6.2 a. $U_1 = 3Y$

$$P(U_1 \le u) = P(3Y \le u) = P\left(Y \le \frac{u}{3}\right)$$

$$= \int_{-1}^{\frac{u}{3}} \frac{3}{2} y^2 \cdot dy$$

$$= \frac{y^3}{2} \Big|_{-1}^{\frac{u}{3}}$$

$$= \frac{u^3}{54} + \frac{1}{2}, \quad -1 \le \frac{u}{3} \le 1$$

$$\Rightarrow f(u) = \frac{u^2}{18}, \quad -3 \le u \le 3$$

b. $P(U_2 \le u) = P(3 - Y \le u) = P(Y \ge 3 - u)$

$$= \int_{3-u}^{1} \frac{3}{2} y^2 \cdot dy$$

$$= \frac{y^3}{2} \Big|_{3-u}^{1}$$

$$= \frac{1}{2} - \frac{(3-u)^3}{2}, \quad -1 \le 3 - u \le 1$$

$$\Rightarrow f(u) = \frac{3(u-3)^2}{2}, \quad 2 \le u \le 4.$$

c. $P(U_3 \le u) = P(Y^2 \le u) = P(-\sqrt{u} \le Y \le \sqrt{u})$

$$= \frac{y^3}{2} \Big|_{-\sqrt{u}}^{\sqrt{u}}$$

$$= u^{3/2}, \quad -1 \le -\sqrt{u} < \sqrt{u} \le 1$$

$$\Rightarrow f(u) = \frac{3}{2} u^{\frac{1}{2}}, \quad 0 \le u \le 1$$

6.26 a. $U = Y^m$

$\Rightarrow Y = U^{\frac{1}{m}}$

$$\left|\frac{d u^{\frac{1}{m}}}{du}\right| = \frac{1}{m} u^{\frac{1}{m}-1}$$

$$\Rightarrow f(u) = \frac{1}{\alpha^m}\left(u^{\frac{1}{m}}\right)^{m-1} e^{-u/\alpha} \left|\frac{1}{m} u^{\frac{1}{m}-1}\right|$$

$$= \frac{1}{\alpha^m} u^{1-\frac{1}{m}} e^{-u/\alpha} \cdot u^{\frac{1}{m}-1}.$$

$$= \frac{1}{\alpha^m} e^{-u/\alpha}, \quad u > 0$$

b. ~~Since $U \sim \exp(\alpha)$~~

~~$\Rightarrow$ If $U = Y^k$~~

$$Y^k = \left(Y^m\right)^{k/m} = U^{k/m}$$

$$\Rightarrow \mathbb{E}Y^k = \int \frac{1}{\alpha^m} e^{-u/\alpha} u^{k/m} \cdot du.$$

$$= \int \frac{1}{\Gamma\left(\frac{k}{m}+1\right)\alpha^{k/m+1}} e^{-u/\alpha} u^{(k/m+1)-1} \cdot du$$

$$\times \left[\frac{1}{\alpha} \cdot \alpha^{k/m+1} \cdot \Gamma\left(\frac{k}{m}+1\right)\right]$$

$$= \alpha^{k/m} \Gamma\left(\frac{k}{m}+1\right)$$