AdvStDaAn, Worksheet, Week 1

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Contents

Exercise 1

```r
path <- file.path('Datasets', 'Softdrink.dat')
df <- read.table(path, header=TRUE)

summary(df)
```

Dataset loading and sanity check:

```
##       Time           volume         distance       location        
##  Min.   : 8.00   Min.   : 2.00   Min.   : 10.8   Length:25         
##  1st Qu.:13.75   1st Qu.: 4.00   1st Qu.: 45.0   Class :character  
##  Median :18.11   Median : 7.00   Median : 99.0   Mode  :character  
##  Mean   :22.38   Mean   : 8.76   Mean   :122.8                     
##  3rd Qu.:21.50   3rd Qu.:10.00   3rd Qu.:181.5                     
##  Max.   :79.24   Max.   :30.00   Max.   :438.0                     
```

```r
head(df)
```

```
##    Time volume distance  location
## 1 16.68      7      168 San Diego
## 2 11.50      3       66 San Diego
## 3 12.03      3      102 San Diego
## 4 14.88      4       24 San Diego
## 5 13.75      6       45 San Diego
## 6 18.11      7       99 San Diego
```

```
tail(df)
```

```
##     Time volume distance    location
## 20 35.10     17    231.0      Austin
## 21 17.90     10     42.0      Austin
## 22 52.32     26    243.0      Austin
## 23 18.75      9    135.0      Austin
## 24 19.83      8    190.5 Minneapolis
## 25 10.75      4     45.0 Minneapolis
```

Data looks just fine.

Exercise 1.a)

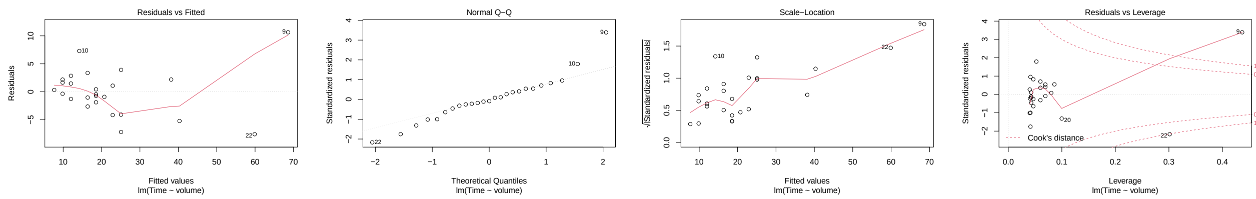
```
mod1.1 <- lm(Time ~ volume, data = df)
summary(mod1.1)
```

```
##
## Call:
## lm(formula = Time ~ volume, data = df)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -7.5811 -1.8739 -0.3493  2.1807 10.6342
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)    3.321      1.371   2.422   0.0237 *
## volume         2.176      0.124  17.546 8.22e-15 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 4.181 on 23 degrees of freedom
## Multiple R-squared:  0.9305, Adjusted R-squared:  0.9275
## F-statistic: 307.8 on 1 and 23 DF,  p-value: 8.22e-15
```

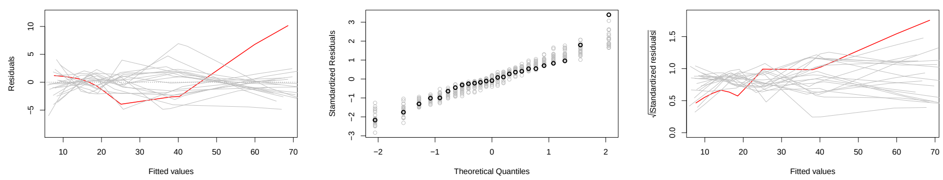
The model looks fine: - Volume is significant on the 5% niveau and the R-squared has a score of 0.93.

