Exemplo 1

$$\underbrace{\mathcal{L}^{-1}\left\{\frac{2}{(s-1)(s^2+4)}\right\}} = \mathcal{L}^{-1}\left\{\frac{1}{s-1}\cdot\frac{2}{s^2+4}\right\} = (*)$$

$$\mathcal{L}\{e^t\} = \frac{1}{s-1} \quad ; \quad \mathcal{L}\{\operatorname{sen}(2t)\} = \frac{2}{s^2+4}$$

$$(*) = \mathcal{L}^{-1}\left\{\frac{1}{s-1}\right\} * \mathcal{L}^{-1}\left\{\frac{2}{s^2+4}\right\} = \underbrace{e^t * \operatorname{sen}(2t)} = (*)$$

$\longrightarrow$

②

$$e^t * \operatorname{sen}(2t) = \int_0^t \operatorname{sen}(2\tau) \cdot e^{(t-\tau)}\, d\tau.$$

$$(f*g)(t) = \int_0^t f(\tau)\, g(t-\tau)\, dt$$

$$\int_0^t \operatorname{sen}(2\tau) \cdot e^{(t-\tau)}\, d\tau = -\frac{e^{(t-\tau)}}{2}\cos(2\tau)\Big|_0^t - \frac{1}{2}\int_0^t \cos(2\tau) \cdot e^{(t-\tau)}\, d\tau.$$

$u = e^{t-\tau}$ $\quad du = -e^{t-\tau}$

$dv = \operatorname{sen}(2\tau)$ $\quad v = \dfrac{-\cos(2\tau)}{2}$

$u = e^{t-\tau}$ $\quad du = -e^{t-\tau}$

$dv = \cos(2\tau)$ $\quad v = \frac{1}{2}\operatorname{sen}(2\tau)$

$$= -\frac{e^{(t-\tau)}}{2}\cos(2\tau)\Big|_0^t - \frac{1}{2}\left[e^{(t-\tau)}\frac{\operatorname{sen}(2\tau)}{2}\Big|_0^t + \int_0^t \frac{1}{2}\operatorname{sen}(2\tau)\, e^{t-\tau}\, d\tau\right]$$

$$\frac{5}{4}\int_0^t \sin(2\tau)\, e^{(t-\tau)}\, d\tau = -\frac{\cos(2t)}{2} + \frac{e^t}{2} - \frac{1}{2}\,\frac{\sin(2t)}{2}$$

$$\int_0^t \sin(2\tau)\, e^{(t-\tau)} = \underbrace{-\frac{2}{5}\cos(2t) + \frac{2}{5}e^t - \frac{1}{5}\sin(2t)}$$

$$(*) = -\frac{2}{5}\cos(2t) + \frac{2}{5}e^t - \frac{1}{5}\sin(2t)$$

//

Ex 2] (c) $F(s) = \dfrac{1}{s^2+6s+9} = \dfrac{1}{(s+3)^2}$

$$\mathcal{L}^{-1}\left\{\frac{1}{(s+3)^2}\right\} = e^{-3t}\mathcal{L}^{-1}\left\{\frac{1}{s^2}\right\} = e^{-3t}\,t, \quad t \geq 0$$

$$\mathcal{L}\{e^{\lambda t} f(t)\} = F(s-\lambda) \qquad \Bigg| \qquad \mathcal{L}^{-1}\left\{\frac{1}{s^2}\right\} = t.$$

$$\mathcal{L}\{e^{-3t}\, t\} = F(s-(-3))$$

E2 (e)

$$F(s) = \frac{1}{s^2+4s+6} = \frac{1}{\underbrace{s^2+4s+4}+6-4} = \frac{1}{(s+2)^2+2}$$

$$\mathcal{L}^{-1}\left\{\frac{1}{(s+2)^2+\underbrace{2}_{(\sqrt{2})^2}}\right\} = \frac{1}{\sqrt{2}}\mathcal{L}^{-1}\left\{\frac{\sqrt{2}}{(s+2)^2+2}\right\}$$

