Exercises

3.145 If Y has a binomial distribution with n trials and probability of success p, show that the moment-generating function for Y is

$$m(t) = (pe^t + q)^n, \qquad \text{where } q = 1 - p.$$

3.146 Differentiate the moment-generating function in Exercise 3.145 to find $E(Y)$ and $E(Y^2)$. Then find $V(Y)$.

3.147 If Y has a geometric distribution with probability of success p, show that the moment-generating function for Y is

$$m(t) = \frac{pe^t}{1 - qe^t}, \qquad \text{where } q = 1 - p.$$

3.148 Differentiate the moment-generating function in Exercise 3.147 to find $E(Y)$ and $E(Y^2)$. Then find $V(Y)$.

3.149 Refer to Exercise 3.145. Use the uniqueness of moment-generating functions to give the distribution of a random variable with moment-generating function $m(t) = (.6e^t + .4)^3$.

3.150 Refer to Exercise 3.147. Use the uniqueness of moment-generating functions to give the distribution of a random variable with moment-generating function $m(t) = \dfrac{.3e^t}{1 - .7e^t}$.

3.151 Refer to Exercise 3.145. If Y has moment-generating function $m(t) = (.7e^t + .3)^{10}$, what is $P(Y \le 5)$?

3.152 Refer to Example 3.23. If Y has moment-generating function $m(t) = e^{6(e^t - 1)}$, what is $P(|Y - \mu| \le 2\sigma)$?

3.153 Find the distributions of the random variables that have each of the following moment-generating functions:

a $m(t) = [(1/3)e^t + (2/3)]^5$.

b $m(t) = \dfrac{e^t}{2 - e^t}$.

c $m(t) = e^{2(e^t - 1)}$.

3.154 Refer to Exercise 3.153. By inspection, give the mean and variance of the random variables associated with the moment-generating functions given in parts (a), (b), and (c).

3.155 Let $m(t) = (1/6)e^t + (2/6)e^{2t} + (3/6)e^{3t}$. Find the following:

a $E(Y)$

b $V(Y)$

c The distribution of Y

3.156 Suppose that Y is a random variable with moment-generating ...