10.81. (a)

$F < (F^{v_2}_{v_1,\alpha/2})^{-1} \iff F^{-1} = S_2^2/S_1^2 > F^{v_2}_{v_1,\alpha/2}$

(b) (i) if $S_1^2 > S_2^2$, we know $S_1^2/S_2^2 > 1 > (F^{v_2}_{v_1,\alpha/2})^{-1}$

we reject H_0 if $S_1^2/S_2^2 > F^{v_1}_{v_2,\alpha/2}$.

(ii) if $S_1^2 \le S_2^2$, we know $S_2^2/S_1^2 > 1 > (F^{v_1}_{v_2,\alpha/2})^{-1}$

we reject H_0 if $S_2^2/S_1^2 > F^{v_2}_{v_1,\alpha/2}$.

$\Rightarrow P(S_L^2/S_S^2 > F^{v_L}_{v_S,\alpha/2}) = \alpha$

10.95 (a) $n=4$.

$$L(\theta;\vec{y}) = \left(\frac{1}{2\theta^3}\right)^4 \prod_{i=1}^{4} y_i^2 \, e^{-\sum_{i=1}^{4} y_i/\theta}$$

By Neyman-Pearson

$$\frac{L(\theta_0)}{L(\theta_a)} = \left(\frac{2\theta_a^3}{2\theta_0^3}\right)^4 e^{-\sum_{i=1}^{4}(y_i)(1/\theta_0 - 1/\theta_a)} < k$$

$$\Rightarrow -\sum_{i=1}^{4} y_i \left(\frac{1}{\theta_0} - \frac{1}{\theta_a}\right) < k^*$$

as $\theta_a > \theta_0 \Rightarrow \frac{1}{\theta_0} - \frac{1}{\theta_a} > 0$

$$\Rightarrow -\sum_{i=1}^{4} y_i < k^{**}$$

$$\Rightarrow \sum_{i=1}^{4} y_i > k^{***}$$

Each $\frac{2Y_i}{\theta_0} \sim \chi^2_6$ under H_0

$$\Rightarrow \sum_{i=1}^{4} \frac{2Y_i}{\theta_0} \sim \chi^2_{24}. \Rightarrow R = \left\{ \sum \frac{2Y_i}{\theta_0} > \chi^2_{24,\alpha} \right\}$$

(b) since R do not depend on the value of θ_a, it is UMP test.