

$$4t - 2t + 4$$

$$4t - 2s$$

c) kecepatan rata-rata:

$$\bar{r}(1) = 4(1) - 2(1) + 4$$

$$= 2 + 4 = 6 \hat{i}$$

$$\bar{r}(2) = 4(2) - 2(2) + 4$$

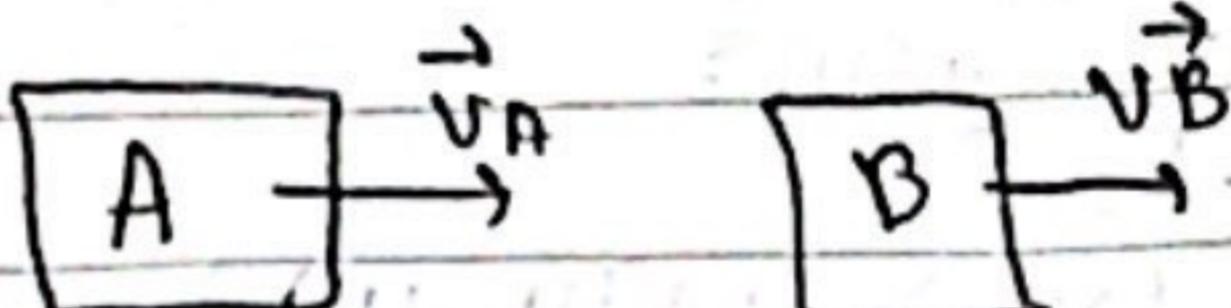
$$= 8 - 4 + 4 = 8 \hat{i}$$

$$\frac{\bar{r}(2) - \bar{r}(1)}{2-1}$$

$$\frac{d\bar{x}_{BA}}{dt} = \frac{d\bar{x}_B}{dt} - \frac{d\bar{x}_A}{dt}$$

$$\bar{v}_{BA} = \bar{v}_B - \bar{v}_A$$

KOC. RELATIF



d) kapan kecepatan benda \perp

terhadap percepatan?

$$\rightarrow \vec{v} \cdot \vec{a} = 0$$

$$v \cdot \frac{d\bar{r}}{dt} = (4t)\hat{i} + (4t - 2t + 4)\hat{j}$$

$$a: \frac{dv}{dt} = 4\hat{i} + (-2)\hat{j}$$

$$\vec{v} \cdot \vec{a} = 0$$

$$(4t)\hat{i} + (4t - 2t + 4)\hat{j} \cdot (4\hat{i} + (-2)\hat{j})$$

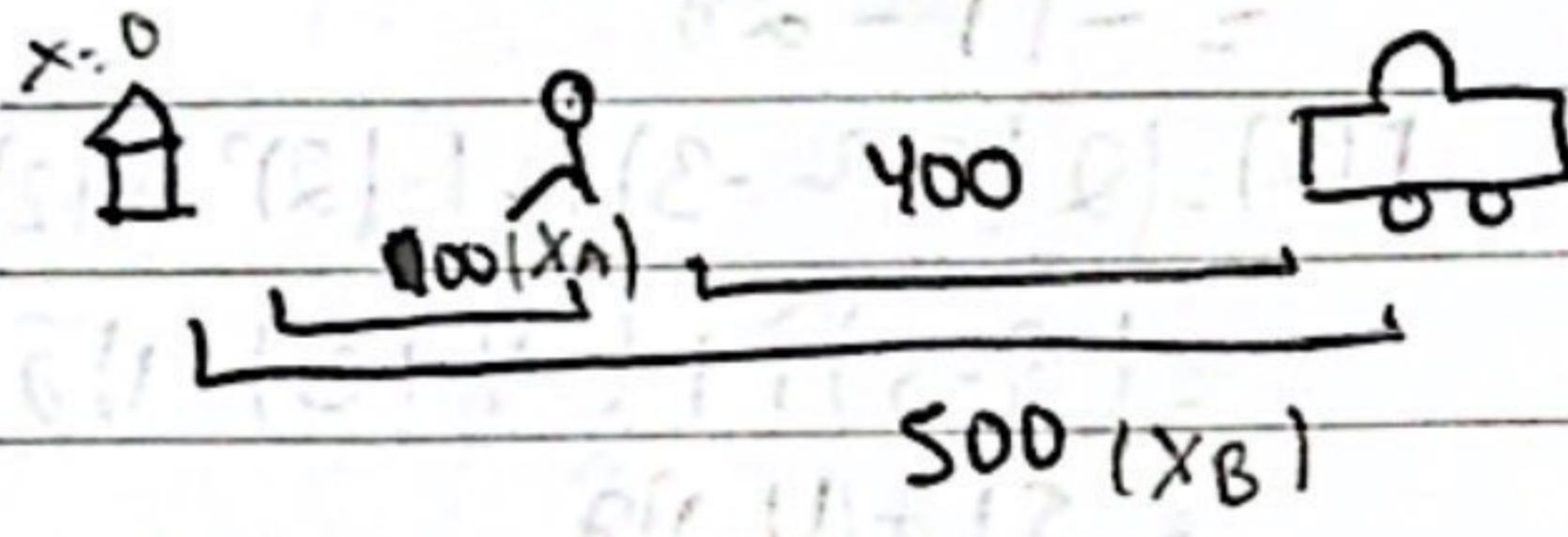
$$16t^2 + 4t - 8 = 0$$

$$= 20t - 8 = 0$$

$$t = \frac{8}{20} = 0,4$$

$$= 0,4 \text{ s}$$

① POSISI RELATIF



$$x_{BA} = x_B - x_A$$

kecepatan B thdp A:

$$\bar{v}_{BA} = \bar{v}_B - \bar{v}_A$$

pengambilan

$$\text{diket } B \text{ ke kiri} = \bar{v}_B = -\bar{v}_B$$

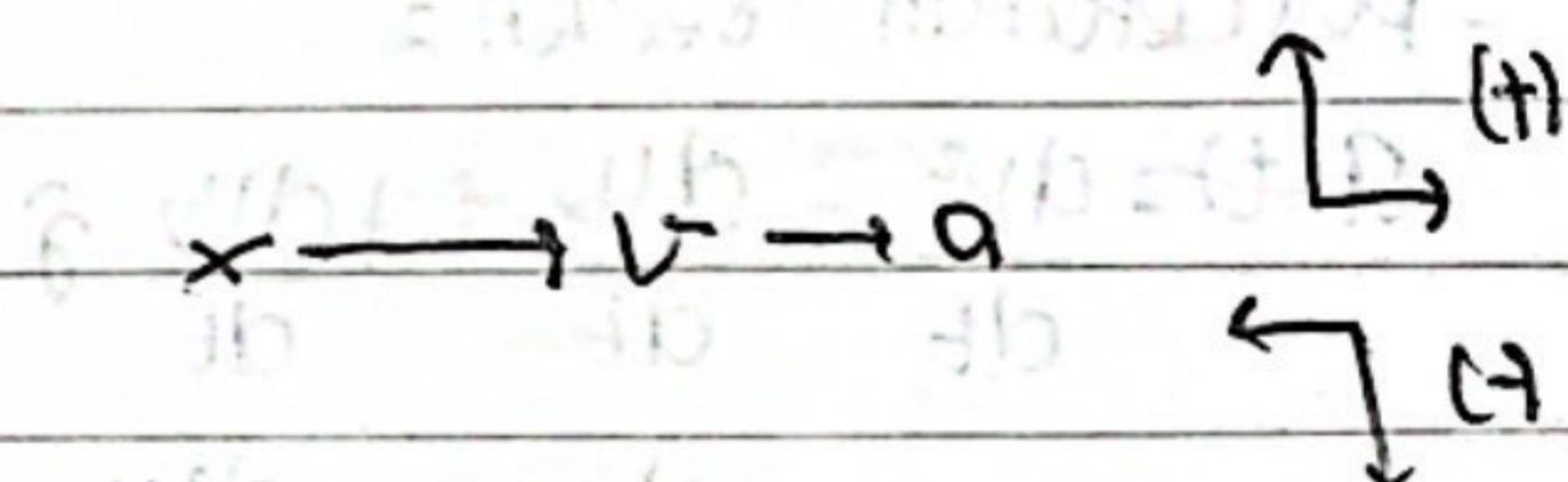
$$\begin{aligned} \bar{v}_{BA} &= -\bar{v}_B - \bar{v}_A \\ &= -(\bar{v}_B + \bar{v}_A) \end{aligned}$$

