ML Evaluation Classification

CMSC 473/673 - NATURAL LANGUAGE PROCESSING

Learning Objectives

Develop an intuition about precision & recall

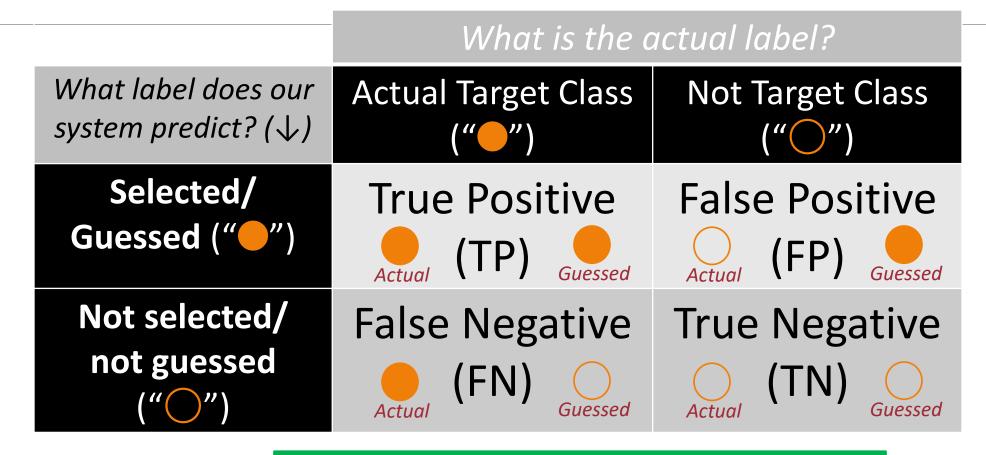
Extend P/R to multi-class problems

Identify when you might want certain evaluation metrics over others

Model classification problems using logistic regression

Define appropriate features for a logistic regression problem

Review: Classification Evaluation: the 2-by-2 contingency table

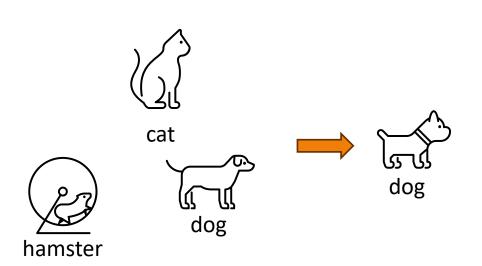


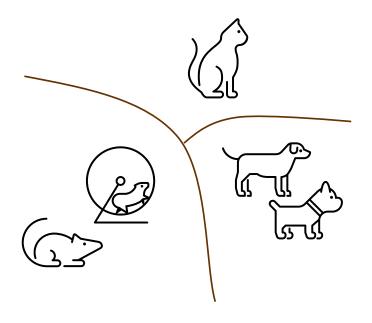
Construct this table by *counting* the number of TPs, FPs, FNs, TNs

Review: Types of Learning

SUPERVISED LEARNING

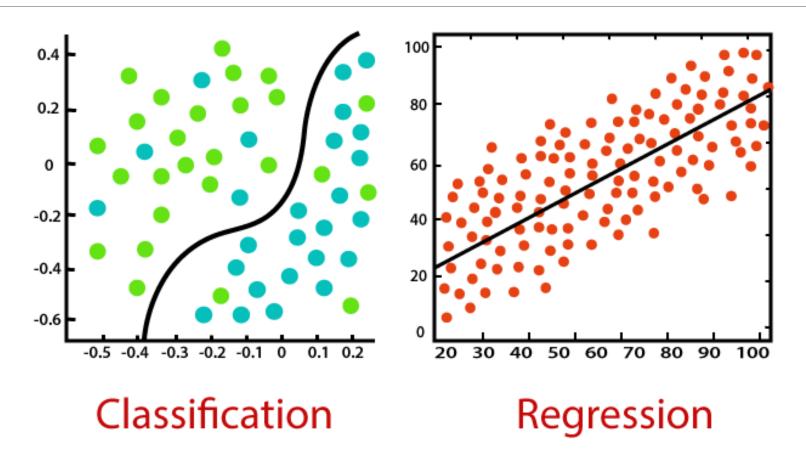
UNSUPERVISED LEARNING





DO NOT ITERATE Review: Steps ON THE TESTING dog SET!!! Training duck Labels **Training Data** perro Word Learned pato **Training Training** model **Features** ... **Dev Set Evaluate Testing Data** Word Learned **Testing** gato Prediction **Features** model

Review: Types of models

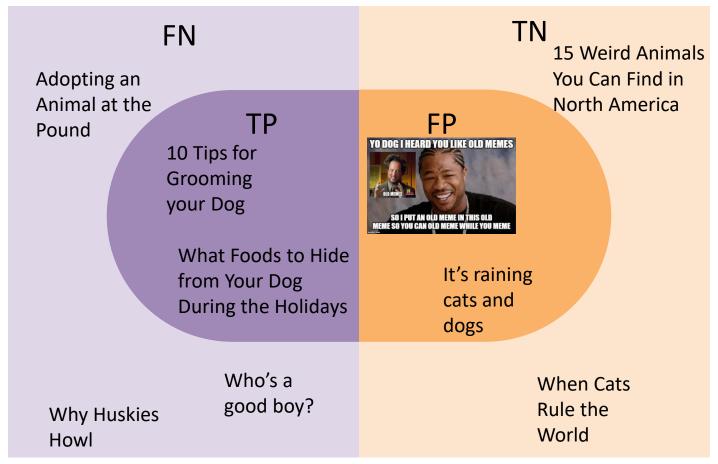


Review: Classification Evaluation: the 2-by-2 contingency table

What is the actual label? What label does our **Actual Target Class Not Target Class** system predict? (\downarrow) Selected/ **False Positive** True Positive Guessed ("
") (FP) Not selected/ False Negative True Negative not guessed (FN) Actual

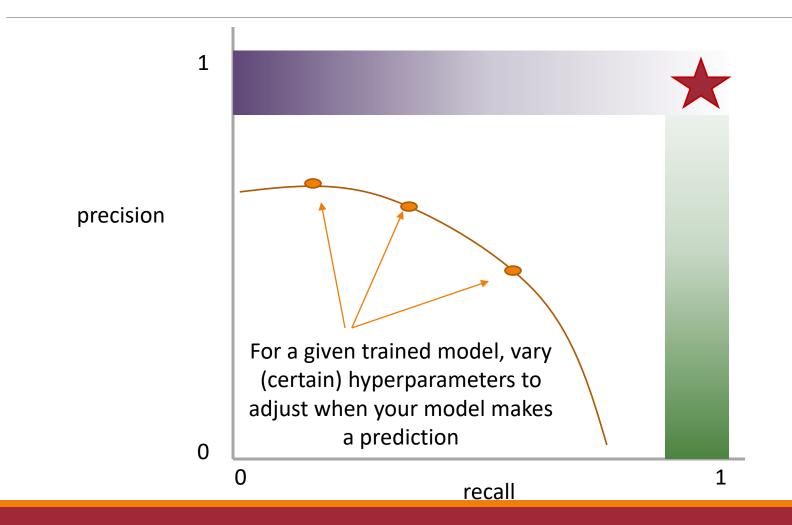
Contingency Table (out of table form)

Query: Articles about dogs



eme from: https://www.reddit.com/r/AdviceAnimals/comments/ck8xh0/yo_dawg_i_heard_you_like_old_memes

Review: Precision and Recall Present a Tradeoff



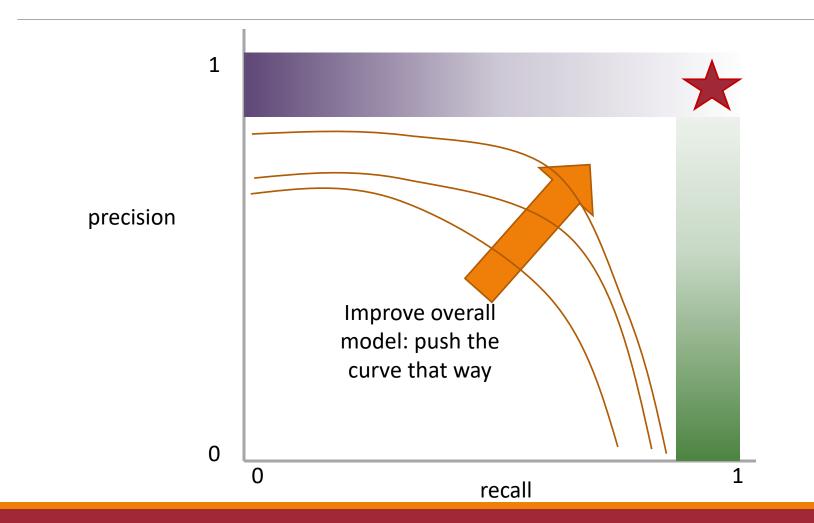
Q: Where do you want your ideal model?

