

5.94. a.)  $\text{cov}(U_1, U_2)$

$$\begin{aligned}
 &= \text{cov}(Y_1, X_1) - \cancel{\text{cov}(Y_1, X_2)} + \cancel{\text{cov}(X_2, Y_1)} \\
 &\quad - \text{cov}(X_2, Y_2) \\
 &= \text{var}(Y_1) - \text{var}(X_2)
 \end{aligned}$$

b).  $\text{var}(U_1)$

$$= \text{var}(X_1) + \text{var}(X_2) + 2\text{cov}(X_1, X_2)$$

$$= \text{var}(X_1) + \text{var}(X_2)$$

$$\text{var}(U_2)$$

$$= \text{var}(X_1) + \text{var}(X_2) - 2\text{cov}(X_1, X_2)$$

$$= \text{var}(X_1) + \text{var}(X_2)$$

$$\Rightarrow \rho = \frac{\text{cov}(U_1, U_2)}{\sqrt{\text{var}(U_1) \text{var}(U_2)}} = \frac{\text{var}(X_1) - \text{var}(X_2)}{\text{var}(X_1) + \text{var}(X_2)}$$

c) when  $\text{var}(X_1) = \text{var}(X_2)$

5.141.

$$Y_2 | Y_1 \sim \text{Uniform}(0, y_1)$$

$$\Rightarrow E(Y_2 | Y_1 = y_1) = \frac{y_1}{2}$$

$$\text{var}(Y_2 | Y_1 = y_1) = \frac{y_1^2}{12}$$

$$E(Y_2) = E E(Y_2 | Y_1) = \frac{E Y_1}{2} = \frac{\lambda}{2}$$

$$\text{var}(Y_2) = E \text{var}(Y_2 | Y_1) + \text{var}(E(Y_2 | Y_1))$$

$$= E \frac{y_1^2}{12} + \text{var} \frac{y_1}{2}$$

$$= \frac{E Y_1^2}{12} + \frac{[\text{var} Y_1]}{4}$$

$$\text{small } Y_1 \sim \text{Exp}(\lambda) \Rightarrow \text{var}(Y_1) = \lambda^2$$

$$E Y_1^2 - (E Y_1)^2 = \lambda^2$$

$$\Rightarrow E Y_1^2 - \lambda^2 = \lambda^2$$

$$\Rightarrow E Y_1^2 = 2\lambda^2$$

$$\Rightarrow \text{var}(Y_2) = \frac{2\lambda^2}{12} + \frac{\lambda^2}{4} = \frac{5}{12} \lambda^2$$