We have to do a residual and sensitivity analysis with stochastic simulation to investigate the correctness of the model.

```r
plot(mod1.1)
```




```r
plot.lmSim(mod1.1, SEED = 1)
```



Interpretation:

1. Tukey-Anscombe plot: Shows outlier with index i=9 which affects the smoother. In the simulation it is visible that the original curve is extreme.
   => The expectation of the residuals cannot be constant.
2. Q-Q plot: In the lower as well as in the higher part of the plot some points differ from the straigt line. Most of them are within the stochastic fluctuation except i=9.
   => The assumption of normal distributed residuals seems violated.
3. Scale-location plot: Shows a clear upwards trend. In the simulation it is visible that the original curve is extreme.
   => The variance of the residuals is not constant.
4. Residuals vs. Leverage: Observations $i = 9$ & 22 have Cooke's Distance > 1 and are therefore too influentious. Both observations have addationally too much leverage.
   => Residuals are not normally distributed.

CONCLUSION: The fit is not satisfactory. Trying transformations of response and explanatory variable. Since the noncanstant variance seems to be the most severe problem, log-transformations might help.

Exercise 1.b)

```r
df$lVolume <- log(df$volume)
df$lTime <- log(df$Time)

head(df)
```

Tukey's first-aid transformations:

```
##    Time volume distance  location  lVolume    lTime
## 1 16.68      7      168 San Diego 1.945910 2.814210
```

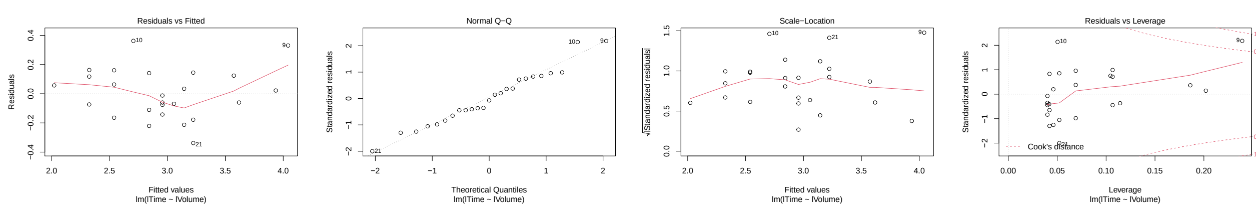
```
## 2 11.50      3       66 San Diego 1.098612 2.442347
## 3 12.03      3      102 San Diego 1.098612 2.487404
## 4 14.88      4       24 San Diego 1.386294 2.700018
## 5 13.75      6       45 San Diego 1.791759 2.621039
## 6 18.11      7       99 San Diego 1.945910 2.896464
```

Model with transformed variables:

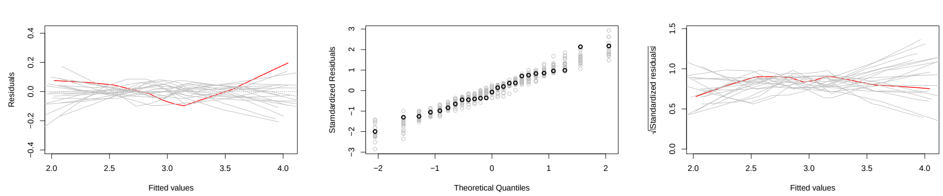
```r
mod1.2 <- lm(lTime ~ lVolume, data = df)
summary(mod1.2)
```

```
##
## Call:
## lm(formula = lTime ~ lVolume, data = df)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -0.33794 -0.11068 -0.01232  0.12385  0.36222
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)  1.50560    0.10897   13.82 1.26e-12 ***
## lVolume      0.74575    0.05317   14.03 9.25e-13 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.1738 on 23 degrees of freedom
## Multiple R-squared:  0.8953, Adjusted R-squared:  0.8908
## F-statistic: 196.7 on 1 and 23 DF,  p-value: 9.252e-13
```

```r
plot(mod1.2)
```




```r
plot.lmSim(mod1.2, SEED = 1)
```



Interpretation:

1. Tukey-Anscombe plot: The smoother shows a somewhat strange banana form with the low in the middle which is outside the stochastic fluctuation.
   => The assumption of constant expactaion is therefore violated.
2. Q-Q plot: The data scatters nicely around the straight line and seems to be within the stochastic fluctuation.
   => The assumption of Gaussian distributed errors seems not violated.
3. Scale-location plot: The smoother shows a slightly decreasing trend but seems to be ok and lies within the stochastic fluctuation.
   => There is no evidence against the assumption of constant variance of the residuals.
4. Residuals vs. Leverage: All observations have Cook's Distance <1 and therewith no too influential points.
   => No too influential (dangerous) observations

CONCLUSION: The model does still not fit adequately the data, although it is much better than the one before.

