$\boxed{\text{Ex13}}$

$\boxed{\text{Folha 5}}$

b) $x^3 y' - y - 1 = 0$

$$y' - \frac{1}{x^3} y = \frac{1}{x^3} \qquad (*)$$

$$\rightarrow \mu(x) = e^{\frac{1}{2}x^{-2}}$$

Multiplicando (*) por $\mu(x)$

$$\underbrace{e^{\frac{x^{-2}}{2}} \cdot y' - \frac{e^{x^{-2}/2}}{x^3} y}_{(e^{x^{-2}/2}\, y)'} = \frac{e^{\frac{x^{-2}}{2}}}{x^3}$$

$$\left(e^{x^{-2}/2}\, y\right)' = \frac{e^{x^{-2}/2}}{x^3}$$

$$e^{\frac{-x^{-2}}{2}} y = \int \frac{e^{x^{-2}/2}}{x^3} dx \qquad (**)$$

$y' + p(x) y = q(x)$ $\quad \mu(x) = e$

$p(x) = -\frac{1}{x^3}$ $\qquad q(x) = \frac{1}{x^3}$

$$\int p(x) dx = -\int \frac{1}{x^3} dx = \frac{1}{2x^2}$$

CA

$$\int \frac{e^{x^{-2}/2}}{x^3} dx = \int -e^u du = -$$

$$= -e^{\frac{1}{2}x^{-2}}$$

$u = \frac{1}{2} x^{-2}$

$du = \frac{-2}{2} x^{-3} = -x^{-3} = -\frac{1}{x^3} dx$

$-du = \frac{1}{x^3} dx$

(**)

$$e^{\frac{x^{-2}}{2}} y = \int \frac{e^{x^{-2}/2}}{x^3}\, dx$$

$$e^{\frac{x^{-2}}{2}} y = -e^{x^{-2}/2} + C \,,\; C \in \mathbb{R}.$$

$$y = \frac{-e^{x^{-2}/2}}{e^{x^{-2}/2}} + \frac{C}{e^{x^{-2}/2}} \,,\; C \in \mathbb{R}$$

$$\boxed{y = -1 + C\, e^{-\frac{1}{2}x^{-2}} \,,\; C \in \mathbb{R}.}$$

$\boxed{\text{Ex7}}$

F5

b) $xy + x + y'\underbrace{\sqrt{4+x^2}} = 0$, $y(0) = 1$.

$y' + p(x)\,y = g(x)$

$$y' + \frac{x}{\sqrt{4+x^2}}\,y = -\frac{x}{\sqrt{4+x^2}}$$

$\boxed{\text{CA}}$

$p(x) = \frac{x}{\sqrt{4+x^2}}$, $g(x) =$

$\int p(x)\,dx = \int \frac{x}{\sqrt{4+x^2}}\,dx = \frac{1}{2}$

$u = 4+x^2$

$du = 2x\,dx$

$\frac{du}{2} = x\,dx$

$= \frac{1}{2}\,\frac{u^{1/2}}{\cancel{1/2}} =$

$= (4+x^2)^{1/2}$

$= \sqrt{4+x^2}$

$$\mu(x) = e^{\sqrt{4+x^2}}$$

$$\underbrace{e^{\sqrt{4+x^2}}\,y' + \frac{x\,e^{\sqrt{4+x^2}}}{\sqrt{4+x^2}}\,y} = \frac{-x\,e^{\sqrt{4+x^2}}}{\sqrt{4+x^2}}$$

$$\left(e^{\sqrt{4+x^2}}\cdot y\right)' = -\frac{x\,e^{\sqrt{4+x^2}}}{\sqrt{4+x^2}}$$

$$e^{\sqrt{4+x^2}}\cdot y = \int -x\,e^{\sqrt{4+x^2}} \quad dx$$

$$e^{\sqrt{4+x^2}}\, y = \int \frac{-x\, e^{\sqrt{4+x^2}}}{\sqrt{4+x^2}}\, dx$$

$$\underbrace{e^{\sqrt{4+x^2}}}\, y = -\, e^{\sqrt{4+x^2}} + C, \quad C \in \mathbb{R}$$

