AdvStDaAn, Worksheet, Week 2

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Contents

Exercise 1

```r
path <- file.path('Datasets', 'sniffer.dat')
df <- read.table(path, header=TRUE)

summary(df)
```

Dataset loading and sanity check:

```
##    Temp.Tank        Temp.Gas       Vapor.Tank    Vapor.Dispensed
##  Min.   :31.00   Min.   :35.00   Min.   :2.590   Min.   :2.590
##  1st Qu.:37.00   1st Qu.:41.00   1st Qu.:3.290   1st Qu.:3.373
##  Median :60.00   Median :60.00   Median :4.285   Median :4.090
##  Mean   :57.91   Mean   :55.91   Mean   :4.422   Mean   :4.324
##  3rd Qu.:62.00   3rd Qu.:62.00   3rd Qu.:4.630   3rd Qu.:4.540
##  Max.   :92.00   Max.   :92.00   Max.   :7.450   Max.   :7.450
##        Y
##  Min.   :16.00
```

```
##   1st Qu.:23.75
##   Median :31.50
##   Mean   :31.12
##   3rd Qu.:34.50
##   Max.   :55.00
```

```r
dim(df)
```

```
## [1] 32  5
```

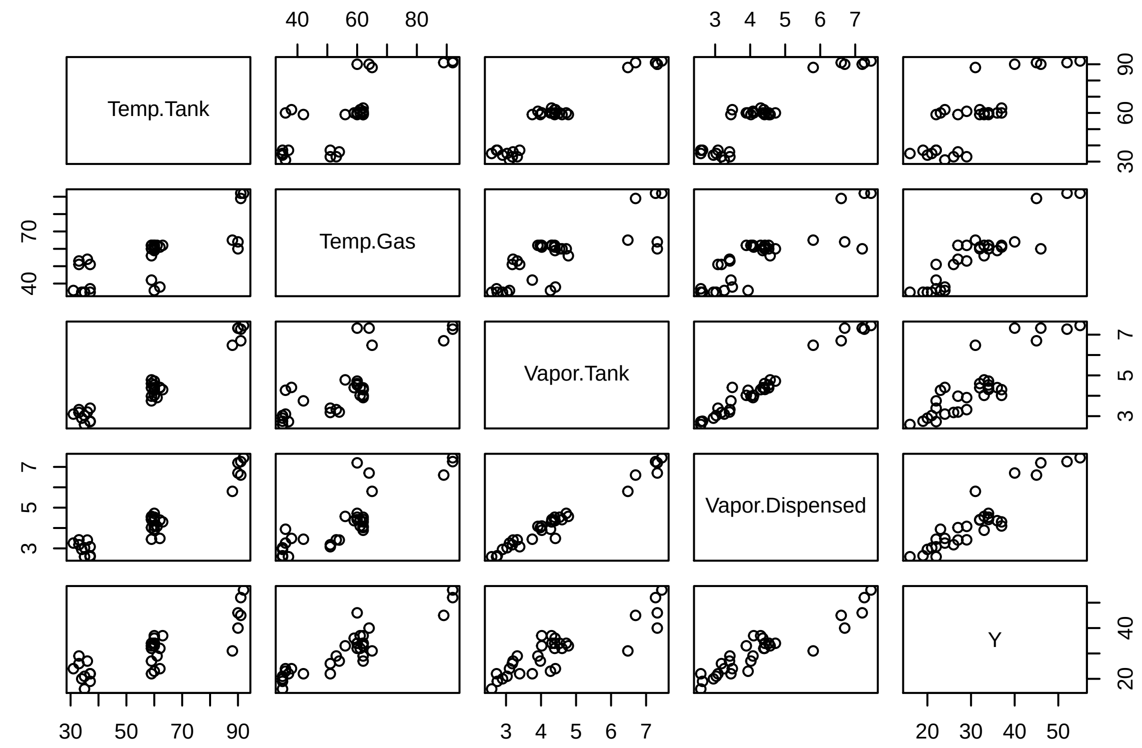
```r
head(df)
```

```
##   Temp.Tank Temp.Gas Vapor.Tank Vapor.Dispensed  Y
## 1        33       53       3.32            3.42 29
## 2        31       36       3.10            3.26 24
## 3        33       51       3.18            3.18 26
## 4        37       51       3.39            3.08 22
## 5        36       54       3.20            3.41 27
## 6        35       35       3.03            3.03 21
```

```r
tail(df)
```

```
##    Temp.Tank Temp.Gas Vapor.Tank Vapor.Dispensed  Y
## 27        60       62       4.02            3.89 33
## 28        59       62       3.98            4.02 27
## 29        59       62       4.39            4.53 34
## 30        37       35       2.75            2.64 19
## 31        35       35       2.59            2.59 16
## 32        37       37       2.73            2.59 22
```

```r
plot(df)
```



Data looks like it is highly correlated with each other. But we keep it this way for the first exercises.

Exercise 1.a)

Fitting a first model without any transformations to the data:

```
lm1.1 <- lm(Y ~ ., data = df)
```

The model looks initially not too bad. For a proper evaluation one would need to perform a residual and sensitivity analysis to investigate the adequacy of the model. But for this exercise we keep the track of the worksheet.

E1.a)(I) Estimated coefficients

```
coef(lm1.1)
```

```
##     (Intercept)       Temp.Tank        Temp.Gas      Vapor.Tank Vapor.Dispensed
##      1.01501756     -0.02860886      0.21581693     -4.32005167      8.97488928
```

E1.a)(II) F-statistic

```
summary(lm1.1)
```

```
##
```

```
## Call:
## lm(formula = Y ~ ., data = df)
##
## Residuals:
##    Min     1Q Median     3Q    Max
## -5.586 -1.221 -0.118  1.320  5.106
##
## Coefficients:
##                 Estimate Std. Error t value Pr(>|t|)
## (Intercept)      1.01502    1.86131   0.545  0.59001
## Temp.Tank       -0.02861    0.09060  -0.316  0.75461
## Temp.Gas         0.21582    0.06772   3.187  0.00362 **
## Vapor.Tank      -4.32005    2.85097  -1.515  0.14132
## Vapor.Dispensed  8.97489    2.77263   3.237  0.00319 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 2.73 on 27 degrees of freedom
## Multiple R-squared:  0.9261, Adjusted R-squared:  0.9151
## F-statistic: 84.54 on 4 and 27 DF,  p-value: 7.249e-15
```

The p-value of the F-statistic is « 0.05 indicating that at least one of the variables can not be 0 and therfore are important to describe the response value. Even though, the p-values of the t-test indicate that not all of them are of the same importance. In this case are only 2 explanatory variables significantly important (Temp.Gas & Vapor.Dispensed).

E1.a)(III) Variance Inflation Factor (VIF)

Inspecting multicollinearity with the Variance Inflation Factor (VIF):

```
library(car)
```

```
## Loading required package: carData
```

```
vif(lm1.1)
```

```
##       Temp.Tank        Temp.Gas      Vapor.Tank Vapor.Dispensed
##       12.997379        4.720998       71.301491       61.932647
```

A vif above 5 to 10 indicates problems with multicollinearity. According to this guideline all variables but Temp.Gas have too high vif factors and therewith problems with multicollinearity. Vapor.Tank is affected the most.

Exercise 1.b)

Performing a variable selection using the AIC stepwise from the model fitted in Exercise 1.a):

```
step(lm1.1)
```

```
## Start:  AIC=68.84
## Y ~ Temp.Tank + Temp.Gas + Vapor.Tank + Vapor.Dispensed
##
```

```
##                   Df Sum of Sq    RSS    AIC
## - Temp.Tank        1     0.743 201.97 66.956
## <none>                         201.23 68.838
## - Vapor.Tank       1    17.113 218.34 69.450
## - Temp.Gas         1    75.698 276.93 77.056
## - Vapor.Dispensed  1    78.090 279.32 77.332
##
## Step:  AIC=66.96
## Y ~ Temp.Gas + Vapor.Tank + Vapor.Dispensed
##
##                   Df Sum of Sq    RSS    AIC
## <none>                         201.97 66.956
## - Vapor.Tank       1    36.416 238.39 70.261
## - Temp.Gas         1    78.831 280.80 75.501
## - Vapor.Dispensed  1    91.850 293.82 76.952
```

```
##
## Call:
## lm(formula = Y ~ Temp.Gas + Vapor.Tank + Vapor.Dispensed, data = df)
##
## Coefficients:
##     (Intercept)         Temp.Gas       Vapor.Tank  Vapor.Dispensed
##          1.0655           0.2091          -4.8882           9.2480
```

The best model with the stepwise variable selection from the model in Exercise 1.a) is
$Y \sim Temp.Gas + Vapor.Tank + Vapor.Dispensed$
Temp.Tank gets not included. This would be due to multicollinearity with other variables.

