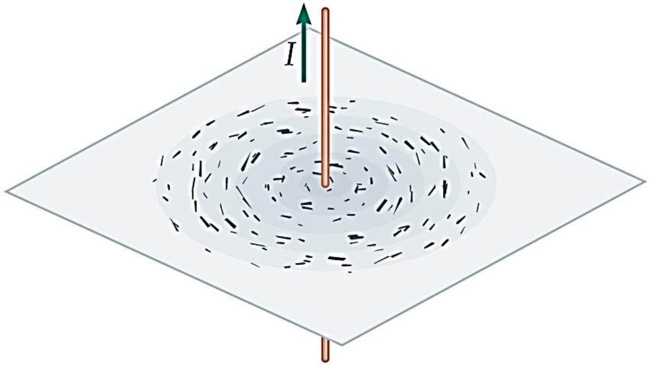
Sources of Magnetic Fields

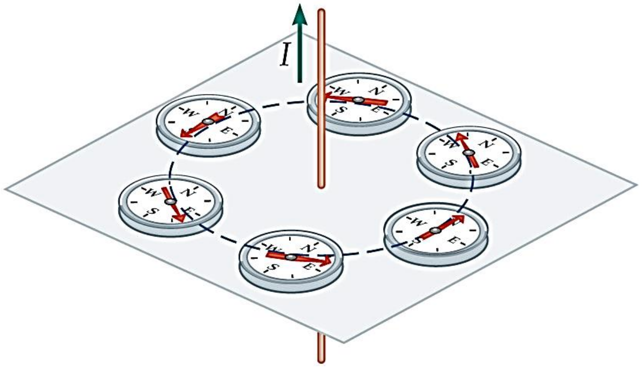
*Electric Currents and Magnetism

Experimental observation:

- ***Electric currents can produce magnetic fields.***



(a)

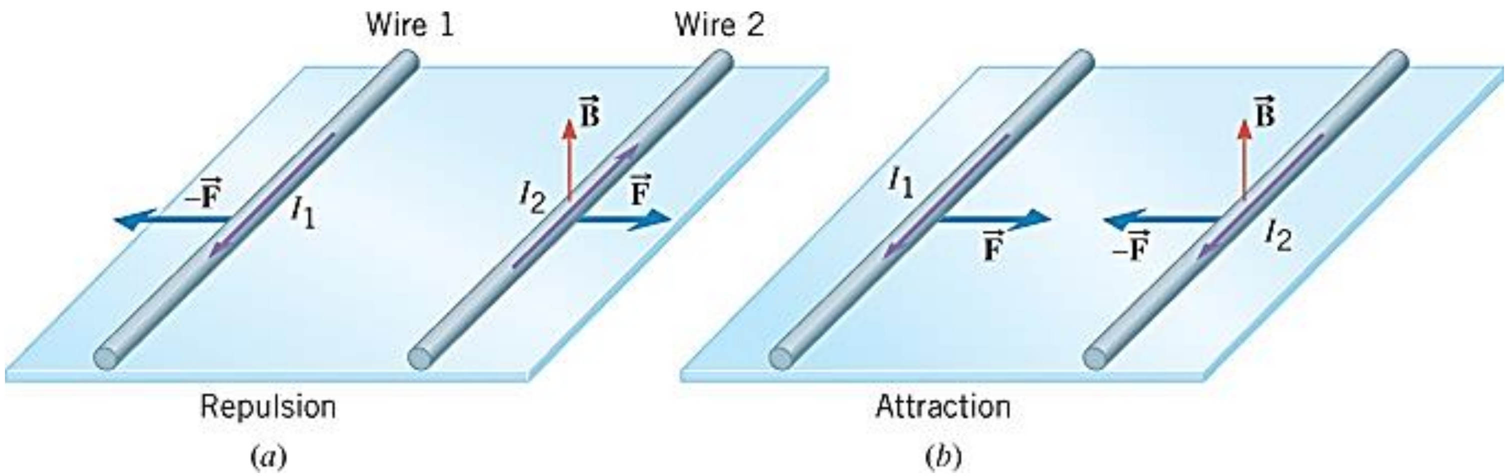


(b)

*Electric Currents and Magnetism

Experimental observation:

- ***Current carrying wires can exert forces on each other*.**



  - The force between parallel wires

$$F = 2k_m \frac{I_1 I_2}{d} L = I_1 L \left(2k_m \frac{I_2}{d}\right) = I_2 L \left(2k_m \frac{I_1}{d}\right)$$

$$B = 2k_m \frac{I}{d} \quad ?$$

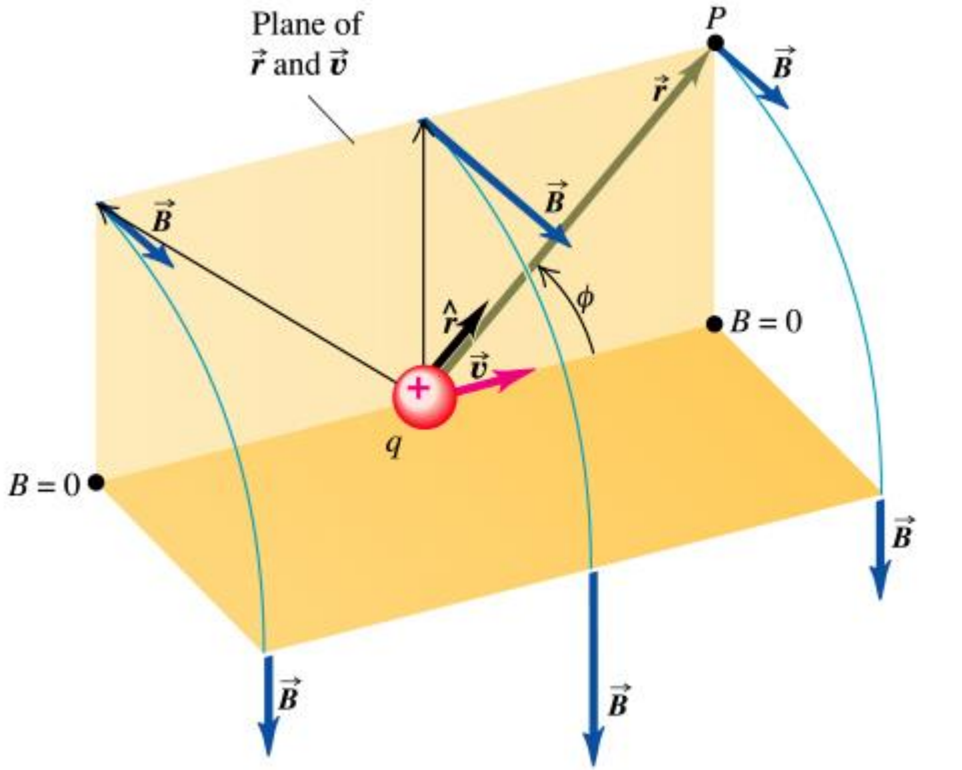
Magnetic Field of a Moving Charge

For a single moving point charge *q*

Magnetic field produced at *P*.

- Force on parallel wires

$$B = k_m \frac{|q| v \sin\phi}{r^2}$$



- In vector form

$$\vec{B} = k_m \frac{q\, \vec{v} \times \vec{r}}{r^3}$$

- The permeability constant:

$$k_m = \frac{\mu_0}{4\pi} = 1 \times 10^{-7}\ \mathrm{T \cdot m/A} \qquad \mu_0 = 4\pi \times 10^{-7}\ \mathrm{T \cdot m/A}$$

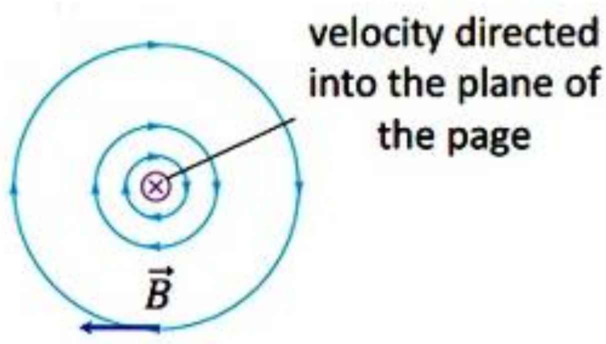
Magnetic Field of a Moving Charge

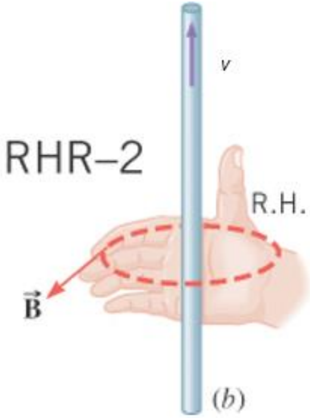
For a single moving point charge *q*

- Vector magnetic field

$$\vec{\boldsymbol{B}} = k_m \frac{q\,\vec{\boldsymbol{v}} \times \vec{\boldsymbol{r}}}{r^3}$$

- Direction by right-hand rule No. 2





Magnetic Field of a Current Element

In a conductor, moving charges gives rise to the current.

- The net magnetic field around the conductor is the superposition of the magnetic field of each charge.

- For a short segment *dL*

$$dQ = q\,n(AdL)$$

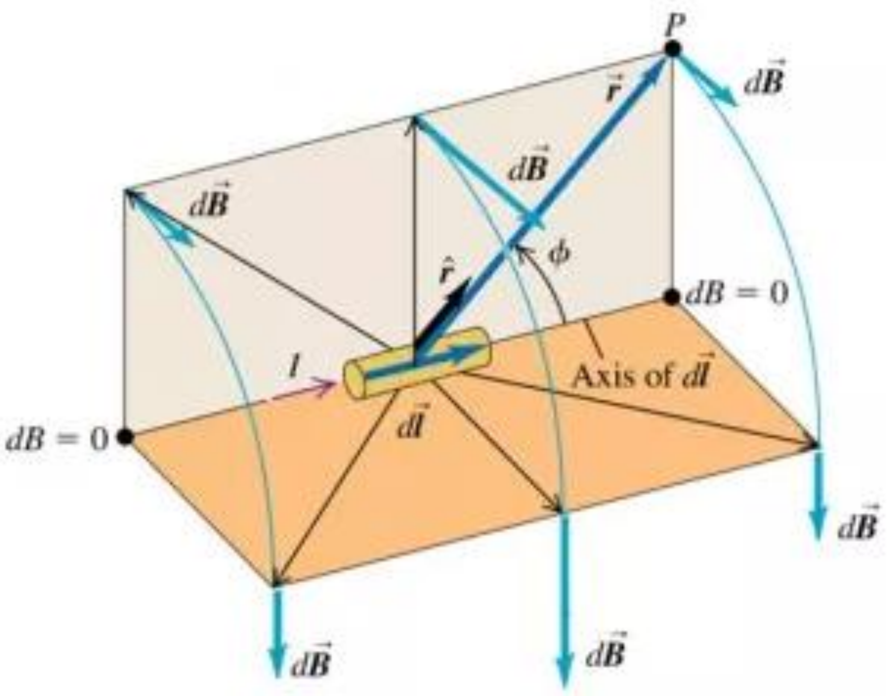
$n =$ charge carrier density

- The field dB

$$dB = k_m \frac{q(nAvdL)\,\sin\phi}{r^2}$$

$$= k_m \frac{IdL\,\sin\phi}{r^2}$$

with $\quad I = nqvA$



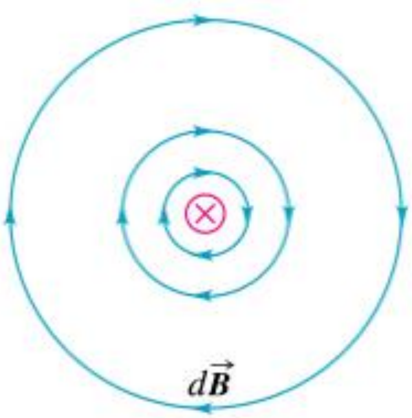
Magnetic Field of a Current Element

In a conductor, moving charges gives rise to the current.

