Folha 5 . Ex 6

a) $x + yy' = 0$

$$x + y\frac{dy}{dx} = 0$$

$$x = -y\frac{dy}{dx}$$

$$\int x\,dx = -\int y\,dx$$

$$\frac{x^2}{2} = -\frac{y^2}{2} + C \text{ ; } C \in \mathbb{R}$$

$$x^2 = -y^2 + C$$

$$x^2 + y^2 = C \in \mathbb{R}.$$

Folha 5 Ex 6

c) $(t^2 - xt^2)\dfrac{dx}{dt} + x^2 = -tx^2$

$(t^2 - xt^2)\dfrac{dx}{dt} = -tx^2 - x^2$

$\dfrac{dx}{dt} = \dfrac{-tx^2 - x^2}{t^2 - xt^2} = \dfrac{tx^2 + x^2}{xt^2 - t^2}$

$\dfrac{dx}{dt} = \dfrac{x^2(t+1)}{(x-1)\,t^2}$

$\displaystyle\int \frac{x-1}{x^2}\,dx = \int \frac{t+1}{t^2}\,dt$

$\ln|x| + \dfrac{1}{x} = \ln|t| - \dfrac{1}{t} + C$

$\ln|x| - \ln|t| = -\dfrac{1}{t} - \dfrac{1}{x} + C$

$\ln\left|\dfrac{x}{t}\right| = -\dfrac{1}{t} - \dfrac{1}{x} + C$

CA

$\displaystyle\int \frac{x-1}{x^2}\,dx = \int \frac{1}{x} - \frac{1}{x^2}\,dx$

$\displaystyle = \int \frac{1}{x}\,dx - \int \frac{1}{x^2}\,dx$

$= \ln|x| + \dfrac{1}{x} + C_1$

$\displaystyle\int \frac{t+1}{t^2}\,dt = \int \frac{1}{t} + \frac{1}{t^2}\,dt$

$= \ln|t| - \dfrac{1}{t} + C_2$

Exercício 6(c) folha 5

continuação (*)
$$\frac{x}{t} = e^{-\frac{1}{t}-\frac{1}{x}+C_1}$$
$$\frac{x}{t} = e^{-\frac{1}{t}-\frac{1}{x}}\cdot C \,,\ C\in\mathbb{R}.$$

$$(t^2 - xt^2)\frac{dx}{dt} = -tx^2 - x^2$$
$$\frac{dx}{dt} = \frac{-tx^2 - x^2}{t^2 - xt^2} = \frac{tx^2 + x^2}{xt^2 - t^2}$$
$$\frac{dx}{dt} = \frac{x^2(t+1)}{(x-1)\,t^2}$$
$$\int \frac{x-1}{x^2}\,dx = \int \frac{t+1}{t^2}\,dt$$
$$\ln|x| + \frac{1}{x} = \ln|t| - \frac{1}{t} + C_1$$
$$\ln|x| - \ln|t| = -\frac{1}{t} - \frac{1}{x} + C_1$$
$$\ln\left|\frac{x}{t}\right| = -\frac{1}{t} - \frac{1}{x} + C_1$$

CA

$$\int \frac{x-1}{x^2}\,dx = \int \frac{1}{x} - \frac{1}{x^2}\,dx$$
$$= \int \frac{1}{x}\,dx - \int \frac{1}{x^2}\,dx$$
$$= \ln|x| + \frac{1}{x} + C_1$$

$$\int \frac{t+1}{t^2}\,dt = \int \frac{1}{t} + \frac{1}{t^2}\,dt$$
$$= \ln|t| - \frac{1}{t} + C_2$$

continua (*)