Exercise 1.c)

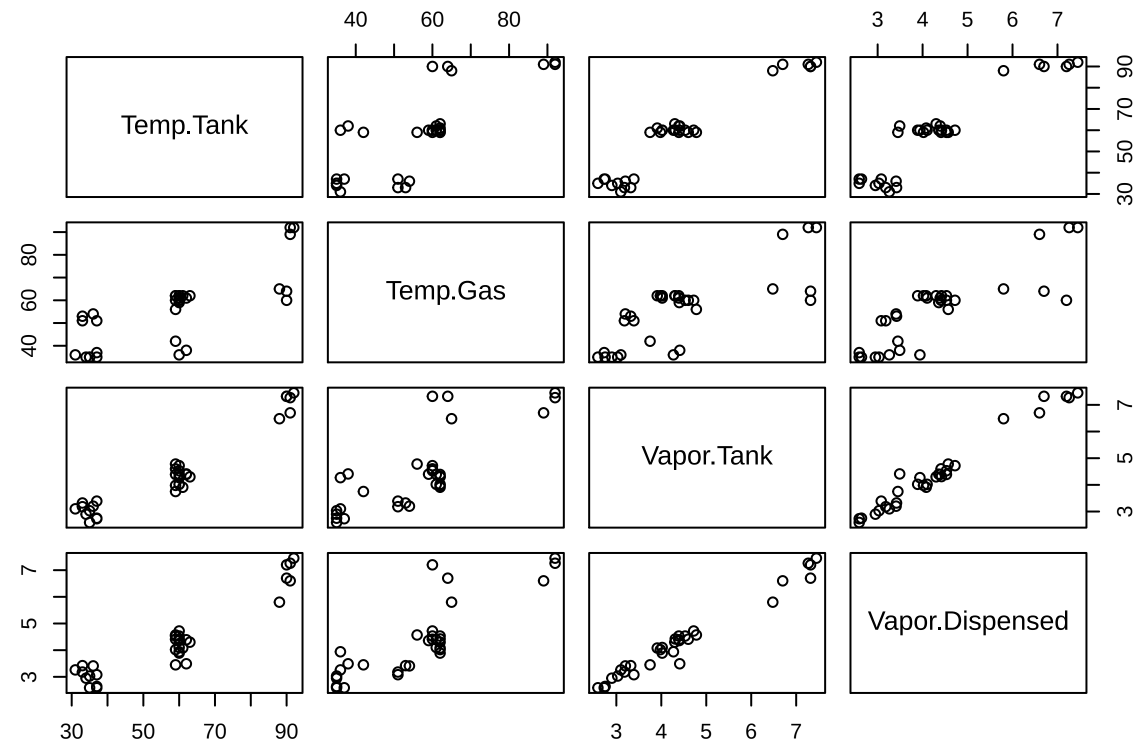
Did we already remedy the initially found multicollinearity with the stepwise variable selection? We can check by performing a vif on the newly found model.

```r
lm1.2 <- lm(Y ~ Temp.Gas + Vapor.Tank + Vapor.Dispensed, data = df)
vif(lm1.2)
```

```
##        Temp.Gas      Vapor.Tank Vapor.Dispensed
##        4.255787       42.899447       55.907555
```

No, Vapor.Tank and Vapor.Dispensed have still vif values from above 5 to 10. Which ones are correlated the most?

```r
pairs(df[,-5])
```



Vapor.Tank and Vapor.Dispensed seem to be correlated the most. So we try transformations of the variables by replacing them by the mean and the difference.

```
df2 <- data.frame(diffVapor = df$Vapor.Tank - df$Vapor.Dispensed,
                  meanVapor = (df$Vapor.Tank + df$Vapor.Dispensed) / 2)

df3 <- cbind(df2, Temp.Tank = df$Temp.Tank, Temp.Gas = df$Temp.Gas, Y = df$Y)

head(df3)
```

```
##   diffVapor meanVapor Temp.Tank Temp.Gas  Y
## 1     -0.10     3.370        33       53 29
## 2     -0.16     3.180        31       36 24
## 3      0.00     3.180        33       51 26
## 4      0.31     3.235        37       51 22
## 5     -0.21     3.305        36       54 27
## 6      0.00     3.030        35       35 21
```

With the newly created data.frame with the transformed variables one can now perform another stepwise variable selection.

```
lm1.3 <- lm(Y ~ ., data = df3)
step(lm1.3)
```

```
## Start:  AIC=68.84
```

```
## Y ~ diffVapor + meanVapor + Temp.Tank + Temp.Gas
##
##             Df Sum of Sq    RSS    AIC
## - Temp.Tank  1     0.743 201.97 66.956
## <none>                   201.23 68.838
## - diffVapor  1    43.585 244.81 73.112
## - Temp.Gas   1    75.698 276.93 77.056
## - meanVapor  1   114.810 316.04 81.284
##
## Step:  AIC=66.96
## Y ~ diffVapor + meanVapor + Temp.Gas
##
##             Df Sum of Sq    RSS    AIC
## <none>                   201.97 66.956
## - diffVapor  1    64.398 266.37 73.813
## - Temp.Gas   1    78.831 280.80 75.501
## - meanVapor  1   265.710 467.68 91.826
```

```
##
## Call:
## lm(formula = Y ~ diffVapor + meanVapor + Temp.Gas, data = df3)
##
## Coefficients:
## (Intercept)    diffVapor    meanVapor     Temp.Gas
##      1.0655      -7.0681       4.3597       0.2091
```

This is the same model as found in Exercise 1.b) but with the transformed variables. Now one can check if the problems with multicollinearity still persists.

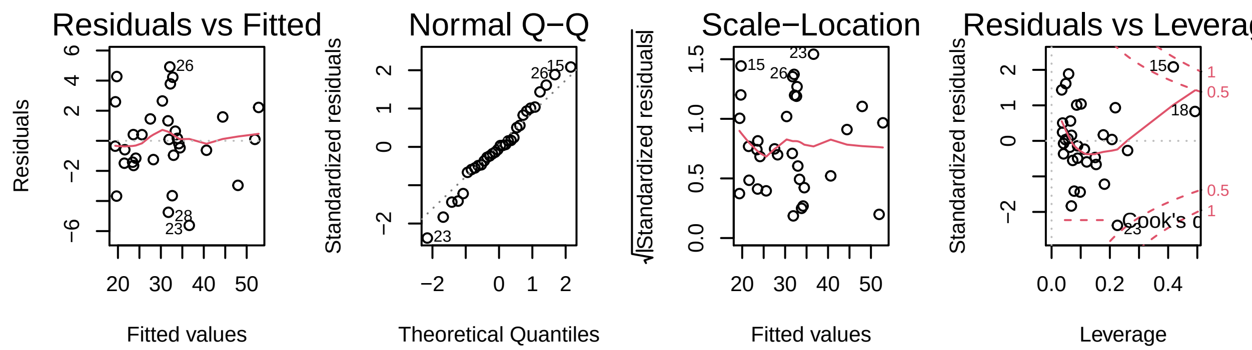
```
lm1.4 <- lm(Y ~ diffVapor + meanVapor + Temp.Gas, data = df3)
vif(lm1.4)
```

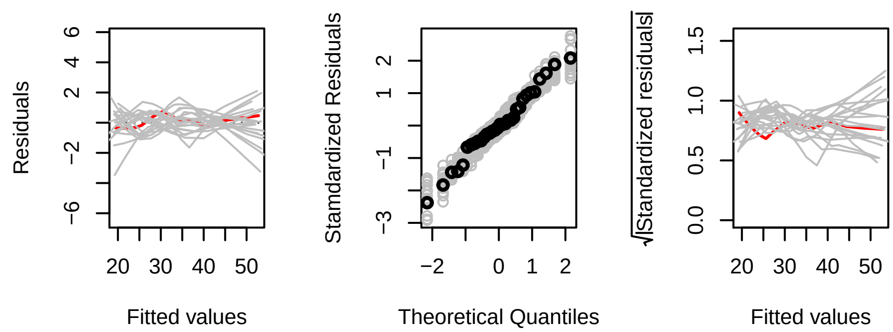
```
## diffVapor meanVapor  Temp.Gas
##  1.538981  4.450470  4.255787
```

All vif values are lower than 5 and therewith the problem with multicollinearity does not persist.

How looks the residual and sensitivity analysis?

```
par(mfrow = c(2, 4))
plot(lm1.4)
plot.lmSim(lm1.4, SEED = 1)
```





leverage points > 0.25

There is no evidence that any of the assumptions is violated.

Exercise 2

```
path <- file.path('Datasets', 'jet.dat')
df <- read.table(path, header=TRUE)

summary(df)
```

Dataset loading and sanity check:

```
##        Y              x1             x2              x3              x4
##  Min.   :3045   Min.   :1388   Min.   :17780   Min.   :28675   Min.   :133.0
##  1st Qu.:3518   1st Qu.:1608   1st Qu.:18880   1st Qu.:29302   1st Qu.:155.2
##  Median :3977   Median :1850   Median :19765   Median :29745   Median :179.0
##  Mean   :3904   Mean   :1810   Mean   :19495   Mean   :29606   Mean   :174.5
##  3rd Qu.:4332   3rd Qu.:2024   3rd Qu.:20286   3rd Qu.:29960   3rd Qu.:193.5
##  Max.   :4833   Max.   :2239   Max.   :20740   Max.   :30250   Max.   :216.0
##        x5              x6
##  Min.   :1522   Min.   : 85.00
##  1st Qu.:1592   1st Qu.: 97.00
```

```
##  Median :1668   Median : 99.00  
##  Mean   :1652   Mean   : 97.42  
##  3rd Qu.:1710   3rd Qu.:100.00  
##  Max.   :1758   Max.   :102.00
```

```r
dim(df)
```

```
## [1] 40  7
```

