Topological State-Space Estimation of Dynamically Changing Functional Brain Networks

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Abstract

A new data-driven topological approach is presented for estimating state spaces in dynamically changing functional brain networks of humans. The approach penalizes the topological distance between networks and clusters, dynamically changing brain networks into topologically distinct states. The method considers the temporal dimension of the data through the Wasserstein distance between networks. The method is shown to outperform the widely used k-means clustering often used in estimating the state space in brain networks. The method is applied to accurately determine the state spaces of dynamically changing functional brain networks. Subsequently, the question of if the overall topology of brain networks is a heritable feature using the twin study design is addressed. The talk is based on arXiv:2201.00087(under review in *PLOS Computational Biology*)

Basics on Neuroimages

Magnetic resonance imaging (MRI)

Wisconsin Twin Brain Imaging Study done here

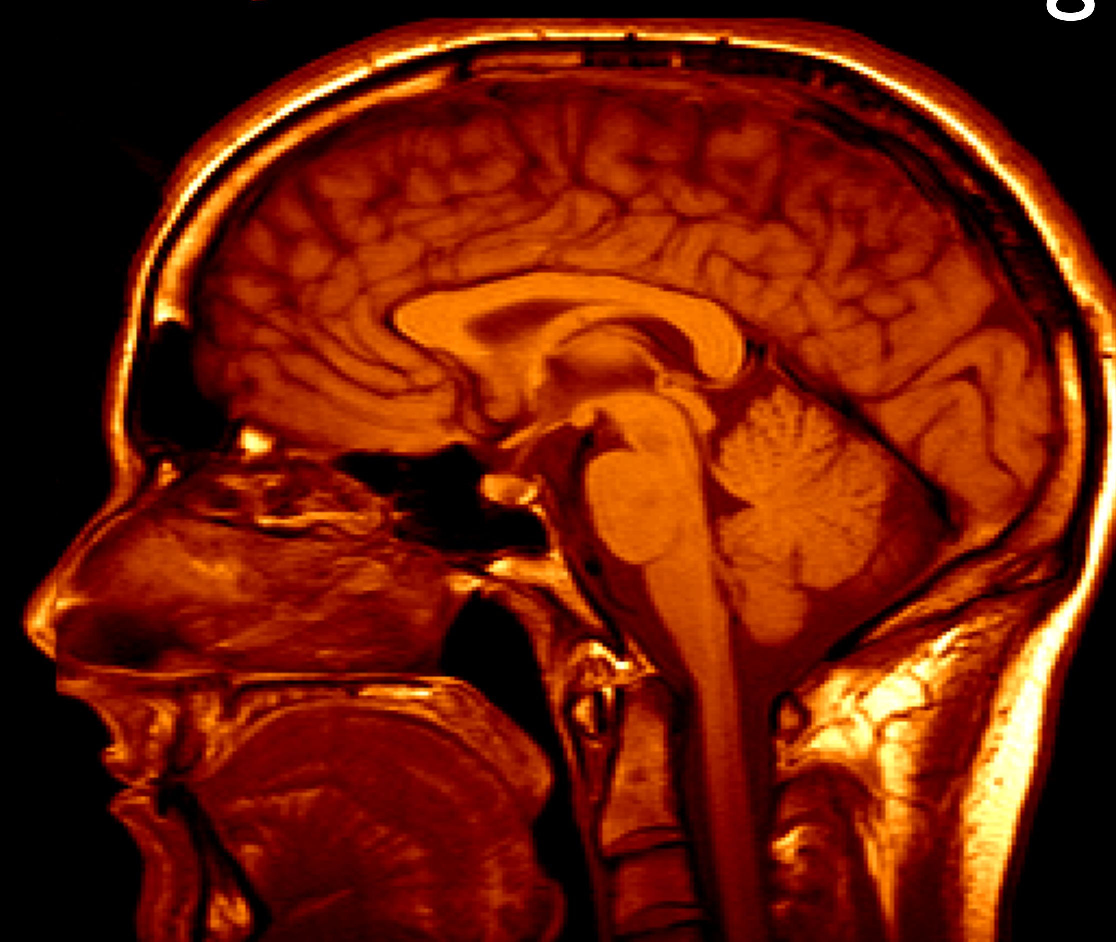
3T GE Discovery X750
Waisman Brain Imaging Laboratory
University of Wisconsin-Madison

Magnetic Resonance Imaging (MRI)

3.0 Tesla GE Scanner

Soft tissues

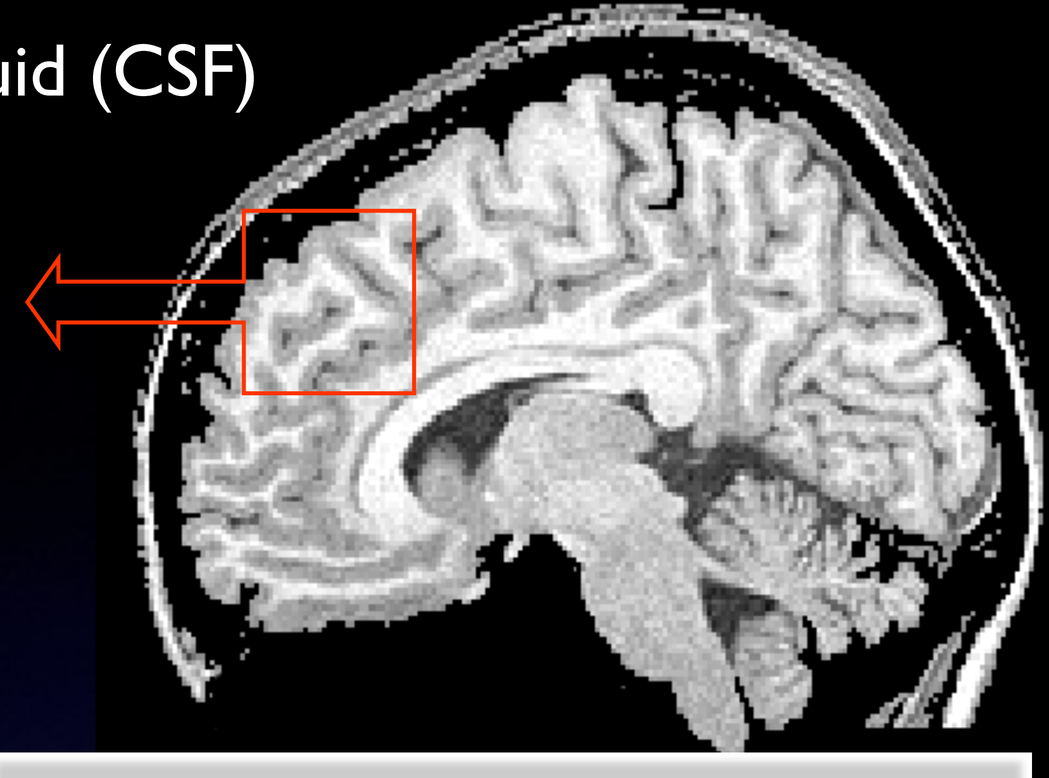
3D image

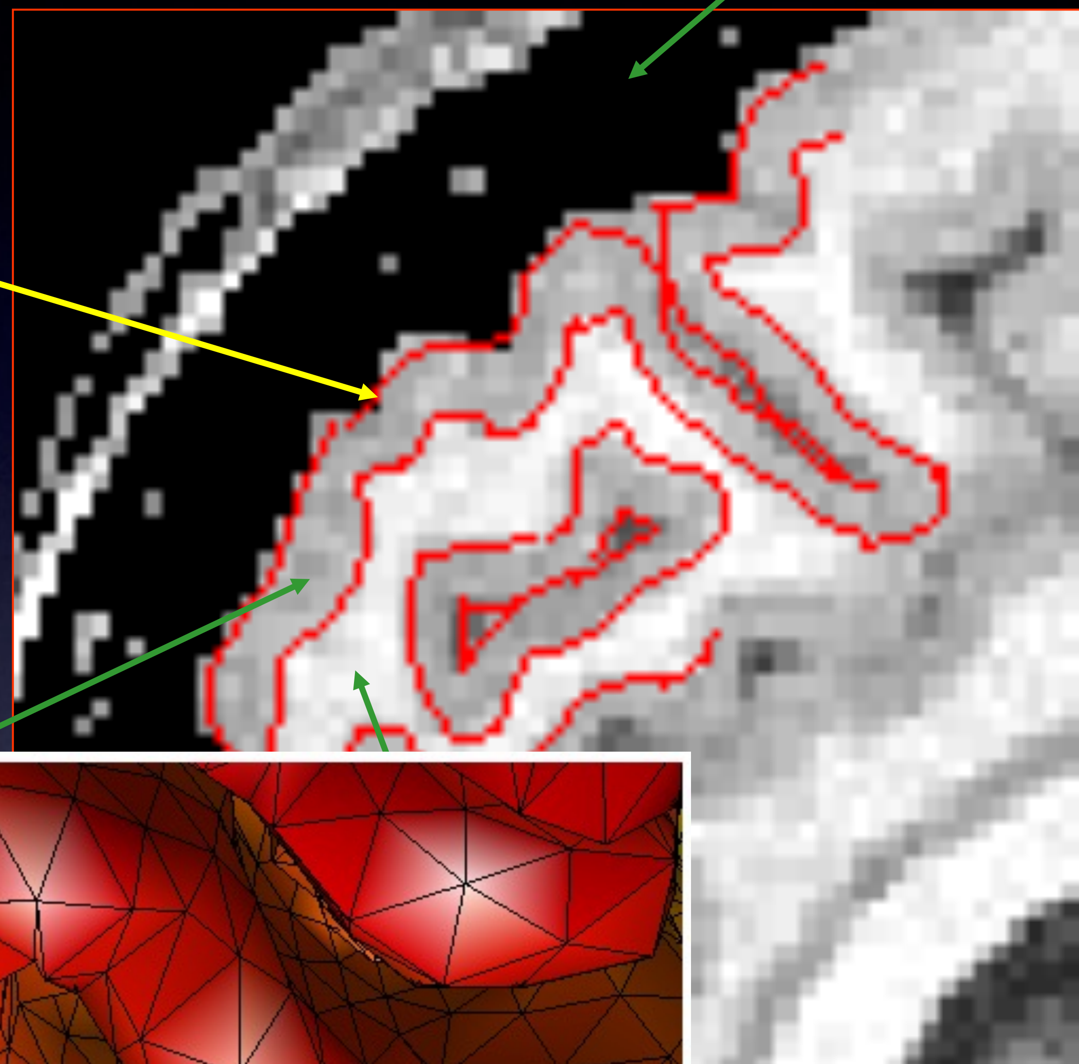


Soft tissues: brain

Basic brain anatomy

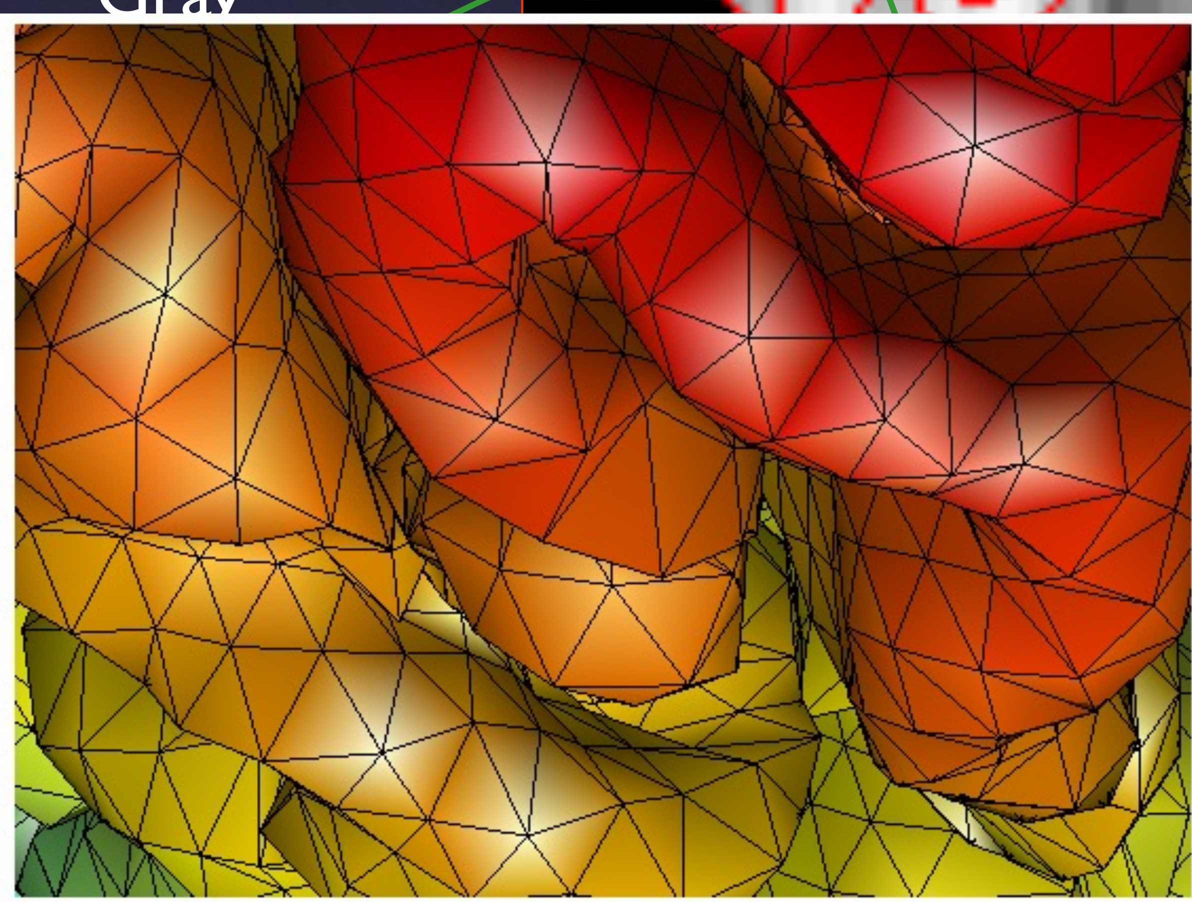
Cerebral Spinal Fluid (CSF)