Q: You have a model that always identifies correct instances. Where on this graph is it?

Q: You have a model that only make correct predictions. Where on this graph is it?

Idea: measure the tradeoff between precision and recall

Review: Precision and Recall Present a Tradeoff



Q: Where do you want your ideal model?

Q: You have a model that always identifies correct instances. Where on this graph is it?

Q: You have a model that only make correct predictions. Where on this graph is it?

Idea: measure the tradeoff between precision and recall

Review: A combined measure: F-score

Weighted (harmonic) average of Precision & Recall

F1 measure: equal weighting between precision and recall

$$F_1 = \frac{2 * P * R}{P + R} = \frac{2 * TP}{2 * TP + FP + FN}$$

(useful when P = R = 0)

Classification Evaluation: Accuracy, Precision, and Recall

Accuracy: % of items correct

$$\frac{TP + TN}{TP + FP + FN + TN}$$

$$F_1 = \frac{2 * P * R}{P + R} = \frac{2 * TP}{2 * TP + FP + FN}$$

When would you want to use accuracy vs F1?

Accuracy works better if the dataset is <u>balanced</u>

Accuracy takes everything in consideration

F-Score is focused on TP

	Actually Target	Actually Not Target
Selected/Guessed	True Positive (TP)	False Positive (FP)
Not select/not guessed	False Negative (FN)	True Negative (TN)

P/R/F in a Multi-class Setting: Micro- vs. Macro-Averaging

Macroaveraging: Compute performance for each class, then average.

macroprecision =
$$\frac{1}{C} \sum_{c} \frac{\text{TP}_{c}}{\text{TP}_{c} + \text{FP}_{c}} = \frac{1}{C} \sum_{c} \text{precision}_{c}$$

macrorecall =
$$\frac{1}{C} \sum_{c} \frac{\text{TP}_{c}}{\text{TP}_{c} + \text{FN}_{c}} = \frac{1}{C} \sum_{c} \text{recall}_{c}$$

Microaveraging: Collect decisions for all classes, compute contingency table, evaluate.

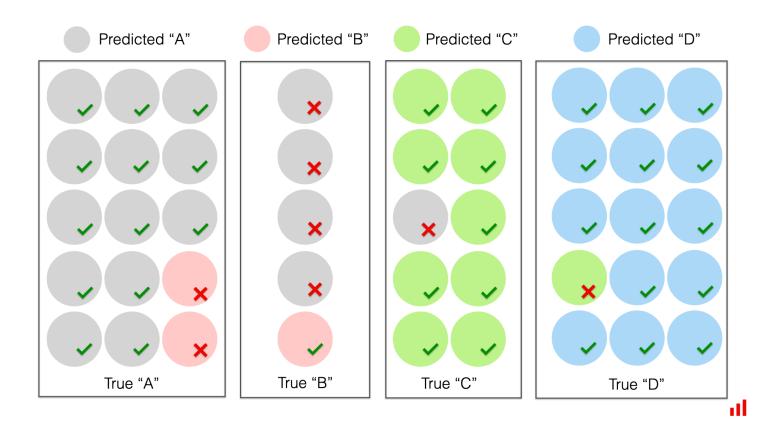
microprecision =
$$\frac{\sum_{c} TP_{c}}{\sum_{c} TP_{c} + \sum_{c} FP_{c}}$$

when to prefer macroaveraging?

when to prefer microaveraging?

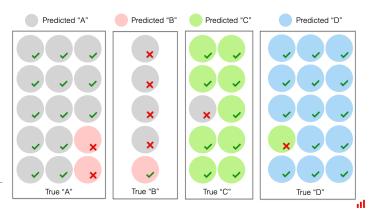
microrecall =
$$\frac{\sum_{c} TP_{c}}{\sum_{c} TP_{c} + \sum_{c} FN_{c}}$$

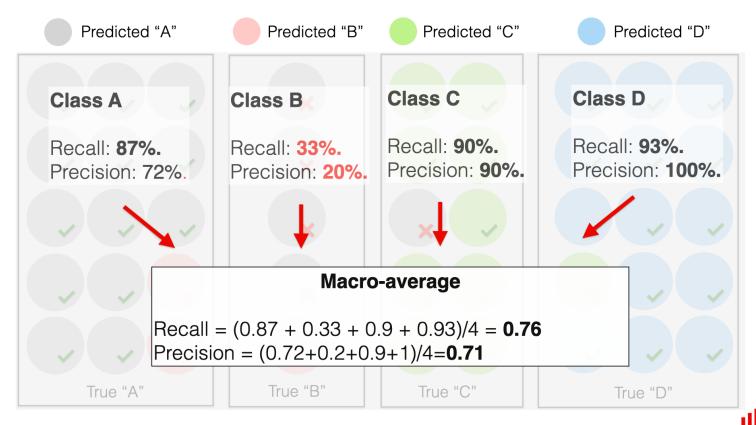
Macro/Micro Example



Each *class* has equal weight

Macro-Average



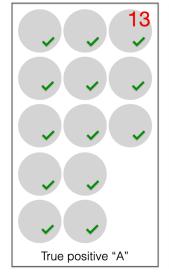


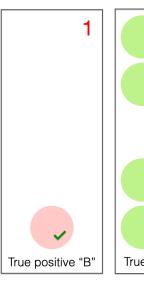
ps://www.evidentlyai.com/classification-metrics/multi-class-metrics

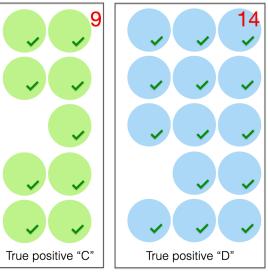
Each *instance* has equal weight

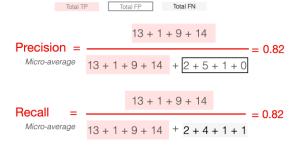
Micro-Average

All true positives

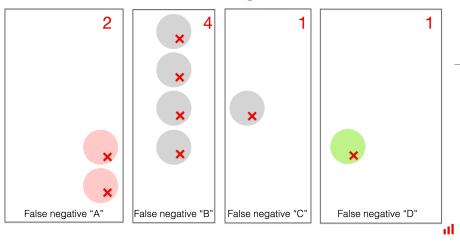




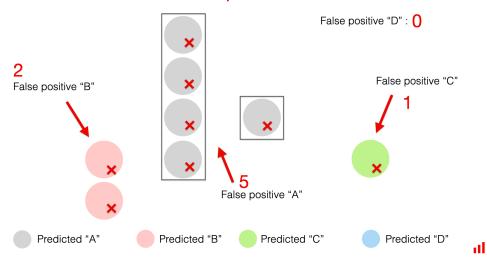




All false negatives



All false positives



s://www.evidentlyai.com/classification-metrics/multi-class-metrics

Micro- vs Macro-Average

So when would we want to prefer micro-averaging vs macro-averaging?

macroprecision =
$$\frac{1}{C} \sum_{c} \frac{\text{TP}_{c}}{\text{TP}_{c} + \text{FP}_{c}} = \frac{1}{C} \sum_{c} \text{precision}_{c}$$

macrorecall =
$$\frac{1}{C} \sum_{c} \frac{\text{TP}_{c}}{\text{TP}_{c} + \text{FN}_{c}} = \frac{1}{C} \sum_{c} \text{recall}_{c}$$

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microrecall =
$$\frac{\sum_{c} TP_{c}}{\sum_{c} TP_{c} + \sum_{c} FN_{c}}$$

But how do we compute stats for multiple classes?