$$0 \cdot 20 = -20$$

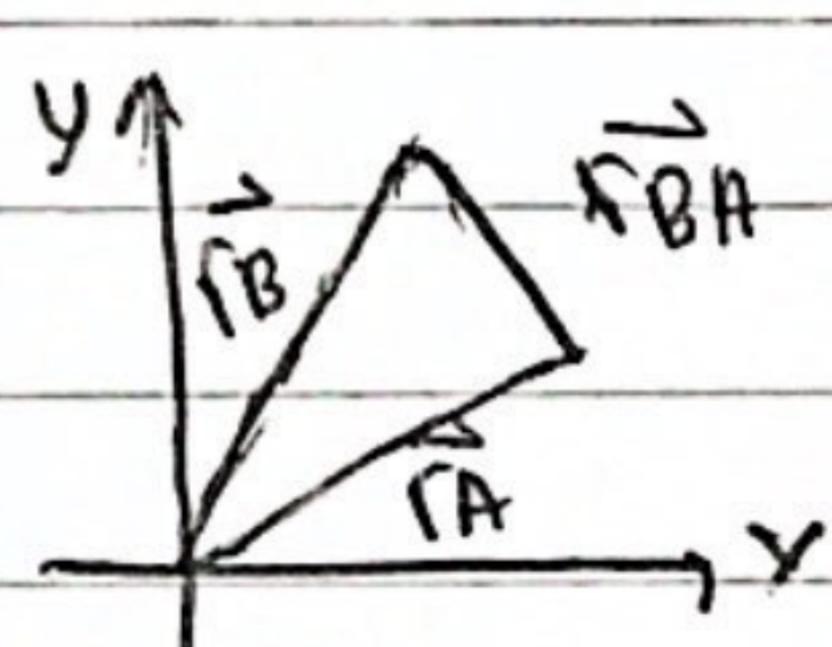
③ PERCEPATAN RELATIF

$$\frac{d^2x_{BA}}{dt^2} = \frac{d^2x_B}{dt^2} - \frac{d^2x_A}{dt^2}$$

$$a_{BA} = a_B - a_A$$



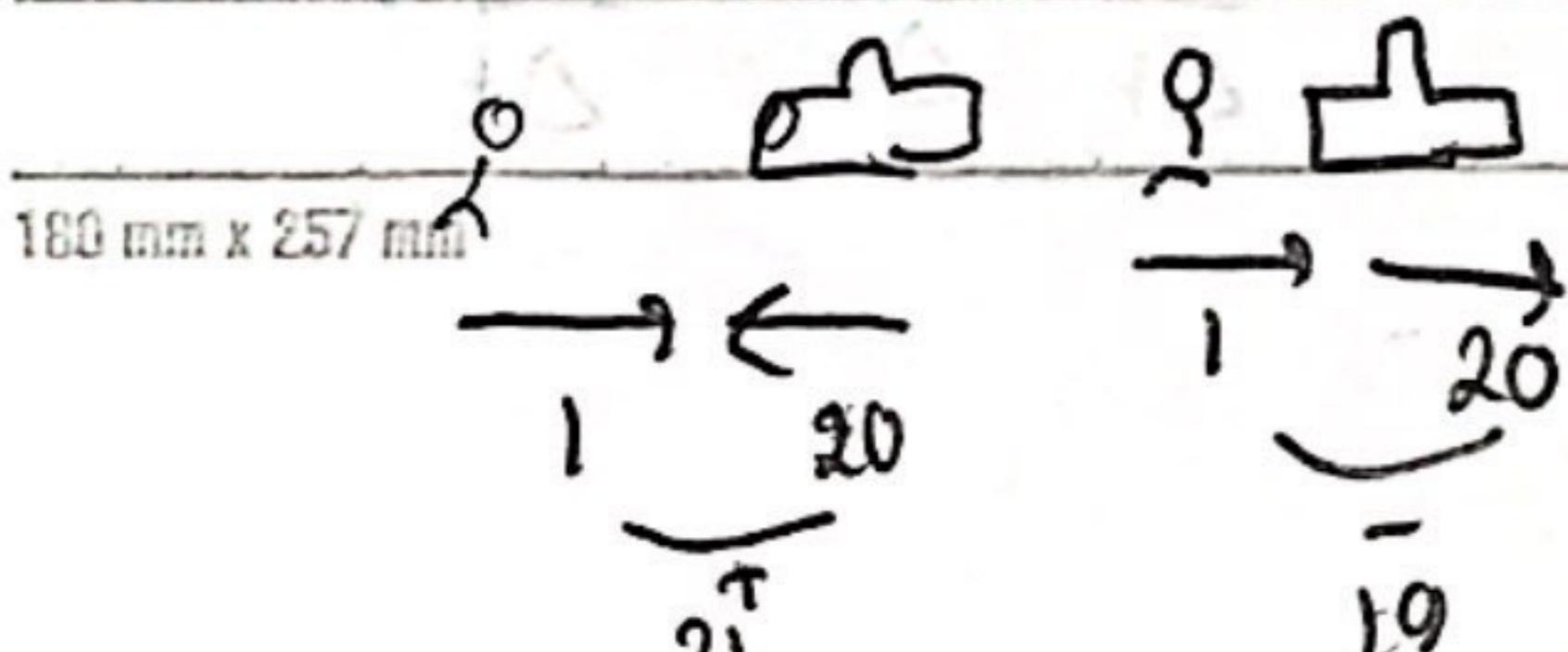
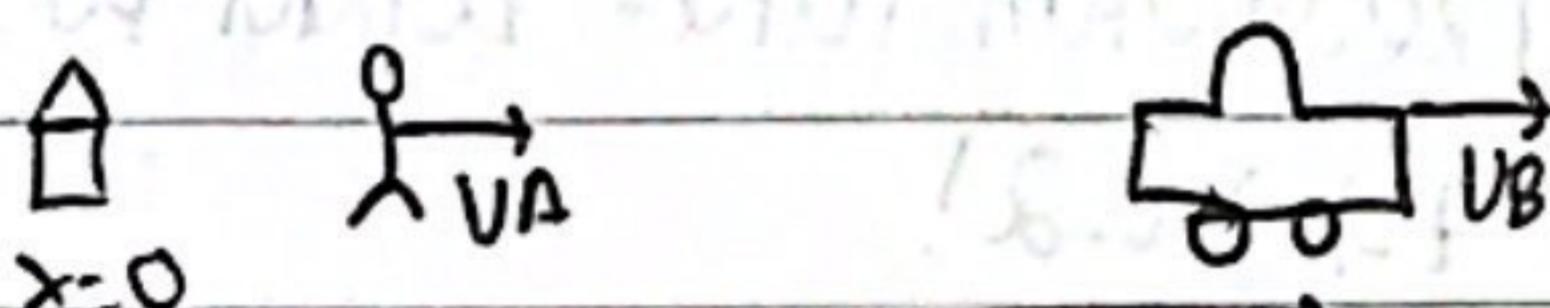
Secara umum:

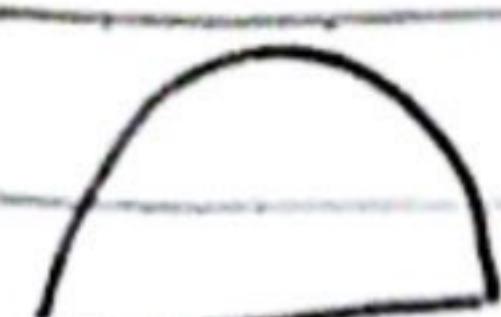


$$\frac{d^2r_B}{dt^2} - \frac{d^2r_A}{dt^2}$$

$$\vec{a}_B - \vec{a}_A$$

② KECEPATAN RELATIF





Posisi setiap saat: Percepatan (a)

$$\vec{F}_A = V_0 \times t \hat{i}$$

$$a_x = 0 \quad a_y = -g$$

$$\vec{r}_B = V_0 x t \hat{i} + \left(V_0 y t - \frac{1}{2} g t^2\right) \hat{j}$$