$$y = -1 + \frac{C}{e^{\sqrt{4+x^2}}}, \quad C \in \mathbb{R}$$

$$y(\overset{x}{0}) = 1$$

$$1 = -1 + \frac{C}{e^{\sqrt{4+0}}}$$

$$1 = -1 + \frac{C}{e^2}$$

$$\boxed{e^2\, 2 = C}$$

$$\boxed{y = -1 + \frac{2e^2}{\sqrt{4+x^2}}}$$ → Solução P.V.I.

C.A)

$$\int \frac{-x\, e^{\sqrt{4+x^2}}\, dx}{\sqrt{4+x^2}} = -\int e^u\, du$$

$$= -e^u$$

$$= -$$

$$u = \sqrt{4+x^2} = (4+x^2)^{1/2}$$

$$\Rightarrow du = \frac{1}{\cancel{2}}(4+x^2)^{-1/2} \cdot \cancel{2}\, x\, dx$$

$$du = \frac{x}{\sqrt{4+x^2}}\, dx$$

Exemplo

$$y' = \underbrace{\frac{y-x}{y+x}}_{f(x,y)}$$

$$f(\lambda x, \lambda y) = \frac{\lambda y - \lambda x}{\lambda y + \lambda x} = \frac{\lambda (y-x)}{\lambda (y+x)} = \frac{y-x}{y+x} = f(x,y), \ \lambda \neq 0$$

$\therefore$ f é uma funç. homogênea de grau zero sempre que $\lambda \neq 0$

$$y' = \underbrace{\frac{x^2+xy+y^2}{x^2}}_{f(x,y)}$$

f é homog. de grau zero.

$\underline{\underline{zx}} = y \Rightarrow y' = z + xz'$

$$z + xz' = \frac{x^2 + x^2 z + x^2 z^2}{x^2} = \frac{x^2(1+z+z^2)}{x^2} = \underbrace{1+z+z^2}_{f(1,z)}$$

$$\cancel{z} + xz' = 1 + \cancel{z} + z^2$$

$$x\frac{dz}{dx} = 1+z^2$$

$$\frac{1}{1+z^2}\,dz = \frac{1}{x}\,dx$$

cont. $\longrightarrow$

$$\frac{1}{1+z^2}\,dz = \frac{1}{x}\,dx$$

$$zx = y$$

$$z = \frac{y}{x}$$

$$\operatorname{arctg} z = \ln|x| + C,\ \in \mathbb{R}$$

$$\cdot\ \operatorname{arctg}\left(\frac{y}{x}\right) = \ln|x| + C\ ,$$

$$\frac{y}{x} = \operatorname{tg}\left(\ln|x| + C\right)$$

$$y = x\operatorname{tg}\left(\ln|x| + C\right)\ ,\ C \in R\ //.$$

Ex8 a) $(x^2+y^2)y' = xy$

F5.

$$y' = \underbrace{\frac{xy}{x^2+y^2}}_{f(x,y)}$$

$y' = \underset{\text{homogéneo}}{f(x,y)}$ 8

✓ f é homogénea de grau zero; $\lambda \neq 0$.

$$f(\lambda x, \lambda y) = \frac{\lambda x \lambda y}{(\lambda x)^2 + (\lambda y)^2} = \frac{\lambda^2 x y}{\lambda^2 x^2 + \lambda^2 y^2} = \frac{\cancel{\lambda^2}\, x y}{\cancel{\lambda^2}\,(x^2+y^2)} = \frac{xy}{x^2+y^2} = f(x,y)$$

cont. $\longrightarrow$

$$y' = \frac{xy}{x^2 + y^2}$$

$zx = y$ $\qquad y' = z + xz'$

$$f(1, z)$$

$$z + xz' = \frac{x^2 z}{x^2 + z^2 x^2} = \frac{x^2 z}{x^2(1 + z^2)} = \overbrace{\frac{z}{1 + z^2}}$$

