№4. $\xi \sim R[\theta; 2\theta]$, $p(x) = \frac{1}{\theta}$ $[\theta; 2\theta]$

а) X_n – выборка

$$\alpha_1(\theta) = M\xi = \int_\theta^{2\theta} x \cdot \frac{1}{\theta} dx = \frac{1}{\theta} \cdot \frac{x^2}{2}\Big|_\theta^{2\theta} = \frac{1}{\theta}\left(\frac{4\theta^2}{2} - \frac{\theta^2}{2}\right) =$$
$$= \frac{3}{2}\theta$$

$$\left.\begin{array}{l} \\ \tilde{\alpha}_1 = \frac{1}{n}\sum x_i \end{array}\right\} \Rightarrow \frac{3}{2}\theta = \bar{X} \Rightarrow \underline{\tilde{\theta}_{омм} = \frac{2}{3}\bar{X}}$$

$$L(\theta) = \prod_{i=1}^{n} p(x_i; \theta) = \frac{1}{\theta^n} \to \max$$

$$L(\theta) = \begin{cases} \frac{1}{\theta^n}, & \min(x_i), \max(x_i) \in [\theta; 2\theta] \\ 0, & \min(x_i), \max(x_i) \notin [\theta; 2\theta] \end{cases} \Rightarrow$$

$$\Rightarrow \underline{\tilde{\theta}_{омп} = \frac{\max(x_i)}{2}}$$

б) $\underline{\tilde{\theta}_{омм} = \frac{2}{3}\bar{X}}$

$$M[\tilde{\theta}_{омм}] = \frac{2}{3} \cdot \frac{1}{n} n \cdot \frac{3}{2}\theta = \theta \text{ – несмещ.}$$

$$D\left[\frac{2}{3}\bar{X}\right] = \left(\frac{2}{3}\right)^2 \cdot D\xi \cdot \frac{1}{n}$$

$$M\xi^2 = \int_\theta^{2\theta} x^2 \cdot \frac{1}{\theta} dx = \frac{1}{\theta}\left(\frac{8\theta^3}{3} - \frac{\theta^3}{3}\right) = \frac{7}{3}\theta^2$$

$$D[\tilde{\theta}_{омм}] = \frac{4}{9n}\left(\frac{7}{3}\theta^2 - \frac{9}{4}\theta^2\right) \underset{n\to\infty}{\Rightarrow} 0 \text{ – состоятельна}$$

$$\underline{\tilde{\theta}_{омп} = \frac{\max(x_i)}{2}}$$

$$M[\tilde{\theta}_{омп}] = \frac{1}{2} \cdot M[x_{max}] = (*)$$

$$p(x) = n \cdot (F(z))^{n-1} \cdot p(z) \Rightarrow F(z) = \int_\theta^x \frac{1}{\theta} dz = \frac{z}{\theta}\Big|_\theta^x =$$
$$= \frac{x}{\theta} - 1$$

$$(*) = \frac{1}{2} \cdot \int_\theta^{2\theta} nx\left(\frac{x}{\theta} - 1\right)^{n-1} \cdot \frac{1}{\theta} dx = \frac{1}{2}\int_0^1 n(t+1) \cdot t^{n-1}\theta\, dt =$$
$$= \frac{n\theta}{2}\int_0^1 (t^n + t^{n-1}) dt = \frac{n\theta}{2}\left(\frac{1}{n+1} + \frac{1}{n}\right) = \frac{2n+1}{n+1} \cdot \frac{\theta}{2} \text{ – смещ.}$$

$\tilde{\theta}'_{омп} = \frac{2(n+1)}{2n+1} \cdot \tilde{\theta}_{омп} = \frac{n+1}{2n+1} X_{max}$ – несмещ

$D\left[\frac{n+1}{2n+1} X_{max}\right] = 4\left(\frac{n+1}{2n+1}\right)^2 \cdot D\left[\frac{X_{max}}{2}\right]$

$M\xi^2 = \frac{1}{4}\int_{\theta}^{2\theta} nx^2 \left(\frac{x}{\theta}-1\right)^{n-1} \frac{1}{\theta} dx = \left\{ \frac{x}{\theta} - 1 = t,\ x = (t+1)\theta \right\} =$

$= \frac{1}{4}\int_0^1 n\theta^2 (t+1)^2 t^{n-1} dt = \frac{1}{4} n\theta^2 \int_0^1 (t^{n+1} + 2t^n + t^{n-1}) dt =$

