***7.30** Suppose that Z has a standard normal distribution and that Y is an independent χ^2-distributed random variable with ν df. Then, according to Definition 7.2,

$$T = \frac{Z}{\sqrt{Y/\nu}}$$

has a t distribution with ν df.[1]

a If Z has a standard normal distribution, give $E(Z)$ and $E(Z^2)$. [*Hint:* For any random variable, $E(Z^2) = V(Z) + (E(Z))^2$.]

b According to the result derived in Exercise 4.112(a), if Y has a χ^2 distribution with ν df, then

$$E(Y^a) = \frac{\Gamma([\nu/2] + a)}{\Gamma(\nu/2)} 2^a, \qquad \text{if } \nu > -2a.$$

Use this result, the result from part (a), and the structure of T to show the following. [*Hint:* Recall the independence of Z and Y.]

i $E(T) = 0$, if $\nu > 1$.

ii $V(T) = \nu/(\nu - 2)$, if $\nu > 2$.

7.57 Twenty-five heat lamps are connected in a greenhouse so that when one lamp fails, another takes over immediately. (Only one lamp is turned on at any time.) The lamps operate independently, and each has a mean life of 50 hours and standard deviation of 4 hours. If the greenhouse is not checked for 1300 hours after the lamp system is turned on, what is the probability that a lamp will be burning at the end of the 1300-hour period?