An alternative transformation for volume could be the square-root transformation. So let's try out:

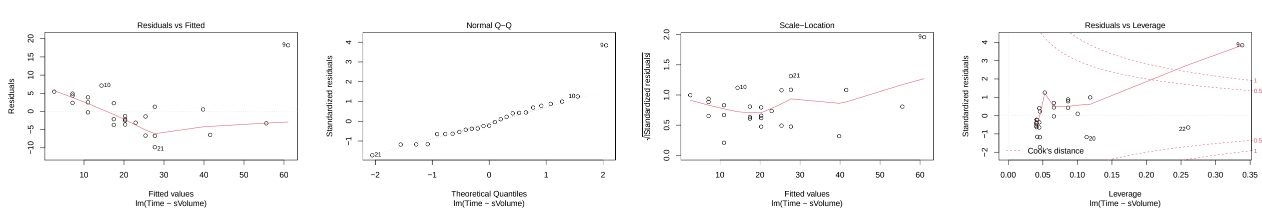
```
df$sVolume <- sqrt(df$volume)

mod1.3 <- lm(Time ~ sVolume, data = df)
summary(mod1.3)
```

```
##
## Call:
## lm(formula = Time ~ sVolume, data = df)
##
## Residuals:
##    Min     1Q Median     3Q    Max
## -9.817 -3.266 -1.284  2.509 18.212
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)  -17.788      3.510  -5.067 3.95e-05 ***
## sVolume       14.390      1.186  12.133 1.77e-11 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 5.83 on 23 degrees of freedom
## Multiple R-squared:  0.8649, Adjusted R-squared:  0.859
## F-statistic: 147.2 on 1 and 23 DF,  p-value: 1.775e-11
```

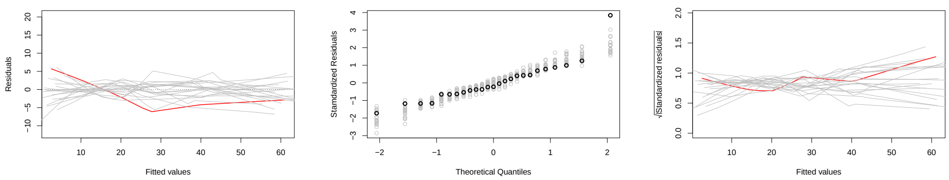
The R^2 is with 0.8649 higher than before.

Residual and Sensitivity Analysis:

```
plot(mod1.3)
```



```r
plot.lmSim(mod1.3, SEED = 1)
```



Interpretation:

1. Tukey-Anscombe plot: The smoother shows still a somewhat strange banana form with the low in the middle but like that it is inside the stochastic fluctuation.
=> The assumption of constant expactation is not violated. 2. Q-Q plot: The data scatters nicely around the straight line (except i=9) and seems to be within the stochastic fluctuation.
=> The assumption of Gaussian distributed errors seems not violated. 3. Scale-location plot: The smoother looks ok and lies within the stochastic fluctuation.
=> There is no evidence against the assumption of constant variance of the residuals. 4. Residuals vs. Leverage: All observations have Cook's Distance <1 and therewith no too influential points.
=> No too influential (dangerous) observations

$CONCLUSION$: The model does still fit adequately the data.

Exercise 1.c)

The fitte model in 1.b) is:

$$Time_i = exp(\beta_0) + exp(\beta_1) * sqrt(volume_i) + exp(E_i)$$

with

$$\mu = 0$$

$$\sigma = \sigma$$

Exercise 1.d)

Extending the model adequately with the second explanatory variable 'distance':