We already saw how the "polarity" affects the stats we compute...

Two main approaches. Either:

- 1. Compute "one-vs-all" 2x2 tables. OR
- 2. Generalize the 2x2 tables and compute per-class TP / FP / FN based on the diagonals and off-diagonals

1. Compute "one-vs-all" 2x2 tables Actual

Look for	Actually Target	Actually Not Target	Look for	Actually Target	Actually Not Target
Selected/G	True	False	Selected/G	True	False
uessed	Positive (TP)	Positive (FP)	uessed	Positive (TP)	Positive (FP)
Not select/not guessed	False	True	Not	False	True
	Negative	Negative	select/not	Negative	Negative
	(FN)	(TN)	guessed	(FN)	(TN)

Look for	Actually Target	Actually Not Target	
Selected/G	True	False	
uessed	Positive (TP)	Positive (FP)	
Not	False	True	
select/not	Negative	Negative	
guessed	(FN)	(TN)	

2/20/2025

1. Compute "one-vs-all" 2x2 tables

























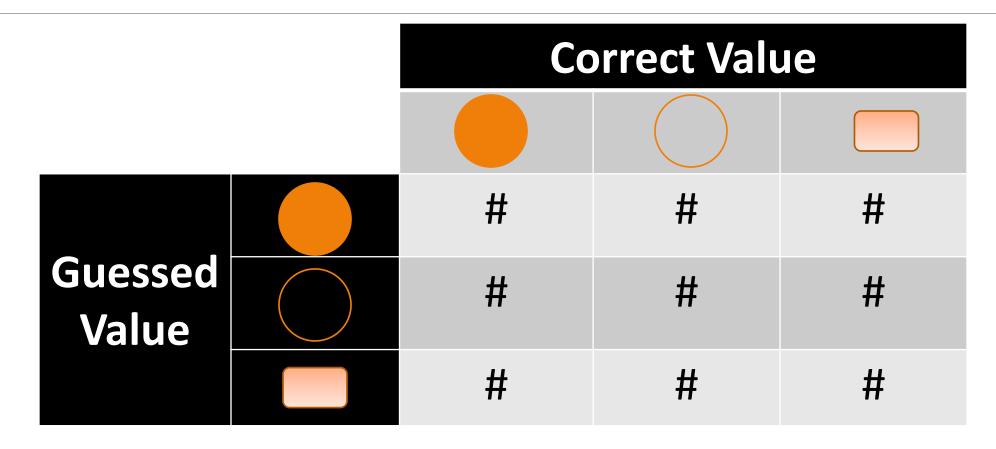




Look for	Actually Target	Actually Not Target	Look for	Actually Target	Actually Not Target
Selected/G uessed	2	1	Selected/G uessed	2	1
Not select/not guessed	2	4	Not select/not guessed	1	5

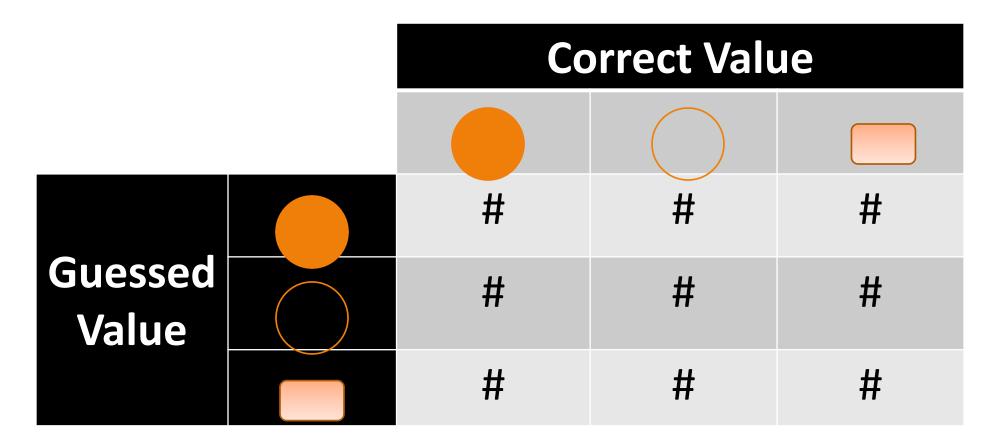
Look for	Actually Target	Actually Not Target
Selected/G uessed	1	2
Not select/not guessed	1 ML EVALUATION	5

2. Generalizing the 2-by-2 contingency table

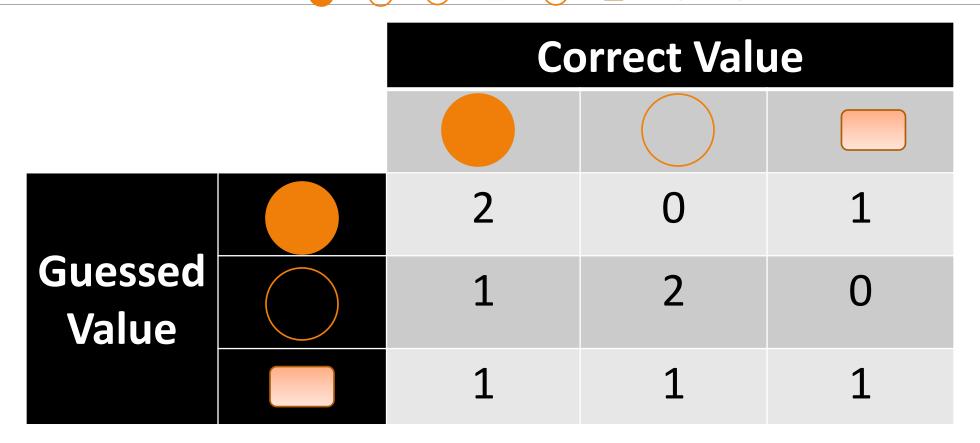


This is also called a **Confusion Matrix**

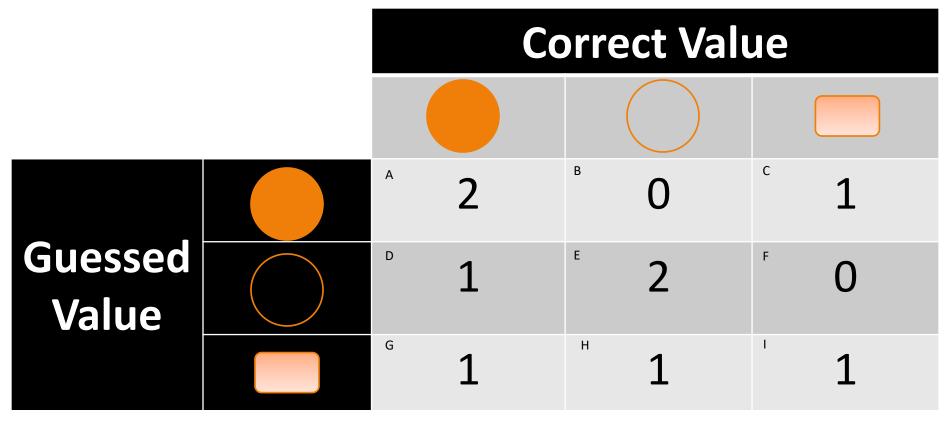
2. Generalizing the 2-by-2 contingency table Actual