$= \frac{1}{4} n\theta^2 \left[\frac{1}{n+2} + \frac{2}{n+1} + \frac{1}{n}\right]$

$D[\tilde{\theta}'_{омп}] = \left(\frac{1}{4} n\theta^2 \left(\frac{1}{n+2} + \frac{2}{n+1} + \frac{1}{n}\right) - \frac{\theta^2}{4} \cdot \frac{(2n+1)^2}{(n+1)^2}\right) \cdot \frac{4(n+1)^2}{(2n+1)^2} =$

$= \theta^2 \frac{n}{4n^3 + 12n^2 + 9n + 2} \xrightarrow[n\to\infty]{} 0 \Rightarrow$ состоятельна

$\tilde{\theta}_3 = \frac{1}{5}(X_{min} + 2X_{max})$

$M[\tilde{\theta}_3] = \frac{1}{5} M[X_{min}] + \frac{2}{5} M[X_{max}] = \frac{1}{5} M[X_{min}] + \frac{2(2n+1)}{n+1} \cdot \frac{\theta}{5}$

$M[X_{min}] = \int_{\theta}^{2\theta} x \frac{1}{\theta} n \left(1 - \left(\frac{x}{\theta} - 1\right)\right)^{n-1} dx = \frac{n}{\theta} \int_{\theta}^{2\theta} x \left(2 - \frac{x}{\theta}\right)^{n-1} dx =$

$= \left\{ 2 - \frac{x}{\theta} = t,\ \frac{x}{\theta} = 2 - t,\ x = \theta(2-t),\ dx = -\theta dt \right\} =$

$= n\int_1^0 (2-t) \cdot t^{n-1} \cdot (-\theta) dt = -\theta n \int_1^0 (2t^{n-1} - t^n) dt =$

$= 2n\theta \int_0^1 t^{n-1} dt - n\theta \int_0^1 t^n dt = 2n\theta \cdot \frac{1}{n} - n\theta \cdot \frac{1}{n+1} =$

$= 2\theta - n\theta \cdot \frac{1}{n+1} = \frac{2\theta n + 2\theta - n\theta}{n+1} = \theta \frac{n+2}{n+1}$

$M[\tilde{\theta}_3] = \frac{\theta}{5}\left(\frac{n+2+4n+2}{n+1}\right) = \frac{\theta}{5} \cdot \frac{5n+4}{n+1}$ – смещ.

$\tilde{\theta}'_3 = \frac{n+1}{5n+4}(X_{min} + 2X_{max})$ – несмещ.

$D[\tilde{\theta}'_3] = D[\tilde{\theta}_3] \cdot \left(\frac{5(n+1)}{5n+4}\right)^2$

$D[\tilde{\theta}_3] = \frac{1}{25}(D[X_{min}] + 4 \cdot D[X_{max}] + 2cov(X_{min}; 2X_{max}))$

$D[X_{min}] = M[X^2_{min}] - \underbrace{M^2[X_{min}]}_{\theta^2\left(\frac{n+2}{n+1}\right)^2}$

$$M[X^2_{min}] = \int_\theta^{2\theta} x^2 \cdot \frac{1}{\theta} n\left(1-\left(\frac{x}{\theta}-1\right)\right)^{n-1} dx =$$

$$n\theta \int_\theta^{2\theta} \frac{x^2}{\theta^2} \left(2-\frac{x}{\theta}\right)^{n-1} dx = -n\theta^2 \int_1^0 (2-t)^2 \cdot t^{n-1} dt =$$

$$= n\theta^2 \int_0^1 \left(4t^{n-1} - 4t^n + t^{n+1}\right) dt = \theta^2 n\left(\frac{4}{n} - \frac{4}{n+1} + \frac{1}{n+2}\right)$$

$$D[X_{min}] = \theta^2 n\left(\frac{4}{n} - \frac{4}{n+1} + \frac{1}{n+2}\right) - \theta^2\left(\frac{n+2}{n+1}\right)^2 =$$

$$= \theta^2\left(\frac{4(n+1)(n+2) - 4n(n+2) + n(n+1)}{(n+1)(n+2)} - \frac{(n+2)^2}{(n+1)^2}\right) =$$

