Solution For Homework 1

7.30 (a) $Z \sim N(0,1)$

$\mathbb{E}Z = 0$

$\mathbb{E}Z^2 = \mathrm{Var}(Z) + (\mathbb{E}Z)^2$

$= 1 + 0^2 = 1$

(b)

Since $T = \frac{Z}{\sqrt{Y/V}}$ and $Z \perp\!\!\!\perp Y$

(i) $\mathbb{E}T = \mathbb{E}Z \cdot \mathbb{E}\frac{1}{\sqrt{Y/V}}$

$= \mathbb{E}Z \cdot \sqrt{V} \cdot \mathbb{E}\frac{1}{\sqrt{Y}}$

$= \mathbb{E}Z \cdot \sqrt{V} \cdot \mathbb{E}\, Y^{-\frac{1}{2}}$

since $\mathbb{E}Z = 0$

$\mathbb{E}\, Y^{-\frac{1}{2}} = \frac{\Gamma(V/2 - \frac{1}{2})}{\Gamma(V/2)} \cdot 2^{-\frac{1}{2}}$

$\Rightarrow \mathbb{E}T = 0.$

(ii) $\mathbb{E}T^2 = \mathbb{E}Z^2\, \mathbb{E}\frac{1}{Y/V}$

$= \mathbb{E}Z^2 \mathbb{E}\frac{V}{Y}$

$= \mathbb{E}Z^2 \cdot V \cdot \mathbb{E}\, Y^{-1}$

$= 1 \cdot V \cdot \frac{\Gamma(V/2 - 1)}{\Gamma(V/2)} \cdot 2^{-1}$

$= \frac{V}{V/2 - 1} \cdot \frac{1}{2} = \frac{V}{V-2}$

$\mathrm{Var}(T) = \mathbb{E}T^2 - (\mathbb{E}T)^2 = V/(V-2)$

7.33 $$T = \frac{Z}{\sqrt{W/V}}$$

$$\Rightarrow T^2 = \frac{Z^2}{W/V}$$

Since

$$Z^2 \sim \chi^2_1$$

$$W \sim \chi^2_V.$$

$$Z^2 \perp\!\!\!\perp W$$

$$\Rightarrow T^2 = \frac{Z^2/1}{W/V} \sim F_{1,V}$$