Dicionários / Tabelas de Símbolos II

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Ficheiro ZIP

- Está disponível no Moodle um ficheiro ZIP de suporte aos tópicos de hoje
- O tipo abstrato Hash Table usando Separate Chaining
- Versão "simples", que permite trabalho autónomo de desenvolvimento e teste

Sumário

- Recap
- Exemplo de aplicação – contagem de ocorrências usando uma Hash Table
- Hash Tables – Representação usando Separate Chaining
- Análise detalhada do TAD Hash Table – Separate Chaining
- Desempenho computacional

Recapitulação

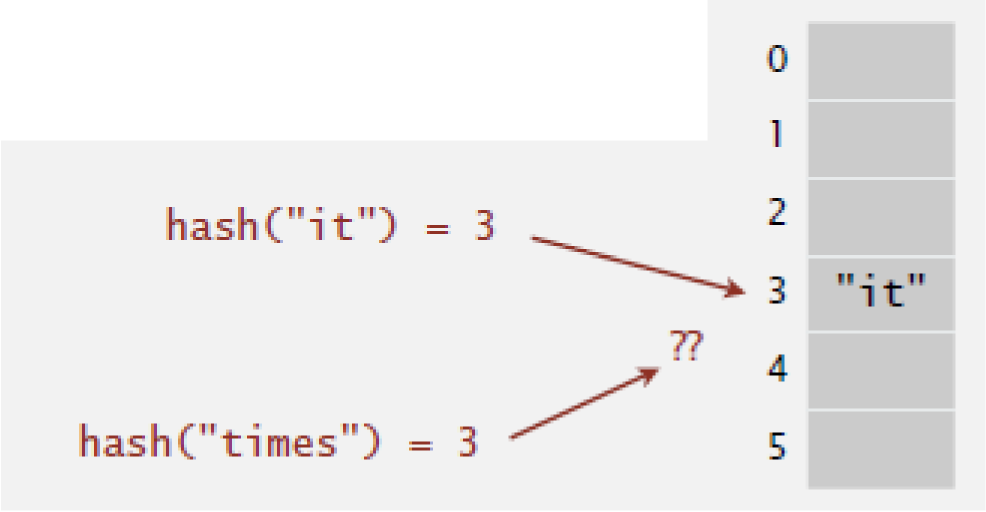
TAD Dicionário / Tabela de Símbolos

- Usar chaves para aceder a itens / valores
- Chaves e itens / valores podem ser de qualquer tipo
- Chaves são comparáveis
- MAS, não há duas chaves iguais !!
- Sem limite de tamanho / do número de pares (chave, valor)
- Chaves não existentes são associadas a um VALOR_NULO

- API simples / Código cliente simples

Hash Tables – Tabelas de Dispersão

- Armazenar itens numa tabela/array indexada pela chave
  - Índice é função da chave
- Função de Hashing: para calcular o índice a partir da chave
  - Rapidez !!
- Colisão: 2 chaves diferentes originam o mesmo resultado da função de hashing



[Sedgewick & Wayne]

Linear Probing

- Aceder à posição i
- Se necessário, tentar em (i + 1) % M, (i + 2) % M, etc.

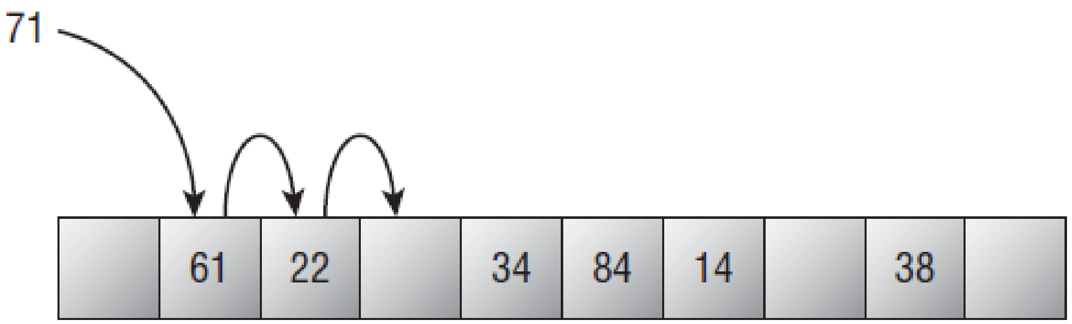


Figure 8-2: In linear probing, the algorithm adds a constant amount to locations to produce a probe sequence.

[Stephens]

Quadratic Probing

- Aceder à posição i
- Se necessário, tentar em (i + 1) % M, (i + 4) % M, (i + 9) % M, etc.