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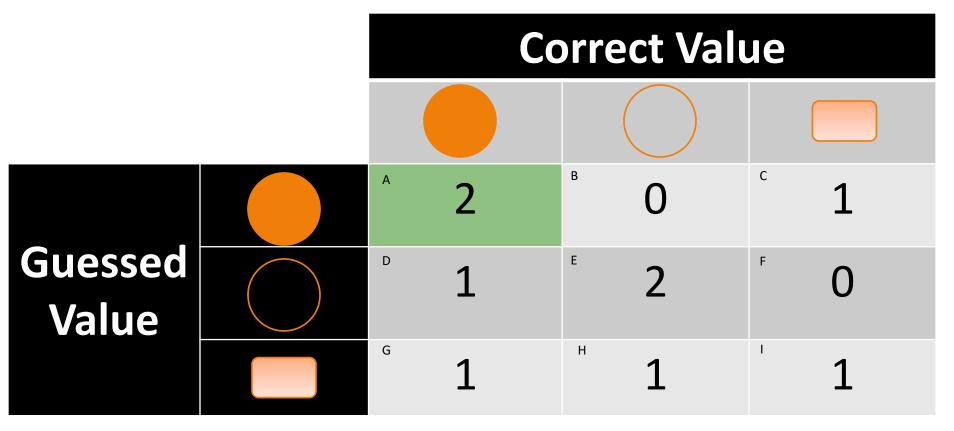






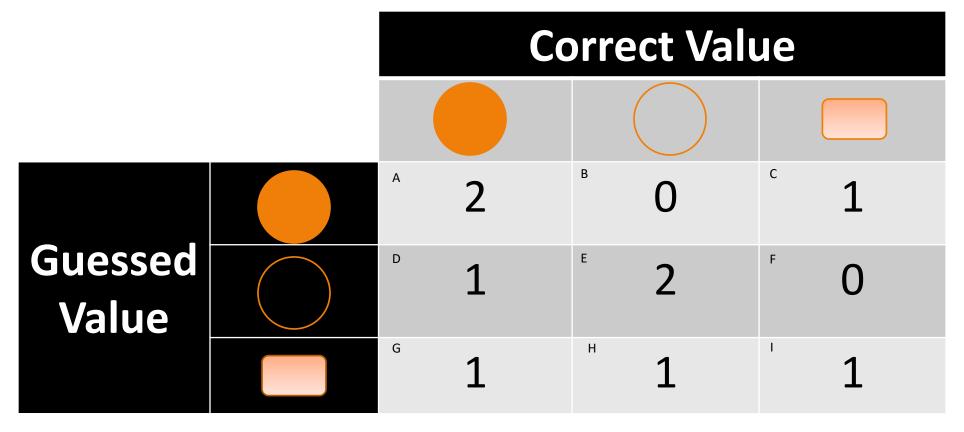
How do you compute TP?





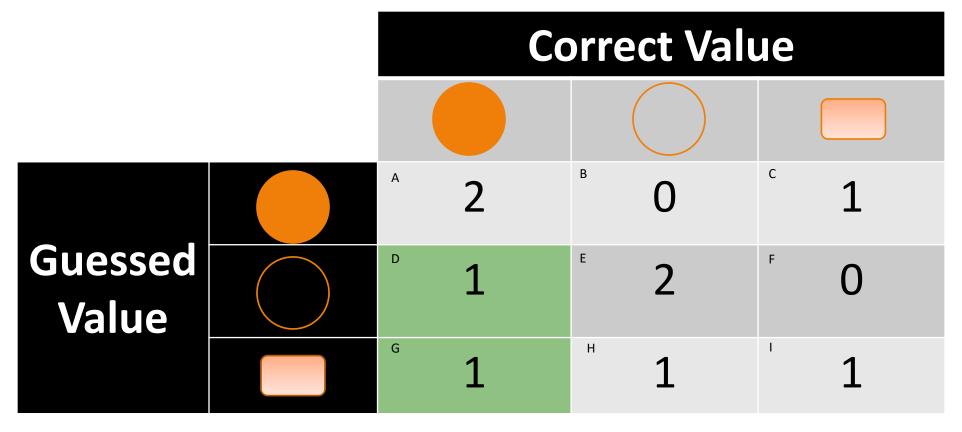
How do you compute TP?





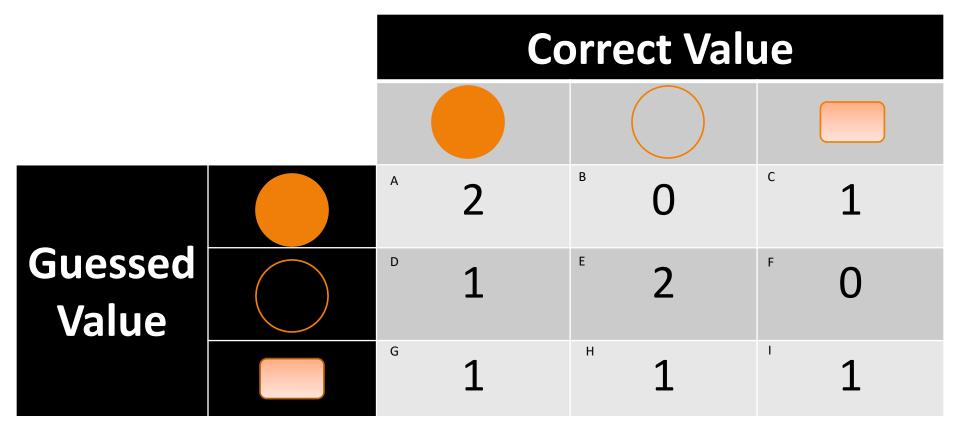
How do you compute FN—?





How do you compute FN—?

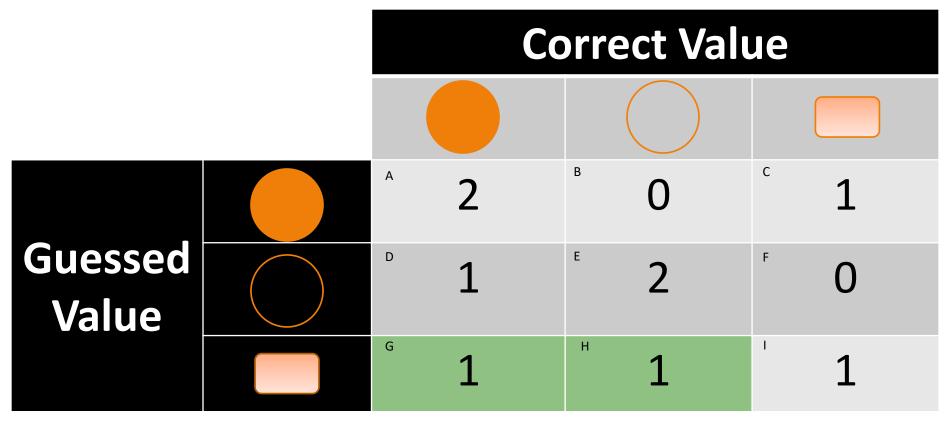




How do you compute FP_{-} ?

