

Ex 10 F5

$$b) \underbrace{y' \left(1 - \ln \frac{y}{x}\right)}_{f(x,y)} = -\frac{y}{x}, \quad x > 0$$

$$(*) \quad y' = \frac{y}{x} \underbrace{\frac{1}{1 - \ln \left| \frac{y}{x} \right|}}_{f(x,y)}$$

$$\underline{z(x) = y} \Rightarrow z = \frac{y}{x}$$

$$z + xz' = z \cdot \frac{1}{1 - \ln z}$$

$$z + x \frac{dz}{dx} = \frac{z}{1 - \ln z}$$

→

$$y' = f(x,y)$$

$$\begin{cases} f(x,y) \text{ é homogênea de grau zero} \\ f(\lambda x, \lambda y) = \dots = \underline{\underline{f(x,y)}}. \end{cases}$$

①

$$z + x \frac{dz}{dx} = \frac{z}{1 - \ln z}$$

$$x \frac{dz}{dx} = \frac{z}{1 - \ln z} - z = \frac{z - z + z \ln z}{1 - \ln z}$$

$$x \frac{dz}{dx} = \frac{z \ln z}{1 - \ln z}$$

$$\frac{1 - \ln z}{z \ln z} dz = \frac{1}{x} dx$$

integrando

$$\ln|\ln(z)| = \ln|z| = \ln|x| + C_1$$

$$\ln|\ln(z)| = \ln|xz| + C_1$$

$$\boxed{\begin{aligned} z x &= y \\ z &= \frac{y}{x} \end{aligned}}$$

$$\begin{aligned} \int \frac{1 - \ln z}{z \ln z} dz &= \int \frac{1}{z \ln z} dz - \int \frac{1}{z} dz \\ \int \frac{1}{z \ln z} dz &= \int \frac{1}{u} du = \ln|u| = \ln|\ln z| \\ \int \frac{1}{z} dz &= \ln|z| \end{aligned}$$

$$\ln |\ln |z|| = \ln |xz| + C_1$$

$$\ln \left| \ln \frac{y}{x} \right| = \ln |y| + C_1 \rightarrow \left\{ \begin{array}{l} \ln \left| \ln \frac{y}{x} \right| = \ln |y| + C_1 \\ e^{\ln \left| \ln \frac{y}{x} \right|} = e^{\ln |y| + C_1} \\ = e^{\ln |y|} \cdot e^{C_1} \end{array} \right.$$

$$\ln \left| \frac{y}{x} \right| = \underbrace{y \cdot e^{C_1}}$$

$$\rightarrow \ln \left| \frac{y}{x} \right| = y C$$

$$\frac{y}{x} = e^{y C}$$

$$y = x e^{C y}, \quad C \in \mathbb{R}.$$

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Exercício:

$$\begin{cases} x^2 y' - 2xy = 3y^4 \\ y(1) = 1/2 \end{cases} \leftarrow$$

④

$$x^2 y' - 2xy = 3y^4$$

$$y' - \frac{2}{x}y = \frac{3}{x^2}y^{\textcircled{4}}$$



$$\alpha = 4$$

$$\underbrace{y^{-4}y'} - \frac{2}{x}y^{-3} = \frac{3}{x^2}$$

$$z = y^{1-4} = y^{-3}$$

$$z = y^{-3}$$

$$z' = -3y^{-4} \cdot y' \Leftrightarrow \frac{z'}{-3} = y^{-4}y'$$

$$\frac{z'}{-3} - \frac{2}{x}z = \frac{3}{x^2}$$

$$\boxed{z' + \frac{6}{x}z = \frac{-9}{x^2}}$$

(5)

$$z' + \frac{6}{x} z = -\frac{9}{x^2}$$

$$p(x) = \frac{6}{x}$$

$$q(x) = -\frac{9}{x^2}$$

$$\mu(x) = e^{\int \frac{6}{x} dx} = e^{6 \ln|x|} = e^{\ln|x|^6} = x^6$$

$$z' \cdot x^6 + 6x^5 z = -9x^4$$

$$(x^6 \cdot z)' = -9x^4$$

$$x^6 \cdot z = -\frac{9}{5} x^5 + C, C \in \mathbb{R}$$

$$z = -\frac{9}{5} \frac{1}{x} + \frac{C}{x^6}, C \in \mathbb{R}.$$

$$z = -\frac{9}{5} \frac{1}{x} + \frac{C}{x^6}$$

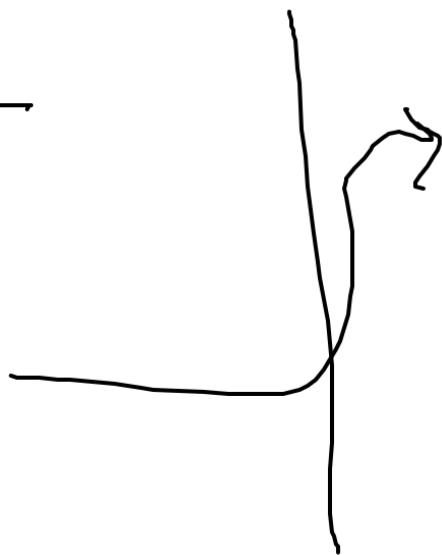
$$\frac{1}{y^3} = -\frac{9}{5} \frac{1}{x} + \frac{C}{x^6} \leftarrow$$

• $y(1) = 1/2 \leftarrow$

x=1

$$\frac{1}{(y(1))^3} = -\frac{9}{5} \frac{1}{1} + \frac{C}{1}$$

$$\frac{1}{(1/2)^3} = -\frac{9}{5} + C$$



$$z = y^{-3}$$

$$z = \frac{1}{y^3}$$

$$y = y(x)$$

$$8 + \frac{9}{5} = C$$

$$\frac{49}{5} = C$$

TPC: Ex 12
- FS.