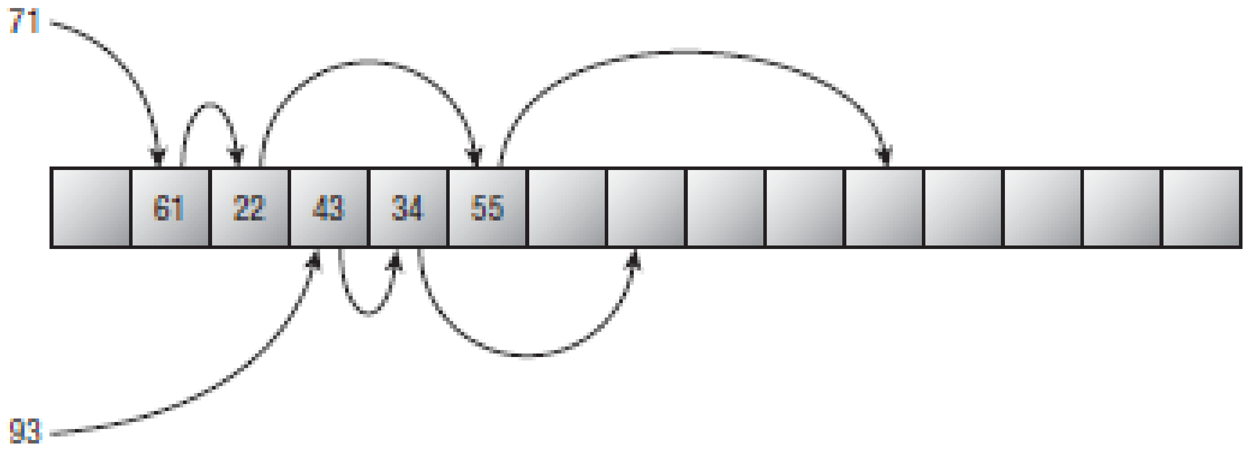


Figure 8-4: Quadratic probing reduces primary clustering.

[Stephens]

Análise – Linear Probing – Knuth, 1963

- Fator de carga – Load Factor $\lambda = N / M$
- Nº médio de tentativas para encontrar um item

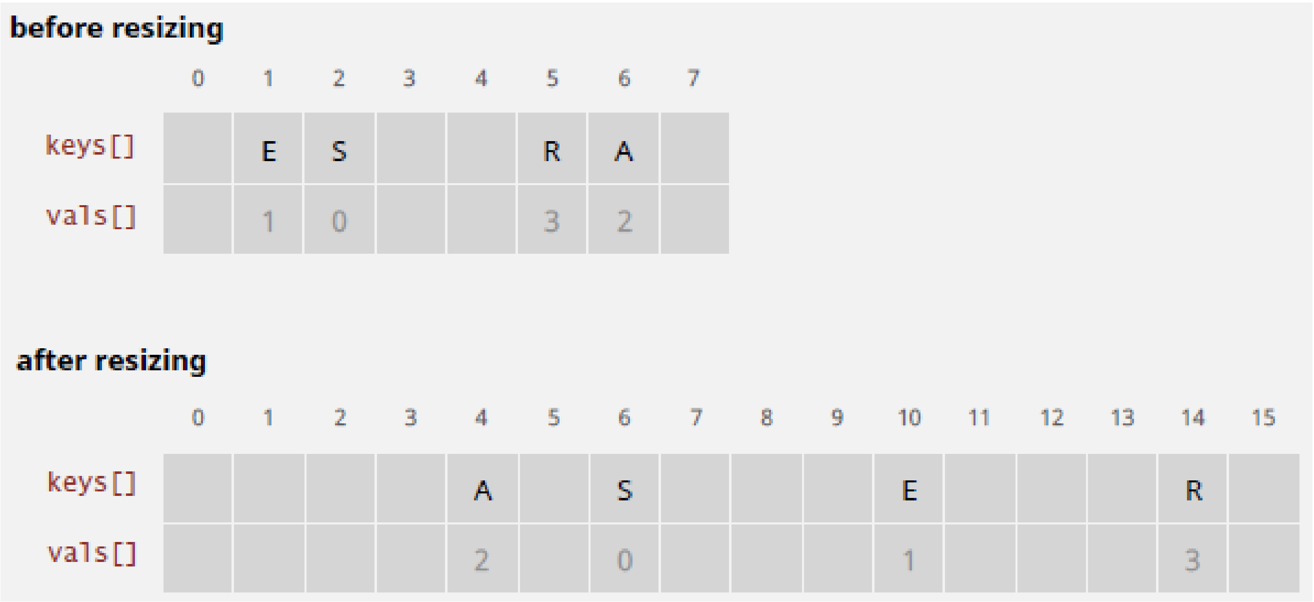
  $1/2 \times (1 + 1/(1 - \lambda))$ -> 1.5, se $\lambda = 50\%$ -> 3, se $\lambda = 80\%$

- Nº médio de tentativas para inserir um item ou concluir que não existe

  $1/2 \times (1 + 1/(1 - \lambda)^2)$ -> 2.5, se $\lambda = 50\%$ -> 13, se $\lambda = 80\%$

Resizing + Rehashing

- Objetivo : fator de carga ≤ 1/2
- Duplicar o tamanho do array quando fator de carga ≥ 1/2
- Reduzir para metade o tamanho do array quando fator de carga ≤ 1/8
- Criar a nova tabela e adicionar, um a um, todos os itens



[Sedgewick & Wayne]

Apagar um item (chave,valor) ?

before deleting S

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
keys[]	P	M			A	C	S	H	L		E				R	X
vals[]	10	9			8	4	0	5	11		12				3	7

doesn't work, e.g., if hash(H) = 4

after deleting S ?