Posisi relatif:

$$\vec{r}_{BA} = \vec{r}_B - \vec{r}_A$$

$$= \left(V_0 y t - \frac{1}{2} g t^2\right) \hat{j}$$

Kecerapan (v)

$$V_x(t) = V_0 x$$

$$V_y(t) = V_0 y t - g t$$

$$V_y(t) = V_0 y t - g t$$

*Kecerapan relatif =

$$V_{BA} = \frac{d^2 \vec{r}_{BA}}{dt^2} = [V_0 y - g t] \hat{j}$$

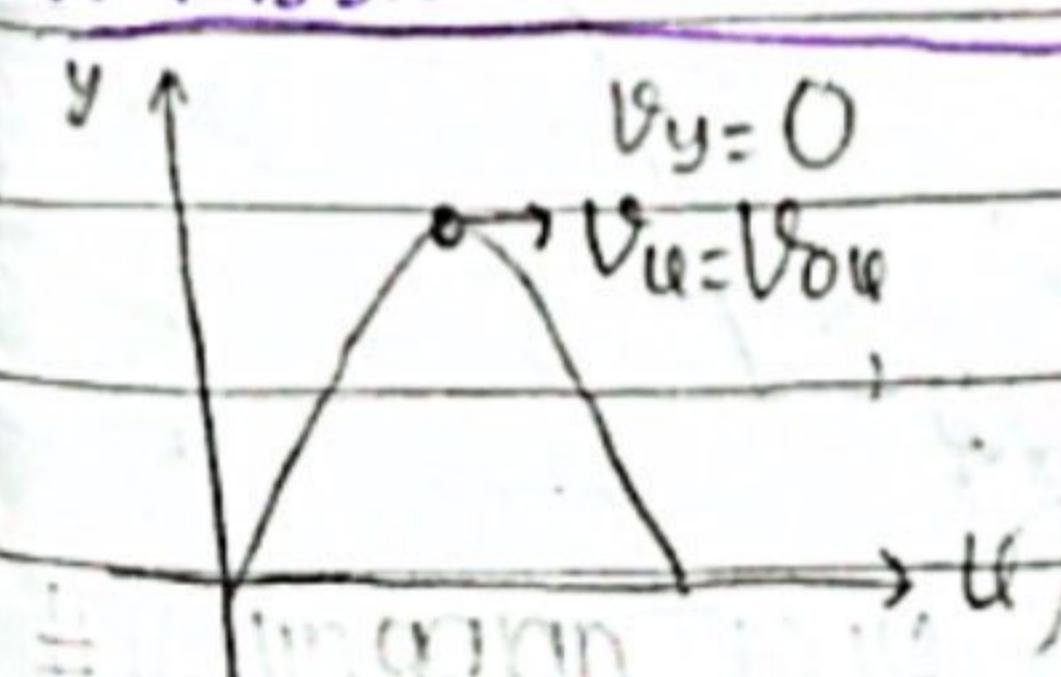
Posisi (x,y)

$$x(t) = V_0 x t \quad y(t) = y_0 + V_0 y t - \frac{1}{2} g t^2$$

*Percepatan relatif =

$$a_{BA} = \frac{d^2 \vec{r}_{BA}}{dt^2} = -g \hat{j}$$

Ketinggian maksimum



Grafik ketinggian

di bawahnya

Arah percepatan & kecerapan gerak parabola

Kapan benda mencapai ketinggian maksimum

$$V_y = 0$$

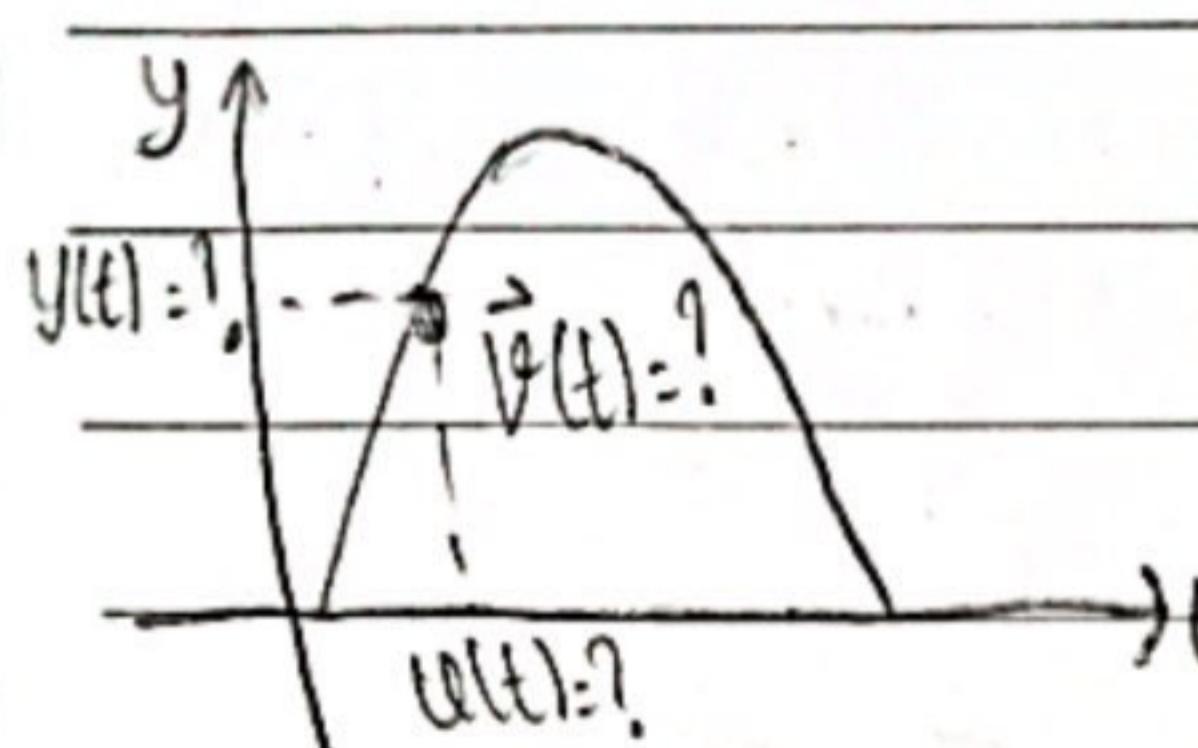
$$V_0 y - g t = 0$$

$$V_0 y = g t$$

Benda mencapai titik tertinggi =

$$t_{max} = \frac{V_0 y}{g}$$

Pada gerak parabola, percepatan hanya ada pada arah vertikal.



Berapa ketinggian max +de ketinggian awal?

Substitusi nilai tmax :

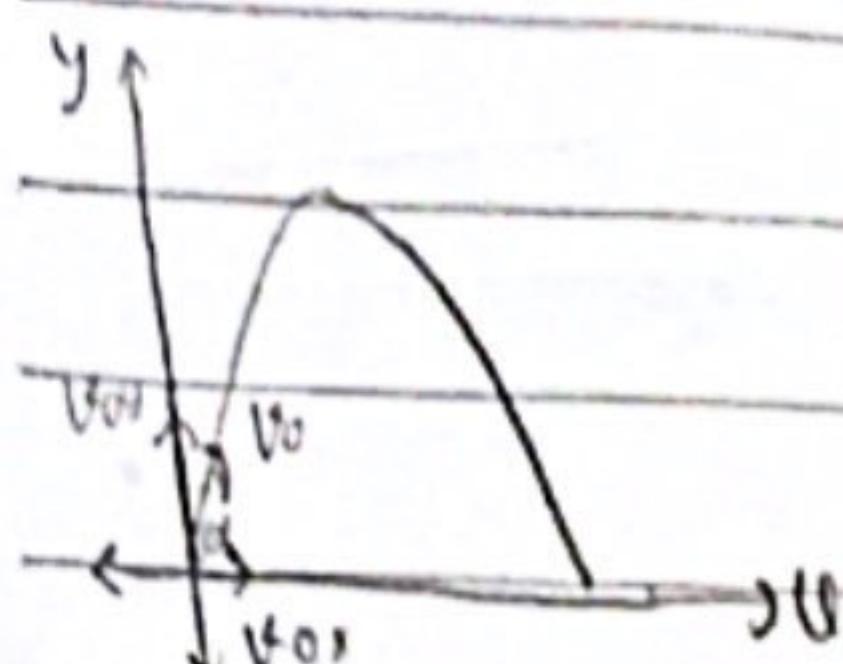
$$y(t) = y_0 + V_0 y t - \frac{1}{2} g t^2$$

