Ex 10 F5

①

b) $\underbrace{y'\left(1-\ln\frac{y}{x}\right)}=\frac{y}{x}$, $x>0$

(*) $y'=\underbrace{\frac{y}{x}\cdot\frac{1}{1-\ln\left|\frac{y}{x}\right|}}_{f(x,y)}$

$y'=f(x,y)$

$\begin{cases} f(x,y) \text{ é homogenea de grau zero} \\ f(\lambda x,\lambda y)= \cdots = \underline{\underline{f(x,y)}}. \end{cases}$

$\underline{zx=y}\Rightarrow z=\frac{y}{x}$

$z+xz'=z\cdot\frac{1}{1-\ln z}$

$z+x\frac{dz}{dx}=\frac{z}{1-\ln z}$

$\longrightarrow$

$$z + x\frac{dz}{dx} = \frac{z}{1-\ln z}.$$

$$\boxed{\begin{array}{c} zx = y \\ z = \dfrac{y}{x} \end{array}}$$

$$x\frac{dz}{dx} = \frac{z}{1-\ln z} - z = \frac{\cancel{z}-\cancel{z}+z\ln z}{1-\ln z}$$

$$x\frac{dz}{dx} = \frac{z\ln z}{1-\ln z}.$$

$$\rightarrow \quad \frac{1-\ln z}{z\ln z}dz = \frac{1}{x}dx$$

integrando

$$\ln|\ln(z)| - \ln|z| = \ln|x| + C_1$$

$$\ln|\ln(z)| = \ln|xz| + C_1$$

$\boxed{CA}$

$$\int \frac{1-\ln z}{z\ln z}dz = \int \frac{1}{z\ln z}dz - \int \frac{1}{z}dz$$

$$u = \ln z \qquad du = \frac{1}{z}dz$$

$$= \int \frac{1}{u} - \ln|z|$$

$$= \ln|u| - \ln|z|$$

$$= \ln|\ln z| - \ln|z|$$

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$$\ln|\ln|z|| = \ln|xz| + C_1$$

$$\ln\left|\ln\frac{y}{x}\right| = \ln|y| + C_1 \longrightarrow \quad e^{\ln|\ln|\frac{y}{x}||} = e^{\ln|y| + C_1} = e^{\ln|y|} \cdot e^{C_1}$$

$$\ln\left|\frac{y}{x}\right| = y \cdot \underbrace{e^{C_1}}$$

$$\longrightarrow \quad \ln\left|\frac{y}{x}\right| = yC$$

$$\frac{y}{x} = e^{yC}$$

$$y = xe^{Cy}, \quad C \in \mathbb{R}.$$

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Exercício:

$$\begin{cases} x^2 y' - 2xy = 3y^4 \\ y(1) = 1/2 \end{cases}$$

$$x^2 y' - 2xy = 3y^4$$

$$y' - \frac{2}{x} y = \frac{3}{x^2} y^4 \longrightarrow \alpha = 4$$

$$\underbrace{y^{-4} y'} - \frac{2}{x} y^{-3} = \frac{3}{x^2}$$

$$z = y^{1-4} = y^{-3}$$

$$z = y^{-3}$$

$$z' = -3 \overbrace{y^{-4} \cdot y'} \Leftrightarrow \frac{z'}{-3} = y^{-4} y'$$

$$\frac{z'}{-3} - \frac{2}{x} z = \frac{3}{x^2}$$

$$\boxed{z' + \frac{6}{x} z = \frac{-9}{x^2}}$$

$$z' + \frac{6}{x} z = -\frac{9}{x^2} \qquad p(x) = \frac{6}{x} \qquad q(x) = \frac{-9}{x^2}$$

