

5.22 a. marginal for Y_1

$$p_1(0) = .76 \quad p_1(1) = .24$$

marginal for Y_2

$$p_2(0) = .55 \quad p_2(1) = .16 \quad p_2(2) = .29$$

b.

$$P(Y_2 | Y_1=0) = \frac{P(Y_2=Y_2, Y_1=0)}{P(Y_1=0)}$$

$$\Rightarrow P(Y_2 | Y_1=0) = \begin{cases} .38 / .76 = .5, & Y_2=0 \\ .14 / .76 = .184, & Y_2=1 \\ .24 / .76 = .316, & Y_2=2 \end{cases}$$

$$c. \quad P(Y_1=0 | Y_2=2) = \frac{.24}{.29} = .82$$

5.32 a.

$$\begin{aligned} f(y_1) &= \int_{y_1}^{2-y_1} f(y_1, y_2) dy_2 \\ &= \int_{y_1}^{2-y_1} 6y_1^2 y_2 dy_2 \\ &= 3y_1^2 \cdot y_2^2 \Big|_{y_1}^{2-y_1} \\ &= 3y_1^2 [(2-y_1)^2 - y_1^2] \\ &= 3y_1^2 (4 - 4y_1) \\ &= 12y_1^2 (1 - y_1) \end{aligned}$$

This is a Beta (3, 2)

$$\begin{aligned}
 b. \quad f(y_2) &= \int_0^{\min(y_2, 2-y_2)} 6y_1^2 y_2 \cdot dy_1 \\
 &= 2y_2 y_1^3 \Big|_0^{\min(y_2, 2-y_2)} \\
 &= 2y_2 [\min(y_2, 2-y_2)]^3,
 \end{aligned}$$

$$\begin{aligned}
 c. \quad f(y_2|y_1) &= \frac{f(y_1, y_2)}{f(y_1)} = \frac{6y_1^2 y_2}{12y_1^2(1-y_1)} \quad 0 \leq y_2 \leq 2 \\
 &= \frac{1}{2} \frac{y_2}{1-y_1} \\
 &\quad , \quad y_1 \leq y_2 \leq 2-y_1
 \end{aligned}$$

$$\begin{aligned}
 d. \quad P(X_2 < 1.1 | X_1 = .6) \\
 &= \int_{.6}^{1.1} \frac{1}{2} \frac{y_2}{1-.6} \\
 &= \frac{1}{.8} \frac{y_2^2}{2} \Big|_{.6}^{1.1} \\
 &= .53125
 \end{aligned}$$