technische universität dortmund

fakultät statistik

Project “Computer Experiments”

Fallstudien II / Case Studies II

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Model Input Parameters

Part	Quantity	Symbol	Unit	Distrib.	Mean	Std.
Membrane	Young's modulus	3 → E_{mem}	[GPa]	Lognormal	0.6	0.09 $\times 10^6$
	Poisson's ratio	7 → ν_{mem}		Uniform	0.4	0.0115
	thickness	t_{mem}	[mm]	Deterministic	1	
	pre-stress	2 → σ_{mem}	[MPa]	Lognormal	4	0.8 $\times 10^3$
	surface loading	6 → f_{mem}	[kPa]	Gumbel	0.4	0.12
	rupture stress[1]	1 → $\sigma_{\mathrm{mem,\,y}}$	[MPa]	Lognormal	11	1.650 $\times 10^3$
Truss	Young's modulus	E_{tru}	[GPa]	Deterministic	205	
	cross sectional area	A_{tru}	[cm^2]	Deterministic	25	
Edge cable	Young's modulus	E_{edg}	[GPa]	Deterministic	205	
	diameter	d_{edg}	[mm]	Deterministic	12	
	pre-stress	4 → σ_{edg}	[MPa]	Lognormal	353.678	70.735 $\times 10^3$
Support cable	Young's modulus	E_{sup}	[GPa]	Deterministic	205	
	diameter	d_{sup}	[mm]	Deterministic	12	
	pre-stress	5 → σ_{sup}	[MPa]	Lognormal	400.834	80.166 $\times 10^3$

[1] No parameter of the structural analysis model.

Normal Distribution

$X \sim \mathcal{N}(\mu, \sigma^2)$ with probability density function

$$f_X(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right) \mathbf{1}_{(-\infty,\infty)}(x) \text{ where } \mu \in \mathbb{R},\ \sigma^2 > 0$$

$$\mathrm{E}(X) = \mu \quad = \text{mean}$$

$$\mathrm{Var}(X) = \sigma^2 \quad = (\text{std})^2$$

(parameters)

Uniform Distribution

$X \sim \mathcal{U}(a, b)$ with probability density function

$$f_X(x) = \frac{1}{b-a}\mathbf{1}_{[a,b]}(x) \text{ where } a, b \in \mathbb{R},\ a < b$$

$$\mathrm{E}(X) = \frac{1}{2}(a+b)$$

$$\mathrm{Var}(X) = \frac{1}{12}(b-a)^2$$

Given in the table of the slides

$$a = \mathrm{E}(X) - \sqrt{3 \cdot \mathrm{Var}(X)}$$

$$b = \mathrm{E}(X) + \sqrt{3 \cdot \mathrm{Var}(X)}$$

find parameters of the pdf

Log-normal Distribution $\ln X \sim N(\mu, \sigma^2)$

$X \sim \mathcal{LN}(\mu, \sigma^2)$ with probability density function

$\Rightarrow \ln(x) \sim N(\mu, \sigma^2)$

$$f_X(x) = \frac{1}{\sqrt{2\pi\sigma^2 x}} \exp\left(-\frac{(\log(x)-\mu)^2}{2\sigma^2}\right) \mathbf{1}_{(0,\infty)}(x) \quad \text{where } \mu \in \mathbb{R},\ \sigma > 0$$

$$\left.\begin{aligned} E(X) &= \exp\left(\mu + \frac{1}{2}\sigma^2\right) \\ \mathrm{Var}(X) &= \exp\left(2\mu + \sigma^2\right)\left(\exp(\sigma^2) - 1\right) \end{aligned}\right\} \text{Given:-}$$

$$\mu = \log\left(\frac{E(X)^2}{\sqrt{E(X)^2 + \mathrm{Var}(X)}}\right)$$

$$\sigma^2 = \log\left(1 + \frac{\mathrm{Var}(X)}{E(X)^2}\right)$$

$E[\ln(x)]$ & $V(\ln(x))$

$$\mu = \log(E[x]) - \frac{1}{2}\sigma^2$$

$$\sigma = \log\left(\sqrt{1 + \left(\frac{std(x)}{E[x]}\right)^2}\right)$$

Implimented

Wiki

$$\sigma = \sqrt{\ln\left(1 + \frac{V(x)}{E[x]^2}\right)}$$

$$\exp(\mu) = \exp\left(\log(E[x]) - \frac{1}{2}\sigma^2\right)$$

$$= \frac{E[x]}{\exp\left(\frac{1}{2}\sigma^2\right)}$$

Gumbel Distribution

$X \sim \mathcal{G}(\mu, \beta)$ with probability density function

$$f_X(x) = \frac{1}{\beta} \exp\left(-z - \exp\left(-z\right)\right) \mathbf{1}_{(-\infty,\infty)}(x) \text{ where } z = \frac{x - \mu}{\beta},\ \mu \in \mathbb{R},\ \beta > 0$$

$\mathrm{E}(X) = \mu + \beta\gamma$ where $\gamma = -\Gamma'(1) \approx$ 0.5772 is the Euler-Mascheroni constant

$$\mathrm{Var}(X) = \frac{\pi^2\beta^2}{6}$$

$$\mu = \mathrm{E}(X) - \frac{\sqrt{6 \cdot \mathrm{Var}(X)}\gamma}{\pi} = E[X] - \beta\gamma$$

$$\beta = \frac{\sqrt{6 \cdot \mathrm{Var}(X)}}{\pi}$$