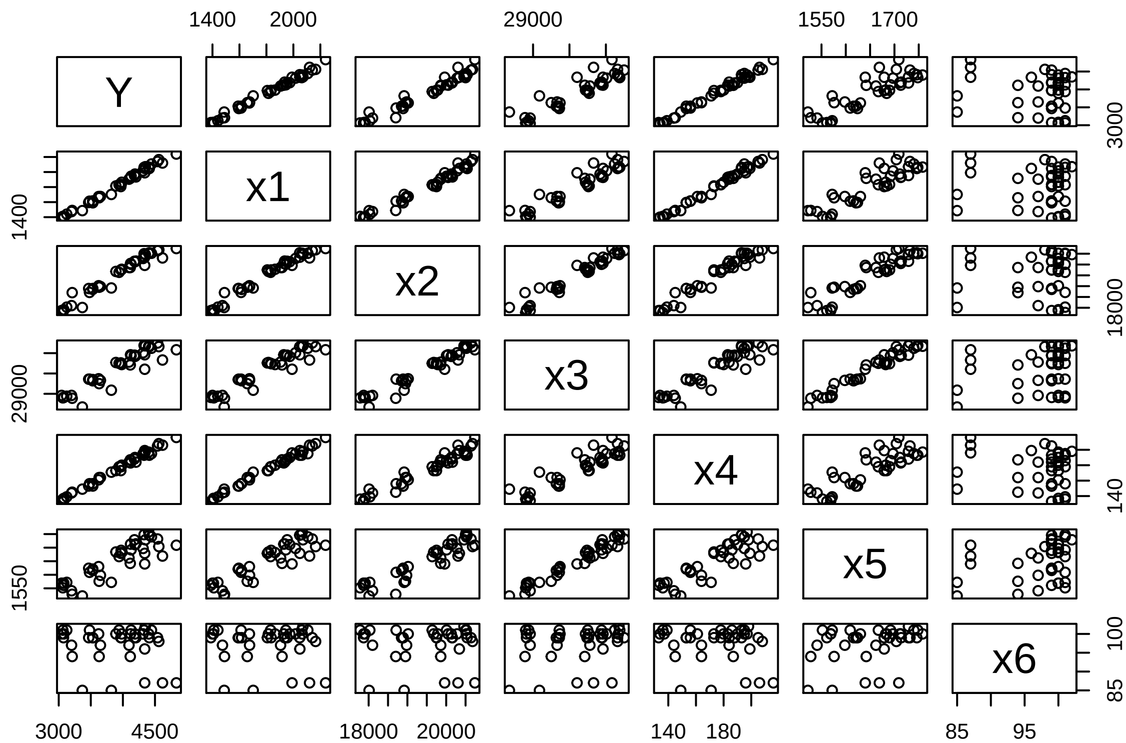
```r
head(df)
```

```
##      Y   x1    x2    x3  x4   x5  x6
## 1 4540 2140 20640 30250 205 1732  99
## 2 4315 2016 20280 30010 195 1697 100
## 3 4095 1905 19860 29780 184 1662  97
## 4 3650 1675 18980 29330 164 1598  97
## 5 3200 1474 18100 28960 144 1541  97
## 6 4833 2239 20740 30083 216 1709  87
```

```r
tail(df)
```

```
##       Y   x1    x2    x3  x4   x5  x6
## 35 3064 1410 17780 28900 136 1552 101
## 36 4402 2066 20520 30170 197 1758 100
## 37 4180 1954 20150 29950 188 1729  99
## 38 3973 1835 19750 29740 178 1690  99
## 39 3530 1616 18850 29320 156 1616  99
## 40 3080 1407 17910 28910 137 1569 100
```

```r
plot(df)
```



There seems to be an issue with multicollinearity as can be seen in the pairsplot. But lets first transform first the variables according to Tukey's first-aid transformations:

```
dft1 <- data.frame(lX1 = log(df$x1),
                   lX2 = log(df$x2),
                   lX3 = log(df$x3),
                   lX4 = log(df$x4),
                   x5 = df$x5,
                   x6 = df$x6,
                   lY = log(df$Y))

head(dft1)
```

```
##        lX1      lX2      lX3      lX4   x5  x6       lY
## 1 7.668561 9.934986 10.31725 5.323010 1732  99 8.420682
## 2 7.608871 9.917390 10.30929 5.273000 1697 100 8.369853
## 3 7.552237 9.896463 10.30159 5.214936 1662  97 8.317522
## 4 7.423568 9.851141 10.28637 5.099866 1598  97 8.202482
## 5 7.295735 9.803667 10.27367 4.969813 1541  97 8.070906
## 6 7.713785 9.939819 10.31172 5.375278 1709  87 8.483223
```

-> x5 and x6 are not transformed because temperature can be negeative (do not transform variables which could be negative numbers according to Tukey's first-aid transformations).

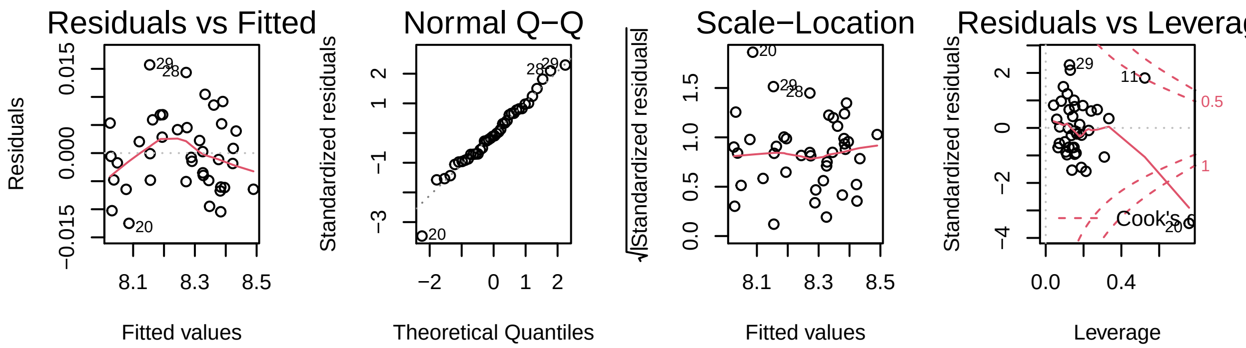
With the transformed dataset one can now start modeling a linear model. Let's start with a full model which includes all the explanatory variables.

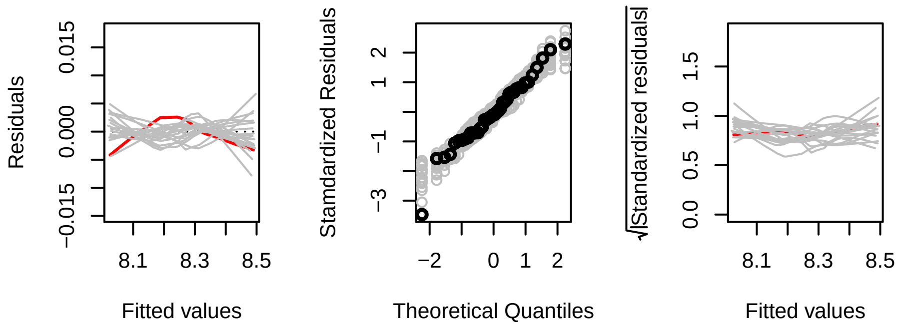
```r
lm2.1 <- lm(lY ~ ., data = dft1)
summary(lm2.1)
```

```
##
## Call:
## lm(formula = lY ~ ., data = dft1)
##
## Residuals:
##        Min         1Q     Median         3Q        Max
## -0.0125101 -0.0049270 -0.0006753  0.0047059  0.0157080
##
## Coefficients:
##               Estimate Std. Error t value Pr(>|t|)
## (Intercept) -9.704e+00  8.614e+00  -1.127   0.2681
## lX1          4.090e-01  1.766e-01   2.316   0.0269 *
## lX2         -6.751e-02  2.043e-01  -0.330   0.7431
## lX3          1.364e+00  9.452e-01   1.443   0.1584
## lX4          2.897e-01  1.389e-01   2.086   0.0448 *
## x5           2.494e-04  9.415e-05   2.648   0.0123 *
## x6          -3.925e-03  7.019e-04  -5.592 3.21e-06 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.007329 on 33 degrees of freedom
## Multiple R-squared:  0.9974, Adjusted R-squared:  0.997
## F-statistic:  2143 on 6 and 33 DF,  p-value: < 2.2e-16
```

The R^2 looks actually pretty good. But not all the variables seem to be relevant and we have to do a residual and sensitivity analysis first.

```r
par(mfrow=c(2,4))
plot(lm2.1)
plot.lmSim(lm2.1, SEED = 1)
```





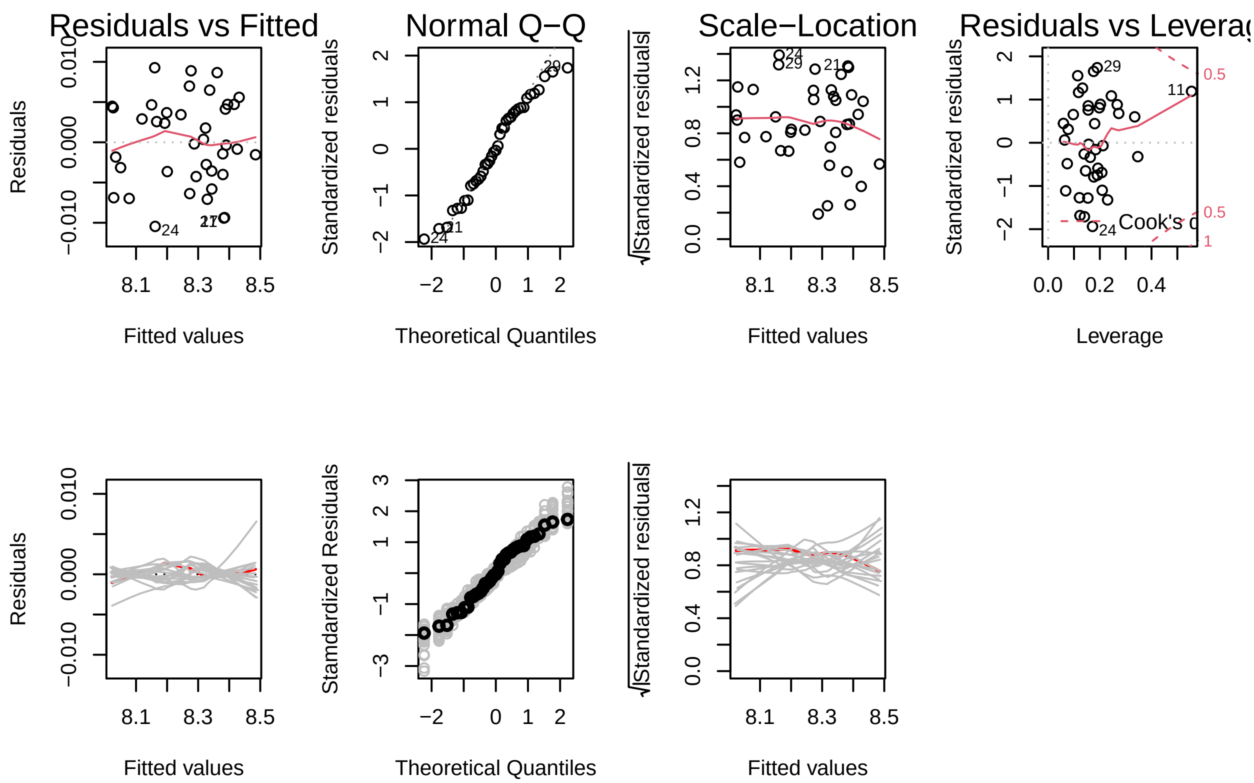
Observation i=20 is an outlier. We remove it and build a new model without it and analyze it.

```
ind <- 20
lm2.2 <- lm(lY ~ ., data = dft1, subset = -ind)
summary(lm2.2)
```

```
##
## Call:
## lm(formula = lY ~ ., data = dft1, subset = -ind)
##
## Residuals:
##        Min         1Q     Median         3Q        Max
## -0.0104560 -0.0038227 -0.0001954  0.0043872  0.0092726
##
## Coefficients:
##               Estimate Std. Error t value Pr(>|t|)
## (Intercept) -3.117e+00  7.138e+00  -0.437  0.66527
## lX1          4.957e-01  1.444e-01   3.434  0.00166 **
## lX2          1.018e+00  3.024e-01   3.367  0.00199 **
## lX3         -2.428e-01  8.519e-01  -0.285  0.77746
## lX4          5.443e-02  1.251e-01   0.435  0.66643
## x5           9.169e-05  8.461e-05   1.084  0.28658
## x6          -3.281e-03  5.876e-04  -5.585 3.62e-06 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
```

```
## Residual standard error: 0.005931 on 32 degrees of freedom
## Multiple R-squared:  0.9983, Adjusted R-squared:  0.998
## F-statistic:  3104 on 6 and 32 DF,  p-value: < 2.2e-16
```

```
par(mfrow=c(2,4))
plot(lm2.2)
plot.lmSim(lm2.2, SEED = 1)
```



