Problem: Graph Traversals

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The report should be generated in LaTex. Do not use Overleaf and must be able to compile LaTex from your computer. The codes must be written in MATLAB, utilizing a main script `main.m` that calls the tasks outlined below and produces the corresponding results. Each line of code should include adequate comments for clarity. Sample codes are given (for different project) in https://github.com/laplcebeltrami/PH-STAT.

A graph is denoted as $G = (V, w)$, where $V = \{1, \dots, p\}$ represents a set of vertices consisting of nodes from 1 to p, and $w = (w_{ij})$ is the $p \times p$ edge weight matrix. The edge between nodes i and j is denoted as (i, j). The edge weight w_{ij} is between nodes i and j, and $w_{ij} \in [0, 1]$. We will assume that the nodes are uniformly distributed along the unit circle.

1. Create a MATLAB function `graph_circle.m` that takes the edge weight matrix w as input and displays the graph. Nodes should be depicted as black dots, while the edges should be color-coded based on their respective weights.

2. Implement a function `graph_traverse.m` to traverse all nodes with the minimum cost. If E is the set of all the edges that the path traversed, the total cost is given by $\sum_{(i,j)\in E} w_{ij}$. The traversed path should be represented as a matrix. For example,
$$\begin{matrix} 1 & 5 & w_{15} \\ 5 & 7 & w_{57} \\ 7 & 8 & w_{78} \end{matrix}$$
represents a traversal from nodes 1 to 5 to 7 to 8.

3. Mathematically prove the correctness of your algorithm.

4. Write a function `graph_traverse_display.m` that visualizes the traversed path using arrows, overlaid on the output from `graph_circle.m`.

Solve the problem with two randomly generated graphs with 10 and 100 nodes respectively.