10.100

$$L(Y_1 \ldots Y_n, X_1 \ldots X_m)$$

$$= \frac{\lambda_1^{\Sigma Y_i} e^{-n\lambda_1}}{\prod_{i=1}^{n} Y_i!} \cdot \frac{\lambda_2^{\Sigma X_j} e^{-m\lambda_2}}{\prod_{j=1}^{m} X_j}$$

By Neyman - Pearson, we need

$$\frac{L(Y_1 \ldots Y_n, X_1 \ldots X_m; \lambda_1 = \lambda_2 = 2)}{L(Y_1 \ldots Y_n, X_1 \ldots X_m; \lambda_1 = 1/2, \lambda_2 = 3)} \leq K.$$

$$\Rightarrow \quad \frac{2^{\Sigma Y_i}}{(1/2)^{\Sigma Y_i}} \cdot \frac{2^{\Sigma X_i}}{3^{\Sigma X_i}} \leq K^*$$

$$\Rightarrow \quad (\log 4) \; \Sigma Y_i + (\log \tfrac{2}{3}) \cdot \Sigma X_i \leq K^{**}$$

Let $U = \log 4 \; \Sigma Y_i + \log \frac{2}{3} \; \Sigma X_i$

a sum of two scaled Poissons

We find c, such that

$$P(U < c) = \alpha.$$

the rejection region is

$$\{U < c\}$$

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10.10 (c) $f(y_i) = \frac{1}{\theta} \exp(-y_i/\theta)$, $y_i > 0$

$$\Rightarrow \frac{L(y_1 \dots y_n; \theta_0)}{L(y_1 \dots y_n; \theta_a)} = \left(\frac{\theta_a}{\theta_0}\right)^n \exp\left(-\sum y_i \left[\frac{1}{\theta_0} - \frac{1}{\theta_a}\right]\right)$$

$$\leq K.$$

$$\Rightarrow \quad -\sum y_i \left[\frac{1}{\theta_0} - \frac{1}{\theta_a}\right] \leq K^*$$

$$\xrightarrow{\theta_a < \theta_0} \quad \sum y_i \leq K^{**}$$

since

$$\sum_i \frac{2Y_i}{\theta_0} \sim \chi^2_n.$$

rejection region

$$\Rightarrow \quad R = \left\{ \sum \frac{2}{\theta_0} Y_i < \chi^2_{n, 1-\alpha} \right\}$$

produce the MP test

(b) Since R does not depend on specific θ_a,

it is the UMP test.

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