



University of Wisconsin
SCHOOL OF MEDICINE
AND PUBLIC HEALTH

Unraveling dynamic Brain Networks: A topological data analysis framework for state space estimation

Moo K. Chung
University of Wisconsin-Madison
www.stat.wisc.edu/~mchung

June 17, 2024 1:20 pm Ocean Way (Mezzanine Level)
S13: Novel statistical methods for imaging data analysis and beyond
Organizer Panpan Zhang

Abstract

We present a novel topological data analysis (TDA) framework for modeling state spaces in dynamic functional brain networks. Core principles of TDA are explored, particularly highlighting how topological distances can be harnessed to cluster time-evolving brain networks. The approach is centered on the use of Wasserstein metrics to measure distances between 0D and 1D topological features, which significantly enhances the ability to track the brain's connectivity patterns over time. Our approach outperforms traditional methods such as k-means and hierarchical clustering by capturing the hidden topological patterns in the data, which are ignored in existing methods. The analysis reveals that resting-state brain networks oscillate primarily between two major states, with an additional noisy intermediate state. These fluctuations are linked to the complex sulcal and gyral cortical patterns, where significant differences in functional and structural connectivity are observed. The variability in connectivity introduces a dynamic gradient across the network, influencing the overall dynamics of brain connectivity changes. The talk is partially based on [arXiv:2201.00087](https://arxiv.org/abs/2201.00087) (PLOS Computational Biology).

Acknowledgement

D. Vijay Anand, Soumya Das, Ian C. Carroll, H. Hill Goldsmith,
Seth Pollack, Andrew Alexander, Richard Davidson **Univ. of
Wisconsin-Madison**

Felipe Branco De Paiva, Camille Garcia Ramos, Veena A. Nair,
Vivek Prabharakaren, Elizabeth Meyerand, Bruce P. Hermann,
Aaron F. Struck, **Univ. of Wisconsin-Madison**

Jedidiah Ray Mathis, Jeffrey R. Binder, **Medical College of
Wisconsin (MCW)**

Jeffrey S. Phillips, James Gee, Li Shen **University of Pennsylvania**

Grants: NIH U01NS093650, NS117568, EB022856, EB028753, MH133614,
MH101504, P30HD003352, U54HD09025, UL1TR002373, NSF DMS-2010778, 2112455

Acknowledgement

Shih-Gu Huang, Anqi Qiu **National University of Singapore**

Anass El Yaagoubi Bourakna, Chee-Ming Ting, Hernando
Ombao **KAUST, Saudi Arabia**

Sunah Choi, Minah Kim, Hyekyoung Lee, Dong Soo Lee,
Jun Soo Kwon **Seoul National University, Korea**
Jong Chul Ye, **KAIST, Korea**
Ilwoo Lyu **POSTECH, Korea**

Grants: NIH U01NS093650, NS117568, EB022856, EB028753, MH133614,
MH101504, P30HD003352, U54HD09025, UL1TR002373, NSF DMS-2010778, 2112455

Magnetic resonance imaging (MRI)

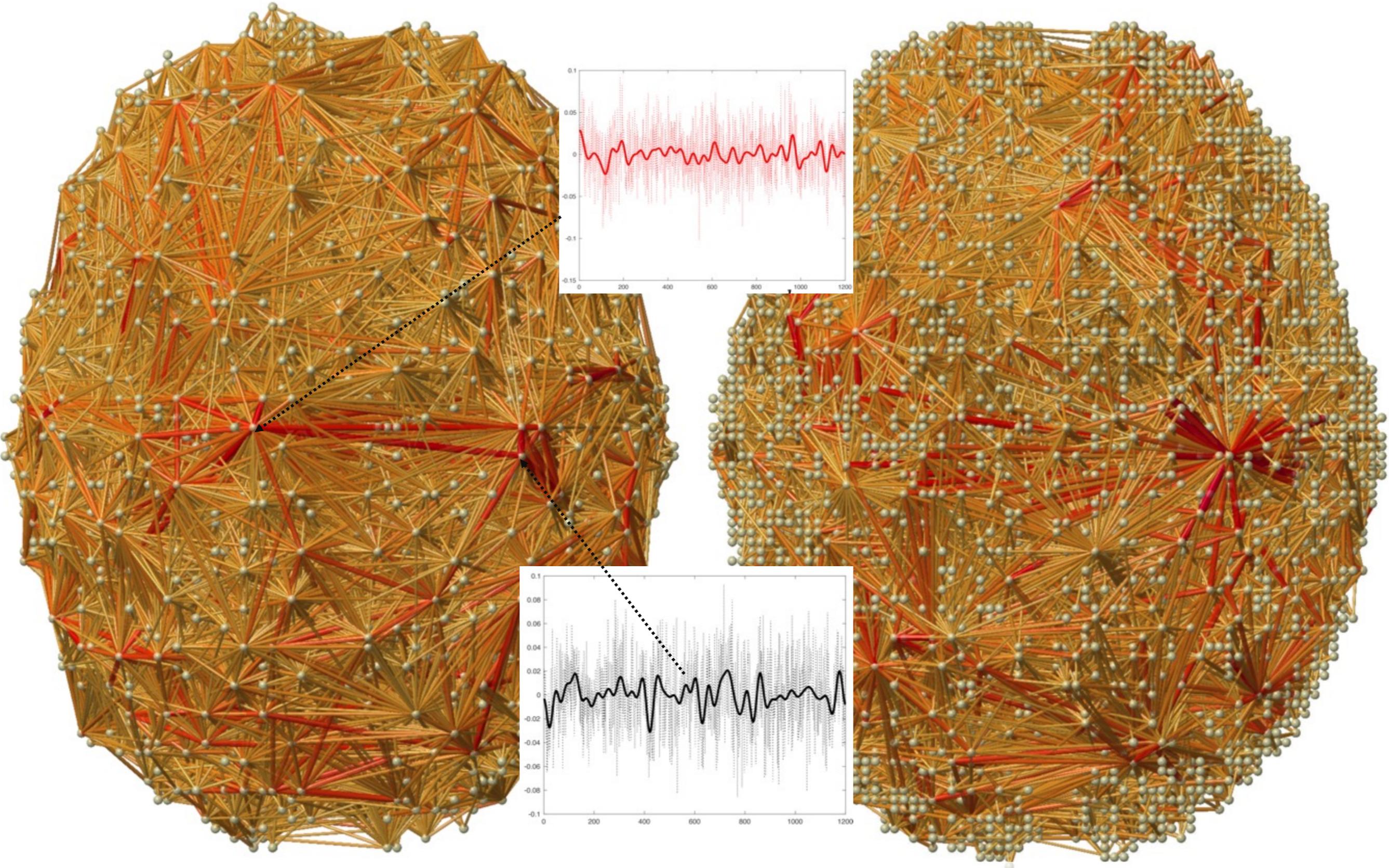


3T GE Discovery X750
Waisman Brain Imaging Laboratory
University of Wisconsin-Madison



3T GE Discovery MR750
Center for Imaging Research
Medical College of
Wisconsin, Milwaukee, WI

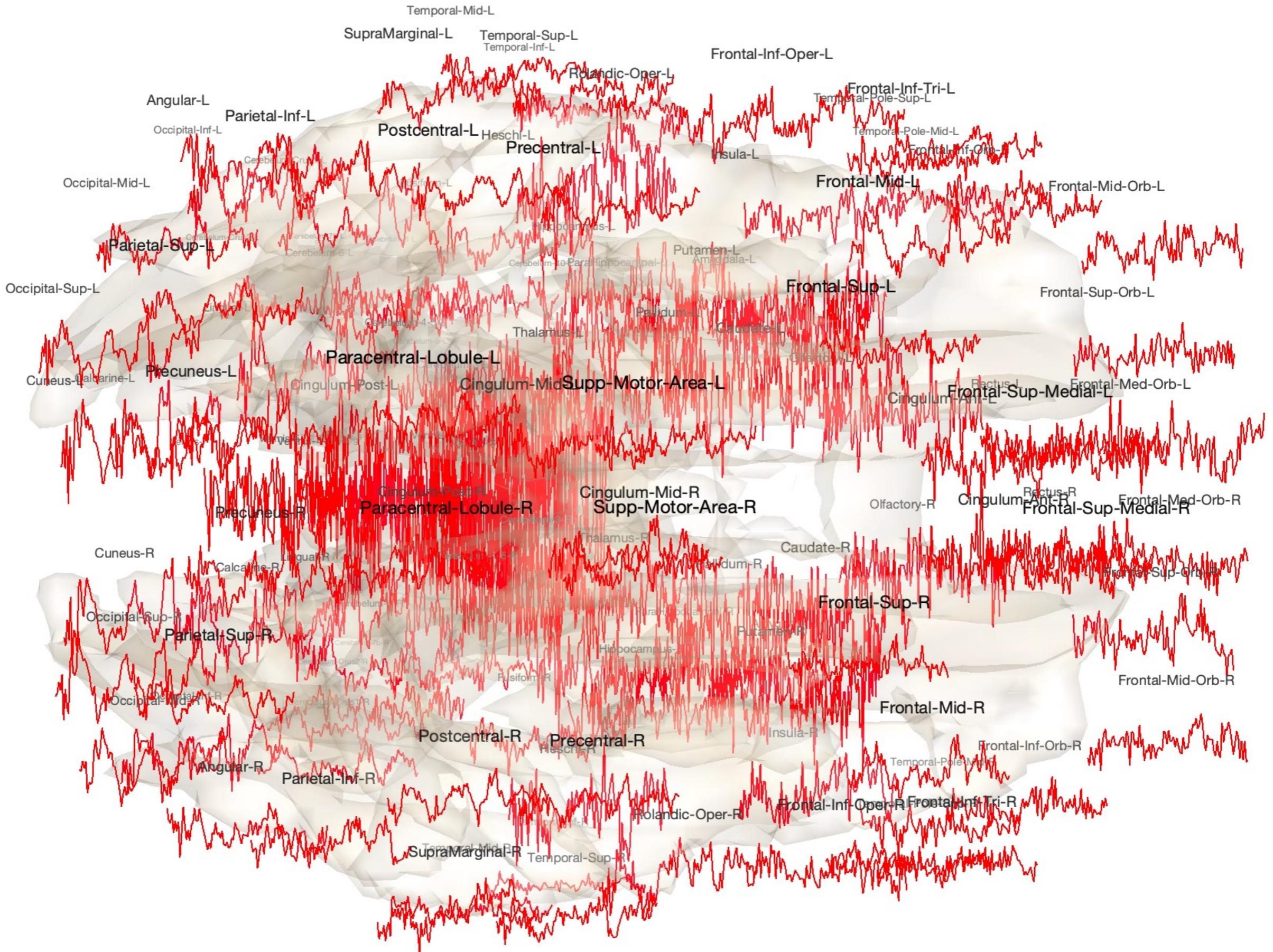
Dynamically changing correlation brain network

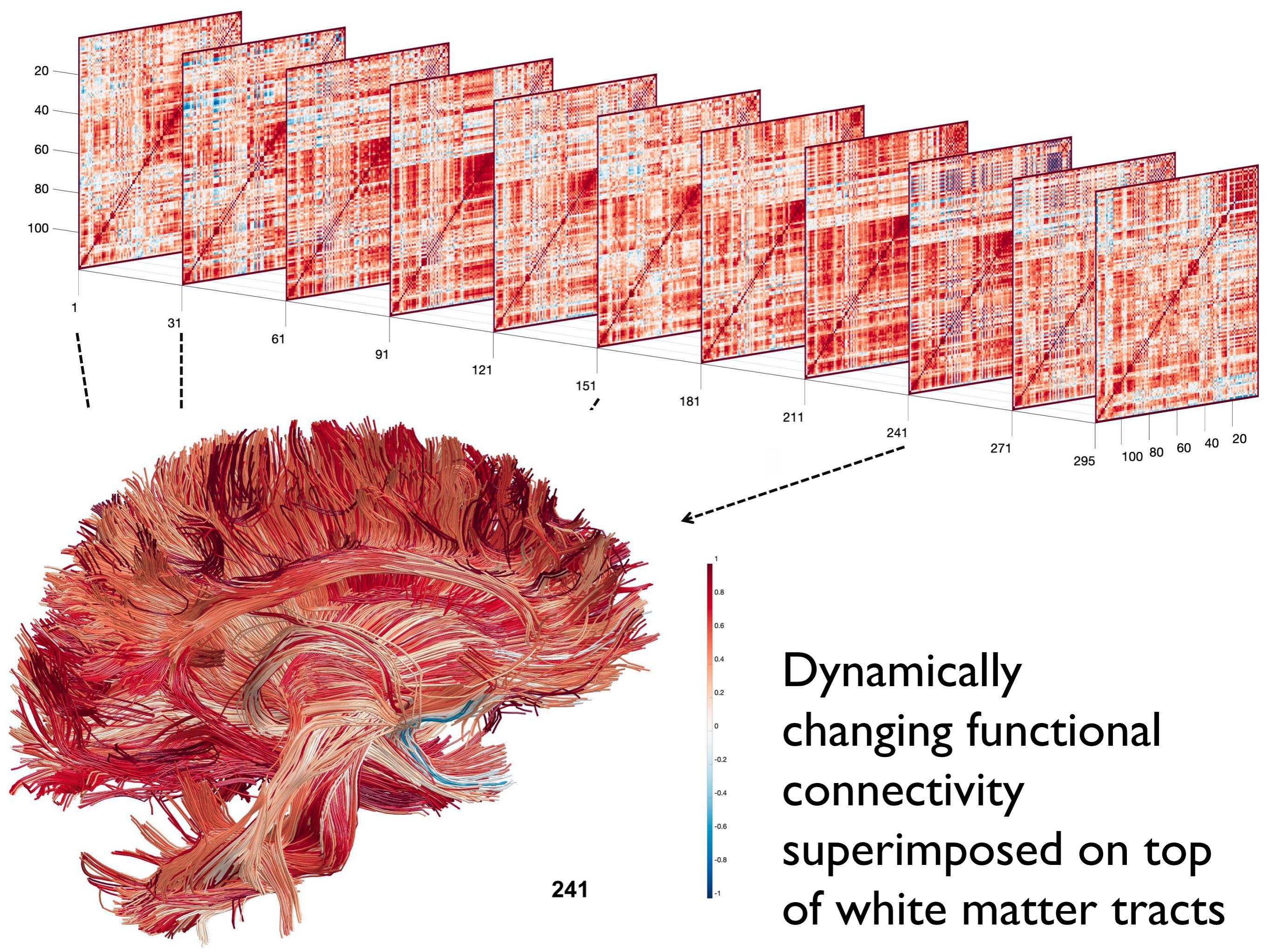


Correlation network of 300000 time series

Dynamically changing complete graph with about $300000^2/2$ cycles.

