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Tayya
b

Determine whether each of these compound propositions is satisfiable.

a) $(p \vee q \vee \neg r) \wedge (p \vee \neg q \vee \neg s) \wedge (p \vee \neg r \vee \neg s) \wedge (\neg p \vee \neg q \vee \neg s) \wedge (p \vee q \vee \neg s)$

b) $(\neg p \vee \neg q \vee r) \wedge (\neg p \vee q \vee \neg s) \wedge (p \vee \neg q \vee \neg s) \wedge (\neg p \vee \neg r \vee \neg s) \wedge (p \vee q \vee \neg r) \wedge (p \vee \neg r \vee \neg s)$

c) $(p \vee q \vee r) \wedge (p \vee \neg q \vee \neg s) \wedge (q \vee \neg r \vee s) \wedge (\neg p \vee r \vee s) \wedge (\neg p \vee q \vee \neg s) \wedge (p \vee \neg q \vee \neg r) \wedge (\neg p \vee \neg q \vee s) \wedge (\neg p \vee \neg r \vee \neg s)$

a) It is satisfiable when p=F, q=F, r=F, s=F .

Part (a)

p	q	r	s	C1	C2	C3	C4	C5	Formula
FALSE	FALSE	FALSE	FALSE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE
FALSE	FALSE	FALSE	TRUE	TRUE	TRUE	TRUE	TRUE	FALSE	FALSE
FALSE	FALSE	TRUE	FALSE	FALSE	TRUE	TRUE	TRUE	TRUE	FALSE
FALSE	FALSE	TRUE	TRUE	FALSE	TRUE	FALSE	TRUE	FALSE	FALSE
FALSE	TRUE	FALSE	FALSE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE
FALSE	TRUE	FALSE	TRUE	TRUE	FALSE	TRUE	TRUE	TRUE	FALSE
FALSE	TRUE	TRUE	FALSE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE
FALSE	TRUE	TRUE	TRUE	TRUE	FALSE	FALSE	TRUE	TRUE	FALSE
TRUE	FALSE	FALSE	FALSE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE
TRUE	FALSE	FALSE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE
TRUE	FALSE	TRUE	FALSE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE
TRUE	FALSE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE
TRUE	TRUE	FALSE	FALSE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE
TRUE	TRUE	FALSE	TRUE	TRUE	TRUE	TRUE	FALSE	TRUE	FALSE
TRUE	TRUE	TRUE	FALSE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE
TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	FALSE	TRUE	FALSE

b) It is satisfiable when p=F, q=F, r=F, s=F .

Part (b)

p	q	r	s	C1	C2	C3	C4	C5	C6	Formula
FALSE	FALSE	FALSE	FALSE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE
FALSE	FALSE	FALSE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE
FALSE	FALSE	TRUE	FALSE	TRUE	TRUE	TRUE	TRUE	FALSE	TRUE	FALSE
FALSE	FALSE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	FALSE	FALSE	FALSE
FALSE	TRUE	FALSE	FALSE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE
FALSE	TRUE	FALSE	TRUE	TRUE	TRUE	FALSE	TRUE	TRUE	TRUE	FALSE
FALSE	TRUE	TRUE	FALSE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE
FALSE	TRUE	TRUE	TRUE	TRUE	TRUE	FALSE	TRUE	TRUE	FALSE	FALSE
TRUE	FALSE	FALSE	FALSE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE
TRUE	FALSE	FALSE	TRUE	TRUE	FALSE	TRUE	FALSE	TRUE	TRUE	FALSE
TRUE	FALSE	TRUE	FALSE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE
TRUE	FALSE	TRUE	TRUE	TRUE	FALSE	TRUE	TRUE	TRUE	TRUE	FALSE
TRUE	TRUE	FALSE	FALSE	FALSE	TRUE	TRUE	TRUE	TRUE	TRUE	FALSE
TRUE	TRUE	FALSE	TRUE	FALSE	TRUE	TRUE	FALSE	TRUE	TRUE	FALSE
TRUE	TRUE	TRUE	FALSE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE
TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE

c) It is satisfiable when p=F, q=F, r=T, s=t .