There is no evidence that any of the assumptions is violated and no outlier is visible. Lets now perform a variable selection with the step() function.

```
step(lm2.2, scope = list(upper =~ lX1 + lX2 + lX3 + lX4 + x5 + x6, lower =~ 1))
```

```
## Start:  AIC=-393.66
## lY ~ lX1 + lX2 + lX3 + lX4 + x5 + x6
##
##         Df  Sum of Sq       RSS     AIC
## - lX3    1 0.00000286 0.0011285 -395.57
## - lX4    1 0.00000666 0.0011323 -395.43
## - x5     1 0.00004132 0.0011670 -394.26
## <none>                0.0011257 -393.66
## - lX2    1 0.00039868 0.0015244 -383.84
## - lX1    1 0.00041485 0.0015405 -383.43
## - x6     1 0.00109718 0.0022229 -369.13
##
```

```
## Step:  AIC=-395.57
## lY ~ lX1 + lX2 + lX4 + x5 + x6
##
##        Df  Sum of Sq       RSS     AIC
## - lX4   1 0.00000992 0.0011385 -397.22
## - x5    1 0.00003846 0.0011670 -396.26
## <none>               0.0011285 -395.57
## + lX3   1 0.00000286 0.0011257 -393.66
## - lX1   1 0.00046212 0.0015907 -384.18
## - lX2   1 0.00050646 0.0016350 -383.11
## - x6    1 0.00283395 0.0039625 -348.58
##
## Step:  AIC=-397.22
## lY ~ lX1 + lX2 + x5 + x6
##
##        Df Sum of Sq       RSS     AIC
## - x5    1 0.0000286 0.0011671 -398.25
## <none>              0.0011385 -397.22
## + lX4   1 0.0000099 0.0011285 -395.57
## + lX3   1 0.0000061 0.0011323 -395.43
## - lX2   1 0.0005920 0.0017305 -382.89
## - lX1   1 0.0019052 0.0030437 -360.87
## - x6    1 0.0032661 0.0044046 -346.46
##
## Step:  AIC=-398.25
## lY ~ lX1 + lX2 + x6
##
##        Df Sum of Sq       RSS     AIC
## <none>              0.0011671 -398.25
## + x5    1 0.0000286 0.0011385 -397.22
## + lX4   1 0.0000001 0.0011670 -396.26
## + lX3   1 0.0000000 0.0011671 -396.26
## - lX2   1 0.0010830 0.0022501 -374.65
## - lX1   1 0.0019053 0.0030725 -362.50
## - x6    1 0.0046090 0.0057761 -337.89
```

```
##
## Call:
## lm(formula = lY ~ lX1 + lX2 + x6, data = dft1, subset = -ind)
##
## Coefficients:
## (Intercept)          lX1          lX2           x6
##   -6.524816     0.521784     1.133755    -0.003275
```

```
step(lm(lY ~ 1, data = dft1[-ind,]),
     direction = 'both',
     scope = list(upper =~  lX1 + lX2 + lX3 + lX4 + x5 + x6,
                  lower =~ 1))
```

```
## Start:  AIC=-157.3
## lY ~ 1
##
##        Df Sum of Sq     RSS     AIC
```

```
## + lX4   1   0.65059 0.00578 -339.85
## + lX1   1   0.65016 0.00621 -337.08
## + lX2   1   0.63390 0.02247 -286.91
## + lX3   1   0.56517 0.09120 -232.27
## + x5    1   0.49540 0.16097 -210.11
## <none>              0.65637 -157.30
## + x6    1   0.01927 0.63710 -156.46
##
## Step:  AIC=-339.85
## lY ~ lX4
##
##        Df Sum of Sq     RSS     AIC
## + lX1   1   0.00166 0.00412 -351.09
## + lX2   1   0.00146 0.00432 -349.19
## + x5    1   0.00115 0.00464 -346.46
## + lX3   1   0.00062 0.00516 -342.27
## <none>              0.00578 -339.85
## + x6    1   0.00000 0.00578 -337.88
## - lX4   1   0.65059 0.65637 -157.30
##
## Step:  AIC=-351.09
## lY ~ lX4 + lX1
##
##        Df  Sum of Sq       RSS     AIC
## + x6    1 0.00187618 0.0022413 -372.81
## + lX3   1 0.00054043 0.0035770 -354.57
## <none>               0.0041174 -351.09
## + lX2   1 0.00004121 0.0040762 -349.48
## + x5    1 0.00000879 0.0041087 -349.17
## - lX1   1 0.00166441 0.0057819 -339.85
## - lX4   1 0.00208945 0.0062069 -337.08
##
## Step:  AIC=-372.81
## lY ~ lX4 + lX1 + x6
##
##        Df Sum of Sq       RSS     AIC
## + lX2   1 0.0010743 0.0011670 -396.26
## + x5    1 0.0006063 0.0016350 -383.11
## + lX3   1 0.0005090 0.0017323 -380.85
## - lX4   1 0.0000088 0.0022501 -374.65
## <none>              0.0022413 -372.81
## - x6    1 0.0018762 0.0041174 -351.09
## - lX1   1 0.0035363 0.0057776 -337.88
##
## Step:  AIC=-396.26
## lY ~ lX4 + lX1 + x6 + lX2
##
##        Df  Sum of Sq       RSS     AIC
## - lX4   1 0.00000011 0.0011671 -398.25
## <none>               0.0011670 -396.26
## + x5    1 0.00003846 0.0011285 -395.57
## + lX3   1 0.00000000 0.0011670 -394.26
## - lX1   1 0.00063695 0.0018040 -381.27
## - lX2   1 0.00107426 0.0022413 -372.81
```

```
## - x6    1 0.00290924 0.0040762 -349.48
##
## Step:  AIC=-398.25
## lY ~ lX1 + x6 + lX2
##
##          Df Sum of Sq       RSS     AIC
## <none>                0.0011671 -398.25
## + x5    1 0.0000286 0.0011385 -397.22
## + lX4   1 0.0000001 0.0011670 -396.26
## + lX3   1 0.0000000 0.0011671 -396.26
## - lX2   1 0.0010830 0.0022501 -374.65
## - lX1   1 0.0019053 0.0030725 -362.50
## - x6    1 0.0046090 0.0057761 -337.89
```

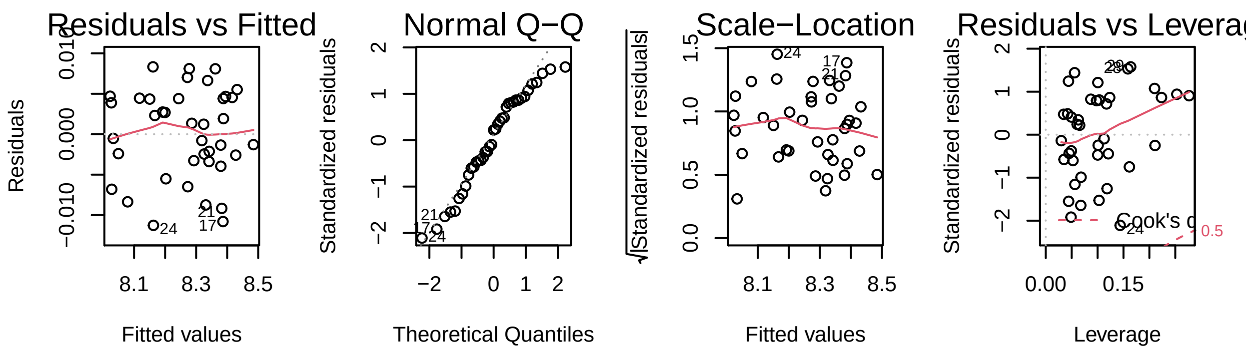
```
##
## Call:
## lm(formula = lY ~ lX1 + x6 + lX2, data = dft1[-ind, ])
##
## Coefficients:
## (Intercept)          lX1           x6          lX2
##   -6.524816     0.521784    -0.003275     1.133755
```

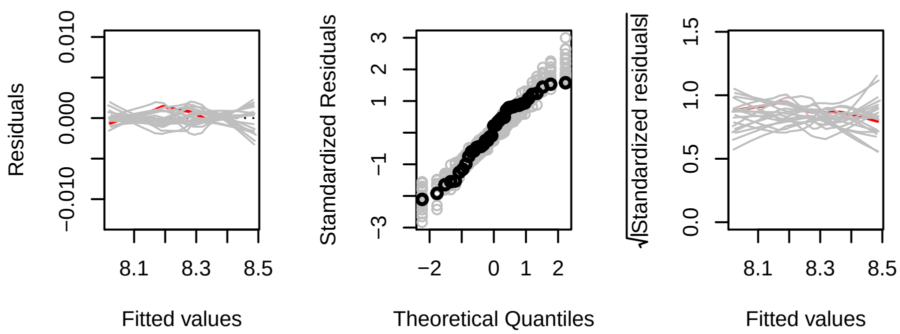
In both cases the final suggested model with the lowest AIC is lY = lX1 + lX2 + x6 without the observation i=20.

