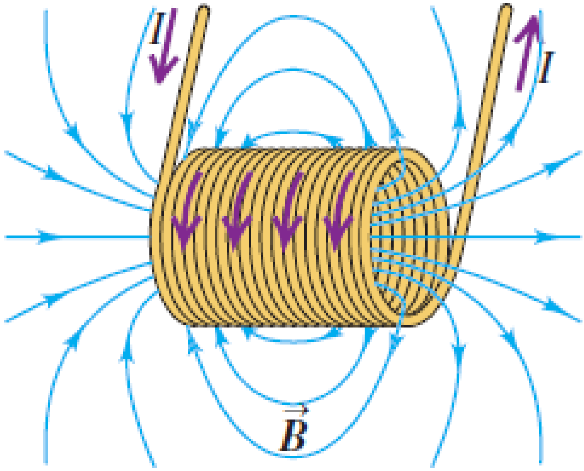
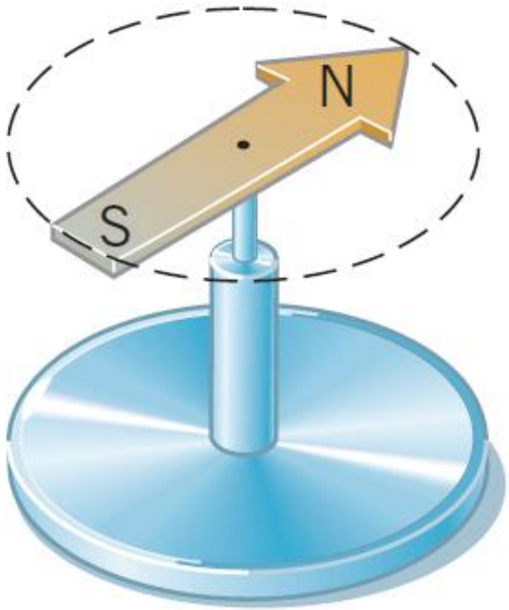
Magnetic Field and Magnetic Forces



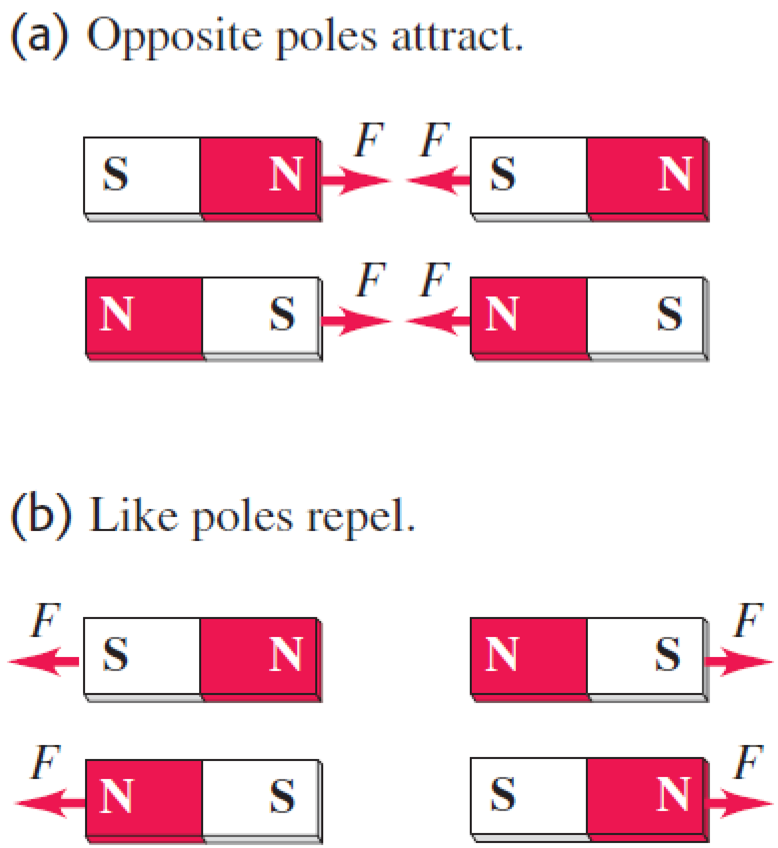
Magnetism

The needle of a *compass* is permanent magnet that has a *north* magnetic pole (N) at one end and a *south* magnetic pole (S) at the other.



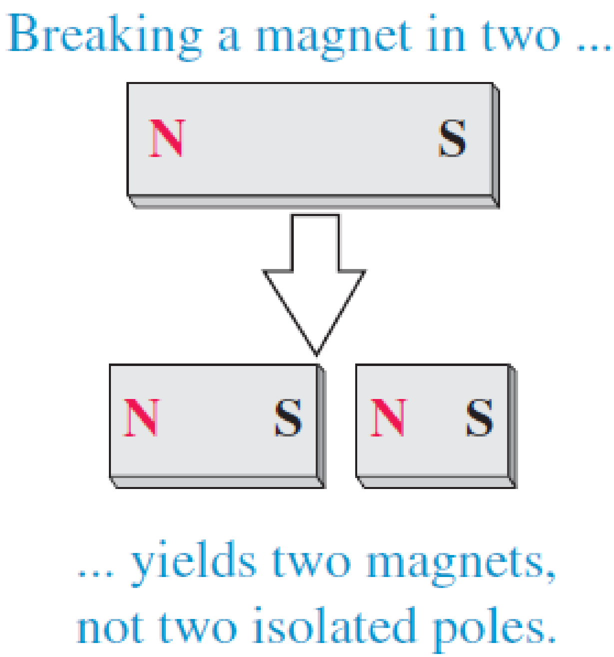
Magnetism

The behavior of magnetic poles is similar to that of like and unlike electric charges.



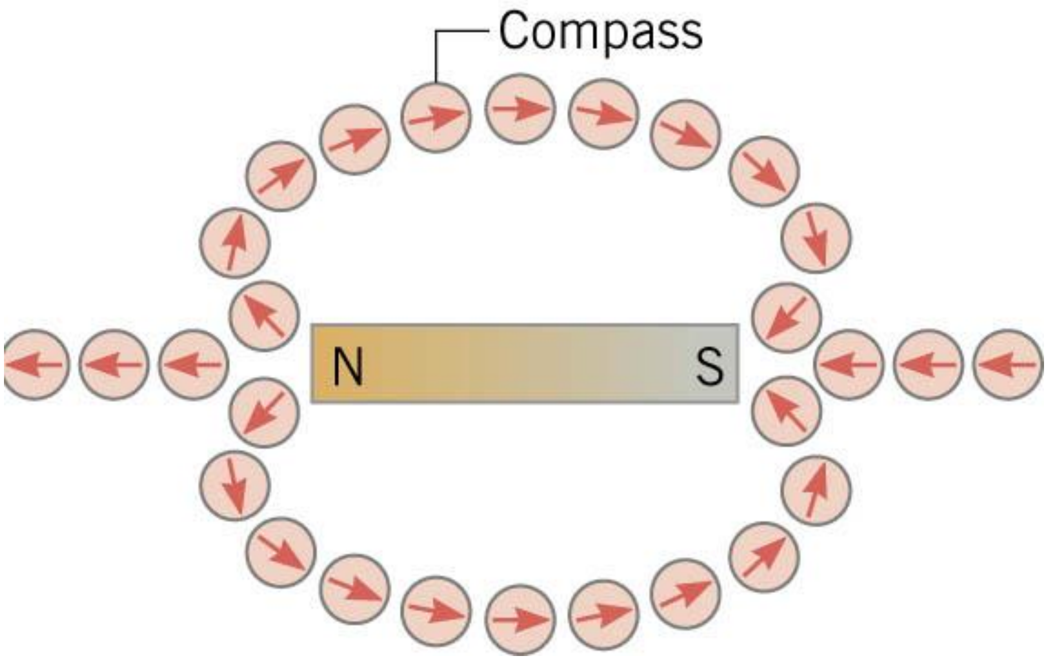
Magnetism

While isolated positive and negative charges exist, there is *no* experimental evidence that a single isolated magnetic pole exists; poles always appear in pairs.



Magnetism

Surrounding a magnet there is a ***magnetic field***. The direction of the magnetic field at any point in space is the direction indicated by the north pole of a small compass needle placed at that point.

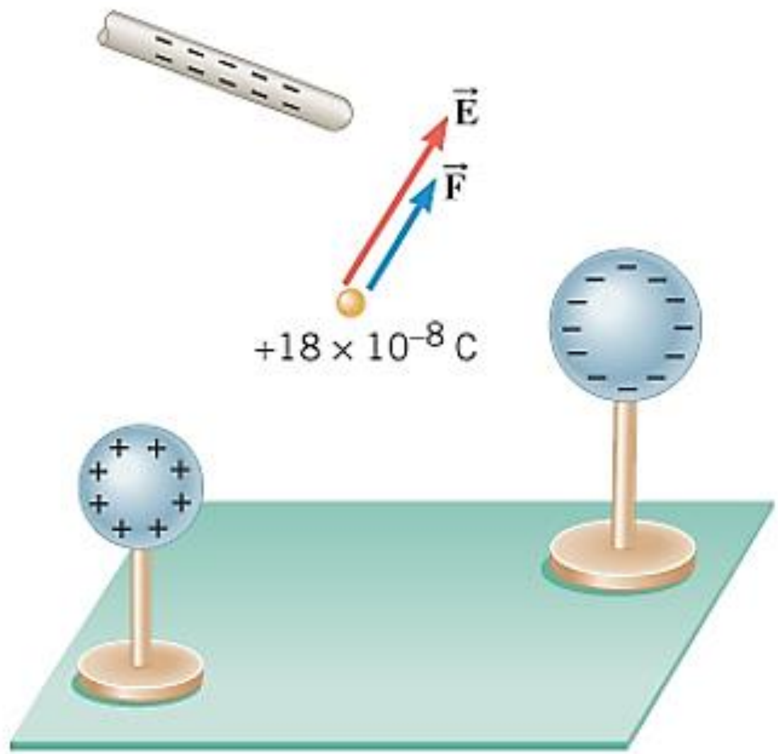


Magnetic Field

Magnetic force on charged particle

When a charge is placed in an **electric field**, it experiences a force, according to