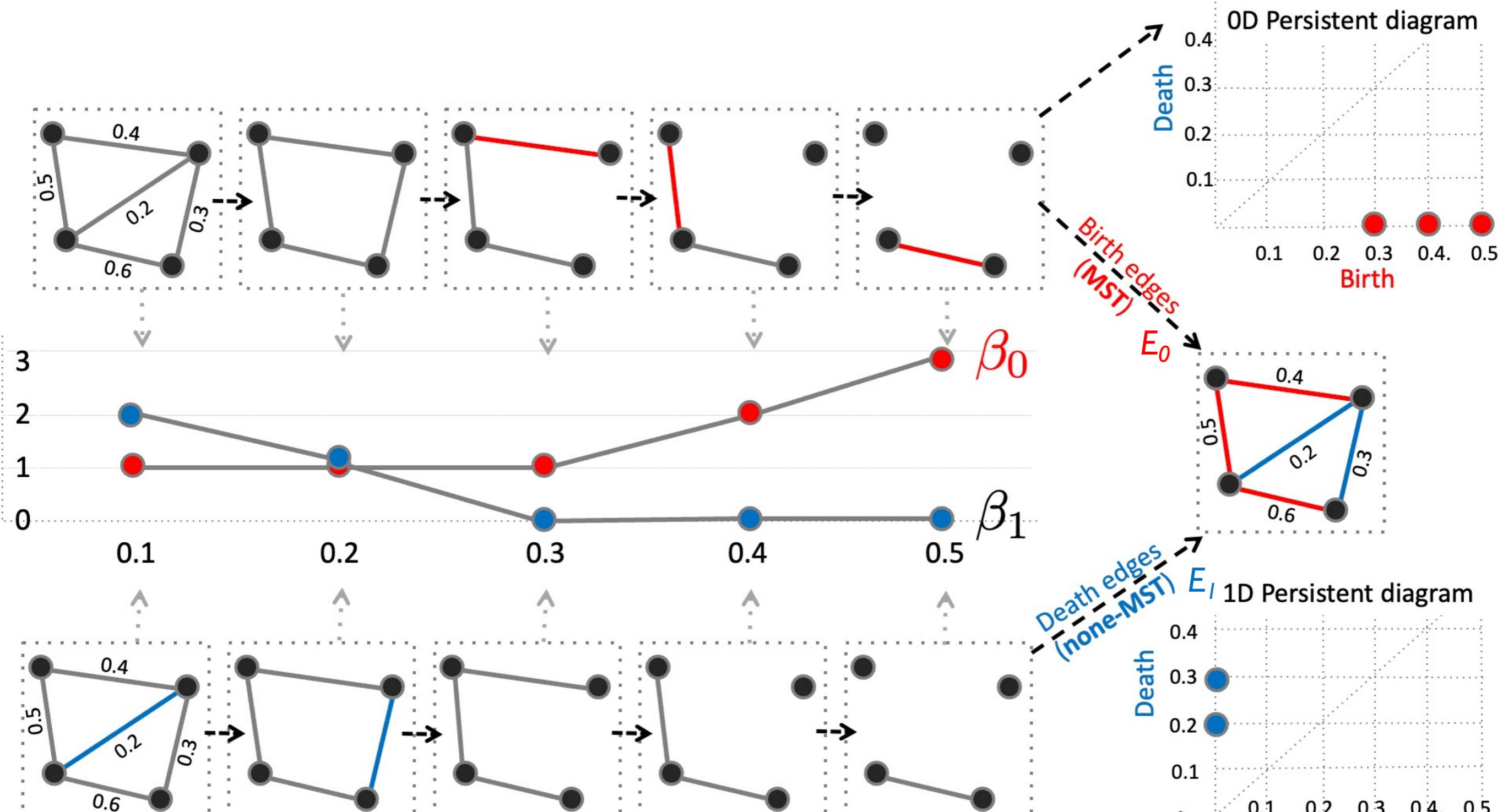
Time series averaged into parcellations





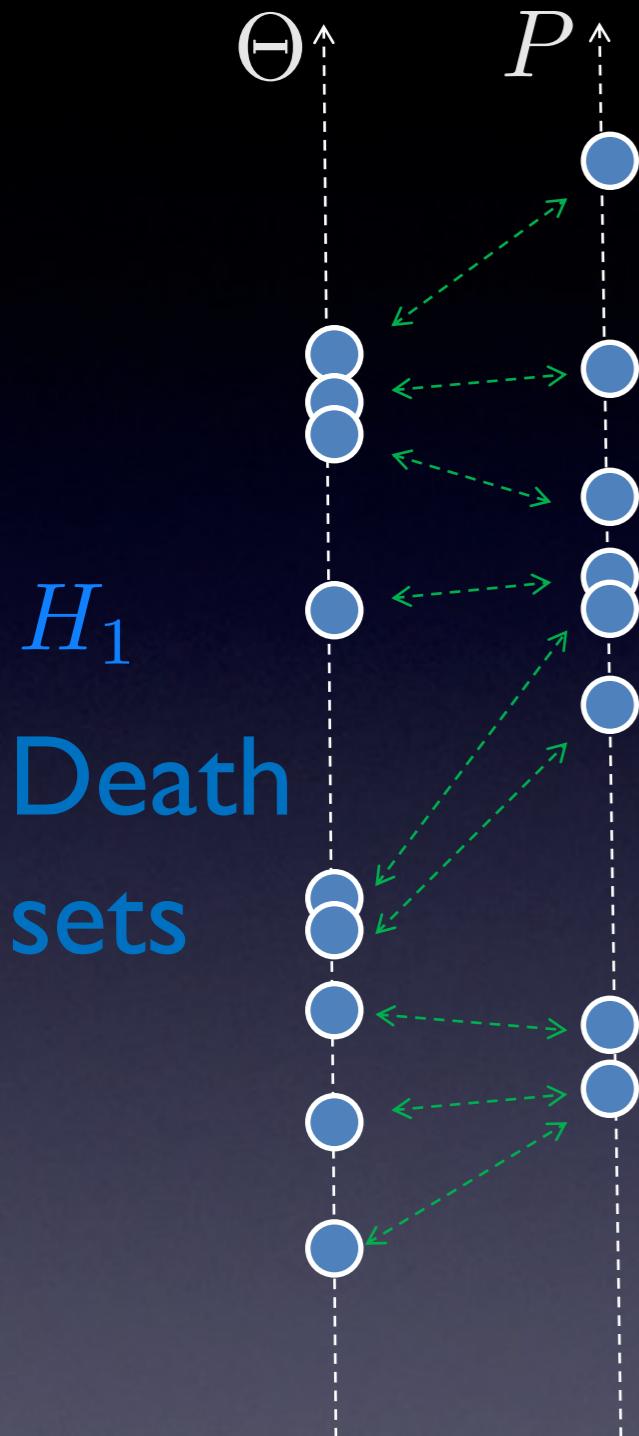
Topological decomposition

Theorem: Birth & death decomposition

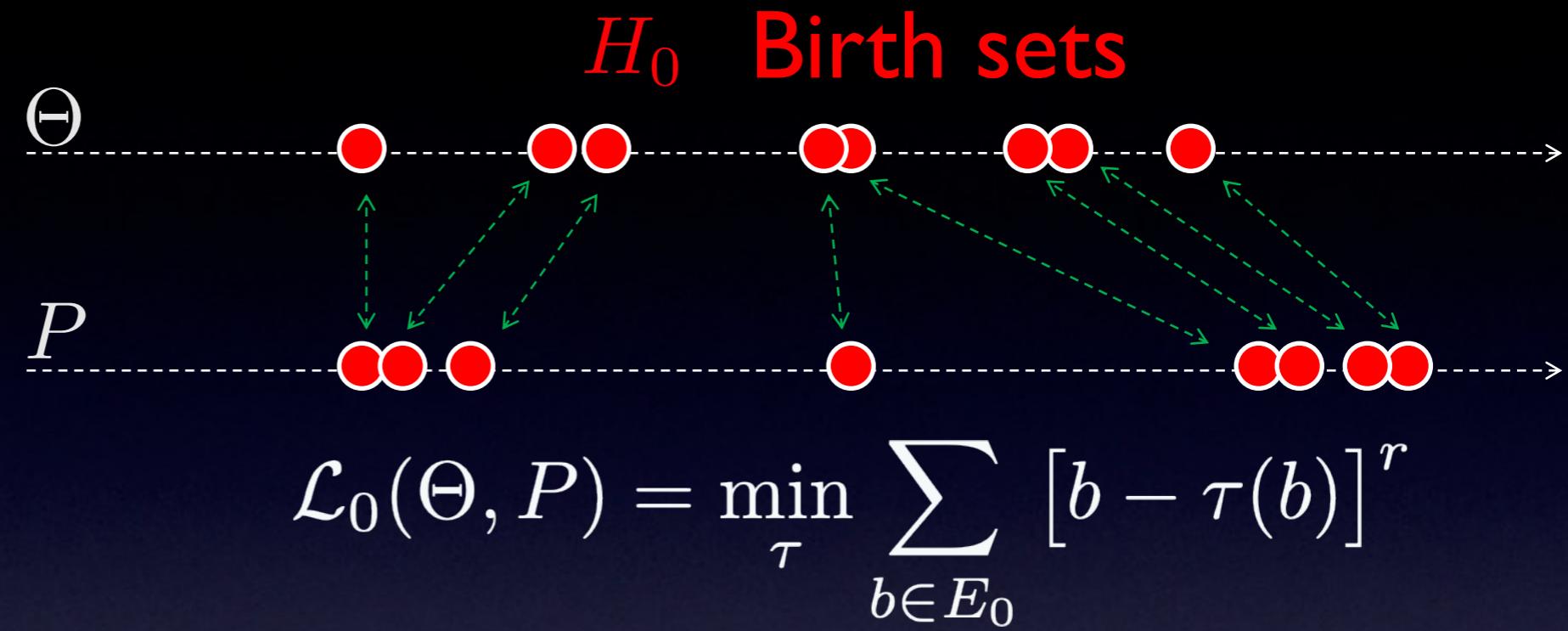


$$\text{Edge set } E = E_0 + E_1$$

Theorem: Minimization of the r-Wasserstein distance



$$\mathcal{L}_1(\Theta, P) = \min_{\tau} \sum_{d \in E_1} [d - \tau(d)]^r$$



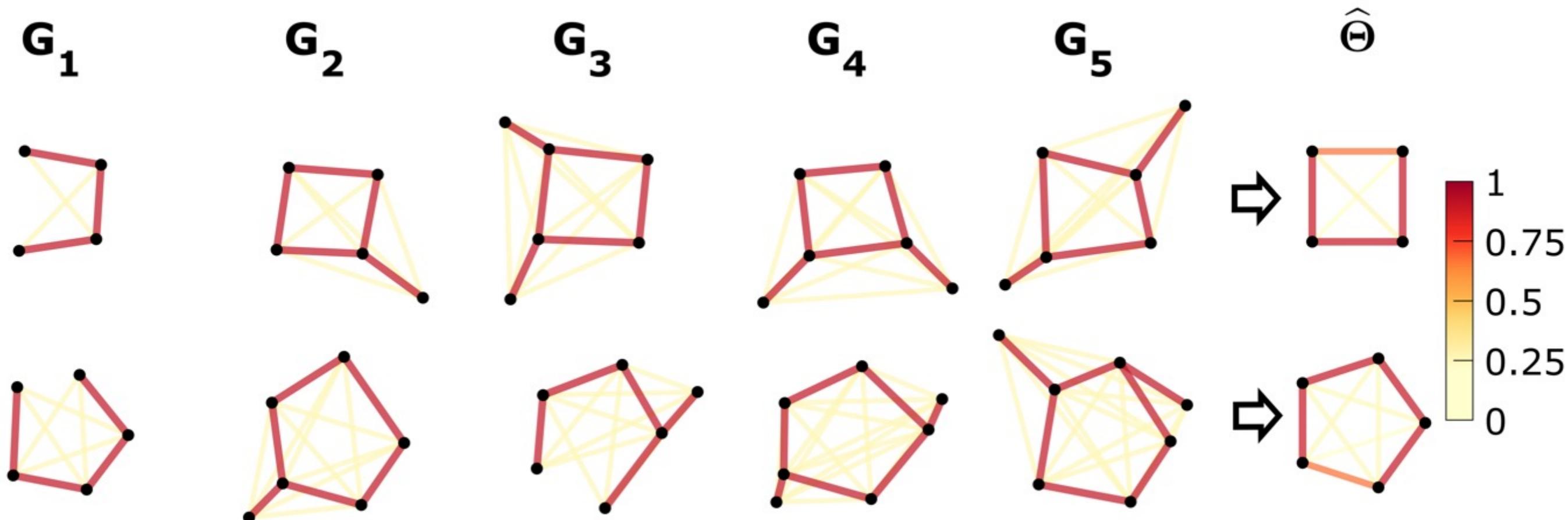
$$\mathcal{L}_0(\Theta, P) = \min_{\tau} \sum_{b \in E_0} [b - \tau(b)]^r$$

runtime $\mathcal{O}(q \log q)$

Proof in
Chung et al. 2024,
Foundations of Data Science

Nonlinear registration of topological objects

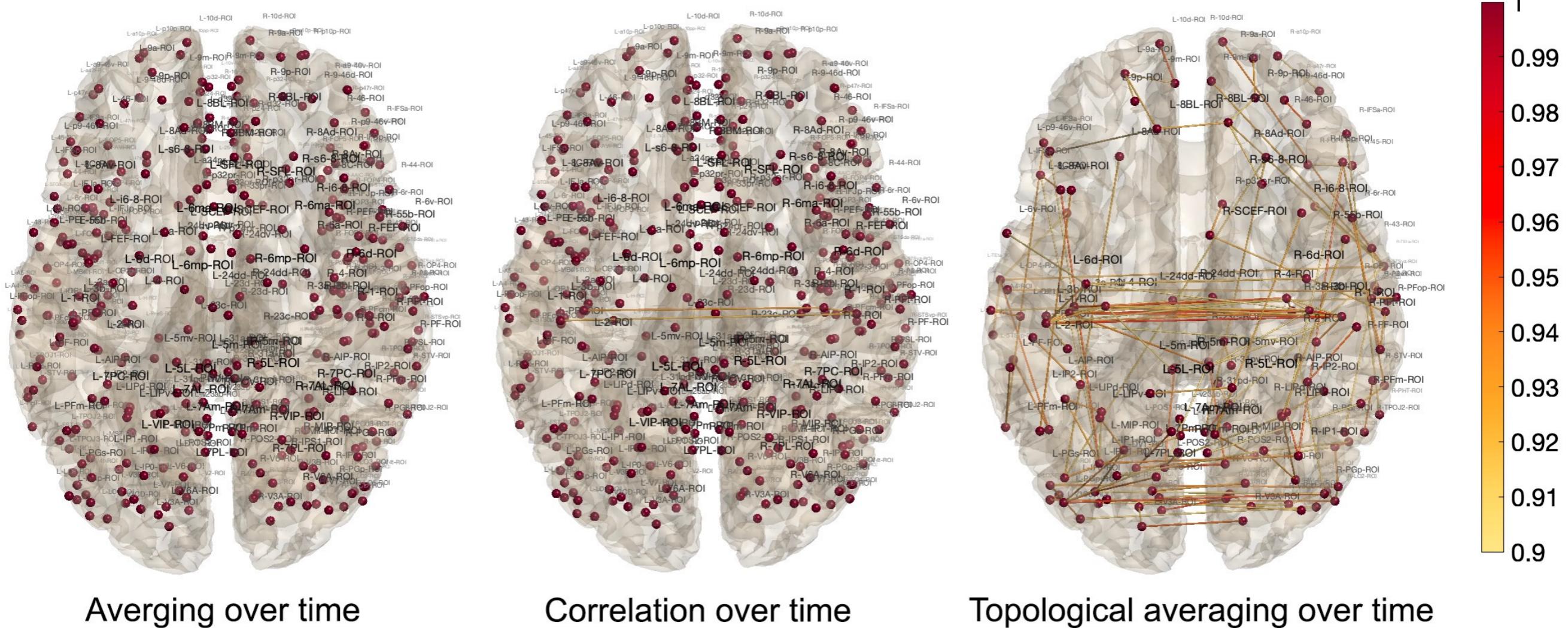
Topological mean
= minimizer of Wasserstein distance



Average sorted edge weights over MST and none-MST
separately and reconstruct \rightarrow minimizer of topological distance

Unequivocal demonstration

Superiority of topological averaging in rs-fMRI connectivity



Topological clustering = topological inference

Within-group distance

$$l_W \propto \sum_k \sum_{i,j \in C_k} \mathcal{L}(\mathcal{X}_i, \mathcal{X}_j)$$