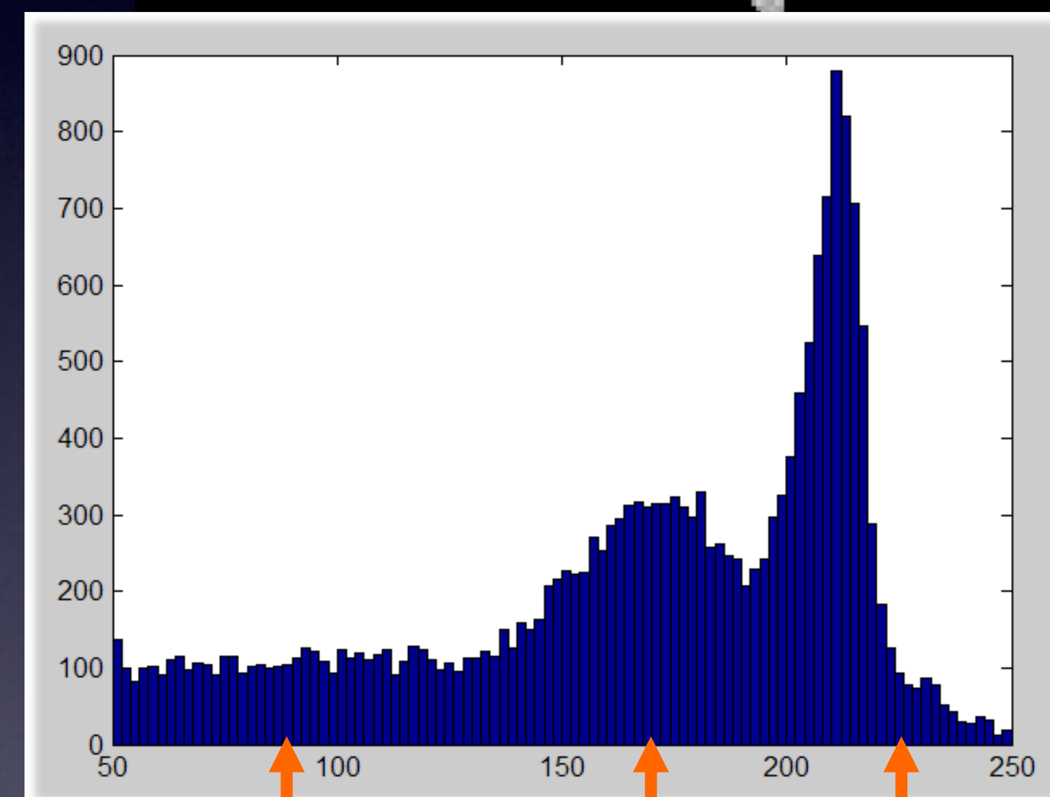




Outer Cortical Surface

Gray

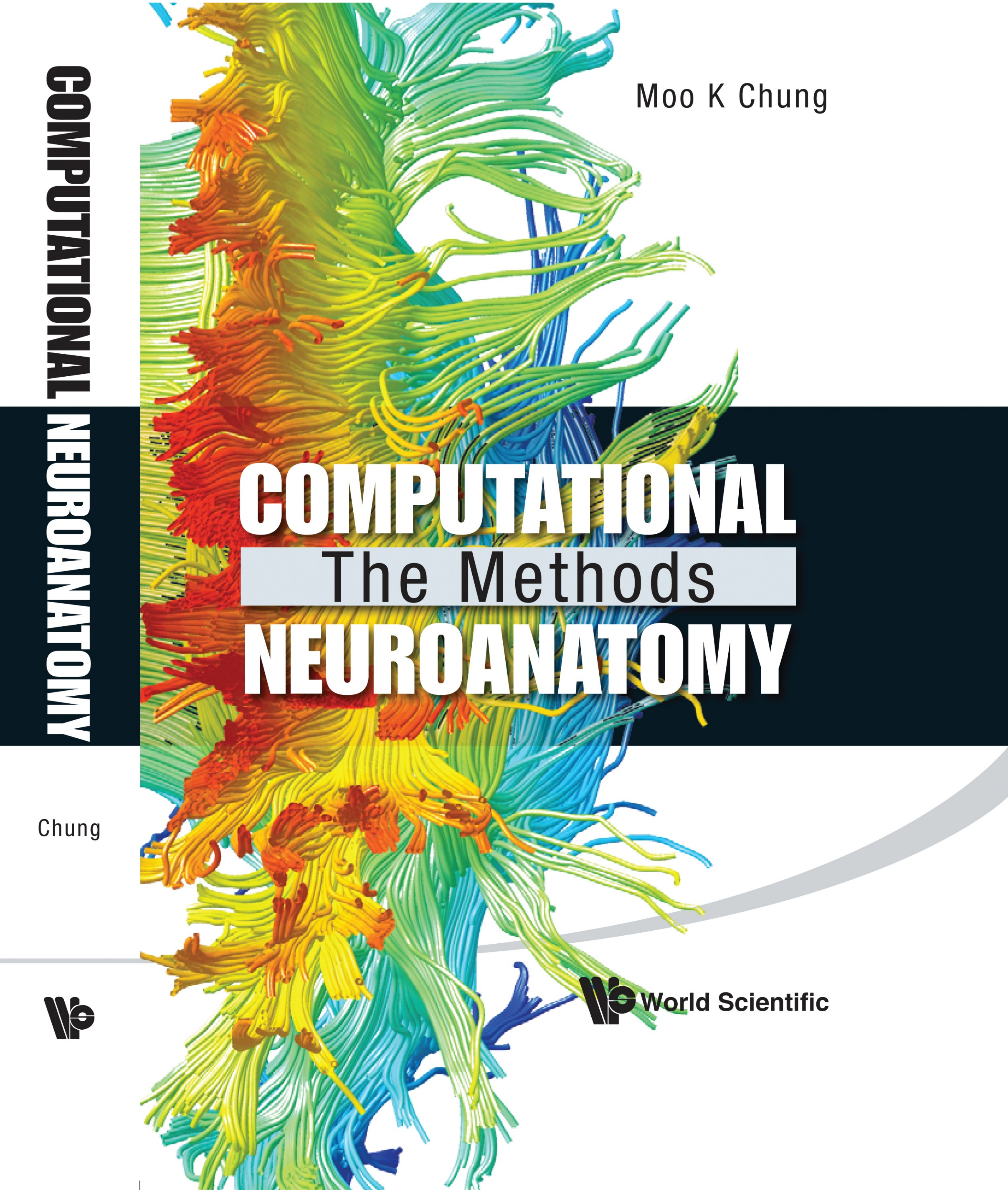




CSF Gray White

3 component Gaussian mixture

computational neuroanatomy



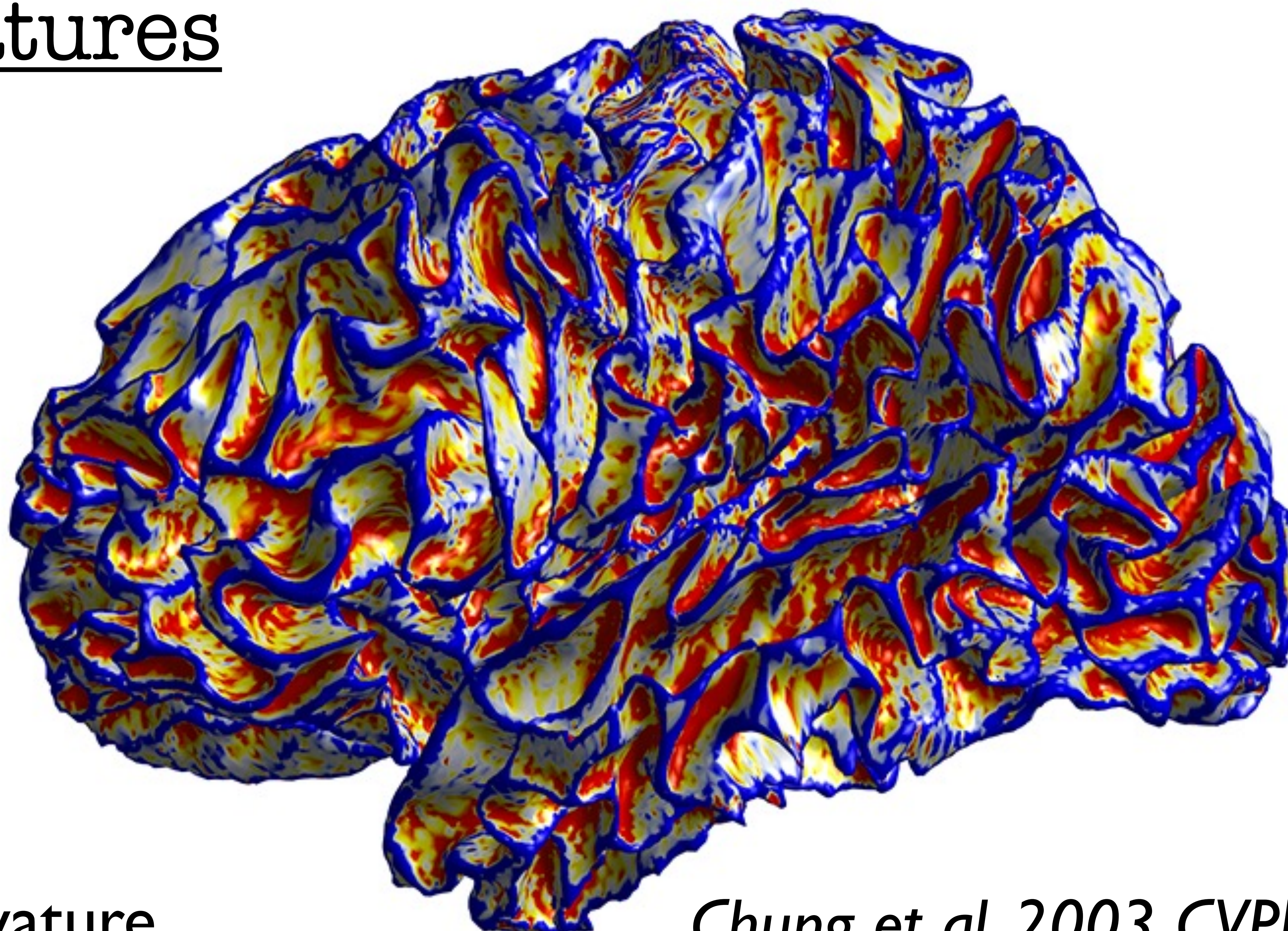
Mathematics
Computer Science
Statistics
Neuroimaging

↓

Quantification of spatiotemporal dynamics of anatomical shapes and objects biomedical images through differential geometry

Code and sample data
https://github.com/laplcebeltrami/curvatures

MATLAB



Mean curvature

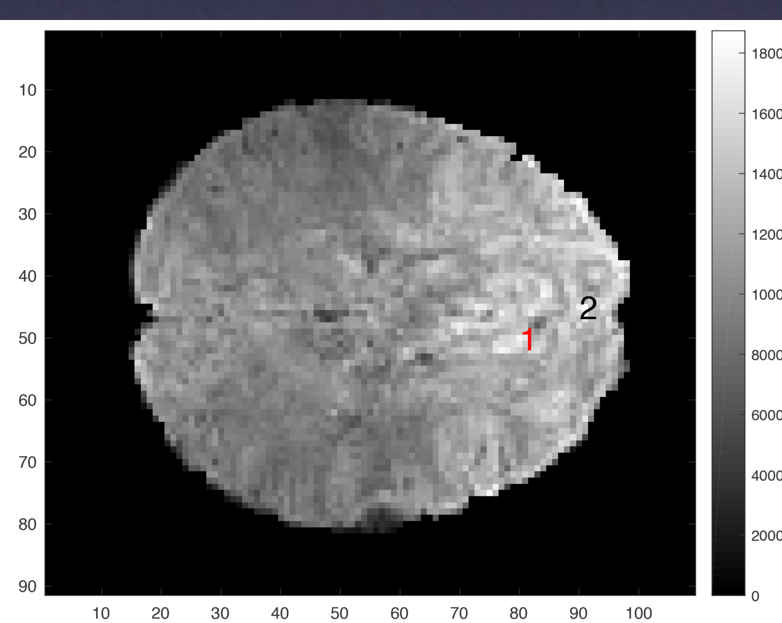
Chung et al. 2003 CVPR

Functional Magnetic Resonance Imaging (fMRI)

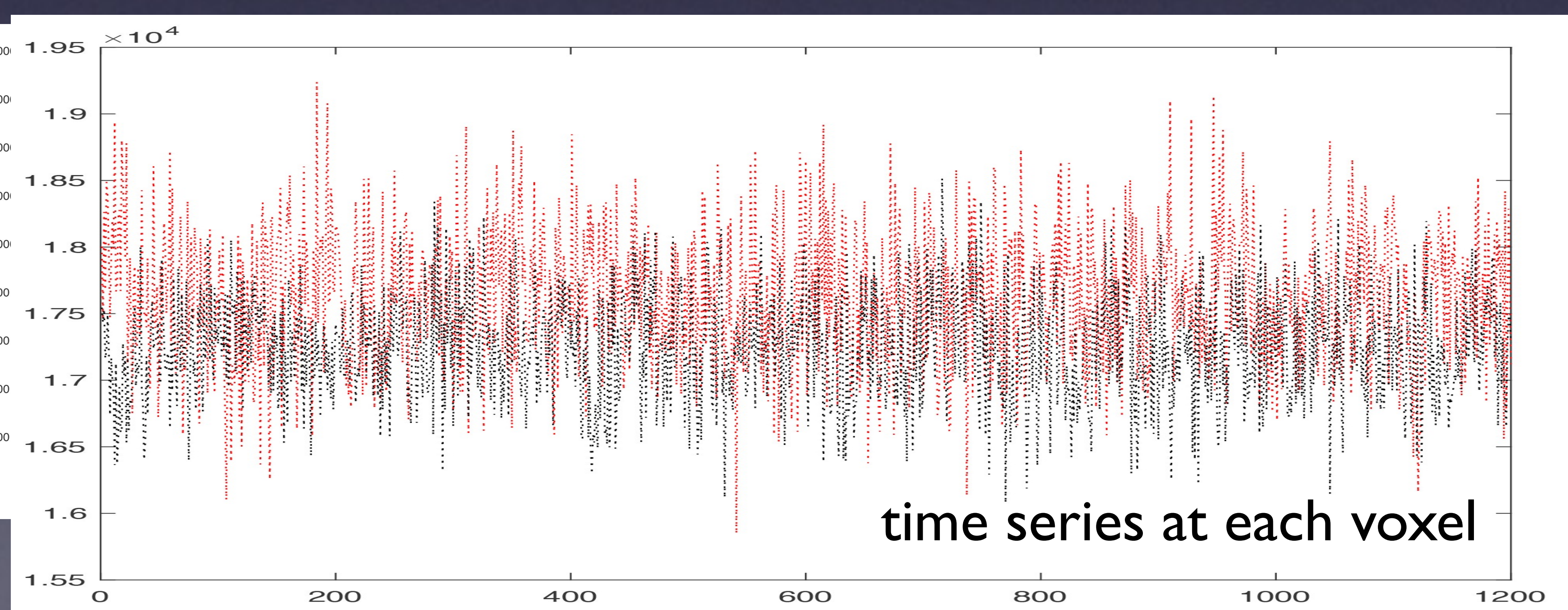
Resting-state fMRI at 300000 voxels per subject measured over 14min 33 seconds inside a scanner

416 subjects (131 MZ twins 77 DZ twins) x 2GB = 832GB data

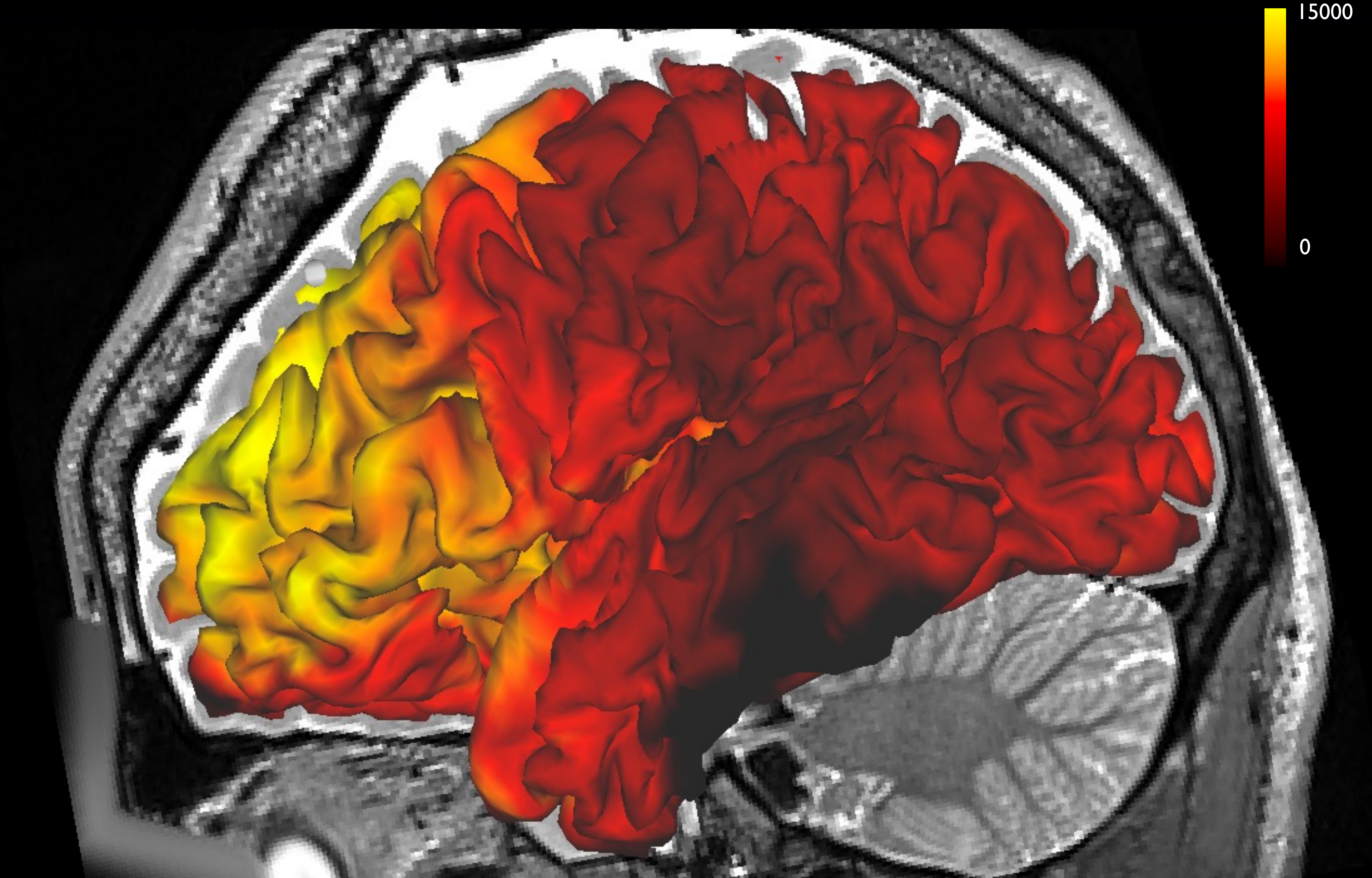
3.0 Tesla GE Scanner



Blood oxygenation level

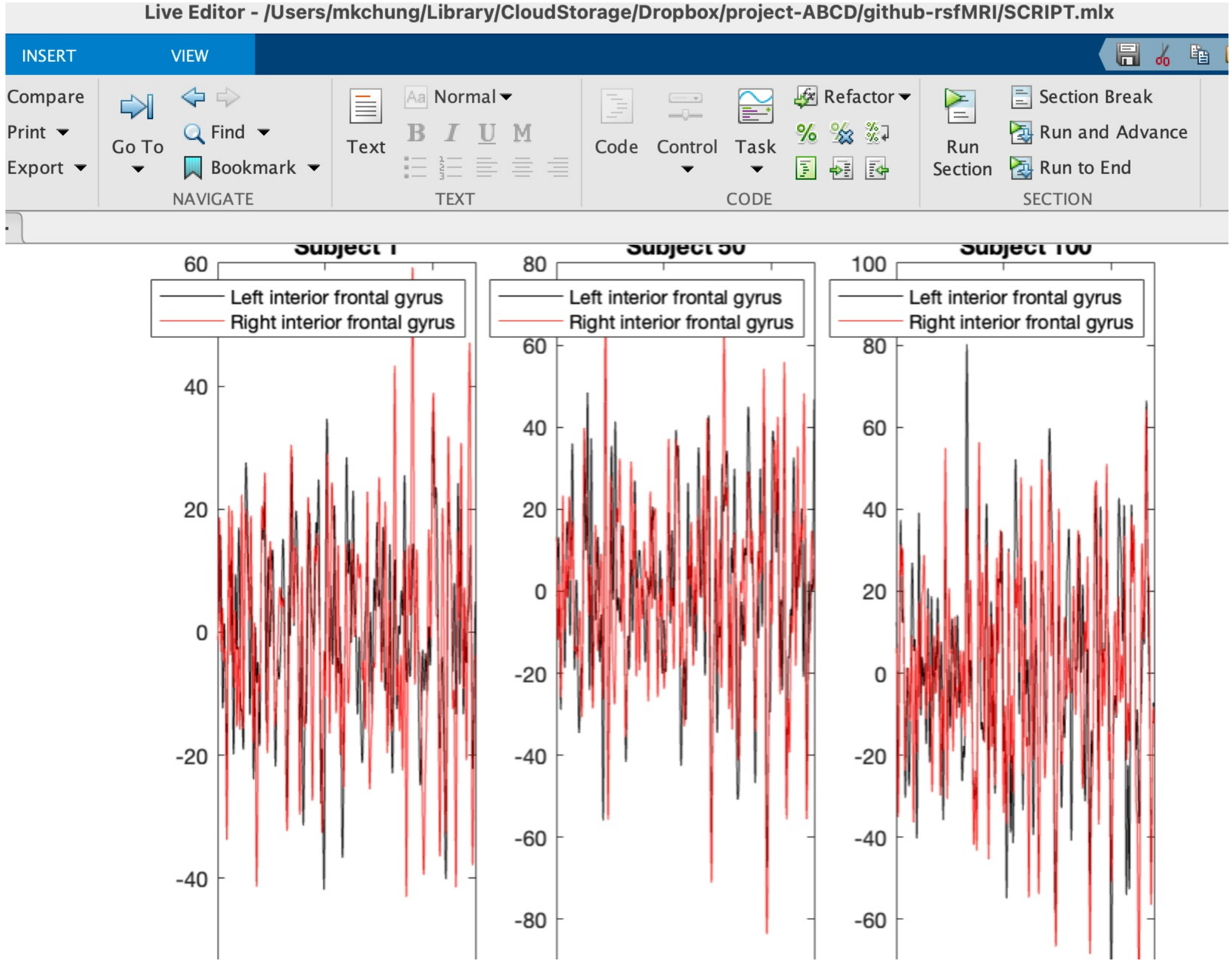


Typical rs-fMRI (every 30 second)