$$\vec{\mathbf{F}} = q\vec{\mathbf{E}}$$



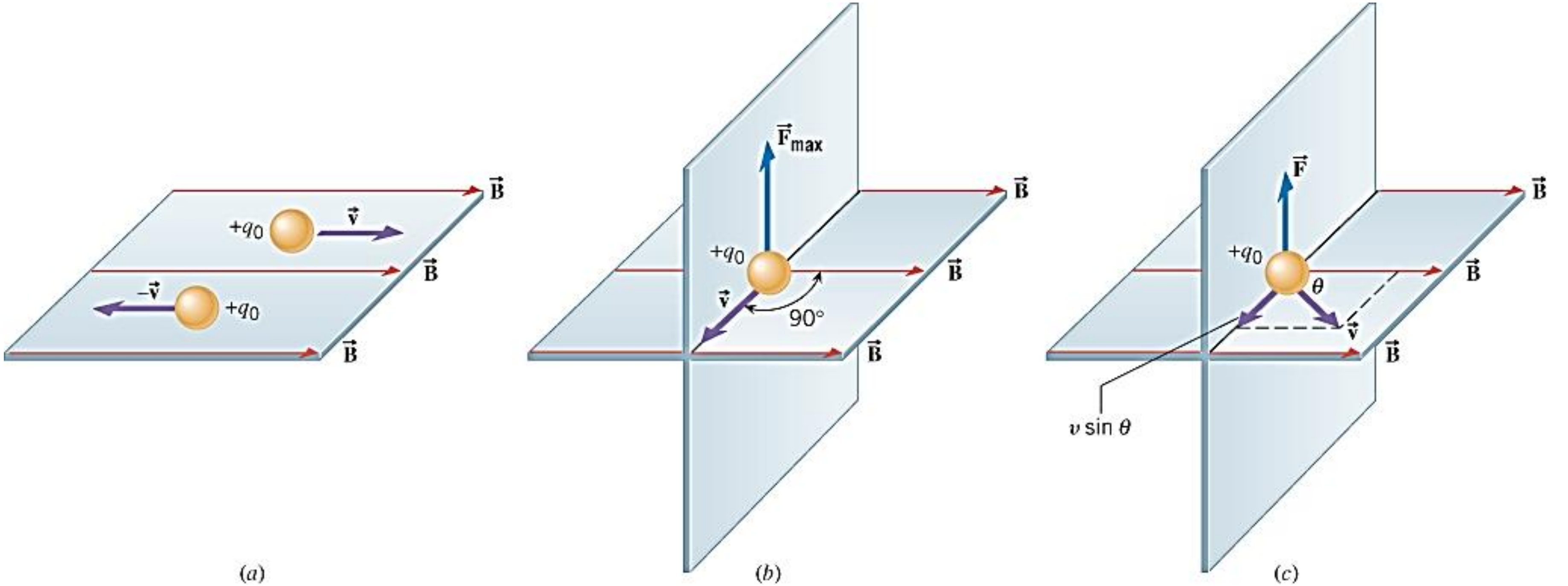
(*a*)

Magnetic Field

Magnetic force on charged particle

For a charge to experience a **magnetic force** when placed in a **magnetic field**

- The charge must be ***moving***.
- The velocity of the charge must have a *component* that is *perpendicular* to the direction of the magnetic field.



Magnetic Field

Magnetic force on charged particle

The magnitude of the magnetic force:

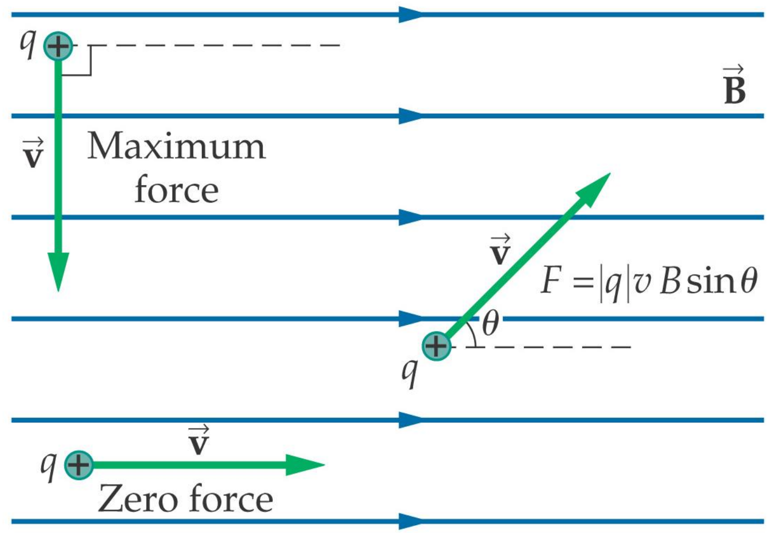
$$F = |q|\, vB \sin\theta$$

- The maximum possible force

$$F_{max} = |q|\, v\, B$$

- This is an experimental result – we observe it to be true.
- The **magnetic field** is *defined* from this relation.

$$B = \frac{F_{max}}{|q|\, v}$$



Magnetic Field

Magnetic force on charged particle

SI Unit of Magnetic Field:

$$\frac{\text{newton}\cdot\text{second}}{\text{coulomb}\cdot\text{meter}} = 1\,\text{tesla}\,(\text{T})$$

Typical Values:

Earth's magnetic field	0.00005 Tesla	0.5 Gauss
Small bar magnet	0.01 Tesla	100 Gauss
Within a sunspot	0.15 Tesla	1500 Gauss
Big electromagnet	1.5 Tesla	15,000 Gauss
Strong lab magnet	10 Tesla	100,000 Gauss

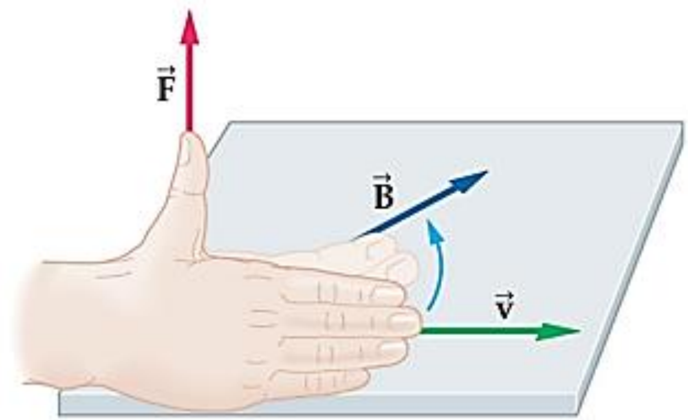
$$1\,\text{gauss} = 10^{-4}\ \text{tesla}$$

Magnetic force on charged particle

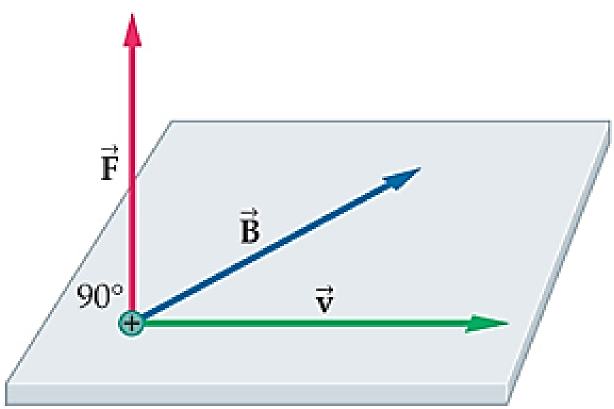
The direction of the **magnetic field**:

Right Hand Rule No. 1.

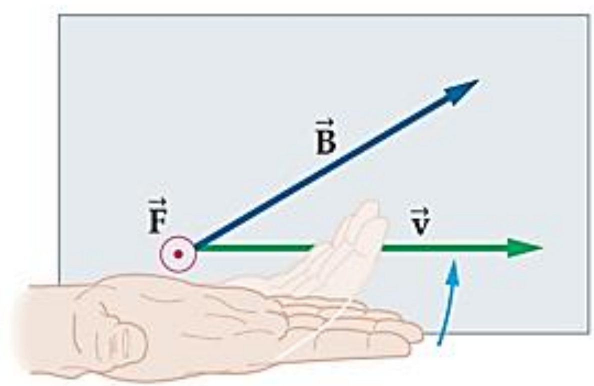
- Gives the direction of the force on a positive charge.
- The force on a negative charge would be in the opposite direction.



(b)



(a)



(c) Top view

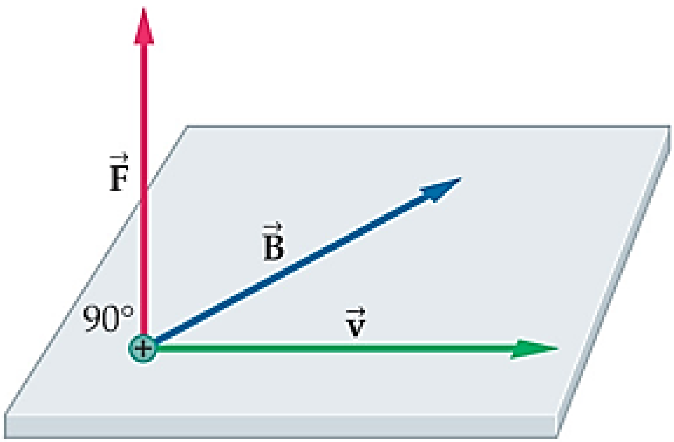
Magnetic Field

Magnetic force on charged particle

The direction of the **magnetic field**:

- This relationship between the three vectors – magnetic field, velocity, and force – can also be written as a **vector cross product**:

$$\vec{\mathbf{F}} = q\vec{\mathbf{v}} \times \vec{\mathbf{B}}$$



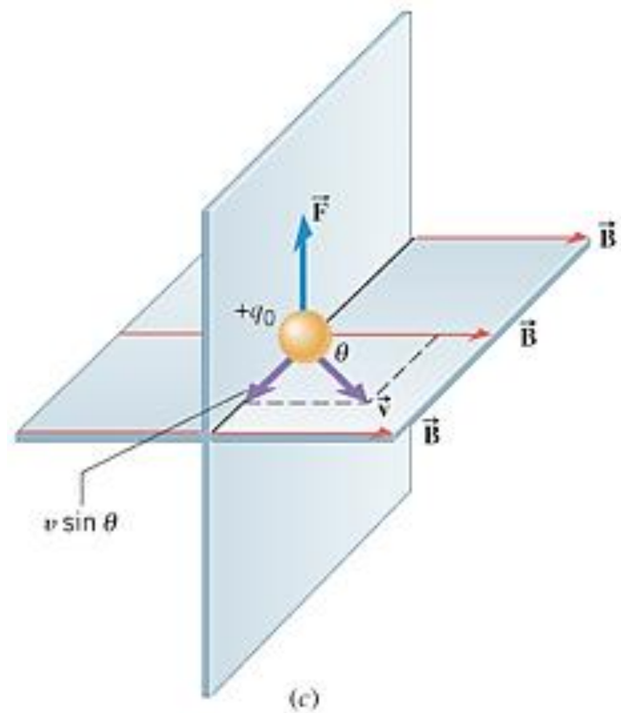
(a)

Magnetic Field

Example 1: Magnetic Force on Charged Particles

A proton in a particle accelerator has a speed of $5.0\text{x}10^6$ m/s. The proton encounters a magnetic field whose magnitude is 0.40 T and whose direction makes and angle of 30.0º with respect to the proton's velocity.

Find (a) the magnitude and direction of the force on the proton and (b) the acceleration of the proton. (c) What would be the force and acceleration of the particle were an electron?