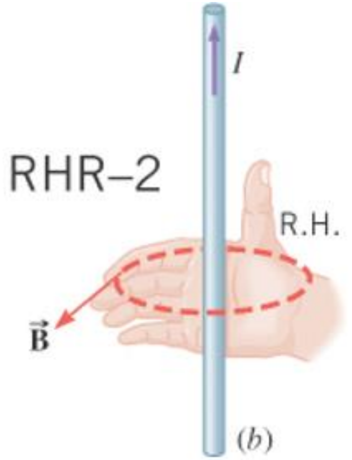
- The field $d\vec{\boldsymbol{B}}$

$$dB = k_m \frac{IdL\,\sin\phi}{r^2}$$

$$d\vec{\boldsymbol{B}} = k_m \frac{I\, d\vec{\boldsymbol{L}} \times \vec{\boldsymbol{r}}}{r^3}$$



⊗ *Current into page*



Magnetic Field of a Current Element

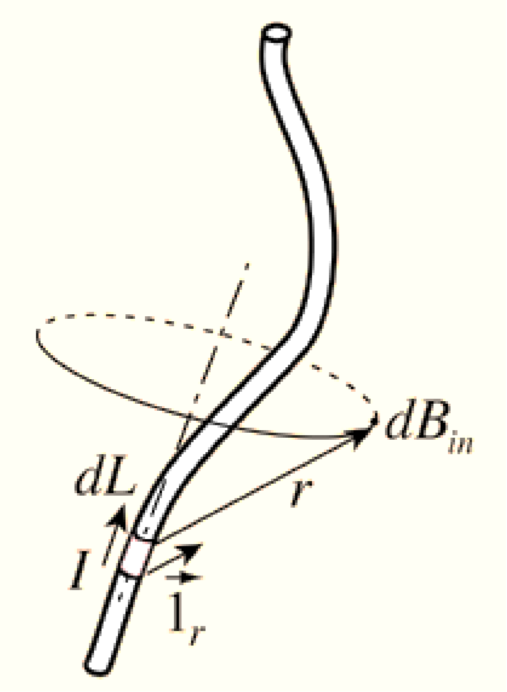
Biot-Savart Law

- The field $d\vec{\boldsymbol{B}}$ due to a length element

$$d\vec{\boldsymbol{B}} = k_m \frac{I\, d\vec{\boldsymbol{L}} \times \vec{\boldsymbol{r}}}{r^3}$$

- For a finite length of wire

$$\vec{\boldsymbol{B}} = k_m \int \frac{I\, d\vec{\boldsymbol{L}} \times \vec{\boldsymbol{r}}}{r^3}$$



Magnetic Field of a Straight Current

Using Biot-Savart Law

Magnetic field produced by a straight current-carrying conductor of length 2*a*.

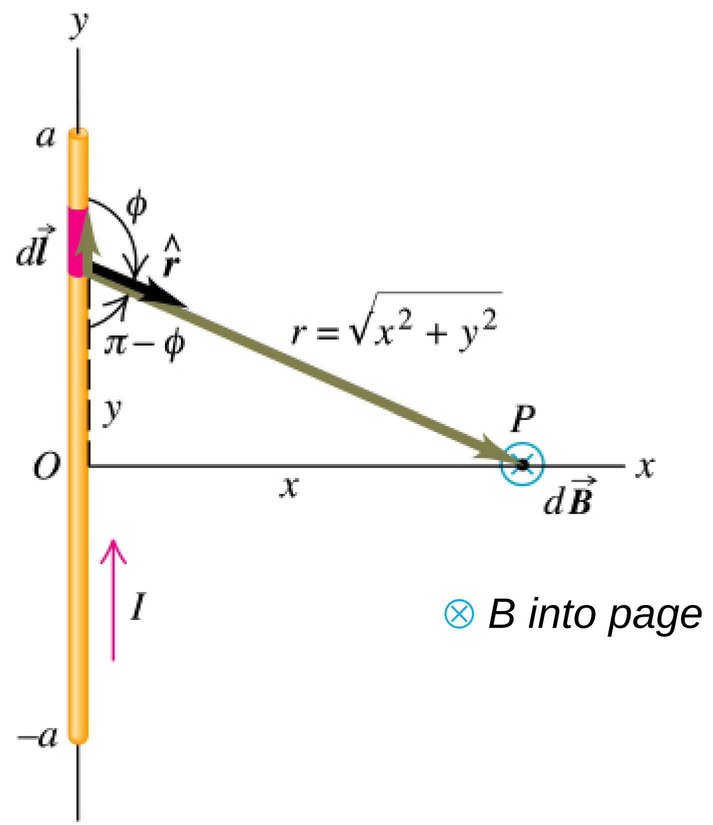
$$\vec{\boldsymbol{B}} = k_m \int \frac{I\, d\vec{\boldsymbol{L}} \times \vec{\boldsymbol{r}}}{r^3}$$

- For a finite length of wire

$$dL = dy \qquad r = \sqrt{x^2 + y^2}$$

$$\sin\phi = x/\sqrt{x^2 + y^2}$$

$$B = k_m I \int \frac{x\,dy}{(x^2 + y^2)^{3/2}}$$



Magnetic Field of a Straight Current

Using Biot-Savart Law

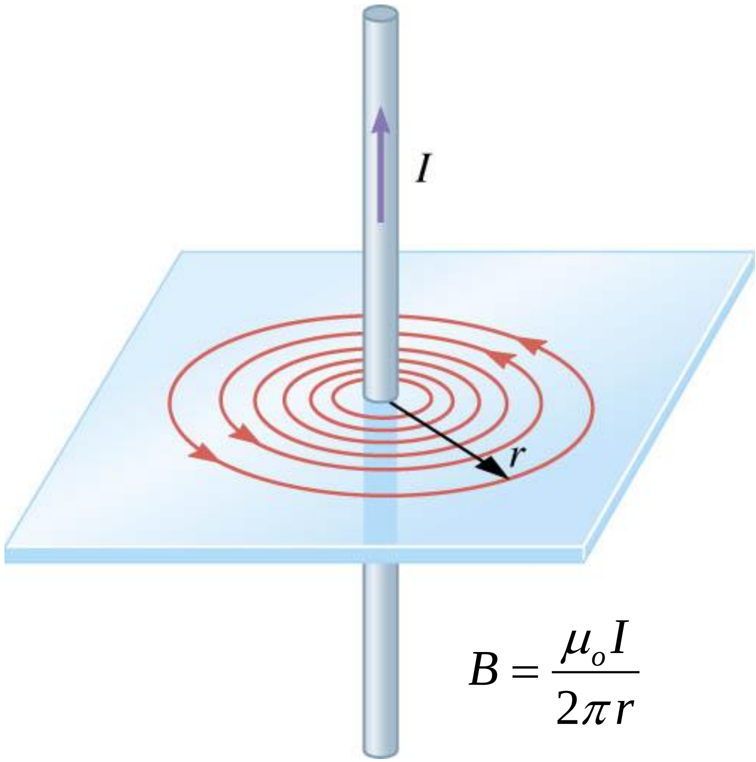
Magnetic field produced by a straight current-carrying conductor of length 2*a*.

- On the perpendicular bisector

$$B = k_m I \frac{2a}{x\sqrt{(x^2 + a^2)}}$$

- In the limit $a \gg x$

$$B = 2k_m \frac{I}{x}$$



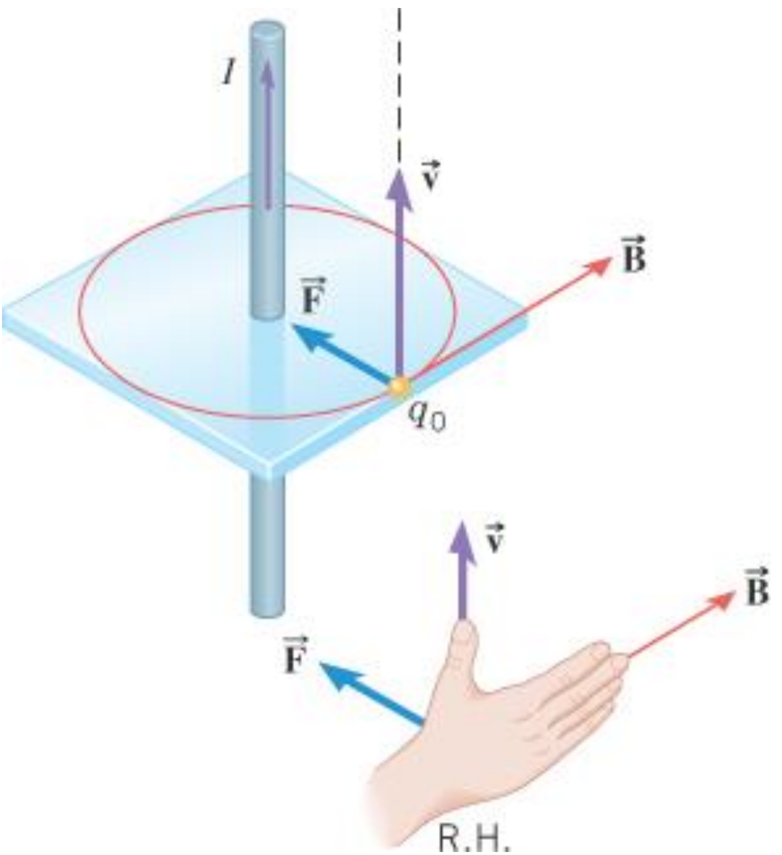
$$B = \frac{\mu_o I}{2\pi r}$$

Magnetic Field of a Straight Current

Example: A Current Exerts a Force on a Moving Charge

The long straight wire carries a current of 3.0 A. A particle has a charge of $+6.5 \times 10^{-6}$ C and is moving parallel to the wire at a distance of 0.050 m. The speed of the particle is 280 m/s.

