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$$L(X_1, \dots, X_n, X_1, \dots, X_m) \\ = \frac{\lambda_1^{\sum X_i} e^{-n\lambda_1}}{\prod_{i=1}^n X_i!} \cdot \frac{\lambda_2^{\sum X_j} e^{-m\lambda_2}}{\prod_{j=1}^m X_j!}$$

By Neyman-Pearson, we need

$$\frac{L(X_1, \dots, X_n, X_1, \dots, X_m; \lambda_1 = \lambda_2 = 2)}{L(X_1, \dots, X_n, X_1, \dots, X_m; \lambda_1 = 1/2, \lambda_2 = 3)} \leq k$$

$$\Rightarrow \frac{2^{\sum X_i}}{(1/2)^{\sum X_i}} \cdot \frac{2^{\sum X_i}}{3^{\sum X_i}} \leq k^*$$

$$\Rightarrow (\log 4) \sum X_i + (\log \frac{2}{3}) \sum X_i \leq k^{**}$$

Let $U = \log 4 \sum X_i + \log \frac{2}{3} \sum X_i$
a sum of two scaled Poissons

We find c , such that

$$P(U < c) = \alpha$$

the rejection region is

$$\{U < c\}$$

10/12/ (c). $f(y_i) = \frac{1}{\theta} \exp(-y_i/\theta)$, $y_i > 0$

$$\Rightarrow \frac{L(y_1, \dots, y_n; \theta_0)}{L(y_1, \dots, y_n; \theta_a)} = \left(\frac{\theta_a}{\theta_0} \right)^n \exp\left(-\sum y_i \left[\frac{1}{\theta_0} - \frac{1}{\theta_a} \right]\right) \leq K.$$

$$\Rightarrow -\sum y_i \left[\frac{1}{\theta_0} - \frac{1}{\theta_a} \right] \leq K^*$$

$$\theta_a < \theta_0 \Rightarrow \sum y_i \leq K^{**}$$

Since

$$\sum_i \frac{y_i}{\theta_0} \sim \chi^2_{2n}.$$

rejection region

$$\Rightarrow R = \left\{ \sum \frac{y_i}{\theta_0} < \chi^2_{2n, 1-\alpha} \right\}$$

produce the MP test

(b) Since R does not depend on specific θ_a ,
it is the UMP test.