d) $(x^2-1)y' + 2xy^2 = 0$

$$(x^2-1)\frac{dy}{dx} + 2xy^2 = 0$$

$$\frac{dy}{dx} = \frac{-2x}{x^2-1} \cdot y^2$$

$$\frac{1}{y^2}\,dy = \frac{-2x}{x^2-1}\,dx$$

$$\int \frac{1}{y^2}\,dy = \int \frac{-2x}{x^2-1}\,dx$$

$$-\frac{1}{y} = -\ln|x^2-1| + C_1 \quad C \in \mathbb{R}$$

$$\frac{1}{y} = \ln|x^2-1| - C$$

$$\int \frac{-2x}{x^2-1}\,dx = -\int \frac{1}{u}\,du$$

$u = x^2-1$

$du = 2x\,dx$

$$= -\ln|u| + C$$

$$= -\ln|x^2-1| + C$$

$$y = \frac{1}{\ln|x^2-1| - C}, \quad C \in \mathbb{R}$$

Slide 17/45: resolução de uma EDO Linear de 1ª ordem usando fator integrante

$$y' + p(x)\, y = g(x) \quad (1)$$

1) Determinar $\mu(x) = e^{\int p(x)\,dx}$

2) Multiplicar a EDO por $\mu(x)$

$$\underbrace{\mu(x)\, y' + \mu(x)\, p(x)\, y}_{} = \mu(x)\, g(x) \Longrightarrow (\mu(x)\, y)' = \mu(x)\, g(x)$$

$$\mu(x)\, y = \int \mu(x)\, g(x)\, dx$$

$$\boxed{y = \frac{1}{\mu(x)} \int \mu(x)\, g(x)\, dx}$$

Observemos que

$$(\mu(x)\, y)' = \mu'(x)\, y + \mu(x)\, y'$$

$$\mu'(x) = \left(e^{\int p(x)\,dx}\right)' = e^{\int p(x)\,dx} \cdot p(x)$$

$$\mu'(x) = \mu(x) \cdot p(x)$$

$y' + xy = 0$, $p(x) = x$ $q(x) = 0$.

1) $\underline{\mu(x)} = e^{\int x\,dx} = \underline{e^{\frac{x^2}{2}}}$

$$\underbrace{e^{\frac{x^2}{2}} y' + e^{x^2/2} xy}_{} = 0$$

$$\left(e^{x^2/2} \cdot y\right)' = 0$$

$$e^{\frac{x^2}{2}} y = 0 + C$$

$$e^{x^2/2} y = C \longrightarrow y = \frac{C}{e^{x^2/2}} = C e^{-x^2/2}$$

$$\boxed{\begin{aligned} y &= \frac{1}{\mu(x)} \int \underbrace{\mu(x) q(x)}_{0} \, dx \\ y &= \frac{1}{e^{x^2/2}} \cdot C. \end{aligned}}$$

$$\to y' - y = -e^{x}, \quad p(x) = -1, \quad q(x) = -e^{x}$$

$$\mu(x) = e^{\int -1\,dx} = \underline{\underline{e^{-x}}}$$

$$\underbrace{e^{-x}y' - e^{-x}y} = \underbrace{-e^{x}e^{-x}}$$

$$(e^{-x}\cdot y)' = -1$$

$$e^{-x}\cdot y = -x + C, \quad C\in\mathbb{R}$$

$$y = \frac{C-x}{e^{-x}} = (C-x)e^{x}, \quad C\in\mathbb{R}. \quad //$$

$$\to y' - y = -e^x \quad , \quad p(x) = -1 \quad , \quad q(x) = -e^x$$

$$\mu(x) = e^{\int -1\,dx} = \underline{\underline{e^{-x}}}$$

$$\underbrace{e^{-x}y' - e^{-x}y} = \underbrace{-e^x e^{-x}}$$

$$(e^{-x} \cdot y)' = -1$$

$$e^{-x} \cdot y = -x + C \, , \quad C \in \mathbb{R}$$

$$y = \frac{C-x}{e^{-x}} = (C-x)e^x \, , \quad C \in \mathbb{R} \, . \quad //$$

Resolver a seguinte EDO

$$\boxed{y' + p(x)y = q(x)}$$

1) $xy' - y = x - 1, \quad \underline{x > 0}$

$$y' - \frac{1}{x}y = 1 - \frac{1}{x} \quad \rightarrow \quad p(x) = -\frac{1}{x}, \quad q(x) = 1 - \frac{1}{x}$$