$$= \theta^2\left(\frac{4(n^2+2n+n+2) - 4n^2 - 8n + n^2 + n}{(n+1)(n+2)} - \frac{(n^2+4n+4)}{(n+1)^2}\right) =$$

$$= \theta^2\left(\frac{(5n+n^2+8)(n+1) - (n^2+4n+4)(n+2)}{(n^2+2n+1)(n+2)}\right) =$$

$$= \theta^2 \cdot \frac{5n^2 + n^3 + n^2 + 5n + 8n + 8 - n^3 - 2n^2 - 4n^2 - 8n - 4n - 8}{(n+1)^2(n+2)} =$$

$$= \theta^2 \cdot \frac{n}{(n+1)^2(n+2)}$$

$$D[X_{max}] = n\theta^2\left[\frac{1}{n+2} + \frac{2}{n+1} + \frac{1}{n}\right] - \theta^2\frac{(2n+1)^2}{(n+1)^2} =$$

$$= \theta^2\left(\frac{n(n+1) + 2n(n+2) + (n+1)(n+2)}{(n+1)(n+2)} - \frac{(4n^2+4n+1)}{(n+1)^2}\right) =$$

$$= \theta^2\left(\frac{n^2+n+2n^2+4n+n^2+2n+n+2}{(n+1)(n+2)} - \frac{(4n^2+4n+1)}{(n+1)^2}\right) =$$

$$= \theta^2\left(\frac{(4n^2+8n+2)(n+1) - (4n^2+4n+1)(n+2)}{(n+1)^2(n+2)}\right) =$$

$$= \theta^2\left(\frac{4n^3+4n^2+8n^2+8n+2n+2-4n^3-8n^2-4n^2-8n-n-2}{(n+1)^2(n+2)}\right)$$

$$= \theta^2 \frac{n}{(n+1)^2(n+2)}$$

$2 \cdot cov(X_{min}; 2X_{max}) = 4 \cdot cov(X_{min}; X_{max})$

$cov(X_{max}; X_{min}) = M[X_{min} \cdot X_{max}] - M[X_{min}] \cdot M[X_{max}]$

$M[X_{min} \cdot X_{max}] = n(n-1)\int\limits_{\theta}^{2\theta} dz \int\limits_{\theta}^{z} yz\left(\frac{z}{\theta} - \frac{y}{\theta}\right)^{n-2} \frac{1}{\theta^2} dy =$

$= \frac{n(n-1)}{\theta^2}\int\limits_{\theta}^{2\theta} dz \int\limits_{\theta}^{z} yz\left(\frac{z}{\theta} - \frac{y}{\theta}\right)^{n-2} dy = \left\{\frac{z}{\theta} - \frac{y}{\theta} = t,\ z-y = t\theta,\right.$

$\left. y = z - t\theta,\ dy = -\theta dt,\ z = t\theta + y\right\} =$

$= -\frac{n(n-1)}{\theta^2}\int\limits_{\theta}^{2\theta} dz \int (z - t\theta) z t^{n-2} \theta dt = \frac{n(n-1)}{\theta}\int\limits_{\theta}^{2\theta} z dz \int\limits_{0}^{\frac{z}{\theta}-1} (z t^{n-2} -$

$- \theta t^{n-1}) dt = \frac{n(n-1)}{\theta}\int\limits_{\theta}^{2\theta} z dz \left(\frac{z t^{n-1}}{n-1} - \frac{\theta t^n}{n}\right)\Bigg|_0^{\frac{z}{\theta}-1} =$

$= \frac{n(n-1)}{\theta}\int\limits_{\theta}^{2\theta}\left(\frac{z^2\left(\frac{z}{\theta}-1\right)^{n-1}}{n-1} - \frac{z\theta\left(\frac{z}{\theta}-1\right)^n}{n}\right) dz = \left\{\frac{z}{\theta} - 1 = k,\ z = \theta(k+1),\right.$

$\left. dz = \theta dk\right\} = n(n-1)\int\limits_{0}^{1}\left(\frac{\theta^2(k+1)^2}{n-1} \cdot k^{n-1} - \frac{\theta^2(k+1)}{n} k^n\right) dk =$

$= \theta^2 \cdot n\left(\frac{1}{n+2} + \frac{2}{n+1} + \frac{1}{n}\right) - \theta^2(n-1)\left(\frac{1}{n+2} + \frac{1}{n+1}\right)$