Determine the magnitude and direction of the magnetic force on the particle.



Magnetic Field of a Straight Current

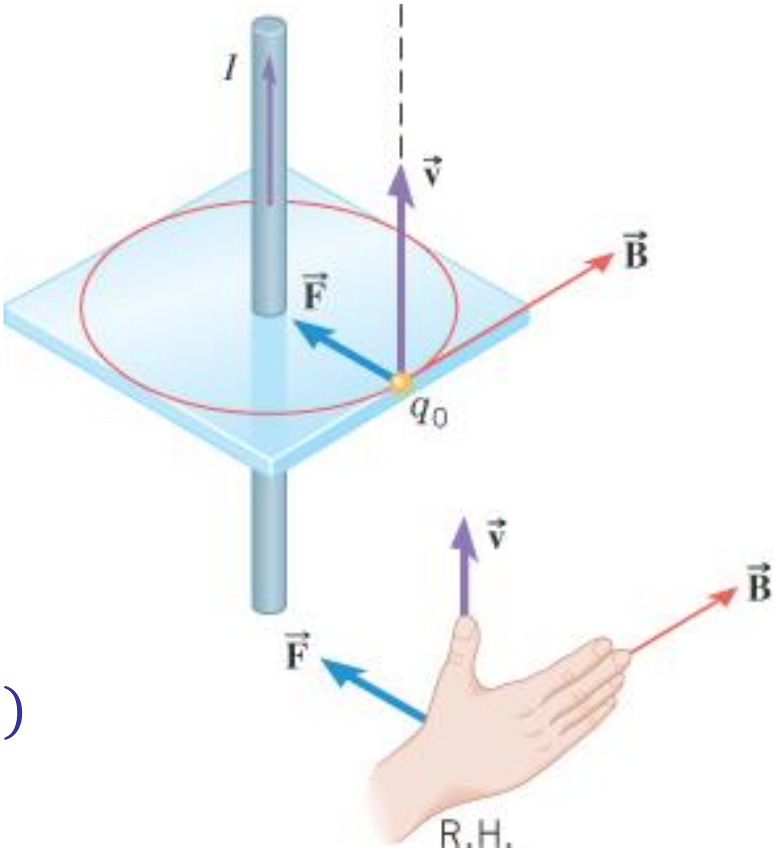
Example: A Current Exerts a Force on a Moving Charge

- The magnetic field.

$$B = 2k_m \frac{I}{r}$$

$$= \frac{(2 \times 10^{-7})(3.0)}{(0.050)}$$

$$= 1.2 \times 10^{-5}\ \mathrm{T}$$



- The magnetic force

$$F = qvB\sin 90°$$

$$= (6.5 \times 10^{-6})(280)(1.2 \times 10^{-5})$$

$$= 2.18 \times 10^{-8}\ \mathrm{N}$$

Force Between Parallel Currents

The magnetic force parallel current carrying wires

- Magnetic field due to I_1:

$$B_1 = 2k_m \frac{I_1}{r}$$

- The force on I_2:

$$F_2 = I_2 L B_1$$

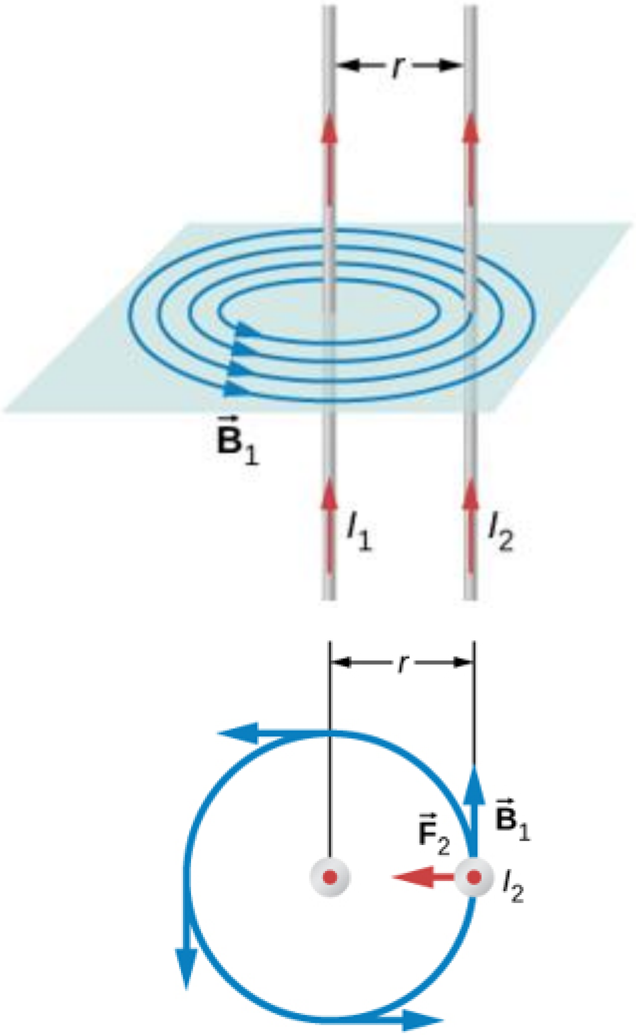
$$= 2k_m \frac{I_1 I_2}{r} L$$

- The force on I_1:

$$F_1 = I_1 L B_2$$

$$= 2k_m \frac{I_1 I_2}{r} L$$

By Newton's 3rd law



Force Between Parallel Currents

Example:

Two straight parallel wires 4.5 mm apart carry equal currents of 15.0 A in opposite directions.

- The magnetic force:

$$F = 2k_m \frac{I_1 I_2}{r} L = L(2 \times 10^{-7})(15.0)^2/(0.0045)$$

$$= 0.01\, L$$

- The force per unit length:

$$F/L = 0.01\ \mathrm{N/m}$$

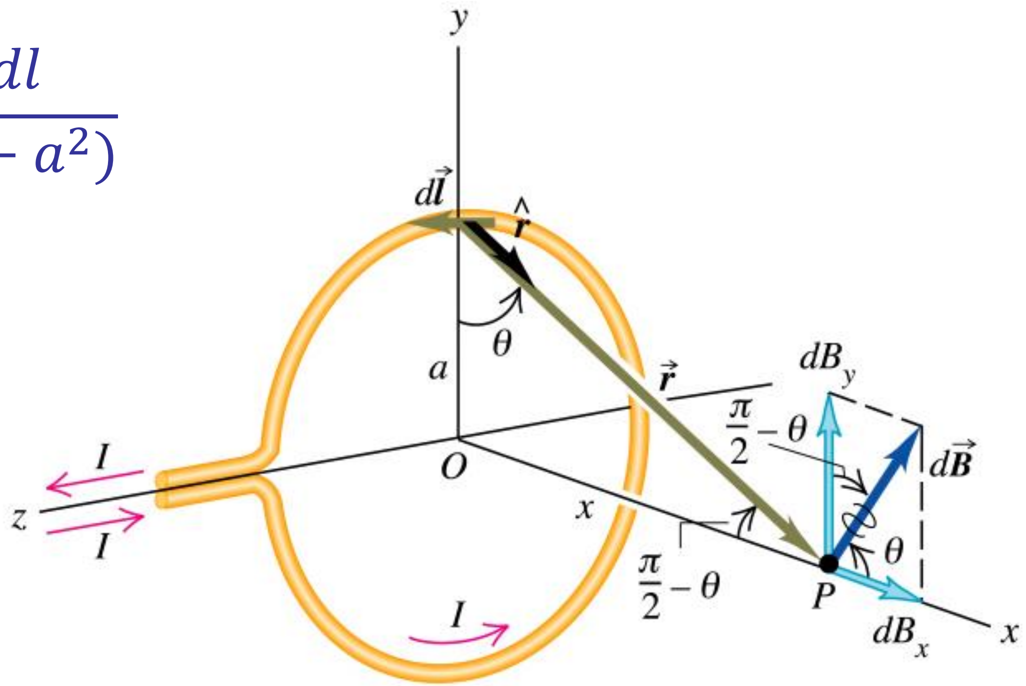
Magnetic Field of a Circular Current Loop

Magnetic field on the axis of circular loop of radius *a*.

- The field $d\vec{\boldsymbol{B}}$ due to element *dL*

$$d\vec{\boldsymbol{B}} = k_m \frac{I\, d\vec{\boldsymbol{L}} \times \vec{\boldsymbol{r}}}{r^3}$$

$$dB = k_m \frac{I\, dl}{r^2} = k_m \frac{I\, dl}{(x^2 + a^2)}$$



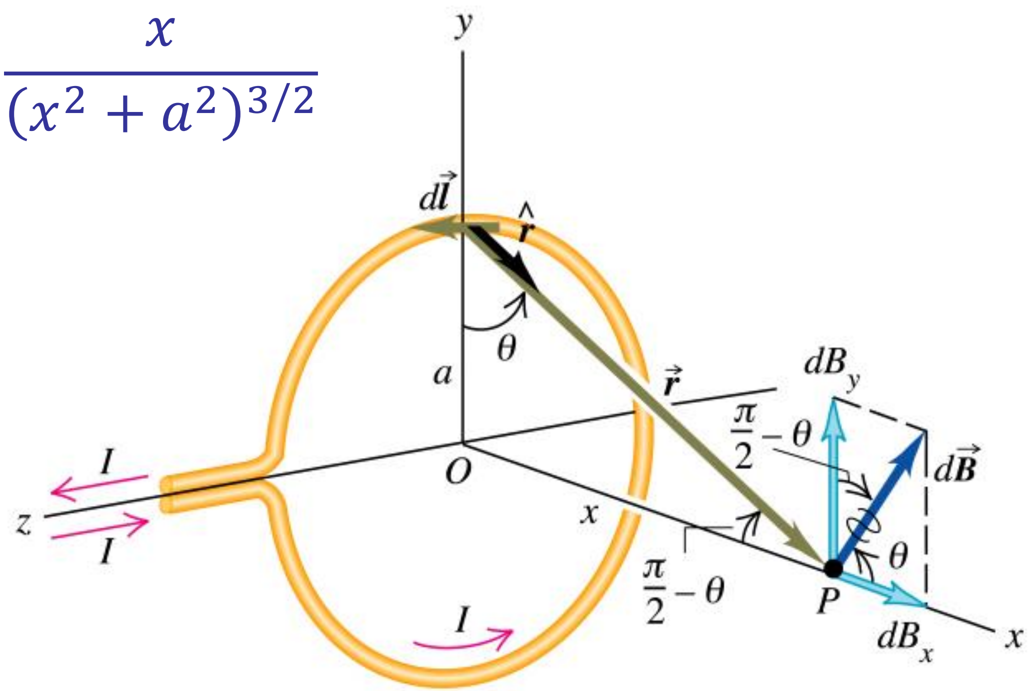
Magnetic Field of a Circular Current Loop

Magnetic field on the axis of circular loop of radius *a*.