Magnetic Field

Example 1: Magnetic Force on Charged Particles

(a) the magnitude and direction of the force on the proton

$$F = |q_o| vB\sin\theta = \left(1.60\times10^{-19}\,\mathrm{C}\right)\left(5.0\times10^{6}\ \mathrm{m/s}\right)\left(0.40\mathrm{T}\right)\sin\left(30.0^{\circ}\right)$$

$$= 1.6\times10^{-13}\,\mathrm{N}$$

(b) the acceleration of the proton.

$$a = \frac{F}{m_{\mathrm{p}}} = \frac{1.6\times10^{-13}\,\mathrm{N}}{1.67\times10^{-27}\,\mathrm{kg}} = 9.6\times10^{13}\ \mathrm{m/s^2}$$

(c) The force is the same, but direction is opposite

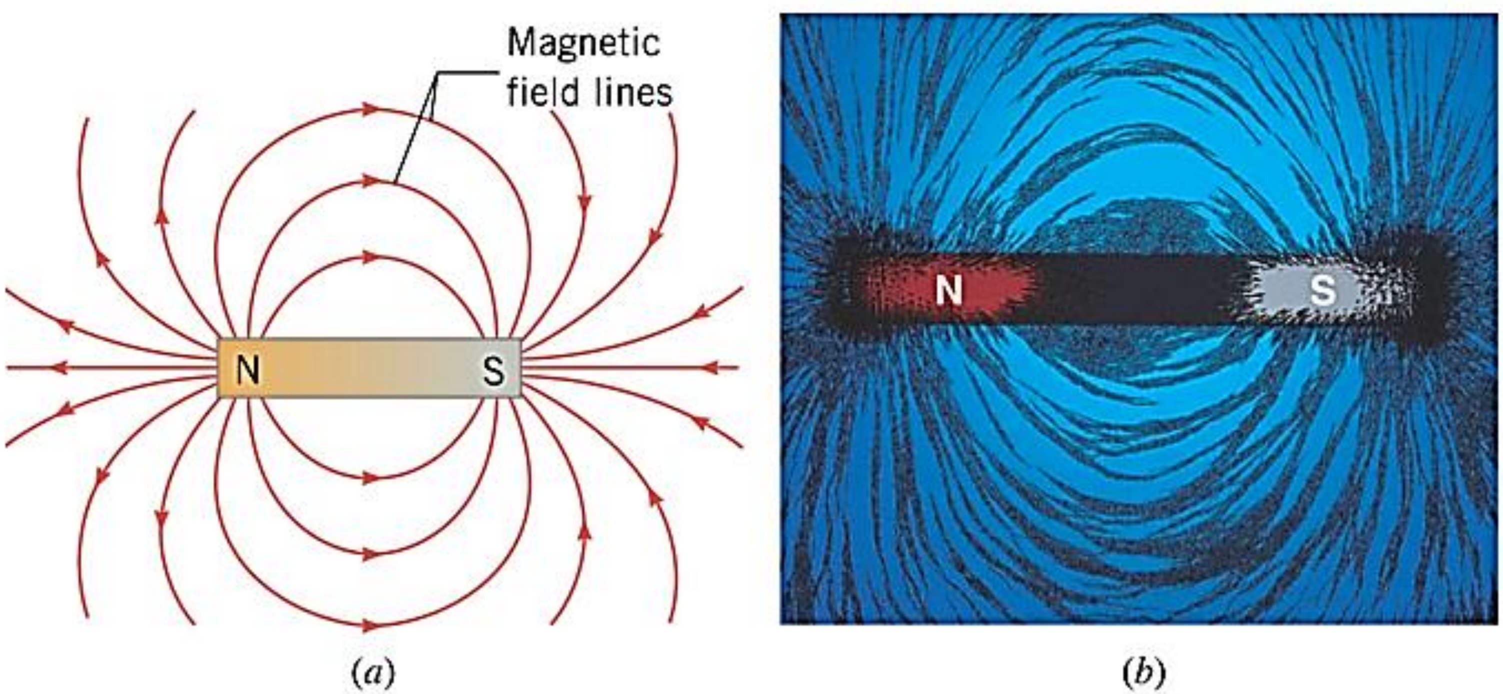
$$a = \frac{F}{m_{\mathrm{e}}} = \frac{1.6\times10^{-13}\,\mathrm{N}}{9.11\times10^{-31}\,\mathrm{kg}} = 1.8\times10^{17}\ \mathrm{m/s^2}$$

Magnetic Field Lines and Magnetic Flux

The magnetic field can be visualized using **magnetic field lines**, similar to the electric field.

(a) The magnetic field lines.

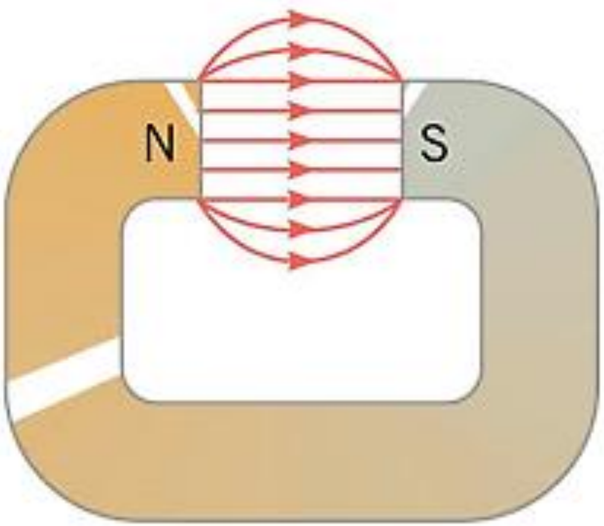
(b) Pattern of iron filings in the vicinity of a bar magnet.



Magnetic Field Lines and Magnetic Flux

The magnetic field can be visualized using **magnetic field lines**, similar to the electric field.

(c) The magnetic field lines in the gap of a horseshoe magnet.

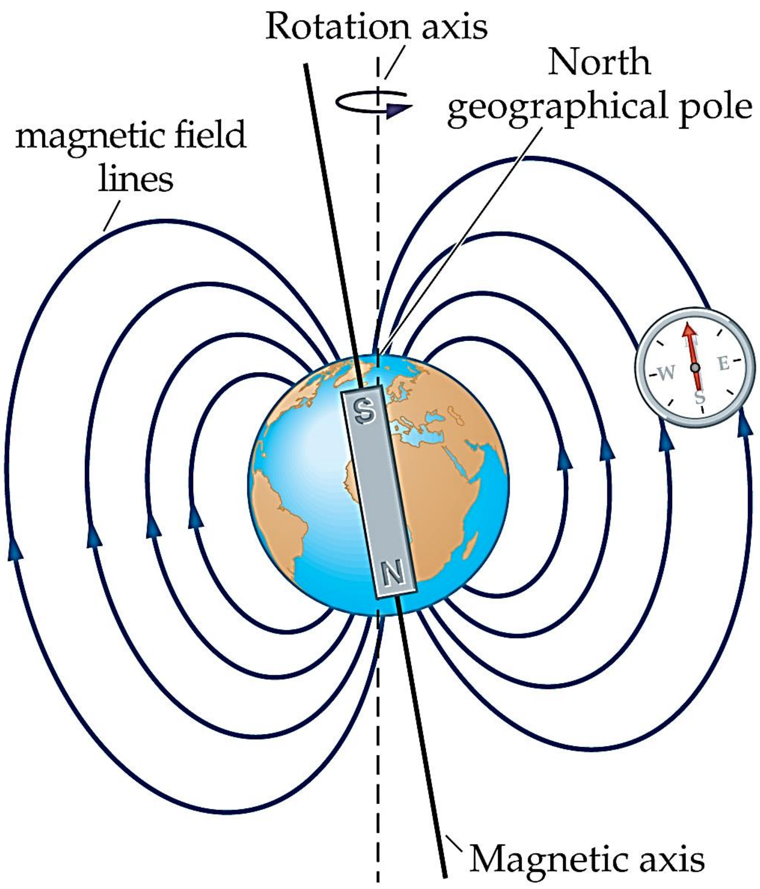


(c)

Magnetic Field Lines and Magnetic Flux

The Earth's magnetic field

- The *Earth's magnetic field* resembles that of a bar magnet.
- However, since the north poles of compass needles point towards the north, the magnetic pole there is actually a south pole.



Magnetic Field Lines and Magnetic Flux

Magnetic Flux

$$\Phi_B = B_\perp A = BA\cos\phi$$

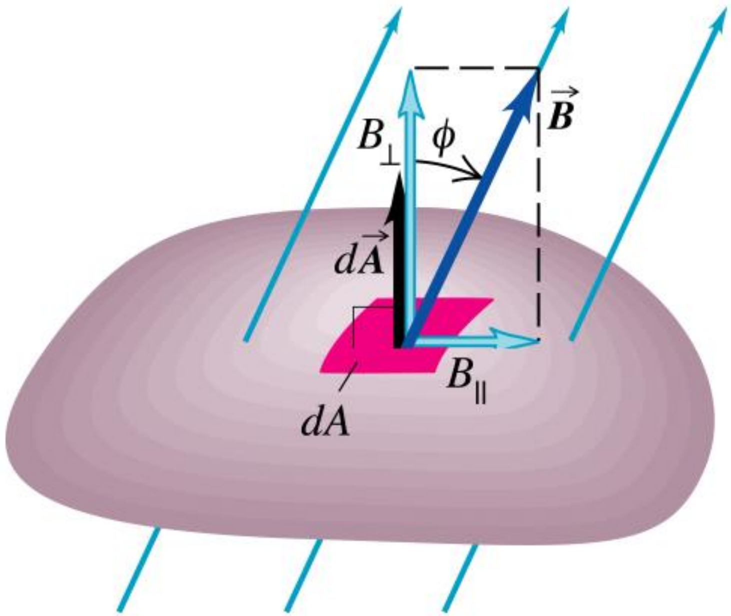
- If **B** is perpendicular to the surface area.

$$B = \frac{\Phi_B}{A}$$

- *Unit of Magnetic Flux:*

$$\text{weber} = \text{Tesla} \cdot \text{m}^2$$

$$\text{Tesla} = \frac{\text{weber}}{\text{m}^2}$$



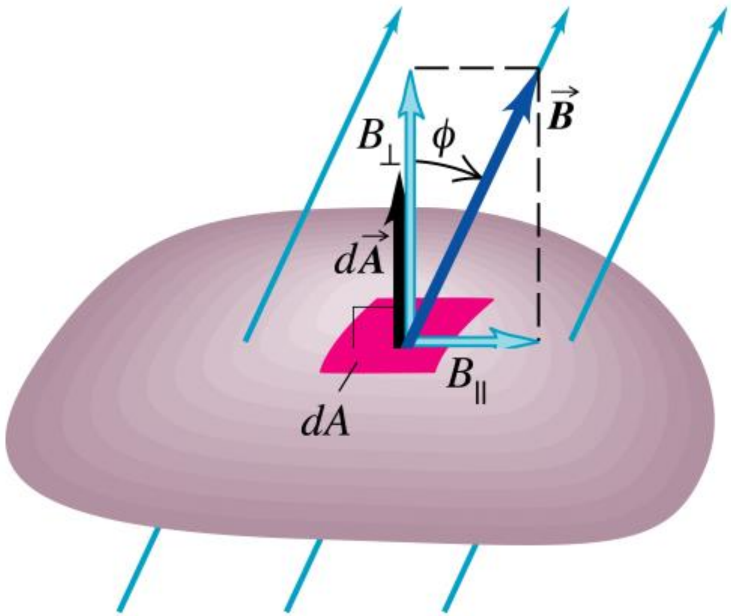
Magnetic Field Lines and Magnetic Flux

Magnetic Flux

- For non-uniform magnetic field and/or irregular surface area.

$$d\Phi_B = B_\perp dA = B\cos\phi \, dA$$

$$= \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}}$$



- Total flux

$$\Phi_B = \iint \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}}$$

- Gauss' law for magnetism

$$\Phi_B = \iint_{\substack{closed \\ surface}} \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}} = 0$$

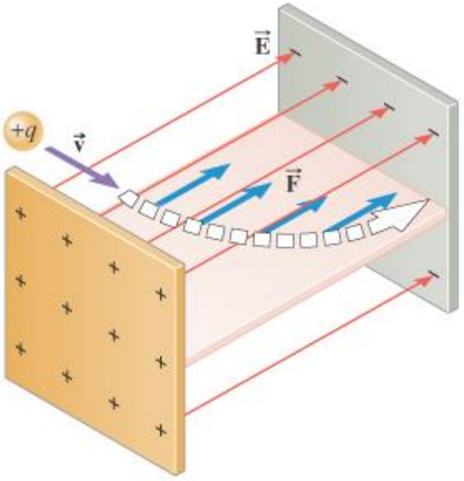
Motion of Charged Particles in a Magnetic Field

The motion of a charged particle in an ***electric*** field

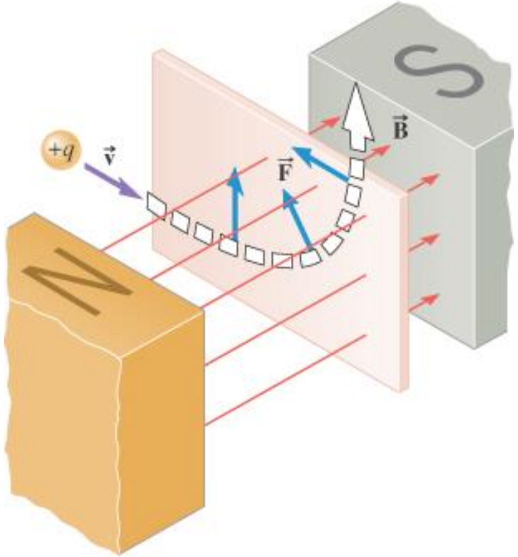
- The electrical force ***can*** do work on a charged particle.

