NOTES ON THRESHOLD UPDATES FOR ORDINAL PROBIT

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The proposed algorithm for ordinal probit is described as:
CDA first integrates the joint distribution

$$\pi(z, y, x, \beta, \gamma, y) = \prod_i \mathcal{N}(z_i | x_i^T \beta, 1) \prod_{j=1} \frac{1\{\max\limits_{i:y_i=j}(z_i) < \gamma_j < \min\limits_{i:y_i=j+1}(z_i)\}}{\min\limits_{i:y_i=j+1}(z_i) - \max\limits_{i:y_i=j}(z_i)}$$

over $\gamma_2, \ldots \gamma_{k-1}$*, where each uniform simply integrates to* 1.

This was bothering me, since the MCMC algorithm is then identical to the binary probit algorithm. I think I have now found the problem. You need

$$\begin{aligned} p(z_i \mid x_i, \beta, y_i) &= \int_{\gamma_1, \ldots, \gamma_{K-1}} p(z_i \mid \beta, y_i, \gamma) p(\gamma \mid X, y, \beta) \\ &= \int_{\gamma_1, \ldots, \gamma_{K-1}} \frac{\exp(-(z_i - x_i\beta)^2/2)}{\sqrt{2\pi}\{\Phi(\gamma_{y_i} - x_i\beta) - \Phi(\gamma_{y_i-1} - x_i\beta)\}} \\ &\qquad \times \mathbb{1}_{\{z_i \in [\gamma_{y_i-1}, \gamma_{y_i}]\}} p(\gamma \mid y, X, \beta) d\gamma, \end{aligned}$$

where $p(\gamma \mid X, y, \beta) = p(\gamma_1, \ldots, \gamma_{K-1} \mid X, y, \beta)$ is the distribution of γ given data and β, which we don't know. Regardless of what that is, this is not a normal distribution with mean $x_i\beta$ and variance 1, since you have to integrate the normalizing constant

$$\{\Phi(\gamma_{y_i} - x_i\beta) - \Phi(\gamma_{y_i-1} - x_i\beta)\}^{-1}$$

over γ to obtain $p(z_i \mid x_i, \beta, y_i)$, which is what you're trying to use in deriving your algorithm.