$$\mu(x) = e^{\int p(x)\,dx} = e^{-\ln|x|} = \frac{1}{x}$$

$$\underbrace{\frac{1}{x}y' - \frac{1}{x^2}y}_{} = \frac{1}{x} - \frac{1}{x^2}$$

$$\left(\frac{1}{x}y\right)' = \frac{1}{x} - \frac{1}{x^2}$$

$$\frac{1}{x}y = \int \frac{1}{x} - \frac{1}{x^2}\,dx$$

$$\frac{1}{x}y = \ln x + \frac{1}{x} + C, \quad C \in \mathbb{R}$$

$$y = x\ln x + 1 + Cx, \quad C \in \mathbb{R} \quad //$$

2) $xy' + y - e^x = 0 \,, \quad x > 0$

$$y' + \frac{1}{x} y = \frac{1}{x} e^x \longrightarrow p(x) = \frac{1}{x} \quad \wedge \; q(x) = \frac{e^x}{x}$$

$$\mu(x) = e^{\int p(x)\,dx} = e^{\int \frac{1}{x} dx} = e^{\ln|x|} = x \,.$$

$$xy' + y = e^x$$

$$(x \cdot y)' = e^x$$

$$xy = \int e^x \, dx$$

$$xy = e^x + C \implies y = \frac{e^x}{x} + \frac{C}{x} \quad , C \in R \; //.$$

Folha 5 | 13

a) $y' + 2y = \cos x$

$p(x) = 2 \qquad q(x) = \cos x$

$$\mu(x) = e^{\int 2\,dx} = e^{2x}.$$

$$e^{2x} y' + 2e^{2x} y = e^{2x} \cos x$$

$$(e^{2x} y)' = e^{2x} \cos x$$

$$e^{2x} y = \int e^{2x} \cos x \, dx$$

$$\int e^{2x} \cos x = \frac{1}{5}\left(e^{2x} \operatorname{sen} x + 2e^{2x} \cos x\right) + C$$

$$e^{2x} y = \frac{1}{5} e^{2x} \operatorname{sen} x + \frac{2}{5} e^{2x} \cos x + C,\ C \in \mathbb{R}.$$

$$\boxed{y = \frac{1}{5} \operatorname{sen} x + \frac{2}{5} \cos x + \frac{C}{e^{2x}},\ C \in \mathbb{R}}$$

$$\int e^{2x}\cos x\,dx = e^{2x}\operatorname{sen}x - \left[\int 2e^{2x}\cdot\operatorname{sen}x\,dx\right]$$

$u = e^{2x}$ $\quad du = 2e^{2x}$

$dv = \cos x$ $\quad v = \operatorname{sen}x$

$$= e^{2x}\operatorname{sen}x - 2\underbrace{\int e^{2x}\operatorname{sen}x\,dx}$$

$u = e^{2x}$ $\quad du = 2e^{2x}$

$dv = \operatorname{sen}x$ $\quad v = -\cos x$

$$= e^{2x}\operatorname{sen}x - 2\left[-e^{2x}\cos x + 2\underbrace{\int e^{2x}\cos x\,dx}\right]$$

$$\int e^{2x}\cos x\,dx = e^{2x}\operatorname{sen}x + 2e^{2x}\cos x - 4\underbrace{\int e^{2x}\cos x}$$

$$5\int e^{2x}\cos x\,dx = e^{2x}\operatorname{sen}x + 2e^{2x}\cos x$$

$$\int e^{2x}\cos x\,dx = \frac{1}{5}e^{2x}\operatorname{sen}x + \frac{2}{5}e^{2x}\cos x.$$