30

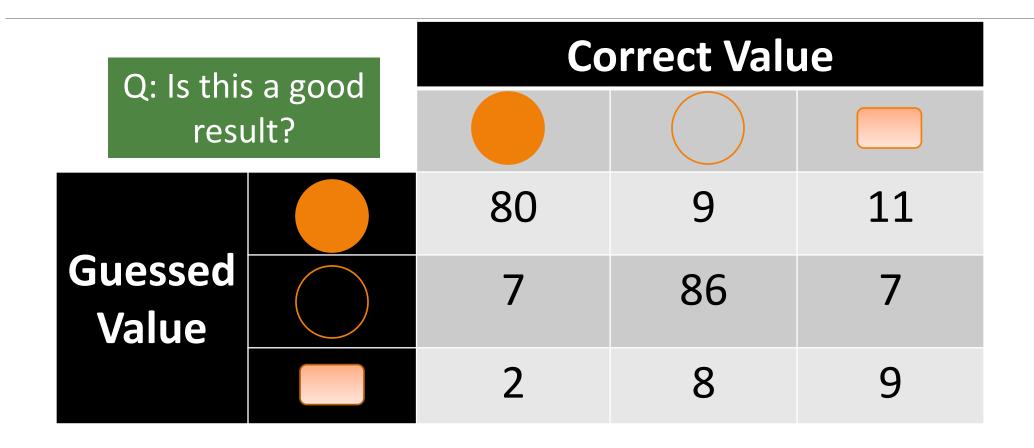




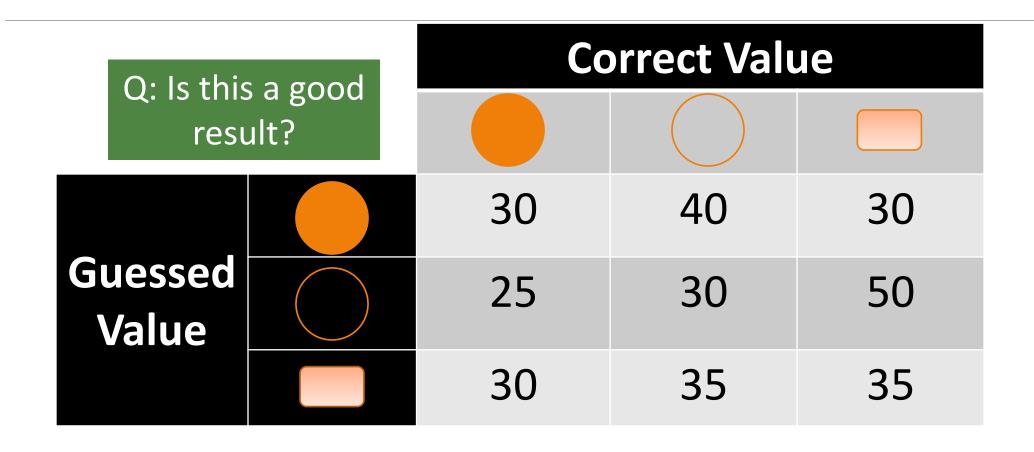
How do you compute FP_{-} ?

31

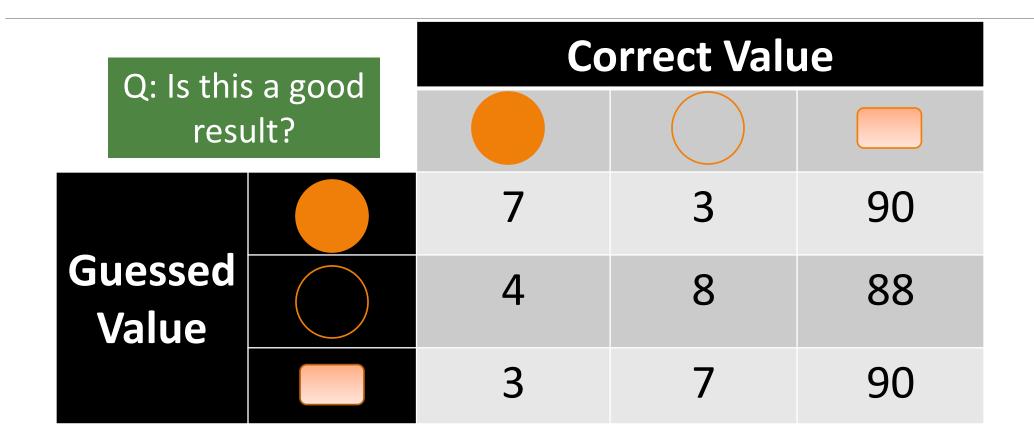
Generalizing the 2-by-2 contingency table



Generalizing the 2-by-2 contingency table

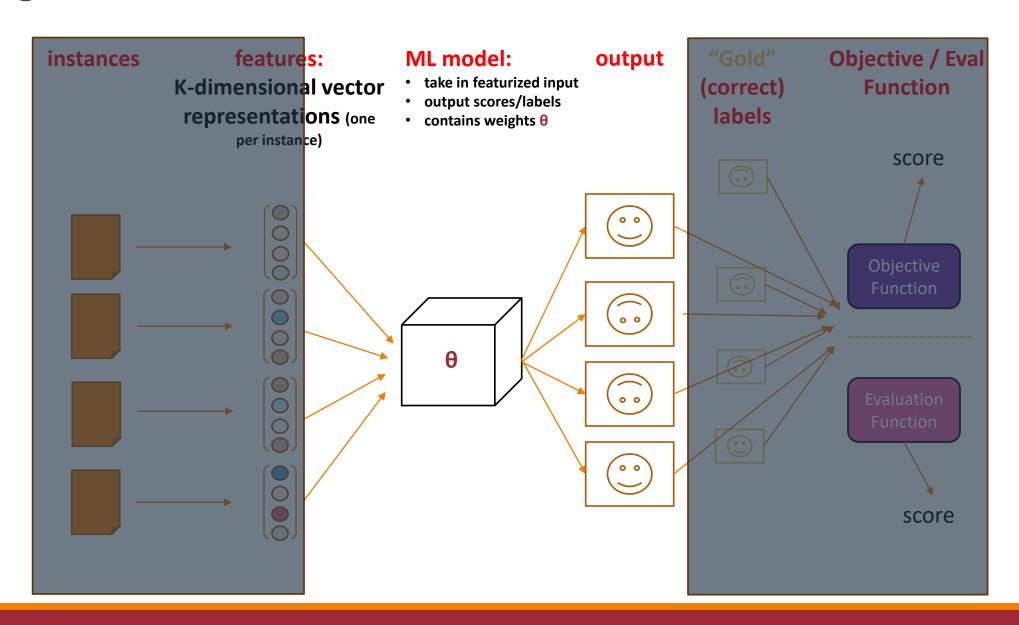


Generalizing the 2-by-2 contingency table



Classification

Defining the Model



Terminology

common NLP

Log-Linear Models

term

(Multinomial) logistic regression

as statistical regression

Softmax regression

based in

information theory Maximum Entropy models (MaxEnt)

Generalized Linear Models a form of

viewed as

Discriminative Naïve Bayes

to be cool

Very shallow (sigmoidal) neural nets

today

Maxent Models are Flexible

Maxent models can be used:

- to design discriminatively trained classifiers, or
- to create featureful language models

(among other approaches in NLP and ML more broadly)

Examining Assumption 3 Made for Classification Evaluation

Given X, our classifier produces a score for each possible label

Normally (*but this can be adjusted!)

best label =
$$\underset{\text{label}}{\operatorname{arg max}} P(|\text{label}||\text{example})$$

Terminology: Posterior Probability

Posterior probability:

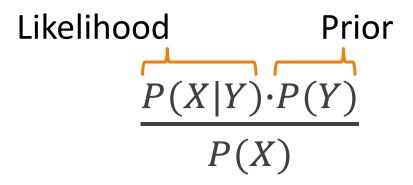
These are conditional probabilities

• If and are the only two options:

$$p(|X) + p(|X) = 1$$

$$p(|X| \ge 0, p(|X| \ge 0)$$

Bayes' Rule



Posterior: P(Y|X)

Posterior probability:

probability of event Y

with knowledge that X

has occurred

NLP pg. 450

Terminology (with variables)

Posterior probability:

$$p(Y = label_1 | X) vs. p(Y = label_0 | X)$$

Conditional probabilities:

$$p(Y = label_1 | X) + p(Y = label_0 | X) = 1$$

 $p(Y = label_1 | X) \ge 0,$
 $p(Y = label_0 | X) \ge 0$

Conditional probability:
probability of event Y, assuming event X happens too

NLP pg. 449



Yey Take-away



We will *learn* this $p(Y \mid X)$

Maxent Models for Classification: Discriminatively or ...

