Homework 2 Solution

7.52. a. $\mu = 200$ $\quad \sigma = 10$

$n = 25$

Since $\bar{X} \overset{approx}{\sim} N(\mu, \frac{\sigma^2}{n})$

$P[\bar{X} \in (199, 202)]$

$= P[\frac{\bar{X}-\mu}{\sigma/\sqrt{n}} \in (\frac{199-200}{10/\sqrt{25}}, \frac{202-200}{10/\sqrt{25}})]$

$= P[Z \in (-0.5, 1)]$

$= \cancel{0.669} \quad 0.533$

b. $\sum X_i \overset{approx}{\sim} N(n\mu, n\sigma^2)$

$P(\sum X_i \leq 5100)$

$= P(\frac{\sum X - n\mu}{\sqrt{n}\sigma} \leq \frac{5100 - 25 \times 200}{\sqrt{25} \cdot 10})$

$= P(Z \leq 2)$

$= 0.977$

7.96

$\mu = \mathbb{E}Y = \int_0^1 y \cdot 3y^2 \, dy = \frac{3}{4} y^4 \Big|_0^1 = \frac{3}{4} = 0.75$

$\mathbb{E}Y^2 = \int_0^1 y^2 \, 3y^2 \, dy = \frac{3}{5} y^5 \Big|_0^1 = \frac{3}{5}$

$\Rightarrow \sigma^2 = Var(Y) = \mathbb{E}Y^2 - (\mathbb{E}Y)^2 = \frac{3}{5} - (\frac{3}{4})^2 = 0.0375$

Use CLT. $\bar{Y} \overset{approx}{\sim} N(\mu, \frac{\sigma^2}{n})$

$P(\bar{Y} > .7) = P(\frac{\bar{Y}-\mu}{\sigma/\sqrt{n}} > \frac{0.7 - 0.75}{\sqrt{0.0375}/\sqrt{n}})$

$= P(Z > -1.633) = 0.949$

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