The motion of a charged particle in a ***magnetic*** field

- The magnetic force ***cannot*** do work on a charged particle.



(*a*)



(*b*)

Motion of Charged Particles in a Magnetic Field

Motion in a uniform magnetic field:

- The magnetic force always remains perpendicular to the velocity and is directed toward the center of the circular path.

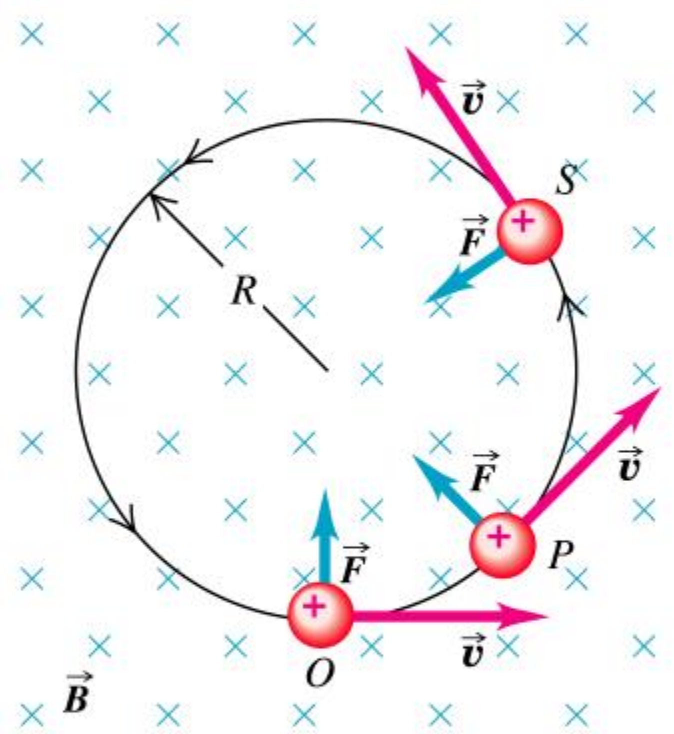
$$F_c = m\frac{v^2}{R} = F_M$$

$$\Downarrow$$

$$qvB = m\frac{v^2}{R}$$

$$\Downarrow$$

$$R = \frac{mv}{qB}$$

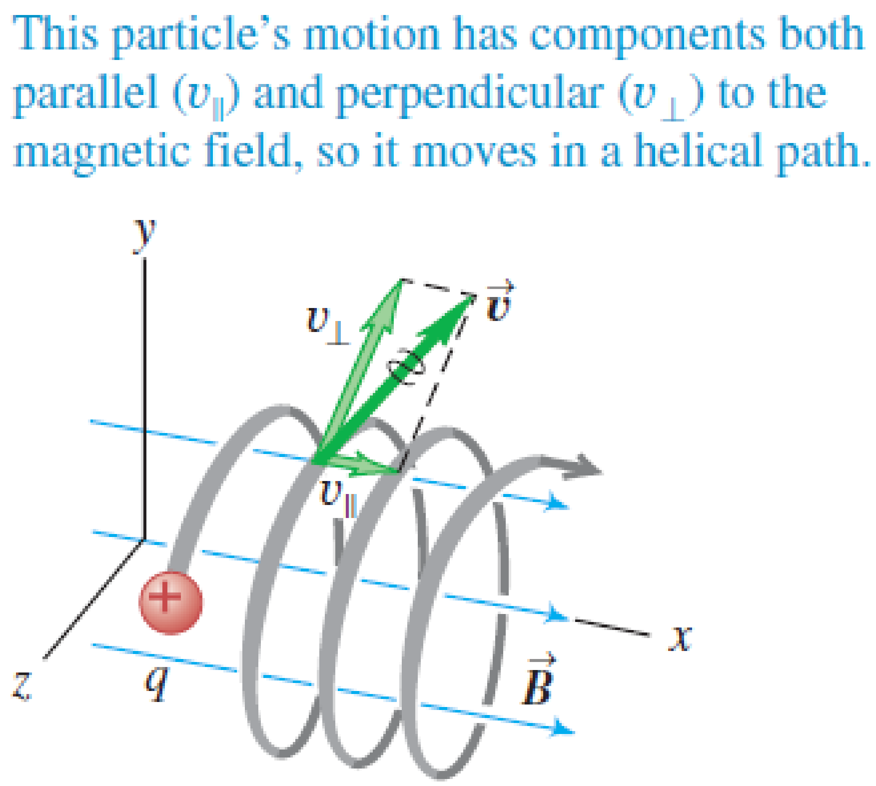


(a)

Motion of Charged Particles in a Magnetic Field

Motion in a uniform magnetic field:

- When the velocity is in some arbitrary direction, a *helical path* results.



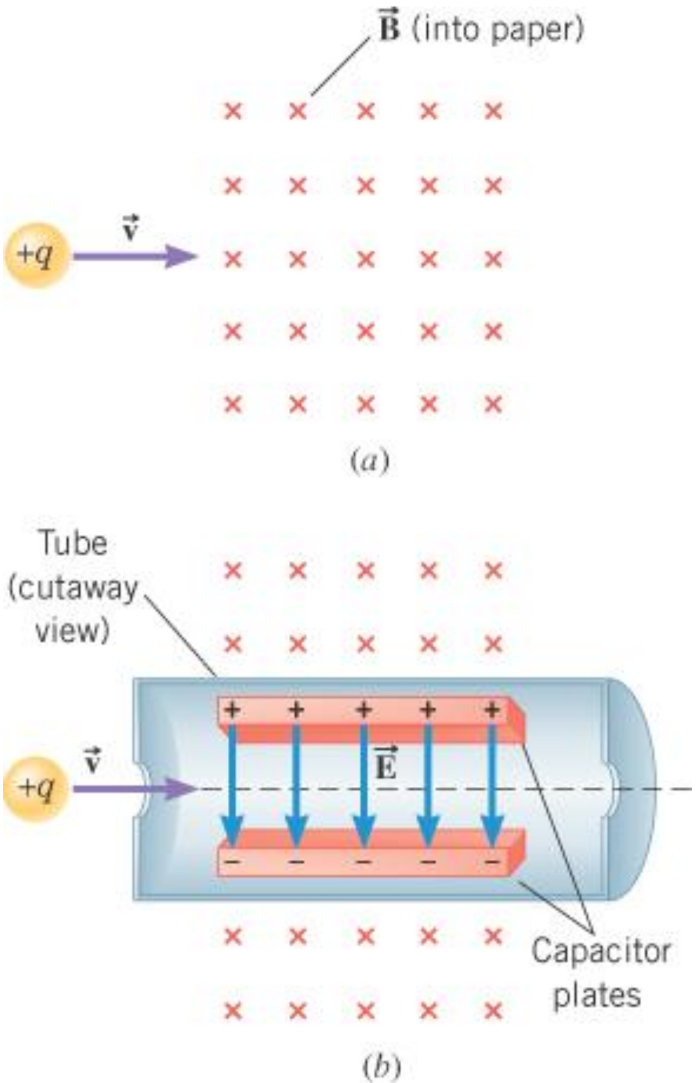
Applications of Motion of Charged Particles

A Velocity Selector

- A velocity selector is a device for measuring the velocity of a charged particle. The device operates by applying electric and magnetic forces to the particle in such a way that these forces balance.

$$F_E = qE = F_M = qvB$$

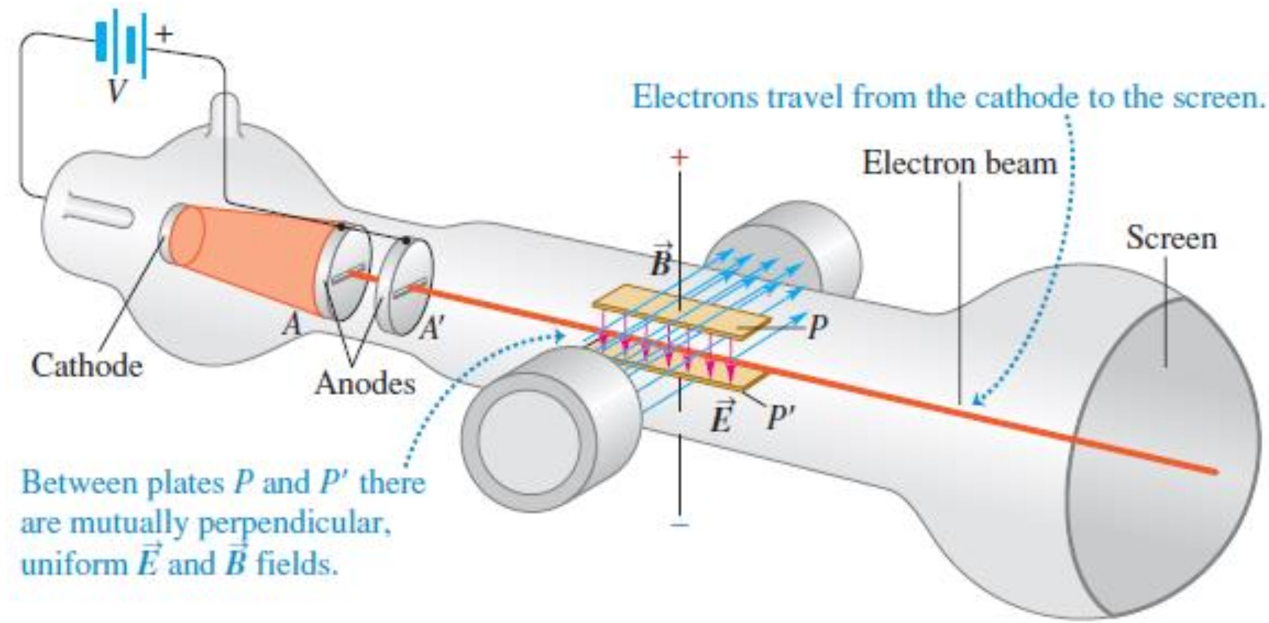
$$v = \frac{E}{B}$$



Applications of Motion of Charged Particles

Thomson's Experiment

- Using the velocity selector principle to determine the *charge-to-mass ratio* of the electron.