So lets investigate this model with a residual and sensitivity analysis.

```
lm2.3 <- lm(lY ~ lX1 + lX2 + x6, data = dft1[-20,])

par(mfrow = c(2,4))
plot(lm2.3)
plot.lmSim(lm2.3, SEED = 1)
```





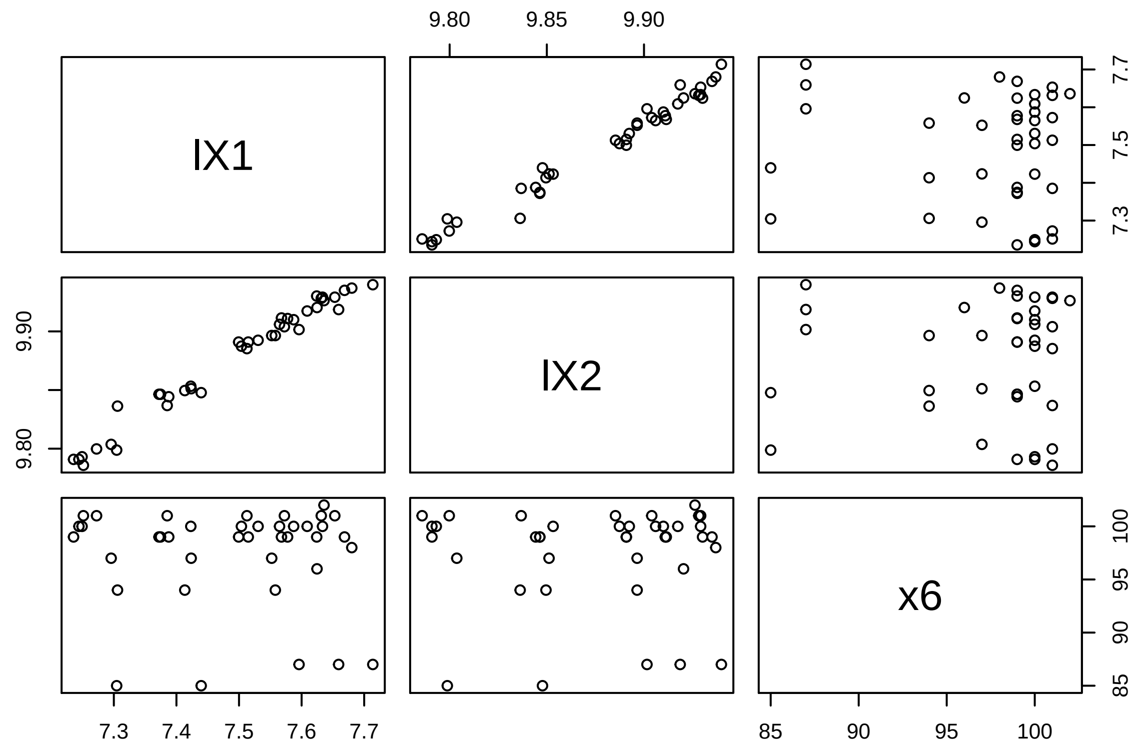
There is no evidence of any violation of the model assumptions. So lets now investigate the multicollinearity with the Variance Inflation Factor (vif)

```
library(car)
vif(lm2.3)
```

```
##        lX1        lX2         x6
## 109.721932 108.742539   1.991382
```

There seems to be a problem with multicollinearity for the variables lX1 and lX2. Lets look at it:

```
plot(dft1[, c('lX1', 'lX2', 'x6')])
```



Indeed, lX1 and lX2 are highliy correlated. Lets transform them to the mean and their difference and check if the plot looks better:

```r
dft1$dlRoSp <- dft1$lX2 - dft1$lX1
dft1$mlRoSp <- (dft1$lX1 + dft1$lX2)/2
head(dft1)
```

```
##        lX1      lX2      lX3      lX4   x5  x6       lY   dlRoSp   mlRoSp
## 1 7.668561 9.934986 10.31725 5.323010 1732  99 8.420682 2.266425 8.801774
## 2 7.608871 9.917390 10.30929 5.273000 1697 100 8.369853 2.308520 8.763131
## 3 7.552237 9.896463 10.30159 5.214936 1662  97 8.317522 2.344226 8.724350
## 4 7.423568 9.851141 10.28637 5.099866 1598  97 8.202482 2.427573 8.637355
## 5 7.295735 9.803667 10.27367 4.969813 1541  97 8.070906 2.507932 8.549701
## 6 7.713785 9.939819 10.31172 5.375278 1709  87 8.483223 2.226035 8.826802
```

```r
lm2.4 <- lm(lY ~ dlRoSp + mlRoSp + x6, data = dft1[-20,])
summary(lm2.4)
```

```
##
## Call:
## lm(formula = lY ~ dlRoSp + mlRoSp + x6, data = dft1[-20, ])
##
## Residuals:
##       Min        1Q    Median        3Q       Max
## -0.011274 -0.003329  0.001228  0.004430  0.008328
```

```
##
## Coefficients:
##               Estimate Std. Error t value Pr(>|t|)
## (Intercept) -6.5248156  1.4320310  -4.556 6.08e-05 ***
## dlRoSp       0.3059856  0.1338667   2.286   0.0284 *
## mlRoSp       1.6555392  0.1304000  12.696 1.17e-14 ***
## x6          -0.0032750  0.0002786 -11.757 1.04e-13 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.005775 on 35 degrees of freedom
## Multiple R-squared:  0.9982, Adjusted R-squared:  0.9981
## F-statistic:  6550 on 3 and 35 DF,  p-value: < 2.2e-16
```

```
vif(lm2.4)
```

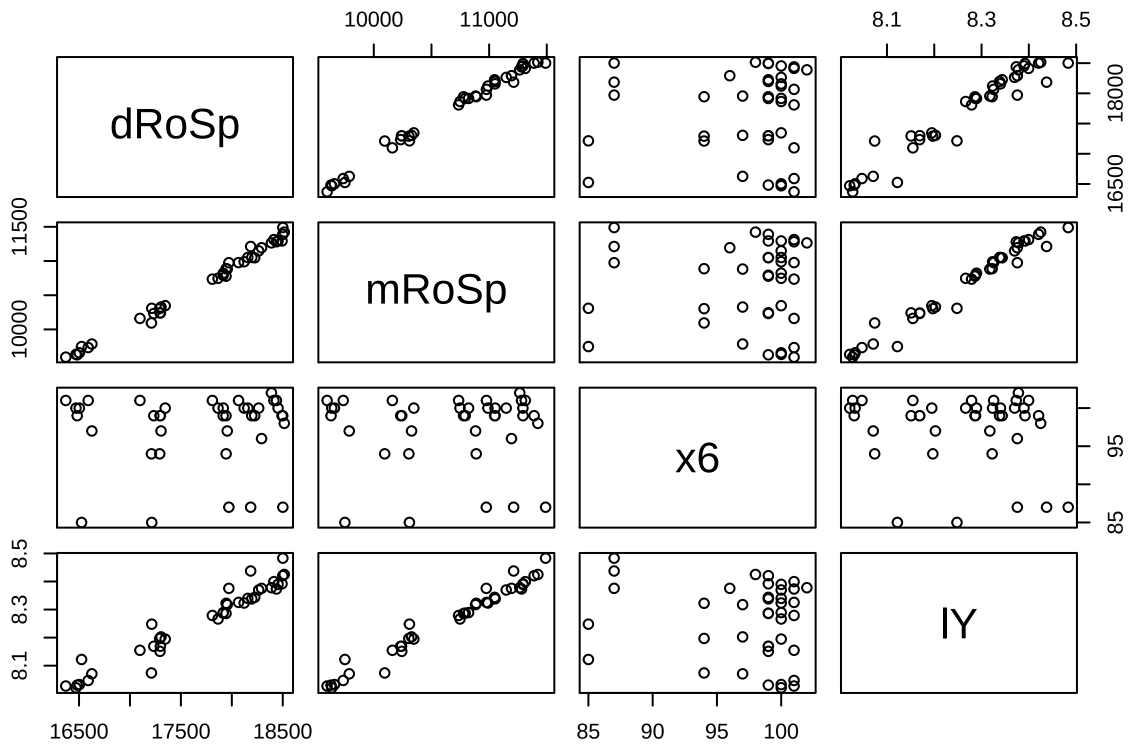
```
##     dlRoSp     mlRoSp         x6
## 179.409858 176.580766   1.991382
```

This does not get any better: Still looks like a very strong correlation. So one could try the transformaion with the untransformed variables.

```
dft2 <- data.frame(dRoSp = df$x2 - df$x1,
                   mRoSp = (df$x1 + df$x2)/2,
                   x6 = df$x6,
                   lY = dft1$lY)
head(dft2)
```

```
##   dRoSp   mRoSp  x6       lY
## 1 18500 11390.0  99 8.420682
## 2 18264 11148.0 100 8.369853
## 3 17955 10882.5  97 8.317522
## 4 17305 10327.5  97 8.202482
## 5 16626  9787.0  97 8.070906
## 6 18501 11489.5  87 8.483223
```

```
plot(dft2)
```




```
lm2.5 <- lm(lY ~ ., data = dft2[-20,])
summary(lm2.5)
```

```
##
## Call:
## lm(formula = lY ~ ., data = dft2[-20, ])
##
## Residuals:
##       Min        1Q    Median        3Q       Max
## -0.014169 -0.004725  0.000420  0.005255  0.012592
##
## Coefficients:
##               Estimate Std. Error t value Pr(>|t|)
## (Intercept)  6.499e+00  1.088e-01  59.737  < 2e-16 ***
## dRoSp       -3.152e-05  2.365e-05  -1.333    0.191
## mRoSp        2.539e-04  2.773e-05   9.158 8.04e-11 ***
## x6          -3.931e-03  3.137e-04 -12.532 1.70e-14 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.00709 on 35 degrees of freedom
## Multiple R-squared:  0.9973, Adjusted R-squared:  0.9971
## F-statistic:  4340 on 3 and 35 DF,  p-value: < 2.2e-16
```

```r
vif(lm2.5)
```

```
##      dRoSp      mRoSp         x6 
## 206.481767 206.281338   1.674926
```