Part c

p	q	r	s	C1	C2	C3	C4	C5	C6	C7	C8	Formula
FALSE	FALSE	FALSE	FALSE	FALSE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	FALSE
FALSE	FALSE	FALSE	TRUE	FALSE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	FALSE
FALSE	FALSE	TRUE	FALSE	TRUE	TRUE	FALSE	TRUE	TRUE	TRUE	TRUE	TRUE	FALSE
FALSE	FALSE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE
FALSE	TRUE	FALSE	FALSE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE
FALSE	TRUE	FALSE	TRUE	TRUE	FALSE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	FALSE
FALSE	TRUE	TRUE	FALSE	TRUE	TRUE	TRUE	TRUE	TRUE	FALSE	TRUE	TRUE	FALSE
FALSE	TRUE	TRUE	TRUE	TRUE	FALSE	TRUE	TRUE	TRUE	FALSE	TRUE	TRUE	FALSE
TRUE	FALSE	FALSE	FALSE	TRUE	TRUE	TRUE	FALSE	TRUE	TRUE	TRUE	TRUE	FALSE
TRUE	FALSE	FALSE	TRUE	TRUE	TRUE	TRUE	TRUE	FALSE	TRUE	TRUE	TRUE	FALSE
TRUE	FALSE	TRUE	FALSE	TRUE	TRUE	FALSE	TRUE	TRUE	TRUE	TRUE	TRUE	FALSE
TRUE	FALSE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	FALSE	TRUE	TRUE	FALSE	FALSE
TRUE	TRUE	FALSE	FALSE	TRUE	TRUE	TRUE	FALSE	TRUE	TRUE	FALSE	TRUE	FALSE
TRUE	TRUE	FALSE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE
TRUE	TRUE	TRUE	FALSE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	FALSE	TRUE	FALSE
TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	FALSE	FALSE

Assignment-2

. For each of these statements find a domain for which the statement is true and a domain for which the statement is false.

a) Everyone speaks Hindi.
b) There is someone older than 21 years.
c) Every two people have the same first name.
d) Someone knows more than two other people.

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a) “Everyone speaks Hindi.”

Domain where the statement is TRUE

- **Domain:** All people in a small Hindi-medium classroom in India.
- In this group, everyone speaks Hindi → **True**.

Domain where the statement is FALSE

- **Domain:** All people in the world.
- Not everyone in the world speaks Hindi → **False**.

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b) “There is someone older than 21 years.”

Domain where the statement is TRUE

- **Domain:** All adults living in a city.
- There will definitely be many people older than 21 → **True**.

Domain where the statement is FALSE

- **Domain:** All students in a kindergarten class.
- All children are younger than 21 → **False**.

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c) “Every two people have the same first name.”

Domain where the statement is TRUE

- **Domain:** A group of three boys all named “Ali.”
- Any two of them have the same first name → **True**.

Domain where the statement is FALSE

- **Domain:** Students in a typical school class.
- They have different names, so not every two have the same first name → **False**.

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d) “Someone knows more than two other people.”

Domain where the statement is TRUE

- **Domain:** Members of a large company with many employees.
- At least one person knows more than two others → **True**.

Domain where the statement is FALSE

- **Domain:** A group of only two people.
- No one can know “more than two people” because there are only 2 → **False**.

Assignment-3

Prove or disprove each of these statements about the floor and ceiling functions.

a) $\lfloor \lceil x \rceil \rfloor = \lceil x \rceil$ for all real numbers x.
b) $\lfloor x + y \rfloor = \lfloor x \rfloor + \lfloor y \rfloor$ for all real numbers x and y.
c) $\lceil \lceil x/2 \rceil /2 \rceil = \lceil x/4 \rceil$ for all real numbers x.
d) $\lfloor \sqrt{\lceil x \rceil} \rfloor = \lfloor \sqrt{x} \rfloor$ for all positive real numbers x.
e) $\lfloor x \rfloor + \lfloor y \rfloor + \lfloor x + y \rfloor \le \lfloor 2x \rfloor + \lfloor 2y \rfloor$ for all real numbers x and y.

(a) ⌊⌈x⌉⌋=⌈x⌉\lfloor\lceil x\rceil\rfloor=\lceil x\rceil⌊⌈x⌉⌋=⌈x⌉ for all real xxx.