Directly model the posterior

$$p(Y | X) = maxent(X; Y)$$

Discriminatively trained classifier

Maxent Models for Classification: Discriminatively or Generatively Trained

Directly model the posterior

$$p(Y | X) = maxent(X; Y)$$

Discriminatively trained classifier

Model the posterior with Bayes rule

$$p(Y \mid X) \propto \mathbf{maxent}(X \mid Y)p(Y)$$

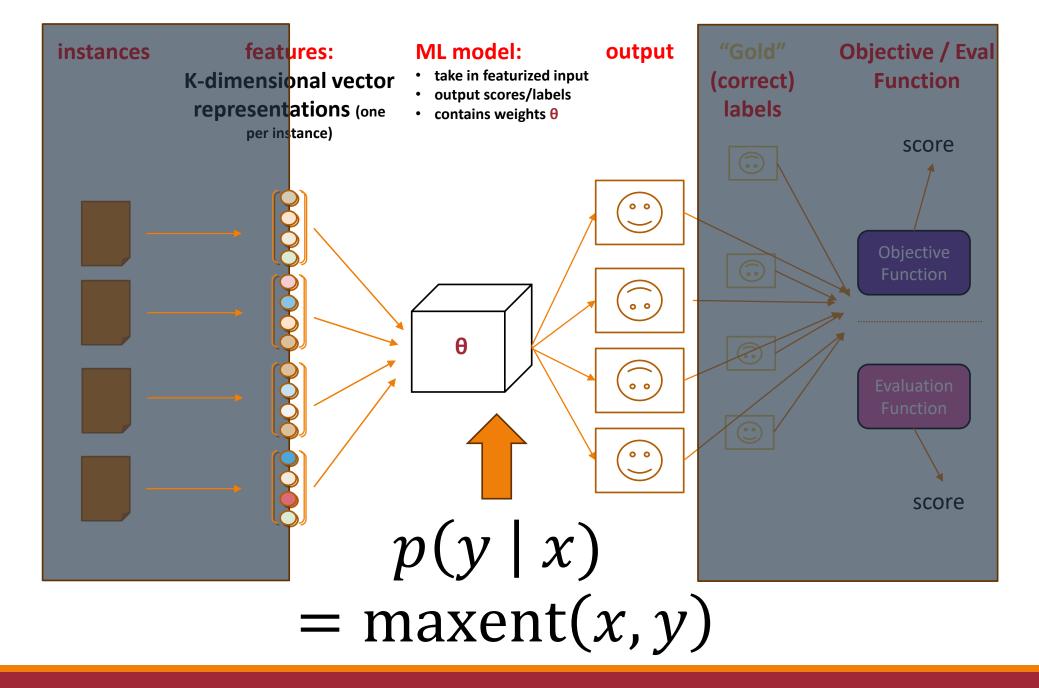
Generatively trained classifier with maxent-based language model

Maximum Entropy (Log-linear) Models For Discriminatively Trained Classifiers

(we'll start with this one)

$$p(y \mid x) = \max(x, y)$$

discriminatively trained: classify in one go



Core Aspects to Maxent Classifier p(y|x)

We need to define:

- features f(x) from x that are meaningful;
- weights θ (at least one per feature, often one per feature/label combination) to say how important each feature is; and
- ullet a way to **form probabilities** from f and heta

Overview of Featurization

Common goal: probabilistic classifier $p(y \mid x)$

Often done by defining **features** between x and y that are meaningful

Denoted by a general vector of K features

$$f(x) = (f_1(x), ..., f_K(x))$$

Features can be thought of as "soft" rules

• E.g., POSITIVE sentiments tweets *may* be more likely to have the word "happy"

Review: Document Classification via Bagof-Words Features (Example)

Amazon acquired MGM in 2022, taking over a sprawling library that includes more than 4,000 feature films and 17,000 television shows. The tech behemoth also earned the rights to distribute all the Bond movies, but the new deal solidifies the company's oversight of Bond's big-screen future.

With V word types, define V feature functions $f_i(x)$ as

 $f_i(x) = \#$ of times word type i appears in document x

$$f(x) = (f_i(x))_i^V$$

TECH

NOT TECH

Core assumption:
the label can be
predicted from
counts of individual
word types

feature $f_i(x)$	value
Amazon	1
acquired	1
behemoth	1
Bond	2
sniffle	0

3/4/2024

Example Classification Tasks



GLUE

https://gluebenchmark.com/

e datasets: glue

	GLUE Tasks
Name	Download
The Corpus of Linguistic Acceptability	±
The Stanford Sentiment Treebank	<u></u>
Microsoft Research Paraphrase Corpus	<u></u>
Semantic Textual Similarity Benchmark	<u></u>
Quora Question Pairs	<u></u>
MultiNLI Matched	<u></u>
MultiNLI Mismatched	<u></u>
Question NLI	<u></u>
Recognizing Textual Entailment	<u></u>
Winograd NLI	*
Diagnostics Main	<u>*</u>

· _
Identifier

SuperGLUE 1

Name	Identifier
Broadcoverage Diagnostics	AX-b
CommitmentBank	СВ
Choice of Plausible Alternatives	COPA
Multi-Sentence Reading Comprehension	MultiRC
Recognizing Textual Entailment	RTE
Words in Context	WiC
The Winograd Schema Challenge	WSC
BoolQ	BoolQ
Reading Comprehension with Commonsense Reasoning	ReCoRD
Winogender Schema Diagnostics	AX-g



https://super.gluebenchmark.com/

🥰 datasets: super_glue



Given a premise sentence s and hypothesis sentence h, determine if h "follows from" s

ENTAILMENT (yes):

NOT ENTAILED (no):



Given a premise sentence s and hypothesis sentence h, determine if h "follows from" s

ENTAILMENT (yes):

s: Michael Jordan, coach Phil Jackson and the star cast, including Scottie Pippen, took the Chicago Bulls to six National Basketball Association championships.

h: The Bulls basketball team is based in Chicago.

NOT ENTAILED (no):



Given a premise sentence s and hypothesis sentence h, determine if h "follows from" s

ENTAILMENT (yes):

s: Michael Jordan, coach Phil Jackson and the star cast, including Scottie Pippen, took the Chicago Bulls to six National Basketball Association championships.

h: The Bulls basketball team is based in Chicago.

NOT ENTAILED (no):

s: Based on a worldwide study of smoking-related fire and disaster data, UC Davis epidemiologists show smoking is a leading cause of fires and death from fires globally.

h: Domestic fires are the major cause of fire death.

RTE

s: Michael Jordan, coach Phil Jackson and the star cast, including Scottie Pippen, took the Chicago Bulls to six National Basketball Association championships.

h: The Bulls basketball team is based in Chicago.



ENTAILED

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These extractions are all **features** that have **fired** (likely have some significance)

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ENTAILED

These extractions are all **features** that have **fired** (likely have some significance)

We need to *score* the different extracted clues.



Score and Combine Our Clues

```
score<sub>1, Entailed</sub>(((a)))
score<sub>2, Entailed</sub>(((a)))
score<sub>3, Entailed</sub>(((a)))
...
score<sub>k, Entailed</sub>(((a)))
```



posterior probability of ENTAILED

Scoring Our Clues

score

s: Michael Jordan, coach Phil Jackson and the star cast, including Scottie Pippen, took the Chicago Bulls to six National Basketball Association championships.

h: The Bulls basketball team is based in Chicago.

(ignore the feature indexing for now)

•••

Turning Scores into Probabilities

s: Michael Jordan, coach
Phil Jackson and the star
cast, including Scottie
Pippen, took the Chicago
Bulls to six National
Basketball Association
championships.
h: The Bulls basketball
team is based in Chicago.

, entailed) > score(

s: Michael Jordan, coach Phil Jackson and the star cast, including Scottie Pippen, took the Chicago Bulls to six National Basketball Association championships. h: The Bulls basketball team is based in Chicago.

Not)

p(entailed

s: Michael Jordan, coach Phil Jackson and the star cast, including Scottie Pippen, took the Chicago Bulls to six National Basketball Association championships.

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> p(NOT ENTAILED

s: Michael Jordan, coach Phil Jackson and the star cast, including Scottie Pippen, took the Chicago Bulls to six National Basketball Association championships.

