16 h) $\boxed{y'' + 9y = \underline{\text{sen}\, x} - e^{-x}}$ (*)

- Solução homogenea

$$y'' + 9y = 0$$

$$r^2 + 9 = 0 \Rightarrow \underbrace{r = \pm 3i}$$ $\to \alpha = 0$, $\to \beta = 3.$

SFS : $\{\cos(3x), \text{sen}(3x)\}$

Solução geral da homogenea é : $y_h = A\cos(3x) + B\,\text{sen}(3x)$, $A, B \in \mathbb{R}$.

- Solução particular de (*)

  - Solução particular $\boxed{y'' + 9y = \underline{\operatorname{sen} x}}$

$$\operatorname{sen} x = \underbrace{B_{\overset{\swarrow \text{grau}}{m}}(x)}\, e^{\alpha x} \operatorname{sen}(\beta x) \quad , \quad \beta = 1 \ , \ \alpha = 0 \ , \ \underline{\underline{m = 0}}$$

Como $\alpha + \beta i = 0 + i = i$ não é raiz do polinomio caracteristico

então $\underline{k = 0}$

$$y_p = x^0 e^{0x} \left( \overbrace{C(x)}^{\to \text{grau } 0} \cos(x) + \overbrace{D(x)}^{\to \text{grau } 0.} \operatorname{sen}(x) \right)$$

$$\longrightarrow y_p = C \cos(x) + D \operatorname{sen} x.$$

$$\boxed{y_p = \underbrace{C\cos x + D\sin x}.}$$

$$y'' + 9y = \sin x.$$

$$y_p' = -C\sin x + D\cos x$$

$$y_p'' = -C\cos x - D\sin x$$

$$\underline{-C\cos x} \; \underline{-D\sin x} + \underline{9C\cos x} + \underline{9D\sin x} = \sin x.$$

$$8C\cos x + 8D\sin x = \underline{\sin x}$$

$$8C = 0$$

$$\boxed{C = 0}$$

$$8D = 1 \Rightarrow \boxed{D = \frac{1}{8}}.$$

$$\boxed{y_p = \frac{1}{8} \operatorname{sen} x}.$$

- Solução particular de: $y'' + 9y = -e^{-x}$

$$-e^{-x} = \underbrace{B_m(x)} e^{\alpha x} \cos(\beta x) \quad , \quad \alpha = -1 \ , \ \beta = 0 \ , \ m = 0$$

Como $\alpha + \beta i = -1$ não é raiz pol. característica então $K=0$

$$y_p = x^0 e^{-x} \left( \underbrace{E(x)}_{\text{grau } 0} \cos(0) + \underbrace{F(x)}_{\text{grau } 0} \cancel{\operatorname{sen}(0)} \right)$$

$$\boxed{y_p = e^{-x} \cdot E} \quad , \ E \text{ constante a determinar}$$

$$y_p = Ee^{-x}$$

$$\underbrace{y'' + 9y = -e^{-x}}$$

$$y'_p = -Ee^{-x}$$

$$y''_p = Ee^{-x}$$

$$Ee^{-x} + 9Ee^{-x} = -e^{-x}$$

$$10Ee^{-x} = -e^{-x}$$

$$10E = -1$$

$$E = -\frac{1}{10}$$

$$\boxed{y_p = -\frac{1}{10}e^{-x}}$$

Uma solução particular de $y'' + 9y = \operatorname{sen} x - e^{-x}$ é

$$y_p = \frac{1}{8} \operatorname{sen} x - \frac{1}{10} e^{-x}$$ (usando princípio de sobreposição)

Logo, uma solução geral de $y'' + 9y = \operatorname{sen} x - e^{-x}$

$$y = y_h + y_p$$

$$y = A\cos(3x) + B\operatorname{sen}(3x) + \frac{1}{8}\operatorname{sen} x - \frac{1}{10} e^{-x}$$ //

Ficha 6

Ex 1 (g) $f(t) = (t-2)^2 e^{2(t-2)} H_2(t)$



$$\mathcal{L}\{H_2(t)\underbrace{(t-2)^2 e^{2(t-2)}}_{g(t-2)}\} = \mathcal{L}\{H_2(t)\, g(t-2)\} = e^{-2s} G(s)$$

$$g(t) = e^{2t} t^2$$

$$G(s) = \mathcal{L}\{g(t)\} =$$

$$\mathcal{L}\{e^{2t} t^2\} = \underbrace{F(s-2)}$$

onde $F(s) = \mathcal{L}\{t^2\} = \dfrac{2}{s^3}, \; s>0$

$$= e^{-2s} \underbrace{F(s-2)}$$

$$= e^{-2s} \frac{2}{(s-2)^3}, \quad s > 0+2$$

$$s > 2$$

$$\mathcal{L}\{f(t)\} = e^{-2s} \frac{2}{(s-2)^3}, \; s > 2$$

//

Exemplo: Derivada da transformada

$$\mathcal{L}\{t^2 \cos t\} = (-1)^2 F''(s)$$

$$F(s) = \mathcal{L}\{\cos t\}$$

$$F(s) = \frac{s}{s^2+1}$$

$$= \left(\frac{s}{s^2+1}\right)''$$

$$= \left(\frac{s^2+1-2s^2}{(s^2+1)^2}\right)' = \left(\frac{-s^2+1}{(s^2+1)^2}\right)'$$

$$= \ldots$$

Terminem

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**Ex 3** b) $\displaystyle\int_0^{+\infty} e^{-3t}\, \underbrace{t \sin t}\, dt.$

$$F(s) = \int_0^{+\infty} e^{-st}\, \underbrace{f(t)}\, dt.$$

$$F(3) = \int_0^{+\infty} e^{-3t} f(t)\, dt$$

$$\underbrace{\mathcal{L}\{t \sin t\}(3)} = \int_0^{+\infty} e^{-3t}\, t \sin t\, dt$$

$$\mathcal{L}\{t^1 \sin t\} = (-1)^1 \left(\mathcal{L}\{\sin t\}\right)' = -\left(\frac{1}{s^2+1}\right)' = \frac{2s}{(s^2+1)^2}$$

$$\int_0^{+\infty} e^{-3t}\, t \sin t\, dt = \frac{2(3)}{(3^2+1)^2} = \frac{6}{100} = \frac{3}{50}.$$

Ex 3 (a)
Ex 5 (a)

SL15) Determinar $y = y(t)$ tal que

$$\underbrace{y'' + 2y' + 10y = 1}\ (*),\quad y(0) = 0,\quad y'(0) = 0$$

$\mathcal{L}\{y\} = Y(s)$

Aplicando a transformada de Laplace a ambos os membros de (*)

$$\mathcal{L}\{y'' + 2y' + 10y\} = \mathcal{L}\{1\}$$

$$\mathcal{L}\{y''\} + 2\,\mathcal{L}\{y'\} + 10\,\mathcal{L}\{y\} = \mathcal{L}\{1\}$$

$$s^2Y(s) + 2sY(s) + 10\,Y(s) = \frac{1}{s}$$

continua →

- $\mathcal{L}\{y''\} = s^2Y(s) - s\cancel{y(0)}^{0} - \cancel{y'(0)}^{0} = \underline{\underline{s^2Y(s)}}$

(A)

$$\mathcal{L}\{y'\} = sY(s) - y(0) = sY(s)$$

continue

$$(s^2 + 2s + 10)\,Y(s) = \frac{1}{s}.$$

$$Y(s) = \frac{1}{s(s^2+2s+10)}, \quad s > 0$$

$$=$$