Introduction to Probability (Spring 2019)

Exam 1

Name: _____

ATTENTION!!

- Show clearly how you derive the result. Only having the right answer without support reasoning $\Rightarrow$ 1 pt only.

1. Without using the Venn diagram, find the complement of $(A \cap B) \cup C$. (5 pt) What law/theorem did you use? (5 pt)

$$[(A \cap B) \cup C]^c$$
$$= (A \cap B)^c \cap C^c$$
$$= A^c \cup B^c \cap C^c$$

De Morgan's Law.

2. $P(A) = 0.5, P(A \cup B) = 0.6, P(A \cap B) = 0.1$, find $P(B)$. (10 pt)

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$
$$\Rightarrow P(B) = P(A \cup B) + P(A \cap B) - P(A)$$
$$= 0.6 + 0.1 - 0.5$$
$$= 0.2$$

3. If $P(A) > 0$, $P(B) > 0$ and $P(A) < P(A \mid B)$, prove that $P(B) < P(B \mid A)$. (10 pt)

$$P(B|A) = \frac{P(A|B)P(B)}{P(A)}$$

$$> \frac{P(A)P(B)}{P(A)} = P(B)$$

4. Two methods, A and B, are available for teaching a certain industrial skill. The failure rate is 20% for A and 10% for B. However, B is more expensive hence is used only 30% of the time, while A is used 70% of the time. What is the probability that a person fails to learn the skill? (10 pt)

By Law of Total Probability

$$P(F|A)P(A) + P(F|B)P(B)$$

$$= 0.2 \times 0.7 + 0.1 \times 0.3$$

$$= 0.17$$

5. A discrete random variable Y has the follow probability mass function

$$p(y) = \begin{cases} c/y^2 & \text{for } y = 1, 2, 3, 4 \\ 0 & \text{otherwise,} \end{cases}$$

where c is a constant. Using the properties of $p(y)$, find the value of c. (10 pt)

Since $\sum p(y) = 1$

$$c\left[1 + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2}\right] = 1$$

$$c = 0.702$$