9.2

a. $E\,\frac{1}{2}(Y_1+Y_2) = \frac{1}{2}(\mu+\mu) = \mu$.

$E\ \frac{1}{4}Y_1 + \frac{Y_2+\cdots+Y_{n-1}}{2(n-2)} + \frac{1}{4}Y_n$

$= \frac{1}{4}\mu + \frac{(n-2)\mu}{2(n-2)} + \frac{1}{4}\mu = \mu$

$E\ \bar{Y} = \mu$

b. $var(\bar{Y}) = \frac{\sigma^2}{n}$

$var(\frac{1}{2}(Y_1+Y_2)) = \frac{\sigma^2}{2}$.

$var(\frac{1}{4}Y_1 + \frac{Y_2+\cdots+Y_{n-1}}{2(n-2)} + \frac{1}{4}Y_n)$

$= \frac{1}{16}\sigma^2 + \frac{1}{4}\frac{\sigma^2}{n-2} + \frac{1}{16}\sigma^2$

$= [\frac{1}{8} + \frac{1}{4(n-2)}]\sigma^2$

$\Rightarrow\ RE(\hat{\mu}_3, \hat{\mu}_1) = \frac{\sigma^2/2}{\sigma^2/n} = n/2$

$RE(\hat{\mu}_3, \hat{\mu}_2) = \frac{[1/8 + 1/4(n-2)]\cdot\sigma^2}{\sigma^2/n} = \frac{n}{(1/8+1/4(n-2))}$

9.5 $\frac{(n-1)\cdot\hat{\sigma}_1^2}{\sigma^2} \sim \chi^2_{n-1}$. $var(\hat{\sigma}_1^2) = \sigma^4\cdot\frac{2(n-1)}{(n-1)^2}$

$= \frac{2\sigma^4}{n-1}$

$\hat{\sigma}^2 = \frac{1}{2}(Y_1-\mu+\mu-Y_2)^2 = \frac{1}{2}[(Y_1-\mu)^2 + (Y_2-\mu)^2 - 2(Y_1-\mu)(Y_2-\mu)]$

$var(\hat{\sigma}^2) = var(\frac{(Y_1-\mu)^2+(Y_2-\mu)^2}{2}) + var((Y_1-\mu)(Y_2-\mu))$

$- 2\,cov[\frac{(Y_1-\mu)^2+(Y_2-\mu)^2}{2}, (Y_1-\mu)(Y_2-\mu)]$

as $\dfrac{(Y_1-\mu)^2+(Y_2-\mu)^2}{\sigma^2} \sim \chi^2_2$.

$var\left(\dfrac{(Y_1-\mu)^2+(Y_2-\mu)^2}{2}\right) = \dfrac{\sigma^4}{4}\cdot 4 = \sigma^4$

$var\left((Y_1-\mu)(Y_2-\mu)\right) = \mathbb{E}(Y_1-\mu)^2(Y_2-\mu)^2$

$- \left[\mathbb{E}(Y_1-\mu)(Y_2-\mu)\right]^2$

$= \mathbb{E}(Y_1-\mu)^2\,\mathbb{E}(Y_2-\mu)^2 - 0^2$

$= \sigma^4$

$cov\left[\dfrac{(Y_1-\mu)^2+(Y_2-\mu)^2}{2}, (Y_1-\mu)(Y_2-\mu)\right]$

$= \mathbb{E}\,\dfrac{(Y_1-\mu)^2+(Y_2-\mu)^2}{2}\cdot(Y_1-\mu)\cdot(Y_2-\mu)$

$- \mathbb{E}\,\dfrac{(Y_1-\mu)^2+(Y_2-\mu)^2}{2}\,\mathbb{E}\left(Y_1-\mu\right)(Y_2-\mu)$

$= \mathbb{E}\dfrac{(Y_1-\mu)^3}{2}\mathbb{E}(Y_2-\mu) + \mathbb{E}\dfrac{(Y_2-\mu)^3}{2}\mathbb{E}(Y_1-\mu) - 0$

$= 0$

$\Rightarrow var(\hat{\sigma}_2^2) = 2\sigma^4$

$\Rightarrow RE(\hat{\sigma}_1, \hat{\sigma}_2^2) = \dfrac{2\sigma^4}{2\sigma^4(n-1)} = n-1$

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