True.
Reason: ⌈x⌉\lceil x\rceil⌈x⌉ is an integer by definition. The floor of any integer equals the integer itself, so ⌊⌈x⌉⌋=⌈x⌉.\lfloor\lceil x\rceil\rfloor=\lceil x\rceil.⌊⌈x⌉⌋=⌈x⌉. ■

(b) ⌊x+y⌋=⌊x⌋+⌊y⌋\lfloor x+y\rfloor=\lfloor x\rfloor+\lfloor y\rfloor⌊x+y⌋=⌊x⌋+⌊y⌋ for all real x,yx,yx,y.

False.
Counterexample: x=y=0.5x=y=0.5x=y=0.5. Then ⌊x+y⌋=⌊1⌋=1\lfloor x+y\rfloor=\lfloor1\rfloor=1⌊x+y⌋=⌊1⌋=1 but ⌊x⌋+⌊y⌋=0+0=0\lfloor x\rfloor+\lfloor y\rfloor=0+0=0⌊x⌋+⌊y⌋=0+0=0. So equality fails.

Related correct fact (useful):

⌊x⌋+⌊y⌋≤⌊x+y⌋≤⌊x⌋+⌊y⌋+1,\lfloor x\rfloor+\lfloor y\rfloor \le \lfloor x+y\rfloor \le \lfloor x\rfloor+\lfloor y\rfloor+1,⌊x⌋+⌊y⌋≤⌊x+y⌋≤⌊x⌋+⌊y⌋+1,

which you can prove by writing x=⌊x⌋+{x}x=\lfloor x\rfloor+\{x\}x=⌊x⌋+{x} and y=⌊y⌋+{y}y=\lfloor y\rfloor+\{y\}y=⌊y⌋+{y} where 0≤{x},{y}<10\le\{x\},\{y\}<10≤{x},{y}<1.

(c) ⌊⌊x/2⌋/2⌋=⌊x/4⌋\big\lfloor \lfloor x/2\rfloor /2\big\rfloor = \lfloor x/4\rfloor⌊⌊x/2⌋/2⌋=⌊x/4⌋ for all real xxx.

True.
Proof: Let k=⌊x/2⌋k=\lfloor x/2\rfloork=⌊x/2⌋. Then k≤x/2<k+1k\le x/2<k+1k≤x/2<k+1. Dividing by 222 gives

k2≤x4<k+12.\frac{k}{2}\le\frac{x}{4}<\frac{k+1}{2}.2k≤4x<2k+1.

Now consider the integer ⌊k/2⌋\lfloor k/2\rfloor⌊k/2⌋. If k=2mk=2mk=2m then k/2=mk/2=mk/2=m and x4∈[m,m+0.5)\frac{x}{4}\in[m,m+0.5)4x∈[m,m+0.5) so ⌊x/4⌋=m=⌊k/2⌋\lfloor x/4\rfloor=m=\lfloor k/2\rfloor⌊x/4⌋=m=⌊k/2⌋. If k=2m+1k=2m+1k=2m+1 then k/2=m+12k/2=m+\tfrac12k/2=m+21 and x4∈[m+12,m+1)\frac{x}{4}\in[m+\tfrac12,m+1)4x∈[m+21,m+1) so again ⌊x/4⌋=m=⌊k/2⌋\lfloor x/4\rfloor=m=\lfloor k/2\rfloor⌊x/4⌋=m=⌊k/2⌋. In both cases ⌊x/4⌋=⌊k/2⌋=⌊⌊x/2⌋/2⌋.\lfloor x/4\rfloor=\lfloor k/2\rfloor=\big\lfloor\lfloor x/2\rfloor/2\big\rfloor.⌊x/4⌋=⌊k/2⌋=⌊⌊x/2⌋/2⌋. ■

(d) ⌈x⌉=⌊x⌋\lceil\sqrt{x}\rceil=\lfloor\sqrt{x}\rfloor⌈x⌉=⌊x⌋ for all positive real xxx.

False.
Counterexample: x=2x=2x=2. 2≈1.414\sqrt2\approx1.4142≈1.414 so ⌊2⌋=1\lfloor\sqrt2\rfloor=1⌊2⌋=1 and ⌈2⌉=2\lceil\sqrt2\rceil=2⌈2⌉=2. They are equal only when x\sqrt{x}x is an integer (i.e. when xxx is a perfect square).

