$$Z \sim N(0,1)$$

7.30. a. $\mathbb{E} Z = 0$

$$V(Z) = 1$$

$$\mathbb{E} Z^2 = V(Z) + (\mathbb{E} Z)^2 = 1$$

b.

$$T = \frac{Z}{\sqrt{Y/v}}.$$

$$\mathbb{E} T \overset{\text{indep}}{=} \mathbb{E} Z \, \mathbb{E}\left(\sqrt{\frac{Y}{v}}\right)^{-1}$$

$$= \mathbb{E} Z \cdot \mathbb{E} Y^{-\frac{1}{2}} \cdot v^{+\frac{1}{2}}$$

$$= 0 \times \frac{\Gamma([v/2] - \frac{1}{2})}{\Gamma(v/2)} \cdot 2^{-\frac{1}{2}} v^{+\frac{1}{2}}$$

$v > 1.$

$$= 0.$$

$$\mathbb{E} T^2 = \mathbb{E} Z^2 \, \mathbb{E} Y^{-1} \cdot v$$

$$= 1 \times \frac{\Gamma(v/2 - 1)}{\Gamma(v/2)} \cdot 2^{-1} \cdot v^{1}$$

$$= \frac{v}{(v/2 - 1) \times 2} = \frac{v}{v-2}.$$

$v > 2.$

7.57. $n = 25$

$\mu = 50$

$\sigma^2 = 4^2$

$$\sum Y_i \overset{approx}{\sim} N(n\mu, n\sigma^2)$$

$$P(\sum Y_i \geq 1300) = P\left(\frac{\sum Y_i - 25 \times 50}{\sqrt{25 \times 4^2}} \geq \frac{1300 - 25 \times 50}{\sqrt{25 \times 4^2}} \right)$$

$$= P(Z \geq 2.5)$$

$$= .0062$$