$cov(X_{max}; X_{min}) = n\theta^2\left(\frac{1}{n+2} + \frac{2}{n+1} + \frac{1}{n}\right) - \theta^2(n-1)\left(\frac{1}{n+2} + \frac{1}{n+1}\right) -$

$- \theta^2\left(\frac{n+2}{n+1}\right)\left(\frac{2n+1}{n+1}\right)$

$D[\tilde{\theta}_3] = \frac{1}{25}\left(\theta^2 \frac{n}{(n+1)^2(n+2)} + 4 \cdot \theta^2 \frac{n}{(n+1)^2(n+2)}\right) + 4 \cdot cov(X_{min}; X_{max})$

$D[\tilde{\theta}_3'] = \left(\frac{5(n+1)}{5n+4}\right)^2 D[\tilde{\theta}_3] = \frac{4\theta^2 + 5n}{25n^3 + 90n^2 + 96n + 32} \xrightarrow[n \to \infty]{} 0$ –

– состоятельна

c) $D[\tilde{\theta}_{омм}] = \frac{\theta^2}{n}\left(\frac{28}{27} - 1\right) = \frac{\theta^2}{27n}$

$D[\tilde{\theta}_{омп}'] = \theta^2 \cdot \frac{n}{4n^3 + 12n^2 + 9n + 2} \sim \frac{1}{4n^2}$

$D[\tilde{\theta}_3'] = \frac{4\theta^2 + 5n}{25n^3 + 90n^2 + 96n + 32} \sim \frac{1}{5n^2}$

$\tilde{\theta}_3'$ – эфф. всех и асимпт. эфф всех

d) $\xi \sim R[\theta, 2\theta]$, $h = \theta$

и f) $\tilde{h} = \frac{X_{max}}{2}$

$$f = \frac{X_{max}}{2\theta}$$

$$F_f = P(f < t) = P(X_{max} < 2\theta t) = P(X_i < 2\theta t \;\forall i) =$$

$$= (P(X_1 < 2\theta t))^n = (F_\theta(2\theta t))^n$$

$$F_\theta(t) = \begin{cases} 0, & t < \theta \\ t/\theta, & t \in (\theta; 2\theta) \\ 1, & t > 2\theta \end{cases} \qquad F_f(t) = \begin{cases} 0, & t \le \frac{1}{2} \\ (2t-1)^n, & t \in (\frac{1}{2}; 1) \\ 1, & t \ge 1 \end{cases}$$

$$p_f(t) = \begin{cases} 0, & t \le \frac{1}{2} \\ 2n(2t-1)^{n-1}, & t \in (\frac{1}{2}; 1) \\ 0, & t \ge 1 \end{cases}$$

$$p\left(a < \frac{X_{max}}{2\theta} < b\right) = 0{,}95$$

$$\int_{a_0}^{a} p_f(t)\,dt = 0{,}025 \Rightarrow \int_{1/2}^{a} 2n(2t-1)^{n-1}\,dt = 0{,}025$$

$$n = 100: \int_{\frac{1}{2}}^{a} 100 \cdot 2(2t-1)^{99}\,dt = 0{,}025 \Rightarrow a = 0{,}971$$

$$\int_{b}^{1} 200(2t-1)^{99}\,dt = 0{,}025 \Rightarrow b = 0{,}99987$$

$$\frac{1}{2} \cdot \frac{X_{max}}{0{,}971} > \theta > \frac{1}{2} \cdot \frac{X_{max}}{0{,}99987}$$

$$X_{max} = 239{,}44;\ \theta = 121$$

$$\theta \in (119{,}73;\ 123{,}29)$$

e) $\tilde{\theta}_{ОММ} = \frac{2}{3}\bar{\alpha}_1 = \frac{2}{3}\bar{X}$

и 1)

$$f(\alpha) = \frac{2}{3}\alpha_1, \quad \alpha_k = \frac{1}{n}\sum_{i=1}^{n} x_i^k$$

$$f'(\alpha) = \frac{2}{3}, \quad K_{11} = \alpha_2 - \alpha_1 \cdot \alpha_1$$

$$\sigma = \sqrt{\nabla^T f(\alpha) \cdot K \nabla f(\alpha)} = \sqrt{\frac{4}{9}(\alpha_2 - \alpha_1^2)}$$

$$\tilde{\sigma} \to \sigma \Rightarrow \frac{f(\bar{\alpha}) - f(\alpha)}{\tilde{\sigma}}\sqrt{n} \rightsquigarrow N(0;1)$$

$$\frac{\breve{\theta}-\theta}{\frac{2}{3}\sqrt{\tilde{\alpha}_2-\tilde{\alpha}_1^2}}\sqrt{n} \rightsquigarrow N(0;1)$$

