

9.17

$$E \bar{X} - \bar{Y} = \mu_1 - \mu_2$$

$$\text{Var}(\bar{X} - \bar{Y}) = \frac{\sigma_1^2}{n} + \frac{\sigma_2^2}{n} \rightarrow 0 \quad \text{as } n \rightarrow \infty$$

$\Rightarrow \bar{X} - \bar{Y}$ is a consistent estimator for $\mu_1 - \mu_2$

9.18

$$S_1^2 = \frac{\sum (x_i - \bar{x})^2}{n-1}$$

$S_2^2 = \frac{\sum (x_i - \bar{x})^2}{n-1}$ and both consistent estimator for σ^2 .

$$\frac{(n-1)S_1^2 + (n-1)S_2^2}{2n-2}$$

$$= \frac{S_1^2}{2} + \frac{S_2^2}{2} \xrightarrow{P} \sigma^2,$$

which shows that it is a consistent estimator.