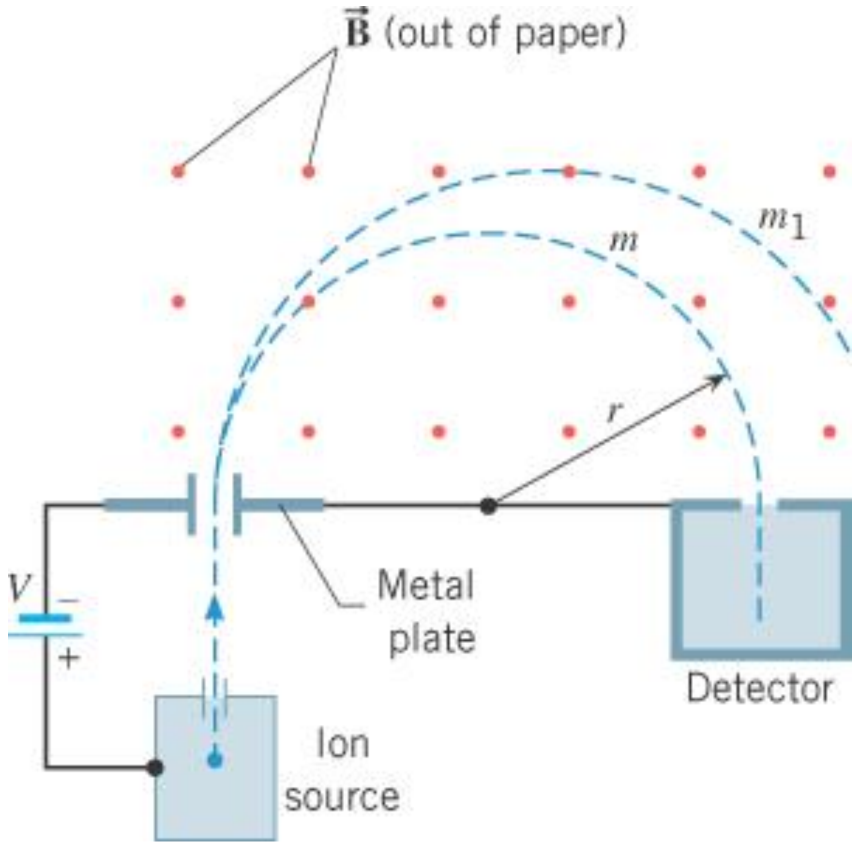


$$\frac{e}{m} = \frac{E^2}{2VB^2}$$

Applications of Motion of Charged Particles

Mass Spectrometer

- In a mass spectrometer, ions of different mass and charge move in circles of different radii, allowing separation of different isotopes of the same element.



Applications of Motion of Charged Particles

Mass Spectrometer

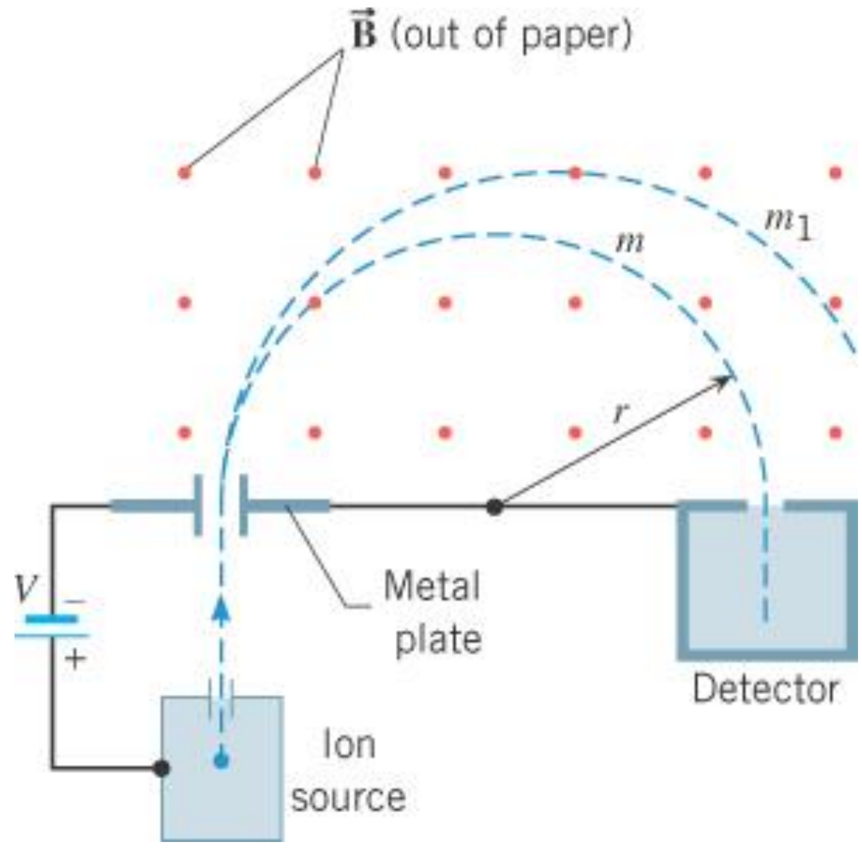
- For circular path

$$r = \frac{mv}{qB}$$

- Energy of particle

$$\frac{1}{2}mv^2 = qV$$

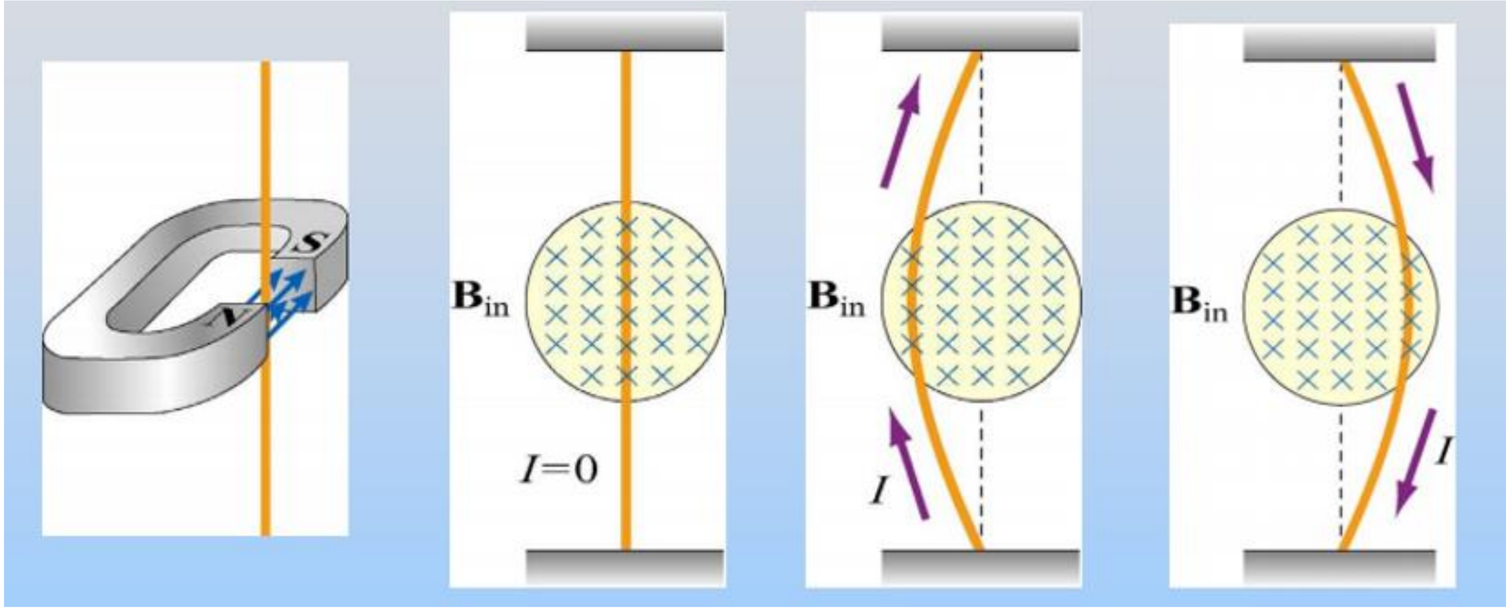
$$m = \left(\frac{qr^2}{2V}\right)B^2$$



Magnetic Force on a Current-carrying Conductor

Electric current in a magnetic field

- The magnetic force on the moving charges pushes the wire.



Magnetic Force on a Current-carrying Conductor

Electric current in a magnetic field

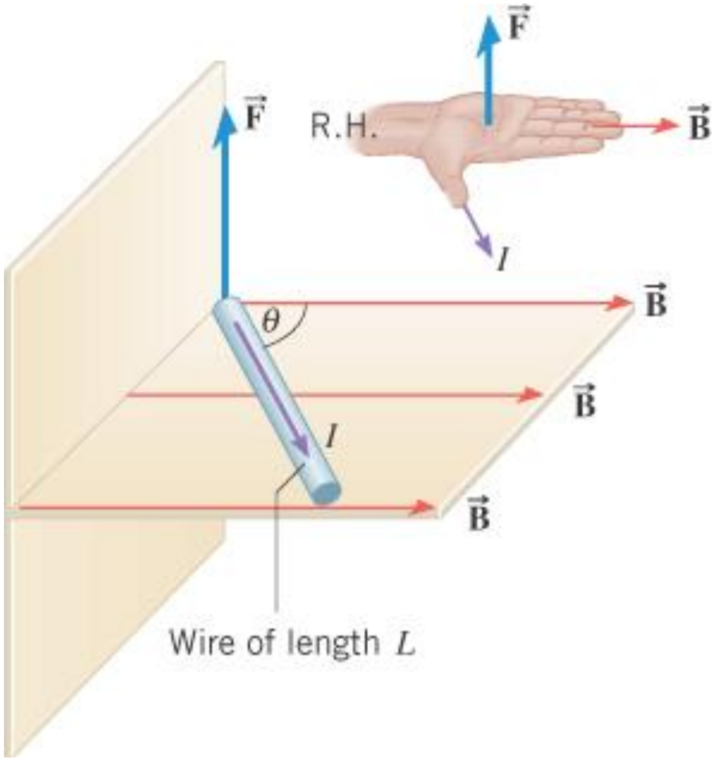
- The force on a segment *L* of a current-carrying wire in a magnetic field is given by:

$$F_M = \Delta q\, vB \sin\theta$$

$$= \left(\frac{\Delta q}{\Delta t}\right)(v\Delta t)B \sin\theta$$

$$\Downarrow$$

$$F_M = I(\Delta L)B \sin\theta$$



Magnetic Force on a Current-carrying Conductor

Electric current in a magnetic field

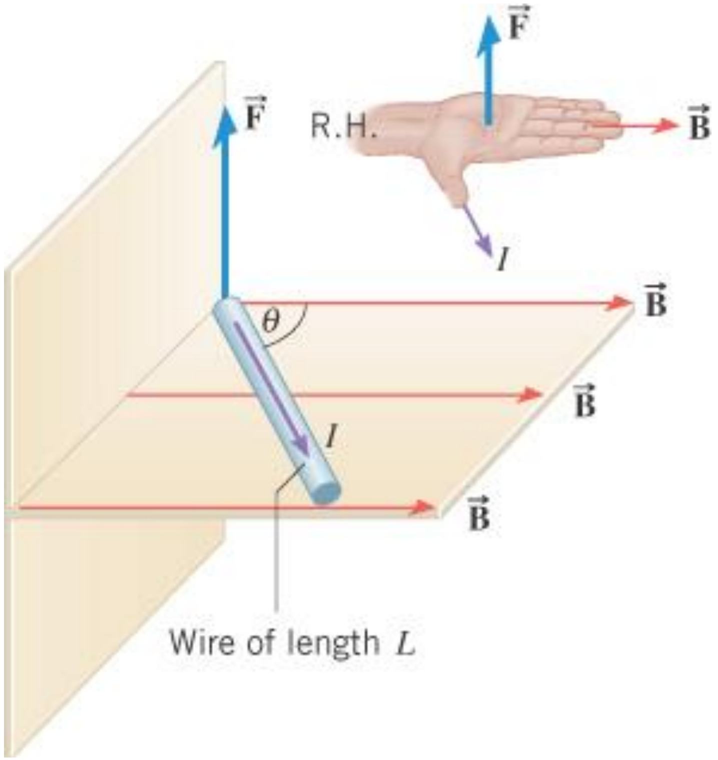
- The force on a straight segment *L*

$$F_M = ILB\,\sin\theta$$

- In vector form

$$\vec{\mathbf{F}} = I\left(\vec{\mathbf{L}} \times \vec{\mathbf{B}}\right)$$

(The current *I* is not a vector!)