Solução do P.V I

$$\frac{1}{y^3} = -\frac{9}{5} \frac{1}{x} + \frac{49}{5x^6}$$

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SL 28

Exemplo : $y' - 2y = \underbrace{e^{5x}}_{b(x)}$

TPC:
(fator integrante)

Equação homogênea associada é .

$$\boxed{y' - 2y = 0} \quad (**)$$

Solução geral de (**)

$$\frac{dy}{dx} - 2y = 0$$

$$\frac{dy}{dx} = 2y$$

$$\frac{1}{2y} dy = dx$$

$$\rightarrow \frac{1}{2} \ln|u| = x + C_1, C_1 \in \mathbb{R}$$

$$\ln|y| = 2x + C_2, C_2 \in \mathbb{R}$$

$$y = e^{2x} \cdot e^{C_2}$$

$$\rightarrow \boxed{y_h = e^{2x} C}, C \in \mathbb{R}.$$

S(28)
Cont

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$$y_p = \frac{1}{3} e^{5x} \text{ é } \underline{\text{solução particular}} \text{ de } y' - 2y = e^{5x}$$

Verificar!

$$y'_p - 2y_p = \frac{5}{3} e^{5x} - \frac{2}{3} e^{5x} = \frac{3}{3} e^{5x} = e^{5x} \checkmark$$

Pelo tmg (S(28))

Solução geral da eq. completa

$$y = \underbrace{\frac{1}{3} e^{5x}}_{y_p} + \underbrace{C e^{2x}}_{y_h}, \quad C \in \mathbb{R}$$

Exer 13
F5

Considere a EDO linear homogênea (de coef. não constantes)

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$$(1-x)y'' + xy' - y = 0 \quad , \quad \underbrace{x \in]1, +\infty[}$$

a) Mostrar que $\{e^x, x\}$ é um S.F.S. da equação

$$w(x, e^x) = \begin{bmatrix} x & e^x \\ 1 & e^x \end{bmatrix}$$

$$\det(w) = xe^x - e^x = \underbrace{e^x}_{\neq 0} \cdot (x-1) \neq 0$$

$\{e^x, x\}$ é um S.F.S. da equação

cont
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b) Solução geral da EDO :

$\{x, e^x\}$

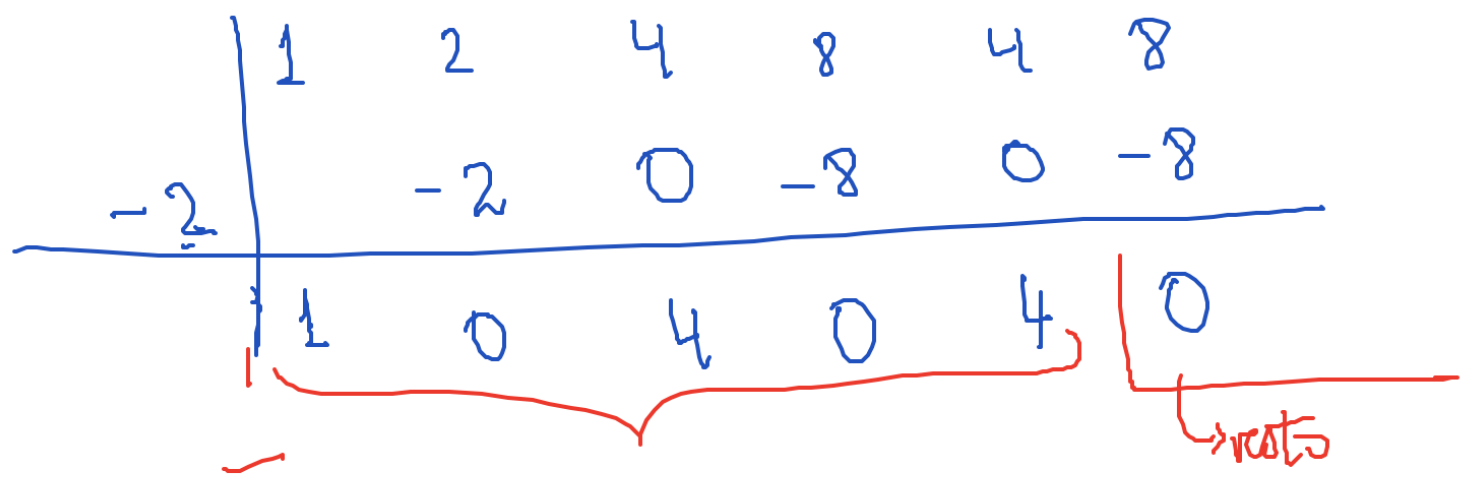
$$y = C_1 x + C_2 e^x, \quad C_1, C_2 \in \mathbb{R}.$$

SL36] Exemplo : $y^{(5)} + 2y^{(4)} + 4y^{(3)} + 8y^{(2)} + 4y' + 8y = 0$

Polinômio característico

$$p(r) = r^5 + 2r^4 + 4r^3 + 8r^2 + 4r + 8$$

$$p(r) = 0$$



$$p(r) = (r+2)(r^4 + 4r^2 + 4)$$

$$r^4 + 4r^2 + 4 = 0$$

$$(r^2 + 2)^2 = 0 \Rightarrow r^2 + 2 = 0$$

$$r^2 = -2$$

$$r = \pm i\sqrt{2}$$

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