Exercises

11.15 a Derive the following identity:

$$\begin{aligned} \text{SSE} &= \sum_{i=1}^{n}(y_i - \hat{y}_i)^2 = \sum_{i=1}^{n}(y_i - \hat{\beta}_0 - \hat{\beta}_1 x_i)^2 \\ &= \sum_{i=1}^{n}(y_i - \overline{y})^2 - \hat{\beta}_1 \sum_{i=1}^{n}(x_i - \overline{x})(y_i - \overline{y}) = S_{yy} - \hat{\beta}_1 S_{xy}. \end{aligned}$$

Notice that this provides an easier computational method of finding SSE.

b Use the computational formula for SSE derived in part (a) to prove that $\text{SSE} \le S_{yy}$. [*Hint:* $\hat{\beta}_1 = S_{xy}/S_{xx}$.]

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11.20 Suppose that $Y_1, Y_2, \ldots, Y_n$ are independent normal random variables with $E(Y_i) = \beta_0 + \beta_1 x_i$ and $V(Y_i) = \sigma^2$, for $i = 1, 2, \ldots, n$. Show that the maximum-likelihood estimators (MLEs) of β_0 and β_1 are the same as the least-squares estimators of Section 11.3.

11.21 Under the assumptions of Exercise 11.20, find $\text{Cov}(\hat{\beta}_0, \hat{\beta}_1)$. Use this answer to show that $\hat{\beta}_0$ and $\hat{\beta}_1$ are independent if $\sum_{i=1}^{n} x_i = 0$. [*Hint:* $\text{Cov}(\hat{\beta}_0, \hat{\beta}_1) = \text{Cov}(\overline{Y} - \hat{\beta}_1\overline{x}, \hat{\beta}_1)$. Use Theorem 5.12 and the results of this section.]

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