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
keys[]	P	M			A	C		H	L		E				R	X
vals[]	10	9			8	4		5	11		12				3	7

[Sedgewick & Wayne]

Lazy Deletion

- Marcar inicialmente todos elementos da tabela como livres
- Ao inserir um item, o correspondente elemento fica ocupado
- Ao apagar um item, marcar esse elemento da tabela como apagado
- Para que qualquer cadeia que o use não seja quebrada !!
- E se possa continuar a procurar uma chave usando probing

- Quando termina uma procura ?
- Ao encontrar a chave procurada ou um elemento marcado como livre

Exemplo
– Hash Table (String, String)

TAD Hash Table

```c
HashTable* HashTableCreate(unsigned int capacity, hashFunction hashF,
                           probeFunction probeF, unsigned int resizeIsEnabled);

void HashTableDestroy(HashTable** p);
```

```c
int HashTableContains(const HashTable* hashT, const char* key);

char* HashTableGet(HashTable* hashT, const char* key);

int HashTablePut(HashTable* hashT, const char* key, const char* value);

int HashTableReplace(const HashTable* hashT, const char* key,
                     const char* value);

int HashTableRemove(HashTable* hashT, const char* key);
```

Estrutura de dados

```c
struct _HashTableHeader {
  unsigned int size;
  unsigned int numActive;
  unsigned int numUsed;
  hashFunction hashF;
  probeFunction probeF;
  unsigned int resizeIsEnabled;
  struct _HashTableBin* table;
};
```

```c
struct _HashTableBin {
  char* key;
  char* value;
  unsigned int isDeleted;
  unsigned int isFree;
};
```

Funções auxiliares para testes

```c
unsigned int hash1(const char* key) {
  assert(strlen(key) > 0);
  return key[0];
}

unsigned int hash2(const char* key) {
  assert(strlen(key) > 0);
  if (strlen(key) == 1) return key[0];
  return key[0] + key[1];
}
```

```c
unsigned int linearProbing(unsigned int index, unsigned int i,
                           unsigned int size) {
  return (index + i) % size;
}
```

```c
unsigned int quadraticProbing(unsigned int index, unsigned int i,
                              unsigned int size) {
  return (index + i * i) % size;
}
```

HashTableCreate

```
HashTable* HashTableCreate(unsigned int size, hashFunction hashF,
                           probeFunction probeF, unsigned int resizeIsEnabled) {
  assert(size > 0);
  HashTable* hTable = (HashTable*)malloc(sizeof(struct _HashTableHeader));
  assert(hTable != NULL);
  hTable->table =
      (struct _HashTableBin*)malloc(size * sizeof(struct _HashTableBin));
  assert(hTable->table != NULL);
```

HashTableCreate

```
  hTable->size = size;
  hTable->numActive = 0;
  hTable->numUsed = 0;
  hTable->hashF = hashF;
  hTable->probeF = probeF;
  hTable->resizeIsEnabled = resizeIsEnabled;

  for (int i = 0; i < size; i++) {
    hTable->table[i].key = NULL;
    hTable->table[i].value = NULL;
    hTable->table[i].isFree = 1;
    hTable->table[i].isDeleted = 0;
  }

  return hTable;
}
```

HashTableDestroy

```c
void HashTableDestroy(HashTable** p) {
  assert(*p != NULL);
  HashTable* t = *p;

  for (int i = 0; i < t->size; i++) {
    if (t->table[i].key) free(t->table[i].key);
    if (t->table[i].value) free(t->table[i].value);
  }
  free(t->table);
  free(t);
  *p = NULL;
}
```

Procura de uma chave

```
for (unsigned int i = 0; i < hashT->size; i++) {
  index = hashT->probeF(hashKey, i, hashT->size);

  bin = &(hashT->table[index]);

  if (bin->isFree) {
    // Not in the table !
    return index;
  }

  if ((bin->isDeleted == 0) && (strcmp(bin->key, key) == 0)) {
    // Found it !
    return index;
  }
}
```

HashTableContains

```c
int HashTableContains(const HashTable* hashT, const char* key) {
  int result = _searchHashTable(hashT, key);
  if (result == -1 || hashT->table[result].isFree == 1) {
    // NOT FOUND
    return 0;
  }
  return 1;
}
```

HashTableGet

```c
char* HashTableGet(HashTable* hashT, const char* key) {
  int index = _searchHashTable(hashT, key);
  if (index == -1 || hashT->table[index].isFree == 1) {
    // NOT FOUND
    return NULL;
  }

  struct _HashTableBin* bin = &(hashT->table[index]);
  char* result = (char*)malloc(sizeof(char) * (1 + strlen(bin->value)));
  strcpy(result, bin->value);

  return result;
}
```

HashTablePut

```
int HashTablePut(HashTable* hashT, const char* key, const char* value) {
  int result = _searchHashTable(hashT, key);

  if (result == -1) {
    // NO PLACE AVAILABLE
    return 0;
  }

  if (hashT->table[result].isFree == 0) {
    // ALREADY IN THE TABLE
    return 0;
  }

  // Does NOT BELONG to the table
  // See if it can be stored earlier in the chain, by starting again
  // Losing some efficiency here
```

HashTablePut

```c
unsigned int hashKey = hashT->hashF(key);

unsigned int index;
struct _HashTableBin* bin;

for (unsigned int i = 0; i < hashT->size; i++) {
  index = hashT->probeF(hashKey, i, hashT->size);

  bin = &(hashT->table[index]);

  if (bin->isFree) {
    bin->key = (char*)malloc(sizeof(char) * (1 + strlen(key)));
    strcpy(bin->key, key);
    bin->value = (char*)malloc(sizeof(char) * (1 + strlen(value)));
    strcpy(bin->value, value);
    bin->isFree = bin->isDeleted = 0;

    hashT->numActive++;
    hashT->numUsed++;
```