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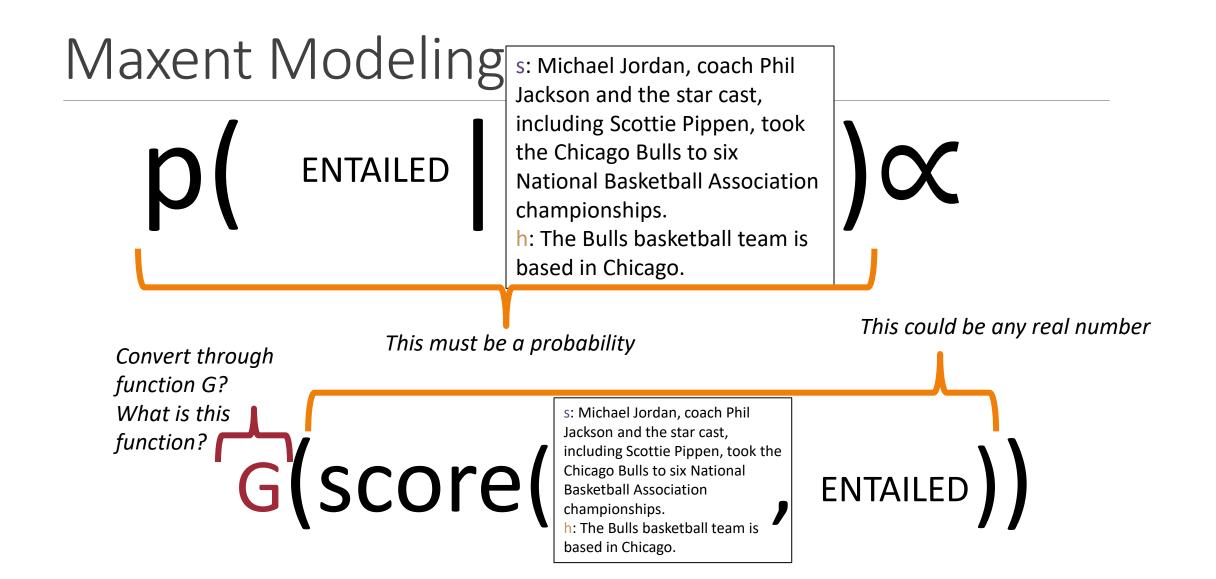
KEY IDEA

Turning Scores into Probabilities (More Generally)

score(x, y₁) > score(x, y₂)

$$p(y_1|x) > p(y_2|x)$$

KEY IDEA



What function G...

operates on any real number?

is never less than 0?

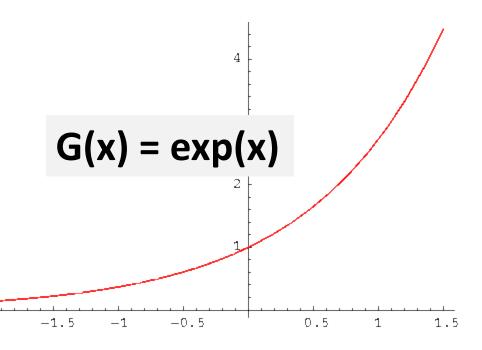
monotonic? (a < b \rightarrow G(a) < G(b))

What function G...

operates on any real number?

is never less than 0?

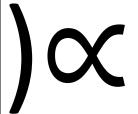
monotonic? (a < b \rightarrow G(a) < G(b))



D ENTAILED

s: Michael Jordan, coach Phil Jackson and the star cast, including Scottie Pippen, took the Chicago Bulls to six National Basketball Association championships.

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exp(score(

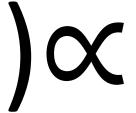
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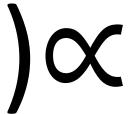
```
exp(score<sub>1, Entailed</sub>() + ))

score<sub>2, Entailed</sub>() + ))

score<sub>3, Entailed</sub>() +
```

s: Michael Jordan, coach Phil Jackson and the star cast, including Scottie Pippen, took
the Chicago Bulls to six
National Basketball Association
championships

h: The Bulls basketball team is based in Chicago.



```
weight<sub>1, Entailed</sub> * applies<sub>1</sub>( ) +

weight<sub>2, Entailed</sub> * applies<sub>2</sub>( ) +

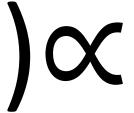
weight<sub>3, Entailed</sub> * applies<sub>3</sub>( ) +
```

3/4/2024

s: Michael Jordan, coach Phil Jackson and the star cast, ENTAILED

including Scottie Pippen, took
the Chicago Bulls to six
National Basketball Association
champiages.

h: The Bulls basketball team is based in Chicago.



```
weight<sub>1, Entailed</sub> * applies<sub>1</sub>( ) +

weight<sub>2, Entailed</sub> * applies<sub>2</sub>( ) +

weight<sub>3, Entailed</sub> * applies<sub>3</sub>( ) +
```

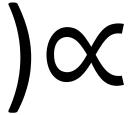
weights...

K different for K different features

s: Michael Jordan, coach Phil Jackson and the star cast, ENTAILED

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```
weight<sub>1, Entailed</sub> * applies<sub>1</sub>( ) +
weight<sub>2, Entailed</sub> * applies<sub>2</sub>( ) +
weight<sub>3, Entailed</sub> * applies<sub>3</sub>( ) +
```

weights...

K different for K different features

multiplied and then summed

ENTAILED

s: Michael Jordan, coach Phil Jackson and the star cast, including Scottie Pippen, took the Chicago Bulls to six **National Basketball Association** championships.

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```
EXD Dot_product of Entailed weight_vec feature_vec(
)
```

weights...

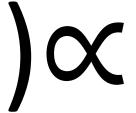
K different for K different features

multiplied and then summed

D ENTAILED

s: Michael Jordan, coach Phil Jackson and the star cast, including Scottie Pippen, took the Chicago Bulls to six National Basketball Association championships.

h: The Bulls basketball team is based in Chicago.







Implementation Check

$$exp(\theta_{ENTAILED}^Tf(\mathbf{b}))$$

Assume we can compute
$$\mathbf{x} = f(\mathbf{x})$$
 (a one-dimensional tensor)

How can we implement the above computation in Pytorch?

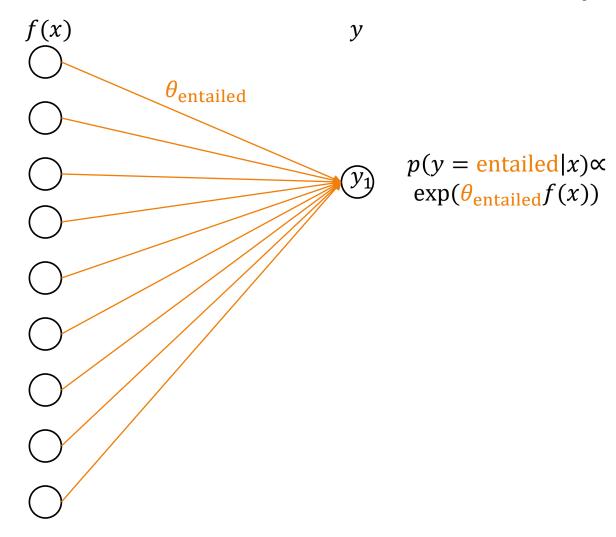
Knowledge Check: Data Prep

https://colab.research.google.com/drive/19yg0EUXQtHozBiSuO6cKOBhoSPzQHg

ug?usp=sharing



Maxent Classifier, schematically



Maxent Modeling

D ENTAILED

s: Michael Jordan, coach Phil Jackson and the star cast, including Scottie Pippen, took the Chicago Bulls to six National Basketball Association championships.

h: The Bulls basketball team is based in Chicago.

Q: How do we define Z?