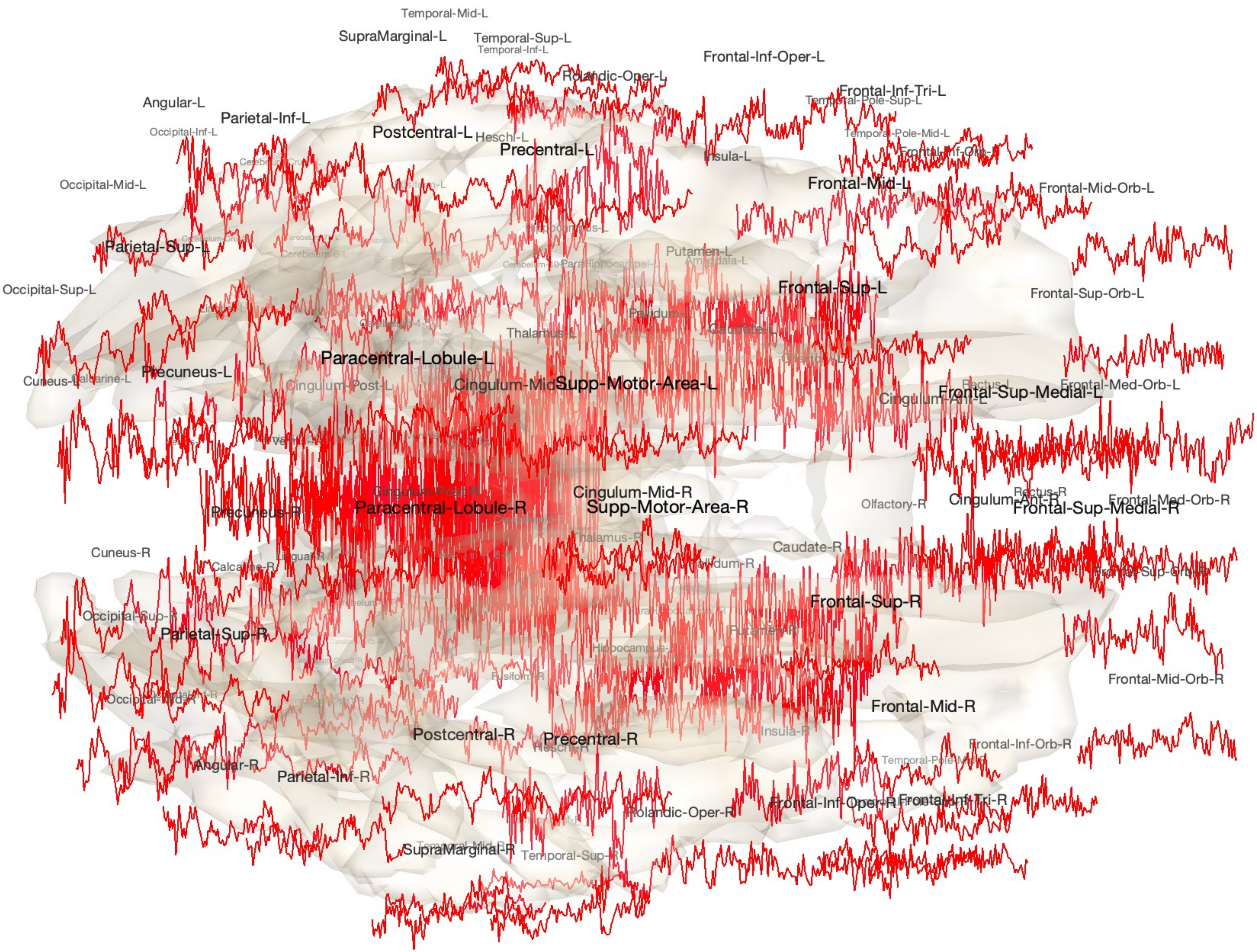
https://github.com/laplcebeltrami/rsfMRI

MATLAB

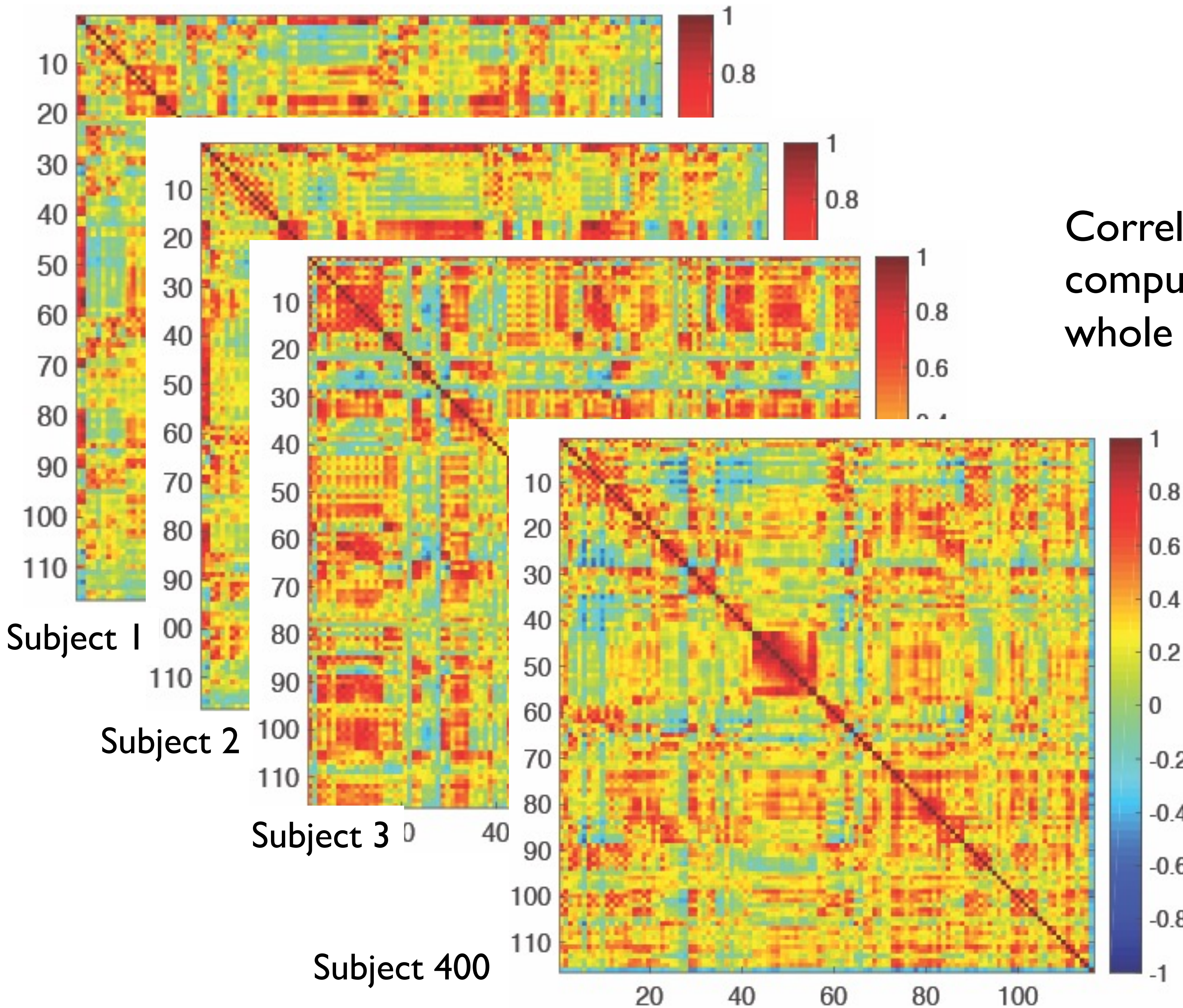


Huang et al. 2020 Neuroscience Methods

Time series averaged into 116 brain regions

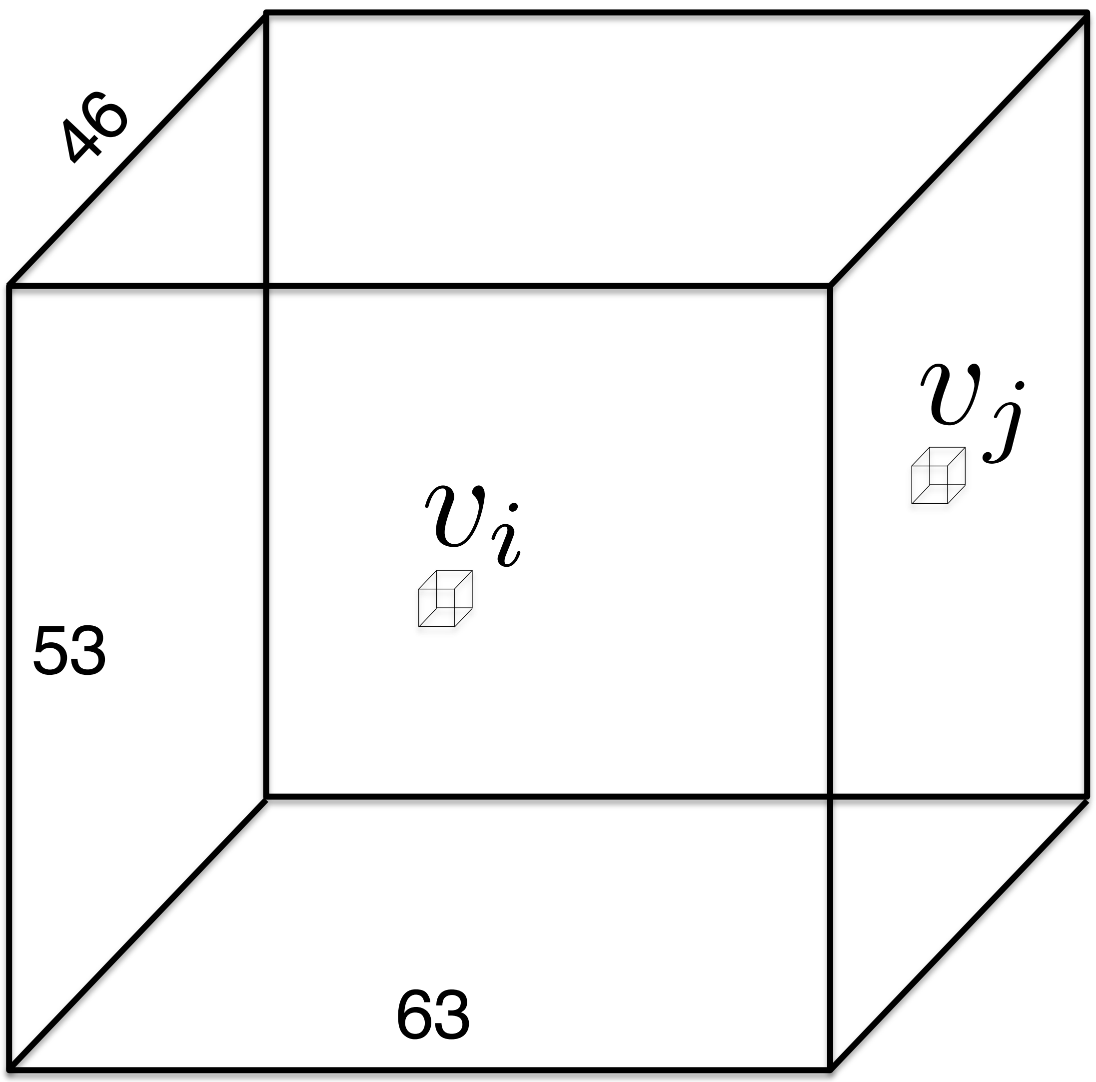


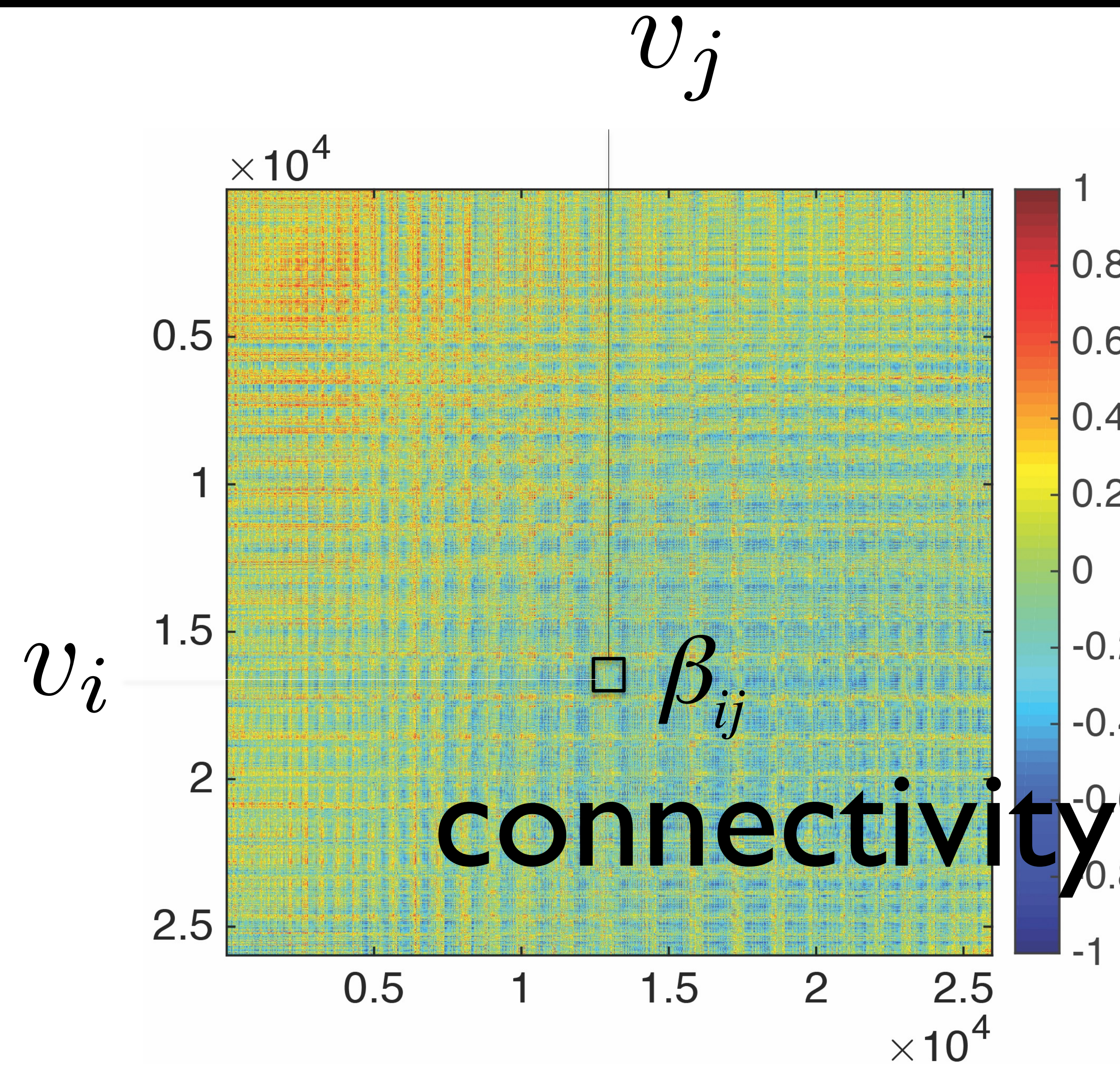
Subject level brain connectivity matrix



Correlation computed over whole time points

How big is fMRI connectivity?