HashTablePut

```
    if (hashT->resizeIsEnabled && HashTableGetLoadFactor(hashT) > 0.5) {
      _resizeHashTable(hashT, hashT->size * 2);
    }

    return 1;
  }

  if (bin->isDeleted) {
    bin->key = (char*)malloc(sizeof(char) * (1 + strlen(key)));
    strcpy(bin->key, key);
    bin->value = (char*)malloc(sizeof(char) * (1 + strlen(value)));
    strcpy(bin->value, value);
    bin->isFree = bin->isDeleted = 0;

    hashT->numActive++;

    return 1;
  }
```

HashTableRemove

```c
int HashTableRemove(HashTable* hashT, const char* key) {
  int index = _searchHashTable(hashT, key);
  if (index == -1 || hashT->table[index].isFree == 1) {
    // NOT FOUND
    return 0;
  }

  // Mark as deleted to keep the chain

  hashT->table[index].isDeleted = 1;
  hashT->numActive--;
```

HashTableRemove

```
    free(hashT->table[index].key);
    free(hashT->table[index].value);
    hashT->table[index].key = hashT->table[index].value = NULL;

    if (hashT->resizeIsEnabled && HashTableGetLoadFactor(hashT) < 0.125) {
      _resizeHashTable(hashT, hashT->size / 2);
    }

    return 1;
}
```

Exemplo – M = 17 – N = 12

```
size = 17 | Used = 12 | Active = 12
  0 - Free = 0 - Deleted = 0 - Hash =   68, 1st index =   0, (December, The last month of the year)
  1 - Free = 1 - Deleted = 0 -
  2 - Free = 0 - Deleted = 0 - Hash =   70, 1st index =   2, (February, The second month of the year)
  3 - Free = 1 - Deleted = 0 -
  4 - Free = 1 - Deleted = 0 -
  5 - Free = 1 - Deleted = 0 -
  6 - Free = 0 - Deleted = 0 - Hash =   74, 1st index =   6, (January, 1st month of the year)
  7 - Free = 0 - Deleted = 0 - Hash =   74, 1st index =   6, (June, 6th month)
  8 - Free = 0 - Deleted = 0 - Hash =   74, 1st index =   6, (July, 7th month)
  9 - Free = 0 - Deleted = 0 - Hash =   77, 1st index =   9, (March, 3rd month)
 10 - Free = 0 - Deleted = 0 - Hash =   77, 1st index =   9, (May, 5th month)
 11 - Free = 0 - Deleted = 0 - Hash =   79, 1st index =  11, (October, 10th month)
 12 - Free = 0 - Deleted = 0 - Hash =   78, 1st index =  10, (November, Almost at the end of the year)
 13 - Free = 1 - Deleted = 0 -
 14 - Free = 0 - Deleted = 0 - Hash =   65, 1st index =  14, (April, 4th month)
 15 - Free = 0 - Deleted = 0 - Hash =   65, 1st index =  14, (August, 8th month)
 16 - Free = 0 - Deleted = 0 - Hash =   83, 1st index =  15, (September, 9th month)
```

Exemplo

– Contagem de Ocorrências

– Hash Table (String, Int)

Aplicação – Contagem

- Dado um ficheiro de texto
- Contar o nº de ocorrências de cada palavra

- Não se conhece, à partida, qual o nº de palavras distintas !!

- Chave : palavra
- Valor : nº de ocorrências

Exemplo

```
Conan 2
Arthur 38
Doyle 2
Table 8
Scarlet 10
In 505
Four 14
Holmes 2913
Scandal 2
Sherlock 411
The 2777
Sign 6
Red 18
League 15
Boscombe 15
```

```
Life 6
Avenging 3
Angels 3
Continuation 2
Reminiscences 2
Watson 1028
Conclusion 2
Being 5
reprint 1
from 2780
reminiscences 3
late 156
Army 6
Medical 5
```

Tarefas

- Analisar as funções desenvolvidas
- E o programa de aplicação

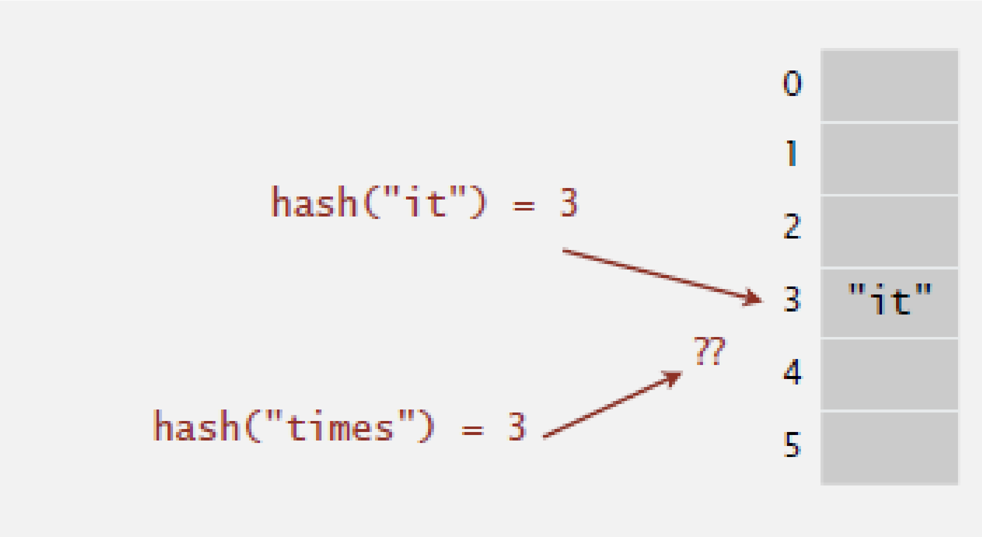
- Escolher vários textos e contar as suas palavras distintas
- Melhorar o processamento das palavras lidas
  - Por exemplo, converter maiúsculas em minúsculas
- Não contar “stop words”
- Obter uma listagem ordenada – Como fazer ??

