Folha 2 - parte 2: Soluções

1. (a)
$$y = \frac{x^4}{2} + K x^2$$
, $K \in \mathbb{R}$;
(b) $y = \frac{x}{2} \csc x - \frac{\cos x}{2} + K \csc x$, $K \in \mathbb{R}$.

2. (a)
$$y = C_1 e^{-x} + \frac{\sin x}{2} - \frac{\cos x}{2}$$
;

(b)
$$y = C_1 e^x + C_2 e^{-x} + \cos x;$$

(c)
$$y = C_1 e^x + C_2 e^{-2x} + 3x$$
;

(d)
$$y = \left(C_1 + C_2 x + \frac{x^3}{6}\right) e^{2x};$$

(e)
$$y = C_1 + (C_2 - x) e^{-x}$$
;

(f)
$$y = C_1 \sin(2x) + C_2 \cos(2x) - \frac{1}{4} \cos(2x) \ln|\sec(2x) + \tan(2x)|$$
; também se pode escrever na forma $y = C_1 \sin(2x) + C_2 \cos(2x) - \frac{1}{4} \cos(2x) \ln \left| \frac{1 + \tan x}{1 - \tan x} \right|$;

(g)
$$y = C_1 + C_2 \cos x + C_3 \sin x - \frac{x}{2} \sin x;$$

(h)
$$y = C_1 \operatorname{sen}(3x) + C_2 \cos(3x) + \frac{\operatorname{sen} x}{8} - \frac{e^{-x}}{10}$$

 $(C_1, C_2, C_3 \text{ são constantes reais arbitrárias}).$

3.
$$y = \frac{3}{4}(x-\pi)e^{2(\pi-x)} + \frac{\sin(2x)}{8}$$

4. (a)
$$x(t) = \frac{3}{10} \operatorname{sen} t - \frac{1}{10} \cos t - \frac{9}{10} e^{\frac{t}{3}};$$

(b)
$$y(t) = \frac{1}{3} \operatorname{sen}(6t) - \cos(6t);$$

(c)
$$y(t) = t - \frac{2}{3} + \frac{2}{3\sqrt{2}} e^{-t} \operatorname{sen}(\sqrt{2}t) + \frac{2}{3} e^{-t} \cos(\sqrt{2}t);$$

(d)
$$y(t) = (t - \pi)^2 + 2\pi(t - \pi) + \pi^2 - 1 + \cos(t - \pi) = t^2 - 1 - \cos t$$
.

5. (a)
$$y = \frac{K}{(x^2 + 1)^2}$$
, $K \in \mathbb{R}$;

(b)
$$y = C_1 \cos x + C_2 \sin x + x \cos x$$
, $C_1, C_2 \in \mathbb{R}$.

6.
$$y = 1 + e^{-\operatorname{sen} x}, \quad x \in \mathbb{R}.$$

7. (a)
$$y = C e^{\operatorname{arctg} x}$$
, $C \in \mathbb{R}$;

(b)
$$y = C_1 + C_2 \cos(2x) + C_3 \sin(2x) + \frac{1}{3} \sin x$$
, $C_1, C_2 \in \mathbb{R}$.

8.
$$y = K e^{x^3} - \frac{1}{3}, \quad K \in \mathbb{R}.$$

9.
$$y = C_1 e^{-2x} + (C_2 + C_3 x + 2x^2) e^x$$
, $C_1, C_2, C_3 \in \mathbb{R}$.