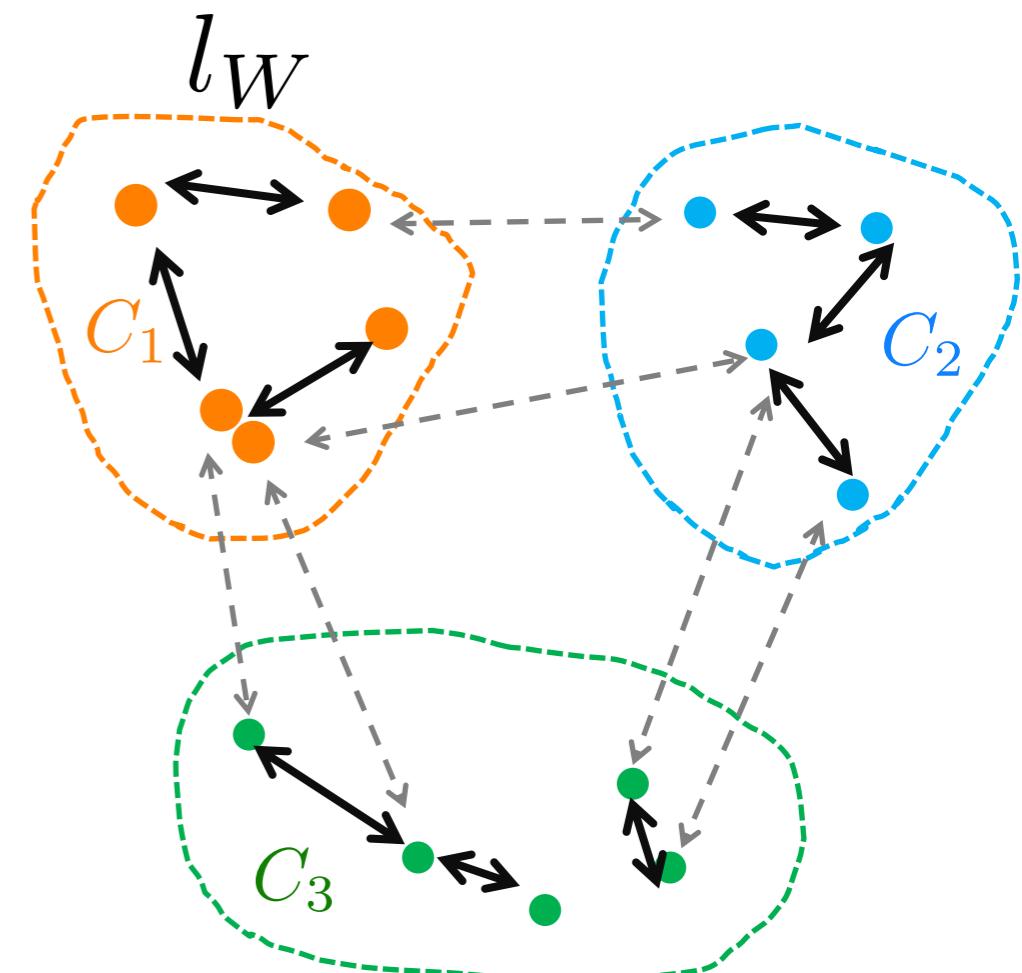
0D and 1D combined distances

Clustering accuracy

$$\frac{dA}{dp} = \frac{dA}{d\phi} \cdot \frac{d\phi}{dp} \leq 0$$

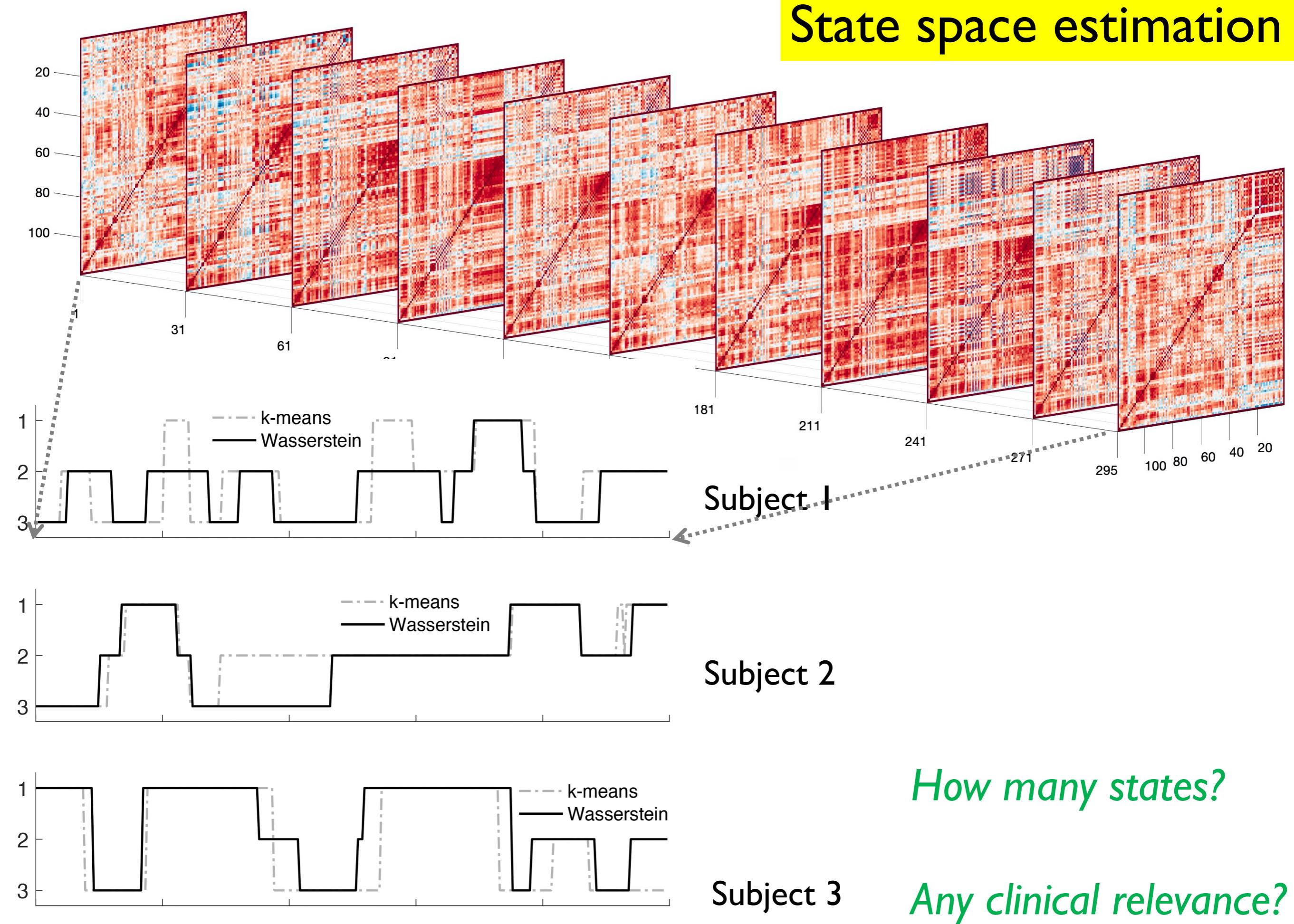
p-value

There exists a monotone
function f satisfying $p = f(A)$

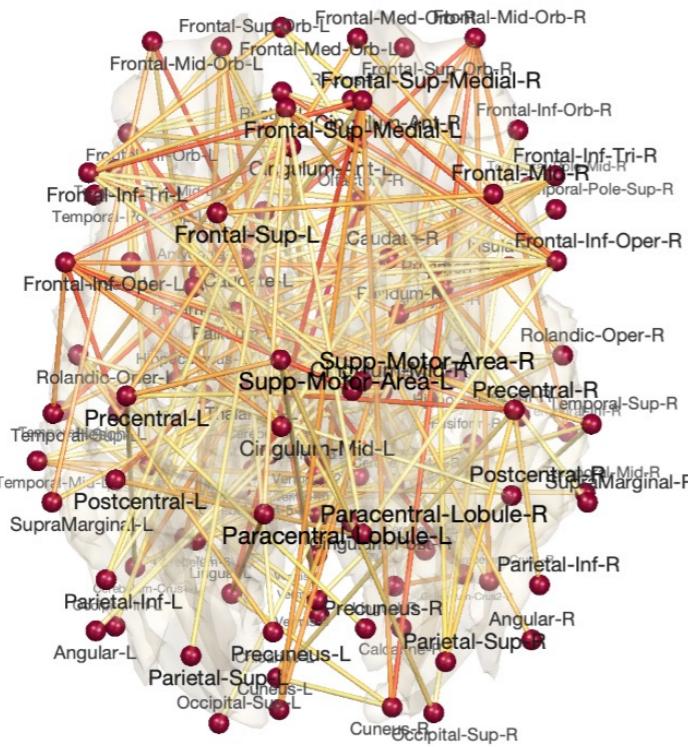


