AdvStDaAn, Worksheet, Week 11

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Contents

Task 1

You can choose from two different types of light-emitting diodes.

You order 100 diodes of type 1 and 100 diodes of type 2. 8 diodes of type 1 and 12 diodes of type 2 were insufficient for your purposes. However diodes of type 2 are 10% cheaper.

Is it beneficial to buy diodes of type 2? Reject rates usually vary around 10% ± 5%. Propose an appropriate prior and perform a Bayesian analysis.

First one can calculate a and b for the beta prior.

$E(X) = \frac{a}{a+b} = 0.1$ b = 9a

$Var(X) = \frac{ab}{(a+b+1)(a+b)^2} = 0.05^2$

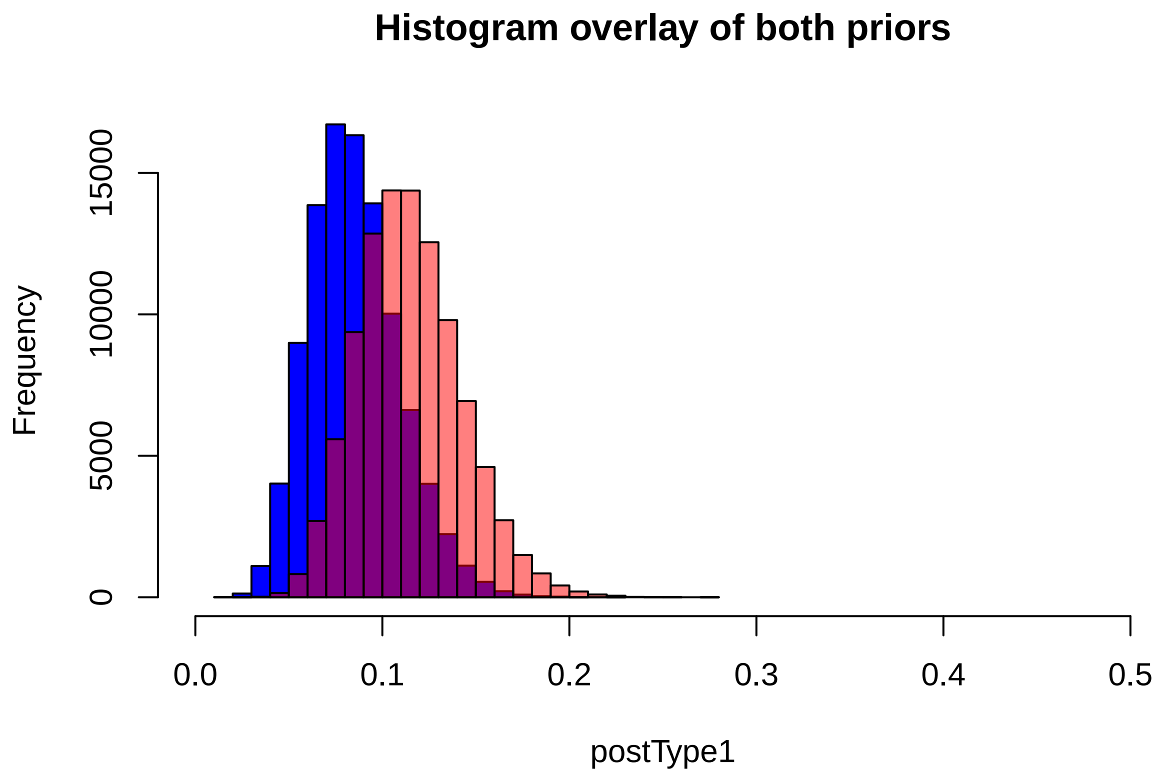
set b = 9a

$\frac{a*9a}{(a+9a+1)(a+9a)^2} = 0.05^2$

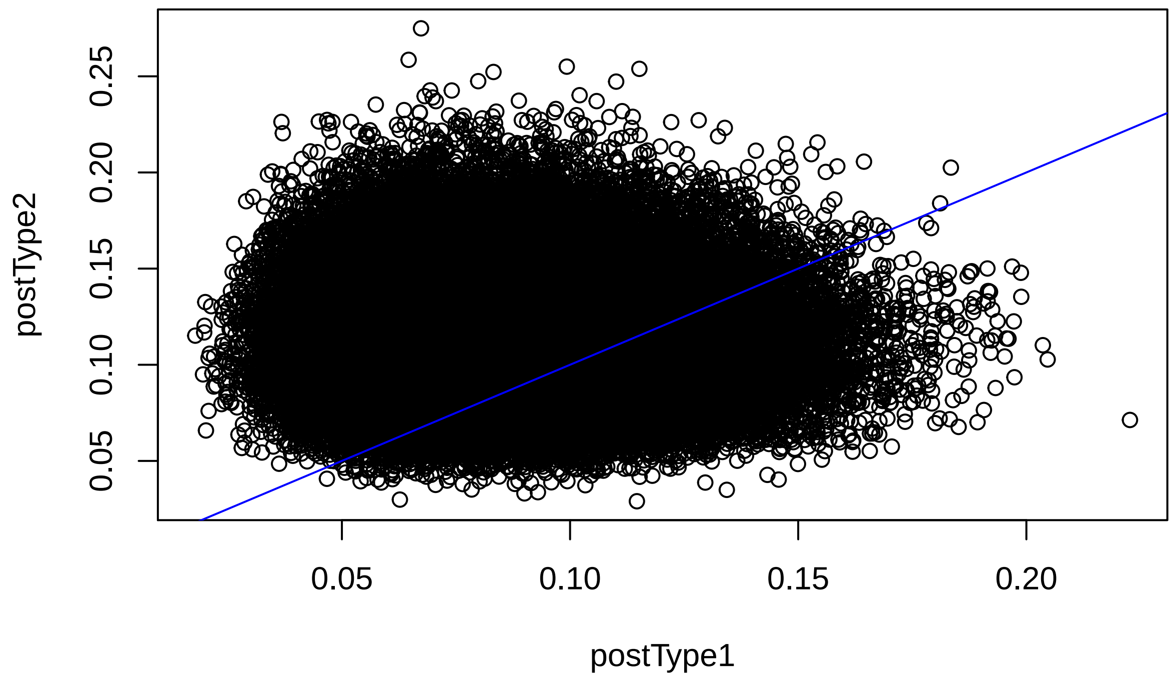
a = 3.5
b = 31.5

```
# Simulate reject rates for diodes of type 1 and type 2
nSamples = 100000
postType1 = rbeta(nSamples, 3.5 +  8, 31.5 + 92)
postType2 = rbeta(nSamples, 3.5 + 12, 31.5 + 88)

hist(postType1, xlim = c(0, .5), col = 'blue', main = 'Histogram overlay of both priors')
hist(postType2, xlim = c(0, .5), col = alpha('red', 0.5), add = TRUE)
```




```r
plot(postType1, postType2)
abline(0,1,col="blue")
```




```
sum(postType1< postType2)/nSamples
```

```
## [1] 0.79644
```

```
# Calculate expected reject rates
expType1 = (3.5 +   8)/ (3.5 +   8 +31.5 + 92) # -> E(X)
expType2 = (3.5 +  12)/ (3.5 +  12 +31.5 + 88) # -> E(X)

# To compare, include that diodes of type 2 are 10% cheaper
(1-expType1)
```

```
## [1] 0.9148148
```

```
(1-expType2)/.9
```

```
## [1] 0.9835391
```

=> Type 2 is better than Type 1.

Task 2

Given are 6 trials and their numbers of trys before a success occurs: 5, 8, 3, 9, 2 and 6.

