R is the Perfect Gas Constant and n is the number of moles of the component in the system.

In the printed volume, the parametric domains for P' , T' and V' are:

$$0 < P' < 5$$

$$0 < V' < 10$$

$$0 < T' < 2$$

Owing to the definition of P' , V' and T' , the coordinates of the critical point on the printed surface are (1, 1, 1). A video dedicated to this 3D printed piece and its use with students can be found in my YouTube Channel, La Physique avec les Mains (https://www.youtube.com/channel/UCO6Mecxua_W7SR-KwNwyRdw).

Lesson Plan and Activity

The printed surface gives access to a "physical" and "sensorial" view of what is otherwise merely an abstract equation in the mind of students. Simply touching and manipulating the printed piece, in relation to the underlying formula, yields a deeper understanding and a better memorisation of what the equation stands for. As a more advanced exercise, students are given laser pointers equipped with small lenses that distort the laser beam into a laser plane. By appropriately orienting the laser plane with respect to the printed surface, students may make iso-lines visible, such as isothermal curves, isobaric curves and isochoric curves. By relying on the graduations present on the base of the piece, and with the help of a second laser plane, more complex iso-lines can be constructed point by point, by triangulation. For instance, students can be asked to construct and draw a few isentropic curves directly on the piece's surface.

Materials Needed

None in principle.

A means to produce laser plane allows to explore the printed surface in great depths. This can be achieved by relying on cylindrical lenses, on a series of parallel, small optic fibres placed perpendicular to the laser beam, or on a diffraction grating.

References

P. Carles, *A brief review of the thermophysical properties of supercritical fluids*, Journal of Supercritical Fluids 53, 2-11 (2010)