Problem: Graph Traversals

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The report should be generated in LaTex. Do not use Overleaf and must be able to compile LaTex from your computer. The codes must be written in MATLAB, utilizing a main script `main.m` that calls the tasks outlined below and produces the corresponding results. Each line of code should include adequate comments for clarity. Sample codes are given (for different project) are given in https://github.com/laplcebeltrami/PH-STAT.

A graph is denoted as $G = (V, w)$, where $V = \{1, \dots, p\}$ represents a set of vertices consisting of nodes from 1 to p, and $w = (w_{ij})$ is the $p \times p$ edge weight matrix. The edge between nodes i and j is denoted as (i, j). The edge weight w_{ij} is between nodes i and j, and $w_{ij} \in [0, 1]$. We will assume that the nodes are uniformly distributed along the unit circle.

1. Create a MATLAB function `graph_circle.m` that takes the edge weight matrix w as input and displays the graph. Nodes should be depicted as black dots, while the edges should be color-coded based on their respective weights.

2. Implement a function `graph_traverse.m` to traverse all nodes with the minimum cost. If E is the set of all the edges that the path traversed, the total cost is given by $\sum_{(i,j)\in E} w_{ij}$. The traversed path should be represented as a matrix. For example,
$$\begin{matrix} 1 & 5 & w_{15} \\ 5 & 7 & w_{57} \\ 7 & 8 & w_{78} \end{matrix}$$
represents a traversal from nodes 1 to 5 to 7 to 8.

3. Mathematically prove the correctness of your algorithm.

4. Write a function `graph_traverse_display.m` that visualizes the traversed path using arrows, overlaid on the output from `graph_circle.m`.

Solve the problem with two randomly generated graphs with 10 and 100 nodes respectively.