Ingin persamaan gerak:

$$V_2 = V_1 + a t$$

$$x_2 = x_1 + V_1 t + \frac{1}{2} a t^2$$

Persamaan gerak parabola



- PERCEPATAN

Anitayi

$$\vec{a}(t) = -g\hat{y}$$

- KECEPATAN

$$V(t) = V_0\hat{x} + V_0y\hat{y}$$

$$\vec{V}(t) = V_0\hat{x} + (V_0y - gt)\hat{y}$$

- POSISI

$$r(t) = V_0t + y_0\hat{y}$$

$$\vec{r}(t) = (V_0t + V_0y t)\hat{x} +$$

$$(y_0 + V_0y - \frac{1}{2}gt^2)\hat{y}$$

Ambil kasus -

$$V_0 = 0, y_0 = 0$$

$$V_0 = V_0 y t \Leftrightarrow t = \frac{V_0}{V_0 y}$$

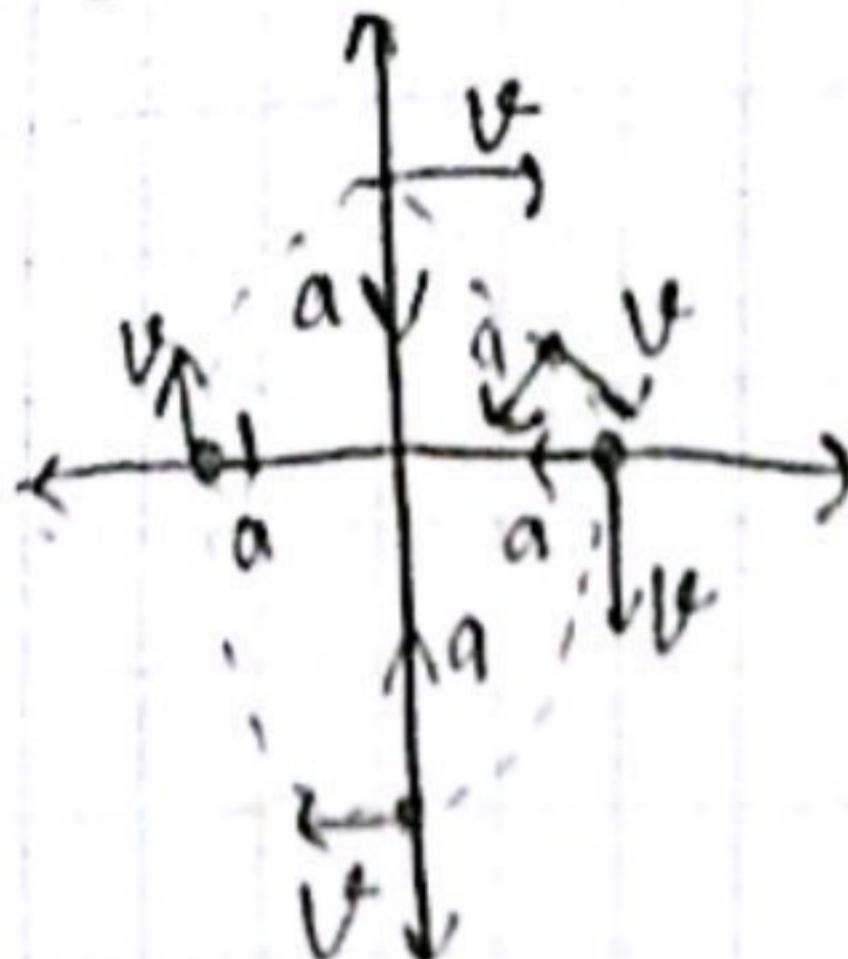
$$y = V_0 y t - \frac{1}{2}gt^2$$

$$y = V_0 y \left(\frac{V_0}{V_0 y} \right) - \frac{1}{2}g \left(\frac{V_0}{V_0 y} \right)^2$$

$$\boxed{y = V_0 y \left(1 - \frac{g}{2V_0^2} V_0^2 \right)}$$

Gorak Nelingkar !

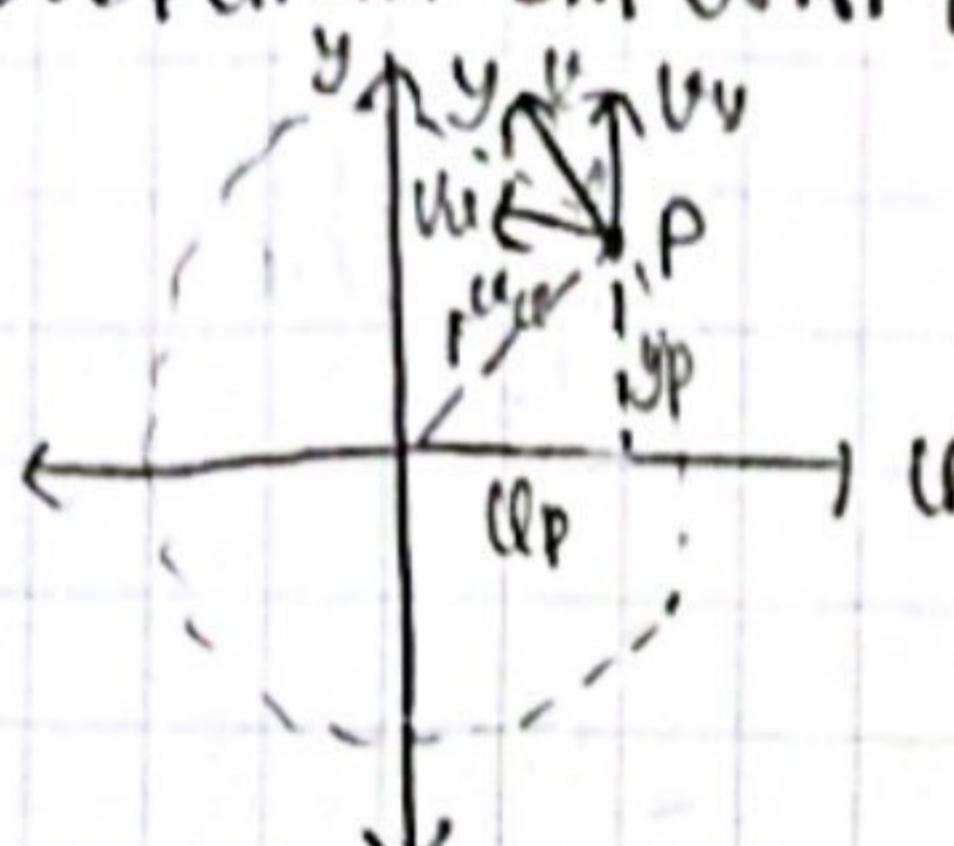
a beraturan



- $\langle v \rangle$ konstan, arahnya berubah

- Percepatan (a) konstan, menuju lintasan \langle per. sentripeti

b. Percepatan diposisi tertentu



Tinjau percepatan di titik P =

$$\begin{array}{l} \text{dengan } \theta \\ \text{dapat } v = v_u i + v_y j \\ \vec{v} = (-v \sin \theta) i + \end{array}$$

$$\begin{array}{l} \text{dengan } \\ \sin \theta = \frac{v_y}{v} \\ \cos \theta = \frac{v_x}{v} \end{array}$$

$$\sin \theta = \frac{v_y}{v}$$

$$\cos \theta = \frac{v_x}{v}$$

$$\sin \theta = \frac{y_p}{r}$$

$$\cos \theta = \frac{x_p}{r}$$

$$\vec{v} = (-v \sin \theta) i + (v \cos \theta) j$$

$$= (-v \left(\frac{y_p}{r} \right)) i + (v \left(\frac{x_p}{r} \right)) j$$

c. Percepatan sentripetal (1)

$$\begin{array}{l} \text{hubungan :} \\ \vec{v} = v_u i + v_y j \\ a_s = \frac{d\vec{v}}{dt} \end{array}$$