- The components of $d\vec{\boldsymbol{B}}$

$$dB_x = dB\cos\theta = k_m I dl \frac{a}{(x^2 + a^2)^{3/2}}$$

$$dB_y = dB\sin\theta = k_m I dl \frac{x}{(x^2 + a^2)^{3/2}}$$



Magnetic Field of a Circular Current Loop

Magnetic field on the axis of circular loop of radius *a*.

- Integrate

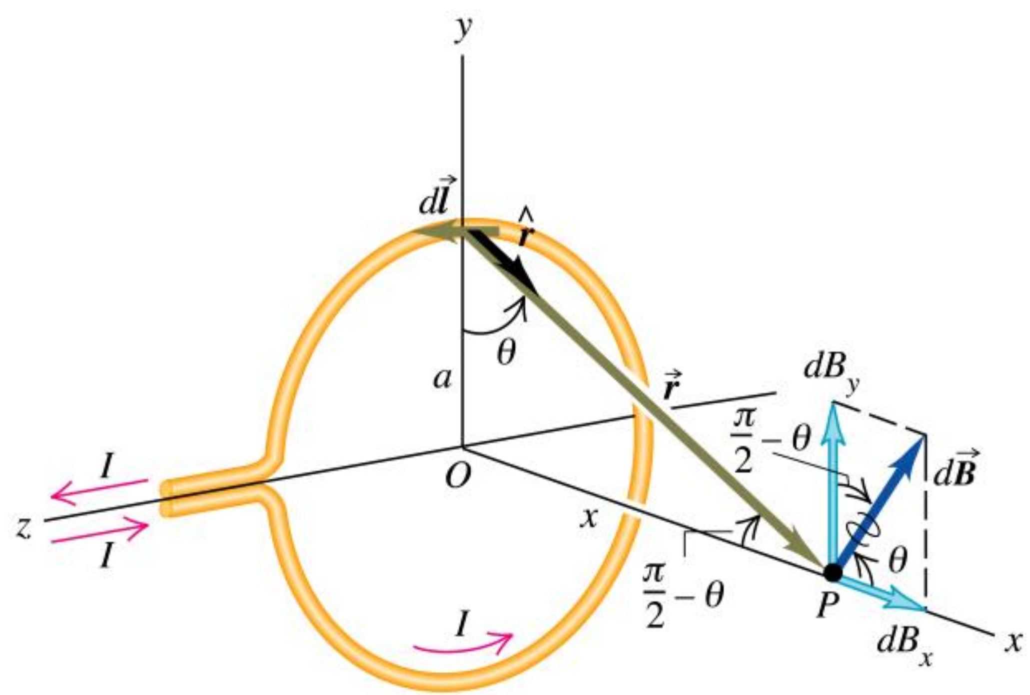
$$B_x = k_m I a \int \frac{dl}{(x^2 + a^2)^{3/2}}$$

$$= k_m \frac{Ia}{(x^2 + a^2)^{3/2}} \int dl$$

$$= 2\pi k_m \frac{Ia^2}{(x^2 + a^2)^{3/2}}$$

$$B_y = \int dB_y = 0$$

(*By symmetry*)

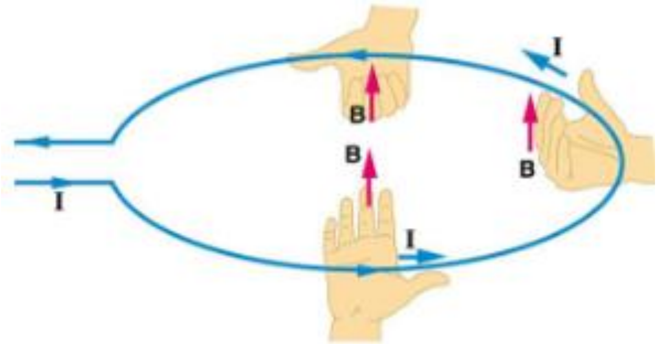


Magnetic Field of a Circular Current Loop

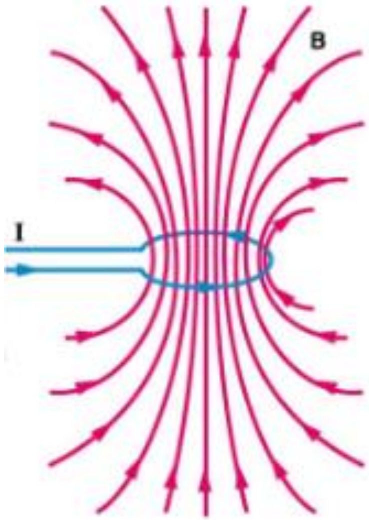
Magnetic field on the axis of circular loop of radius *a*.

$$B_x = 2\pi k_m \frac{Ia^2}{(x^2 + a^2)^{3/2}}$$

Direction by RHR-2



(a)



(b)

- At the center (*x*=0)

$$B_0 = 2\pi k_m \frac{I}{a}$$

Magnetic Field of a Circular Current Loop

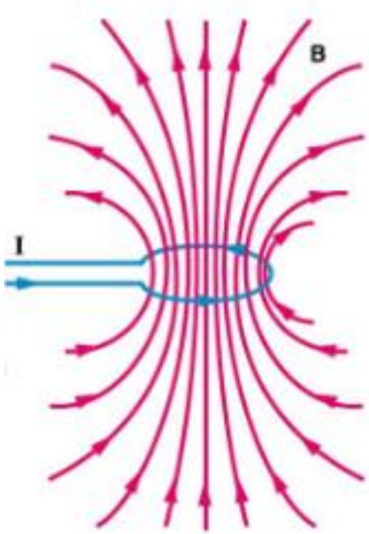
Example:

A coil consisting of 100 circular loops with radius 0.60 m carries a 5.0-A current.

- At the center (*x*=0)

$$B_0 = N 2\pi k_m \frac{I}{a} = \frac{100(2\pi \times 10^{-7})(5.0)}{(0.060)}$$

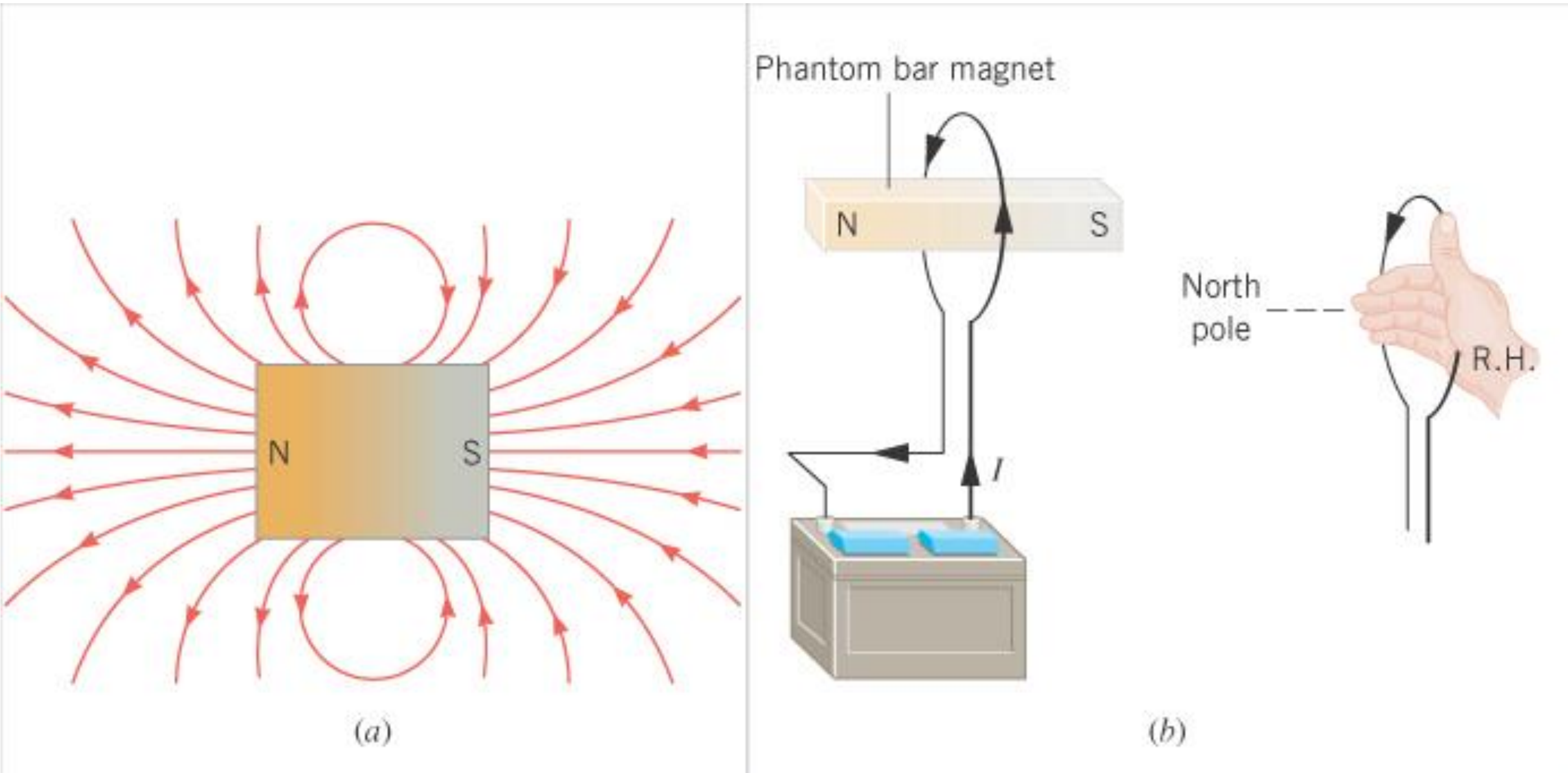
$$= 5.2 \times 10^{-3}\ \mathrm{T}$$



(b)

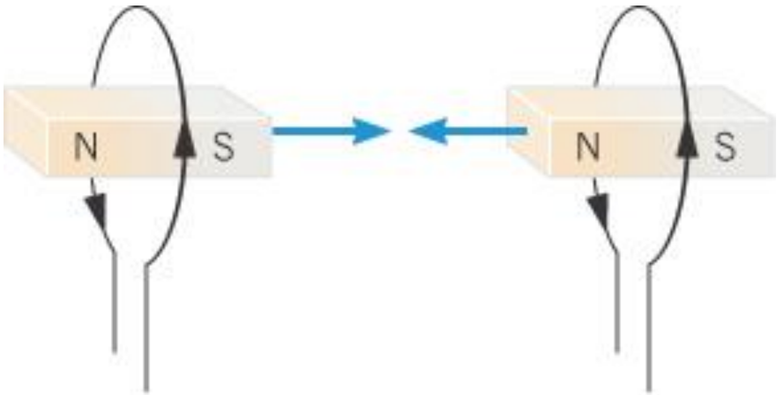
Magnetic Field of a Circular Current Loop

Magnetic field lines around a *circular loop* resemble those around a *bar magnet*.

