Definisi Integral Tentu (Limit Jumlah Riemann)

$$\int_a^b f(x)\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(\bar{x}_i)\Delta x = \text{Luas di atas sb.x} - \text{Luas di bawah sb.x}$$

$\Delta x = \frac{b-a}{n}$, $x_i = a + i\Delta x$

Riemann Kiri : $\bar{x}_i = x_{i-1}$

Riemann Kanan: $\bar{x}_i = x_i$

TDK 1

$$\frac{d}{dx}\int_a^x f(t)\,dt = f(x)$$

TDK 2

$$\int f(x)\,dx = F(x) \Rightarrow \int_a^b f(x)\,dx = F(x)\Big|_a^b = F(b) - F(a)$$

Contoh: $\int_1^4 x^2\,dx = \frac{x^3}{3}\Big|_1^4 = \frac{4^3 - 1^3}{3} = 21$

$$\int_{x=0}^{x=9} \frac{\sqrt{1+\sqrt{x}}}{\sqrt{x}}\,dx = \int_0^9 \sqrt{1+\sqrt{x}}\left(\frac{1}{\sqrt{x}}\,dx\right) = \int_{u=1}^{u=4} \sqrt{u}\;2\,du = 2\cdot\frac{u^{3/2}}{3/2}\Big|_1^4 = \frac{4}{3}\left(4^{3/2} - 1^{3/2}\right) = \frac{28}{3}$$

$u(x) = 1+\sqrt{x} \Rightarrow du = \frac{1}{2\sqrt{x}}\,dx$

$u(0) = 1$, $u(9) = 4$

Masalah: $\int_0^2 \frac{1}{1+x^2}\,dx$

SULIT DICARI ANTITURUNANNYA.

Kita cuma bisa cari HAMPIRANNYA

$$\int_0^2 \frac{x}{1+x^2}\,dx = \int_1^5 \frac{1}{u}\,\frac{du}{2}$$

$u(x) = 1+x^2 \Rightarrow du = 2x\,dx$

$u(0) = 1$, $u(2) = 5$

Solusi: Integral Numerik.

Integral Numerik

1. Jumlah Riemann Kiri (atau Kanan atau Titik Tengah)

   Mengestimasi luas dengan persegi panjang

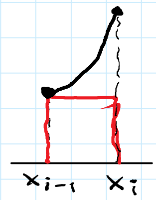
2. Aturan Trapesium

   Mengestimasi luas dengan trapesium

3. Aturan Simpson

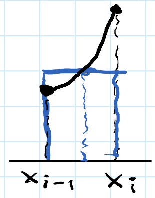
   Mengestimasi luas dengan daerah yang dibatasi parabola

$$L_n = \sum_{i=1}^{n} f(x_{i-1})\Delta x = \Delta x[f(x_0) + f(x_1) + \cdots + f(x_{n-1})]$$

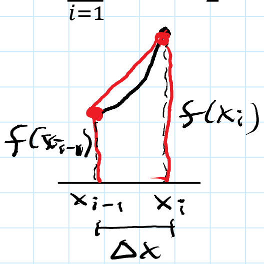


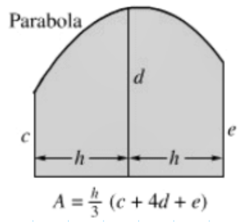
$$R_n = \sum_{i=1}^{n} f(x_i)\Delta x = \Delta x[f(x_1) + f(x_2) + \cdots + f(x_n)]$$

$$M_n = \sum_{i=1}^{n} f\left(\frac{x_{i-1} + x_i}{2}\right)\Delta x$$



$$T_n = \sum_{i=1}^{n} \frac{f(x_{i-1}) + f(x_i)}{2}\Delta x = \frac{\Delta x}{2}[f(x_0) + 2f(x_1) + 2f(x_2) + \cdots + 2f(x_{n-1}) + f(x_n)]$$





$n = 2k, k$ bulat positif

$$P_n = \sum_{i=1}^{k} \frac{\Delta x}{3}[f(x_{2i-2}) + 4f(x_{2i-1}) + f(x_{2i})]$$

$$= \frac{\Delta x}{3}[f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + 2f(x_4) + \cdots + 2f(x_{n-2}) + 4f(x_{n-1}) + f(x_n)]$$

1. Aproksimasi $\int_1^3 \frac{1}{1+x^2} dx$ dengan menggunakan jumlah Riemann kiri, aturan trapesium, dan Simpson dengan $n = 4$. Kemudian tentukan galat mutlak maksimum.