dengan: $\vec{v} = (-v \frac{y_p}{r}) i + (v \frac{x_p}{r}) j$

sehingga:

$$\vec{a} = -\frac{v}{r} v_y i + \frac{v}{r} v_x j$$

atau

$$\vec{a} = -\frac{v}{r} (v \cos \theta) i + \frac{v}{r} (-v \sin \theta) j$$

STOP yg di tulip :

$$\frac{d}{dt} \left(-v \frac{y_p}{r} i \right) + \frac{d}{dt} \left(v \frac{x_p}{r} j \right)$$

$$\vec{a} = \frac{v}{r} \frac{dy_p}{dt} i + \frac{v}{r} \frac{dx_p}{dt} j$$

$$\frac{dy_p}{dt} = v_y \quad \frac{dx_p}{dt} = v_u$$

sehingga:

$$\vec{a} = -\frac{v}{r} v_y i + \frac{v}{r} v_u j$$

$$= -\frac{v}{r} (v \cos \theta) i + \frac{v}{r} (-v \sin \theta) j$$

$$= -\frac{v^2}{r} \cos \theta i - \frac{v^2}{r} \sin \theta j$$

(ax) (ay)

$$\Leftrightarrow a = \sqrt{a_x^2 + a_y^2}$$

$$= \sqrt{\left(-\frac{v^2}{r} \cos \theta\right)^2 + \left(\frac{v^2}{r} \sin \theta\right)^2}$$

$$a_{sd} = \frac{v^2}{r}$$

$$a_s = w \cdot r$$

GLB/GLB
4. V konstan \wedge a konstan

① soal up

$x = x_0 + v_0 t$

$y = v_0 y + \frac{1}{2} g L t^2$

$v_y = v_0 y - g L$

$v_y^2 = v_0 y - 2 g L y$

GLB

$$a = \frac{dv}{dt}$$

$$v = \frac{dr}{dt}$$

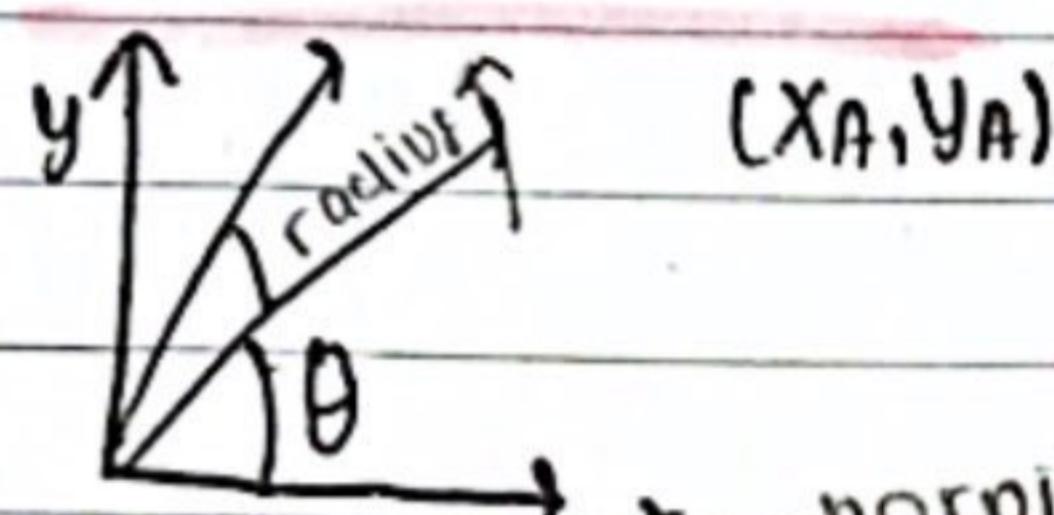
perubahan kecepatan

kecepatan

Gerakan melingkar
4 lintasan melingkar

< posisi tidak >

* posisi melingkar \rightarrow sudut



$$\text{busur} = R \cdot \Delta\theta$$

busur \rightarrow lintasan melingkar

$\Delta\theta$: radian (perpindahan melingkar)

contoh

$$\text{busur} = R \cdot \Delta\theta$$

$$= 1m \cdot \frac{1}{6}\pi \text{ rad}$$

$$x_{\text{rad}} = \frac{1}{6}\pi$$

$$= \frac{\pi}{6} \text{ meter}$$

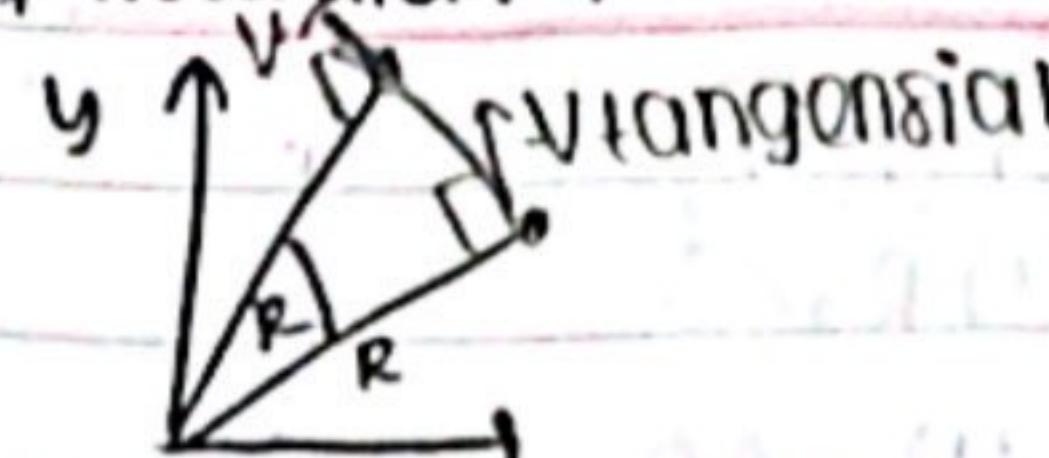
$$30^\circ : \frac{30}{180}\pi$$

$$= \frac{1}{6}\pi$$

$$= \frac{1}{6}\pi \text{ rad}$$

$$= \frac{1}{6}\pi \text{ rad}$$

* kocokan melingkar



* # Melingkar \rightarrow tegak lurus jari-jari

Kec. sudut :

$$W = \frac{v_{\text{tangensial}}}{R}$$

$$\delta\theta = M/R$$

$$W = \text{rad/s}$$

w/ seberapa cepat benda

RPM \rightarrow Rotasi per menit

(satuan lain w)

$$1 \text{ rad} = \frac{\pi}{30} \text{ rad/s}$$

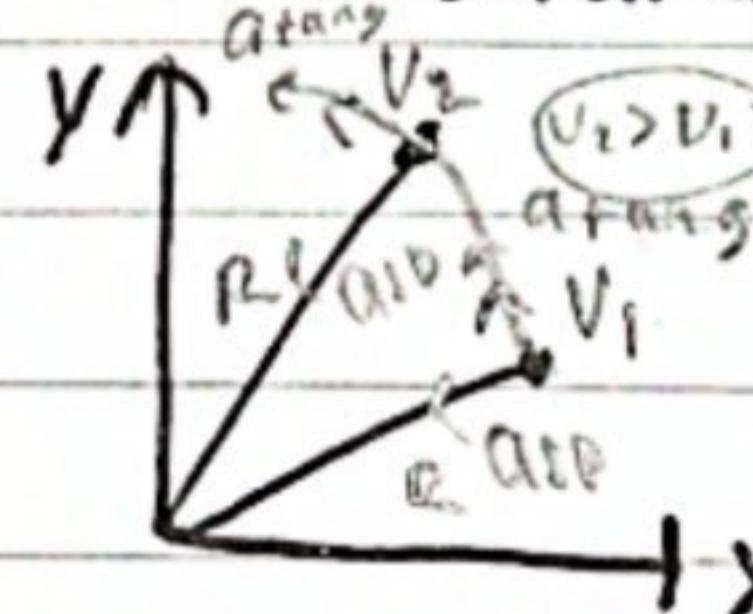
$$1 \text{ rotasi} = \frac{2\pi}{60} \text{ rad}$$

$$* W = \frac{v_{\text{tangensial}}}{R} = \frac{\delta\theta}{\Delta t}$$

* Percepatan melingkar

4 percepatan tangensial (m/s^2)

4 kecepatan tangensial (m/s)



- PERCEPATAN: perubahan kecepatan (waktu)

- PERUBAHAN tangensial merubah ω + tangensial.

