## Exercises

**2.71** If two events, $A$ and $B$, are such that $P(A) = .5$, $P(B) = .3$, and $P(A \cap B) = .1$, find the following:

a   $P(A|B)$
b   $P(B|A)$
c   $P(A|A \cup B)$

a. $P(A|B) = \dfrac{P(A \cap B)}{P(B)} = \dfrac{0.1}{0.3} = \dfrac{1}{3}$

b. $P(B|A) = \dfrac{P(A \cap B)}{P(A)} = \dfrac{0.1}{0.5} = \dfrac{1}{5}$

c. $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$$= 0.7$$

$$P([A \cup B] \cap A) = P(A)$$

$$\Rightarrow P(A|A \cup B) = \dfrac{0.5}{0.7} = \dfrac{5}{7}$$

**d** $P(A|A \cap B)$

**e** $P(A \cap B|A \cup B)$

d. $P[(A \cap B) \cap A] = P(A \cap B)$

$$\Rightarrow P[A|A \cap B] = \frac{0.1}{0.1} = 1$$

e. $P[(A \cap B) \cap (A \cup B)] = P(A \cap B)$

$$= 0.1$$

$$P[A \cap B | A \cup B] = \frac{0.1}{0.7} = \frac{1}{7}$$

**2.83** If $A$ and $B$ are mutually exclusive events and $P(B) > 0$, show that

$$P(A|A \cup B) = \frac{P(A)}{P(A) + P(B)}.$$

$$P(A \cup B) = P(A) + P(B)$$

$$P(A \cap (A \cup B)) = P(A)$$

$$\implies P(A|A \cup B) = \frac{P(A)}{P(A) + P(B)}$$