$\theta = 20$: $\breve{\theta} = 19,64$, $\tilde{\alpha}_1 = 28,93$; $\tilde{\alpha}_2 = 922,04$

$$\frac{+1,96}{10}\cdot 6,15 + 20 > \theta > \frac{-1,96}{10}\cdot 6,15 + 20$$

$$\theta \in (18,8 ; 21,21)$$

T6. $H_0: p(x) \sim B(2)$ $n=2$

$H_1: \overline{H_0}$

$$F(x) = \sum_{k=0}^{n} C_n^k p^k (1-p)^{n-k} \cdot \theta(x-k)$$

$$p(x) = \sum_{k=0}^{n} C_n^k p^k (1-p)^{n-k} \cdot \delta(x-k)$$

$$p(x) = \underbrace{C_2^0 p^0 (1-p)^2}_{p_0 \text{ – ни разу не заболел}} + \underbrace{C_2^1 p^1 (1-p)^1}_{p_1 \text{ – заболел один раз}} + \underbrace{C_2^2 p^2 (1-p)^0}_{p_2 \text{ – заболел 2 раза}}$$

$$p_0 = (1-p)^2, \quad p_1 = 2p(1-p), \quad p_2 = p^2$$

$$L = p_0^{10} \cdot p_1^{181} \cdot p_2^{9}$$

$$\ln L = 10 \cdot \ln(1-p)^2 + 181 \cdot \ln(2p(1-p)) + 9 \cdot \ln(p^2) =$$

$$= 20 \cdot \ln(1-p) + 181 \cdot \ln(2p) + 181 \cdot \ln(1-p) + 18 \cdot \ln(p) =$$

$$= 201 \cdot \ln(1-p) + 181 \cdot \ln(2p) + 18 \cdot \ln(p) \longrightarrow \max$$

$$\frac{-201}{1-p} + \frac{181}{2p} \cdot 2 + \frac{18}{p} = 0$$

$$\frac{-201p + 199(1-p)}{p(1-p)} = 0$$

$$-400p = -199 \Rightarrow p = \frac{199}{400}$$

$$\frac{d^2 \ln L}{dp^2} = \frac{201}{(1-p)^2} \cdot (-1) - \frac{181}{p^2} - \frac{18}{p^2} < 0 \text{ – max}$$

$$\tilde{\Delta} = \frac{\left(10 - 200 \cdot \frac{201}{400}\right)^2}{200 \cdot \frac{201}{400}} + \frac{\left(9 - 200 \cdot \frac{199^2}{400^2}\right)^2}{200 \cdot \frac{199^2}{400^2}} + \frac{\left(181 - 200 \cdot 2 \cdot \frac{199}{400} \cdot \frac{201}{400}\right)^2}{200 \cdot 2 \cdot \frac{199 \cdot 201}{400^2}} = 180,25$$

1640,331

33,137

49,501

8149,503

6561,405

99,998

$\Delta \sim \chi^2(m-1-s) = \chi^2(1)$

кол-во событий -3

кол-во неизвестных -1

пл-ть распр.

$$p\text{-value} = P(\Delta \geq \tilde{\Delta} \mid H_0) = \int_{180,25}^{+\infty} g(x)\,dx < 0,005 < \alpha = 0,05$$

$\Rightarrow H_0$ – отвергаем

T7.

	<	=	>
1 партия	25	50	25
2 партия	52	41	7

H_0: выборки однородны

H_1: $\overline{H_0}$

$p_1 = \frac{77}{200}$ – вер-ть закит.

$p_2 = \frac{91}{200}$ – вер-ть равн.