Hash Tables
– Separate Chaining

Colisões – Como proceder ?

- Duas chaves distintas são mapeadas no mesmo índice da tabela !!

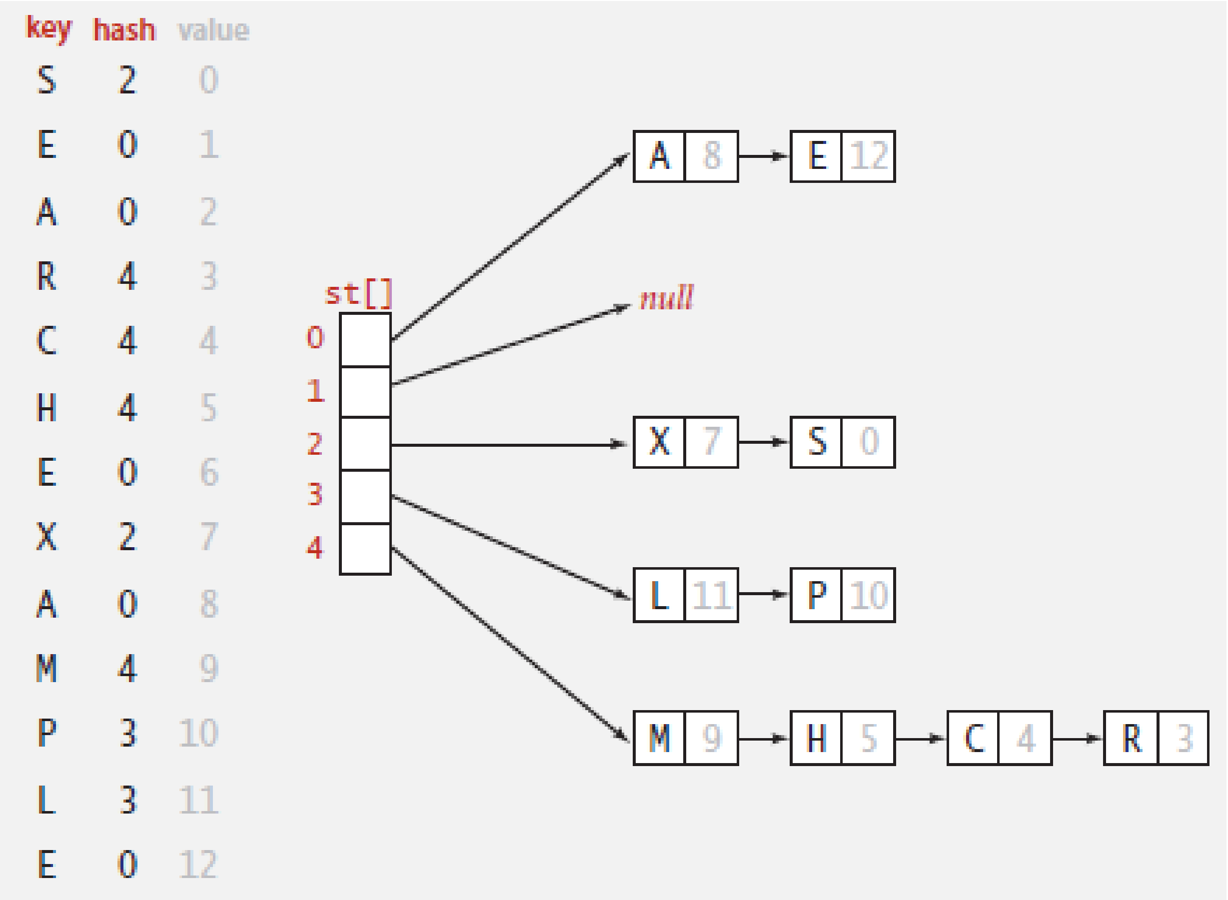
- Como gerir de modo eficiente ?
- Sem usar “demasiada” memória !!
- Alternativa ao Open Addressing ?



