3.145.

$$m(t) = \mathbb{E}\, e^{tY} = \sum_{y=0}^{n} e^{ty} \binom{n}{y} p^y (1-p)^{n-y}$$

$$= \sum_{y=0}^{n} \binom{n}{y} (e^t \cdot p)^y (1-p)^{n-y}$$

$$= [1 - p + pe^t]^n$$

using Binomial theorem

$$(a+b)^n = \sum_{y=0}^{n} \binom{n}{y} a^y b^{n-y}$$

3.155

a. $m'(t) = \frac{1}{6} e^t + \frac{4}{6} e^{2t} + \frac{9}{6} e^{3t}$

set $t=0 \Rightarrow \mathbb{E}Y = \frac{1}{6} + \frac{4}{6} + \frac{9}{6} = \frac{14}{6} = \frac{7}{3}$

b. $m''(t) = \frac{1}{6} + \frac{8}{6} e^{2t} + \frac{27}{6} e^{3t}$

set $t=0 \Rightarrow \mathbb{E}Y^2 = \frac{36}{6} = 6$

$\Rightarrow \operatorname{var}(Y) = \mathbb{E}Y^2 - (\mathbb{E}Y)^2 = 6 - \frac{49}{9}$

$= \frac{5}{9}$

c. Y has pmf

$$p(y) = \begin{cases} 1/6, & \text{if } y = 1 \\ 2/6, & \text{if } y = 2 \\ 3/6, & \text{if } y = 3 \end{cases}$$