Thursday, August 22, 2024 10:57

Baye's theorem

Recall that:

$$\Pr(B\cap A) = \Pr(A\cap B)$$

$$\Pr(B)\Pr(A|B) = \Pr(A)\Pr(B|A)$$

$$\Pr(A|B) = \frac{\Pr(A)}{\Pr(B)}\Pr(B|A)$$

sometimes $\Pr(B)$ is not known

We use,

$$\Pr(B) = \Pr(A)\Pr(B|A) + \Pr(\bar{A})\Pr(B|\bar{A})$$

$$\Pr(A|B) = \frac{\Pr(A)\Pr(B|A)}{\Pr(A)\Pr(B|A)+\Pr(\bar{A})\Pr(B|\bar{A})}$$

Remark:

$$\Pr(A|B) \neq \Pr(B|A)$$

This is why "If A, then B" is NOT necessarily the same as "If B, then A".

In practice (physics),

B - represents some experimental result

A - some theory

$\Pr(B|A)$ - probability of the result occuring if the theory is true

$\Pr(A)$ - some probability we ascribe to the thery (prior probability)

$\Pr(A|B)$ - probability we ascribe to theory in light of the experiment

If the $\Pr(B|A) = 0$, then experimental result is forbidden in the theory

→ if the result is observed, the thery must be discarded

> ►*Suppose that the blood test for some disease is reliable in the following sense: for people who are infected with the disease the test produces a positive result in 99.99% of cases; for people not infected a positive test result is obtained in only 0.02% of cases. Furthermore, assume that in the general population one person in 10 000 people is infected. A person is selected at random and found to test positive for the disease. What is the probability that the individual is actually infected?*

A - event that the individual is infected

B - event that the individual is tests positive for the disease

$$\Pr(B|A) = 0.9999 = \frac{9999}{10{,}000}$$

$$\Pr(B|\bar{A}) = 0.0002 = \frac{2}{10{,}000}$$

Actual population:

$$\Pr(A) = \frac{1}{10{,}000}$$

$$\Pr(\bar{A}) = \frac{9999}{10{,}000}$$

$\Pr(A|B)$ → probability that the person is actually infected when the test result is positive

$\Pr(A|B) =$

Q4. Exercise 30.9 of Riley, et al.

Counting: Permutations & Combinations

Recall that:

$$\Pr(A) = \frac{n_A}{n_S}$$

Permutations

Given n distinct objects, in how many ways we can arrange them?

$$n\cdot(n-1)\cdot(n-2)\cdot(n-3)\ \dots\ (1) = n!$$

1st has n options

2nd choice has 1 less option

nth choice only one option is left.

If we are going to chose only k objects $(k<n)$, then,

$$\underbrace{\sqcup\ \sqcup\ \sqcup \dots \sqcup}_{k \text{ bins}}$$

$$\underbrace{n\cdot(n-1)\cdot(n-2)\dots(n-k+1)}_{k \text{ factors}} = \frac{n!}{(n-k)!} = {}^{n}P_{k}$$

We assumed that the objects are sampled without replacement

If n objects are sampled k times with replacement, then

$$\underbrace{n\cdot n\cdot n \cdots n}_{k\text{ factors}} = n^k$$

Example: How many two-digit numbers can be formed using the set $\{1, 2, 3, 4\}$

solution on the board

Combinations

In permutations, we are strict in the ordering of the arrangement.

→ we are sure that there are n options on the first draw

→ only $n-1$ on the second draw

⋮

There are times when the ordering is immaterial

so,

$$\frac{{}^{n}P_{k}}{k!} = \frac{1}{k!}\cdot\frac{n!}{(n-k)!} = \frac{n!}{k!(n-k)!} = {}^{n}C_{k} = \binom{n}{k}$$

n taken k (binomial coefficient)

Example:

A hand of 13 playing cards is dealt from a well-shuffled pack of 52. What is the probability that the hand contains two aces.

Q5. Exercise 30.11 of Riley et al.

Consider n distinguishable objects that can be divided into m piles, with n_i objects in the ith pile $(i = 1, 2, 3, \dots m)$

$$n = n_1 + n_2 + n_3 + \dots + n_m$$

The order of n_i objects in the ith pile is not relevant ⤳ maybe they are identical

n_1 → identical

n_2 → identical } Distinguishable

⋮

n_m → identical

The number of ways of dividing the original n objects into m piles:

$$\frac{n!}{n_1!\,n_2!\dots n_{m-1}!\,n_m!}$$

Example:

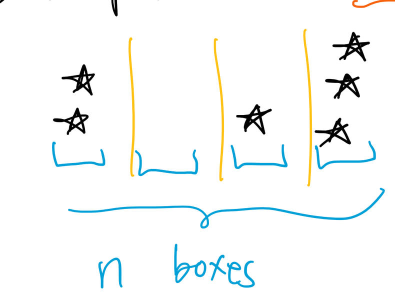
Consider the word ENGINEERING

a) Find the number of permutations of the letters of the word.

b) What if the two Gs are always always together

Indistinguishable particles

How many ways can k indistinguishable particles be placed in n boxes?



total number of ★ → $n_★ = k$

total number of partitions $n_{||} = n-1$

$$N = k + n - 1$$

$$\frac{N!}{n_★!\,n_{||}!} = \frac{(k+n-1)!}{k!(n-1)!}$$