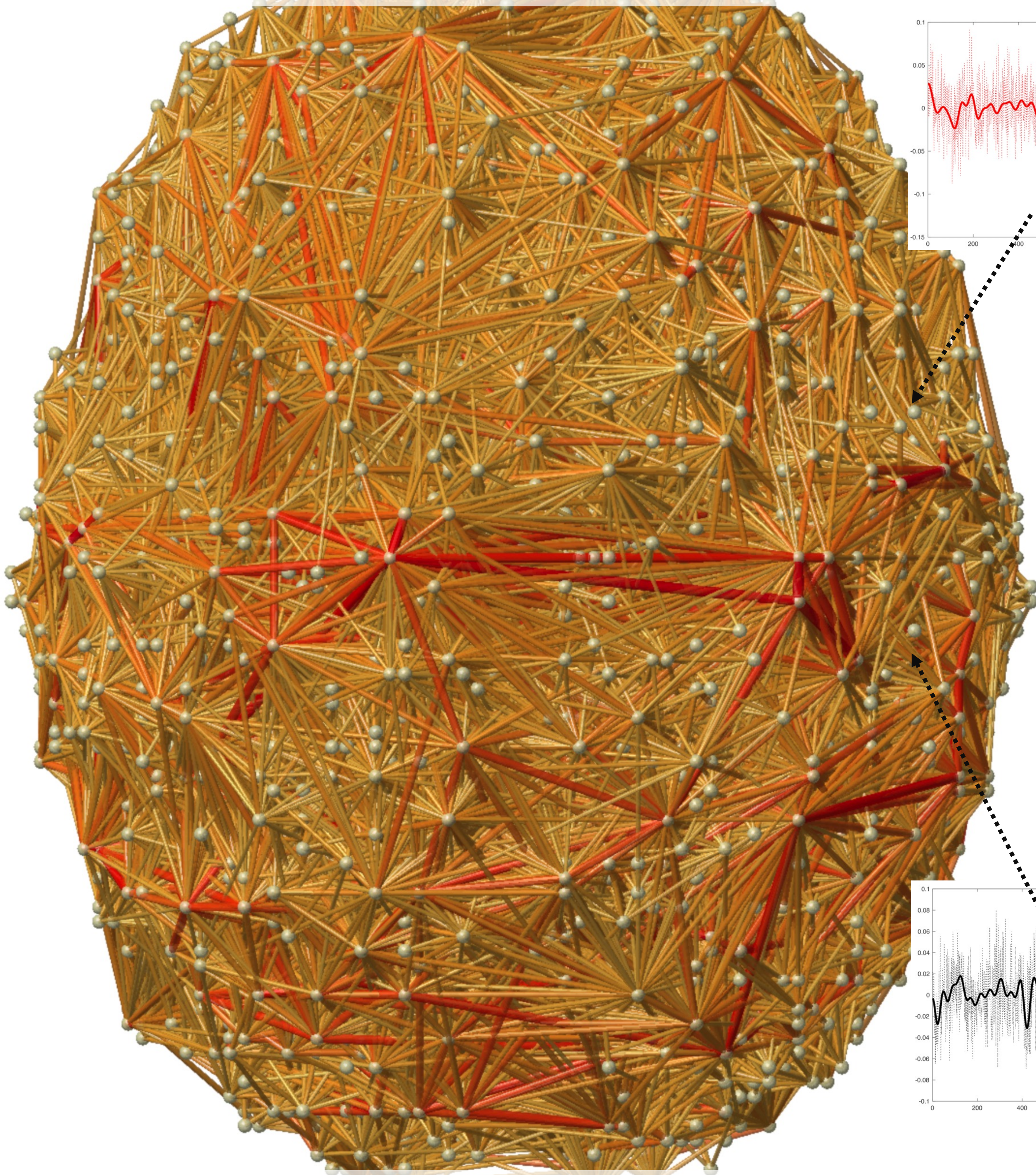


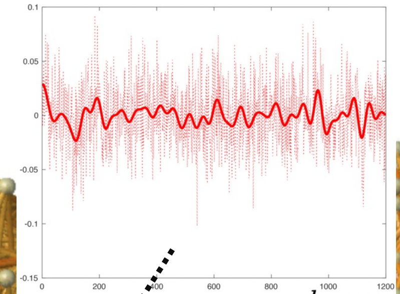
p=25972 voxels (1.5T at 3mm resolution) in the brain

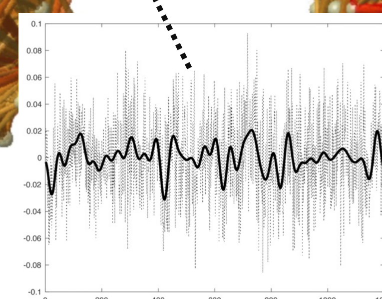
25972 x 25972 = 0.67 billion connections

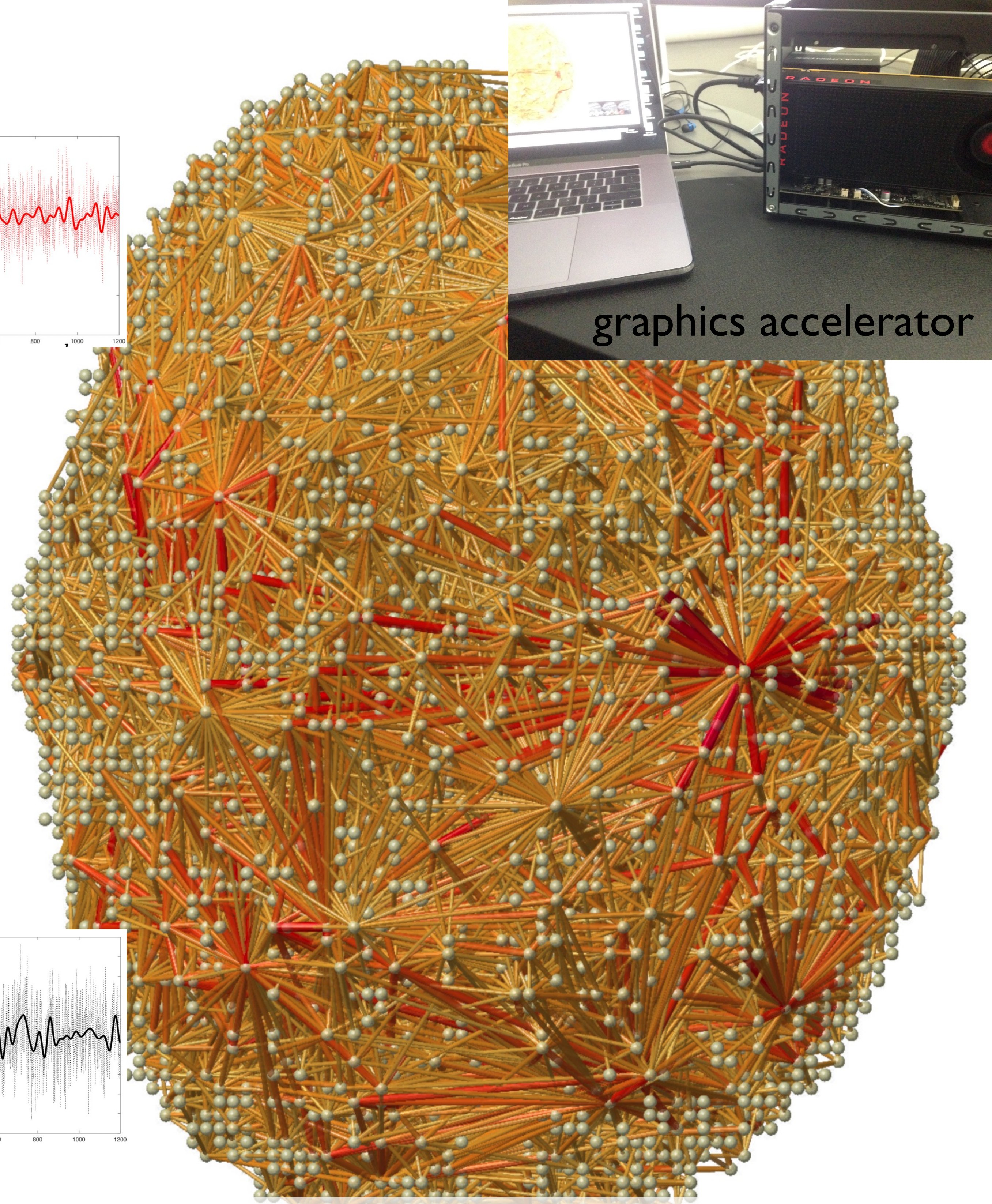
5.2GB data

Big data computation



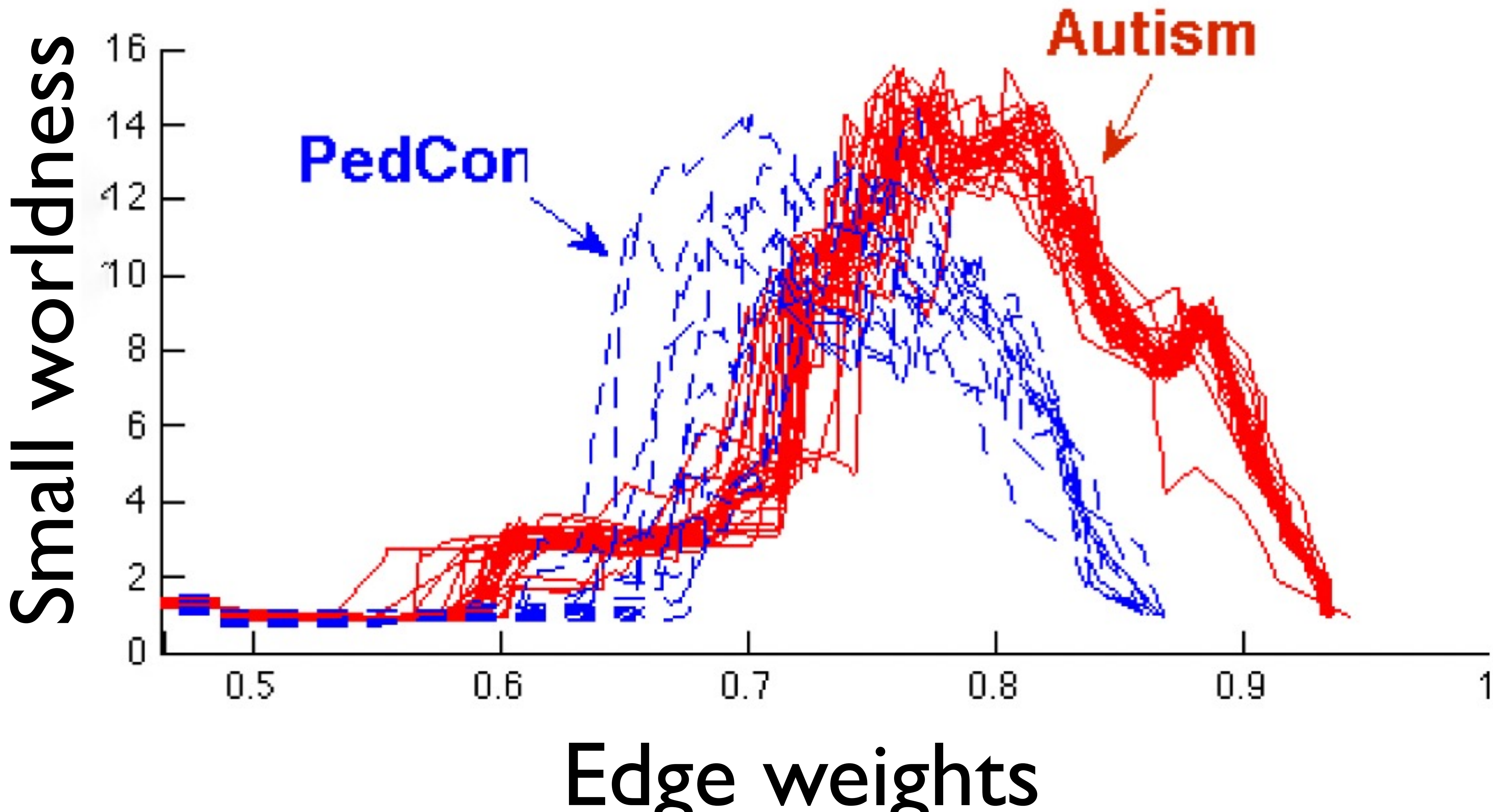


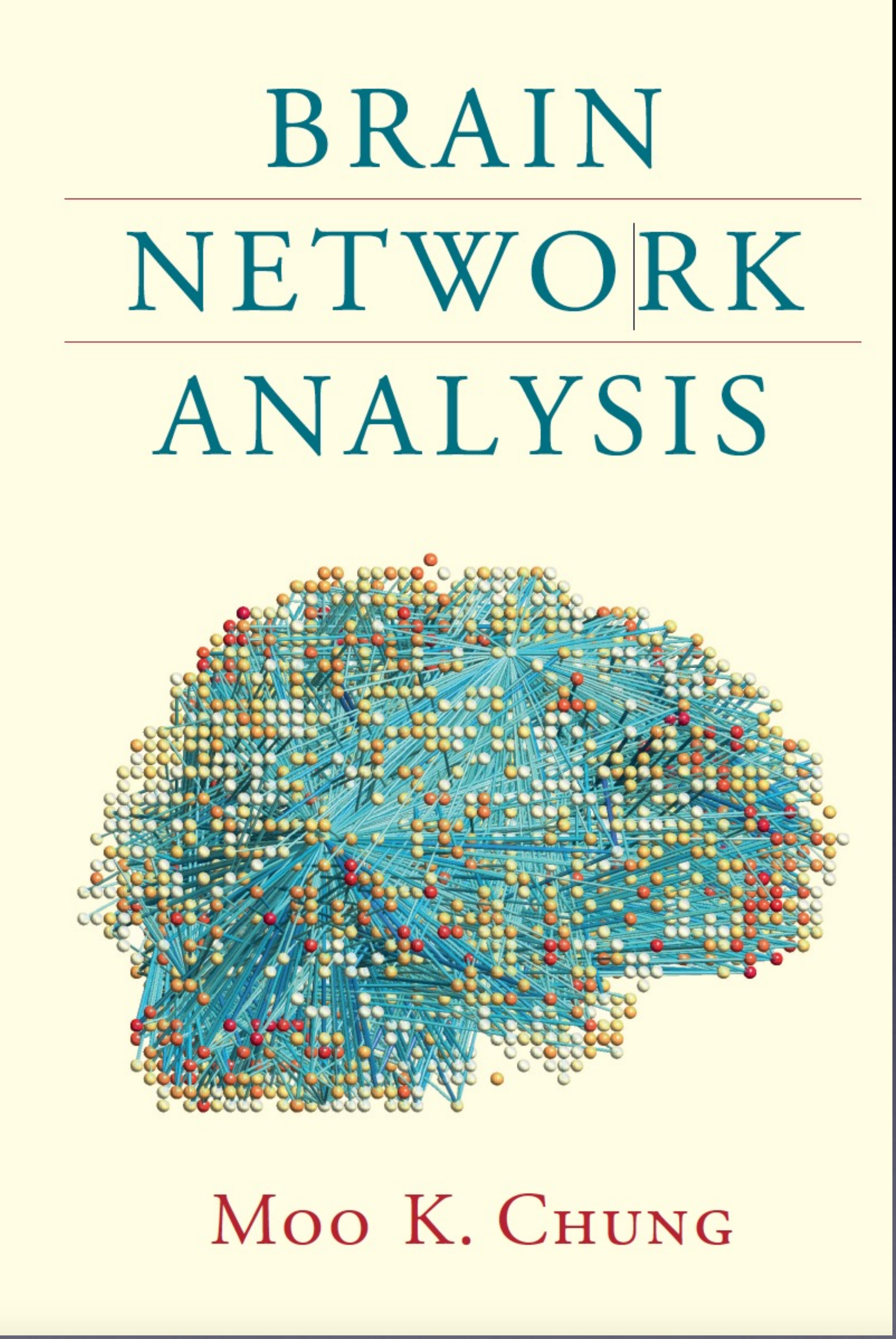




Resting state brain network obtained from functional-MRI.
Correlation of 300000 time series in the brain.
For 3D rendering, subsampled at 25000 nodes.