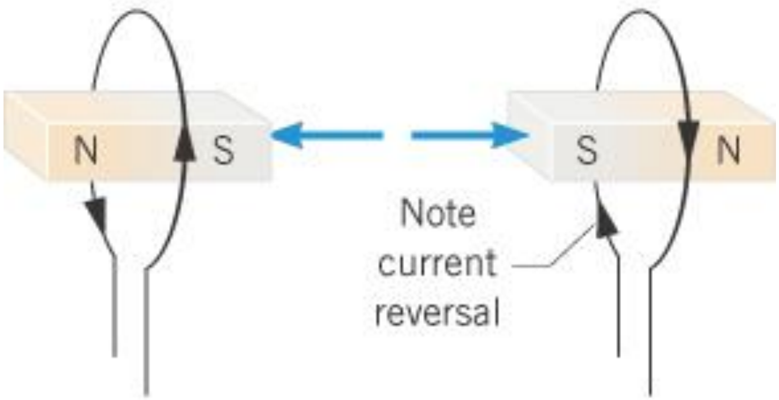


Magnetic Field of a Circular Current Loop

Two *circular loop* interact the same way as two *bar magnets* would.



(*a*) Attraction



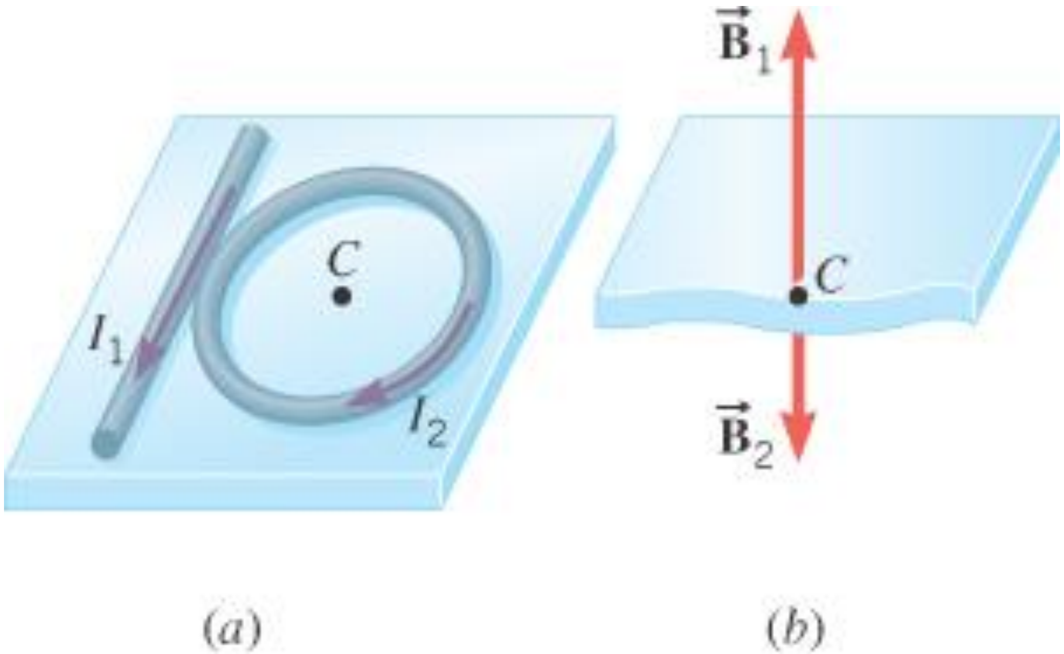
(*b*) Repulsion

*Magnetic Fields - Superposition

Example: Superposition of Magnetic Fields

A long straight wire carries a current of 8.0 A and a circular loop of wire carries a current of 2.0 A and has a radius of 0.030 m. Find the magnitude and direction of the magnetic field at the center of the loop.

- At the center *C*:



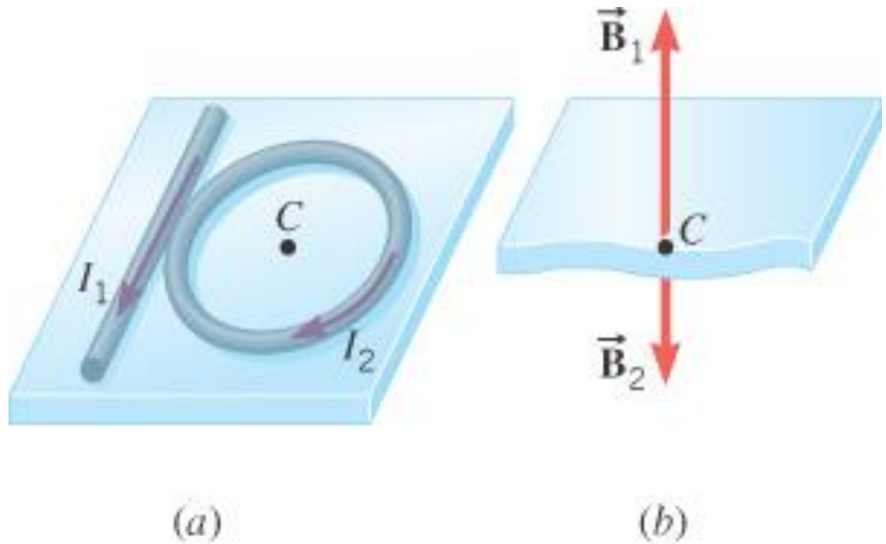
*Magnetic Fields - Superposition

Example: Superposition of Magnetic Fields

- At the center C:

$$B = \frac{\mu_o I_1}{2\pi r} - \frac{\mu_o I_2}{2R} = \frac{\mu_o}{2}\left(\frac{I_1}{\pi r} - \frac{I_2}{R}\right)$$

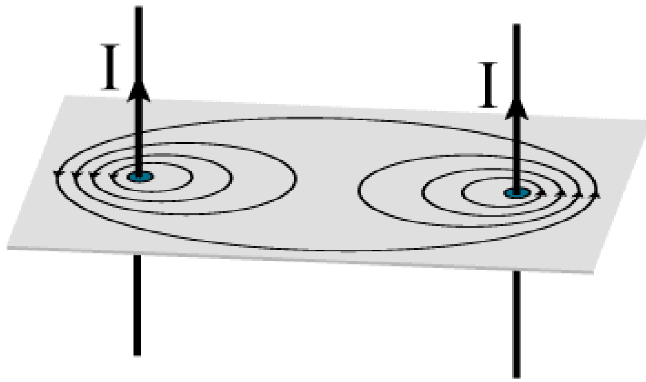
$$B = \frac{\left(4\pi \times 10^{-7}\ \mathrm{T \cdot m/A}\right)}{2}\left(\frac{8.0\ \mathrm{A}}{\pi\left(0.030\,\mathrm{m}\right)} - \frac{2.0\ \mathrm{A}}{0.030\,\mathrm{m}}\right) = 1.1 \times 10^{-5}\,\mathrm{T}$$



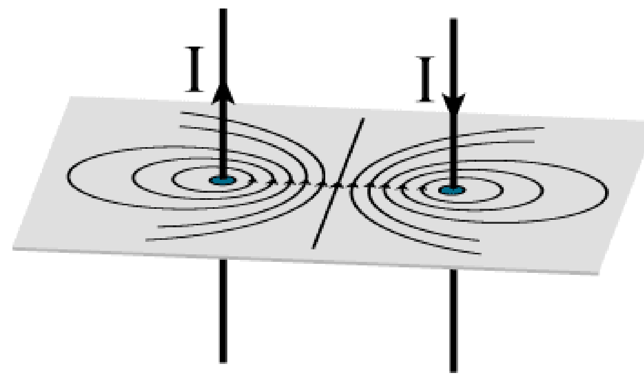
*Magnetic Fields - Superposition

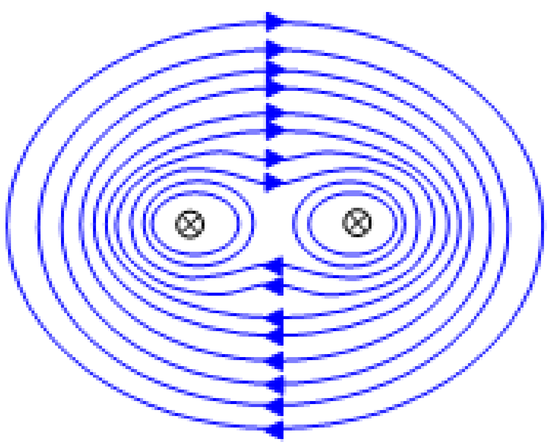
The *magnetic field* due to two parallel currents.

