(1)

Exemplo: $2y^{(5)} - 8y^{(4)} + 8y''' = 0$

$$p(r) = 2r^5 - 8r^4 + 8r^3 = 0$$

$$e^{rx}, xe^{rx}, x^2e^{rx}$$

$$2r^3(r^2 - 4r + 4) = 0$$

$$r^3 = 0 \quad \vee \quad r^2 - 4r + 4 = 0$$

$$\underline{r = 0} \quad \vee \quad (r-2)^2 = 0$$

$$\underline{r = 2}$$

$$S.F.S = \{e^{0x}, xe^{0x}, x^2e^{0x}, e^{2x}, xe^{2x}\} = \{1, x, x^2, e^{2x}, xe^{2x}\}$$

$\longrightarrow$

2

Solução geral da ED : $\boxed{C_1 + C_2 x + C_3 x^2 + C_4 e^{2x} + C_5 x e^{2x}}$

$C_1, C_2, C_3, C_4, C_5 \in \mathbb{R}$.

Exemplo: $y'' + 2y' + 5y = 0$

$$p(r) = r^2 + 2r + 5 = 0$$

$$r = \frac{-2 \pm \sqrt{4-20}}{2} = \frac{-2 \pm \sqrt{-16}}{2} = \frac{-2 \pm 4i}{2} = -1 \pm 2i$$

$$r_1 = -1 + 2i \quad , \quad r_2 = -1 - 2i$$

$$S.F.S = \{ e^{-x}\cos(2x),\ e^{-x}\operatorname{sen}(2x) \}$$

Solução geral: $c_1 e^{-x}\cos(2x) + c_2 e^{-x}\operatorname{sen}(2x)$ //

EX 14 | F5

e) $y'' + 4y = 0$

$$p(r) = r^2 + 4 = 0$$

$$r^2 = -4$$

$$r = \pm 2i$$

$$r_1 = 2i \qquad r_2 = -2i$$

$$S.F.S = \{ e^{0x}\cos(2x),\ e^{0x}\operatorname{sen}(2x) \} = \{ \cos(2x), \operatorname{sen}(2x) \}$$

Solução geral: $C_1\cos(2x) + C_2\operatorname{sen}(2x)$, $C_1, C_2 \in \mathbb{R}$. //

Ex 5 (40)

$$y'' + y = \underbrace{\operatorname{cosec} x}_{b(x)} \quad , \quad x \in \left]0, \pi\right[$$

1) Solução da homogenea

$$y'' + y = 0$$

$$P(r) = r^2 + 1 = 0$$

$$r^2 = -1$$

$$r = \pm i$$

$$SFS = \{ e^{0x} \cos(x), e^{0x} \operatorname{sen}(x) \}$$

$$= \{ \cos(x), \operatorname{sen}(x) \}$$

Sol geral homogenea

$$y_h = C_1 \cos(x) + C_2 \operatorname{sen}(x) \quad , C_1, C_2 \in \mathbb{R}$$

2) $y_p = C_1(x)\cos x + C_2(x)\,\text{sen}\, x$

onde

$$\begin{bmatrix} \cos x & \text{sen}\, x \\ -\text{sen}\, x & \cos x \end{bmatrix} \begin{bmatrix} C_1'(x) \\ C_2'(x) \end{bmatrix} = \begin{bmatrix} 0 \\ \text{cosec}\, x \end{bmatrix} \leftarrow \frac{b(x)}{a_0}$$

$$C_1'(x) = \frac{\begin{vmatrix} 0 & \text{sen}\, x \\ \text{cosec}\, x & \cos x \end{vmatrix}}{\underbrace{\begin{vmatrix} \cos x & \text{sen}\, x \\ -\text{sen}\, x & \cos x \end{vmatrix}}_{\cos^2 x + \text{sen}^2 x = 1}} = \frac{-\text{cosec}\, x \cdot \text{sen}\, x}{1} = -\frac{1}{\text{sen}\, x} \cdot \text{sen}\, x = -1$$

$$\boxed{C_1'(x) = -1}$$

$$C_2'(x) = \frac{\begin{vmatrix} \cos x & 0 \\ -\operatorname{sen} x & \operatorname{cosec} x \end{vmatrix}}{\begin{vmatrix} \cos x & \operatorname{sen} x \\ -\operatorname{sen} x & \cos x \end{vmatrix}} = \frac{\cos x \cdot \operatorname{cosec} x}{1} = \cos x \cdot \frac{1}{\operatorname{sen} x} = \operatorname{ctg} x$$

$$\boxed{C_2'(x) = \operatorname{ctg}(x)}$$

CA

$$\int \operatorname{ctg}(x)\,dx = \int \frac{\cos x}{\operatorname{sen} x}\,dx = \int \frac{1}{u}\,du$$

