

1.25

a. $17^{183} \bmod 256$
 $183 = 10110111$

$$183 = 128 + 32 + 16 + 4 + 2 + 1$$

$$= 2^7 + 2^5 + 2^4 + 2^2 + 2^1 + 2^0$$

$$17^{183} = 17^{(2^0 + 2^1 + 2^2 + 2^4 + 2^5 + 2^7)}$$

$$= 17^{2^0} * 17^{2^1} * 17^{2^2} * 17^{2^4} * 17^{2^5} * 17^{2^7}$$

i	0	1	2	3	4	5	6	7
$17^{2^i} \bmod 256$	17	33	65	97	1	1	1	1

$$= (17 * 33 * 65 * 1 * 1 * 1) \bmod 256$$

$$= 113$$

b. $2^{477} \bmod 1000$
 $477 = 111011101$
 $= 2^8 + 2^7 + 2^6 + 2^4 + 2^3 + 2^2 + 2^0$

$$2^{477} = 2^{2^8} * 2^{2^7} * 2^{2^6} * 2^{2^4} * 2^{2^3} * 2^{2^2} * 2^{2^0}$$

i	0	1	2	3	4	5	6	7	8
$2^{2^i} \bmod 1000$	2	4	16	256	536	296	616	456	936

$$= (2 * 16 * 256 * 536 * 616 * 456 * 936) \bmod 1000$$

$$= 272$$

c. $11^{507} \bmod 1237$
 $507 = 111111011$
 $= 2^8 + 2^7 + 2^6 + 2^5 + 2^4 + 2^3 + 2^1 + 2^0$

$$11^{507} = 11^{2^8} * 11^{2^7} * 11^{2^6} * 11^{2^5} * 11^{2^4} * 11^{2^3} * 11^{2^1} * 11^{2^0}$$

i	0	1	2	3	4	5	6	7	8
$11^{2^i} \bmod 1237$	11	121	1034	388	867	830	1128	748	380

$$= (380 * 748 * 1128 * 830 * 867 * 388 * 121 * 11) \bmod 1237$$

$$= 322$$