Magnetic Force on a Current-carrying Conductor

Electric current in a magnetic field

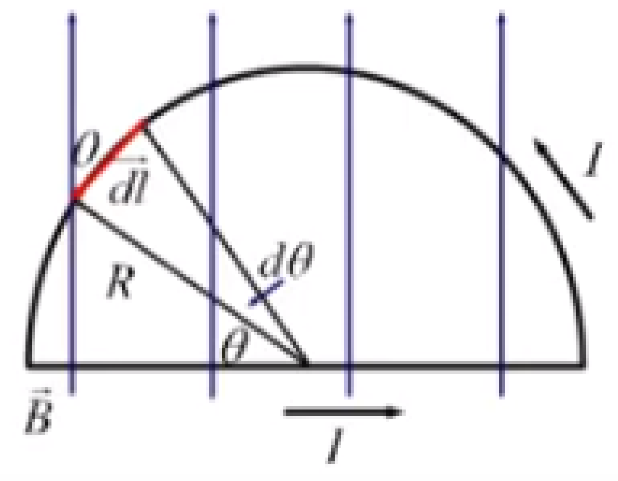
- When the conductor is not a straight, divide the conductor into infinitesimal segments *dL:*

$$d\vec{\mathbf{F}} = I\left(d\vec{\mathbf{L}} \times \vec{\mathbf{B}}\right)$$

- The total force

$$\vec{\mathbf{F}} = \int I\left(d\vec{\mathbf{L}} \times \vec{\mathbf{B}}\right)$$

(*Integrate over the length of the conductor!*)



Magnetic Force on a Current-carrying Conductor

Example: A semi-circular wire

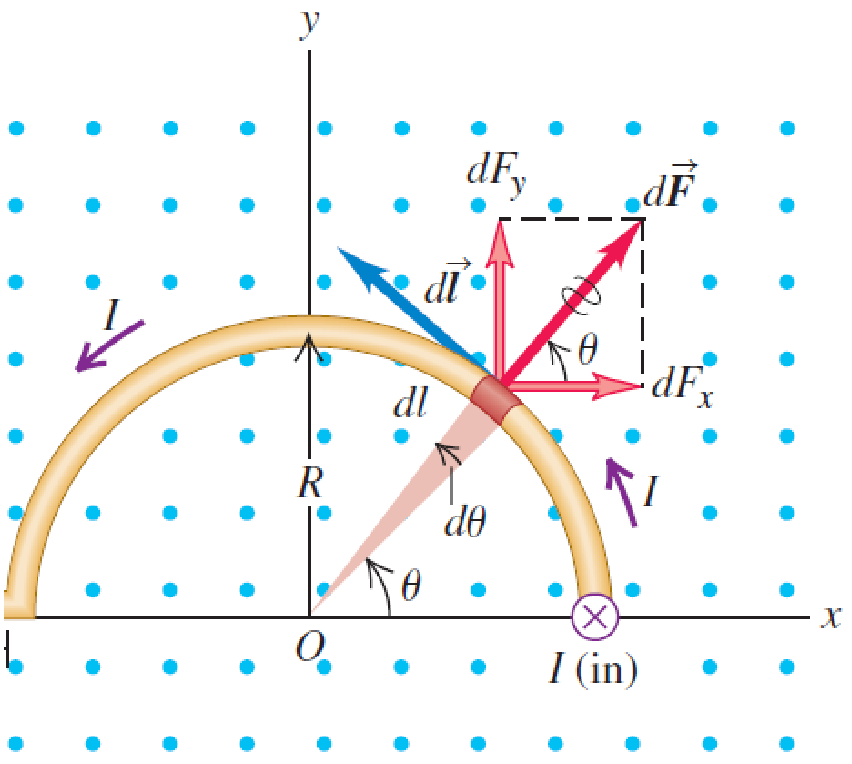
What is the total magnetic force on the conductor?

- For the length element *dL*:

$$\left|d\vec{\boldsymbol{F}}\right| = I(Rd\theta)B$$

$$dF_x = IR(d\theta)B\ \cos\theta$$

$$dF_y = IR(d\theta)B\ \sin\theta$$



Magnetic Force on a Current-carrying Conductor

Example: A semi-circular wire

What is the total magnetic force on the conductor?

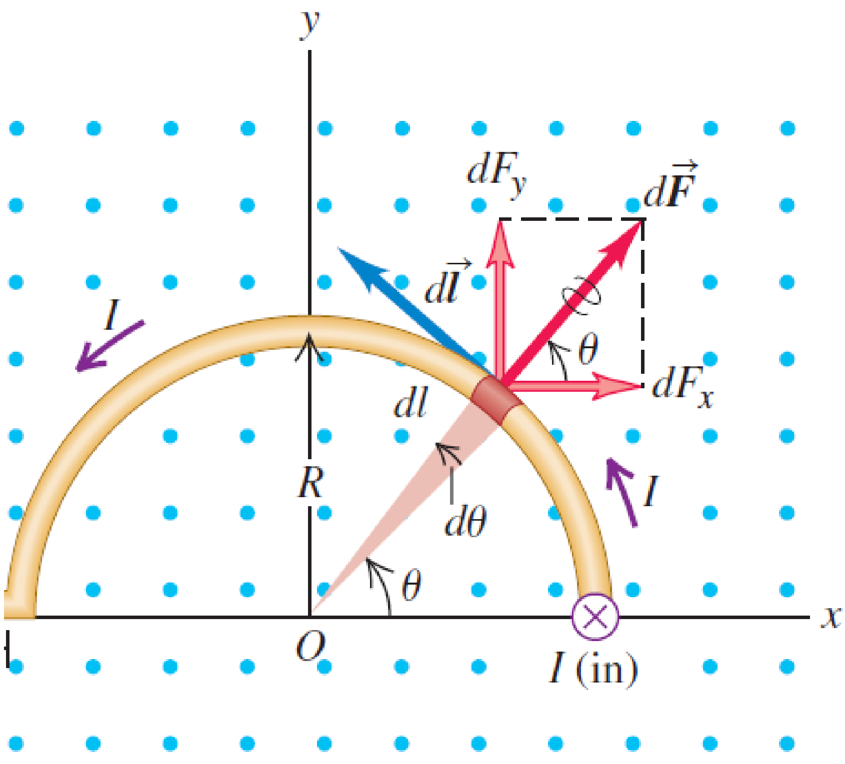
- Integrating:

