8.12.

$Y_1 \dots Y_n \overset{iid}{\sim} Unif(\theta, \theta+1)$

a. $E\bar{Y} = \mu = EY_i = \frac{\theta+(\theta+1)}{2} = \theta + \frac{1}{2}$

$\Rightarrow$ biased.

$B(\bar{Y}) = \theta + \frac{1}{2} - \theta = \frac{1}{2}$

b. $\hat{\theta} = \bar{Y} - \frac{1}{2}$

c. $Var(\bar{Y}) = \frac{\sigma^2}{n} = \frac{1}{12n}$.

$\Rightarrow MSE(\bar{Y}) = Var(\bar{Y}) + B(\bar{Y})^2 = \frac{1}{12n} + \frac{1}{4}$

8.43.

a. $F_U(u) = P(U < u) = P(\frac{Y_{(n)}}{\theta} < u)$

$= P(Y_{(n)} < \theta u)$

$= P(Y_1 < \theta u \dots Y_n < \theta u)$

$= (\frac{\theta u}{\theta})^n = u^n$, for $0 \le u \le 1$

it is known that $F_U(u) = 0$ if $u < 0$

and $F_U(u) = 1$ if $u > 1$.

b. $P(U \le a) = 95\%$

$\Rightarrow P(\frac{Y_{(n)}}{\theta} \le a) = a^n = 95\%$

$\Rightarrow a = (95\%)^{\frac{1}{n}}$ $\Rightarrow P(\theta \ge Y_{(n)}/a) = 95\%$

$\Rightarrow P(\theta \ge Y_{(n)}/(95\%)^{\frac{1}{n}}) = 95\%$

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