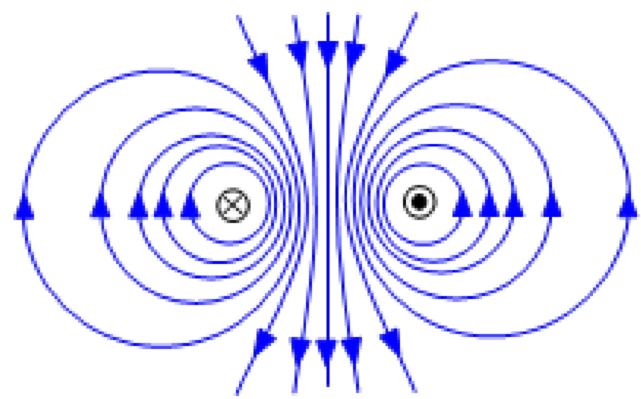
Current in the same direction



Current in opposite direction







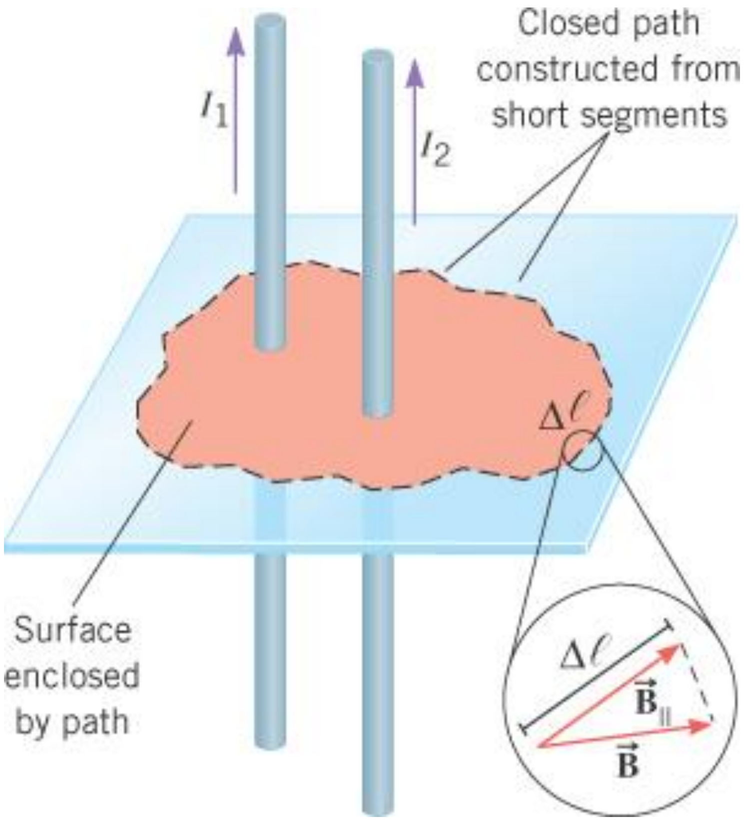
Ampere's Law

The line integral of the *magnetic field*.

$$\sum(\vec{\boldsymbol{B}} \cdot \Delta\vec{\boldsymbol{l}}) = \sum(B_{\parallel}\Delta l)$$

- For infinitesimal line segments.

$$\oint(\vec{\boldsymbol{B}} \cdot d\vec{\boldsymbol{l}}) = \oint(B_{\parallel}dl)$$



Ampere's Law

The line integral of the *magnetic field*.

- Apply to a single straight current (a)

$$B = 2k_m \frac{I}{r}$$

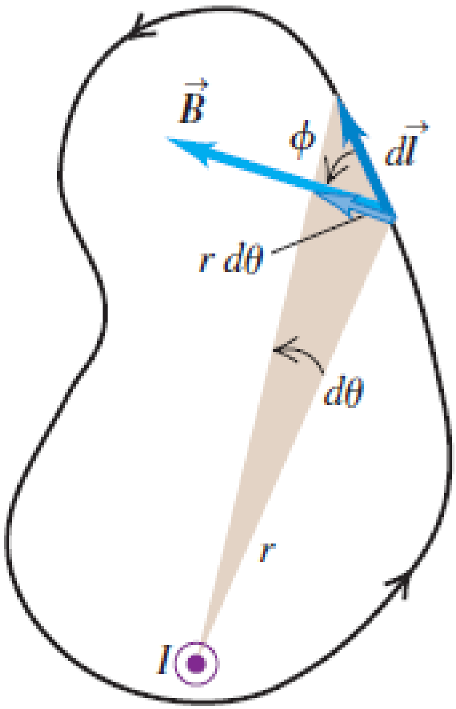
$$\vec{\boldsymbol{B}} \cdot d\vec{\boldsymbol{l}} = B dl \cos\phi \ = B(r d\theta)$$

$$\oint (\vec{\boldsymbol{B}} \cdot d\vec{\boldsymbol{l}}) = \oint \left(2k_m \frac{I}{r}\right)(r d\theta)$$

$$= (2k_m I) \int_0^{2\pi} d\theta$$

$$= 4\pi k_m I = \mu_0 I$$

(a)



Ampere's Law

The line integral of the *magnetic field*.

- Apply to a single straight current (b)

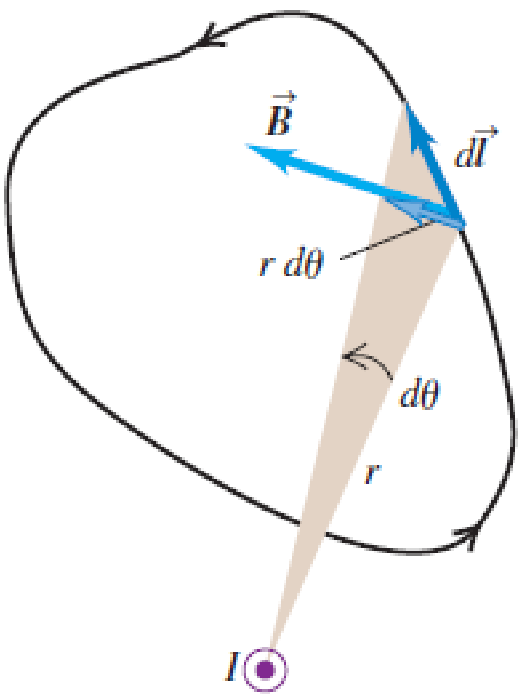
$$\oint (\vec{\boldsymbol{B}} \cdot d\vec{\boldsymbol{l}}) = \oint \left(2k_m \frac{I}{r}\right)(r d\theta)$$

$$= (2k_m I) \int_{\theta_1}^{\theta_1} d\theta = 0$$

- Ampere's law

$$\oint (\vec{\boldsymbol{B}} \cdot d\vec{\boldsymbol{l}}) = \mu_0 I_{enclosed}$$

(b)



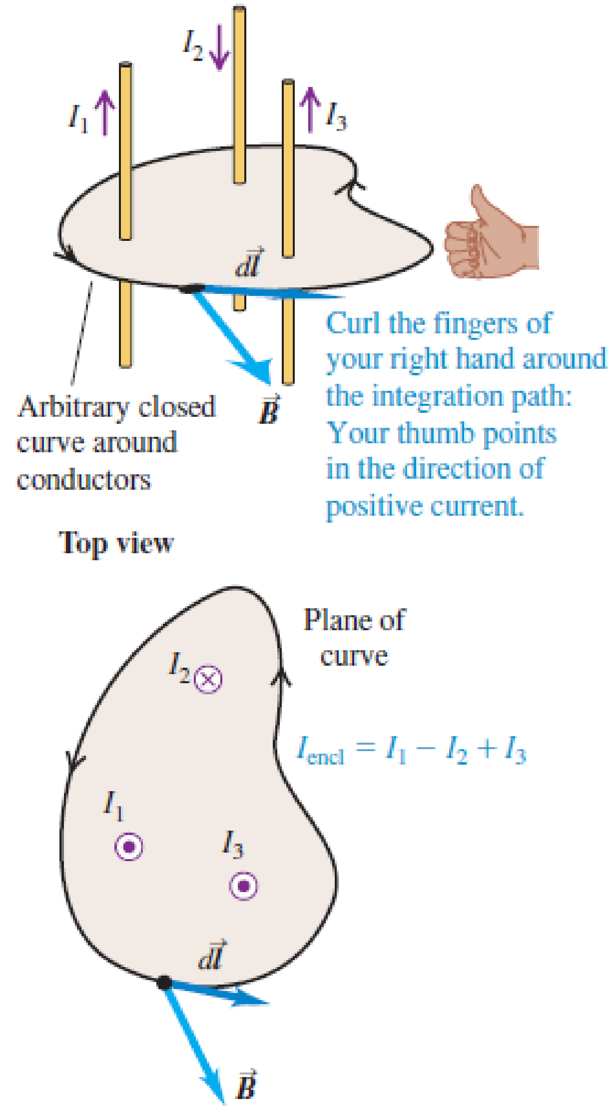
Ampere's Law

The line integral of the *magnetic field*.

- Apply to a system of three straight currents.

$$\oint (\vec{\boldsymbol{B}}_{net} \cdot d\vec{l}) = \mu_0 I_{encl}$$

$$I_{encl} = I_1 - I_2 + I_3$$



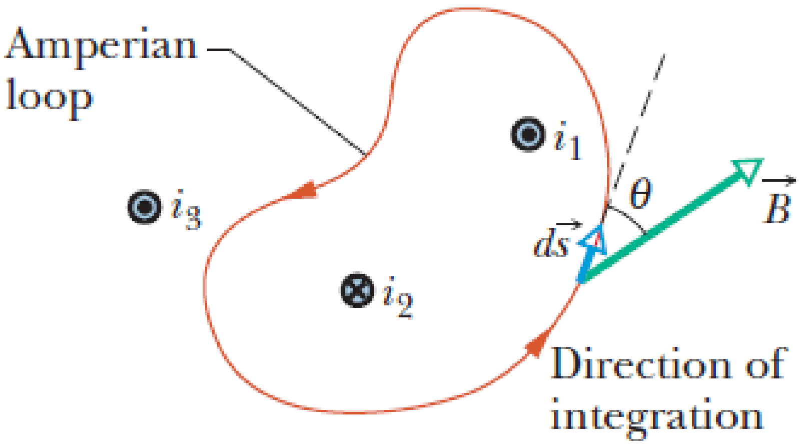
Ampere's Law

The line integral of the *magnetic field*.

- Apply to *any* system of currents.

$$I_{encl} = \sum_k I_k$$

Net current through area bounded by the *Amperian loop*



Only the currents encircled by the loop are used in Ampere's law.

