

# Support Vector Machine for Surrogate Modeling

Group 1

Technische Universität Dortmund

December 2, 2025

## What is SVR?

- Support Vector Regression is the regression form of Support Vector Machines (SVMs). A kernel-based machine learning model that can approximate highly nonlinear functions. Works well even with small-to-medium datasets, making it ideal for engineering simulations.
- SVR can learn a mapping from design variables to simulation output.
- Used when dataset sizes are moderate (100–5000 samples).
- **Core Idea:** SVR fits a hyperplane (higher dim. "line") to the data while ensuring all errors are within a specified margin  $\epsilon$ .

# Finding the hyperplane

**Objective:** Find a function  $f(\mathbf{x}) = \mathbf{w} \cdot \mathbf{x} + b$  that fits the data well, while minimizing the model complexity.

- **Optimization Problem:**

$$\min_{\mathbf{w}, b, \xi_i, \xi_i^*} \frac{1}{2} \|\mathbf{w}\|^2 + C \sum_{i=1}^N (\xi_i + \xi_i^*)$$

- **Subject to Constraints:**

under  
over

$$y_i - f(\mathbf{x}_i) \leq \epsilon + \xi_i$$

$$f(\mathbf{x}_i) - y_i \leq \epsilon + \xi_i^*$$

$$\xi_i, \xi_i^* \geq 0$$

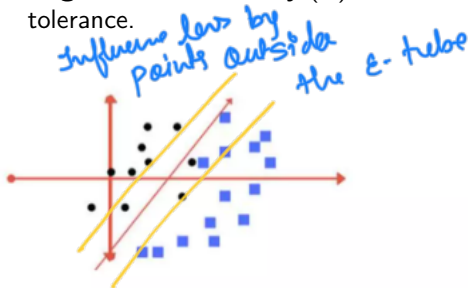
slack variables

- **The Kernel Trick:** Allows operating in a high-dimensional feature space without explicitly computing the coordinates, handling non-linear relationships:  $\mathbf{x} \cdot \mathbf{y} \rightarrow K(\mathbf{x}, \mathbf{y})$ .

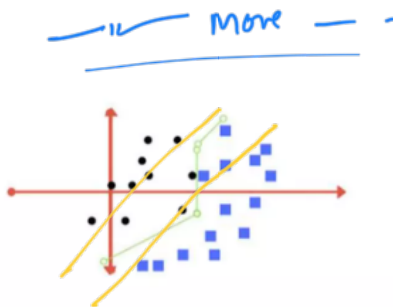
$\phi(\mathbf{x}), \phi(\mathbf{y})$

# Key Hyper-parameters for SVR

- **Regularization Penalty (C):** Trade-off between model flatness and error tolerance.



low regularization value



high regularization value

- Low C  $\rightarrow$  smoother model, more error tolerance
- High C  $\rightarrow$  tighter fit, less error tolerance

# Key Hyper-parameters for SVR

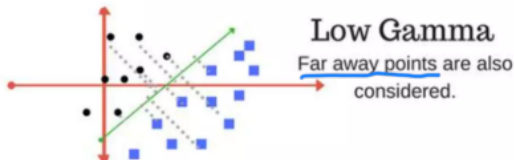
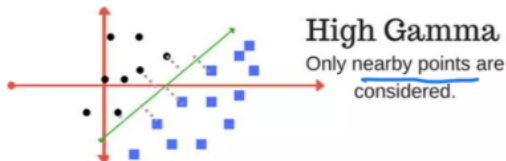
- **Kernel Choice:** Dependent upon the problem we are trying to solve – **RBF** should work for us.

$$K(\mathbf{X}_i, \mathbf{X}_j) = \left\{ \begin{array}{ll} \mathbf{X}_i \cdot \mathbf{X}_j & \text{Linear} \\ (\gamma \mathbf{X}_i \cdot \mathbf{X}_j + C)^d & \text{Polynomial} \\ \exp(-\gamma \|\mathbf{X}_i - \mathbf{X}_j\|^2) & \text{RBF} \\ \tanh(\gamma \mathbf{X}_i \cdot \mathbf{X}_j + C) & \text{Sigmoid} \end{array} \right\}$$

- Allow SVR to approximate complex FEM response surfaces
- Control flexibility vs overfitting

# Key Hyper-parameters for SVR

- **Kernel Coefficient ( $\gamma$ ):** (specifically for RBF) Controls the influence of a single training example.



# SVR as a Surrogate Model: Pros & Cons

## Advantages

$$c \sum_{i=1}^N (\epsilon y_i + \epsilon y_i^*)$$

- **Robustness:** <sup>less</sup> sensitive to outliers due to the  $\epsilon$ -tube.
- **High-Dimensional Data:** Works very well in high-dimensional spaces using the Kernel trick.
- **Efficiency:** The complexity is independent of the input space dimension. <sup>N</sup>

## Disadvantages

- **Computationally Expensive:** Training ( $O(N^2)$  -  $O(N^3)$ ) and hyperparameter optimization are expensive.
- **Lacks built-in uncertainty quantification** and is harder to interpret than simpler models.