$$F_x = IRB\int_0^{\pi}\cos\theta d\theta = 0$$

$$F_y = IRB\int_0^{\pi}\sin\theta d\theta = IRB(2)$$

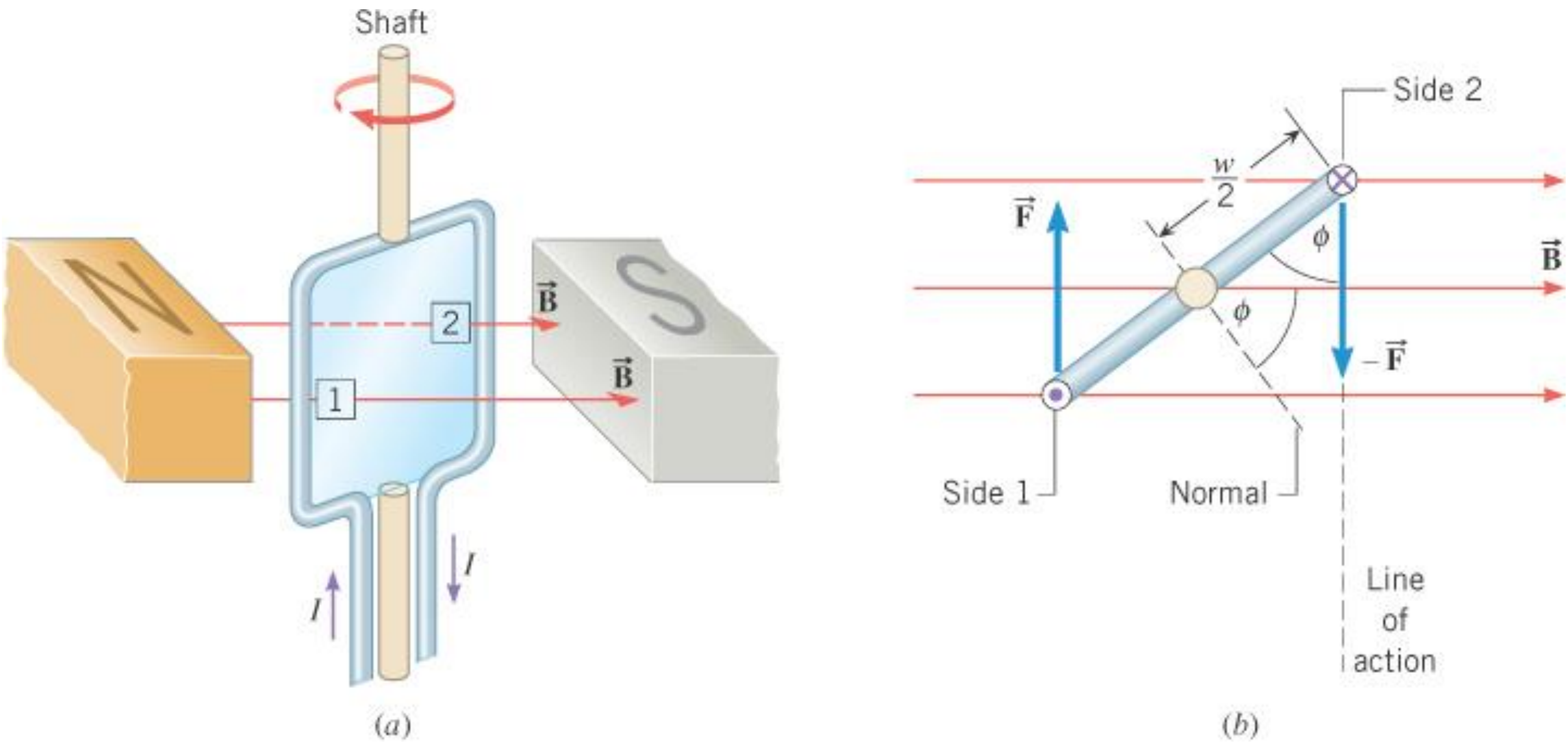
$$F_y = I(2R)B$$

$$\vec{\mathbf{F}} = I(2R)B\,\hat{\boldsymbol{j}}$$



Force and Torque on a Current Loop

A rectangular current loop in a magnetic field

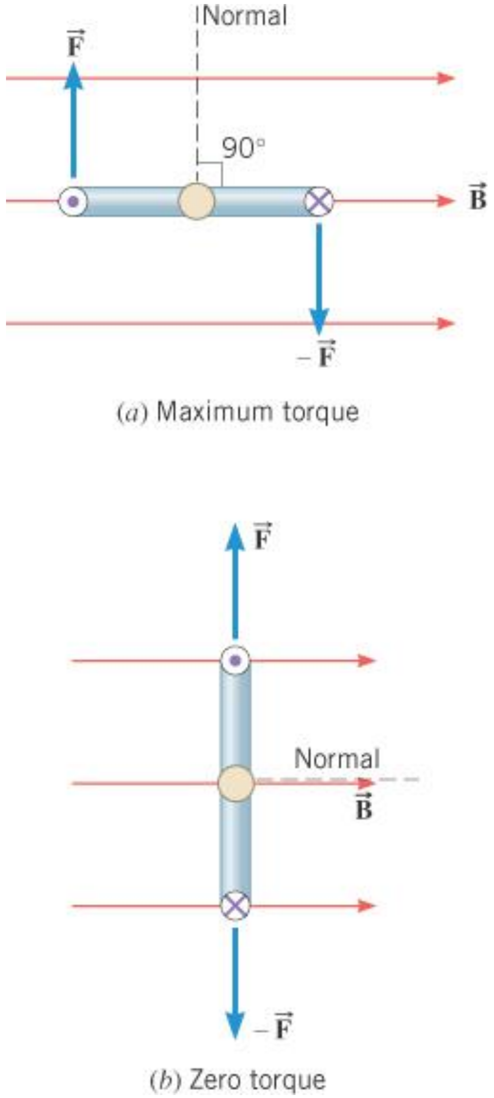


- The two forces on the loop have equal magnitude but are opposite in direction.

Force and Torque on a Current Loop

A rectangular current loop in a magnetic field

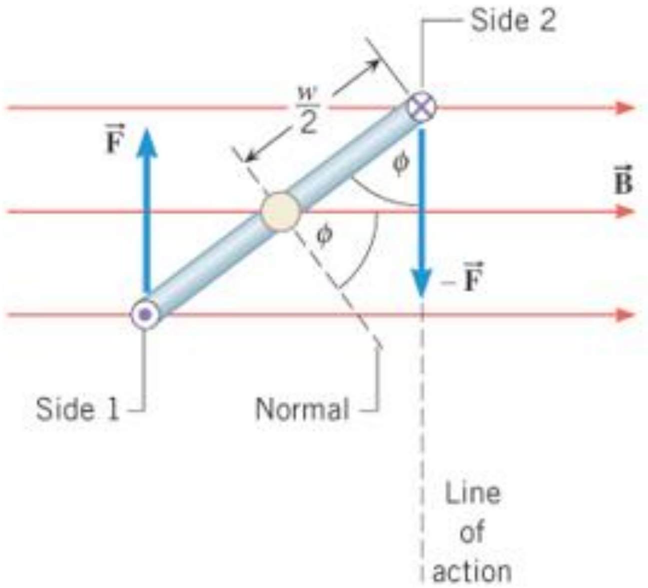
- The loop tends to rotate such that its normal (to the plane of the loop) becomes aligned with the magnetic field.
- Or, the plane of the loop aligns perpendicular to the magnetic field.



Force and Torque on a Current Loop

A rectangular current loop in a magnetic field

- The ***net force*** on the current loop in a uniform magnetic field is ***zero***.
- However, the ***net torque*** in general is *not* zero.



$$\tau = 2(ILB)\left(\tfrac{1}{2}W\sin\phi\right)$$

$$= I(LW)B\sin\phi$$

$$= IAB\sin\phi$$

where $A = (LW)$ is the area of the loop.

Force and Torque on a Current Loop

A rectangular current loop in a magnetic field

- For loop with *N* turns of wire:

$$\tau = NIAB\sin\phi$$

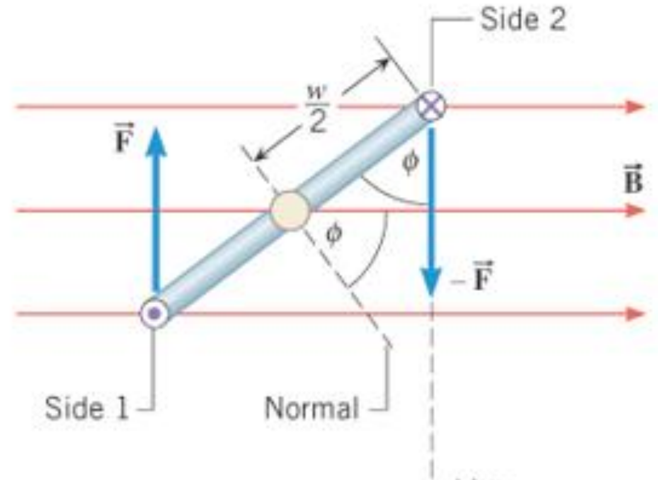
- The *magnetic dipole moment*:

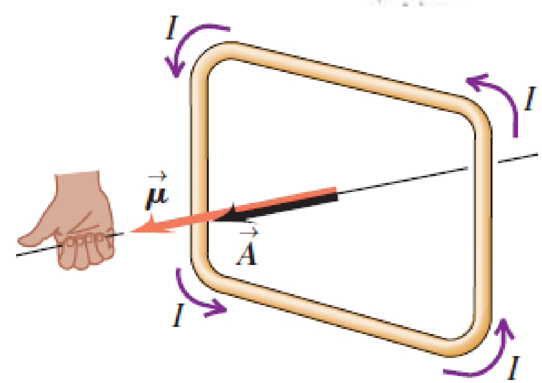
$$\mu = NIA$$

$$\tau = \mu B\sin\phi$$

- In vector form :

$$\vec{\boldsymbol{\tau}} = \vec{\boldsymbol{\mu}} \times \vec{\boldsymbol{B}}$$

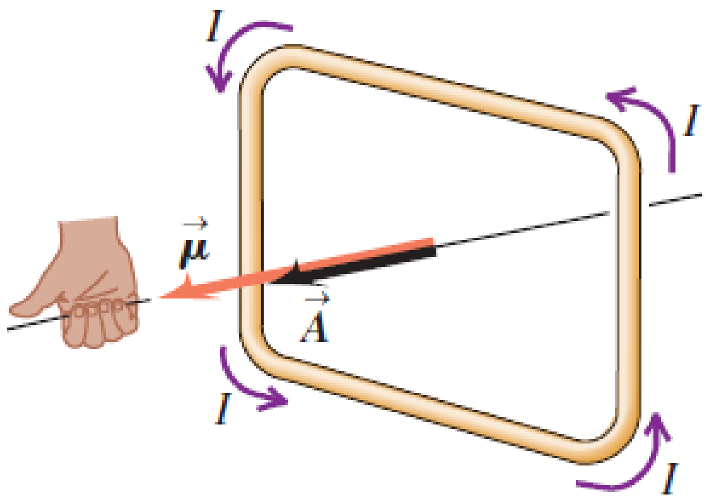




Force and Torque on a Current Loop

Example: The Torque Exerted on a Current-Carrying Coil

A coil of wire has an area of $2.0\text{x}10^{-4}\ \text{m}^2$, consists of 100 turns, and carries a current of 0.045 A. The coil is placed in a uniform magnetic field of 0.15 T. a) Determine the magnetic moment of the coil. b) Find the maximum torque that the magnetic field can exert on the coil.



Force and Torque on a Current Loop

Example: The Torque Exerted on a Current-Carrying Coil

a) the magnetic moment of the coil.

$$\mu = NIA = (100)(0.045\ \text{A})(2.0\times 10^{-4}\ \text{m}^2)$$

$$= 9.0\times 10^{-4}\ \text{A}\cdot\text{m}^2$$

b) the maximum torque

$$\tau = \mu\, B\, \sin 90^\circ = (9.0\times 10^{-4}\ \text{A}\cdot\text{m}^2)(0.15\ \text{T})$$

$$= 1.4\times 10^{-4}\ \text{N}\cdot\text{m}$$

Force and Torque on a Current Loop

Potential energy for a magnetic dipole

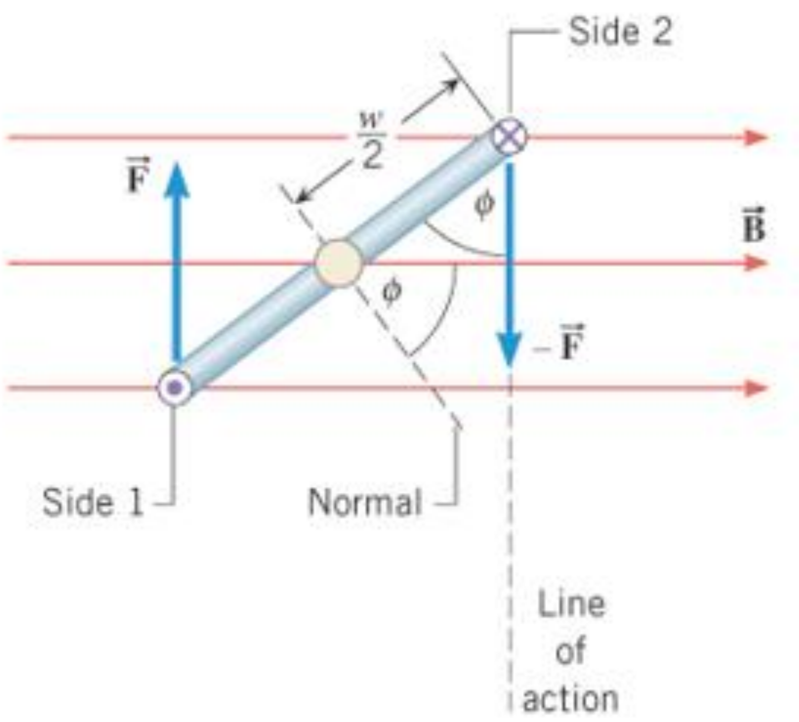
When a magnetic dipole changes orientation in a magnetic field, the field does work on it.

- In an infinitesimal angular displacement $d\phi$,

$$dW = \tau\, d\phi = (\mu B \sin\phi) d\phi$$

- Total *work done*:

$$W = -(\mu B \cos\phi_2 - \mu B \cos\phi_1)$$



Force and Torque on a Current Loop

Potential energy for a magnetic dipole

- Total *work done*:

$$W = -(\mu B \cos\phi_2 - \mu B \cos\phi_1)$$

- The *potential energy*:

$$W = -\Delta U$$

$$U = -\mu B \cos\phi$$

$$= -\vec{\boldsymbol{\mu}} \cdot \vec{\boldsymbol{B}}$$

  - $U_{min} = -\mu B$ occurs when the dipole moment is in the same direction as the field.
  - $U_{max} = +\mu B$ occurs when the dipole moment is in the direction opposite the field.