Proof in Chung et al. 2023 NeuroImage

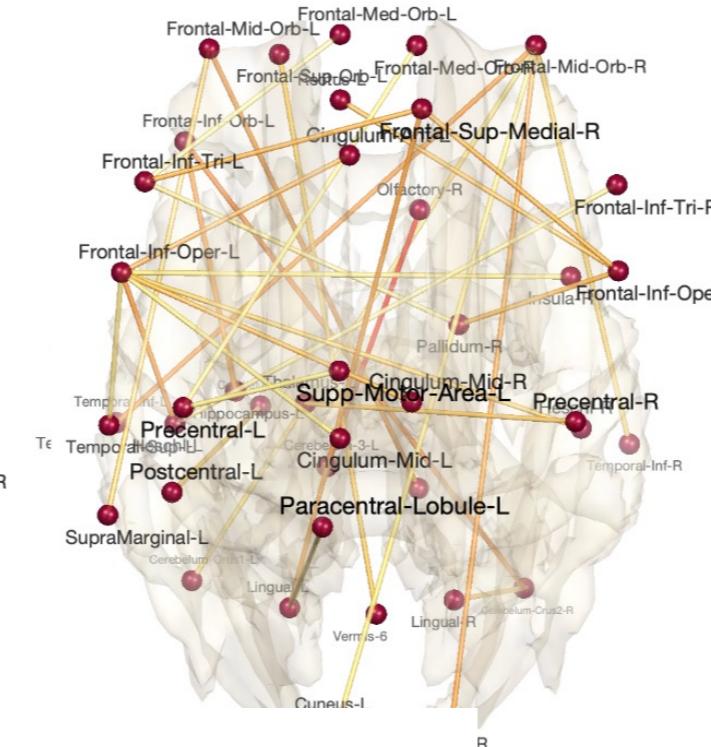
State space estimation



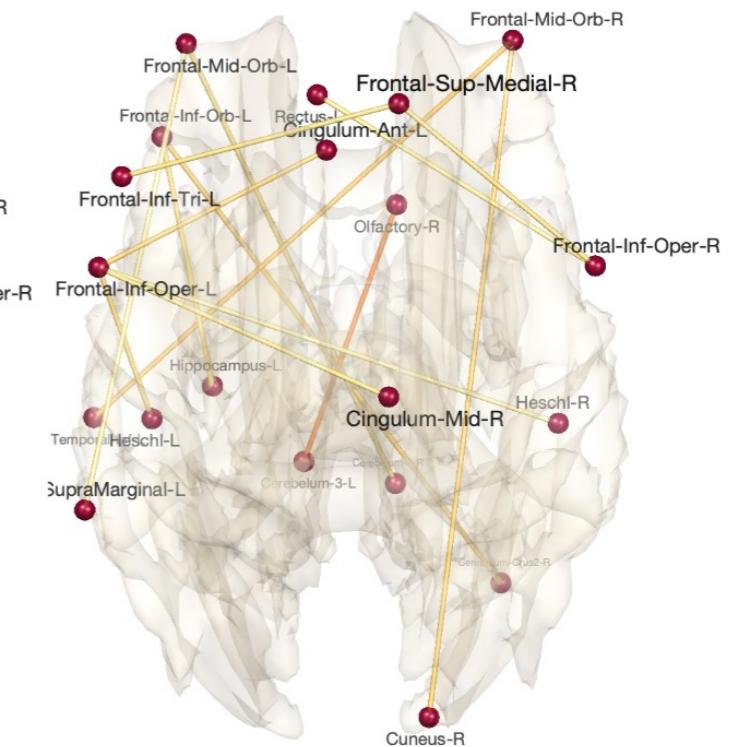
State 1



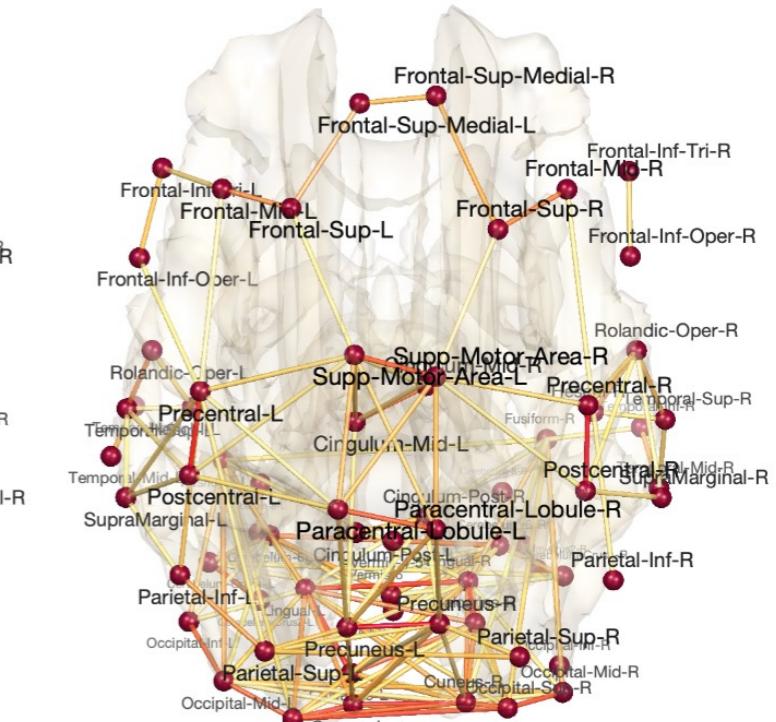
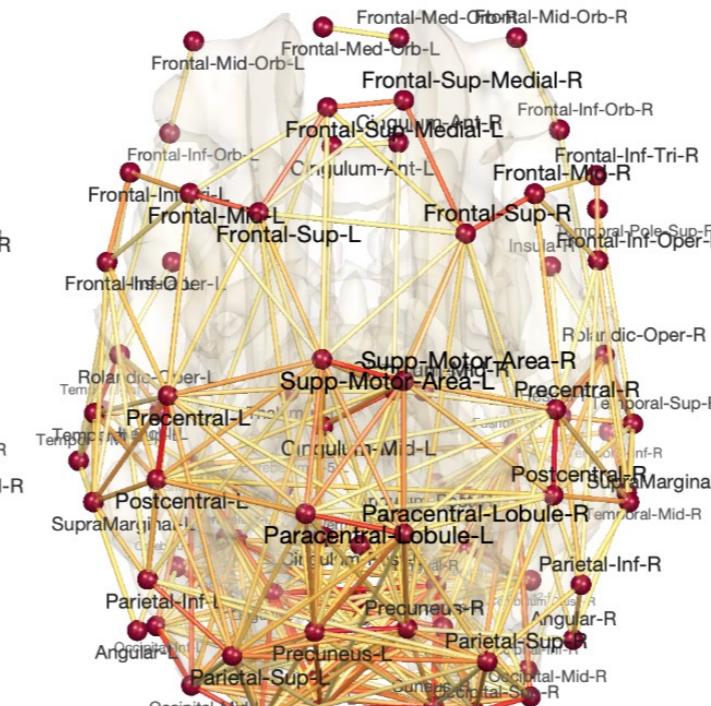
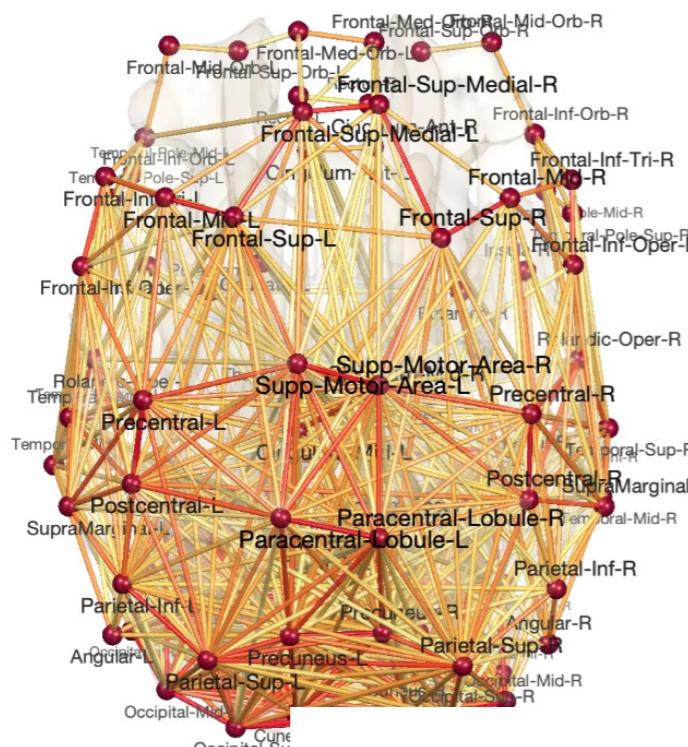
State 2



