University of Wisconsin
SCHOOL OF MEDICINE AND PUBLIC HEALTH

BMI/STAT-768 Statistical Methods for Medical Image Analysis

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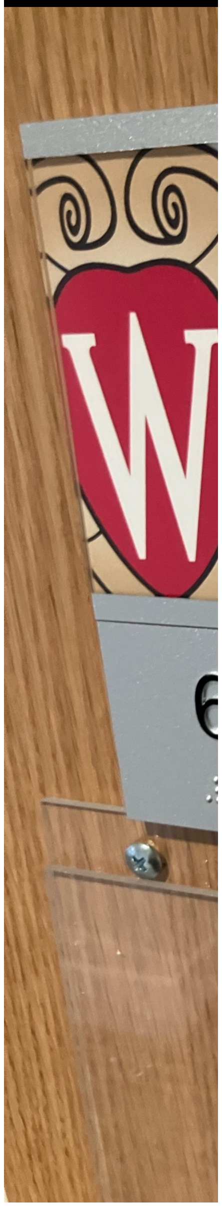
Limitation of LLM and how to fix it



Predictable Verification using Intrinsic Definitions

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We propose a novel mechanism of defining data structures using *intrinsic definitions* that avoids recursion and instead utilizes *monadic maps satisfying local conditions*. We show that intrinsic definitions are a powerful mechanism that can capture a variety of data structures naturally. We show that they also enable a predictable verification methodology that allows engineers to write ghost code to update monadic maps and perform verification using reduction to decidable logics. We evaluate our methodology using Boogie and prove a suite of data structure manipulating programs correct.

CCS Concepts: • **Software and its engineering** → **Formal software verification**; • **Theory of computation** → **Logic and verification**; **Automated reasoning**.

Additional Key Words and Phrases: Predictable Verification, Intrinsic Definitions, Verification of Linked Data Structures, Decidability, Ghost-Code Annotations

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Classical logics used in verification are inherently ***acyclic*** because they rely on well-founded semantics. Frameworks such as first-order logic with **induction** avoids circularity through syntactic acyclicity or semantic mechanisms like ***least fixed points***. Circular definitions are only meaningful when given explicit fixed-point semantics; without them, circular reasoning leads to inconsistency or non-termination.

Monotone function (*adding information*)

$$x \leq y \;\Rightarrow\; F(x) \leq F(y)$$

The least fixed point x* is the smallest element *x* satisfying *F*(x*) = x*.

$$\bot \;\subseteq\; F(\bot) \;\subseteq\; F^2(\bot) \;\subseteq\; F^3(\bot) \;\subseteq\; \cdots$$

Filtration: monotone Chain

Theorem: Monotonicity forces convergence to the least stable point x*

There is no circular logic in this process:

information is added monotonically, and the process eventually reaches a stage where no new information is generated—the least fixed point.

This is the essence of an LLM, which generates outputs through a one-directional, monotone accumulation of context without enforcing global cyclic consistency.

LLM and diffusion models are a long sequence of Markov chain

$$p(x_{1:T}) = \prod_{t=1}^{T} p(x_t \mid x_{1:t-1})$$

All these machine learning models cannot handle cyclic structure.

Potential project topic

Design a Markov model that can handle cyclic data structure.

→ Energy-based method

$$p(x) \propto \exp(-E(x))$$

All tokens interact symmetrically through the energy, so cycles and topological constraints are natural. There is no imposed ordering, no DAG, and no circularity problem—only *equilibrium*.