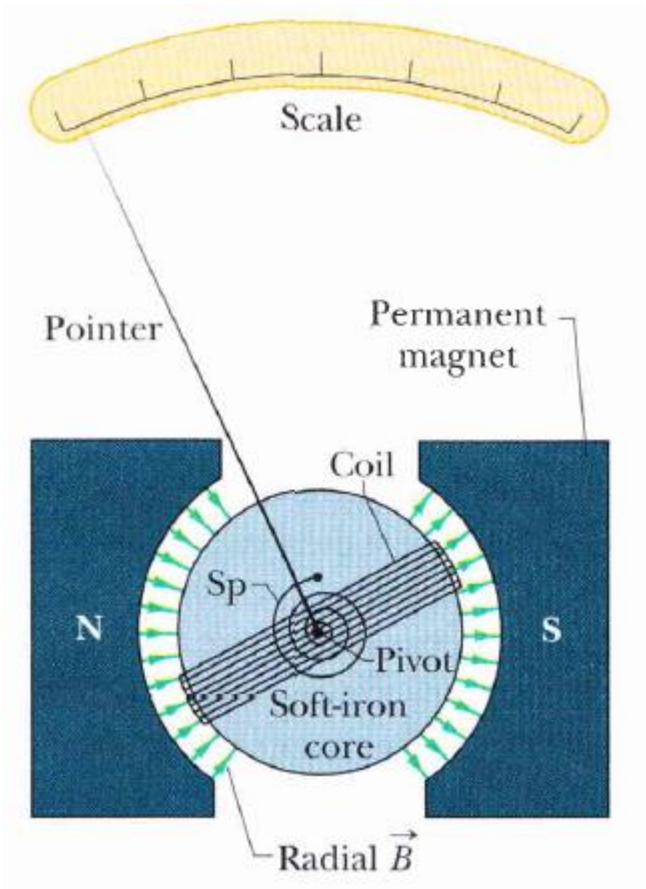
Force and Torque on a Current Loop

Example: The Galvanometer

The torque on a current loop is proportional to the current in it.

- This forms the basis of a variety of useful electrical instruments as a **galvanometer**:

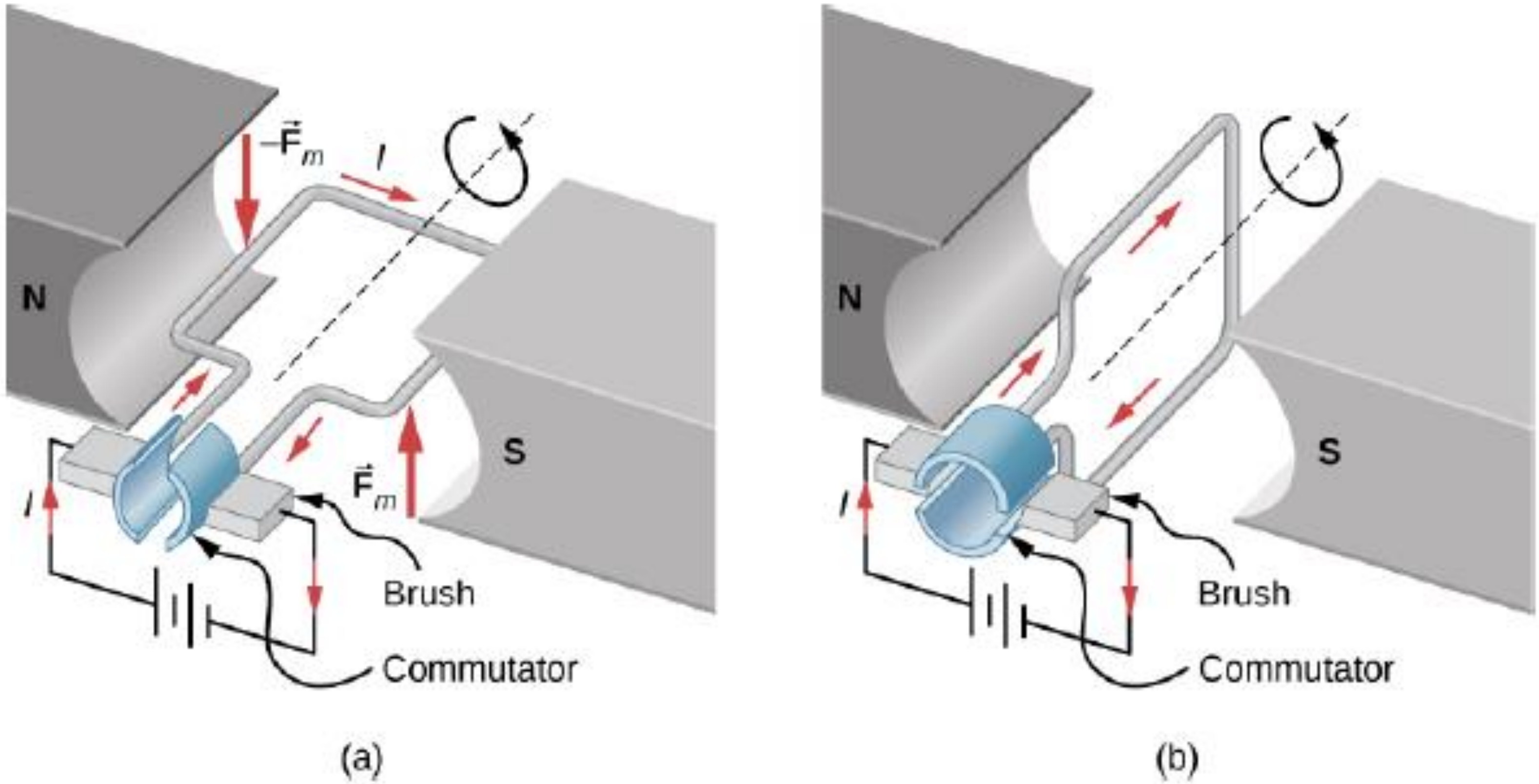
- Depending on the external circuit, this device can be wired up as either a *voltmeter* or an *ammeter*.



Force and Torque on a Current Loop

The DC Electric Motor

The basic components of a DC motor.

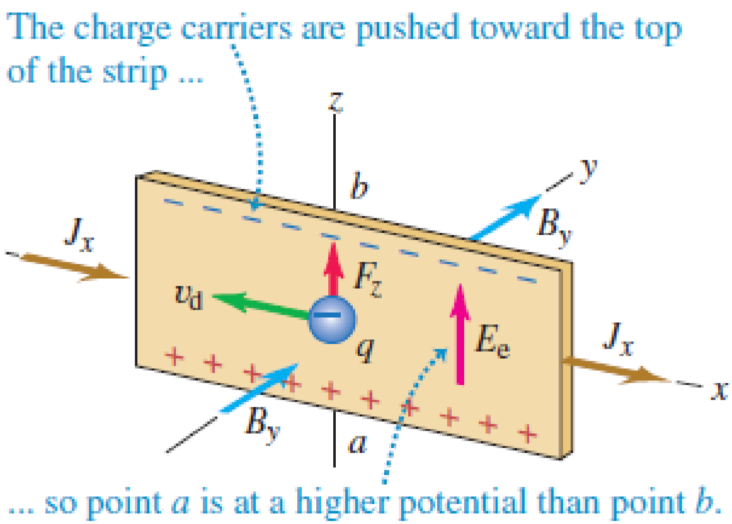


Hall Effect

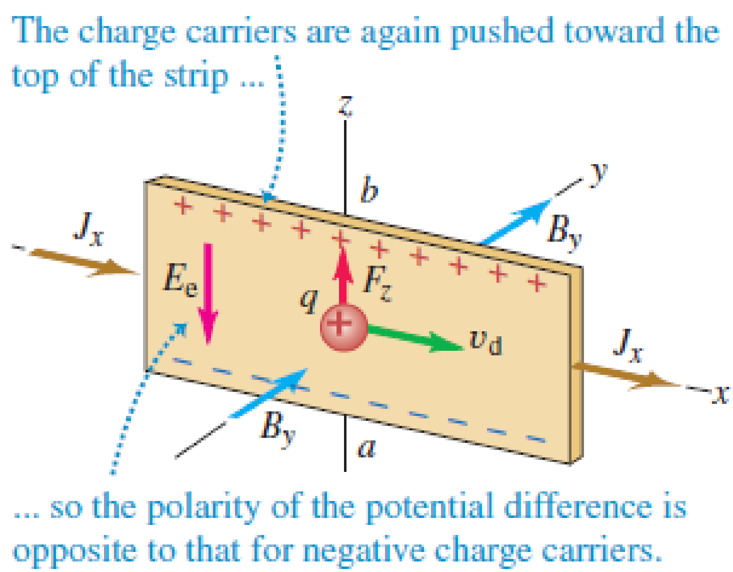
When a current carrying conductor is placed in a magnetic field, a ***potential difference*** is generated in a direction perpendicular to both the current and the magnetic field.

- Arises from the deflection of charge carriers to one side of the conductor as a result of the magnetic forces they experience.
- Gives information regarding the sign of the charge carriers and their density.
- Can also be used to measure magnetic fields.

(a) Negative charge carriers (electrons)



(b) Positive charge carriers



Hall Effect

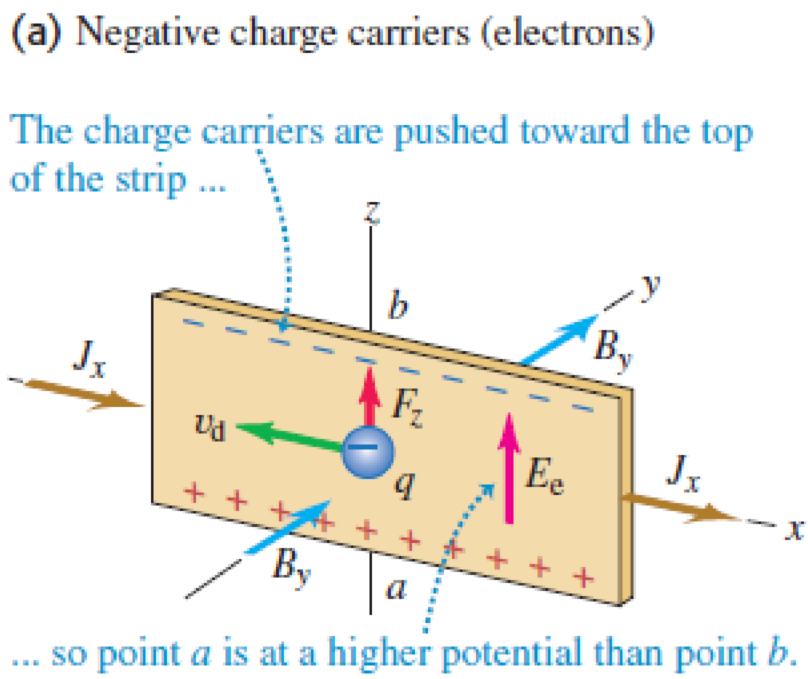
- When the charge carriers are negative, they experience an upward magnetic force, they are deflected upward, an excess of positive charge is left at the lower edge.

- This accumulation of charge establishes an electric field in the conductor and increases until the electric force balances the magnetic force.

- The ***Hall voltage*** for conductor width Δz

$$V_H = E_e \Delta z = v_d B_y \Delta z$$

$$= \frac{J_x B_y \Delta z}{nq}$$



END