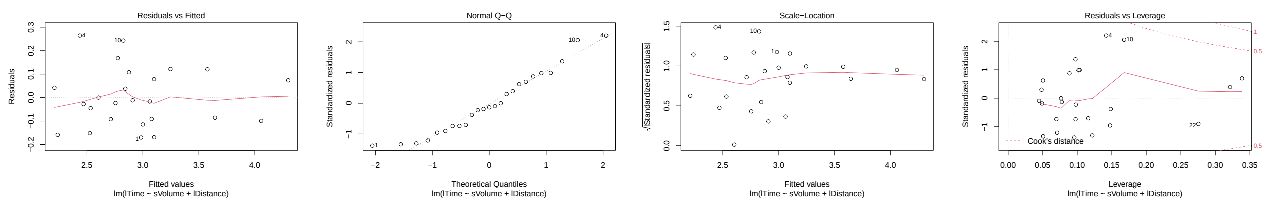
```r
df$lDistance <- log(df$distance)
mod1.4 <- lm(lTime ~ sVolume + lDistance, data = df)
summary(mod1.4)
```

```
##
## Call:
## lm(formula = lTime ~ sVolume + lDistance, data = df)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -0.17012 -0.09203 -0.01658  0.07866  0.26425
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)  1.14704    0.14300   8.021 5.65e-08 ***
## sVolume      0.41553    0.03649  11.389 1.08e-10 ***
```

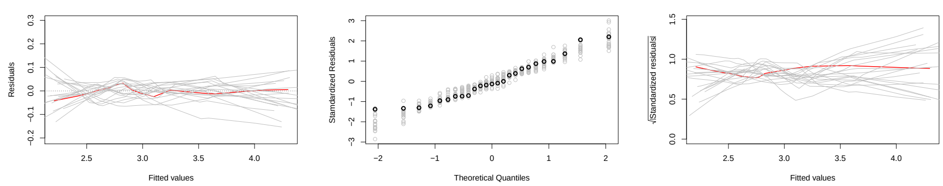
```
## lDistance    0.14401    0.04234   3.401  0.00256 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.1296 on 22 degrees of freedom
## Multiple R-squared:  0.9443, Adjusted R-squared:  0.9393
## F-statistic: 186.6 on 2 and 22 DF,  p-value: 1.587e-14
```

The R^2 increases to 0.9443 which is a really good fit. Lets check the model:

```
plot(mod1.4)
```




```
plot.lmSim(mod1.4, SEED = 1)
```



Interpretation:

1. Tukey-Anscombe plot: The smoother shows a really nice almost straight line which is within the stochastic fluctuation.
   => The assumption of constant expectation is not violated.
2. Q-Q plot: The data scatters nicely around the straight line and seems to be within the stochastic fluctuation.
   => The assumption of Gaussian distributed errors seems not violated.
3. Scale-location plot: The smoother shows a almost straight line and is within the stochastic fluctuation.
   => There is no evidence against the assumption of constant variance of the residuals.
4. Residuals vs. Leverage: All observations have Cook's Distance <1 and therewith no too influential points. Some are leverage point with leverage $> 2 * 3/25 = 0.24$ (25 examples in dataset)
   => No too influential (dangerous) observations

CONCLUSION: The model does fit adequately the data.

Exercise 2

```r
path <- file.path('Datasets', 'Windmill.dat')
df <- read.table(path, header = TRUE)

summary(df)
```

Loading and Checking the data

```
##     velocity        DC.output    
##  Min.   : 5.482   Min.   :0.123  
##  1st Qu.: 8.838   1st Qu.:1.144  
##  Median :13.424   Median :1.800  
##  Mean   :13.720   Mean   :1.610  
##  3rd Qu.:18.235   3rd Qu.:2.166  
##  Max.   :22.822   Max.   :2.386
```

```r
dim(df)
```

```
## [1] 25  2
```

```r
head(df)
```

```
##    velocity DC.output
## 1 11.187073     1.582
## 2 13.424487     1.822
## 3  7.607209     1.057
## 4  6.041019     0.500
## 5 22.374145     2.236
## 6 21.702921     2.386
```

```r
tail(df)
```

```
##     velocity DC.output
## 20 12.193909     1.501
## 21 20.360472     2.303
## 22 22.821628     2.310
## 23  9.173400     1.194
## 24  8.837787     1.144
## 25  5.481666     0.123
```

Exercise 2.a)