1 -exp



K different

for K different

multiplied and

Normalization for Classification

$$\sum \exp(\theta_J^T f(\mathbb{B}))$$

label

$$\theta_J^T f(\mathbb{B})$$

classify doc x with label y in one go

$$p(y \mid x) \propto \exp(\theta_y^T f(x))$$

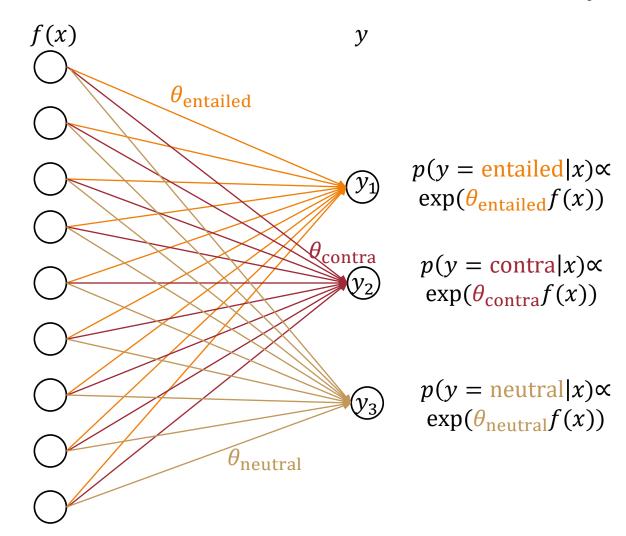
Normalization for Classification (long form)

$$p(y \mid x) \propto \exp(\theta_y^T f(x))$$

classify doc x with label y in one go

Maxent Classifier, schematically

Why would we want to normalize the weights?



output: i = argmax score_i class i

Core Aspects to Maxent Classifier p(y|x)

features f(x) from x that are meaningful;

weights θ (at least one per feature, often one per feature/label combination) to say how important each feature is; and

a way to **form probabilities** from f and θ

$$p(y|x) = \frac{\exp(\theta_y^T f(x))}{\sum_{y'} \exp(\theta_{y'}^T f(x))}$$

Different Notation, Same Meaning

$$p(Y = y \mid x) = \frac{\exp(\theta_y^T f(x))}{\sum_{y'} \exp(\theta_{y'}^T f(x))}$$

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$$p(Y = y \mid x) = \frac{\exp(\theta_y^T f(x))}{\sum_{y'} \exp(\theta_{y'}^T f(x))}$$

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Different Notation, Same Meaning

$$p(Y = y | x) = \frac{\exp(\theta_y^T f(x))}{\sum_{y'} \exp(\theta_{y'}^T f(x))}$$

$$p(Y = y \mid x) \propto \exp(\theta_y^T f(x))$$

$$p(Y \mid x) = \operatorname{softmax}(\theta f(x))$$

Defining Appropriate Features in a Maxent Model

Feature functions help extract useful features (characteristics) of the data

They turn *data* into *numbers*

Features that are not 0 are said to have fired

Generally templated

Binary-valued (0 or 1) or real-valued

Representing a Linguistic "Blob"

Userdefined

Integer representation/on e-hot encoding

Assign each word to some index i, where $0 \le i < V$

Represent each word w with a V-dimensional **binary** vector e_w , where $e_{w,i} = 1$ and 0 otherwise

Modelproduced Dense embedding

Let E be some *embedding size* (often 100, 200, 300, etc.)

Represent each word w with an E-dimensional **real-valued** vector e_w

Featurization is Similar but...

Vocab types (V) / embedding dimension (E) → number of features (number of "clues")

"Linguistic blob" → Instances to represent

Features are extracted on each instance

Review: Bag-of-words as a Function

Based on some tokenization, turn an input document into an array (or dictionary or set) of its unique vocab items

Think of getting a BOW rep. as a function f

input: Document

output: Container of size E, indexable by

each vocab type v

Some Bag-of-words Functions

Kind	Type of $m{f}_{m{v}}$	Interpretation
Binary	0, 1	Did <i>v</i> appear in the document?
Count-based	Natural number (int >= 0)	How often did <i>v</i> occur in the document?
Averaged	Real number (>=0, <= 1)	How often did <i>v</i> occur in the document, normalized by doc length?
TF-IDF (term frequency, inverse document frequency)	Real number (>= 0)	How frequent is a word, tempered by how prevalent it is across the corpus (to be covered later!)
	•••	

Q: Is this a reasonable representation?

Q: What are some tradeoffs (benefits vs. costs)?

Useful Terminology: n-gram

Within a larger string (e.g., sentence), a contiguous sequence of n items (e.g., words)

Colorless green ideas sleep furiously

n	Commonly called	History Size (Markov order)	Example n-gram ending in "furiously"
1	unigram	0	furiously
2	bigram	1	sleep furiously
3	trigram (3-gram)	2	ideas sleep furiously
4	4-gram	3	green ideas sleep furiously
n	n-gram	n-1	$W_{i-n+1} \dots W_{i-1} W_i$

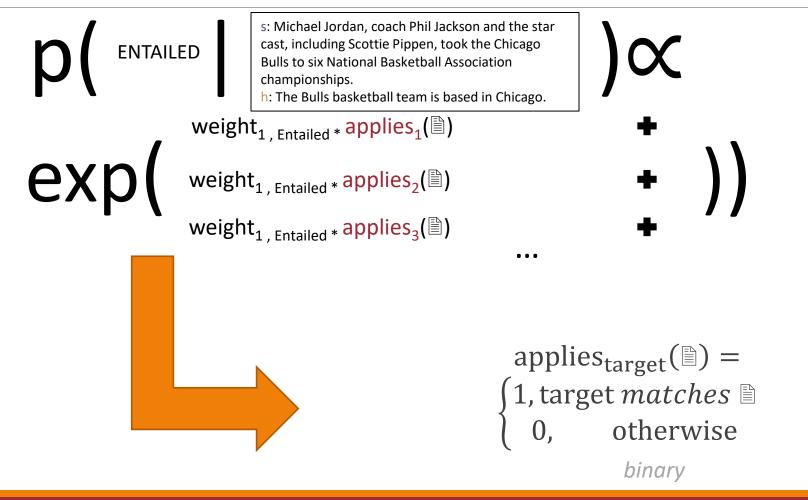
Templated Features

Define a feature fclue(
) for each clue you want to consider

The feature fclue fires if the clue applies to/can be found in 🖹

Clue is often a target phrase (an n-gram)

Maxent Modeling: Templated Binary Feature Functions

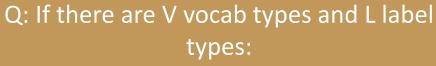


```
applies<sub>target</sub>(\blacksquare) = 
\[ \begin{aligned} 1, \target matches \Bar{\text{\end}} \\ 0, \quad \text{otherwise} \end{aligned}
```



```
applies<sub>ball</sub> (\mathbb{B}) = 
 \begin{cases} 1, \text{ ball } in \text{ both s and h of } \mathbb{B} \\ 0, \text{ otherwise} \end{cases}
```

```
applies<sub>target</sub>(\blacksquare) = (1, target matches \blacksquare 0, otherwise
```



1. How many features are defined if unigram targets are used (w/ each label)?



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applies_{ball} (\mathbb{B}) = \(\) 1, ball *in* both s and h of \mathbb{B} \(\) 0, otherwise Q: If there are V vocab types and L label types:

1. How many features are defined if unigram targets are used (w/each label)?

A1: *VL*

```
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A1: *VL*

2. How many features are defined if bigram targets are used?

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A1: *VL*

2. How many features are defined if bigram targets are used (w/each label)?

A2: V^2L

```
applies<sub>target</sub>(\blacksquare) = 
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applies_{ball} (\mathbb{B}) = $\begin{cases} 1, \text{ ball } in \text{ both s and h of } \mathbb{B} \\ 0, \text{ otherwise} \end{cases}$ Q: If there are V vocab types and L label types:

1. How many features are defined if unigram targets are used (w/ each label)?

A1: *VL*

2. How many features are defined if bigram targets are used (w/each label)?

A2: V^2L

3. How many features are defined if unigram and bigram targets are used (w/ each label)?

```
applies<sub>target</sub>(\blacksquare) = 
\( \) 1, target matches \blacksquare 
\( \) 0, otherwise
```



applies_{ball} (\mathbb{B}) = $\begin{cases} 1, \text{ ball } in \text{ both s and h of } \mathbb{B} \\ 0, \text{ otherwise} \end{cases}$ Q: If there are V vocab types and L label types:

1. How many features are defined if unigram targets are used (w/ each label)?

A1: *VL*

2. How many features are defined if bigram targets are used (w/each label)?

A2: V^2L

3. How many features are defined if unigram and bigram targets are used (w/ each label)?

A2:
$$(V + V^2)L$$