[Sedgewick & Wayne]

Separate Chaining (IBM, 1953)

- Array de M < N itens
- Mapear a chave em [0..(M – 1)]
- Inserir no início de uma cadeia, se não existir
- Procurar só numa cadeia



[Sedgewick & Wayne]

Separate Chaining

```c
struct _HashTableHeader {
  unsigned int size;
  unsigned int numBins;
  hashFunction hashF;
  List** table;
};
```

```c
struct _HashTableBin {
  char* key;
  char* value;
};
```

HashTableCreate

```
HashTable* hTable = (HashTable*)malloc(sizeof(struct _HashTableHeader));
assert(hTable != NULL);
hTable->table = (List**)malloc(size * sizeof(List*));
assert(hTable->table != NULL);

hTable->size = size;
hTable->numBins = 0;
hTable->hashF = hashF;

for (int i = 0; i < size; i++) {
  hTable->table[i] = ListCreate(comparator);
}
```

HashTableDestroy

```c
for (int i = 0; i < t->size; i++) {
  List* l = t->table[i];
  // Free the HT bins of each list
  while (ListIsEmpty(l) == 0) {
    struct _HashTableBin* bin = ListRemoveHead(l);
    free(bin->key);
    free(bin->value);
    free(bin);
  }
  // Destroy the list header
  ListDestroy(&(t->table[i]));
}
free(t->table);
free(t);
```

Procurar

```c
// Search for the key
// If found, the list current node is updated
//
static int _searchKeyInList(List* l, char* key) {
  if (ListIsEmpty(l)) {
    return 0;
  }

  // Needed for the comparator
  // Shallow copy of the key: just the pointer
  struct _HashTableBin searched;
  searched.key = key;

  ListMoveToHead(l);
  return ListSearch(l, &searched) != -1;
}
```

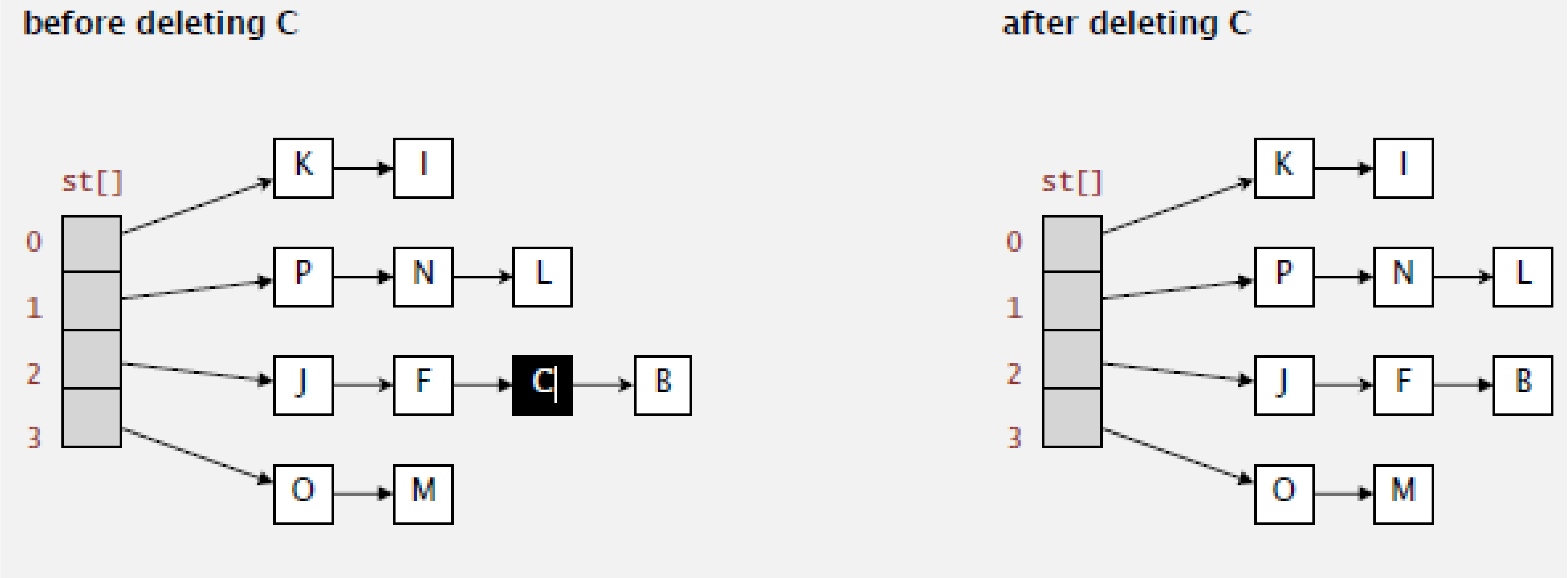
Inserir

```c
int HashTablePut(HashTable* hashT, char* key, char* value) {
  unsigned int index = hashT->hashF(key) % hashT->size;
  List* l = hashT->table[index];

  if (_searchKeyInList(l, key) == 1) {
    // FOUND, cannot be added to the table
    return 0;
  }

  // Does NOT BELONG to the table
  // Insert a new bin in the list

  struct _HashTableBin* bin = (struct _HashTableBin*)malloc(sizeof(*bin));
  bin->key = (char*)malloc(sizeof(char) * (1 + strlen(key)));
  strcpy(bin->key, key);
  bin->value = (char*)malloc(sizeof(char) * (1 + strlen(value)));
  strcpy(bin->value, value);

  ListInsert(l, bin);
  hashT->numBins++;

  return 1;
}
```

Substituir

```c
int HashTableReplace(const HashTable* hashT, char* key, char* value) {
  unsigned int index = hashT->hashF(key) % hashT->size;
  List* l = hashT->table[index];

  // Search and update current, if found
  if (_searchKeyInList(l, key) == 0) {
    return 0;
  }

  struct _HashTableBin* bin = ListGetCurrentItem(l);

  free(bin->value);
  bin->value = (char*)malloc(sizeof(char) * (1 + strlen(value)));
  strcpy(bin->value, value);

  return 1;
}
```

Apagar é fácil !