Start with fitting an ordinary regression model:

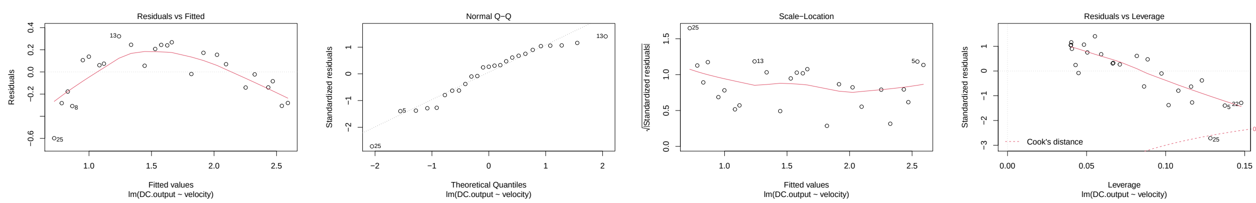
```r
mod2.1 <- lm(DC.output ~ velocity, data = df)
summary(mod2.1)
```

```
## 
## Call:
## lm(formula = DC.output ~ velocity, data = df)
## 
```

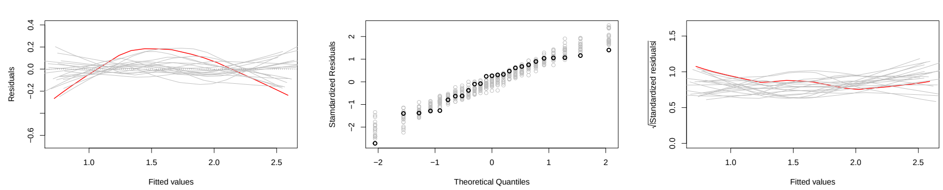
```
## Residuals:
##      Min       1Q   Median       3Q      Max
## -0.59869 -0.14099  0.06059  0.17262  0.32184
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept) 0.130875   0.125989   1.039     0.31
## velocity    0.107780   0.008514  12.659 7.55e-12 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.2361 on 23 degrees of freedom
## Multiple R-squared:  0.8745, Adjusted R-squared:  0.869
## F-statistic: 160.3 on 1 and 23 DF,  p-value: 7.546e-12
```

The model seems to fit not too bad. Lets check this:

```
plot(mod2.1)
```




```
plot.lmSim(mod2.1, SEED = 1)
```



Interpretation:

1. Tukey-Anscombe plot: The smoother shows a banana form which is outside the stochastic fluctuation. => The assumption of constant expectation is violated.
2. Q-Q plot: The data does not scatter nicely around the straight line and but seems to be within the stochastic fluctuation.
   => The assumption of Gaussian distributed errors seems not violated.
3. Scale-location plot: The smoother shows a almost straight line and is within the stochastic fluctuation. => There is no evidence against the assumption of constant variance of the residuals.
4. Residuals vs. Leverage: All observations have Cook's Distance <1 and therewith no too influential points. There are also no leverage points with leverage $> 2 * 2/25 = 0.16$ (25 examples in dataset, 2 variables in model)
   => No too influential (dangerous) observations

CONCLUSION: The model does not fit adequately the data. Maybe some transfromations would help to remedy the inadequacy. -> Exercise 2.b)

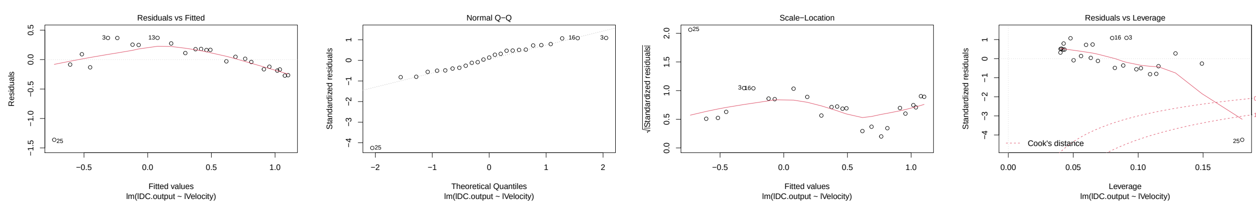
Exercise 2.b)

```r
df$lVelocity <- log(df$velocity)
df$lDC.output <- log(df$DC.output)

mod2.2 <- lm(lDC.output ~ lVelocity, data = df)
summary(mod2.2)
```