- In what range is the rate of success θ?
- What's the probability, that the rate of success is bigger than 25%?

```
n <- 6
obs <- c(5, 8, 3, 9, 2, 6)
obs_sum <- sum(obs)
obs_sum
```

```
## [1] 33
```

```
# calculating the range of the rate of success
qbeta(c(0.05, 0.95), 1 + n, 1 + obs_sum)
```

```
## [1] 0.0851325 0.2747445
```

```
# calculating the probability that the rate of success is bigger than 25%
1 - pbeta(0.25, 1+n, 1+obs_sum)
```

```
## [1] 0.09622459
```

A flat prior seems not very plausible, as data indicates that the success rate should be small. Using a flat prior probably gives too much weight to high success rates and therefore overestimates the real success rate.

Task 3

You have two machines producing a good. Your quality control tracks the number of proper goods until the machine produces a poor good.

- The counts of machine 1 are: 10, 15, 18, 20, 5, 12 and 3.
- The counts of machine 2 are: 23, 16, 19, 28 and 37.

Assume a Beta(1, 20) prior on the failure rate and calculate an equal-tailed 95% credible interval for each machine. What's the expected failure rate for each machine? Calculate the maximum a posteriori probability (MAP) estimate for each failure rate.

```
# number of observations
nM1 <- 7
nM2 <- 5

# sum of failures
(sumObsM1 <- sum(c(10, 15, 18, 20, 5, 12, 3)))
```

```
## [1] 83
```

```r
(sumObsM2 <- sum(c(23, 16, 19, 28, 37)))
```

```
## [1] 123
```

```r
# parameters of both distribution
aM1 <- 1+nM1
bM1 <- 20+sumObsM1
aM2 <- 1+nM2
bM2 <- 20+sumObsM2

# equal-tailed 95% credible interval
qbeta(c(0.025, 0.975), 1+nM1, 20+sumObsM1)
```

```
## [1] 0.03192018 0.12672593
```

```r
qbeta(c(0.025, 0.975), 1+nM2, 20+sumObsM2)
```

```
## [1] 0.01502071 0.07707699
```

```r
# expected failure rate (E(X))
expFailM1 <- (1+nM1) / ((1+nM1) + 20+sumObsM1)
expFailM1
```

```
## [1] 0.07207207
```

```r
expFailM2 <- (1+nM2) / ((1+nM2) + 20+sumObsM2)
expFailM2
```

```
## [1] 0.04026846
```

```r
# maximum a posteriori probability (MAP) -> mode(X)
mapM1 <- ((1+nM1) - 1) / ((1+nM1) + (20+sumObsM1) - 2)
mapM1
```

```
## [1] 0.06422018
```

```r
mapM2 <- ((1+nM2) - 1) / ((1+nM2) + (20+sumObsM2) - 2)
mapM2
```

```
## [1] 0.03401361
```

Task 4

- What's the probability of observing two failures before a success occurs if θ is the probability of success?
- Calculate the maximum likelihood estimate for the parameter θ of the geometric distribution, when you observed two failures before a success occurs.

- What's the posterior distribution of θ conditioned on two failures before a success occurs, when you assume a flat prior?
- What's the posterior mean of θ?
- What's the maximum a posteriori probability estimate of θ?

Probability of observing two failures before a success occurs: $\theta(1-\theta)^2$

mle estimate for θ:

```
1/3
```

```
## [1] 0.3333333
```

Posterior distribution of θ conditioned on two failures before a success occurs with a flat prior? $\theta|D \sim$ beta(1 + 1, 1 + 2)

Posterior mean of θ? $E(X) = \frac{a}{a+b} = \frac{2}{2+3} = 0.4$

Maximum a posteriori probability(MAP -> mode(X) estimate of θ? $mode(X) = \frac{a-1}{a+b-2} = \frac{2-1}{2+3-2}$ 0.3333333

Question Task 1

|>Q1: How do we come up with the equations in the mle calculation? Where comes the argmax argument from?
