of population ...

10.81 From two normal populations with respective variances σ_1^2 and σ_2^2, we observe independent sample variances S_1^2 and S_2^2, with corresponding degrees of freedom $\nu_1 = n_1 - 1$ and $\nu_2 = n_2 - 1$. We wish to test $H_0 : \sigma_1^2 = \sigma_2^2$ versus $H_a : \sigma_1^2 \neq \sigma_2^2$.

a Show that the rejection region given by

$$\left\{ F > F^{\nu_1}_{\nu_2,\alpha/2} \quad \text{or} \quad F < \left(F^{\nu_2}_{\nu_1,\alpha/2}\right)^{-1} \right\},$$

where $F = S_1^2/S_2^2$, is the same as the rejection region given by

$$\left\{ S_1^2/S_2^2 > F^{\nu_1}_{\nu_2,\alpha/2} \quad \text{or} \quad S_2^2/S_1^2 > F^{\nu_2}_{\nu_1,\alpha/2} \right\}.$$

b Let S_L^2 denote the larger of S_1^2 and S_2^2 and let S_S^2 denote the smaller of S_1^2 and S_2^2. Let ν_L and ν_S denote the degrees of freedom associated with S_L^2 and S_S^2, respectively. Use part (a) to show that, under H_0,

$$P\left(S_L^2/S_S^2 > F^{\nu_L}_{\nu_S,\alpha/2} \right) = \alpha.$$

Notice that this gives an equivalent method for testing the equality of two variances.

10.95 Suppose that we have a random sample of four observations from the density function

$$f(y \mid \theta) = \begin{cases} \left(\dfrac{1}{2\theta^3}\right) y^2 e^{-y/\theta}, & y > 0, \\ 0, & \text{elsewhere.} \end{cases}$$

a Find the rejection region for the most powerful test of $H_0 : \theta = \theta_0$ against $H_a : \theta = \theta_a$, assuming that $\theta_a > \theta_0$. [*Hint:* Make use of the χ^2 distribution.]

b Is the test given in part (a) uniformly most powerful for the alternative $\theta > \theta_0$?