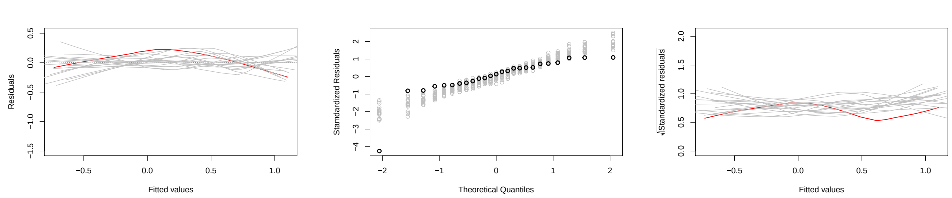
```
##
## Call:
## lm(formula = lDC.output ~ lVelocity, data = df)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -1.36184 -0.13163  0.04707  0.18075  0.36880
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)  -2.9238     0.4112  -7.110 3.05e-07 ***
## lVelocity     1.2872     0.1603   8.031 4.01e-08 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.3537 on 23 degrees of freedom
## Multiple R-squared:  0.7371, Adjusted R-squared:  0.7257
## F-statistic:  64.5 on 1 and 23 DF,  p-value: 4.014e-08
```

Variable seems to be significant and $R^{2} is with 0.7371 not too bad. But we could surely do better with some adjustments. Lets check the model:

```r
plot(mod2.2)
```




```r
plot.lmSim(mod2.2, SEED = 1)
```



Interpretation:
1. Tukey-Anscombe plot: The smoother still shows a banana form which is outside the stochastic fluctuation.

=> The assumption of constant expactation is violated. 2. Q-Q plot: The data does better scatter around the straight line (except outlier i=25) but is outside the stochastic fluctuation.
=> The assumption of Gaussian distributed errors is violated. 3. Scale-location plot: The smoother shows a wavy line and is within the stochastic fluctuation.
=> There is no evidence against the assumption of constant variance of the residuals. 4. Residuals vs. Leverage: Observation i=25 has Cook's Distance >1 and therewith is too influential. But there are no leverage points with leverage $> 2 * 2/25 = 0.16$ (25 examples in dataset, 2 variables in model)
=> i=25 is too influential observations

CONCLUSION: The model does not fit adequately the data. Maybe some transfromations would help to remedy the inadequacy. -> Exercise 2.c)

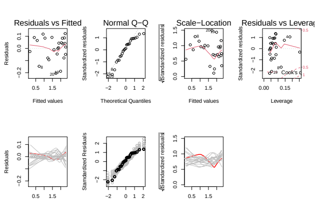
Exercise 2.c)

```r
df$tVelocity <- 1/df$velocity

mod2.3 <- lm(DC.output ~ tVelocity, data = df)
summary(mod2.3)
```

```
##
## Call:
## lm(formula = DC.output ~ tVelocity, data = df)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -0.20547 -0.04940  0.01100  0.08352  0.12204
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)   2.9789     0.0449   66.34   <2e-16 ***
## tVelocity   -15.5155     0.4619  -33.59   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.09417 on 23 degrees of freedom
## Multiple R-squared:   0.98,  Adjusted R-squared:  0.9792
## F-statistic:  1128 on 1 and 23 DF,  p-value: < 2.2e-16
```

This model seems to fit the data way better. The transformation from theroy seems pretty adequat. Lets check the model:

```r
par(mfrow = c(2,4))
plot(mod2.3)
plot.lmSim(mod2.3, SEED = 1)
```



Interpretation:

1. Tukey-Anscombe plot: The smoother shows now no banana form anymore and is within the stochastic fluctuation.
   => The assumption of constant expectation of the errors is not violated.
2. Q-Q plot: The data does scatter around the straight line and is within the stochastic fluctuation.
   => The assumption of Gaussian distributed errors is not violated.
3. Scale-location plot: The smoother shows a little wavy line but is within the stochastic fluctuation.
   => There is no evidence against the assumption of constant variance of the residuals.
4. Residuals vs. Leverage: No Observation has Cook's Distance >1. And there are no leverage points with leverage $> 2 * 2/25 = 0.16$ (25 examples in dataset, 2 variables in model)
   => i=25 is too influential observations

CONCLUSION: The model does fit adequately the data. The transformation of the velocity variable did indeed remedy the model inadequaties.