(e) ⌊x⌋+⌊y⌋+⌊x+y⌋≤⌊2x⌋+⌊2y⌋\lfloor x\rfloor+\lfloor y\rfloor+\lfloor x+y\rfloor \le \lfloor 2x\rfloor+\lfloor 2y\rfloor⌊x⌋+⌊y⌋+⌊x+y⌋≤⌊2x⌋+⌊2y⌋ for all real x,yx,yx,y.

True.
Write x=a+α, y=b+βx=a+\alpha,\;y=b+\betax=a+α,y=b+β with integers a=⌊x⌋, b=⌊y⌋a=\lfloor x\rfloor,\;b=\lfloor y\rfloora=⌊x⌋,b=⌊y⌋ and fractional parts 0≤α,β<10\le\alpha,\beta<10≤α,β<1. Then

⌊x⌋+⌊y⌋+⌊x+y⌋=a+b+(a+b+⌊α+β⌋)=2a+2b+⌊α+β⌋,\lfloor x\rfloor+\lfloor y\rfloor+\lfloor x+y\rfloor = a+b+(a+b+\lfloor\alpha+\beta\rfloor)=2a+2b+\lfloor\alpha+\beta\rfloor,⌊x⌋+⌊y⌋+⌊x+y⌋=a+b+(a+b+⌊α+β⌋)=2a+2b+⌊α+β⌋,

and

⌊2x⌋+⌊2y⌋=(2a+⌊2α⌋)+(2b+⌊2β⌋)=2a+2b+⌊2α⌋+⌊2β⌋.\lfloor 2x\rfloor+\lfloor 2y\rfloor = (2a+\lfloor2\alpha\rfloor)+(2b+\lfloor2\beta\rfloor)=2a+2b+\lfloor2\alpha\rfloor+\lfloor2\beta\rfloor.⌊2x⌋+⌊2y⌋=(2a+⌊2α⌋)+(2b+⌊2β⌋)=2a+2b+⌊2α⌋+⌊2β⌋.

So the inequality is equivalent to

⌊α+β⌋≤⌊2α⌋+⌊2β⌋.\lfloor\alpha+\beta\rfloor \le \lfloor2\alpha\rfloor+\lfloor2\beta\rfloor.⌊α+β⌋≤⌊2α⌋+⌊2β⌋.

But ⌊α+β⌋\lfloor\alpha+\beta\rfloor⌊α+β⌋ is either 000 (if α+β<1\alpha+\beta<1α+β<1) or 111 (if α+β≥1\alpha+\beta\ge1α+β≥1). If α+β<1\alpha+\beta<1α+β<1 the left side is 000 and the inequality holds. If α+β≥1\alpha+\beta\ge1α+β≥1 then at least one of α,β\alpha,\betaα,β is ≥12\ge\tfrac12≥21, so at least one of ⌊2α⌋,⌊2β⌋\lfloor2\alpha\rfloor,\lfloor2\beta\rfloor⌊2α⌋,⌊2β⌋ equals 111; hence the right side is ≥1\ge1≥1. Therefore the inequality holds in all cases. ■

Assignment-4

(i)

There are 18 mathematics majors and 325 computer science majors at a college.

a) In how many ways can two representatives be picked so that one is a mathematics major and the other is a computer science major?

b) In how many ways can one representative be picked who is either a mathematics major or a computer science major?

Solution:

a) Ways to choose math major= 18
   Ways to choose CS major = 325
   Total ways= 325 x 18 = 5850

b) Total ways to choose = 18 + 325 = 343

(ii)

a) Show that if seven integers are selected from the first 10 positive integers, there must be at least two pairs of these integers with the sum 11.

b) Is the conclusion in part (a) true if six integers are selected rather than seven?

Solution:

a) Let 1^{st} 7 integers are selected (1,2,3,4,5,6,7)
   Possible pairs (5,6), (4,7)
   So it is true.

b) Let 1^{st} 6 integers are selected (1,2,3,4,5,6)
   Possible pairs (5,6)
   No further pairs are possible so it is false.