Q39

the p.m.f for $Y_i \overset{iid}{\sim} Poi(\lambda)$

$$f(y_i) = \frac{\lambda^{y_i} e^{-\lambda}}{y_i!}$$

1

$$f(y_1 \cdots y_n) = \prod_{i=1}^{n} \frac{\lambda^{y_i} e^{-\lambda}}{y_i!}$$

$$= \frac{\lambda^{\sum y_i} e^{-n\lambda}}{\prod_{i=1}^{n} y_i!}$$

changing variable to.

$(y_1, \ldots, y_{n-1}, t)$ with $t = \sum y_i$

yields.

$$f(y_1, \ldots y_{n-1}, t)$$

$$= \frac{\lambda^t e^{-n\lambda}}{\left[\prod_{i=1}^{n-1} y_i!\right]\left(t - \sum_{i=1}^{n-1} y_i\right)!}$$

2

the distribution, by the property of iid poisson's.

$T = \sum Y_i \sim Poi(n\lambda)$

$$\Rightarrow f_T(t) = \frac{(n\lambda)^t e^{-n\lambda}}{t!}$$

Combining 1 and 2.

$$f(y_1 \cdots y_n \mid \sum y_i = t) = \frac{t!}{\left[\prod_{i=1}^{n-1} y_i!\right]\left(t - \sum_{i=1}^{n-1} y_i\right)! \, n^t}$$

which is free from λ.

9.64 a. In class we showed that

$\bar{Y}$ is minimum sufficient for μ.

$\Rightarrow$ $\bar{Y}^2$ is minimum sufficient for μ^2.

also, we have

$$\mathbb{E}\hat{\mu}^2 = \mathbb{E}\bar{Y}^2 - \frac{1}{n}$$

$$= \mathrm{Var}\,\bar{Y} + (\mathbb{E}\bar{Y})^2 - \frac{1}{n}$$

$$= \frac{1}{n} + \mu^2 - \frac{1}{n} = \mu^2$$

b. For $X \sim N(\mu, \sigma^2)$

$$\mathbb{E}X^4 = \mu^4 + 6\mu^2\sigma^2 + 3\sigma^4$$

$$\Rightarrow \mathrm{Var}(\hat{\mu}) = \mathrm{Var}\left(\bar{Y}^2 - \frac{1}{n}\right)$$

$$= \mathbb{E}\left[\left(\bar{Y}^2 - \frac{1}{n}\right) - \mu^2\right]^2$$

$$= \mathbb{E}\bar{Y}^4 - 2\left(\frac{1}{n} + \mu^2\right)\mathbb{E}\bar{Y}^2 + \left(\frac{1}{n} + \mu^2\right)^2$$

$$\mathbb{E}\bar{Y}^4 = \mu^4 + 6\cdot\mu^2\frac{1}{n} + 3\cdot\frac{1}{n^2}$$

$$\mathbb{E}\bar{Y}^2 = \mu^2 + \frac{1}{n}$$

$$\Rightarrow \mathrm{Var}(\hat{\mu}) = 4\mu^2\frac{1}{n} + 2\frac{1}{n^2}$$

10.6

(a) $\alpha = P(|y-18| \geq 4 \mid p = .5)$

$= 1 - P(|y-18| < 4 \mid p = .5)$

$= 1 - P(y = 15, 16, 17, 18, 19, 20, 21 \mid p = .5)$

$= 0.243$

(b) $\beta = P(|y-18| < 4 \mid p = 0.7)$

$= 0.092$

10.18 $H_0 : \mu = 13.20$ $H_a : \mu < 13.20$

Rejection region at $\alpha = 0.01$ is

$$\left\{ \frac{\bar{y} - 13.20}{2.5/\sqrt{40}} < z_{0.01} \right\}$$

$\Rightarrow$ putting in the value of $\bar{y} = 12.20$.

$-2.52 < z_{0.01} = -2.33$

Therefore, we reject H_0.