Percepatan sudut

$$\alpha = \frac{a_{\text{tangensial}}}{R}$$

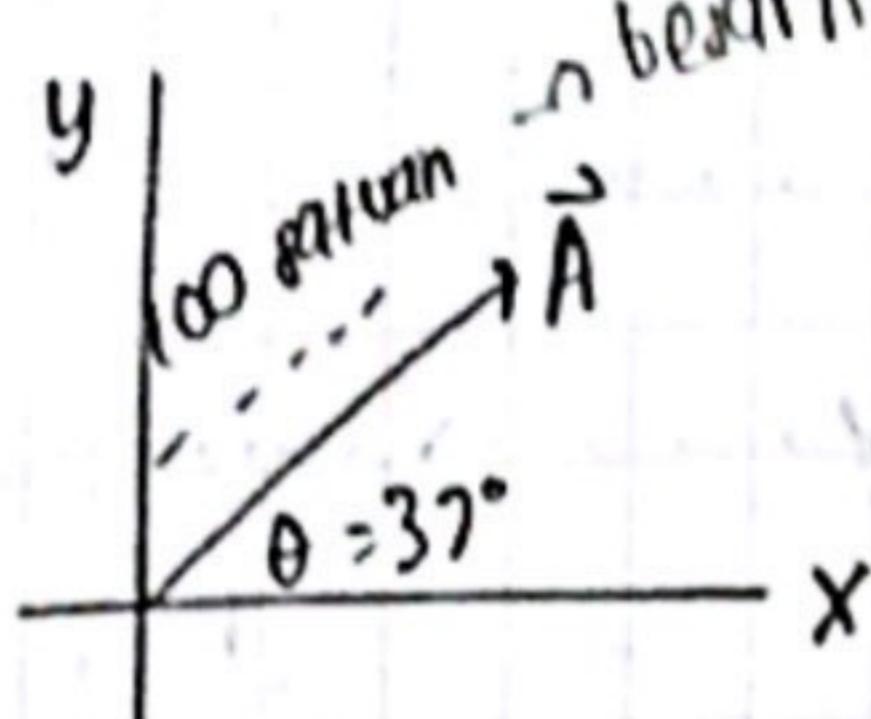
$$\alpha = \frac{\Delta\omega}{\Delta t}$$

$$\alpha_{SP} = \frac{v_{\text{tangensial}}^2}{R} \text{ atau } \omega^2 R$$

Latihan Soal

23-SEPI-2024

① Dalam sebuah koordinat cartesian:



$$a = 100 \text{ satuan}$$

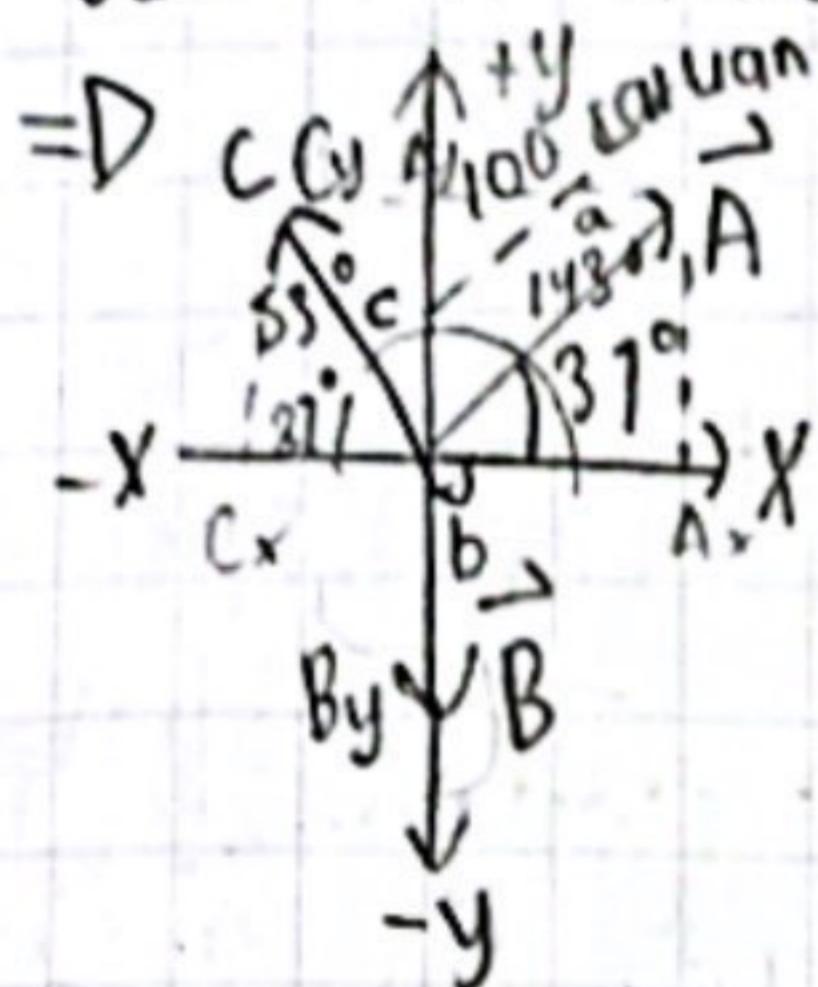
$$B = 80(\hat{i}) \text{ satuan}$$

$$C = 100 \text{ satuan} \\ < 143^\circ (x)$$

② gambaran ketiga vektor & resultan ketiga vektor tsb:

$$\vec{A} + \vec{B} + \vec{C}$$

③ Hitung posisi pd koord vektor tsb ($\vec{A} \times \vec{B} \cdot \vec{C}$)



④ Resultan = ? m/h

$$R_x = Ax + Bx + Cx$$

$$= Ax + 0 - (x)$$

$$= a \cos 37^\circ - c \cos 37^\circ$$

$$= 100 \cos 37^\circ - 100 \cos$$

$$37^\circ = 0$$

$$R_y = Ay + By + Cy$$

$$= Ay - By + Cy$$

$$= a \sin 37^\circ - b \sin 90^\circ +$$

$$c \sin 37^\circ$$

$$= 100 \cdot \frac{3}{5} - 80 \cdot 1 + 100 \cdot \frac{3}{5}$$

$$= 60 - 80 + 60 = 40 \text{ satuan}$$

$$\vec{R} = R_x \hat{i} + R_y \hat{j}$$

$$= 0 + 40 \hat{j} = 40 \hat{j}$$

$$= D (\vec{A} \times \vec{B}) \vec{C}$$

($x(u), y(v), z(w)$)

$$A = (100 \cos 37^\circ \quad 100 \sin 37^\circ)$$

$$B = (80 \cos 90^\circ + 80 \sin 90^\circ)$$

$$C = (100 \cos 143^\circ + 100 \sin 143^\circ)$$

$$= D (\vec{A} \times \vec{B}) \cdot \vec{C}$$

$$= \vec{A} = Ax \hat{i} + Ay \hat{j}$$

$$= a \cos 37^\circ \hat{i} + a \sin 37^\circ \hat{j}$$

$$= 80 \hat{i} + 60 \hat{j}$$

$$\vec{B} = -80 \hat{j}$$

$$\vec{C} = c \cos 143^\circ \hat{i} \Rightarrow c \cos 143^\circ$$

$$= -80 \hat{i} + 60 \hat{j}$$

$$\vec{A} \times \vec{B} = (80 \hat{i} + 60 \hat{j}) \times (-80 \hat{i})$$

$$= -6400 (\hat{i} \times \hat{j}) + (-4800) (\hat{j} \times \hat{i})$$

$$= -6400 \hat{n}$$

$$(\vec{A} \times \vec{B}) \cdot \vec{C} = (-6400 \hat{n}), (-80 \hat{i} + 60 \hat{j})$$

$$= 0$$

② $x_0 = -10 \text{ m}$

$$t = 0$$

(x positif)