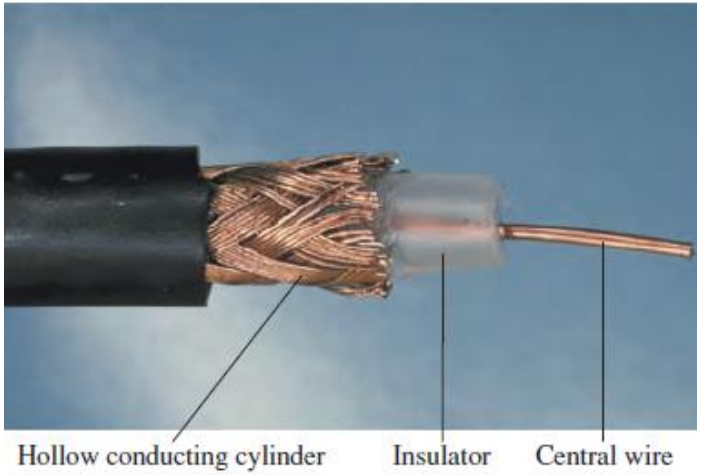
Applications of Ampere's Law

Example: Co-axial cable

- Circular Amperian loop *outside* the cable.

$$B = 0 \quad (\text{ for } r > c\ )$$

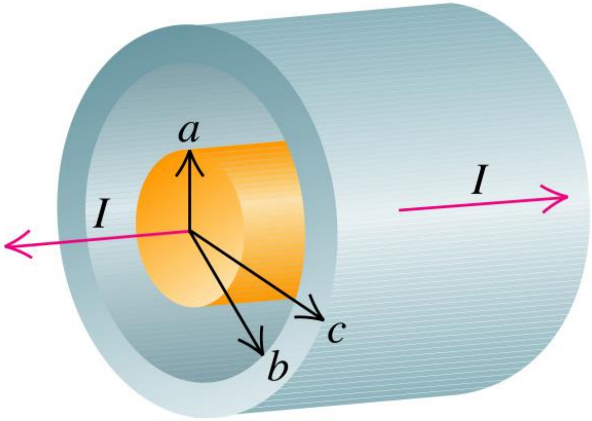
(Net current = 0)



- Circular Amperian loop *inside* the cable.

$$B = \frac{\mu_0 I}{2\pi r}$$

(for $b > r > a$)



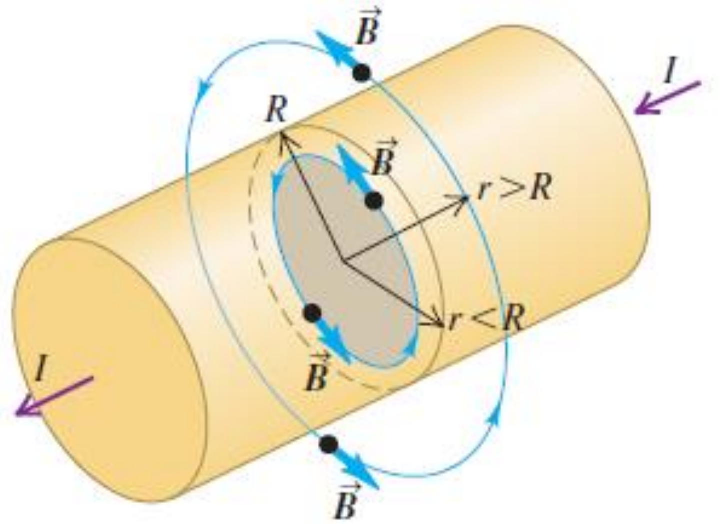
Applications of Ampere's Law

Example: Solid cylindrical conductor

- Circular Amperian loop *outside* the cable. (Symmetry requires B to be uniform along the circle)

$$\oint(\vec{\boldsymbol{B}}\cdot d\vec{\boldsymbol{l}}) = B(2\pi r) = \mu_0 I$$

$$B = \frac{\mu_0 I}{2\pi r} \quad (\text{for } r > R)$$



Applications of Ampere's Law

Example: Solid cylindrical conductor

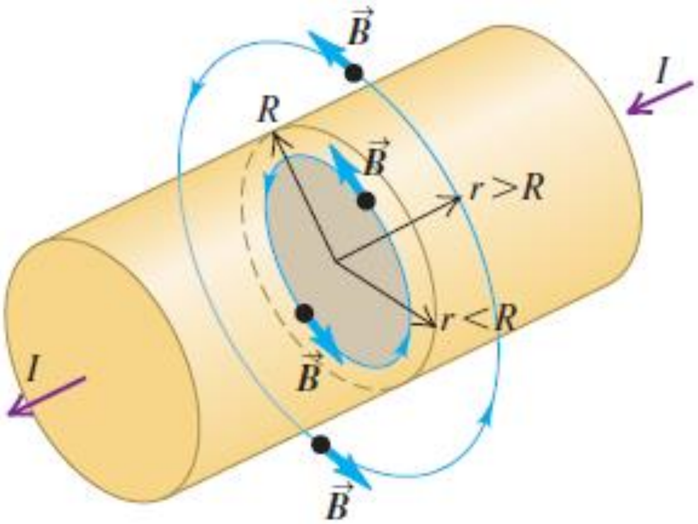
- Circular Amperian loop *inside* the cable. (Symmetry requires B to be uniform along the circle)

Current density $= I/(\pi R^2)$

$$\oint (\vec{\boldsymbol{B}} \cdot d\vec{\boldsymbol{l}}) = B(2\pi r)$$

$$= \mu_0 I \left(\frac{\pi r^2}{\pi R^2}\right)$$

$$B = \frac{\mu_0 I}{2\pi} \frac{r}{R^2} \qquad (\text{for } r < R)$$

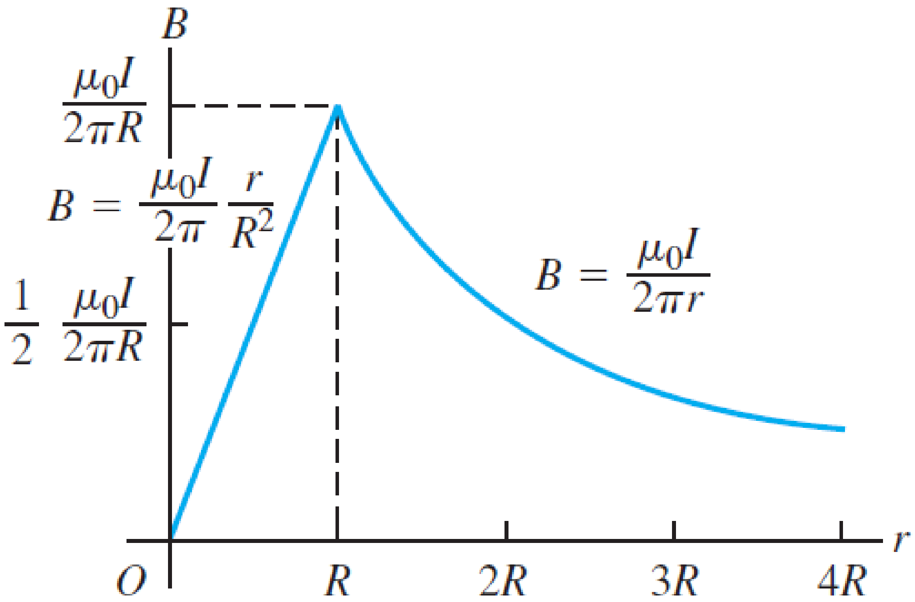


Applications of Ampere's Law

Example: Solid cylindrical conductor

- Solution

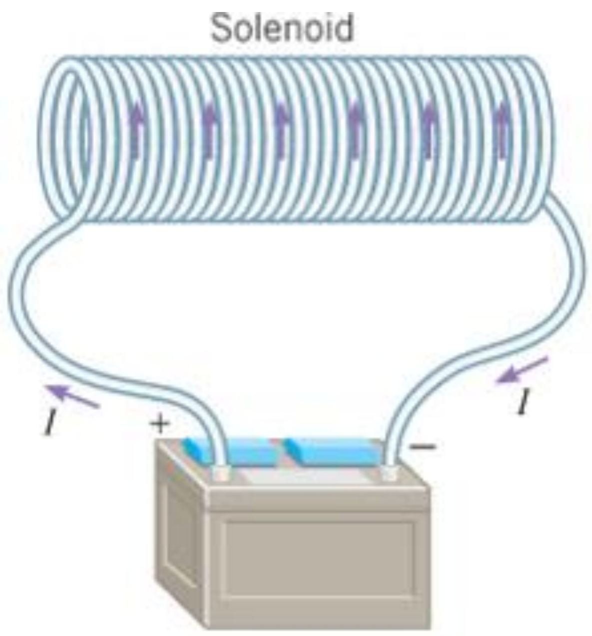
$$B = \begin{cases} \dfrac{\mu_0 I}{2\pi}\dfrac{r}{R^2} & (\text{for } r < R) \\[2ex] \dfrac{\mu_0 I}{2\pi r} & (\text{for } r > R) \end{cases}$$

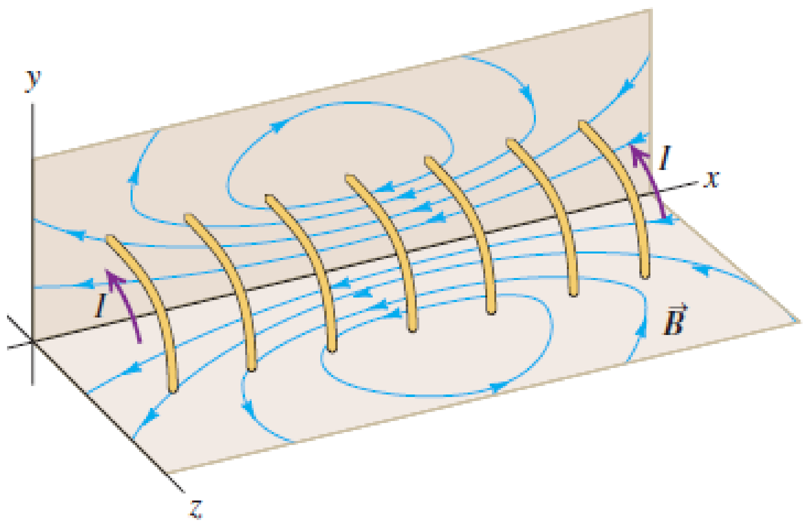


Applications of Ampere's Law

Example: *Ideal* solenoid

N turns of wire around a cylinder of length L





$n = N/L$ = number of turns per unit length

Applications of Ampere's Law

Example: *Ideal* solenoid

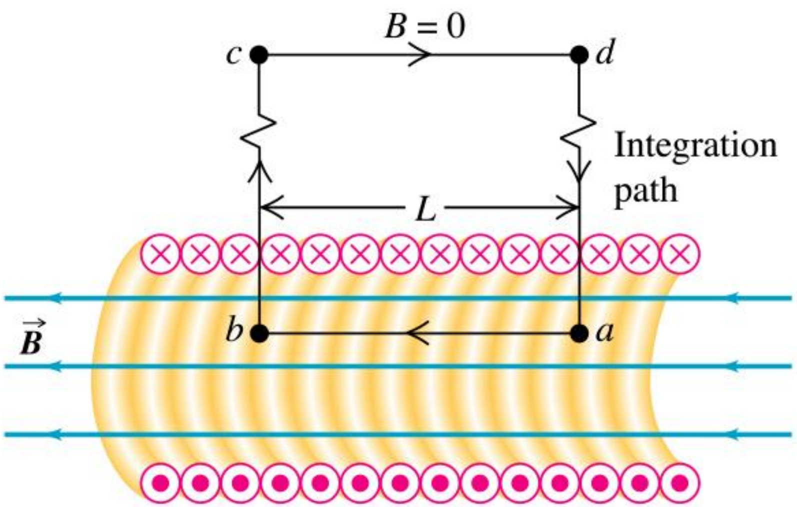
- Amperian loop is rectangular path *abcd.* (Symmetry requires B inside is uniform.)

$$\oint (\vec{\boldsymbol{B}} \cdot d\vec{\boldsymbol{l}}) = B\int_a^b dl = B(L)$$

$$= \mu_0 NI$$

$$B = \mu_0 \left(\frac{N}{L}\right) I = \mu_0 n I$$

Magnetic field inside
the solenoid is *uniform* !