$$= \frac{1}{\sqrt{2}} e^{-2t} \sin(\sqrt{2}t)$$

$$\mathcal{L}\{e^{\lambda t} f(t)\} = F(s-\lambda)$$

$$F(s) = \mathcal{L}\{f(t)\}$$

$$\mathcal{L}\{\sin(at)\} = \frac{a}{s^2+a^2}$$

S 20

$$\mathcal{L}^{-1}\left\{\frac{1}{s(s^2+2s+10)}\right\} = \mathcal{L}^{-1}\left\{\frac{1}{10}\frac{1}{s} - \frac{1}{10}\cdot\frac{s+2}{s^2+2s+10}\right\}$$

CA

$$\frac{1}{s(s^2+2s+10)} = \frac{A}{s} + \frac{\overset{-\frac{1}{10}}{B}s + \overset{-\frac{1}{5}}{C}}{s^2+2s+10} = \frac{\overbrace{(A+B)}^{=0}s^2 + \overbrace{(2A+C)}^{=0}s + \overbrace{10A}^{=1}}{s(s^2+2s+10)}$$

$$10A = 1 \qquad A+B=0 \qquad 2A+C=0$$

$$\boxed{A = 1/10} \qquad B = -A \qquad C = -2A$$

$$\boxed{B = -\frac{1}{10}} \qquad \boxed{C = -\frac{1}{5}}$$

Exemplo: 8(b) $\frac{d^2y}{dt^2} + 36y = 0$, $y(0) = -1$, $\frac{dy}{dt}(0) = 2$

$$\boxed{\mathcal{L}\{y(t)\} = Y(s)}$$

$$\underbrace{y'' + 36y = 0}, \quad y(0) = -1, \quad y'(0) = 2.$$

$$\mathcal{L}\{y'' + 36y\} = 0$$

$$\mathcal{L}\{y''\} + 36\mathcal{L}\{y\} = 0$$

$$s^2Y(s) - sy(0) - y'(0) + 36Y(s) = 0$$

$$s^2Y(s) + s - 2 + 36Y(s) = 0$$

$\longrightarrow$

(8)

$$(s^2+36)\,Y(s) = -s+2$$

$$\mathcal{L}\{\cos at\} = \frac{s}{s^2+a^2}$$

$$Y(s) = \frac{-s+2}{s^2+36}$$

$$\mathcal{L}\{\sin(at)\} = \frac{a}{s^2+a^2}$$

$$\mathcal{L}^{-1}\{Y(s)\} = \mathcal{L}^{-1}\left\{\frac{-s+2}{s^2+36}\right\}$$

$$y(t) = -\mathcal{L}^{-1}\left\{\frac{s}{s^2+36}\right\} + \frac{1}{3}\mathcal{L}^{-1}\left\{\frac{2\cdot 3}{s^2+36}\right\}$$

$$\boxed{y(t) = -\cos(6t) + \frac{1}{3}\sin(6t)}$$ //

8 e) $y'' + y' = \frac{e^{-t}}{2}$, $\underline{y(0) = 0}$, $\underline{y'(0) = 0}$

$$\mathcal{L}\{y(t)\} = Y(s)$$

$$\mathcal{L}\{e^{at}\} = \frac{1}{s-a}$$

$$\mathcal{L}\{y'' + y'\} = \mathcal{L}\left\{\frac{e^{-t}}{2}\right\}$$

$$\mathcal{L}\{y''\} + \mathcal{L}\{y'\} = \frac{1}{2}\mathcal{L}\{e^{-1t}\}$$

$$s^2 Y(s) - s\cancel{y(0)}^{0} - \cancel{y'(0)}^{0} + sY(s) - \cancel{y(0)}^{0} = \frac{1}{2}\frac{1}{s+1}$$

$$(s^2 + s)Y(s) = \frac{1}{2}\frac{1}{s+1}$$

$$Y(s) = \frac{1}{2}\frac{1}{s+1}\frac{1}{s^2+s}$$