$\cdot\,\mu(x)$

$$\mu(x) = e^{\int \frac{6}{x} dx} = e^{6 \ln|x|} = e^{\ln|x|^6} = x^6$$

$$\underbrace{z' \cdot x^6 + 6x^5 z} = -9x^4$$

$$(x^6 \cdot z)' = -9x^4$$

integrando

$$x^6 \cdot z = -\frac{9}{5} x^5 + C, \quad C \in \mathbb{R}$$

$$z = -\frac{9}{5}\frac{1}{x} + \frac{C}{x^6}, \quad C \in \mathbb{R}.$$

$$z = -\frac{9}{5}\frac{1}{x} + \frac{C}{x^6}$$

$$z = y^{-3}$$

$$z = \frac{1}{y^3}$$

$$\frac{1}{y^3} = -\frac{9}{5}\frac{1}{x} + \frac{C}{x^6} \leftarrow$$

TPC: Ex 12 - F5

$$y = y(x)$$

- $y(1) = 1/2 \leftarrow$

$\underline{x=1}$

$$\frac{1}{(y(1))^3} = -\frac{9}{5}\frac{1}{1} + \frac{C}{1}$$

$$\frac{1}{(1/2)^3} = -\frac{9}{5} + C$$

$$8 + \frac{9}{5} = C$$

$$\frac{49}{5} = C$$

Solução do P.V.I

$$\boxed{\frac{1}{y^3} = -\frac{9}{5}\frac{1}{x} + \frac{49}{5x^6}}$$

SL 28

Exemplo: $y' - 2y = \underbrace{e^{5x}}_{b(x)}$

TPC: (fator integrante)

Equação homogenea associada é:

$$\boxed{y' - 2y = 0} \quad (**)$$

Solução geral de $(**)$

$$\frac{dy}{dx} - 2y = 0$$

$$\frac{dy}{dx} = 2y$$

$$\frac{1}{2y}\,dy = dx$$

$\to$ $\frac{1}{2}\ln|y| = x + c_1, \ c_1 \in \mathbb{R}$

$\ln|y| = 2x + c_2, \ c_2 \in \mathbb{R}$

$y = e^{2x} \cdot e^{c_2}$

$\to$ $\boxed{y_h = e^{2x} C}, \ C \in \mathbb{R}.$

SL(28)
cont

$y_p = \frac{1}{3} e^{5x}$ é solução particular de $\to y' - 2y = e^{5x}$

Verificar!

$y_p' - 2y_p = \frac{5}{3} e^{5x} - \frac{2}{3} e^{5x} = \frac{3}{3} e^{5x} = e^{5x}$ ✓

Pelo tma (SL28)

Solução geral da eq. completa

$$y = \underbrace{\frac{1}{3} e^{5x}}_{y_p} + \underbrace{c e^{2x}}_{y_h}, \quad c \in \mathbb{R}$$

Exer 13
F5

Considere a EDO linear homogénea (de coef. não constantes)

$$(1-x)\,y'' + xy' - y = 0 \quad , \quad \underbrace{x \in \,]1, +\infty[}$$

a) Mostrar que $\underbrace{\{e^x, x\}}$ é um S.F.S. da equação

$$W(x, e^x) = \begin{bmatrix} x & e^x \\ 1 & e^x \end{bmatrix}$$

$$\det(W) = xe^x - e^x = \underbrace{e^x}\cdot(x-1) \neq 0$$

cont →

$\{e^x, x\}$ é um S.F.S da equação

b) Solução geral da EDO:

$\{x, e^x\}$

$$y = c_1 x + c_2 e^x \quad , c_1, c_2 \in \mathbb{R}.$$

Exemplo: $y^{(5)} + 2y^{(4)} + 4y^{(3)} + 8y^{(2)} + 4y' + 8y = 0$

Polinômio característico

$$p(r) = r^5 + 2r^4 + 4r^3 + 8r^2 + 4r + 8$$

$$p(r) = 0$$

± 1
± 2
± 4
± 8

	1	2	4	8	4	8
-2		-2	0	-8	0	-8
	1	0	4	0	4	0

resto

$$p(r) = (x+2)(r^4 + 4r^2 + 4)$$

$$r^4 + 4r^2 + 4 = 0$$

$$(r^2+2)^2 = 0 \Rightarrow r^2 + 2 = 0$$

$$r^2 = -2$$

$$\boxed{r = \pm i\sqrt{2}}$$