11.15

a.

$$\sum_i (y_i - \hat{\beta}_0 - \hat{\beta}_1 x_i)^2$$

$$= \sum_i \left[(y_i - \bar{y}) + (\bar{y} - \hat{\beta}_0 - \hat{\beta}_1 x_i)\right]^2$$

$$= \sum \left[(y_i - \bar{y})^2 + (\hat{\beta}_1 \bar{x} - \hat{\beta}_1 x_i)^2 + 2(y_i - \bar{y})(\hat{\beta}_1 \bar{x} - \hat{\beta}_1 x_i)\right]$$

Note $\sum (\hat{\beta}_1 \bar{x} - \hat{\beta}_1 x_i)^2$

$$= \hat{\beta}_1^2 \cdot \sum (x_i - \bar{x})^2$$

$$= \hat{\beta}_1^2 \cdot S_{xx}$$

$$= \hat{\beta}_1 S_{xy}$$

$$\Rightarrow \sum_i (y_i - \hat{\beta}_0 - \hat{\beta}_1 x_i)$$

$$= \sum (y_i - \bar{y})^2 + \hat{\beta}_1 S_{xy} - 2 \hat{\beta}_1 S_{xy}$$

$$= \sum (y_i - \bar{y})^2 - \hat{\beta}_1 S_{xy}$$

b. $SSE = S_{yy} - \hat{\beta}_1 S_{xy} = S_{yy} - \hat{\beta}_1^2 S_{xx}$

since $S_{xx} \geq 0$ $\Rightarrow SSE \leq S_{yy}$

11.21.

$$\mathrm{cov}(\hat{\beta}_0, \hat{\beta}_1) = \mathrm{cov}(\bar{Y} - \hat{\beta}_1 \bar{x}, \hat{\beta}_1)$$

$$= \mathrm{cov}(\bar{Y}, \hat{\beta}_1) - \mathrm{cov}(\bar{x}\hat{\beta}_1, \hat{\beta}_1)$$

$$= \mathrm{cov}\left(\frac{1}{n}\sum_{i=1}^{n} Y_i, \sum_{j=1}^{n} \frac{(x_i - \bar{x})}{S_{xx}} \cdot Y_j\right) - \bar{x} \cdot \mathrm{Var}(\hat{\beta}_1)$$

since $\mathrm{cov}(Y_i, Y_j) = 0$ if $i \neq j$

$$\mathrm{cov}\left(\frac{1}{n}\sum Y_i, \sum \frac{(x_i - \bar{x})}{S_{xx}} \cdot Y_i\right)$$

$$= \sum \frac{1}{n} \cdot \frac{(x_i - \bar{x})}{S_{xx}} \cdot \mathrm{Var}(Y_i)$$

$$= \sum \frac{1}{n} \frac{(x_i - \bar{x})}{S_{xx}} \cdot \sigma^2$$

$$= \frac{1}{n} \cdot \frac{\sigma^2}{S_{xx}} \sum (x_i - \bar{x})$$

$$= 0$$

$$\Rightarrow \mathrm{cov}(\hat{\beta}_0, \hat{\beta}_1) = -\bar{x} \cdot \mathrm{Var}(\hat{\beta}_1)$$

$$= -\bar{x} \frac{\sigma^2}{S_{xx}}.$$

since $\hat{\beta}_0, \hat{\beta}_1$ are jointly normal

$\Rightarrow$ $\hat{\beta}_0$ and $\hat{\beta}_1$ are independent