$$V_m = 20 \text{ m/s}$$

$$t_m = 100 \text{ s}$$

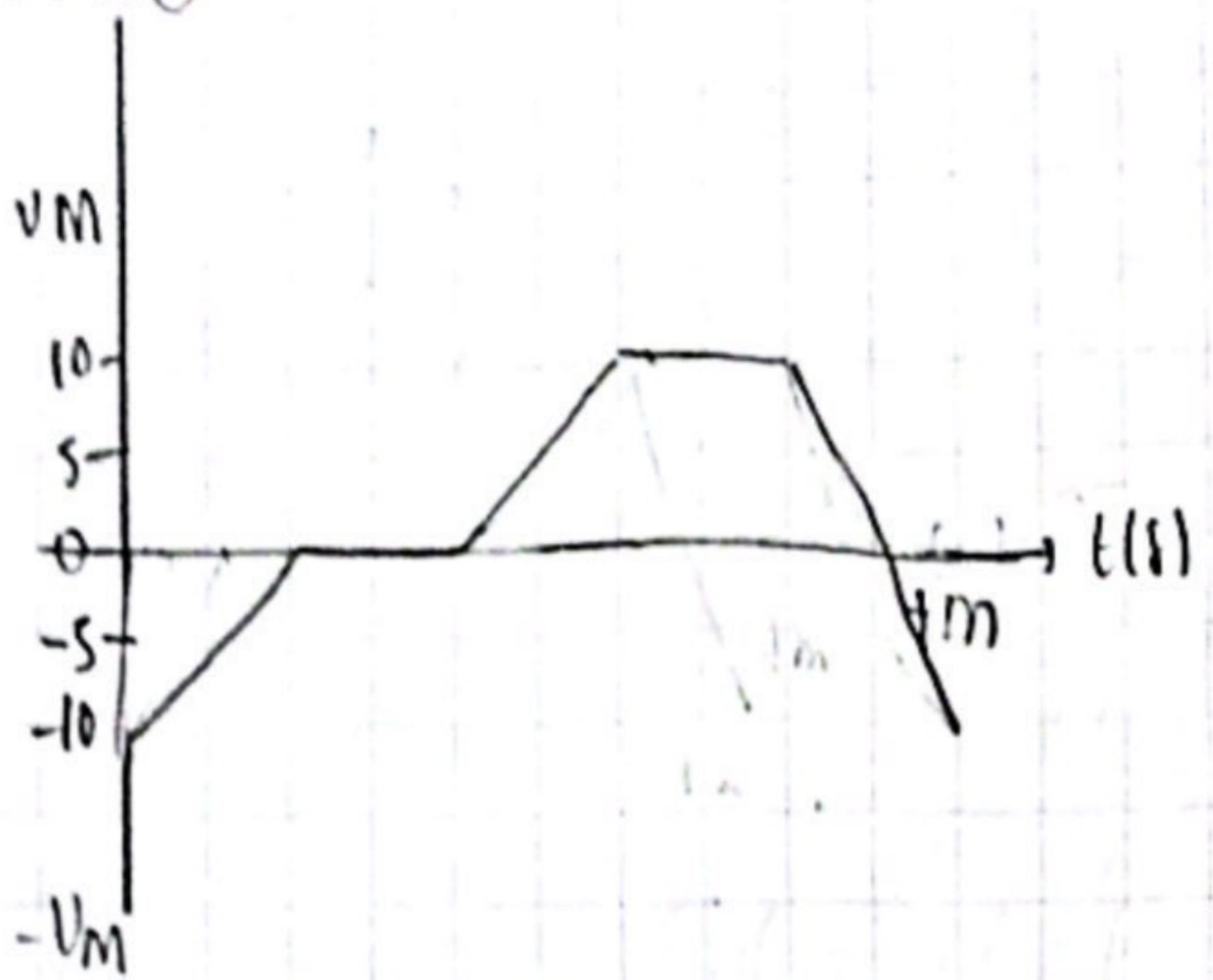
a) Buat suatu grafik posisi, thdp waktu!

b) jarak gerak thdp waktu

c) jarak tempuh D-tm

d) posisi partikel ketika tm

$v_m(m/s)$



a) grafik percepatan-waktu

(a) \rightarrow (+)

$$a = \frac{dv}{dt} \rightarrow \text{gradien kurva } v \text{ vs } t$$

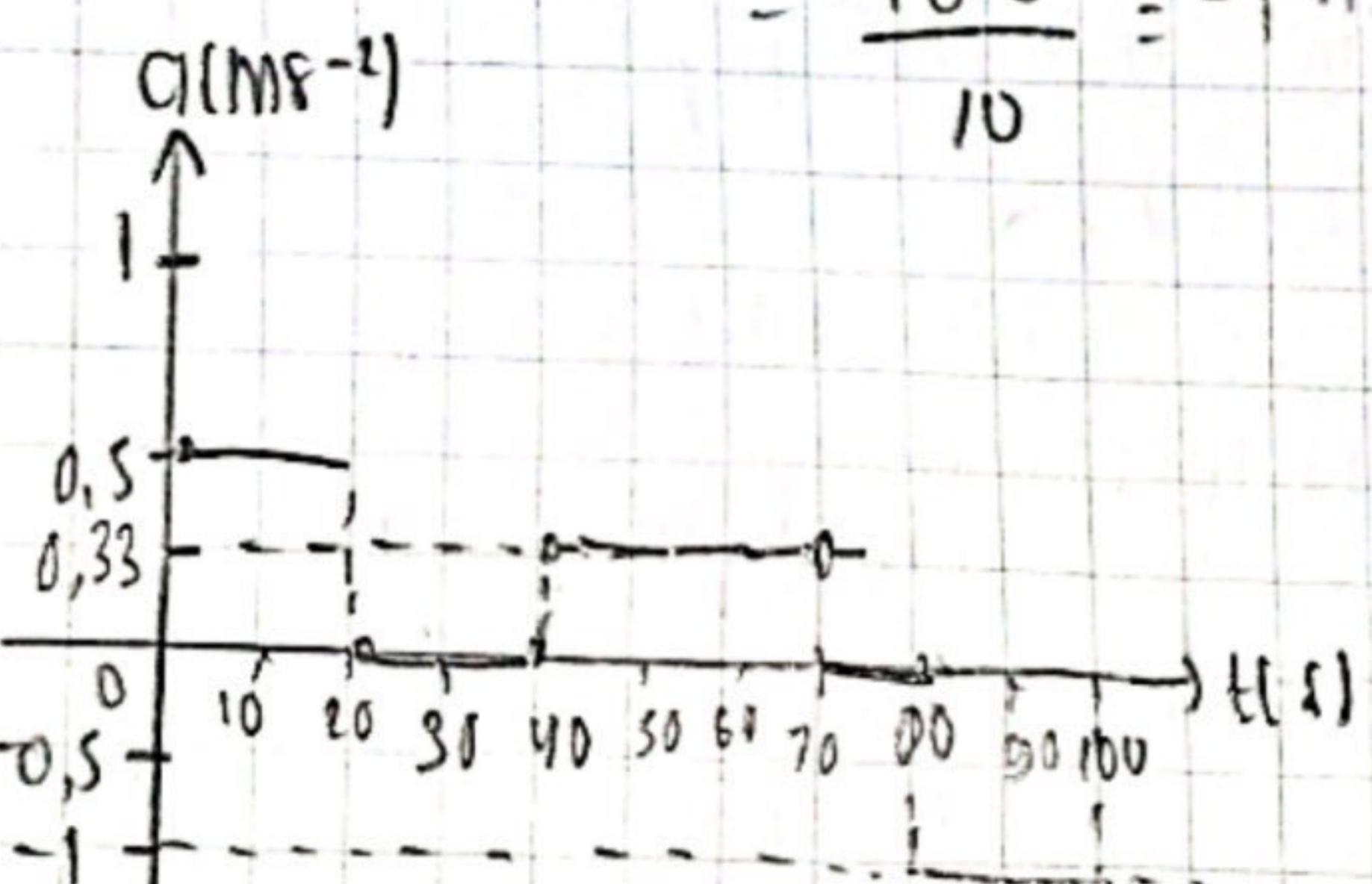
$$0-20s : a_{0-20} = \frac{v_{20}-v_0}{20-0} = \frac{0-(-10)}{20} = 0,5 \text{ ms}^{-2}$$