$$z + xz' = \frac{z}{1 + z^2}$$

$$x\frac{dz}{dx} = \frac{z}{1+z^2} - z = \frac{\cancel{z} - \cancel{z} - z^3}{1 + z^2}$$

$$x\frac{dz}{dx} = \frac{-z^3}{1 + z^2}$$

cont $\longrightarrow$

$$x\frac{dz}{dx} = -\frac{z^3}{1+z^2}$$

$$-\frac{1+z^2}{z^3}\,dz = \frac{1}{x}\,dx$$

integrando

$$\frac{1}{2z^2} - \ln|z| = \ln|x| + c, \quad c \in \mathbb{R}$$

$$\frac{1}{2z^2} = \ln|xz| + c$$

$$\boxed{\frac{x^2}{2y^2} = \ln|y| + C.} \quad C \in \mathbb{R}, \; y \neq 0$$

CA)

$$\int \frac{1+z^2}{z^3}\,dz = \int \frac{1}{z^3} + \frac{1}{z}\,dz = \int z^{-3} + \frac{1}{z}\,dz$$

$$= \frac{z^{-2}}{-2} + \ln|z| + c.$$

$$= -\frac{1}{2z^2} + \ln|z| + C$$

$y = 0$ é solução singular da EDO.

$$y' + a(x)\,y = b(x)\,\underline{y^{\alpha}}$$

$$z = y^{1-\alpha}$$

$$z' = (1-\alpha)\,y^{-\alpha}\cdot y' \Rightarrow \frac{z'}{1-\alpha} = y^{-\alpha}y'$$

$$\underbrace{y^{-\alpha}y'}_{} + a(x)\,\overbrace{y\,y^{-\alpha}}^{y^{1-\alpha}} = b(x)$$

$$\frac{z'}{1-\alpha} + a(x)\,z = b(x)$$

$$\boxed{z' + (1-\alpha)\,a(x)\,z = (1-\alpha)\,b(x)}$$

$$y' + p(x)y = g(x)$$

$$\mu(x)$$

$$y' + y = e^x y^{2} \longrightarrow \underline{\underline{\alpha = 2}}$$

$$y^{-2} y' + y^{-2} y = \underbrace{e^x}_{b(x)}$$

$$\underbrace{y^{-2} y'} + y^{-1} = e^x$$

$$\boxed{z = y^{1-2} = y^{-1}}$$

$$z' = -y^{-2} \cdot y'$$

$$-z' = y^{-2} y'$$

$$\boxed{z' + (1-\alpha)\,\underline{a(x)}\,z = (1-\alpha)\,b(x)}$$

$$-z' + z = e^x$$

$$z' - z = -e^x \longrightarrow p(x) = -1 \qquad q(x) = -e^x$$

cont $\longrightarrow$

$$z' - z = -e^{x}$$

$\cdot\, \mu(x)$

$$\mu(x) = e^{-x}$$

$$\underbrace{e^{-x} z' - z e^{-x}}_{(e^{-x} z)'} = -e^{x} \cdot e^{-x}$$

$$(e^{-x} z)' = -1$$

$$\int -1 \, dx = -x$$

$$z = y^{-1}$$

$$e^{-x} z = -x + c, \quad c \in \mathbb{R}$$

$$z = \frac{(c-x)}{e^{-x}} = (c-x)e^{x}, \in \mathbb{R}$$

$$\frac{1}{y} = (c-x)\, e^{x}$$

$$\boxed{y = \frac{1}{(c-x)\, e^{x}}, \quad c \in \mathbb{R}}$$

Exercício:

$$\begin{cases} x^2 y' - 2xy = 3y^4 \\ y(1) = 1/2 \end{cases}$$

$\alpha = 4$

$z = y^{1-4} = y^{-3}$.