$u = \operatorname{sen} x$ $\quad = \ln|u| = \ln|\operatorname{sen} x|$

$$C_1'(x) = -1$$

$$\boxed{C_1(x) = -x}$$

$$C_2'(x) = \operatorname{ctg}(x)$$

$$C_2(x) = \int \operatorname{ctg}(x)\,dx$$

$$\boxed{C_2(x) = \ln|\operatorname{sen} x|}$$

$, 0 < x < \pi$

Logo, a solução particular.

$$y_p = -x\cos x + \ln|\operatorname{sen} x| \cdot \operatorname{sen} x.$$

Solução geral

$$y = \underbrace{C_1 \cos x + C_2 \operatorname{sen} x}_{y_h} \underbrace{- x\cos x + \ln|\operatorname{sen} x| \operatorname{sen} x}_{y_p.}$$

$C_1, C_2 \in \mathbb{R}$

$0 < x < \pi$

Exercício 15

b) $y' \underbrace{\operatorname{sen} x}_{a_0(x)} + y \cos x = \underbrace{\operatorname{sen}^2 x}_{b(x)}$

1) Solução geral da eq. homogênea

$$y' \operatorname{sen} x + y \cos x = 0$$

$$\frac{dy}{dx} \operatorname{sen} x = -y \cos x$$

$$-\frac{1}{y} dy = \frac{\cos x}{\operatorname{sen} x} dx$$

$$-\ln|y| = \ln|\operatorname{sen} x| + C_1$$

$$y^{-1} = \operatorname{sen} x \, C, \; C \in \mathbb{R}$$

$$\boxed{y_h = \frac{C}{\operatorname{sen} x}, \; C \in \mathbb{R}}$$

2) Solução particular

$$y_p = \frac{C(x)}{\operatorname{sen} x}$$

onde

$$C'(x)\frac{1}{\operatorname{sen} x} = \frac{\operatorname{sen}^2 x}{\operatorname{sen} x}$$

$$C'(x) = \operatorname{sen}^2 x$$

$$C(x) = \int \operatorname{sen}^2 x \, dx$$

$$C(x) = \frac{x}{2} - \frac{\operatorname{sen} x \cos x}{2}$$

$$\int \operatorname{sen}^2 x \, dx = \int \frac{1 - \cos(2x)}{2} \, dx$$

$$= \int \frac{1}{2} - \frac{\cos(2x)}{2} \, dx$$

$$= \frac{x}{2} - \frac{\operatorname{sen}(2x)}{4}$$

$$= \frac{x}{2} - \frac{2 \operatorname{sen} x \cos x}{4}$$

$$y_p = \left[\frac{x}{2} - \frac{\operatorname{sen} x \cos x}{2}\right] \cdot \frac{1}{\operatorname{sen} x} = \frac{x}{2\operatorname{sen} x} - \frac{\cos x}{2}$$

Solução geral:

$$\boxed{y = \frac{C}{\operatorname{sen} x} + \frac{1}{2}\frac{x}{\operatorname{sen} x} - \frac{\cos x}{2}, \quad C \in \mathbb{R}}$$

Ex 2

$$\underbrace{x}_{a_0(x)} y' - y = \underbrace{x - 1}_{b(x)} \quad , \quad x > 0$$

1)

$$x y' - y = 0$$

$$x \frac{dy}{dx} = y$$

$$\frac{1}{y} dy = \frac{1}{x} dx$$

$$\ln |y| = \ln |x| + C_1$$

$$\boxed{y_h = x C}, \quad C \in \mathbb{R}$$

$$y_p = x\,C(x)$$

onde

$$C'(x)\,x = \frac{x-1}{x}$$

$$C'(x) = \frac{x-1}{x^2}$$

$$C(x) = \int \frac{x-1}{x^2}\,dx$$

$$C(x) = \ln|x| + \frac{1}{x}$$

$$y_p = x\left[\ln|x| + \frac{1}{x}\right] = x\ln(x) + 1$$

CA)

$$\int \frac{x-1}{x^2}\,dx = \int \frac{1}{x} - \frac{1}{x^2}\,dx$$

$$\int \frac{x-1}{x^2}\,dx = \ln|x| + \frac{1}{x} + C$$

Solução geral

$$y = \underbrace{xC}_{y_h} + \underbrace{x\ln(x) + 1}_{y_p}, \quad C \in \mathbb{R}$$