8.12 The reading on a voltage meter connected to a test circuit is uniformly distributed over the interval $(\theta, \theta + 1)$, where θ is the true but unknown voltage of the circuit. Suppose that $Y_1, Y_2, \ldots, Y_n$ denote a random sample of such readings.

a Show that $\overline{Y}$ is a biased estimator of θ and compute the bias.

b Find a function of $\overline{Y}$ that is an unbiased estimator of θ.

c Find $\text{MSE}(\overline{Y})$ when $\overline{Y}$ is used as an estimator of θ.

8.43 Let $Y_1, Y_2, \ldots, Y_n$ denote a random sample of size n from a population with a uniform distribution on the interval $(0, \theta)$. Let $Y_{(n)} = \max(Y_1, Y_2, \ldots, Y_n)$ and $U = (1/\theta)Y_{(n)}$.

a Show that U has distribution function

$$F_U(u) = \begin{cases} 0, & u < 0, \\ u^n, & 0 \le u \le 1, \\ 1, & u > 1. \end{cases}$$

b Because the distribution of U does not depend on θ, U is a pivotal quantity. Find a 95% lower confidence bound for θ.