State 3



k-means *Sample mean in each state*



Topological clustering

Topological mean in each state

Topological
embedding

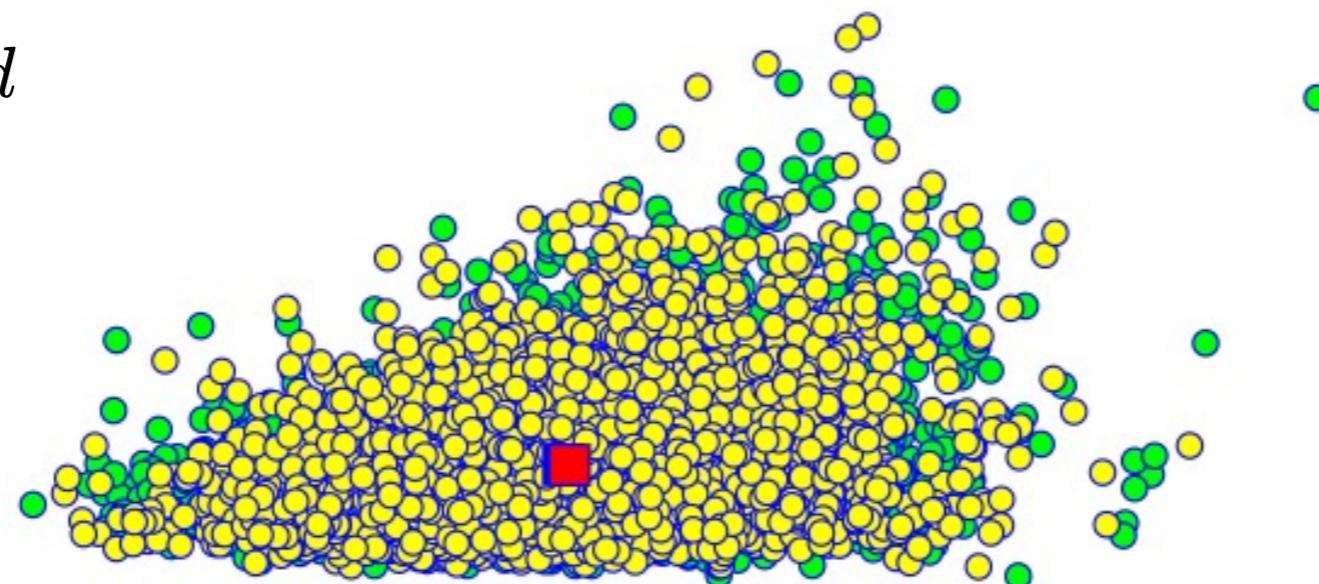
Topological Embedding

- Male
- Female

Deaths

average of
death values

$$\frac{1}{|E_1|} \sum_{d \in E_1} d$$



average of birth values

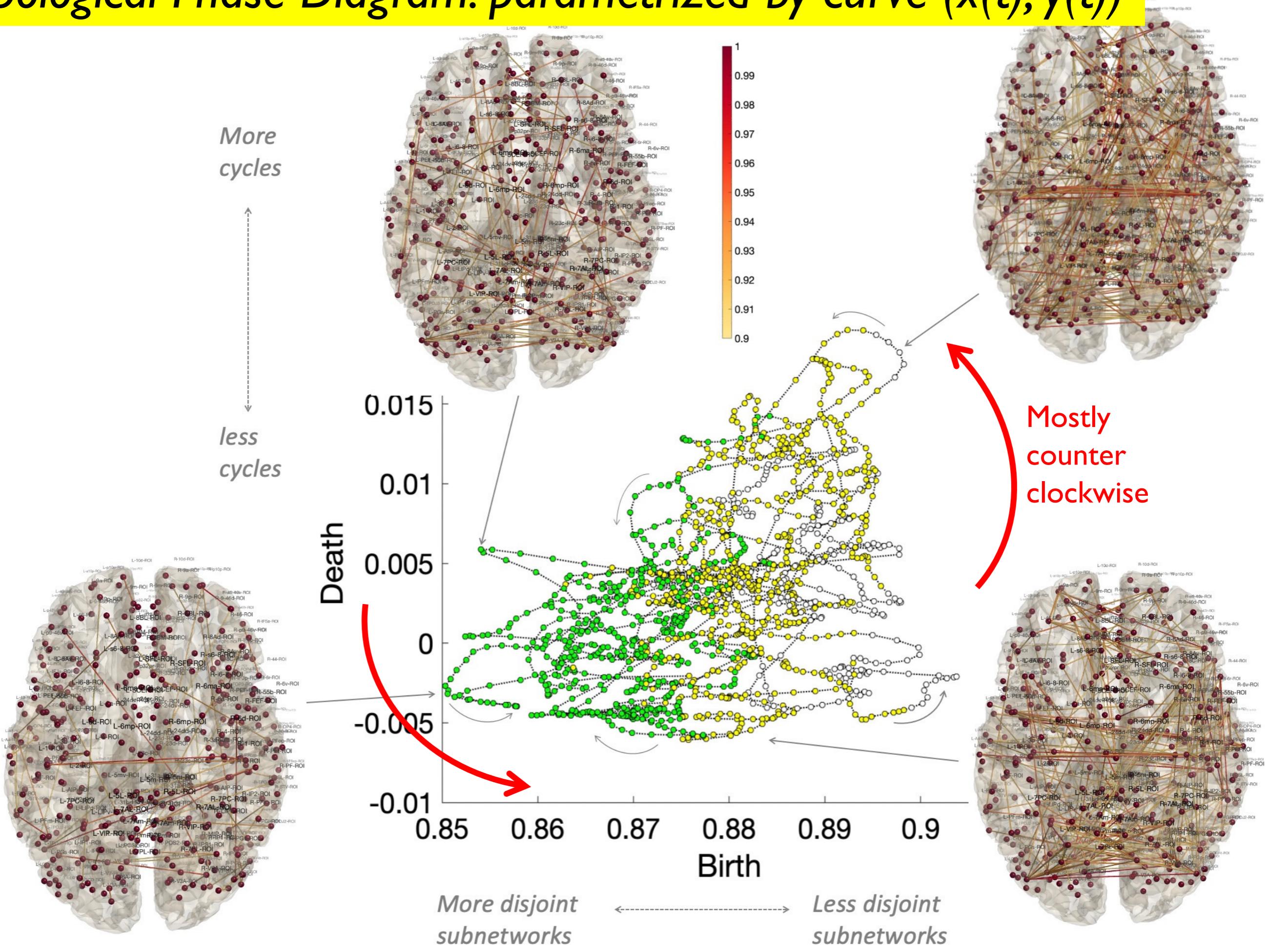
$$\frac{1}{|E_0|} \sum_{b \in E_0} b$$

Births



Male and female
differences are
mostly geometric

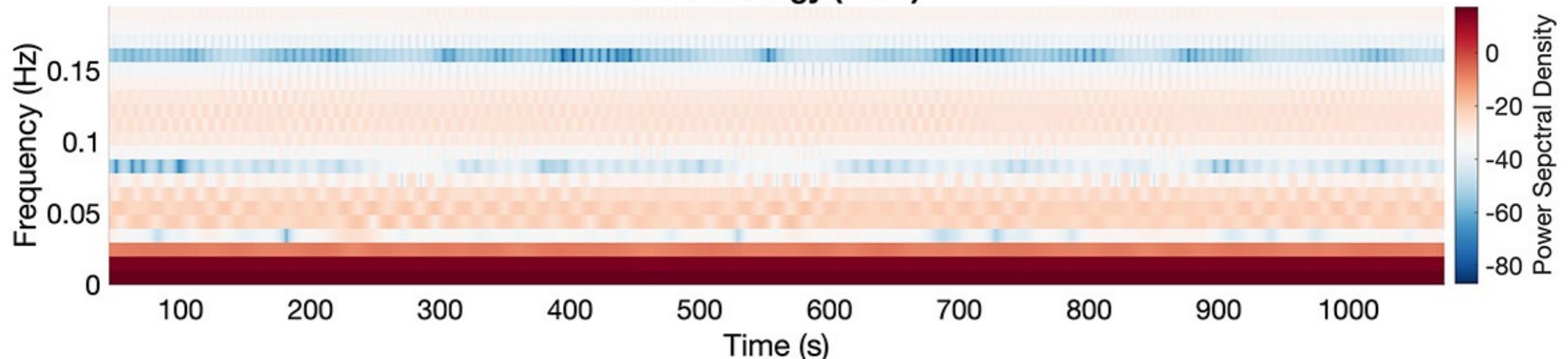
Topological Phase Diagram: parametrized by curve $(x(t), y(t))$



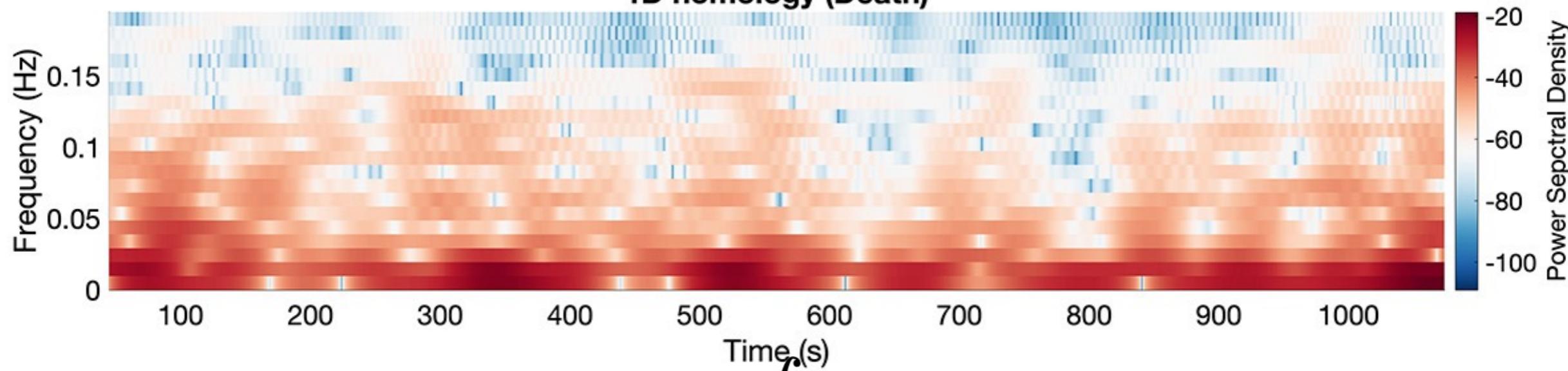
Topological Time frequency analysis: Topological spectrogram

$$X(\tau, \omega) = \int x(t)w(t - \tau)e^{-j\omega t}dt$$

0D homology (Birth)



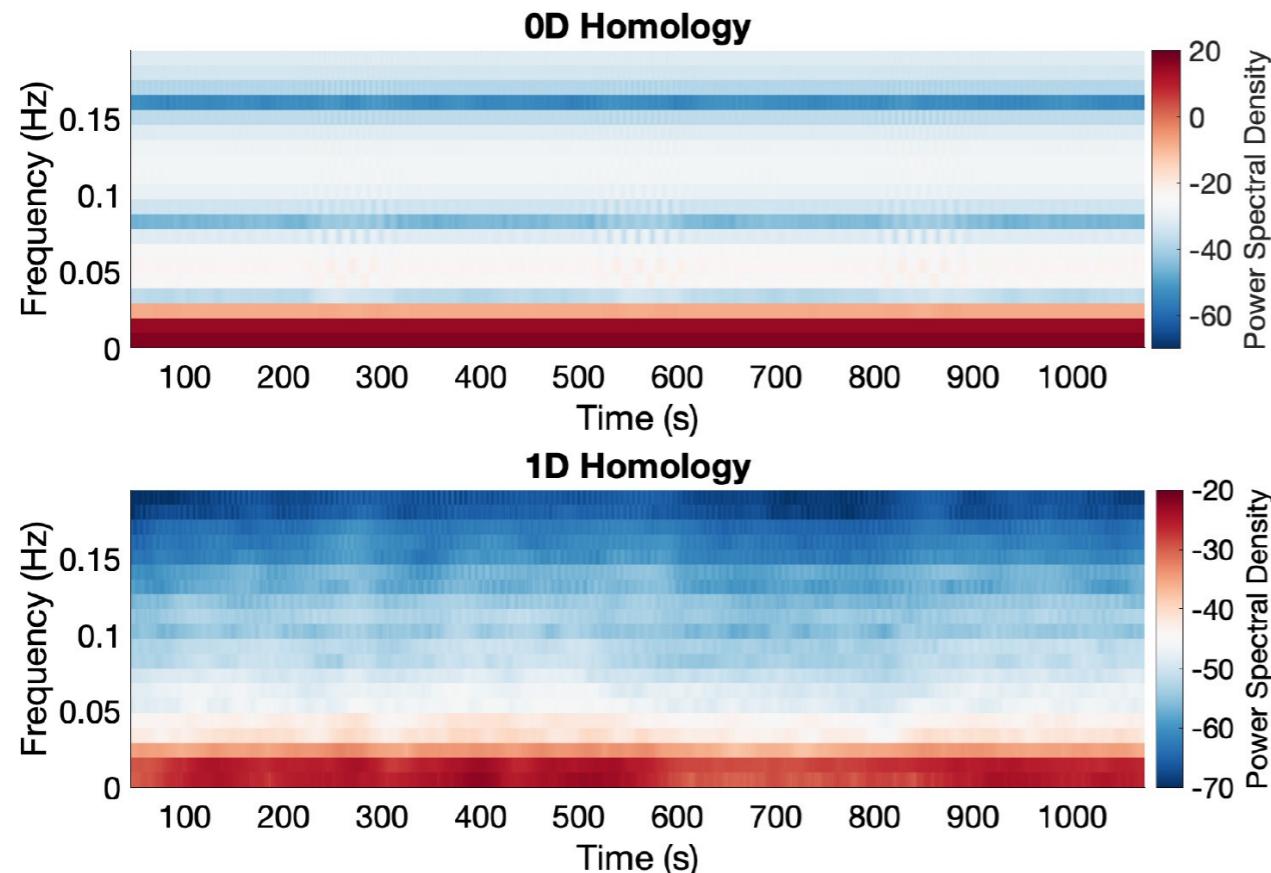
1D homology (Death)



