Q3) the p.m.f for $Y_i \overset{iid}{\sim} Poi(\lambda)$

$$f(y_i) = \frac{\lambda^{y_i} e^{-\lambda}}{y_i!}$$

1 $$f(y_1 \ldots y_n) = \prod_{i=1}^{n} \frac{\lambda^{y_i} e^{-\lambda}}{y_i!}$$

$$= \frac{\lambda^{\sum y_i} e^{-n\lambda}}{\prod_{i=1}^{n} y_i!}$$

~~State~~ changing variable to.

$(y_1, \ldots, y_{n-1}, \cancel{\sum y_i}\ t)$ with $t = \sum y_i$

yields.

$$f(y_1, \ldots y_{n-1}, \cancel{\sum y_i}\ t)$$

$$= \frac{\lambda^{t} e^{-n\lambda}}{\left[\prod_{i=1}^{n-1} y_i!\right]\left(t - \sum_{i=1}^{n-1} y_i\right)!}$$

2 the distribution, by the property of iid poisson's

$T = \sum Y_i \sim Poi(n\lambda)$

$$\Rightarrow f_T(t) = \frac{(n\lambda)^t e^{-n\lambda}}{t!}$$

Combining 1 and 2.

$$f(y_1 \ldots y_n \mid \sum y_i = t) = \frac{t!}{\left[\prod_{i=1}^{n-1} y_i!\right]\left(t - \sum_{i=1}^{n-1} y_i\right)!\, n^t}$$

which is free from λ.

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9.64 a. In class we showed that

$\bar{Y}$ is minimum sufficient for μ.

$\Rightarrow$ $\bar{Y}^2$ is minimum sufficient for μ^2.

also, we have

$$\mathbb{E}\hat{\mu}^2 = \mathbb{E}\,\bar{Y}^2 - \frac{1}{n}$$

$$= \mathrm{Var}\,\bar{Y} + (\mathbb{E}\,\bar{Y})^2 - \frac{1}{n}$$

$$= \frac{1}{n} + \mu^2 - \frac{1}{n} = \mu^2$$

b. For $X \sim N(\mu, \sigma^2)$

$$\mathbb{E}X^4 = \mu^4 + 6\mu^2\sigma^2 + 3\sigma^4$$

$$\Rightarrow \mathrm{Var}(\hat{\mu}) = \mathrm{Var}\left(\bar{Y}^2 - \frac{1}{n}\right)$$

$$= \mathbb{E}\left[\left(\bar{Y}^2 - \frac{1}{n}\right) - \mu^2\right]^2$$

$$= \mathbb{E}\,\bar{Y}^4 - 2\left(\frac{1}{n} + \mu^2\right)\mathbb{E}\,\bar{Y}^2 + \left(\frac{1}{n} + \mu^2\right)^2$$

$$\mathbb{E}\,\bar{Y}^4 = \mu^4 + 6\cdot\mu^2\frac{1}{n} + 3\cdot\frac{1}{n^2}$$

$$\mathbb{E}\,\bar{Y}^2 = \mu^2 + \frac{1}{n}$$

$$\Rightarrow \mathrm{Var}(\hat{\mu}) = 4\mu^2\frac{1}{n} + 2\frac{1}{n^2}$$

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