Step 1 : Define input parameters & Design space

1. Select uncertain input parameters (E_{mem}, ν_{mem}, σ_{mem}, f_{mem} ,σ_{edg}, σ_{sup} , …)

2. Use mean and standard deviation from slides to define distributions

3. Truncate each distribution within a reasonable interval (e.g., ±3σ or quantile range)
   - To avoid extremely unreasonable inputs and maintain simulation stability

4. Construct normalized design space
   - Design the vector $X = (X_1, X_2, \dots, X_d)$, d depends on the number of parameters
   - Normalize all physical parameters to the interval [0,1]
   - Construct input space as a d-dimensional unit hypercube , i.e., $[0,1]^d$

Step 2 : Generate Space Filling Design Points (Latin Hyper-cube Sampling + Maximin Design)

1. Choose sample size $n_{max} < 200$
2. Generate LHS in $[0,1]^d$
   - Divide each dimension into n intervals over [0,1]
   - Randomly select a point within each interval
   - Randomly pair the points across dimensions to obtain the initial n×d design matrix U
3. Optimize using maximin criterion
   - Calculate the Euclidean distance between all pairs of points.
   - Maximize the minimum distance: $\max_{U} min_{i \neq j} \|U_i - U_j\|$
   - Improve the design by swapping rows/columns, simulated annealing, or other heuristic algorithms
4. Map normalized points to physical space
   - Inverse-CDF mapping preserves the stratification property of LHS in each dimension.

   or Quantile function

Step 3 : Coupling FEM Model and Surrogate Model

1. Run FEM model
    - Run Kratos FEM for each design point $x^{(i)}$
    - Obtain maximum Cauchy stress: $y^{(i)} = \sigma_{mem,max}(x^{(i)})$
    - Form the training set: $\mathcal{D} = \{(x^{(i)}, y^{(i)})\}$

2. Build the Surrogate Model (Kriging / GP)
    - Construct a Gaussian Process model: $Y(x) \sim GP\big(m(x), k(x, x')\big)$

3. Assess Design Quality and Surrogate Accuracy
    - Inspect spatial distribution of predictive variance