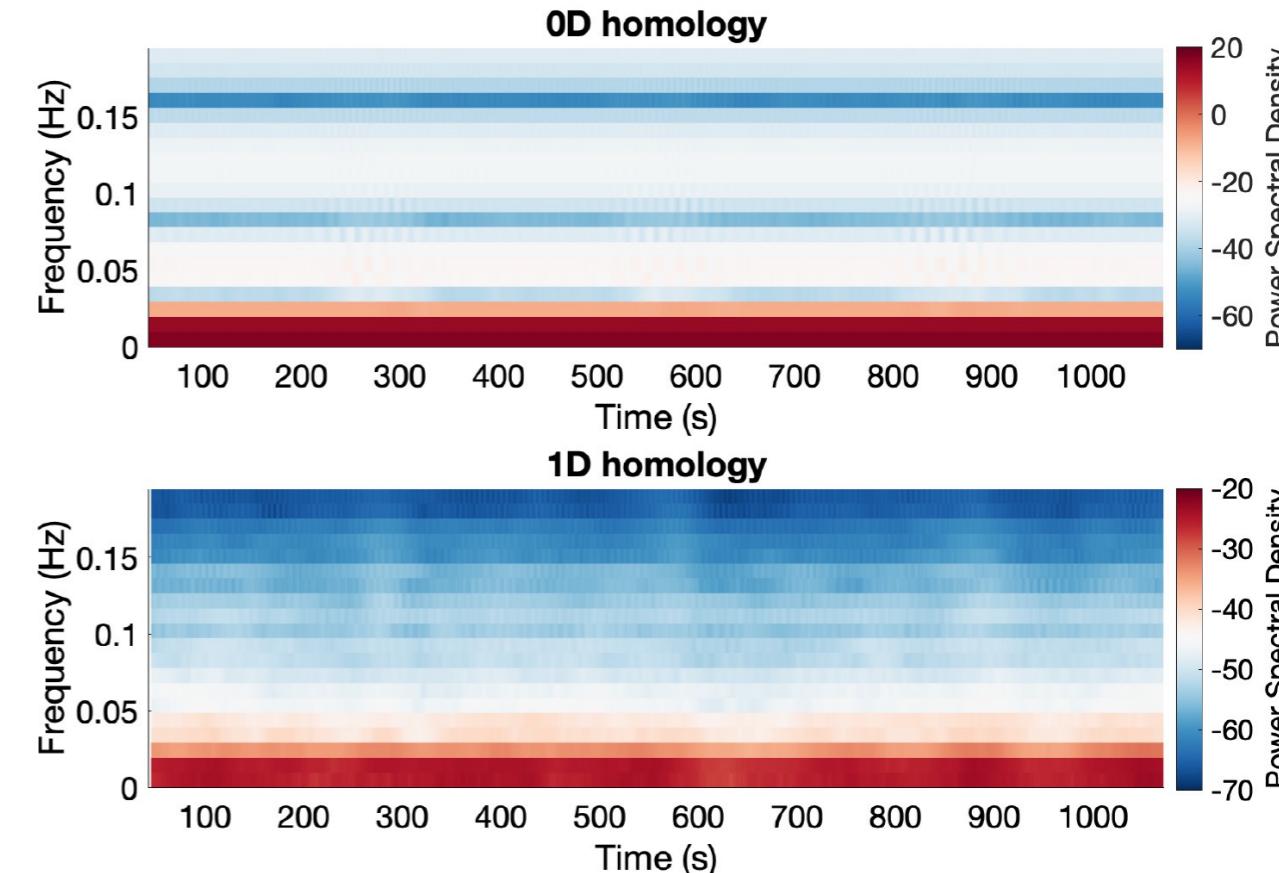
$$Y(\tau, \omega) = \int y(t)w(t - \tau)e^{-j\omega t}dt$$

Window size = 16 or 40 seconds

Mean power spectral density: HC



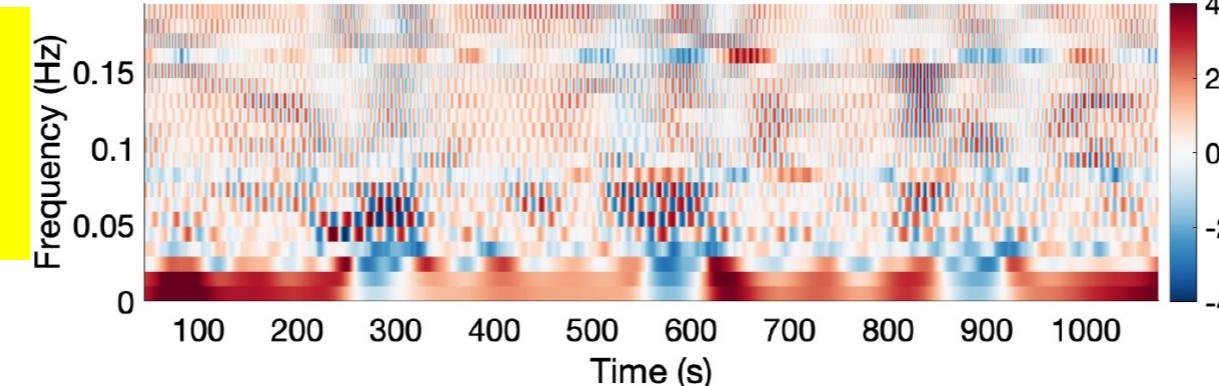
Mean power spectral density: TLE



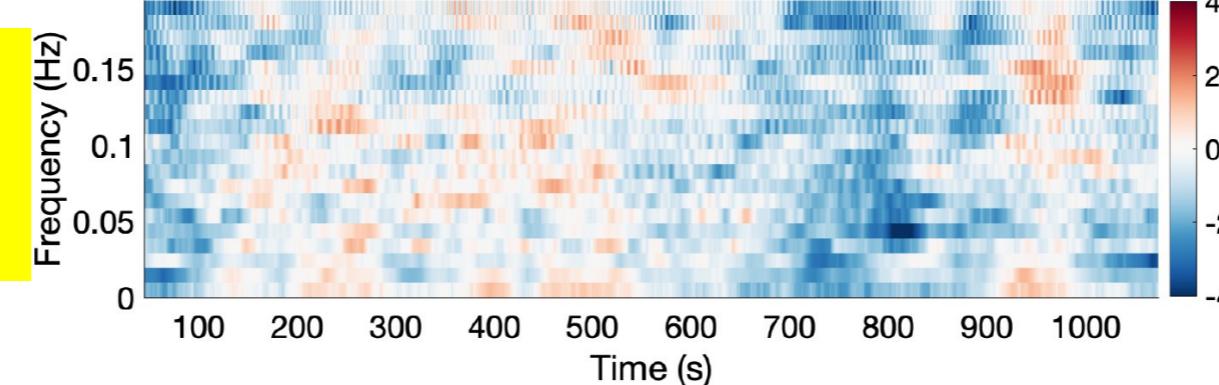
**HC: 0D topology
fluctuates faster.**

**TLE: 1D topology
fluctuates faster.**

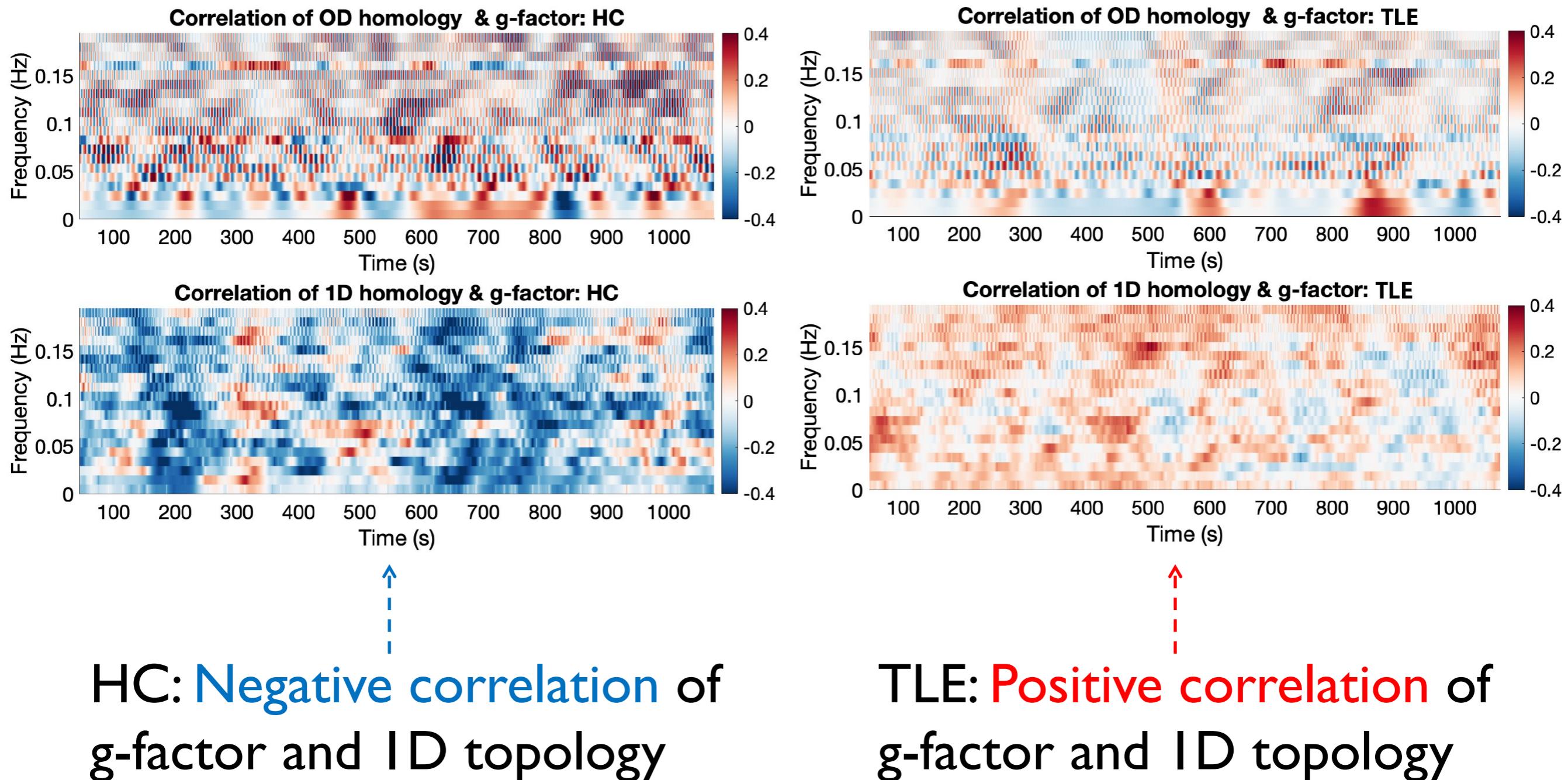
t-stat for 0D homology (HC-TLE)



t-stat for 1D homology (HC-TLE)