$p_3 = \frac{32}{200}$ – вер-ть завыш.

$$\tilde{\Delta}_1 = \frac{\left(25 - 100 \cdot \frac{77}{200}\right)^2}{100 \cdot \frac{77}{200}} + \frac{\left(50 - 100 \cdot \frac{91}{200}\right)^2}{100 \cdot \frac{91}{200}} + \frac{\left(25 - 100 \cdot \frac{32}{200}\right)^2}{100 \cdot \frac{32}{200}} = 10,24$$

$$\tilde{\Delta}_2 = \frac{\left(52 - 100 \cdot \frac{77}{200}\right)^2}{100 \cdot \frac{77}{200}} + \frac{\left(41 - 100 \cdot \frac{91}{200}\right)^2}{100 \cdot \frac{91}{200}} + \frac{\left(7 - 100 \cdot \frac{32}{200}\right)^2}{100 \cdot \frac{32}{200}} = 10,24$$

$\tilde{\Delta} = 20,48$

$\Delta \sim \chi^2((m-1)(k-1)) = \chi^2(2)$

события

партии

$p\text{-value} = P(\Delta \geq \tilde{\Delta} | H_0) = \int_{20,48}^{+\infty} g(x)\,dx < 0,005 < \alpha = 0,05 \Rightarrow$

$\Rightarrow H_0$ отвергаем

T8.

	2	3	4	5
1 поток	33	43	80	144
2 поток	39	35	72	154

H_0: потоки однородные

H_1: $\overline{H_0}$

$p_2 = \frac{72}{600}$, $p_3 = \frac{78}{600}$, $p_4 = \frac{152}{600}$, $p_5 = \frac{298}{600}$

$$\tilde{\Delta}_1 = \frac{\left(33 - 300 \cdot \frac{72}{600}\right)^2}{300 \cdot \frac{72}{600}} + \frac{\left(43 - 300 \cdot \frac{78}{600}\right)^2}{300 \cdot \frac{78}{600}} + \frac{\left(80 - 300 \cdot \frac{152}{600}\right)^2}{300 \cdot \frac{152}{600}} + \frac{\left(144 - 300 \cdot \frac{298}{600}\right)^2}{300 \cdot \frac{298}{600}} = 1,039$$

$\tilde{\Delta}_2 = 1,039$

$\tilde{\Delta} = 2,077$

$\Delta \sim \chi^2((4-1)(2-1)) = \chi^2(3)$

$p\text{-value} = P(\Delta \geq \tilde{\Delta} | H_0) = \int_{2,077}^{+\infty} g(x)\,dx > 0,5 > \alpha = 0,05 \Rightarrow$

$\Rightarrow H_0$ нет оснований отвергн.

T9. $n = 100$

$(-\infty; 0,5)$ 0	$(0,5; 1,5)$ 1	2	3	4	5	6	7	8	9
5	8	6	12	14	18	11	6	13	7

H_0: $\xi \sim R(0; 10)$

a) $\tilde{\Delta} = \underbrace{\frac{\left(5 - 100 \cdot \frac{1}{10}\right)^2}{10}}_{2,5} + \underbrace{\frac{(8-10)^2}{10}}_{0,4} + \underbrace{\frac{(6-10)^2}{10}}_{1,6} + \underbrace{\frac{(12-10)^2}{10}}_{0,4} + \underbrace{\frac{(14-10)^2}{10}}_{1,6} +$

$$+\frac{(18-10)^2}{10}_{6,4} + \frac{(11-10)^2}{10}_{0,1} + \frac{(6-10)^2}{10}_{1,6} + \frac{(13-10)^2}{10}_{0,9} + \frac{(7-10)^2}{10}_{0,9} =$$

$= 16,4$, $\Delta \sim \chi^2(9)$

$p\text{-value} = \int_{16,4}^{+\infty} q(x)\,dx > 0,05 \Rightarrow$ нет оснований отвергать H_0

Колмогоров: $\tilde{\Delta} = \sqrt{n} \max_i \left((\tilde{F}(x_i - 0) - F(x_i)); (\tilde{F}(x_i + 0) - F(x_i)) \right) =$

$= 1,3$ (смотреть на github)

$\Delta \sim K(x) = P(\Delta < x) = 1 + 2\sum_{k=1}^{\infty} (-1)^k e^{-2k^2x^2} \quad (0, +\infty)$

$p\text{-value} = P(\Delta \geq \tilde{\Delta} | H_0) = -2\sum_{k=1}^{\infty} (-1)^k e^{-2k^2\tilde{\Delta}^2} = 0,065 \Rightarrow$

$\Rightarrow$ нет оснований отвергать H_0

b) $p(x) = \frac{1}{\sigma\sqrt{2\pi}} \cdot \exp\left(\frac{-(x-a)^2}{2\sigma^2}\right)$, $\xi \sim N(a, \sigma^2)$