Graph theory features → Often incorrect conclusion



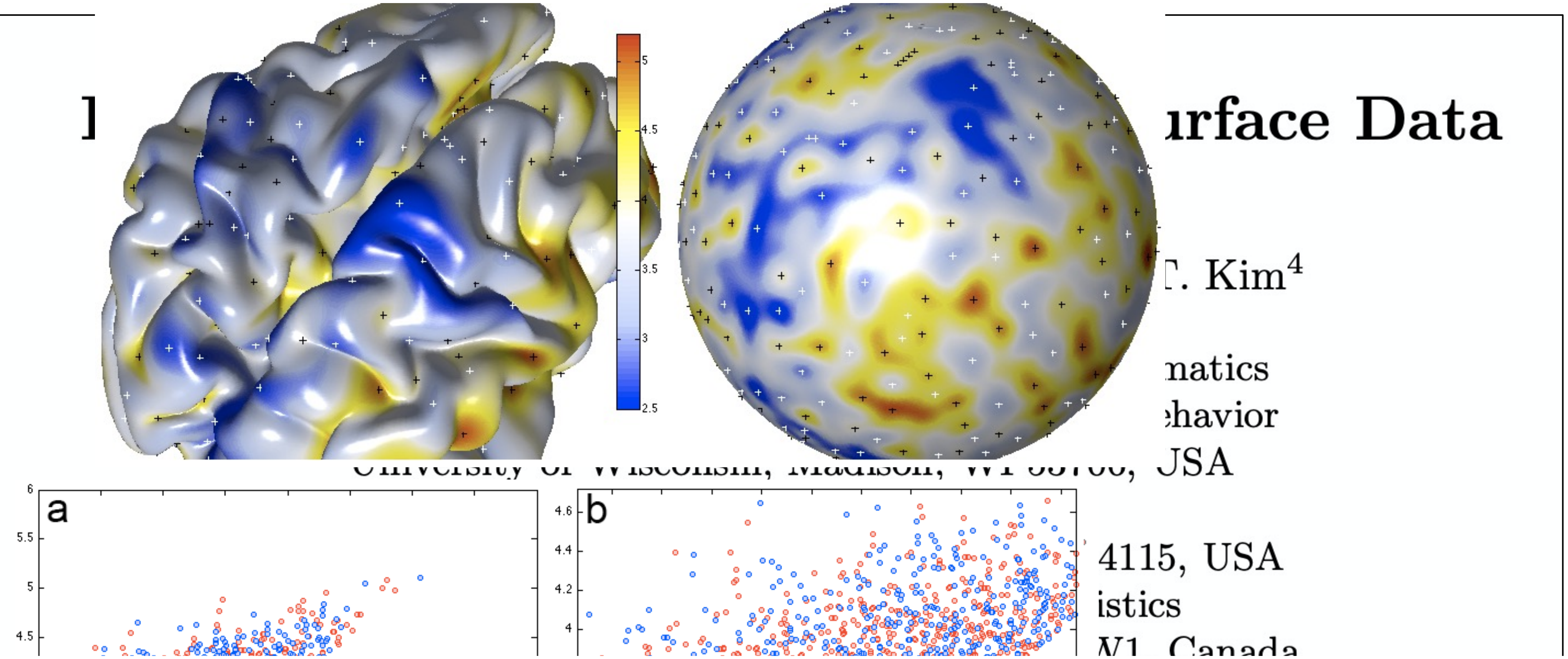


Book writing with Cambridge University Press on tutorial style applied-TDA

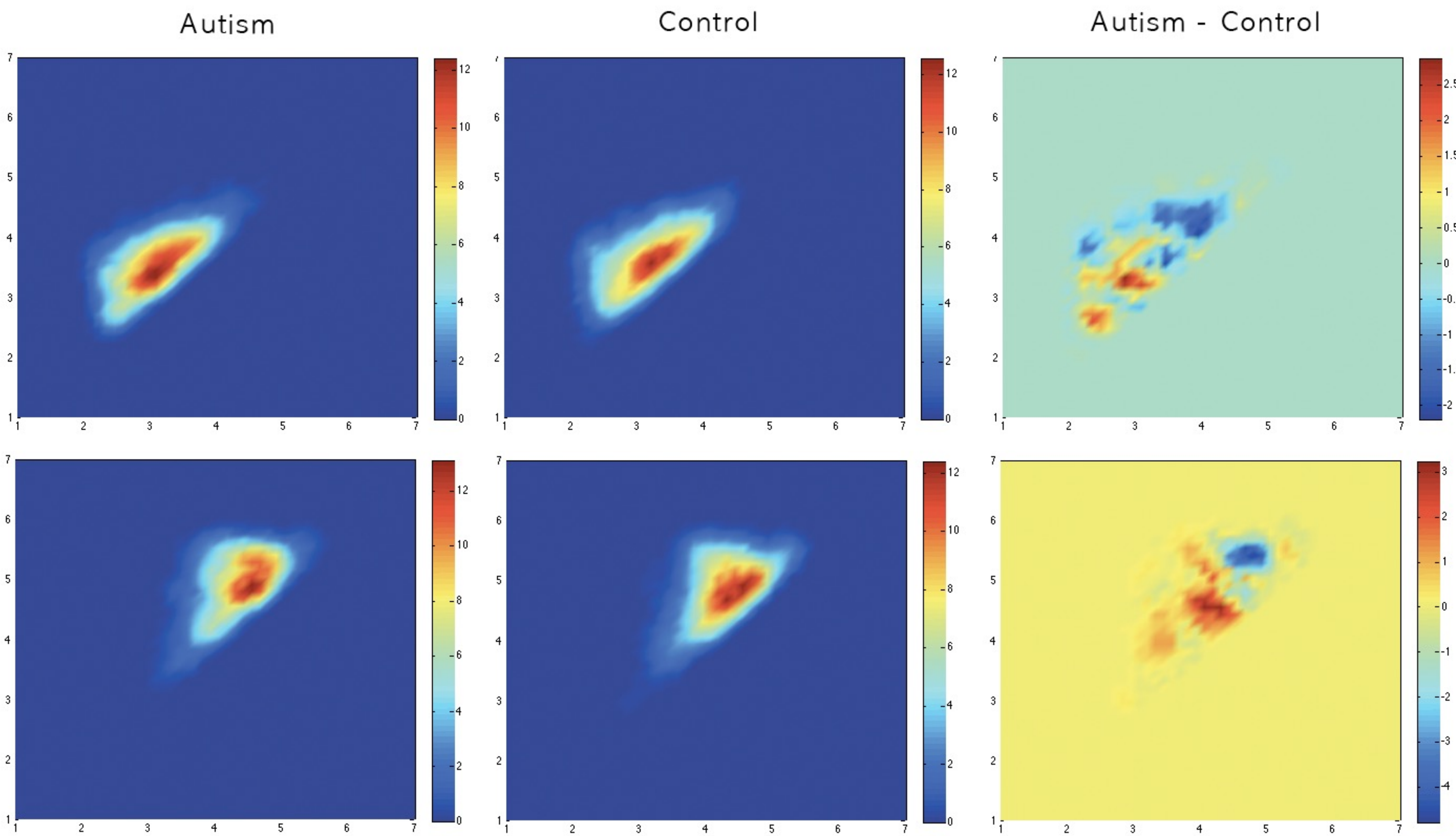
Chung, 2019
Cambridge University Press

Topological data analysis (TDA)

Completely data driven!
No model!
No distributional assumption!



Chung et al., 2009
Information Processing in Medical Imaging (IPMI) 5636:386-397.



First TDA paper in medical imaging

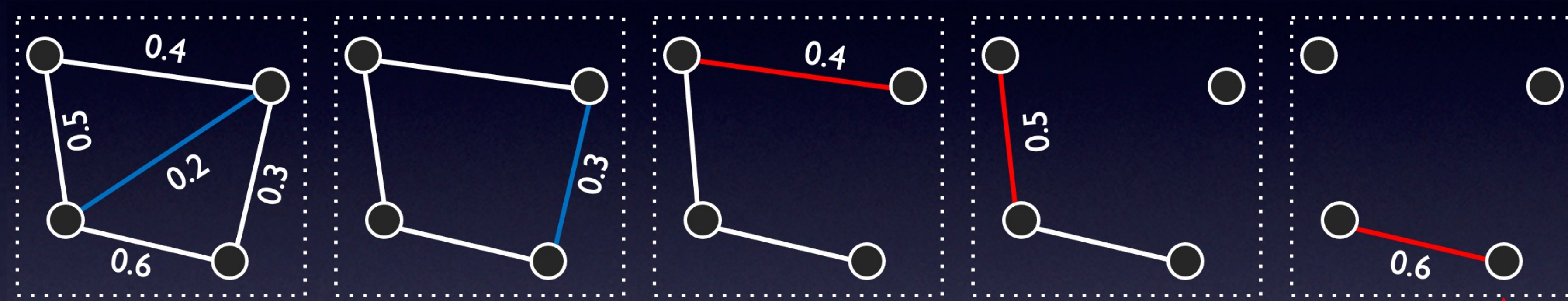
After 14 years and 50 TDA papers later …

Graph Filtrations

Weighted complete graph

$$\mathcal{X} = (V, w) \quad w = (w_{ij})$$

Node set (under V) — Edge weight (under w)



Binary graph

$$\mathcal{X}_\epsilon = (V, w_\epsilon)$$

$$w_{\epsilon,ij} = \begin{cases} 1 & \text{if } w_{ij} > \epsilon; \\ 0 & \text{otherwise.} \end{cases}$$

Graph filtration

$$\mathcal{X}_{\epsilon_0} \supset \mathcal{X}_{\epsilon_1} \supset \mathcal{X}_{\epsilon_2} \supset \cdots$$

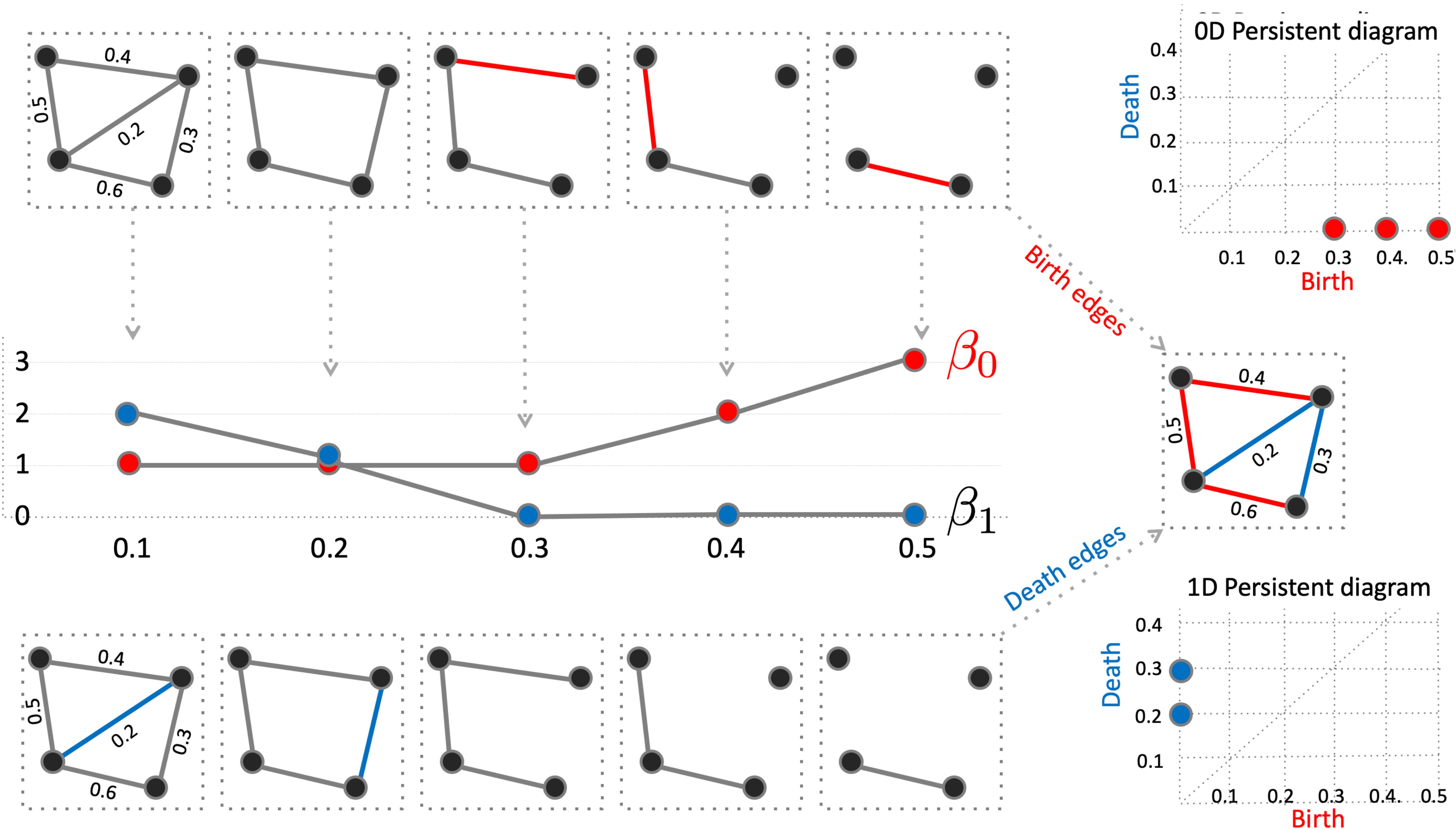
for increased edge weights

$$\epsilon_0 < \epsilon_1 < \epsilon_2 < \cdots$$

Equivalent to Rips filtration on radius 1 – ε on 1-skeleton

Chung et al. 2019 Network Neuroscience 3:674-694

Theorem: Birth & death decomposition

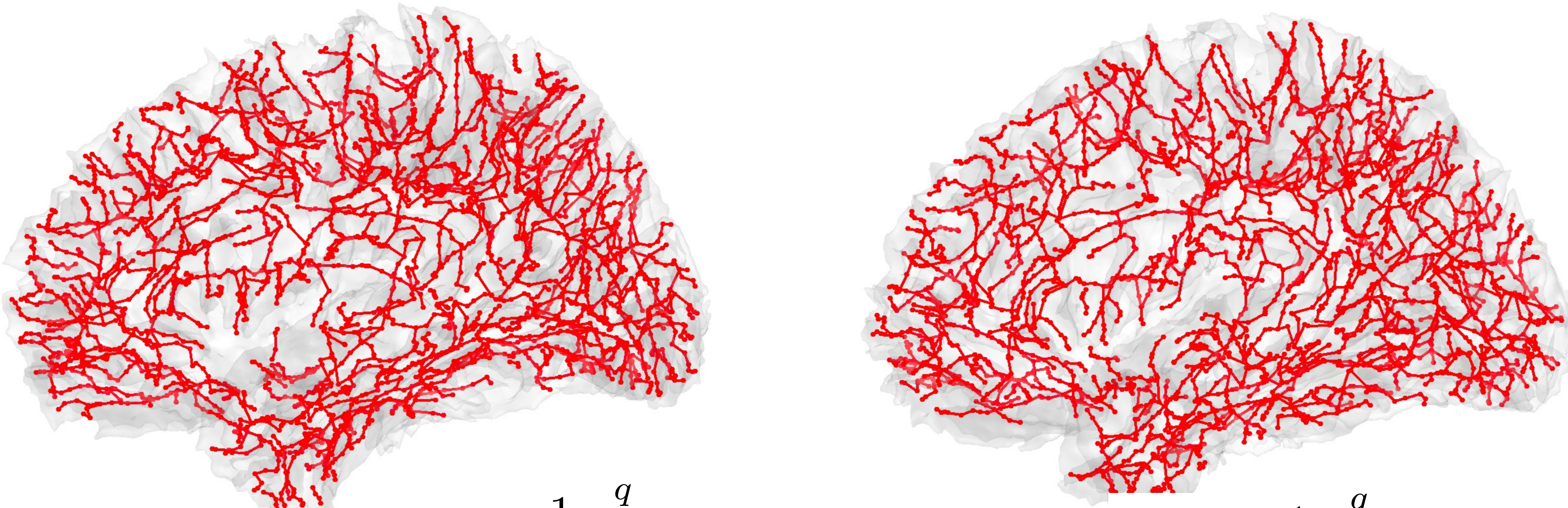


Songdechakraiwut and Chung. 2023, Annals of Appled Statistics 17:403-433

2-Wasserstein distance between scatter points

Random variables: $X \sim f_1 \quad Y \sim f_2$

2-Wasserstein distance: $\mathcal{D}(X,Y) = \left(\inf \mathbb{E}\|X - Y\|^2\right)^{1/2}$

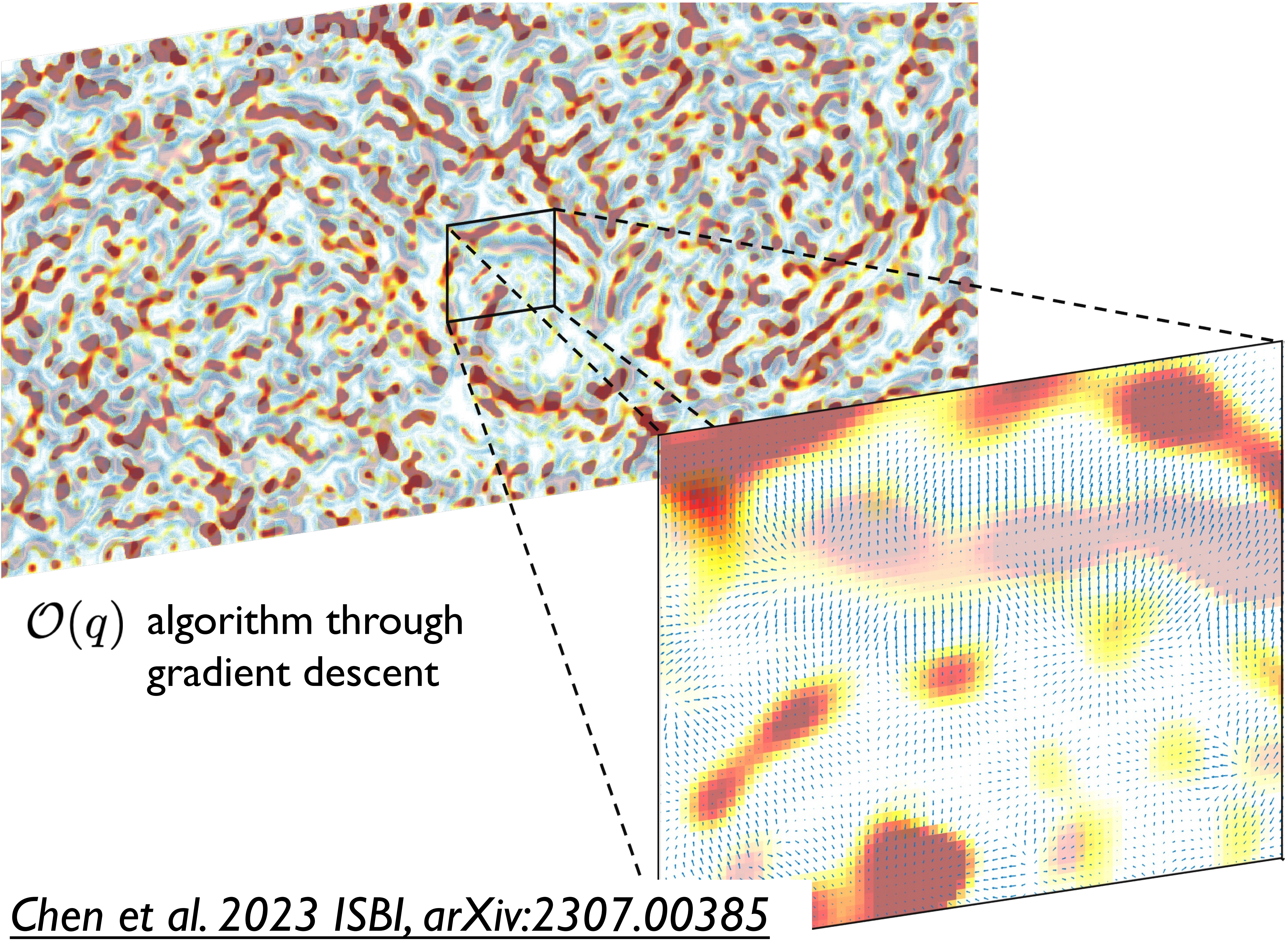


$$f_1(x) = \frac{1}{q}\sum_{i=1}^{q} \delta(x - x_i) \qquad f_2(y) = \frac{1}{q}\sum_{i=1}^{q} \delta(y - y_i)$$

$$\mathcal{L}(P_1, P_2) = \inf_{\psi: P_1 \to P_2} \Big(\sum_{x \in P_1} \|x - \psi(x)\|^2 \Big)^{1/2}$$

Hungarian algorithm in $\mathcal{O}(q^3)$

https://github.com/laplcebeltrami/sulcaltree



$\mathcal{O}(q)$ algorithm through gradient descent

Chen et al. 2023 ISBI, arXiv:2307.00385

Theorem: Wasserstein distance on graph filtrations

$$\mathcal{L}_{0D}(\Theta, P) = \min_{\tau} \sum_{b \in E_0} \left[b - \tau(b) \right]^2$$