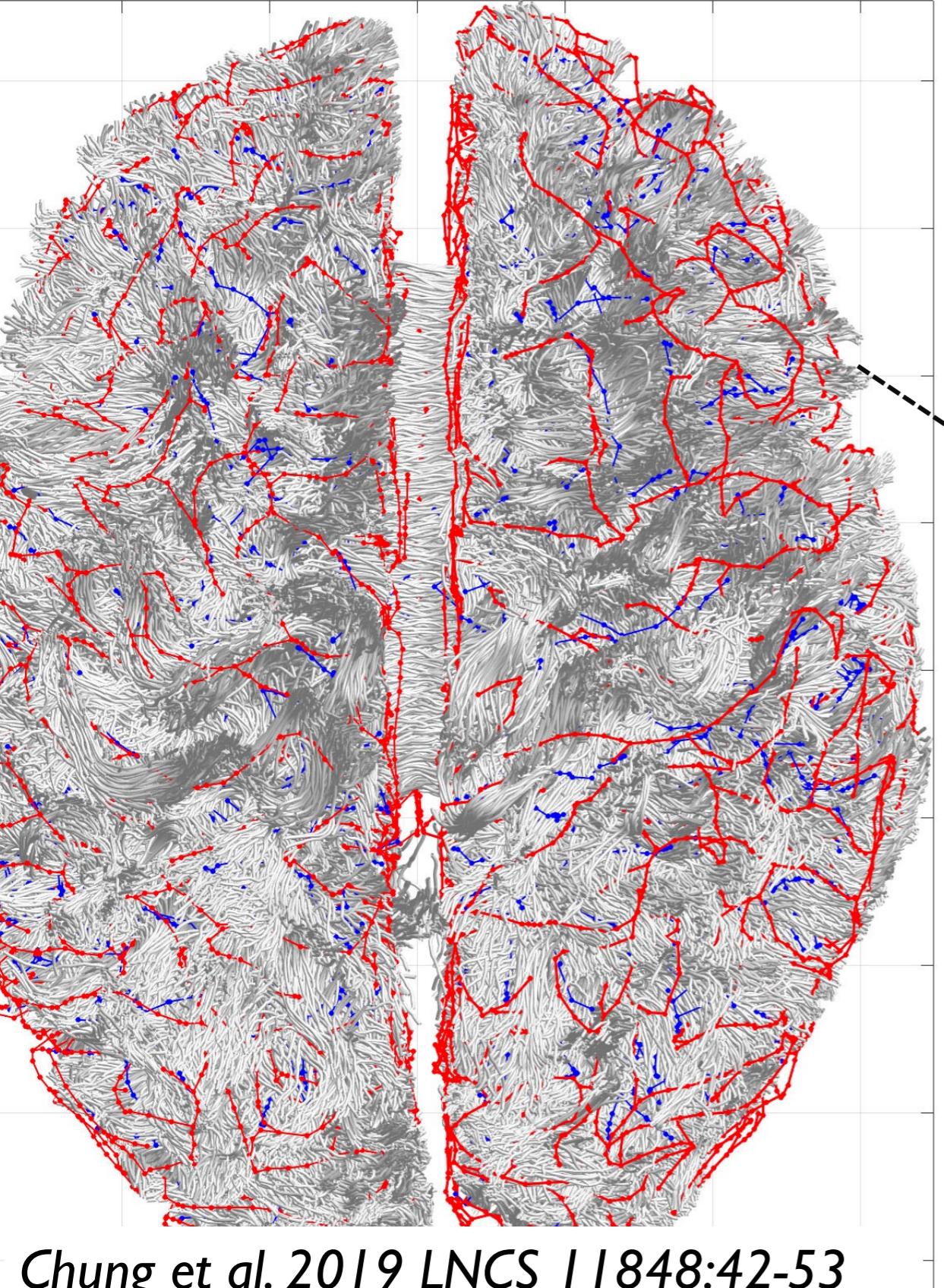


Smarter brain is a slow brain

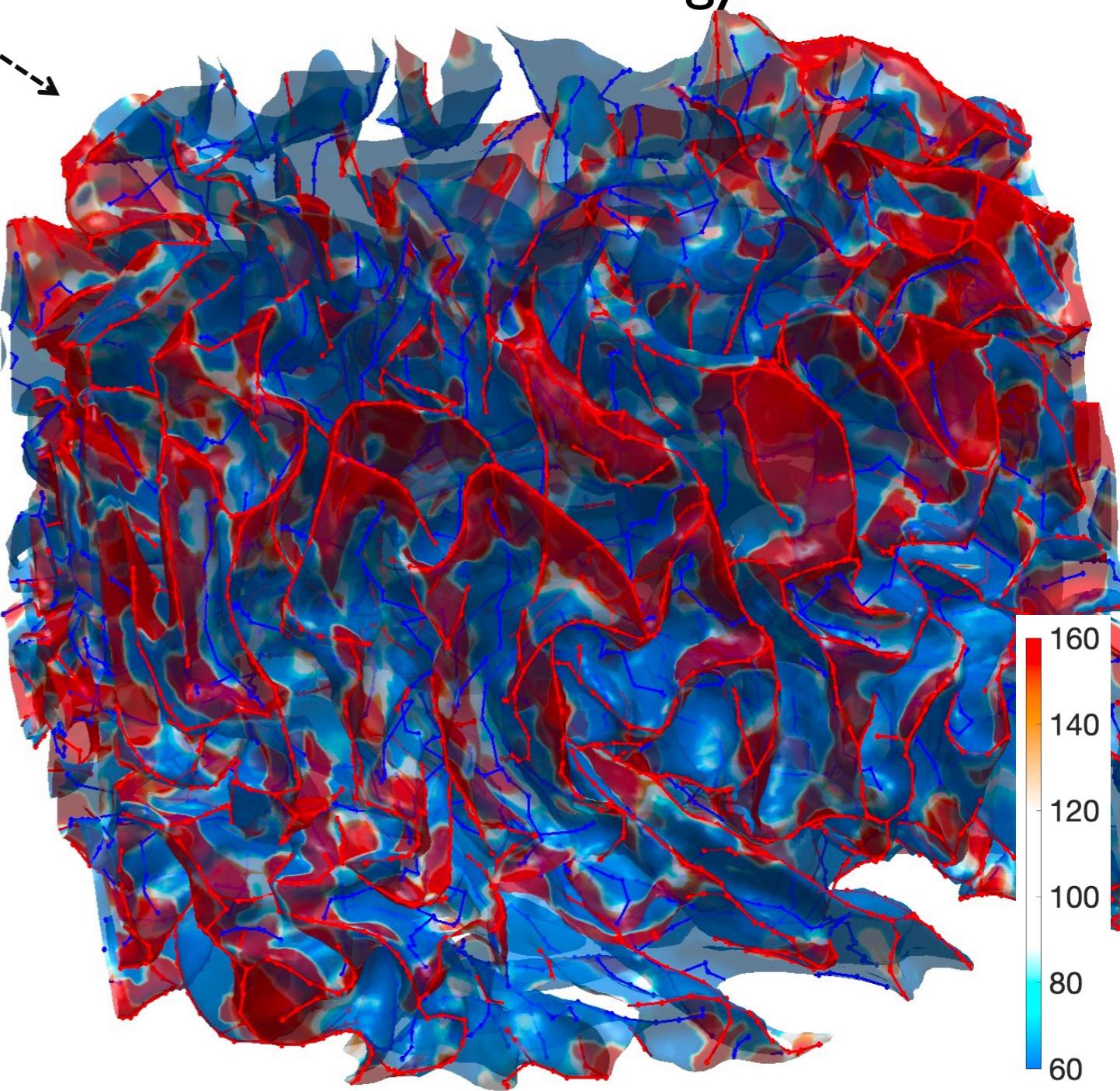


Connectivity gradient
from Gyri to Sulci

diffusionMRI → 1 million white matter tracts per subject



Tract count within 2mm radius
around nodes on sulcal & gyral trees



Chung et al. 2019 LNCS 11848:42-53

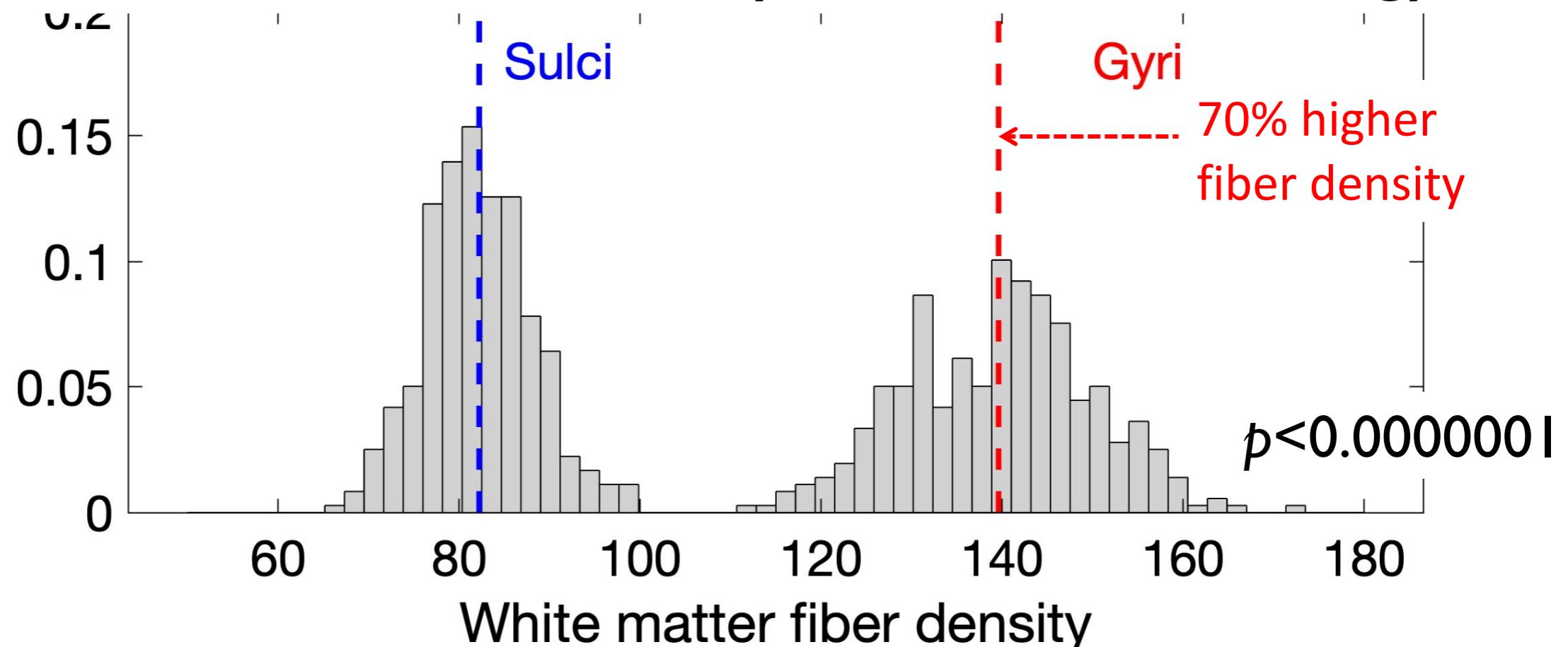
60

Huang et al. 2020 IEEE

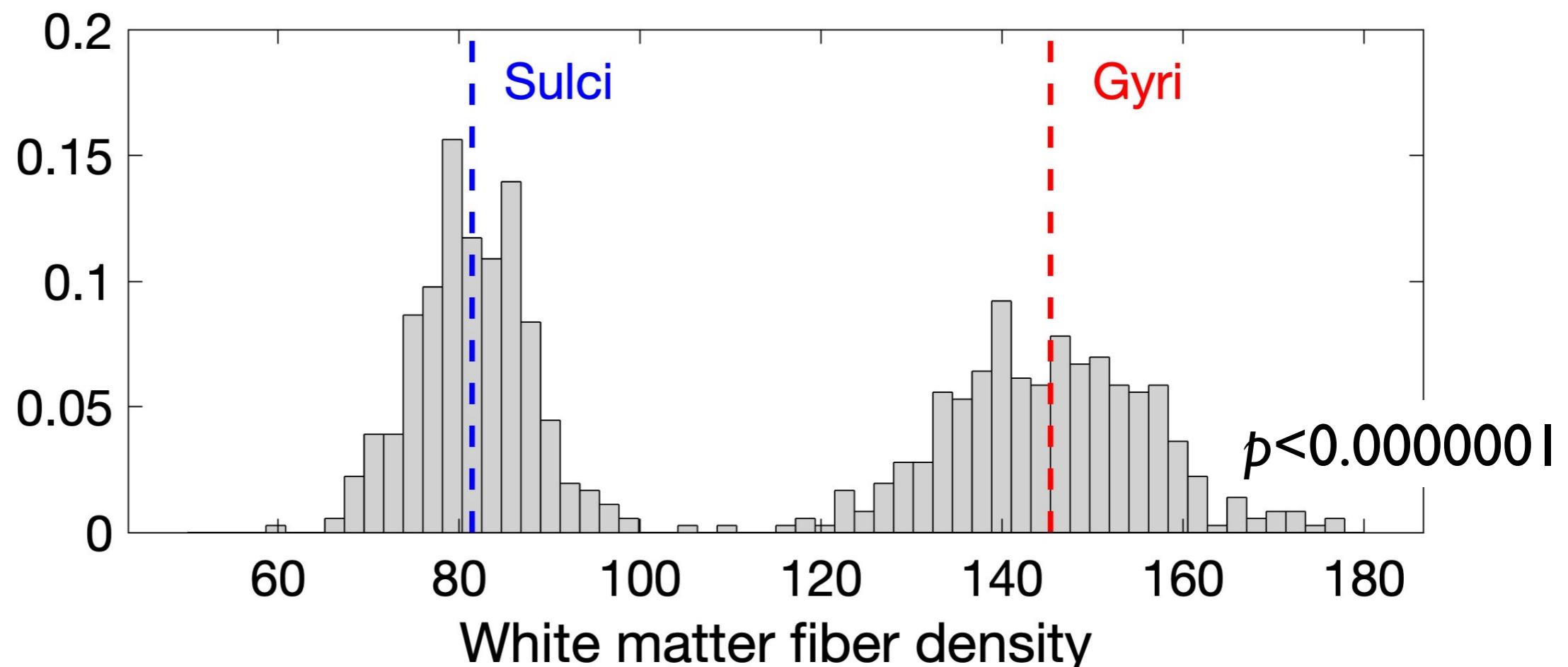
Transactions on Medical Imaging

Differential functional connectivity between sulci and gyri