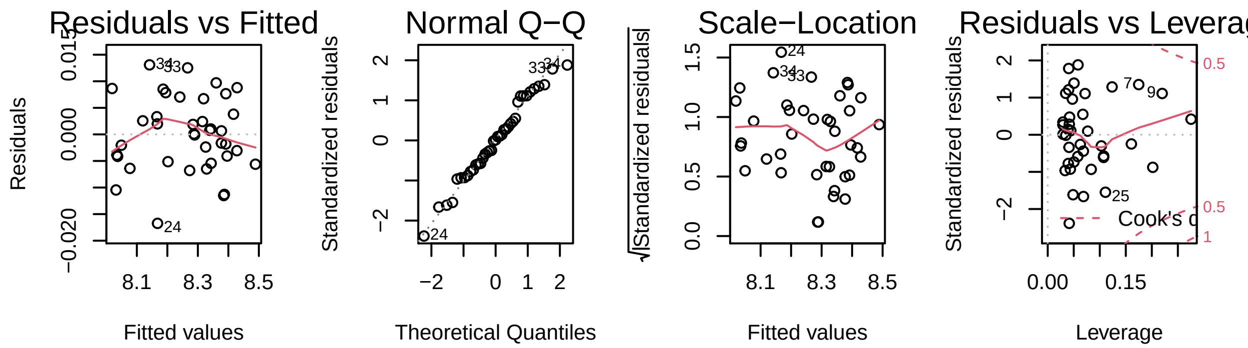
Still problems with the multicollinearity. Because in this model dRoSp is not significant, one can drop this variable.

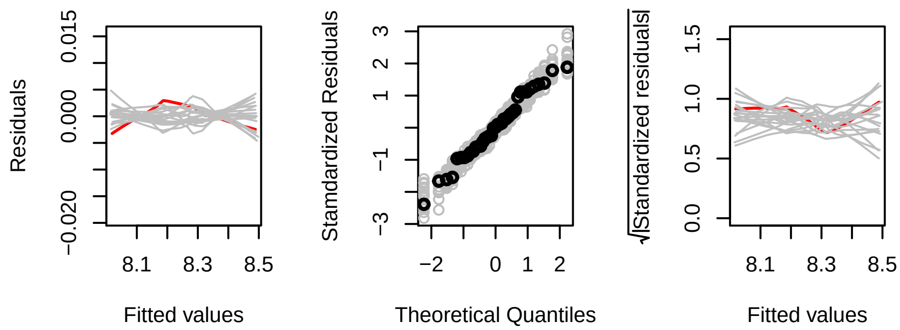
```r
lm2.6 <- lm(lY ~ mRoSp + x6, data = dft2[-20,])
summary(lm2.6)
```

```
## 
## Call:
## lm(formula = lY ~ mRoSp + x6, data = dft2[-20, ])
## 
## Residuals:
##        Min         1Q     Median         3Q        Max 
## -0.0167433 -0.0046339  0.0000973  0.0052560  0.0130837 
## 
## Coefficients:
##               Estimate Std. Error t value Pr(>|t|)    
## (Intercept)  6.360e+00  3.203e-02  198.58   <2e-16 ***
## mRoSp        2.171e-04  1.952e-06  111.22   <2e-16 ***
## x6          -4.196e-03  2.450e-04  -17.13   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
## 
## Residual standard error: 0.007166 on 36 degrees of freedom
## Multiple R-squared:  0.9972, Adjusted R-squared:  0.997 
## F-statistic:  6372 on 2 and 36 DF,  p-value: < 2.2e-16
```

Like that all the variables are significant and the R^2 is still 0.9972 the model performance and suitability looks still very goog. How about the residual and sensitivity analysis?

```r
par(mfrow=c(2,4))
plot(lm2.6)
plot.lmSim(lm2.6, SEED = 1)
```





No model assumptions are violated. What about the multicollinearity problem?

```
vif(lm2.6)
```

```
##    mRoSp       x6
## 1.000405 1.000405
```

Multicollinearity seems also not to be a problem anymore. The model fits the data well like that.

Exercise 3

```
path <- file.path('Datasets', 'windmill.dat')
df <- read.table(path, header=TRUE)

summary(df)
```

Dataset loading and sanity check:

```
##     velocity        DC.output
##  Min.   : 5.482   Min.   :0.123
##  1st Qu.: 8.838   1st Qu.:1.144
##  Median :13.424   Median :1.800
```

```
##   Mean   :13.720   Mean   :1.610
##   3rd Qu.:18.235   3rd Qu.:2.166
##   Max.   :22.822   Max.   :2.386
```

```r
dim(df)
```

```
## [1] 25  2
```

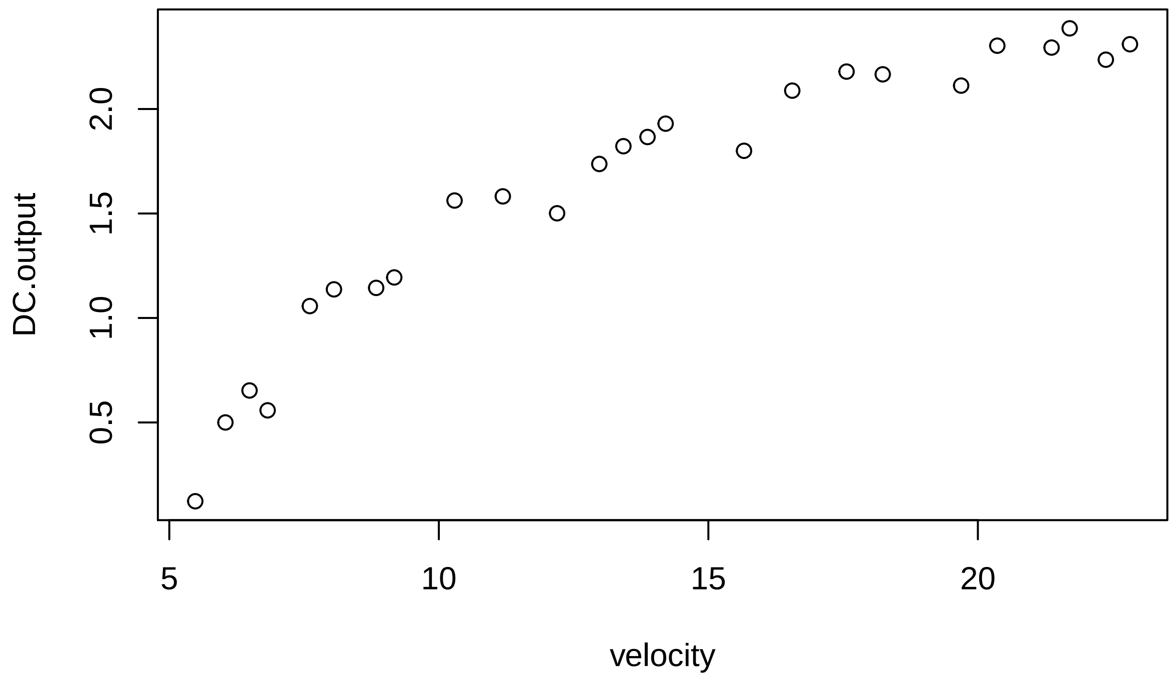
```r
head(df)
```

```
##    velocity DC.output
## 1 11.187073     1.582
## 2 13.424487     1.822
## 3  7.607209     1.057
## 4  6.041019     0.500
## 5 22.374145     2.236
## 6 21.702921     2.386
```

```r
tail(df)
```

```
##     velocity DC.output
## 20 12.193909     1.501
## 21 20.360472     2.303
## 22 22.821628     2.310
## 23  9.173400     1.194
## 24  8.837787     1.144
## 25  5.481666     0.123
```

```r
par(mfrow=c(1,1))
plot(df)
```




```r
df$tVel <- (1 / df$velocity)
head(df)
```

```
##    velocity DC.output       tVel
## 1 11.187073     1.582 0.08938889
## 2 13.424487     1.822 0.07449074
## 3  7.607209     1.057 0.13145426
## 4  6.041019     0.500 0.16553499
## 5 22.374145     2.236 0.04469445
## 6 21.702921     2.386 0.04607675
```

The model was already used in the worksheet of week 1, that is why it is not investigated here but instead used for predictions of DC.output for wind veolicites of one an ten meter per second. So we fit first the known model and used it then for prediciton.

```r
lm3.1 <- lm(DC.output ~ tVel, data = df)

dfPreds <- data.frame(tVel = c(1/1,
                               1/10))
predict(lm3.1, newdata = dfPreds, interval = 'prediction', level = 0.95)
```

```
##          fit        lwr        upr
## 1 -12.536597 -13.430108 -11.643086
## 2   1.427314   1.228331   1.626298
```

The prediciton of the first line (1 meter per second wind velocity) is not usable. One has always to investigate the prediction(-range) and make sure that they are plausible!