Birth set

$$= \sum_{b \in E_0} \left[b - \tau_0^*(b) \right]^2$$

τ_0^* *:The i-th smallest birth value to the i-th smallest birth value*

$$\mathcal{L}_{1D}(\Theta, P) = \min_{\tau} \sum_{d \in E_1} \left[d - \tau(d) \right]^2$$

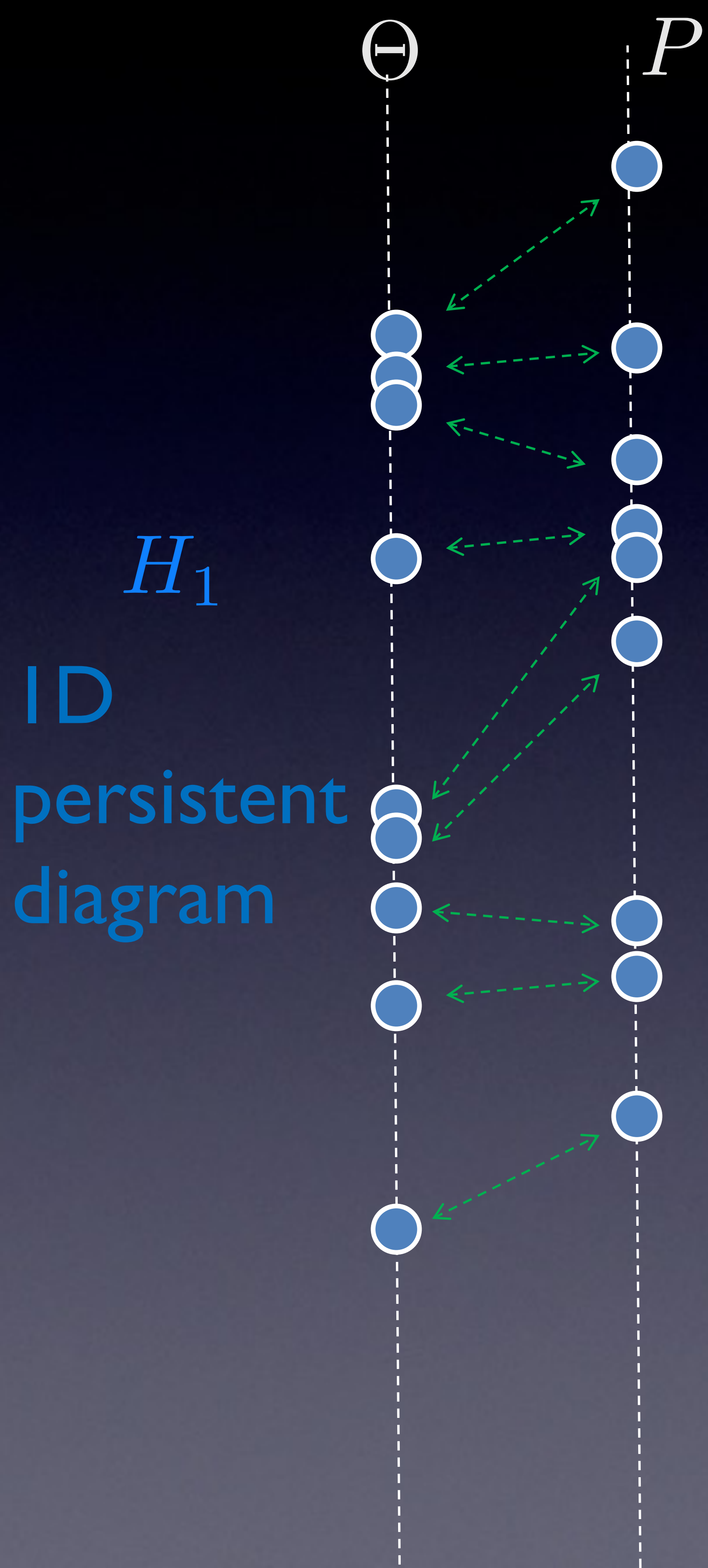
Death set

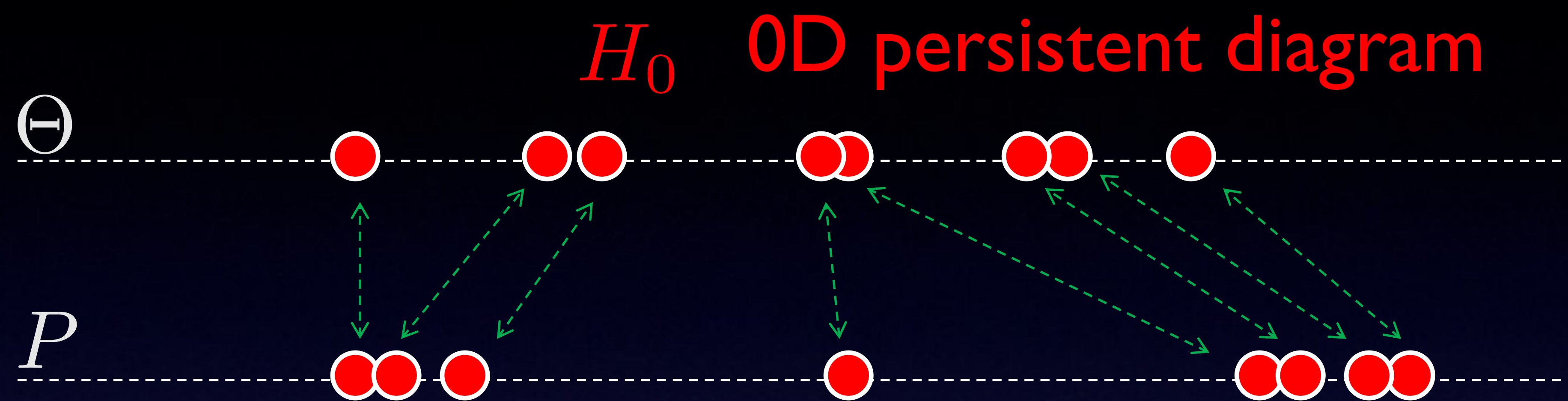
$$= \sum_{d \in E_1} \left[d - \tau_1^*(d) \right]^2$$

τ_1^* *:The i-th smallest death value to the i-th smallest death value*

Songdechakraiwut and Chung. 2023, Annals of Appled Statistics arXiv: 2012.00675

Wasserstein distance for graph filtrations





1D assignment problem has known analytic solution

$\mathcal{O}(q \log q)$

Topological ~~Inference~~ *Clustering*

distance-based inference (our approach)

vs.

feature-based inference (other group's approach)

Songdechakraiwut and Chung 2023 Annals of Appled Statistics

Chung et al. 2023 arXiv:2201.00087

Ratio statistic for Wasserstein distances

$$C_1 \cup C_2 = \{\mathcal{X}_1, \cdots, \mathcal{X}_n\}, \quad C_1 \cap C_2 = \emptyset$$

Between-group distance

$$l_B \propto \sum_{i \in C_1, j \in C_2} \mathcal{L}(\mathcal{X}_i, \mathcal{X}_j)$$ ←---- 0D, 1D and combined distances

Within-group distance

$$l_W \propto \sum_k \sum_{i,j \in C_k} \mathcal{L}(\mathcal{X}_i, \mathcal{X}_j)$$ ------→ Statistic $\phi = \frac{l_B}{l_W}$

Small ϕ → small group separation

Large ϕ → large group separation

More complex ANOVA-type inference can be easily done

Topological clustering

$$C_1 \cup C_2 \cup \cdots \cup C_k = \{\mathcal{X}_1, \cdots, \mathcal{X}_n\}, \quad C_i \cap C_j = \emptyset$$

Minimize the within cluster distance

$$l_W \propto \sum_k \sum_{i,j \in C_k} \mathcal{L}(\mathcal{X}_i, \mathcal{X}_j)$$

Theorem: Topological clustering converges locally.

Algebraic proof: arXiv: 2302.06673

Geometric proof: arXiv:2201.00087

Geometry of Wasserstein distance

Birth values are points in the convex polytope

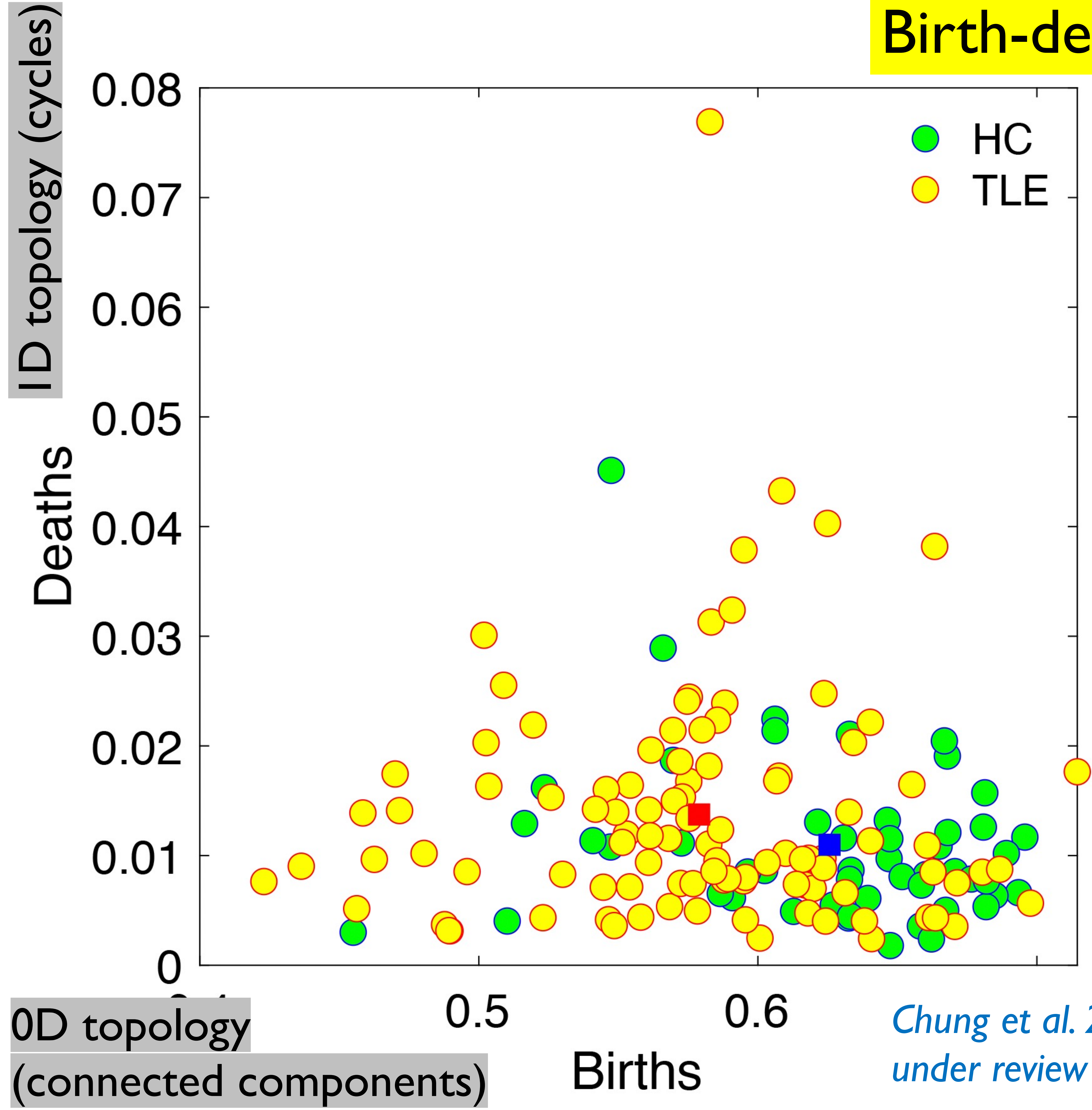
$$\mathcal{T}_0 = \{(x_1, x_2, \cdots, x_{q_0}) | x_1 < x_2 < \cdots < x_{q_0}\} \subset \mathbb{R}^{q_0}$$

Death values are points in the convex polytope

$$\mathcal{T}_1 = \{(x_1, x_2, \cdots, x_{q_1}) | x_1 < x_2 < \cdots < x_{q_1}\} \subset \mathbb{R}^{q_1}$$

The Wasserstein distance is equivalent to the Euclidean distance in the convex set $\mathcal{T}_0 \bigotimes \mathcal{T}_1$

Birth-death embedding

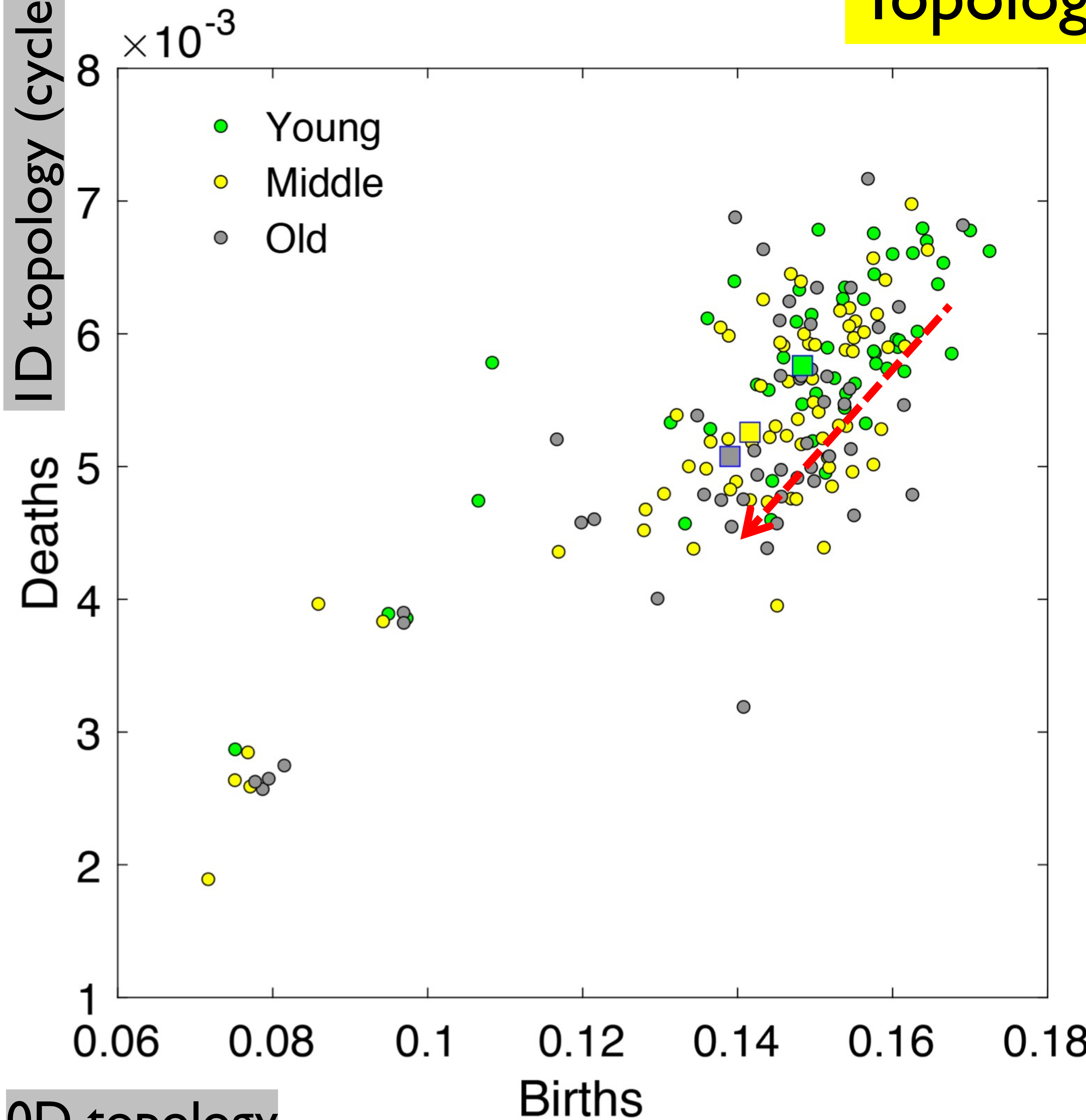


Displays the relative topological spread from the cluster centroid

Chung et al. 2023 arXiv:2302.06673 under review in NeuroImage

Topological gradient in aging

1D topology (cycles)



0D topology
(connected components)