$p_1 = \int_{-\infty}^{0,5} p(x)\,dx \quad \dots \quad p_{10} = \int_{8,5}^{+\infty} p(x)\,dx$

$L = p_1^5 \cdot p_2^8 \cdot \ldots \cdot p_{10}^7 \longrightarrow \max$

на github увидите, что нашли $\sigma = 2,7$, $a = 4,8$

находим $p_1 \dots p_{10}$ и $\hat{\Delta} = 9,8$

$\Delta \sim \chi^2(m-1-\underset{2}{s}) = \chi^2(7)$

$p\text{-value} = \int_{9,8}^{+\infty} q(x)\,dx > 0,1 > \alpha = 0,05 \Rightarrow$ нет оснований отвергать H_0

Колмогоров в коде :)

$p\text{-value} = 1 - \frac{1-k}{N} = 0,53$ — нет оснований отв. H_0

$k = 23\,383$, $N = 50\,000$

c) $\tilde{\alpha}_1 = \frac{1}{n}\sum x_i = a$
$\tilde{\alpha}_2 - (\tilde{\alpha}_1)^2 = \sigma^2$ $\Big\} \Rightarrow$ $a = 4{,}7$, $\sigma = 2{,}5$

получим $k = 39\,359$, $N = 50\,000$ $\Rightarrow$ p-value $= 0{,}21$ – нет оснований отв. H_0

Т12. $n = 3$: $-1{,}11$; $-6{,}10$; $2{,}42$ из $N(a;\sigma^2)$

$H_0: a = 0$
$H_1: a > 0$, $a < 0$; $a \neq 0$

$$s^2 = \frac{1}{n-1}\sum(x_i - \bar{x})^2$$

$\bar{x} = -1{,}596$

$$s^2 = \frac{1}{2}(0{,}237 + 20{,}277 + 16{,}136) = 18{,}325$$

$$\Delta = \frac{\bar{x} - a}{s}\sqrt{n} \sim t(\cancel{n-1}\ 2)$$

$$\tilde{\Delta} = \frac{-1{,}596}{\sqrt{18{,}325}}\sqrt{3} = -0{,}646$$

p-value $= 2\cdot P(\Delta \geq |\tilde{\Delta}|) = 2\int_{0,646}^{+\infty} g(t)\,dt > 0{,}5$ $\Rightarrow$ нет оснований отвергать H_0 при $H_1: a \neq 0$

$H_1: a > 0$ отвергаем, т.к. $\bar{x} < 0$ (оценка)

$H_1: a < 0$

p-value $= \underbrace{1 - P(\Delta \geq \tilde{\Delta})}_{P(\Delta \leq \tilde{\Delta})} = 1 - \int_{-0,646}^{+\infty} g(t)\,dt = \int_{0,646}^{+\infty} g(t)\,dt >$

$> 0{,}25$ – нет оснований отвергать H_0 при $H_1: a < 0$

Т13. $\underset{n=3}{x} = \{-1{,}11; -6{,}10; 2{,}42\}$, a, $\sigma_x^2 = 2$

$\underset{m=2}{y} = \{-2{,}29; -2{,}91\}$, b, $\sigma_y^2 = 1$

$H_0: a = b$, $H_1: a \neq b$, $a > b$, $a < b$

$\bar{x} = -1,596 \ (\tilde{a})$

$\bar{y} = -2,6 \ (\tilde{b})$ => $H_1: a<b$ отвергаем, т.к не наблюдаем

$H_1: a \neq b$

$$\Delta = \frac{\bar{x} - \bar{y}}{\sqrt{\frac{\sigma_x^2}{n} + \frac{\sigma_y^2}{m}}} \sim N(0;1)$$

$$\tilde{\Delta} = \frac{\overbrace{-1,596 + 2,6}^{=1,004}}{\sqrt{\frac{2}{3} + \frac{1}{2}}} = 0,93$$

$$p\text{-value} = 2 \cdot P(\Delta \geq |\tilde{\Delta}|) = 2\int_{0,93}^{+\infty} p(x)\,dx = 0,3524 >$$

$> \alpha = 0,05$ – нет основания отв. H_0

$H_1: a > b$

$$p\text{-value} = \int_{0,93}^{+\infty} p(x)\,dx = 0,1762 > \alpha = 0,05$$ – нет

основания отвергнуть H_0