Exercise 4

```
path <- file.path('Datasets', 'NPScosts.dat')
df <- read.table(path, header=TRUE)

summary(df)
```

Dataset loading and sanity check:

```
##       cost            date             t1              t2
##  Min.   :207.5   Min.   :67.17   Min.   : 7.00   Min.   :44.00
##  1st Qu.:310.3   1st Qu.:67.90   1st Qu.:11.75   1st Qu.:56.50
##  Median :448.1   Median :68.42   Median :13.00   Median :62.50
##  Mean   :461.6   Mean   :68.58   Mean   :13.75   Mean   :62.38
##  3rd Qu.:612.0   3rd Qu.:68.92   3rd Qu.:15.25   3rd Qu.:70.25
##  Max.   :881.2   Max.   :71.08   Max.   :22.00   Max.   :85.00
##       cap               pr               ne              ct
##  Min.   : 457.0   Min.   :0.0000   Min.   :0.00   Min.   :0.0000
##  1st Qu.: 745.0   1st Qu.:0.0000   1st Qu.:0.00   1st Qu.:0.0000
##  Median : 822.0   Median :0.0000   Median :0.00   Median :0.0000
##  Mean   : 825.4   Mean   :0.3125   Mean   :0.25   Mean   :0.4062
##  3rd Qu.: 947.2   3rd Qu.:1.0000   3rd Qu.:0.25   3rd Qu.:1.0000
##  Max.   :1130.0   Max.   :1.0000   Max.   :1.00   Max.   :1.0000
##        bw             cum.n              pt
##  Min.   :0.0000   Min.   : 1.000   Min.   :0.0000
##  1st Qu.:0.0000   1st Qu.: 3.000   1st Qu.:0.0000
##  Median :0.0000   Median : 7.500   Median :0.0000
##  Mean   :0.1875   Mean   : 8.531   Mean   :0.1875
##  3rd Qu.:0.0000   3rd Qu.:12.500   3rd Qu.:0.0000
##  Max.   :1.0000   Max.   :21.000   Max.   :1.0000
```

```
dim(df)
```

```
## [1] 32 11
```

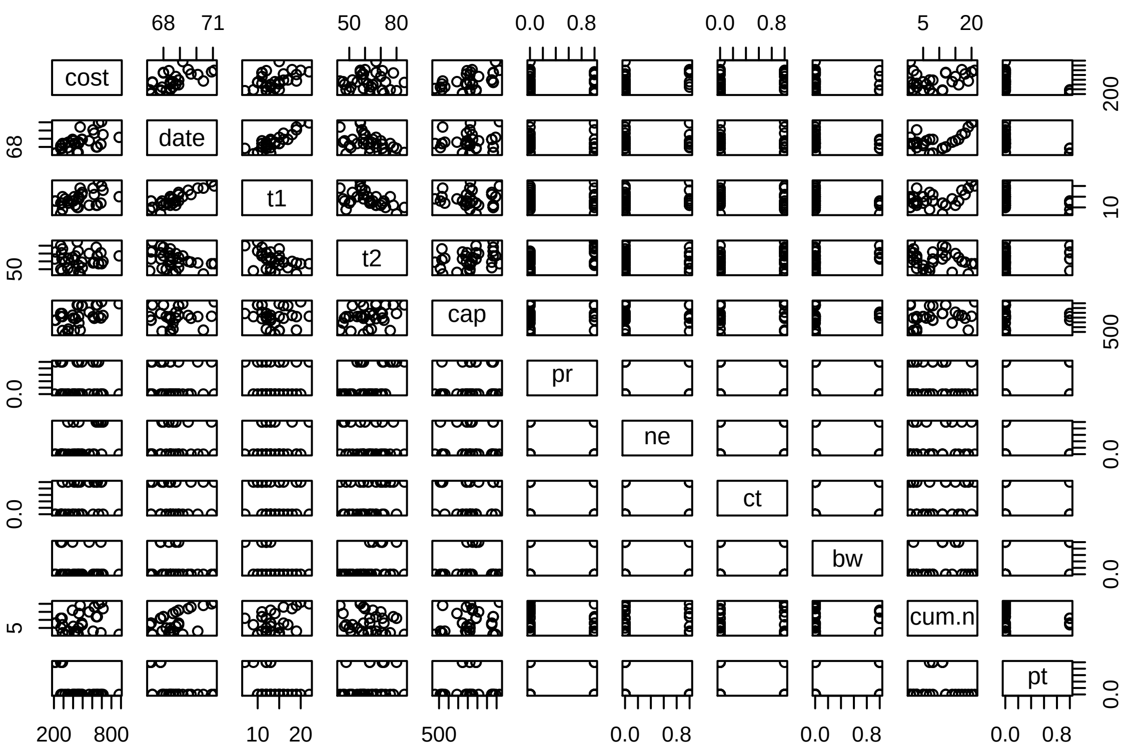
```
head(df)
```

```
##     cost  date t1 t2  cap pr ne ct bw cum.n pt
## 1 460.05 68.58 14 46  687  0  1  0  0    14  0
## 2 452.99 67.33 10 73 1065  0  0  1  0     1  0
## 3 443.22 67.33 10 85 1065  1  0  1  0     1  0
## 4 652.32 68.00 11 67 1065  0  1  1  0    12  0
## 5 642.23 68.00 11 78 1065  1  1  1  0    12  0
## 6 345.39 67.92 13 51  514  0  1  1  0     3  0
```

```
tail(df)
```

```
##      cost  date t1 t2 cap pr ne ct bw cum.n pt
## 27 207.51 67.25 13 63 745  0  0  0  0     8  1
## 28 288.48 67.17  9 48 821  0  0  1  0     7  1
## 29 284.88 67.83 12 63 886  0  0  0  1    11  1
## 30 280.36 67.83 12 71 886  1  0  0  1    11  1
## 31 217.38 67.25 13 72 745  1  0  0  0     8  1
## 32 270.71 67.83  7 80 886  1  0  0  1    11  1
```

```
par(mfrow=c(1,1))
plot(df)
```



The dataset and the model were already partially investigated in Exercise 3 from week 1. We take off where we ended there.

Exercise 4.a)

First a variable selection gets perfomed on the model built in Exercise 3 week 1 (including the performed variable transformations).

```
df$lCost <- log(df$cost)
df$lCap <- log(df$cap)
df$sCum.n <- sqrt(df$cum.n)
```

```r
mod3.1 <- lm(lCost ~ date + t1 + t2 + lCap + pr + ne + ct + bw + sCum.n + pt,
             data = df)
summary(mod3.1)
```

```
##
## Call:
## lm(formula = lCost ~ date + t1 + t2 + lCap + pr + ne + ct + bw +
##     sCum.n + pt, data = df)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -0.29896 -0.10332  0.02118  0.09019  0.26731
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept) -13.875045   5.404826  -2.567  0.01796 *
## date          0.219318   0.082426   2.661  0.01463 *
## t1            0.006067   0.021990   0.276  0.78531
## t2            0.005273   0.004564   1.155  0.26092
## lCap          0.692542   0.137131   5.050 5.32e-05 ***
## pr           -0.105307   0.082004  -1.284  0.21307
## ne            0.254326   0.078075   3.257  0.00377 **
## ct            0.122969   0.068386   1.798  0.08654 .
## bw            0.029418   0.104469   0.282  0.78101
## sCum.n       -0.069016   0.040985  -1.684  0.10700
## pt           -0.229133   0.128059  -1.789  0.08800 .
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.1666 on 21 degrees of freedom
## Multiple R-squared:  0.8684, Adjusted R-squared:  0.8057
## F-statistic: 13.85 on 10 and 21 DF,  p-value: 3.983e-07
```

```r
step(mod3.1, scope = list(upper =~ date + t1 + t2 + lCap + pr + ne + ct + bw + sCum.n + pt,
                          lower =~ 1))
```

```
## Start:  AIC=-106.18
## lCost ~ date + t1 + t2 + lCap + pr + ne + ct + bw + sCum.n +
##     pt
##
##          Df Sum of Sq     RSS      AIC
## - t1      1   0.00211 0.58498 -108.061
## - bw      1   0.00220 0.58507 -108.057
## - t2      1   0.03705 0.61991 -106.205
## <none>                0.58286 -106.177
## - pr      1   0.04577 0.62864 -105.758
## - sCum.n  1   0.07870 0.66157 -104.124
## - pt      1   0.08886 0.67172 -103.637
## - ct      1   0.08974 0.67261 -103.594
## - date    1   0.19650 0.77937  -98.880
## - ne      1   0.29452 0.87738  -95.090
## - lCap    1   0.70790 1.29076  -82.736
```

```
##
## Step:  AIC=-108.06
## lCost ~ date + t2 + lCap + pr + ne + ct + bw + sCum.n + pt
##
##          Df Sum of Sq     RSS      AIC
## - bw      1   0.00094 0.58592 -110.010
## - t2      1   0.03519 0.62017 -108.192
## <none>                0.58498 -108.061
## - pr      1   0.04370 0.62867 -107.756
## + t1      1   0.00211 0.58286 -106.177
## - sCum.n  1   0.08248 0.66746 -105.840
## - pt      1   0.08712 0.67210 -105.619
## - ct      1   0.08804 0.67302 -105.575
## - ne      1   0.29978 0.88476  -96.822
## - date    1   0.60779 1.19276  -87.263
## - lCap    1   0.71127 1.29625  -84.600
##
## Step:  AIC=-110.01
## lCost ~ date + t2 + lCap + pr + ne + ct + sCum.n + pt
##
##          Df Sum of Sq     RSS      AIC
## <none>                0.58592 -110.010
## - pr      1   0.05231 0.63823 -109.273
## - t2      1   0.05233 0.63825 -109.272
## + bw      1   0.00094 0.58498 -108.061
## + t1      1   0.00085 0.58507 -108.057
## - sCum.n  1   0.08294 0.66886 -107.773
## - ct      1   0.08740 0.67332 -107.561
## - pt      1   0.08764 0.67356 -107.549
## - ne      1   0.30004 0.88596  -98.778
## - date    1   0.61189 1.19781  -89.128
## - lCap    1   0.71083 1.29675  -86.588
```

```
##
## Call:
## lm(formula = lCost ~ date + t2 + lCap + pr + ne + ct + sCum.n +
##     pt, data = df)
##
## Coefficients:
## (Intercept)         date           t2         lCap           pr           ne
##  -15.017735     0.237731     0.005466     0.685606    -0.104189     0.256006
##          ct       sCum.n           pt
##    0.119313    -0.068484    -0.216158
```

Starting from the full model the stepwise variable selection suggests the following model: lCost = date + t2 + lCap + pr + ne + ct + sCum.n + pt and therewith drops just 2 variables (t1 & bw) and results in an AIC of -110.01.

