7.9. $\quad Y \sim N(\mu, 1^2)$

$$z = \frac{\bar{Y}-\mu}{\sigma/\sqrt{n}} \sim N(0,1).$$

a) $n=16$, $P\left(-\frac{0.3}{1/\sqrt{16}} \le Z \le \frac{0.3}{1/\sqrt{16}}\right)$

$$= P(-1.2 \le Z \le 1.2)$$
$$= .7699$$

b) $n=25$ $\quad P(-1.5 \le z \le 1.5) = .8664$

$n=36$ $\quad P(-1.8 \le z \le 1.8) = .9281$

$n=49$ $\quad P(-2.1 \le z \le 2.1) = .9643$

$n=64$ $\quad P(-2.4 \le z \le 2.4) = .9836$

c). As n increases, the probability increases.

d). Yes. ~~to~~ have a larger probability for

$P(|\bar{Y}-\mu| \le .3)$,

we need to use a larger n.

7.20. a.

$$E\,U = \int \left(\frac{1}{\Gamma(V/2)\,2^{V/2}} u^{V/2-1} e^{-u/2}\right) u \cdot du$$

$$= \cancel{\frac{1}{\Gamma(V/2+1)2} u^{(V/2+1)-1} e^{-u/2}}$$

$$= \frac{1}{\Gamma(V/2)\,2^{V/2}} \int u^{(V/2+1)-1} e^{-u/2}\,du$$

$$= \frac{\Gamma(V/2+1)\,2^{V/2+1}}{\Gamma(V/2)\;2^{V/2}} = V/2 \times 2 = V.$$

$$\mathbb{E}U^2 = \frac{\Gamma(\nu/2+2)\cdot 2^{\nu/2+2}}{\Gamma(\nu/2)\ 2^{\nu/2}}$$

$$= (\nu/2+1)\ \nu/2 \cdot 2^2$$

$$= (\nu+2)\nu$$

$$\Rightarrow \quad V(U) = \mathbb{E}U^2 - (\mathbb{E}U)^2$$

$$= 2\nu.$$

7.20 b.

$$\frac{(n-1)S^2}{\sigma^2} \sim \chi^2_{n-1}$$

$$\mathbb{E}\frac{(n-1)S^2}{\sigma^2} = (n-1)$$

$$\Rightarrow \quad \mathbb{E}S^2 = \sigma^2$$

$$V\left(\frac{n-1}{\sigma^2}\cdot S^2\right) = 2(n-1).$$

$$V(S^2) = \frac{2\sigma^4}{n-1}.$$