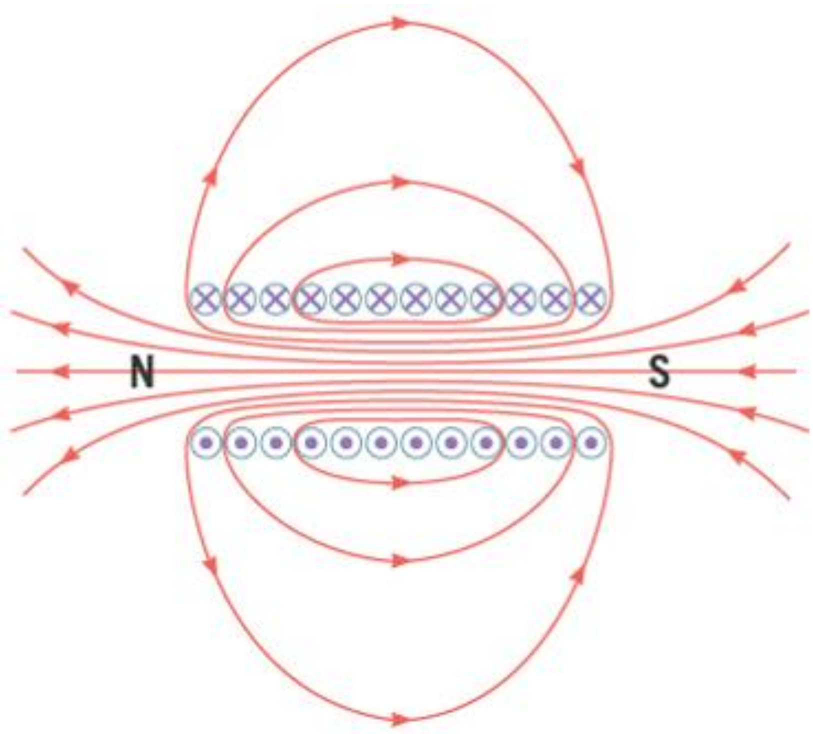


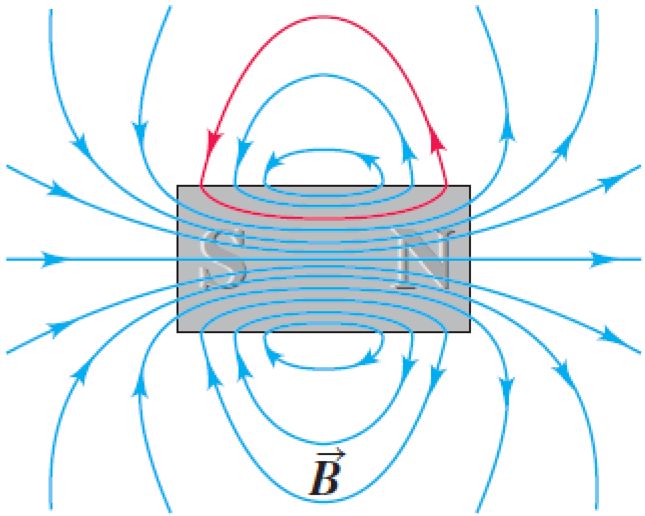
Central part of solenoid

Applications of Ampere’s Law

Example: *Ideal* solenoid

- Real solenoid and bar magnet





Magnetic Materials

Three general types of magnetism **:**

- ***Diamagnetism***: A diamagnetic material placed in an external field $\mathbf{B}_{ext}$ develops a magnetic dipole moment directed opposite $\mathbf{B}_{ext}$.

  When the field is non-uniform, the material is *repelled* from a region of *greater* field toward region of *lesser* field.

- ***Paramagnetism***: A paramagnetic material placed in an external field $\mathbf{B}_{ext}$ develops a magnetic dipole moment in the direction of $\mathbf{B}_{ext}$.

  If the field is non-uniform, the material is *attracted* toward a region of *greater* field from a region of *lesser* field.

Magnetic Materials

Three general types of magnetism :

- ***Ferromagnetism***: A ferromagnetic material placed in an external field $\mathbf{B}_{ext}$ develops a strong magnetic dipole moment in the direction of $\mathbf{B}_{ext}$.

  In a non-uniform field, the material is *attracted* toward a region of *greater* field from a region of *lesser* field.

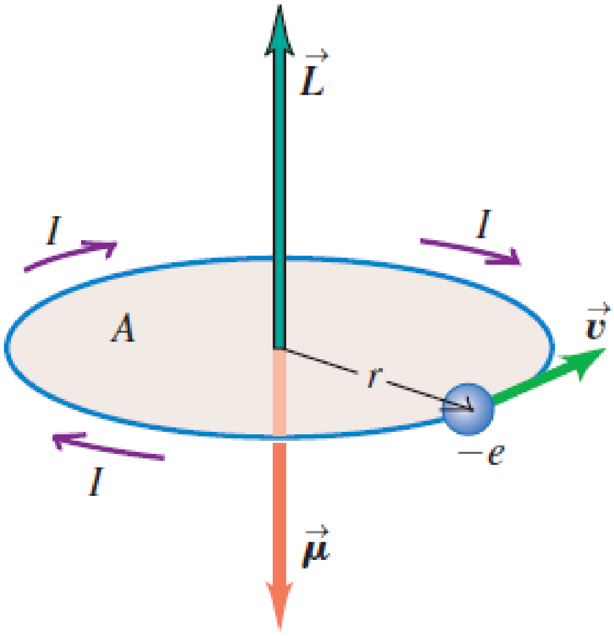
Magnetic Materials

***Origins of magnetism* :**

- ***Orbital spin***:

  In the atom, an electron has an additional angular momentum $\mathbf{L}_{\text{orb}}$. Associated with $\mathbf{L}_{\text{orb}}$ is an **orbital magnetic dipole moment** $\boldsymbol{\mu}_{\text{orb}}$.

$$\vec{\mu}_{\text{orb}} = -\frac{e}{2m}\vec{\mathbf{L}}_{\text{orb}}$$



Fundamental unit:

$$\mu_{orb} = \mu_B = 9.274 \times 10^{-24} \text{ J/T}$$

the ***Bohr magneton***

Magnetic Materials

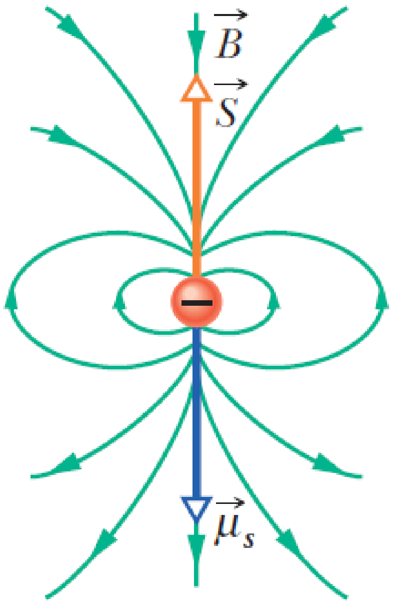
Origins of magnetism :

- ***Intrinsic spin***:

  An electron has an *intrinsic* angular momentum (or spin) **S**. Associated with spin is an intrinsic **spin magnetic dipole moment μ_s**.

  The ***intrinsic "spin"*** and ***orbital motion of electrons*** gives rise to the magnetic properties of materials.

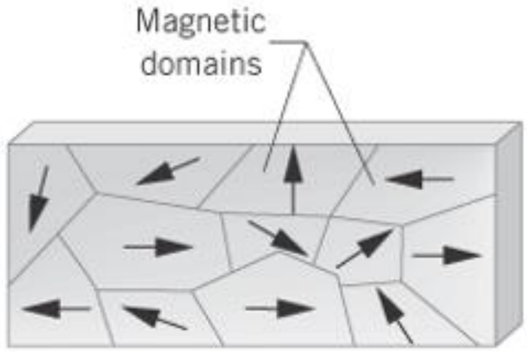
For an electron, the spin is opposite the magnetic dipole moment.



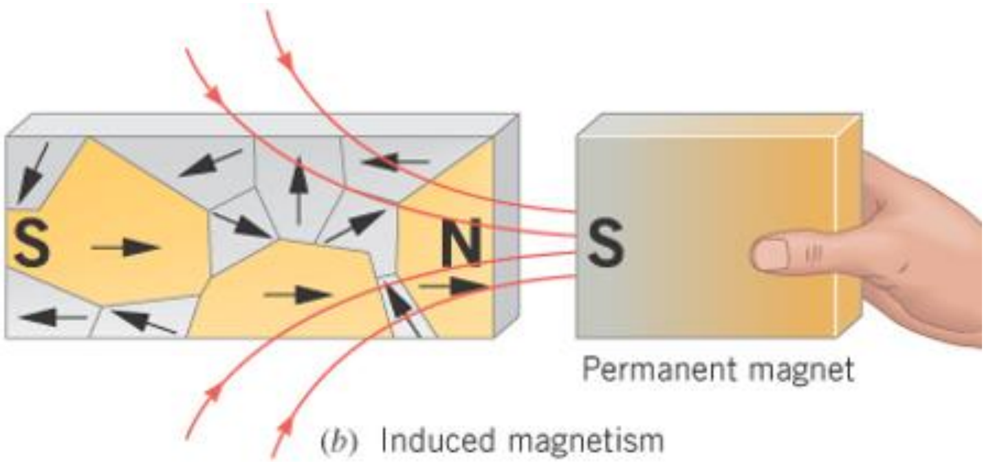
Magnetic Materials

Ferromagnetism

- In ***ferromagnetic materials*** (iron, nickel, cobalt) groups of neighboring atoms, forming ***magnetic domains,*** the spins of electrons are naturally aligned with each other.



(*a*) Unmagnetized iron



(*b*) Induced magnetism

END