Left



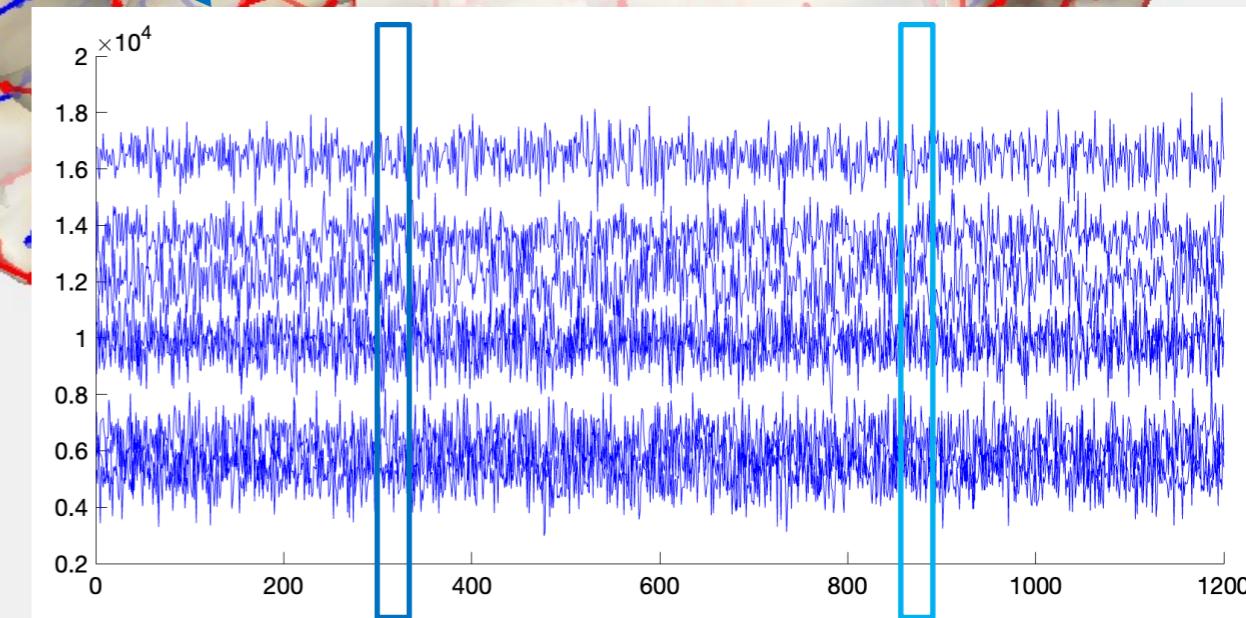
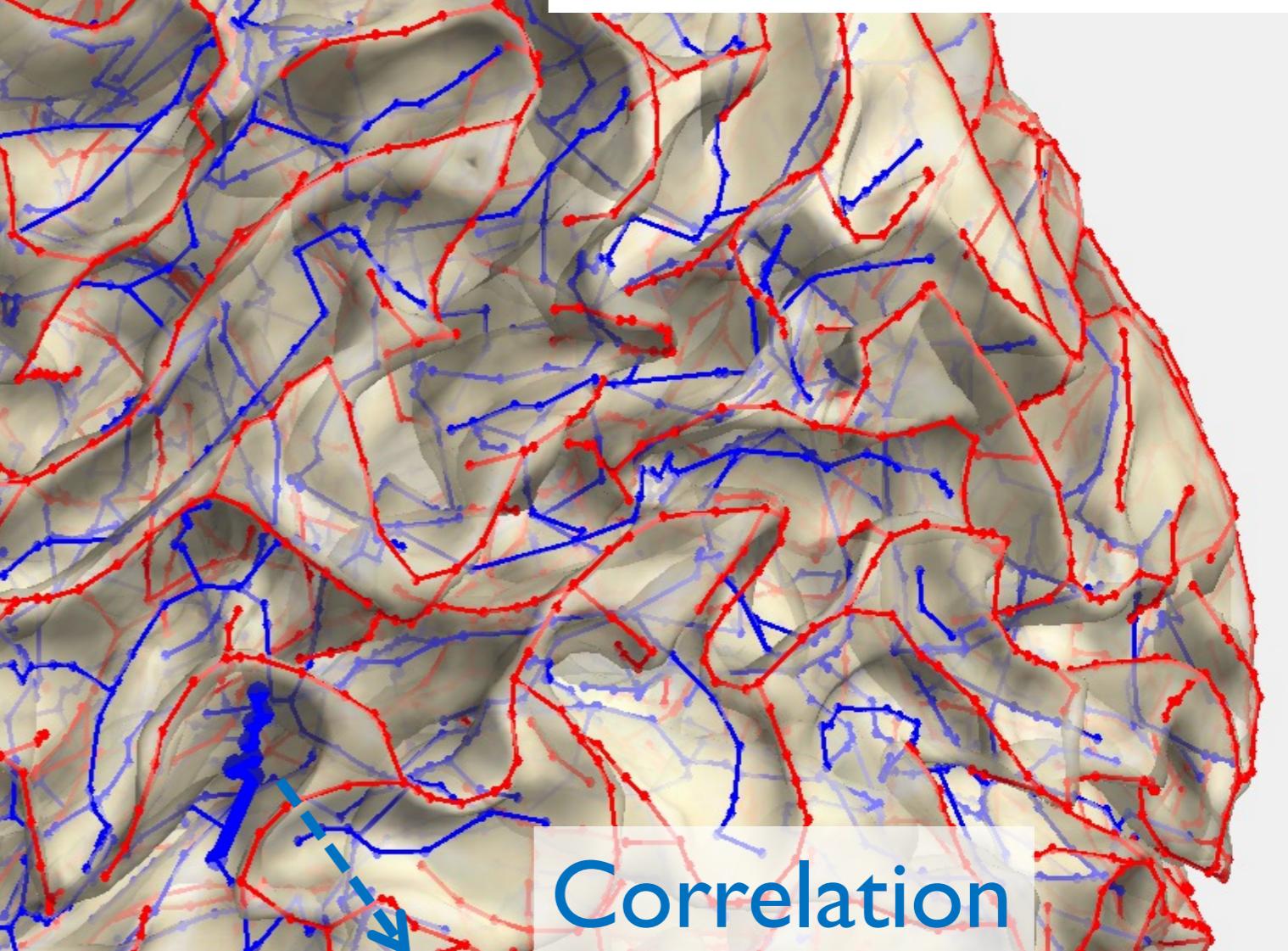
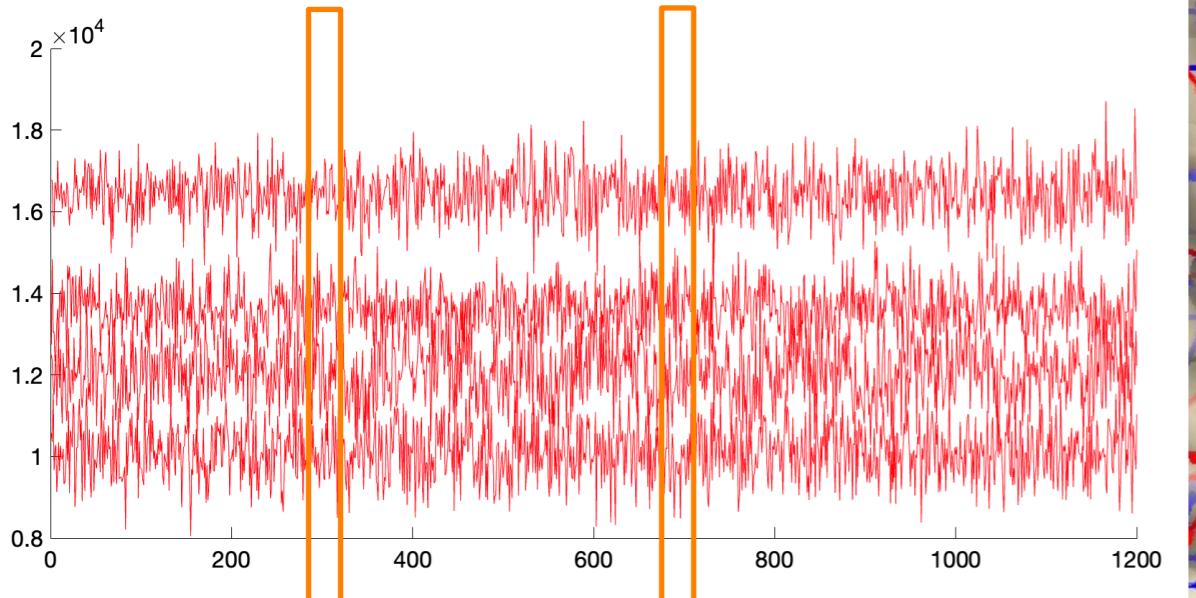
Right



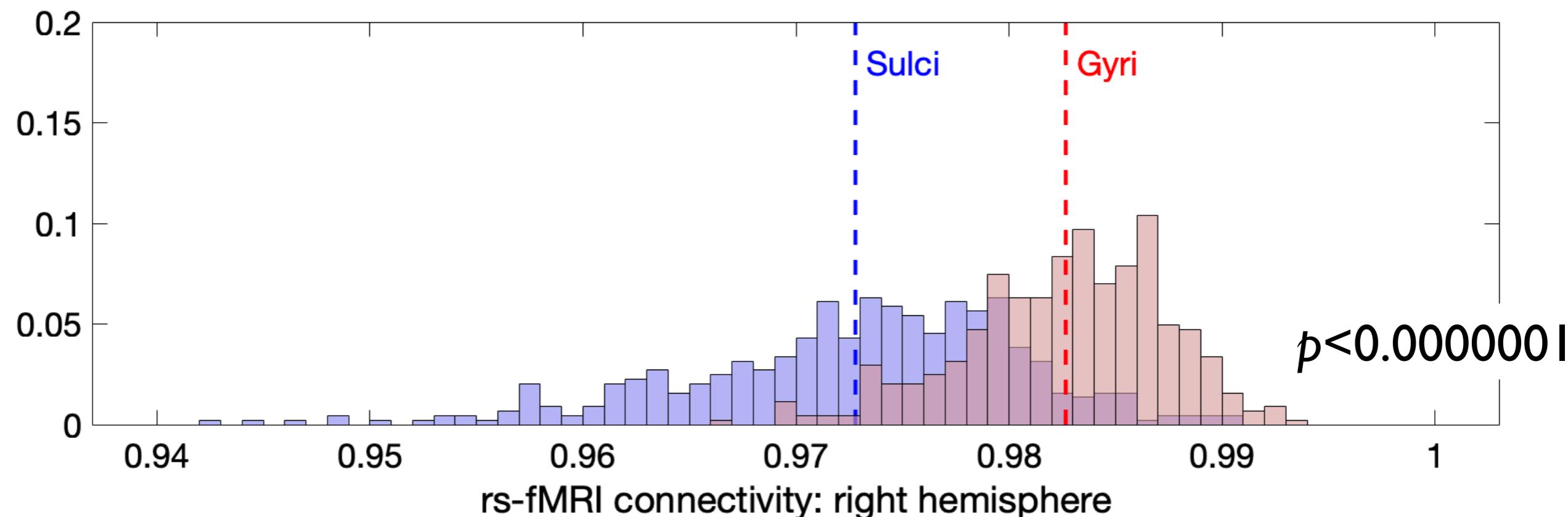
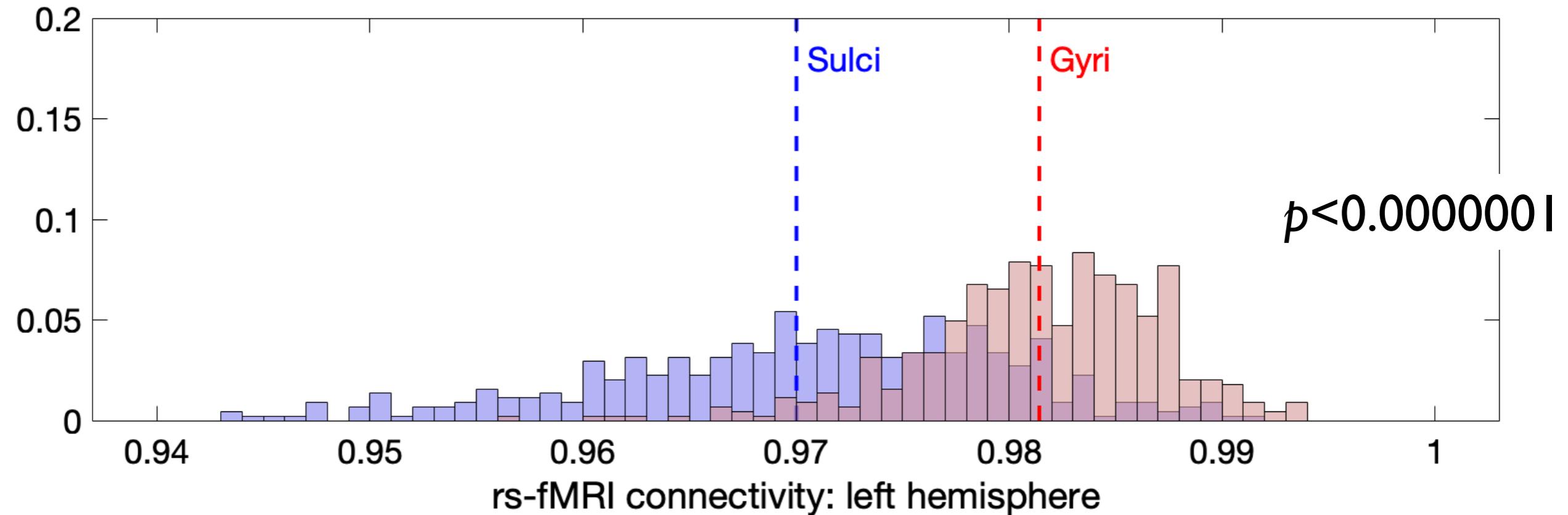
358 subjects

~ 200000 rs-fMRI
time series

Correlation



Differential functional connectivity between sulci and gyri



443 subjects

Thank you.



We are located here. Join us for
postdoc and **graduate research**
if you are better than *chat-GPT*