Decreasing birth values →
more disconnected subnetworks

Decreasing death values →
cycles are disappearing

Joint work with James Gee of University of Pennsylvania

Matlab toolbox PH-STAT

Statistical Inference on Persistent Homology

Code written with Chat-GPT

https://github.com/laplcebeltrami/PH-STAT

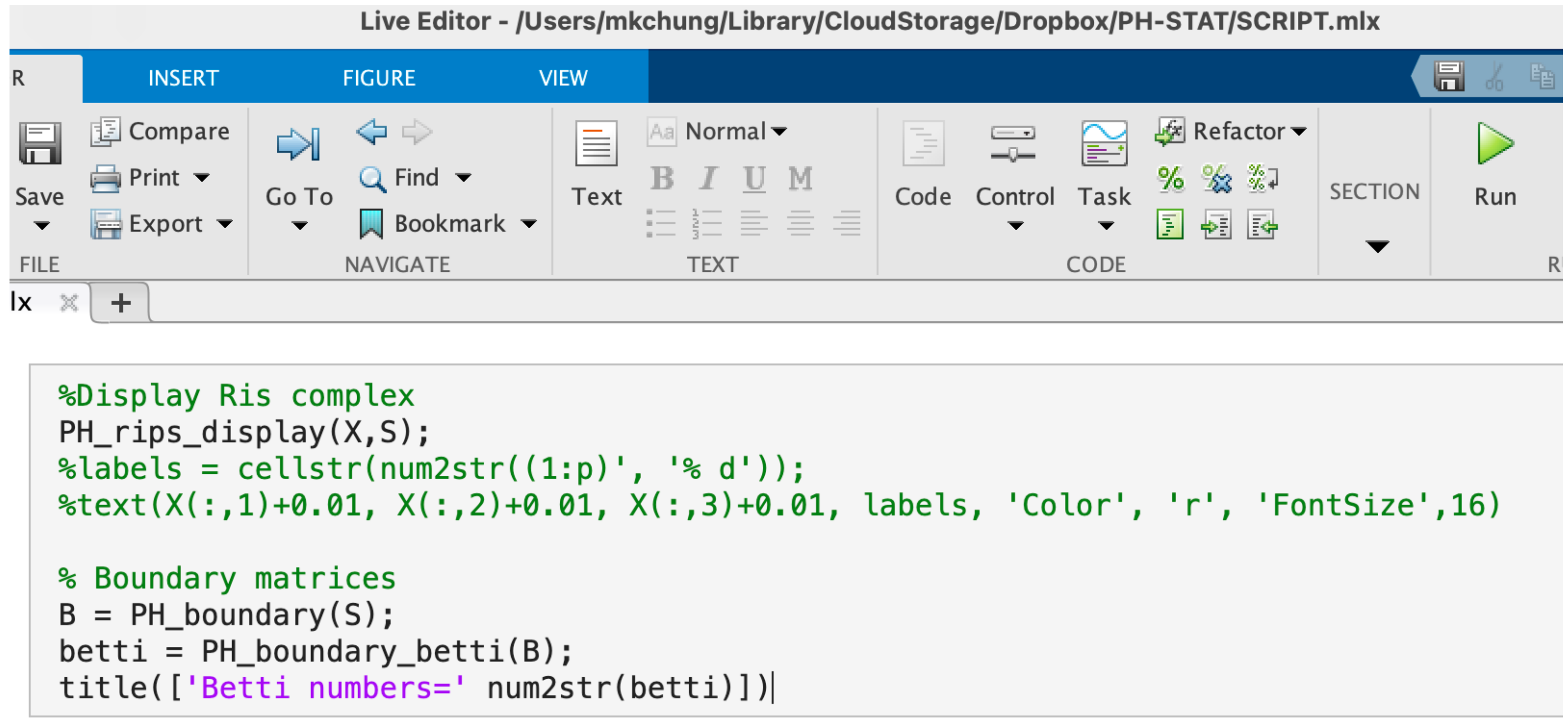
Manual:

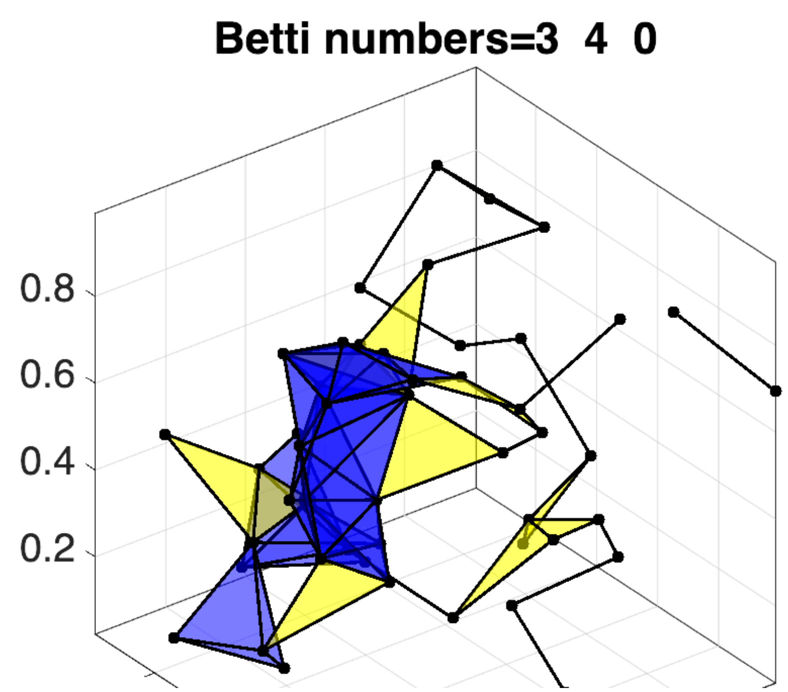
Chung (and CHAT-GPT) 2023, PH-STAT arXIv:2304.05912

The codes are used to publish topological data analysis papers in leading journals and conferences since 2009:

IEEE Transactions on Medical Imaging, NeuroImage, Human Brain Mapping, Network Neuroscience, Annals of Applied Statistics, Information Processing in Medical Imaging (IPMI), MICCAI, ISBI

