Sampling Methods

Random Sampling

Preparation

In the project description, we are given the following details about the unknown distributions of the materials of the sun sail:

Part	Quantity	Unit	Distribution	Mean	Std.
Membrane	Young's modulus	[GPa]	Lognormal	0.6	0.09
	Poisson's ratio		Uniform	0.4	0.0115
	pre-stress	[MPa]	Lognormal	4	0.8
	surface loading	[kPa]	Gumbel	0.4	0.12
	rupture stress	[MPa]	Lognormal	11	1.650
Edge cable	pre-stress	[MPa]	Lognormal	353.678	70.735
Support cable	pre-stress	[MPa]	Lognormal	400.834	80.166

Preparation

To get random samples from the distributions of the sun sail materials, we first need to get their respective parameters from mean and standard deviation.

Lognormal distribution

The lognormal distributions has parameters μ and σ^2 and it holds that

$$\mu = \ln\left(\frac{\mathbb{E}[X]^2}{\sqrt{\mathrm{Var}(X) + \mathbb{E}[X]^2}}\right)$$
$$\sigma^2 = \ln\left(1 + \frac{\mathrm{Var}(X)}{\mathbb{E}[X]^2}\right),$$

where $\mathbb{E}[X]$ is our mean and $\mathrm{Var}(X)$ is the square of our standard deviations. We can then simply calculate the parameters.

Source: https://en.wikipedia.org/wiki/Log-normal_distribution.

Uniform distribution

We have parameters a and b and

$$\begin{aligned}
\mathrm{Var}(X) &= \frac{1}{12}(b-a)^2 \\
\mathbb{E}[X] &= \frac{1}{2}(a+b).
\end{aligned}$$

Uniform distribution

Thus,

$$
\begin{aligned}
b - a = \sqrt{12 \cdot \mathrm{Var}(X)} &\iff b = \sqrt{12 \cdot \mathrm{Var}(X)} + a \\
\mathbb{E}(X) = \frac{1}{2}(a + \sqrt{12 \cdot \mathrm{Var}(X)} + a) &= a + \frac{1}{2}\sqrt{12 \cdot \mathrm{Var}(X)} \\
\Rightarrow a &= \mathbb{E}(X) - \frac{1}{2}\sqrt{12 \cdot \mathrm{Var}(X)} \\
b &= \mathbb{E}(X) + \frac{1}{2}\sqrt{12 \cdot \mathrm{Var}(X)}.
\end{aligned}
$$

See
https://en.wikipedia.org/wiki/Continuous_uniform_distribution.

Gumbel distribution

For the Gumbel distribution, we have parameters μ and β and

$$\mathbb{E}(X) = \mu + \beta \cdot \gamma$$
$$\mathrm{Var}(X) = \frac{\pi^2}{6}\beta^2$$

with γ the Euler-Mascheroni constant.

Gumbel distribution

Thus,

$$\beta = \sqrt{\frac{6}{\pi^2}\mathrm{Var}(X)}$$
$$\mu = \mathbb{E}(X) - \beta \cdot \gamma$$
$$\Rightarrow \mu = \mathbb{E}(X) - \sqrt{\frac{6}{\pi^2}\mathrm{Var}(X)} \cdot \gamma$$

See https://en.wikipedia.org/wiki/Gumbel_distribution.

Generating random samples

With these expressions for the parameters of the respective distributions, we can generate $n = 200$ random samples from each distribution.

Note: For the Gumbel distribution, there is no direct sampling method in base R. It is possible to draw from the Gumbel distribution using `distributions3`. For this to work, we additionally need to install the `revdbayes` package. Here, σ corresponds to our β. The Euler-Mascheroni constant is `-digamma(1)`.

See https://zeileis.github.io/distributions3/.

Generating random samples

```r
gumbel_sigma <- sqrt(6/(pi^2)*0.12^2)*(-digamma(1))
gumbel_mu <- 0.4 - gumbel_sigma

n <- 200

X <- Gumbel(mu = gumbel_mu, sigma = gumbel_sigma)

gumbel_sample <- random(X, n)
```

Advantages and disadvantages

- Advantages:
  1. Very simple to implement
  2. Easy to grasp: we just draw randomly with replacement from the distribution
- Disadvantages:
  1. We miss out on the tails of our distribution (200 draws is fine, but far from perfect)
  2. We draw samples around the mode of the distribution with a very high distribution, which means we get many values that lie closely together (uninteresting, little variability?)