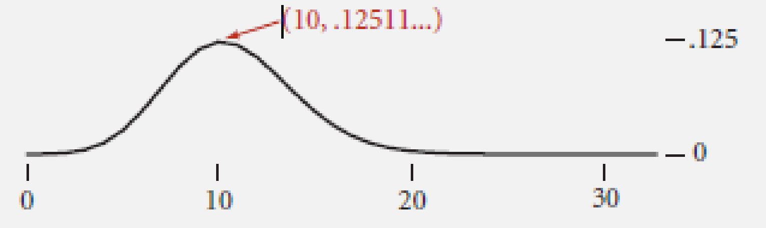
[Sedgewick & Wayne]

Apagar

```c
int HashTableRemove(HashTable* hashT, char* key) {
  unsigned int index = hashT->hashF(key) % hashT->size;
  List* l = hashT->table[index];

  // Search and update current, if found
  if (_searchKeyInList(l, key) == 0) {
    return 0;
  }

  // Get rid of the bin
  struct _HashTableBin* bin = ListGetCurrentItem(l);
  free(bin->key);
  free(bin->value);
  free(bin);

  // Get rid of the list node
  ListRemoveCurrent(l);
  hashT->numBins--;

  return 1;
}
```

Análise

- Em média, N/M itens por cadeia – Load Factor
- Procurar / inserir -> nº de comparações é proporcional a N/M
  - M vezes mais rápido que na procura sequencial



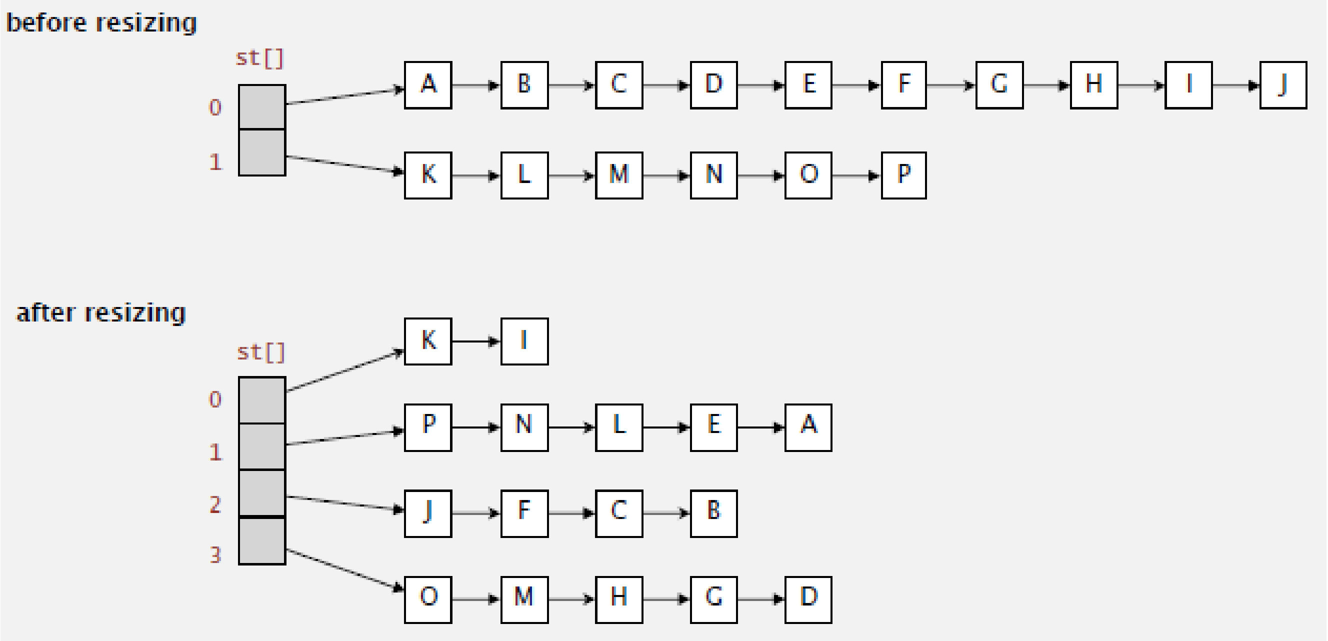
Binomial distribution ($N = 10^4$, $M = 10^3$, $\alpha = 10$)

[Sedgewick & Wayne]

- M muito grande -> demasiadas cadeias vazias
- M demasiado pequeno -> cadeias muito longas
- Escolha habitual : $M \approx N/4$ -> **O(1)**

Resizing + Rehashing

- Objetivo : fator de carga aprox. constante
- Duplicar o tamanho do array quando $N/M \geq 8$
- Reduzir para metade o tamanho do array quando $N/M \leq 2$
- Criar a nova tabela e adicionar, um a um, todos os itens



[Sedgewick & Wayne]

Tarefa

- Implementar uma função para fazer Resizing + Rehasing
- Adaptar o tamanho da tabela à evolução do fator de carga