What if we start with an empty model and perform the variable selection?

```
step(lm(lCost ~ 1, data = df),
     direction = 'both',
     scope = list(upper =~ date + t1 + t2 + lCap + pr + ne + ct + bw + sCum.n + pt,
                  lower =~ 1))
```

```
## Start:  AIC=-61.29
## lCost ~ 1
##
##          Df Sum of Sq    RSS     AIC
## + pt      1   2.01272 2.4153 -78.685
## + date    1   1.75252 2.6755 -75.411
## + t1      1   0.91394 3.5141 -66.686
## + lCap    1   0.76606 3.6620 -65.367
## + ne      1   0.65915 3.7689 -64.446
## + ct      1   0.29142 4.1366 -61.467
## <none>                4.4281 -61.289
## + sCum.n  1   0.15052 4.2775 -60.395
## + bw      1   0.08878 4.3393 -59.937
## + pr      1   0.05087 4.3772 -59.658
## + t2      1   0.00581 4.4223 -59.331
##
## Step:  AIC=-78.68
## lCost ~ pt
##
##          Df Sum of Sq    RSS     AIC
## + lCap    1   0.91498 1.5004 -91.921
## + date    1   0.49197 1.9234 -83.973
## + sCum.n  1   0.36628 2.0491 -81.947
## + ne      1   0.18965 2.2257 -79.301
## + t1      1   0.18163 2.2337 -79.186
## <none>                2.4153 -78.685
## + bw      1   0.07200 2.3433 -77.653
## + ct      1   0.04550 2.3698 -77.293
## + t2      1   0.03212 2.3832 -77.113
## + pr      1   0.00261 2.4127 -76.719
## - pt      1   2.01272 4.4281 -61.289
##
## Step:  AIC=-91.92
## lCost ~ pt + lCap
##
##          Df Sum of Sq    RSS      AIC
## + date    1   0.43560 1.0648 -100.896
## + t1      1   0.26714 1.2332  -96.195
## + ne      1   0.17146 1.3289  -93.804
## + sCum.n  1   0.15713 1.3432  -93.461
## <none>                1.5004  -91.921
## + ct      1   0.04889 1.4515  -90.981
## + bw      1   0.01747 1.4829  -90.296
## + pr      1   0.01151 1.4888  -90.167
## + t2      1   0.01069 1.4897  -90.150
## - lCap    1   0.91498 2.4153  -78.685
## - pt      1   2.16165 3.6620  -65.367
##
## Step:  AIC=-100.9
## lCost ~ pt + lCap + date
##
##          Df Sum of Sq     RSS      AIC
## + ne      1   0.20229 0.86247 -105.638
## + ct      1   0.12776 0.93700 -102.986
```

```
## <none>                1.06476 -100.896
## + t2      1   0.03080 1.03397  -99.835
## + pr      1   0.01883 1.04593  -99.466
## + bw      1   0.01148 1.05328  -99.243
## + t1      1   0.00442 1.06034  -99.029
## + sCum.n  1   0.00120 1.06356  -98.932
## - date    1   0.43560 1.50036  -91.921
## - lCap    1   0.85861 1.92337  -83.973
## - pt      1   0.86610 1.93086  -83.849
##
## Step:  AIC=-105.64
## lCost ~ pt + lCap + date + ne
##
##          Df Sum of Sq     RSS      AIC
## + ct      1   0.11570 0.74677 -108.248
## + t2      1   0.06248 0.79999 -106.045
## + sCum.n  1   0.05322 0.80925 -105.676
## <none>                0.86247 -105.638
## + pr      1   0.01563 0.84684 -104.223
## + bw      1   0.01009 0.85239 -104.014
## + t1      1   0.00688 0.85559 -103.894
## - ne      1   0.20229 1.06476 -100.896
## - date    1   0.46643 1.32890  -93.804
## - pt      1   0.60368 1.46615  -90.659
## - lCap    1   0.83751 1.69998  -85.924
##
## Step:  AIC=-108.25
## lCost ~ pt + lCap + date + ne + ct
##
##          Df Sum of Sq     RSS      AIC
## + sCum.n  1   0.08994 0.65683 -110.354
## <none>                0.74677 -108.248
## + t2      1   0.03416 0.71261 -107.746
## + pr      1   0.00852 0.73825 -106.615
## + bw      1   0.00816 0.73861 -106.599
## + t1      1   0.00074 0.74603 -106.279
## - ct      1   0.11570 0.86247 -105.638
## - ne      1   0.19023 0.93700 -102.986
## - pt      1   0.42372 1.17049  -95.866
## - date    1   0.54164 1.28841  -92.795
## - lCap    1   0.83781 1.58458  -86.173
##
## Step:  AIC=-110.35
## lCost ~ pt + lCap + date + ne + ct + sCum.n
##
##          Df Sum of Sq     RSS      AIC
## <none>                0.65683 -110.354
## + bw      1   0.02032 0.63651 -109.360
## + t2      1   0.01859 0.63823 -109.273
## + pr      1   0.01857 0.63825 -109.272
## + t1      1   0.00639 0.65044 -108.667
## - sCum.n  1   0.08994 0.74677 -108.248
## - pt      1   0.11283 0.76965 -107.282
## - ct      1   0.15243 0.80925 -105.676
```

```
## - ne      1   0.26944 0.92626 -101.355
## - date    1   0.54419 1.20101  -93.042
## - lCap    1   0.92655 1.58338  -84.198
```

```
##
## Call:
## lm(formula = lCost ~ pt + lCap + date + ne + ct + sCum.n, data = df)
##
## Coefficients:
## (Intercept)           pt         lCap         date           ne           ct
##   -13.42364     -0.24042      0.72559      0.21509      0.24059      0.15020
##      sCum.n
##    -0.07013
```

This puts out lCost = pt + lCap + date + ne + ct + sCum.n and drops 4 variables (t1, t2, bw & pr) with an AIC of -110.35. Therewith one can conclude that the second model is more parsimonious than the first because its lower AIC.

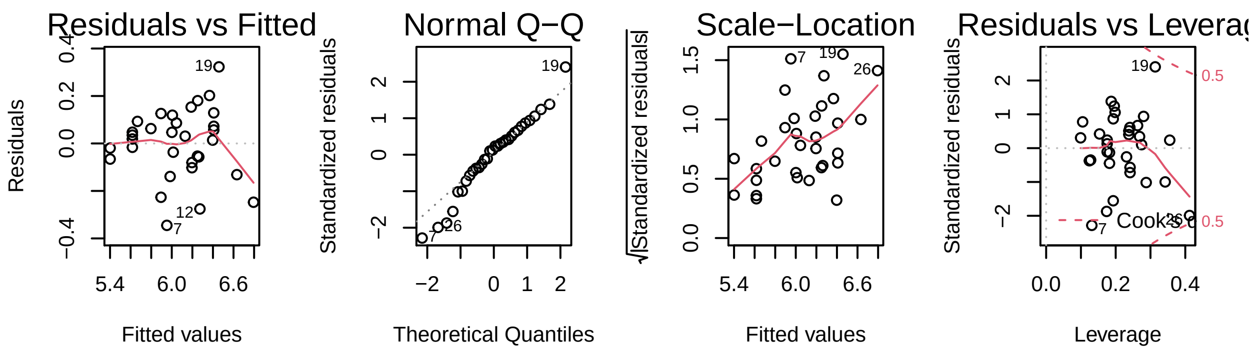
Exercise 4.b)

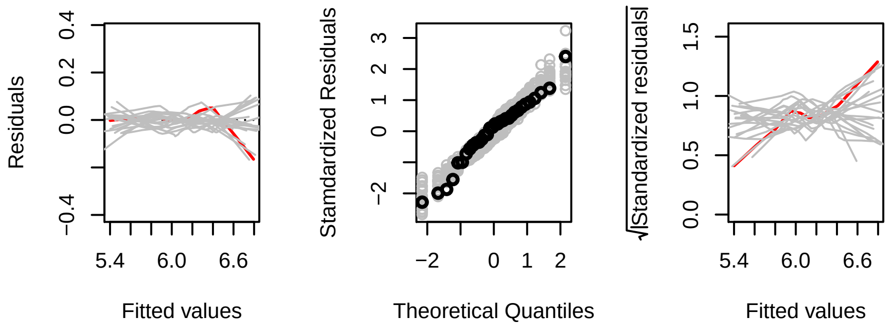
What about the residual and sensitivity analysis?

```r
lm3.2 <- lm(lCost ~ pt + lCap + date + ne + ct + sCum.n, data = df)
summary(lm3.2)
```

```
##
## Call:
## lm(formula = lCost ~ pt + lCap + date + ne + ct + sCum.n, data = df)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -0.34497 -0.06940  0.02474  0.08788  0.32267
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept) -13.42364    3.42411  -3.920 0.000608 ***
## pt           -0.24042    0.11602  -2.072 0.048700 *
## lCap          0.72559    0.12218   5.939 3.37e-06 ***
## date          0.21509    0.04726   4.551 0.000119 ***
## ne            0.24059    0.07513   3.202 0.003694 **
## ct            0.15020    0.06236   2.409 0.023707 *
## sCum.n       -0.07013    0.03790  -1.850 0.076133 .
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.1621 on 25 degrees of freedom
## Multiple R-squared:  0.8517, Adjusted R-squared:  0.8161
## F-statistic: 23.92 on 6 and 25 DF,  p-value: 3.183e-09
```

```r
par(mfrow=c(2,4))
plot(lm3.2)
plot.lmSim(lm3.2, SEED = 1)
```





There is no evidence that any of the model assumptions is violated. What about multicollinearity?

```r
vif(lm3.2)
```

```
##        pt     lCap     date       ne       ct   sCum.n
## 2.497443 1.089288 2.716501 1.289015 1.142437 2.248181
```

There is also no problem with multicollinearity. One can conclude therewith that the model fits the data well.

Exercise 4.c)

```r
confint(lm3.2)
```

```
##                     2.5 %       97.5 %
## (Intercept) -20.47573093 -6.371552102
## pt           -0.47935760 -0.001481439
## lCap          0.47395060  0.977234875
## date          0.11775366  0.312423592
## ne            0.08586099  0.395323607
## ct            0.02177208  0.278632684
## sCum.n       -0.14819161  0.007933141
```

Question to 4.c)

How do we see with the output above, that pt affects the price significantly?