$a=1,\ b=3,\ f(x)=\frac{1}{1+x^2},\ n=4$

$\Delta x = \frac{b-a}{n} = \frac{3-1}{4} = \frac{1}{2}$

$x_i = a + i\Delta x = 1 + i\frac{1}{2} = 1 + \frac{i}{2}$

$L_4 = \Delta x\,[f(x_0)+f(x_1)+f(x_2)+f(x_3)]$
$= \frac{1}{2}[f(1)+f(1.5)+f(2)+f(2.5)] \approx 0.5728$

$R_4 = \frac{1}{2}[f(1.5)+f(2)+f(2.5)+f(3)] \approx 0.3728$

$M_4 = \frac{1}{2}[f(1.25)+f(1.75)+f(2.25)+f(2.75)] \approx 0.4591$

$T_4 = \frac{\Delta x}{2}[f(x_0)+2f(x_1)+2f(x_2)+2f(x_3)+f(x_4)] \approx 0.4728$

$P_4 = \frac{\Delta x}{3}[f(x_0)+4f(x_1)+2f(x_2)+4f(x_3)+f(x_4)] \approx 0.4637$

Nilai sesungguhnya

$$\int_1^3 \frac{1}{1+x^2}\,dx = \tan^{-1}(3) - \frac{\pi}{4} \approx 0.46365$$

2. Tentukan n sehingga aturan trapesium akan mengaproksimasi $\int_1^3 \frac{1}{x}\,dx$ dengan galat E_n yang memenuhi $|E_n| \le 0.01$.