Exercise 2.d)

Plotting the model for the interpretation of the parameters β_0 and β_1.

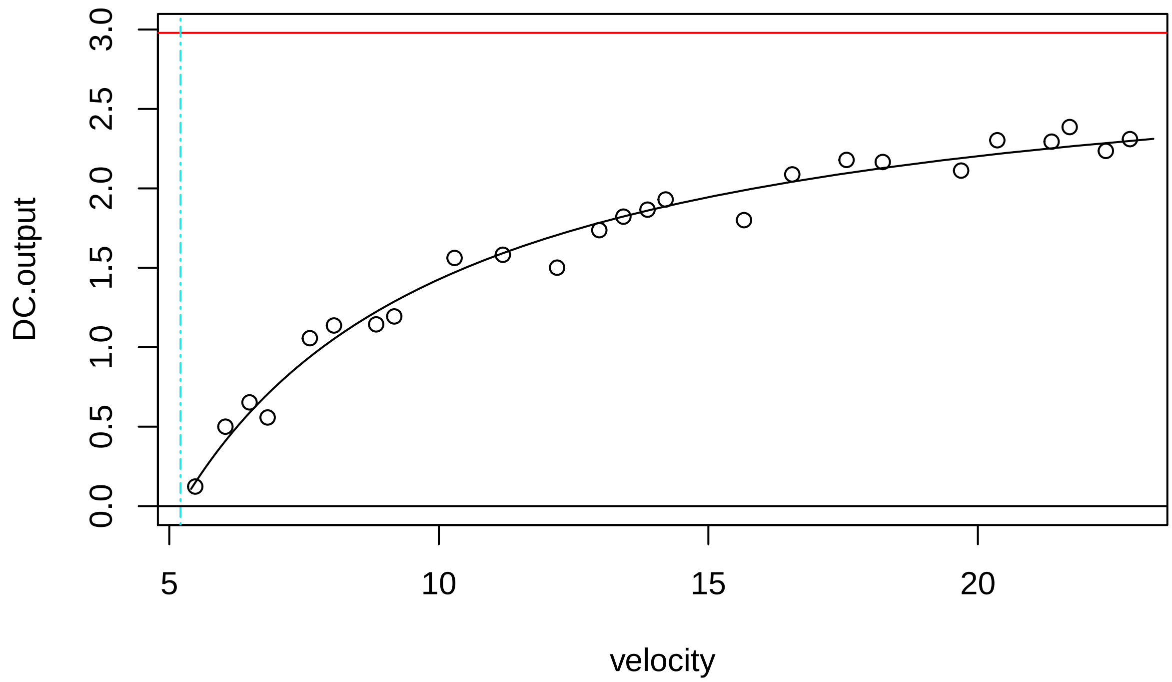
```
par(mfrow=c(1,1))
plot(DC.output ~ velocity, data = df,
     ylim = range(df$DC.output, coef(mod2.3)[1], 0))
# -> The range() takes the larger value of DC.output and the intercept
abline(h = coef(mod2.3)[1], col = "red")

# predicted DC.output
range(df$tVelocity)
```

```
## [1] 0.04381808 0.18242629
```

```
df2 <- data.frame(tVelocity = seq(0.043, 0.185, length = 50))
lines(1/df2$tVelocity, predict(mod2.3, newdata = df2))

# How much wind is needed at least?
(h.minW <- -coef(mod2.3)[2]/coef(mod2.3)[1])  ## = 5.208521 m/s
```

```
## tVelocity
##  5.208521
```

```
abline(v=h.minW, col=5, lty=6)
abline(h=0)
```



DC.output
0.0
0.5
1.0
1.5
2.0
2.5
3.0
5
10
15
20
velocity