8.12. $Y_i \overset{iid}{\sim} U(\theta, \theta+1)$

a. $\mathbb{E} Y_i = \dfrac{\theta + (\theta+1)}{2} = \theta + \frac{1}{2}$

$\Rightarrow \mathbb{E}\bar{Y} = \mathbb{E} Y_i = \theta + \frac{1}{2}$,

which is biased

b. $\bar{Y} - \frac{1}{2}$ is unbiased.

c. $V(Y_i) = \dfrac{[(\theta+1) - \theta]^2}{12} = \dfrac{1}{12}$

$V(\bar{Y}) = \dfrac{V(Y_i)}{n} = \dfrac{1}{12n}$

bias $B(\bar{Y}) = \frac{1}{2}$

$MSE(\bar{Y}) = V(\bar{Y}) + B(\bar{Y})^2$

$= \dfrac{1}{12n} + \dfrac{1}{4}$

8.43. a. $P(U \leq u) = P(\frac{1}{\theta} Y_{(n)} \leq u)$

$= P(Y_{(n)} \leq u\theta)$

$= P(Y_1 \leq u\theta \ldots Y_n \leq u\theta)$

$= \left(\dfrac{u\theta}{\theta}\right)^n = u^n$, $\quad 0 \leq u \leq 1$

b. $P\left(\frac{Y_{(n)}}{\theta} \leq u\right) = u^n \equiv 95\%$

$u = (95\%)^{\frac{1}{n}}$

$\Rightarrow \quad \frac{Y_{(n)}}{\theta} \leq (95\%)^{\frac{1}{n}}$

$\Rightarrow \quad \theta \geq (95\%)^{\frac{1}{n}} \cdot Y_{(n)}$