$$E_n = -\frac{(b-a)^3}{12n^2} f''(c), \text{for } a \le c \le b$$

$$|E_n| \le \left|\frac{(b-a)^3}{12n^2}\right| \cdot \max_{[a,b]}\{|f''(x)|\}$$

$f''(x)$ punya nilai max pada $[a, b]$

$$|f''(c)| \le \max_{[a,b]}\{|f''(x)|\}$$

$f(x) = |x|, [-5, 1]$

max $= 5$, min

$a = 1,\ b = 3,\ f(x) = \frac{1}{x}$

$f'(x) = -\frac{1}{x^2}$

$f''(x) = \frac{2}{x^3},\ [1,3]$

$$\max_{[1,3]}\left\{\left|\frac{2}{x^3}\right|\right\} = 2$$

$$|E_n| \le \frac{8}{12n^2} \cdot 2 = \frac{4}{3n^2} \le 0.01$$

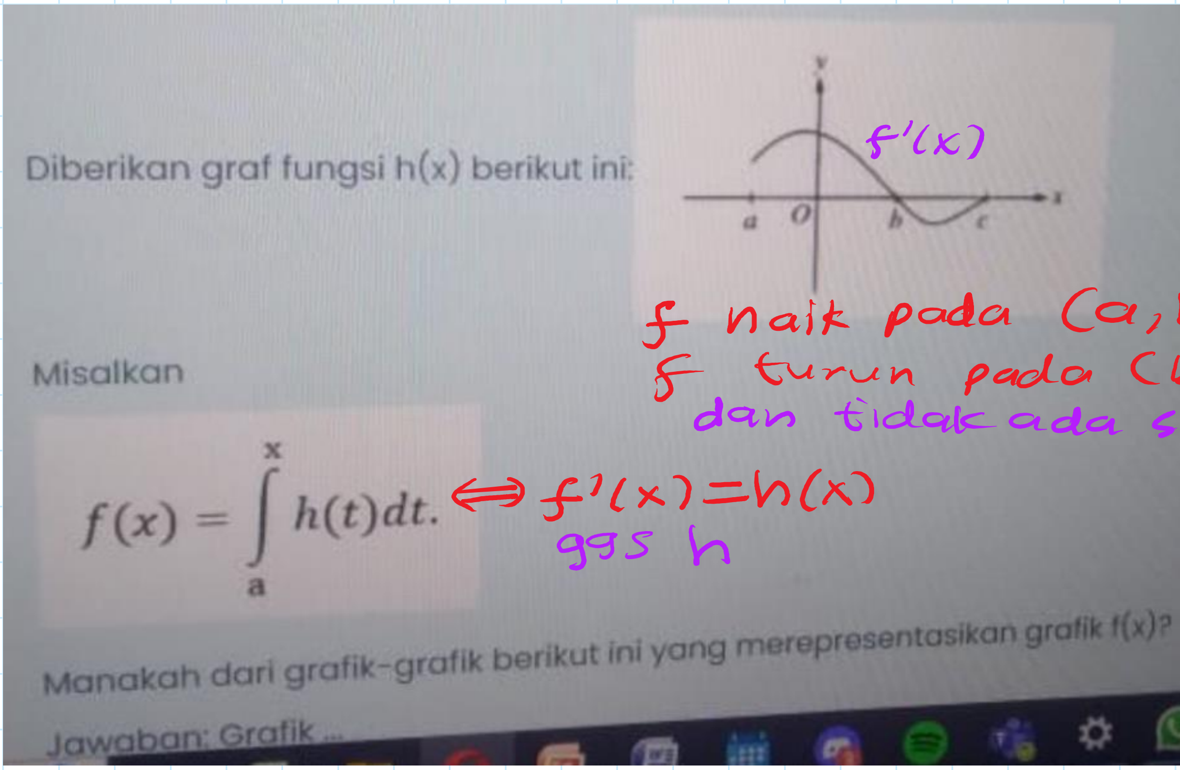
$$\frac{3n^2}{4} \ge 100$$

$$n^2 \ge \frac{400}{3}$$

$$n \ge \frac{20}{\sqrt{3}}$$

$$n \ge \left\lceil \frac{20}{\sqrt{3}} \right\rceil = 12$$

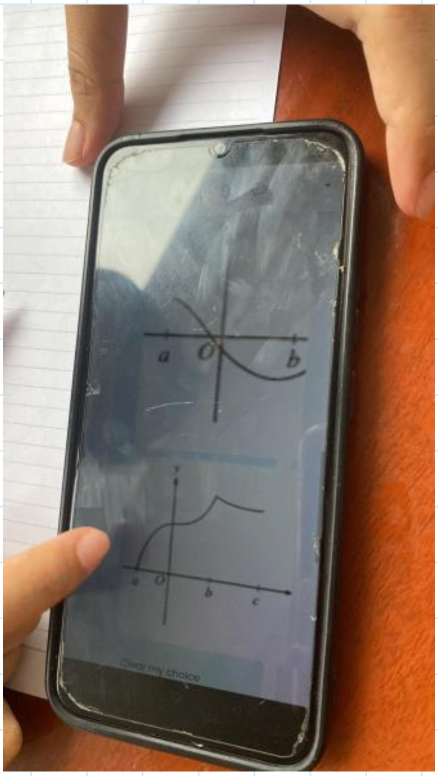
Jadi, agar $|E_n| \le 0.01$, pilih $n = 12$.



$f'(x)$

f naik pada (a,b)
f turun pada (b,c)
dan tidak ada singular

$\Longleftrightarrow f'(x)=h(x)$
ggs h



$f'(b)=0$

$f'(b)$ tidak ada

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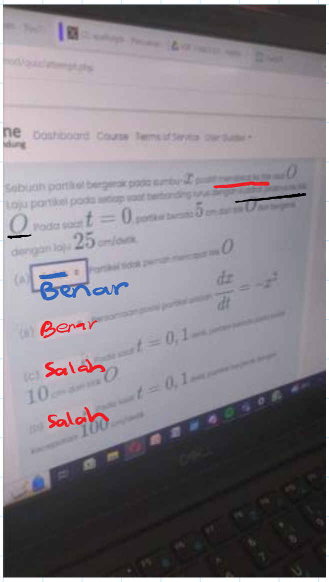

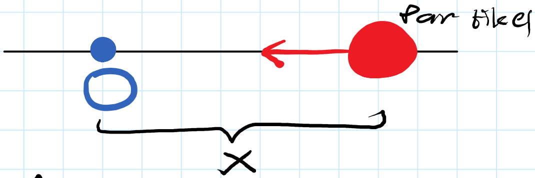

$y = 5x$

$x = 1,\ y = 5$

$x = 2,\ y = 10$

$\frac{5}{1} = \frac{10}{2} = k$

Misal $x(t)$ jarak partikel ke O

$v(t) = x'(t)$ laju partikel

Laju partikel $\sim x^2$

$$v(t) = \frac{dx}{dt} = kx^2$$

$$x(0) = 5,\ v(0) = -25$$

$\bullet)$ $v(0) = k \cdot x(0)^2$

$$-25 = k \cdot 25$$

$$k = -1$$

$\therefore)$ $v(t) = \frac{dx}{dt} = -x^2$

$\bullet)$ $\frac{dx}{x^2} = -dt$

$$\frac{x^{-1}}{-1} = -t + C$$

$$\frac{1}{x} = t + C$$

Karena $t = 0,\ x = 5$

$$\frac{1}{5} = C$$

$$\frac{1}{x} = t + \frac{1}{5} = \frac{5t+1}{5}$$

$$x(t) = \frac{5}{5t+1},\ t \geq 0$$

Karena $t \geq 0$, maka $x(t) > 0$

shg partikel tidak pernah mencapai O

$\bullet)$ $x(0.1) = \frac{5}{0.5+1} = \frac{5}{1.5} = \frac{10}{3}$

$\bullet)$ $v(0.1) = -\left(\frac{10}{3}\right)^2 = -\frac{100}{9}$