Separate Chaining

```
size = 17 | Active = 12
  0 -
      Hash =   68, (December, 12th month)
  1 -
  2 -
      Hash =   70, (February, 2nd month of the year)
  3 -
  4 -
  5 -
  6 -
      Hash =   74, (January, 1st month of the year)
      Hash =   74, (July, 7th month)
      Hash =   74, (June, 6th month)
  7 -
  8 -
  9 -
      Hash =   77, (March, 3rd month)
      Hash =   77, (May, 5th month)
 10 -
      Hash =   78, (November, 11th month)
 11 -
      Hash =   79, (October, 10th month)
 12 -
 13 -
 14 -
      Hash =   65, (April, 4th month)
      Hash =   65, (August, 8th month)
 15 -
      Hash =   83, (September, 9th month)
 16 -
```

Open Addressing + Linear Probing

```
size = 17 | Used = 12 | Active = 12
  0 - Free = 0 - Deleted = 0 - Hash =   68, 1st index =   0, (December, The last month of the year)
  1 - Free = 1 - Deleted = 0 -
  2 - Free = 0 - Deleted = 0 - Hash =   70, 1st index =   2, (February, The second month of the year)
  3 - Free = 1 - Deleted = 0 -
  4 - Free = 1 - Deleted = 0 -
  5 - Free = 1 - Deleted = 0 -
  6 - Free = 0 - Deleted = 0 - Hash =   74, 1st index =   6, (January, 1st month of the year)
  7 - Free = 0 - Deleted = 0 - Hash =   74, 1st index =   6, (June, 6th month)
  8 - Free = 0 - Deleted = 0 - Hash =   74, 1st index =   6, (July, 7th month)
  9 - Free = 0 - Deleted = 0 - Hash =   77, 1st index =   9, (March, 3rd month)
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 11 - Free = 0 - Deleted = 0 - Hash =   79, 1st index =  11, (October, 10th month)
 12 - Free = 0 - Deleted = 0 - Hash =   78, 1st index =  10, (November, Almost at the end of the year)
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```

Eficiência

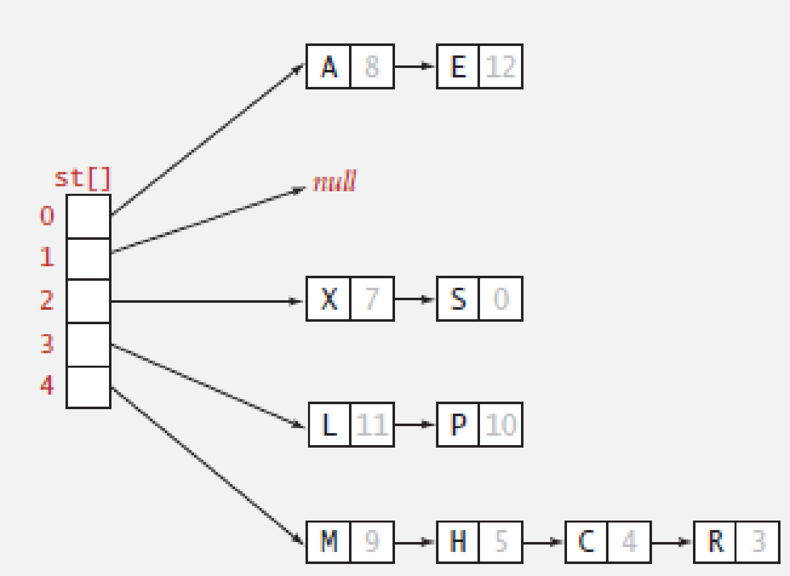
implementation	guarantee			average case			ordered ops?	key interface
	search	insert	delete	search hit	insert	delete		
separate chaining	N	N	N	3-5 *	3-5 *	3-5 *		equals() hashCode()
linear probing	N	N	N	3-5 *	3-5 *	3-5 *		equals() hashCode()

* under uniform hashing assumption

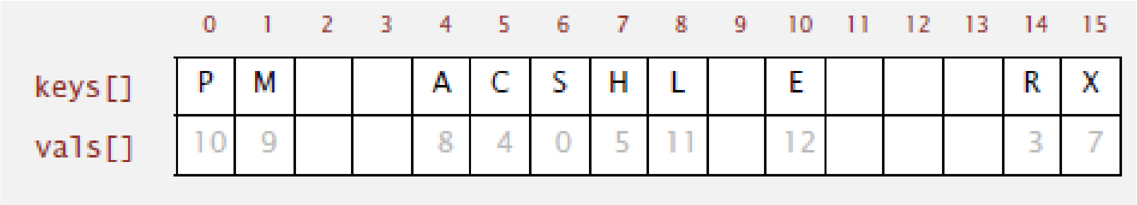
[Sedgewick & Wayne]

Separate Chaining vs Linear Probing

- Separate Chaining
- Desempenho não se degrada abruptamente
- Pouco sensível a funções de hashing menos boas



- Linear Probing
- Menos espaço de memória desperdiçado



[Sedgewick & Wayne]

Hash Tables vs Balanced Search Trees

- Tabelas de Dispersão
- Código mais simples
- Melhor alternativa se não pretendermos ordem
- Mais rápidas, para chaves simples

- Árvores Binárias Equilibradas
- Pior caso : O(log N) vs O(N)
- Suportam ordem
- compareTo() vs equals() + hashCode()