https://github.com/laplcebeltrami/PH-STAT

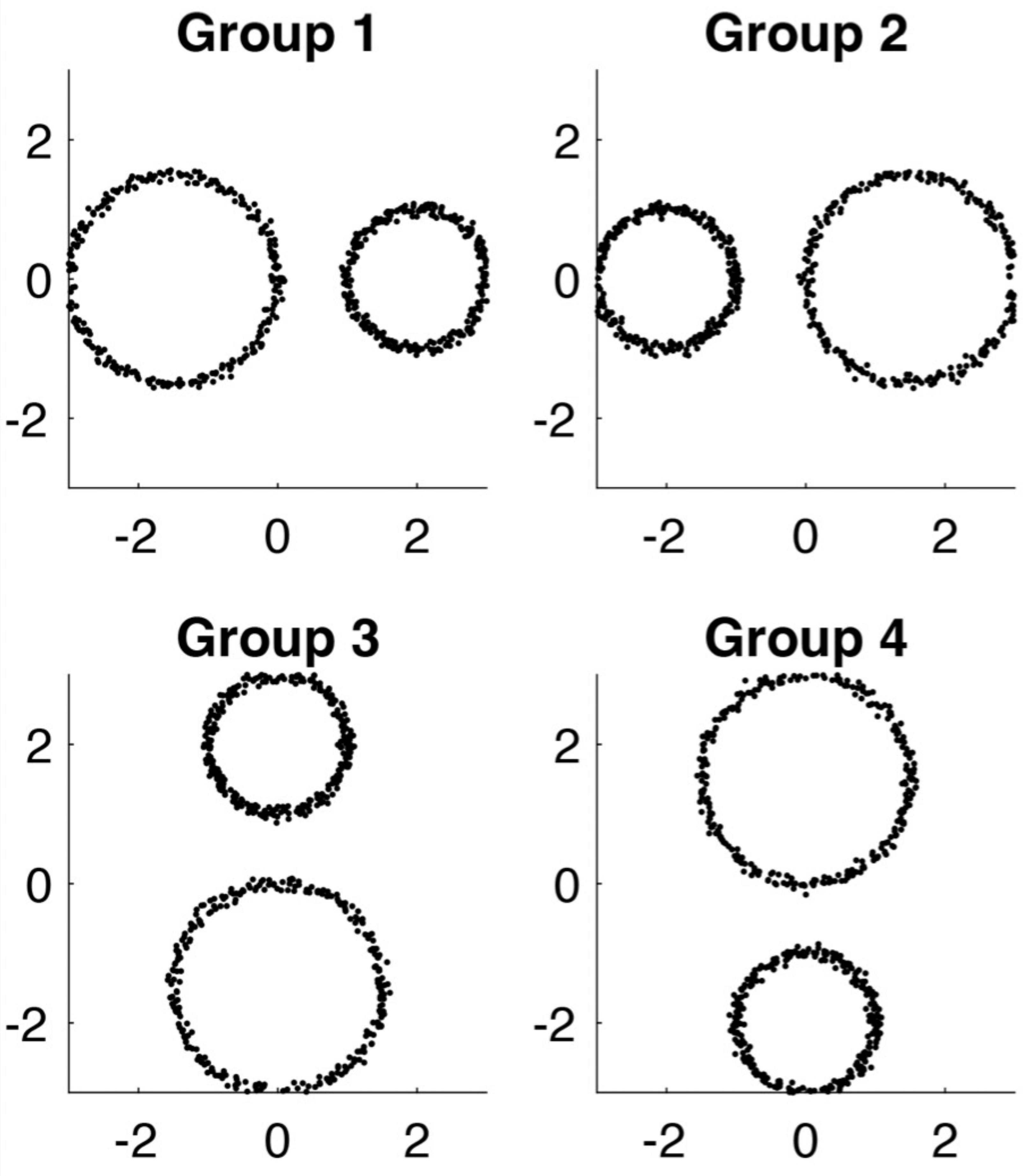




Validation

Testing if we are detecting topological false positives

$\sigma = 0.05$



All false positives

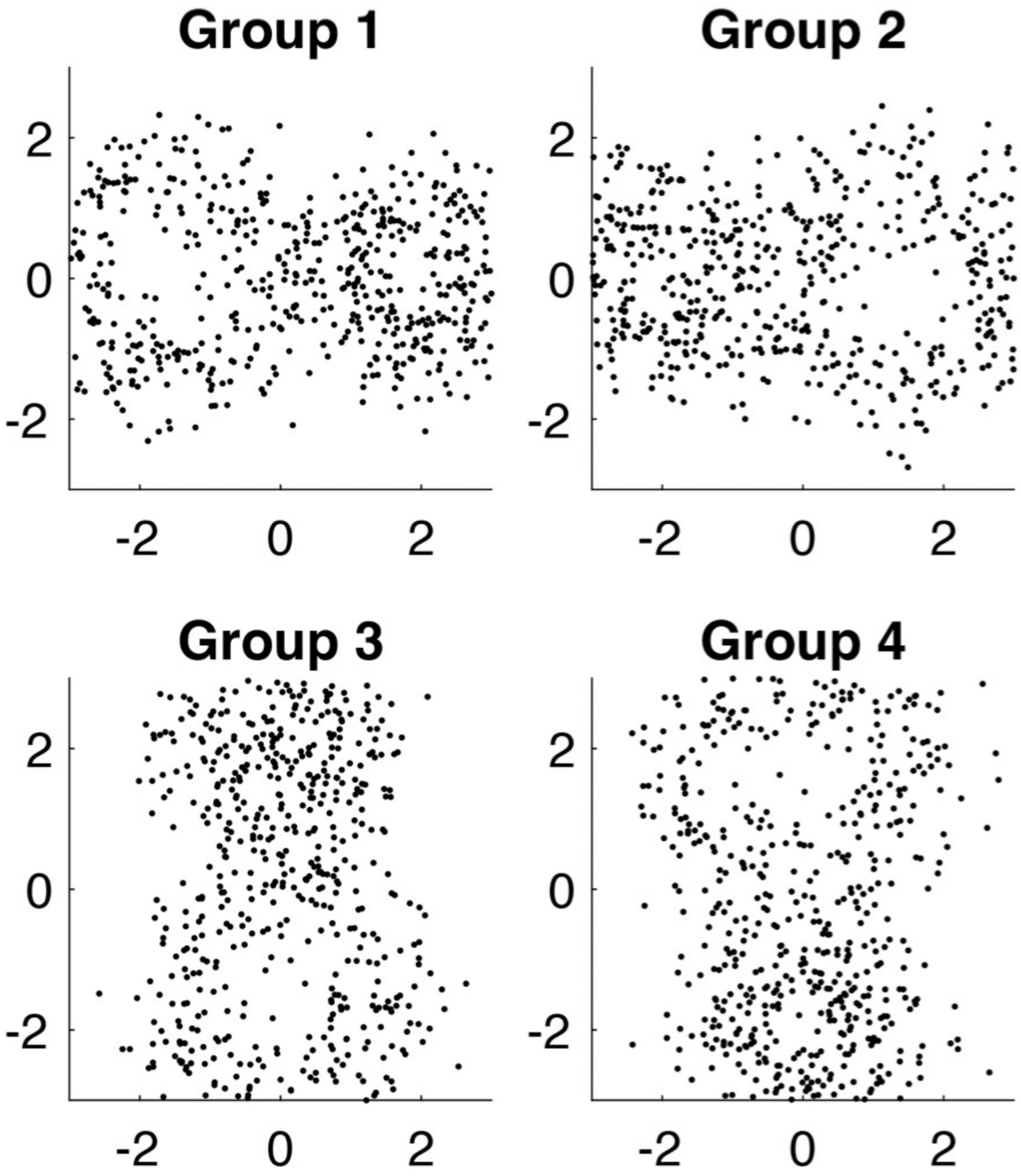
K-means clustering
0.98 +- 0.01

Hierarchical clustering
1.00 +- 0.00

Topological clustering
0.63 +- 0.04

Testing if we are detecting topological false positives

$\sigma = 0.5$



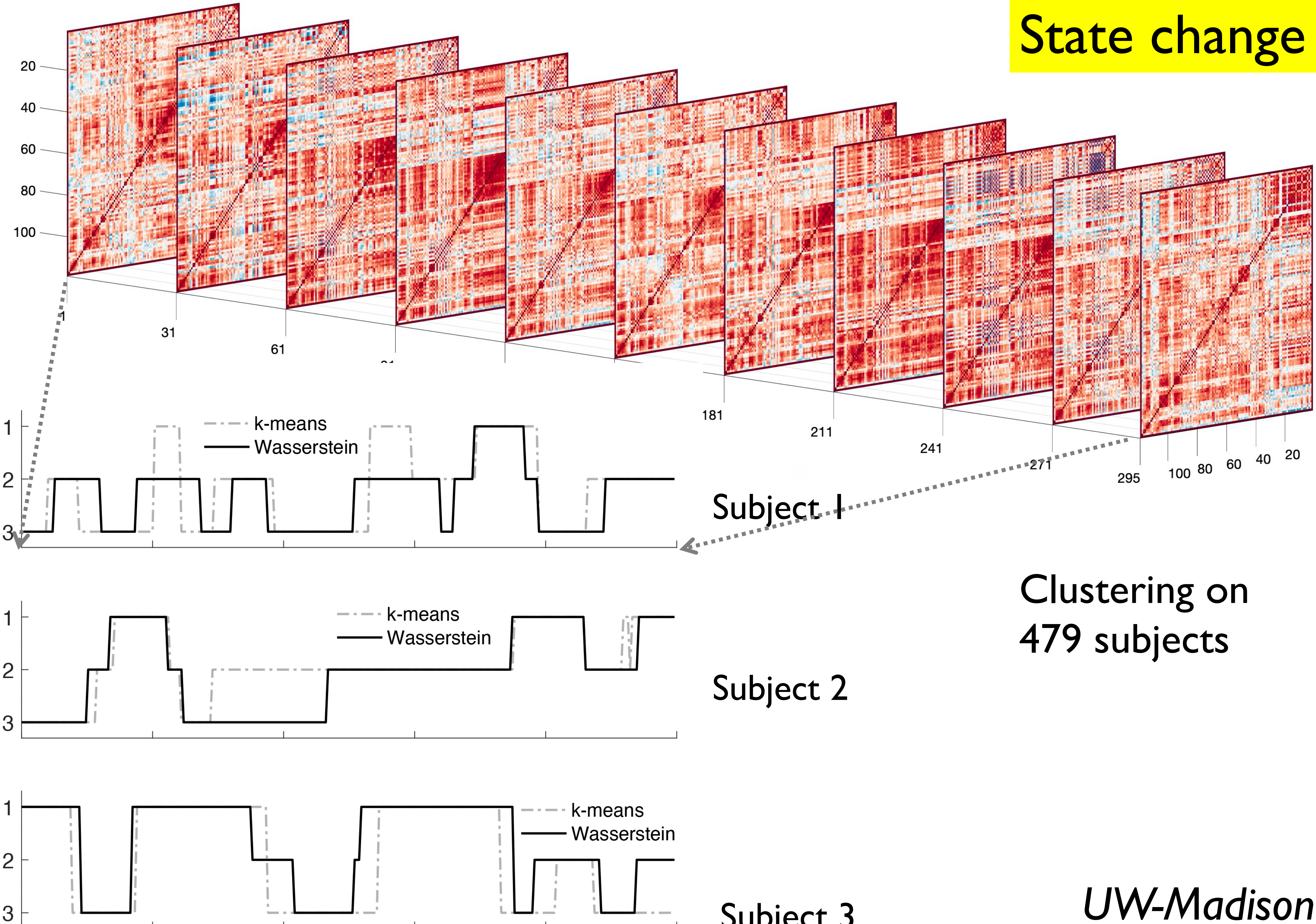
K-means clustering
0.81 +- 0.02

Hierarchical clustering
1.00 +- 0.00

Topological clustering
0.44 +- 0.04

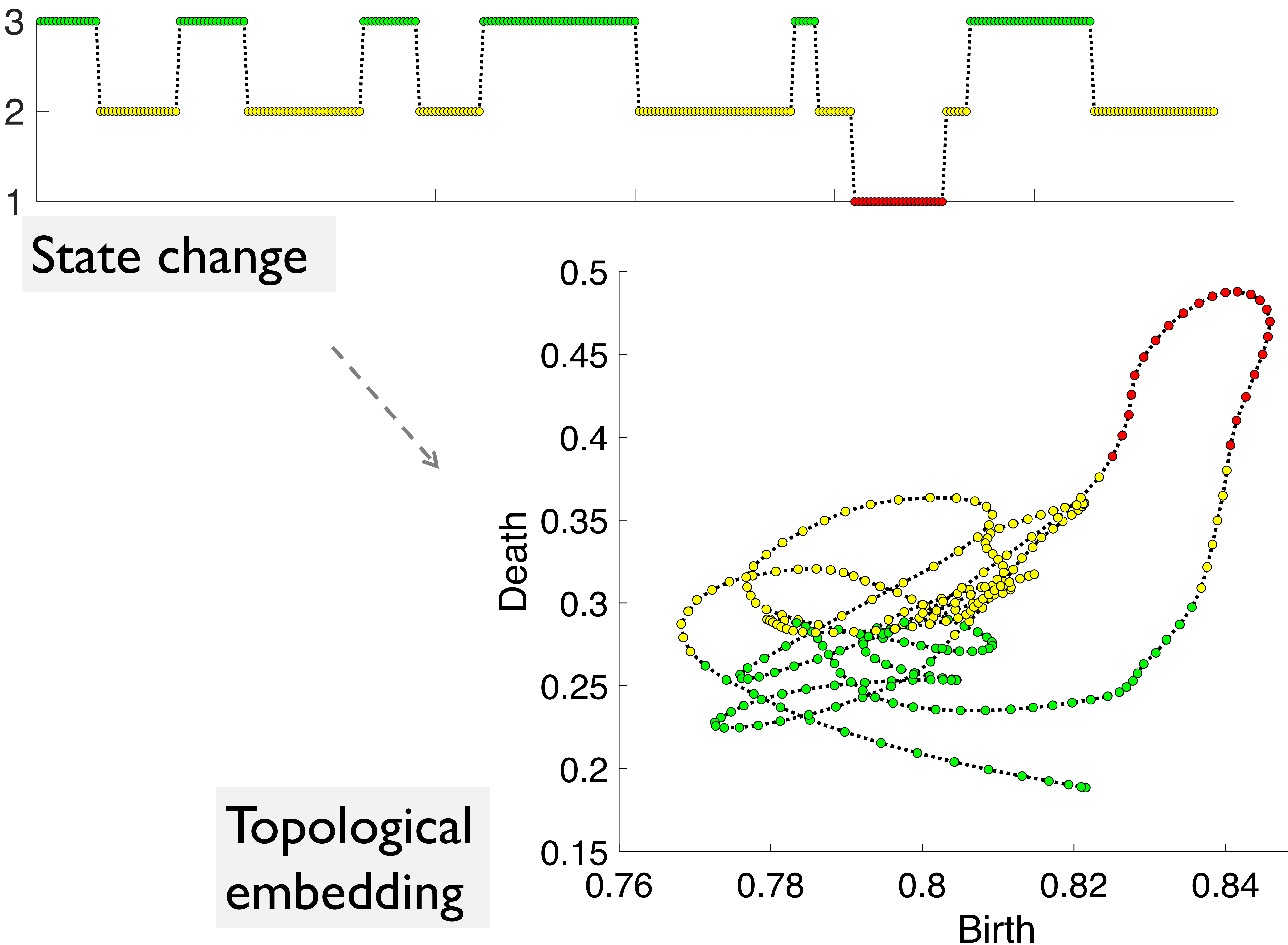
Results

State change

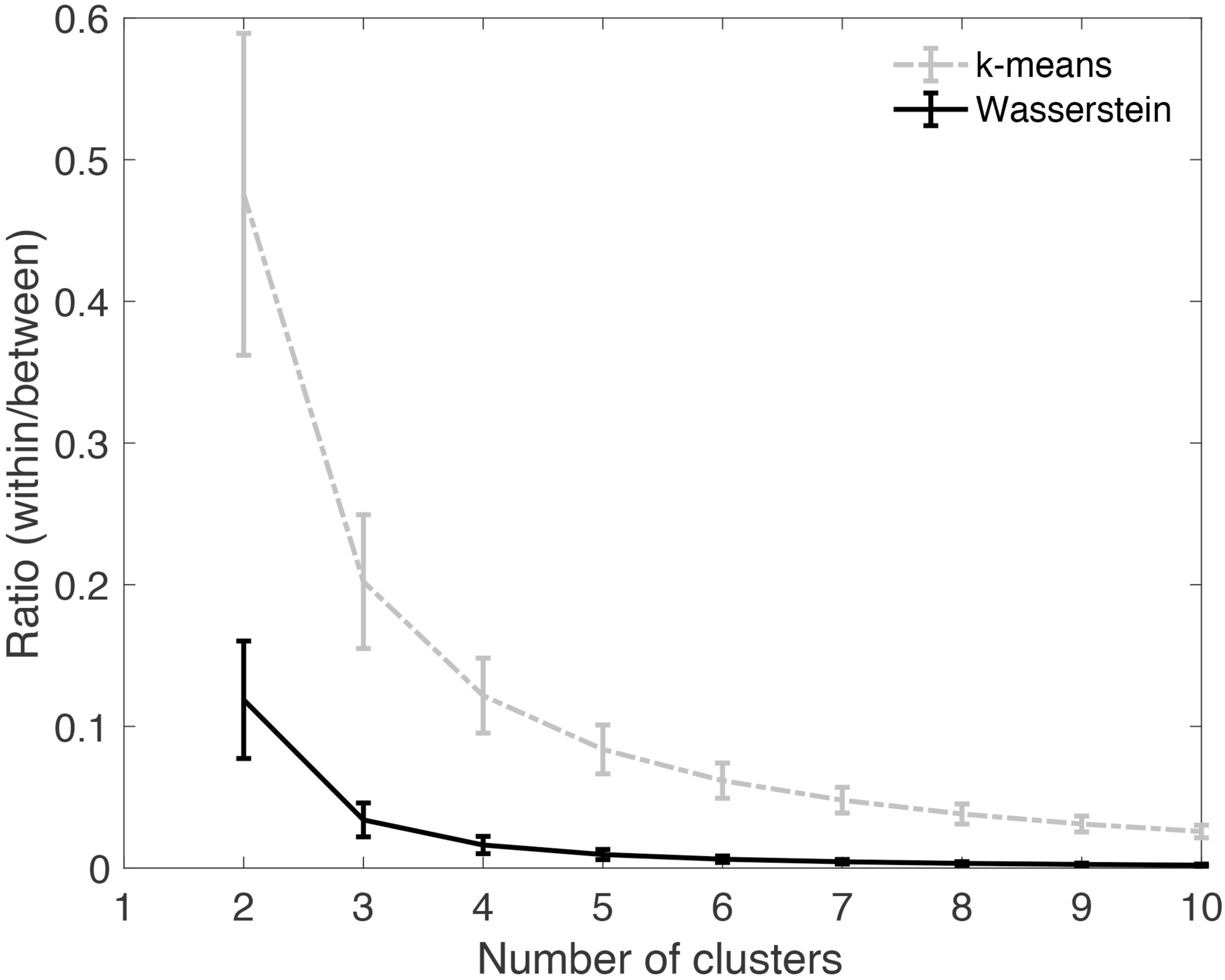


Clustering on 479 subjects

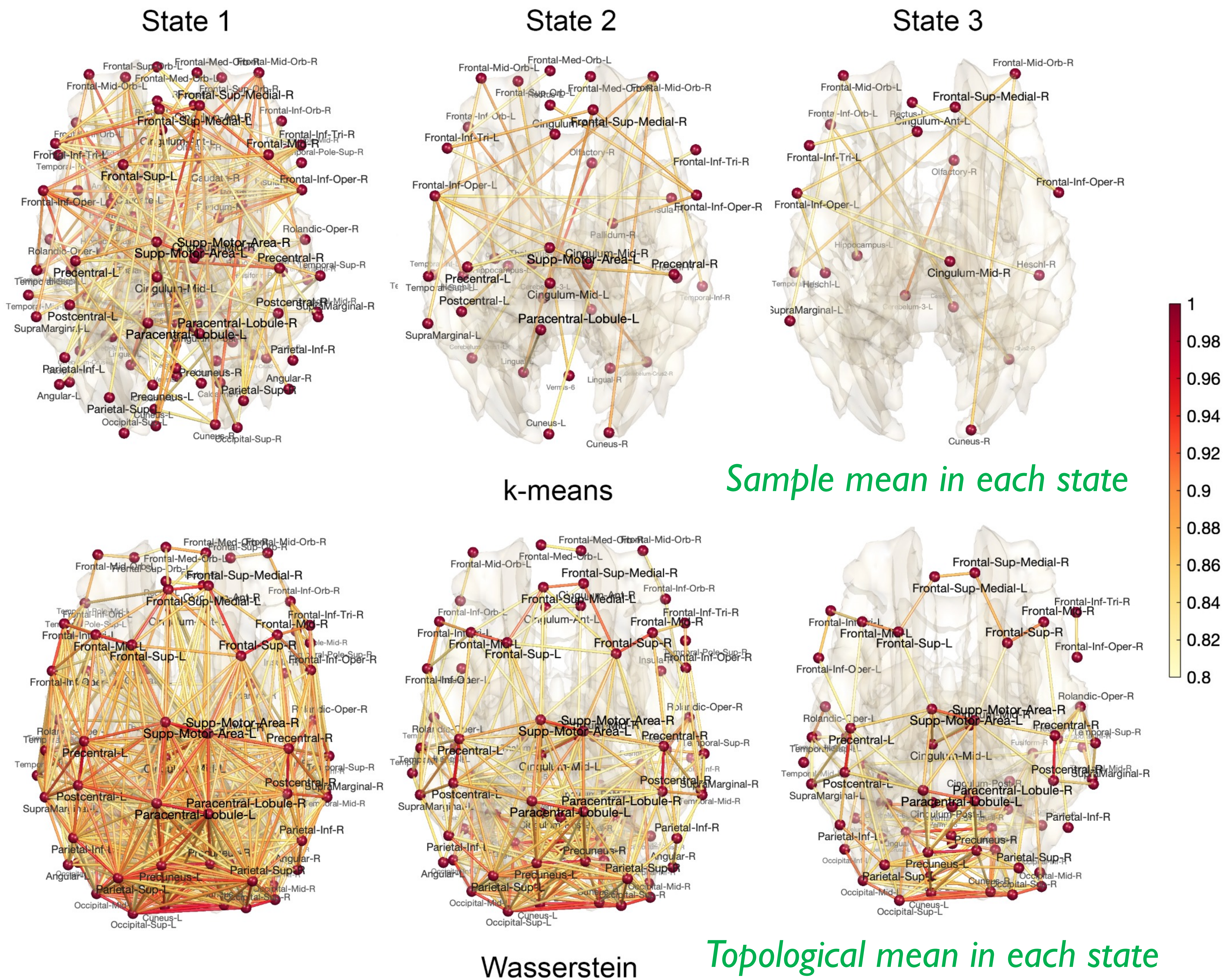
UW-Madison Twin study


3
2
1
State change
0.5
0.45
0.4
0.35
0.3
0.25
0.2
0.15
Death
0.76
0.78
0.8
0.82
0.84
Birth
Topological
embedding

$$\frac{1}{\phi} = \frac{l_W}{l_B}$$

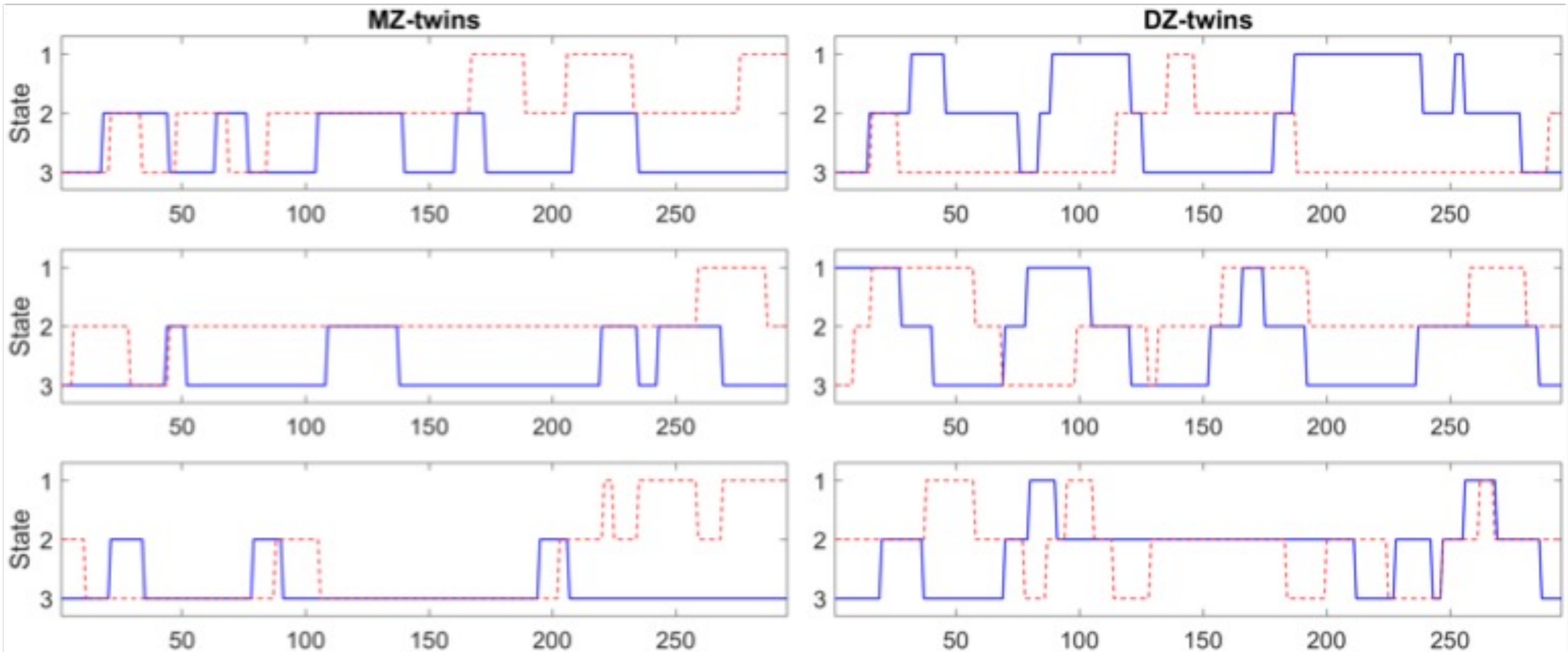


The within cluster variance *6 times* smaller

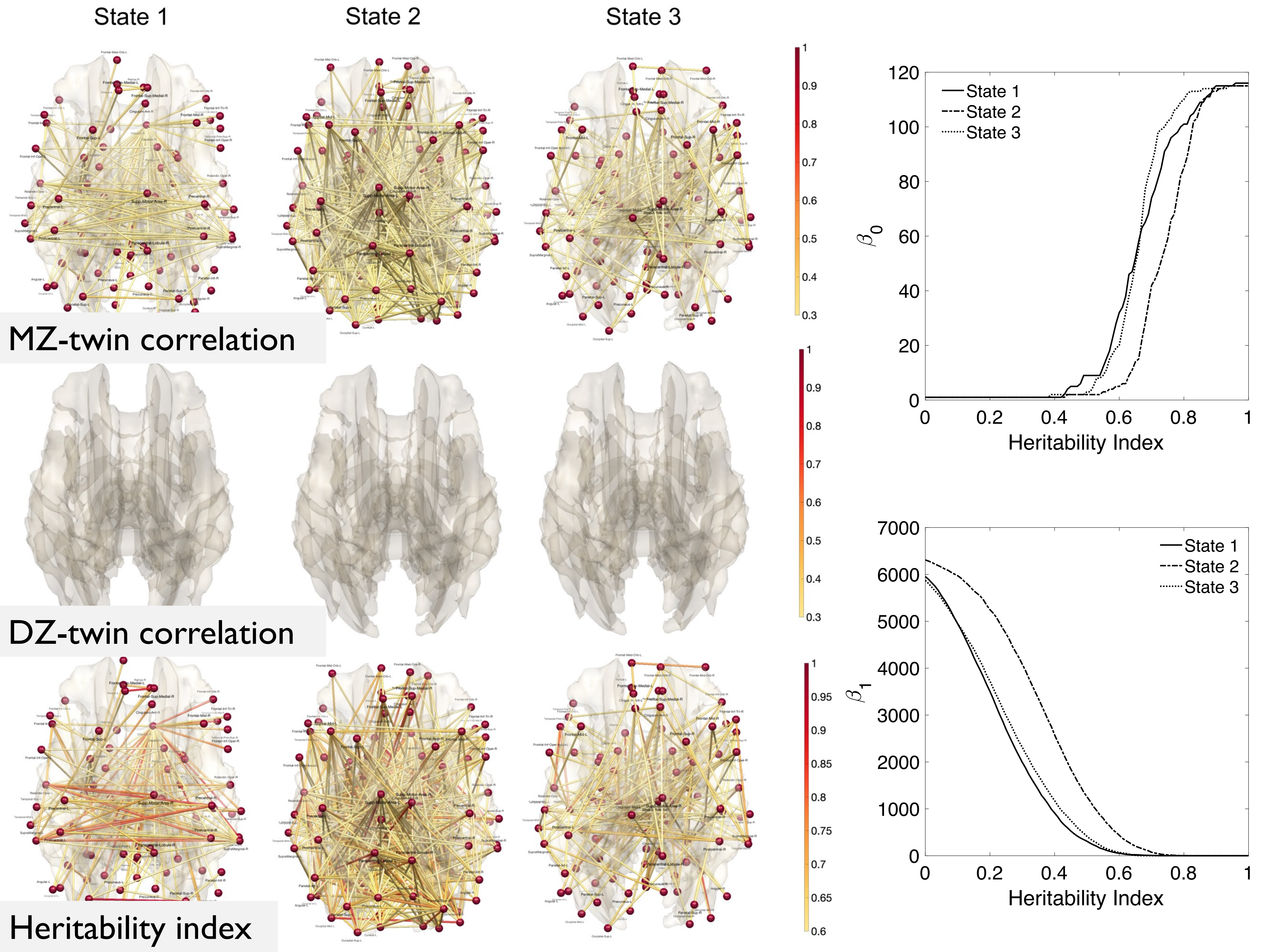


State space estimation on 479 subjects

Is the state-change heritable?



UW-Madison twin study (200 twin pairs)


State 1
State 2
State 3
MZ-twin correlation
DZ-twin correlation
Heritability index
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
1
0.95
0.9
0.85
0.8
0.75
0.7
0.65
0.6
β_0
120
100
80
60
40
20
0
0
0.2
0.4
0.6
0.8
1
Heritability Index
State 1
State 2
State 3
β_1
7000
6000
5000
4000
3000
2000
1000
0
0
0.2
0.4
0.6
0.8
1
Heritability Index
State 1
State 2
State 3

Minisymposium Topological Data Analysis and Machine Learning August 20-25, 2023 Tokyo, Japan

10th International Congress on Industrial and Applied Mathematics

ICIAM 2023 TOKYO

Organizer(s) : Jae-Hun Jung, Shizuo Kaji, Moo K. Chung

Speakers:

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- Jongbaek Song (KIAS)
- Mason Poter (UCLA)
- Keunsu Kim (POSTECH)
- Peter Bubenik (University of Florida)
- Soham Mukherjee (Purdue University)
- Alexander Strang (Chicago University)
- Mathieu Carriere (INRIA)
- Heather Harrington (Oxford University)
- Jae-Hun Jung (POSTECH)
- Shizuo Kaji (Kyushu University)
- Moo K. Chung (UW-Madison)

Thank you.


We are located here. Join us for
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