$$20-40s : a_{20-40} = \frac{v_{40}-v_{20}}{40-20} = 0$$

$$40-70s : a_{40-70} = \frac{v_{70}-v_{40}}{70-40} = \frac{10-0}{30} = 0,33 \text{ ms}^{-2}$$

$$70-80s : a_{70-80} = \frac{v_{80}-v_{70}}{80-70} = \frac{0-10}{10} = -1 \text{ m s}^{-2}$$

$$80-100s : a_{80-100} = \frac{v_{100}-v_{80}}{100-80} = \frac{-10-0}{20} = -0,5 \text{ m s}^{-2}$$



b) kondisi gerak tiap waktunya

- diam $a=0, v=0$

- gerak kecepatan tetap $a=0, v \neq 0$

- gerak dipercantik: tanda a & v sama

- gerak dipelambat=tanda a & v berlawanan

0-20s $\frac{v(t)}{a(t)}$ dipelambat

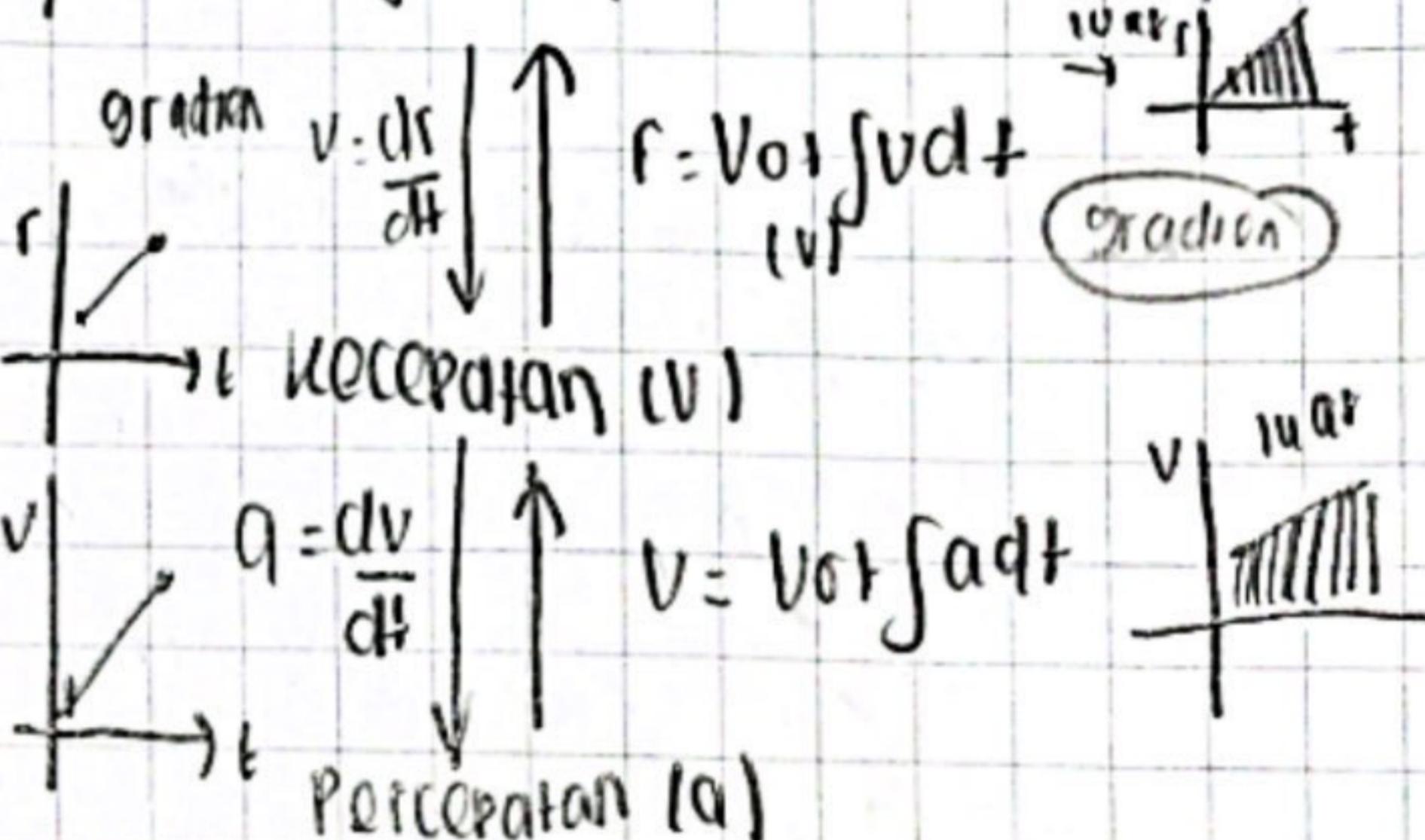
20-40s $a=0, v=0 \rightarrow$ diam

40-70s $\frac{v(t)}{a(t)}$ dipercantik

70-80s $a=0, v \neq 0 \rightarrow$ kec. tetap

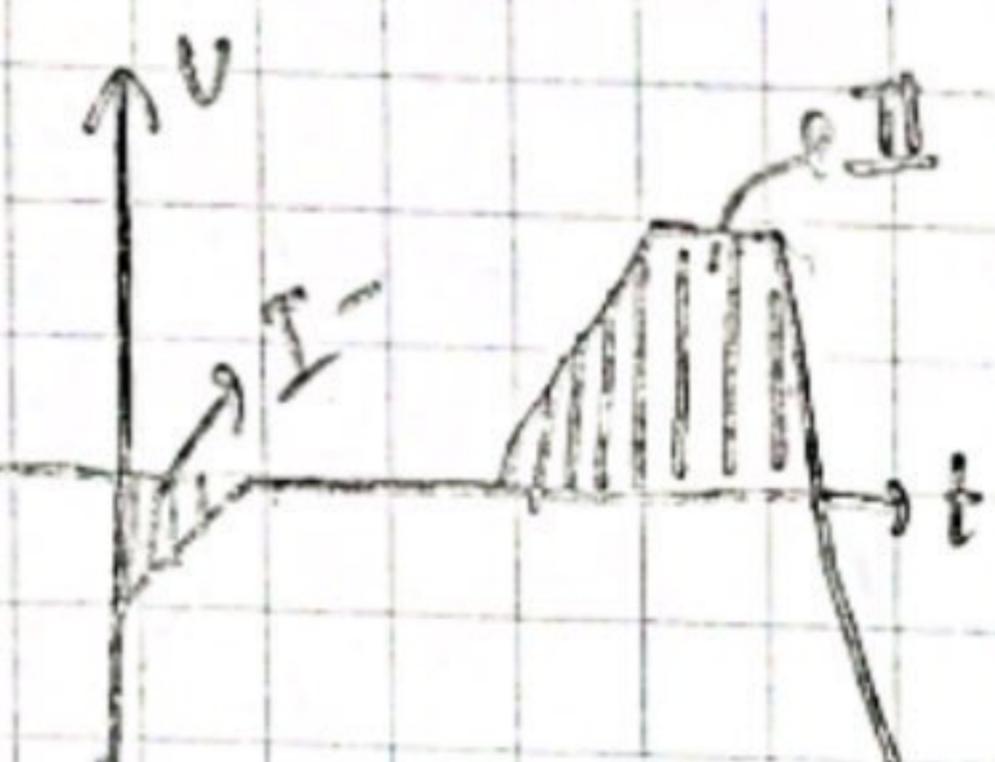
80-90s $= v(t), a(t) \rightarrow$

c) posisi gerak, perpindahan (r)



Jarak tempuh = Dikuriva v vs t

Total nilai muatan mas daerah yg diambil antara v dengan sumbu datar



$$x_{\text{total}} = |I| + |II|$$

$$= \left| \frac{(20 \times 1 - 10)}{2} \right| + \left| \frac{(80 + 10) \cdot 10}{2} \right|$$

$$= 100 + 450$$

$$= 550 \text{ m}$$

d) POSISI PARTIKEL SAAT + WAKTU (x_t)

$$\text{Perpindahan} = \Delta x = x_t - x_0$$

$$x_t = x_0 + \Delta x$$

Perpindahan (vektor)

Total luas daerah yg diapit kurva v dgn sumbu datar "

$$\Delta x = l t + \frac{l}{2}$$

$$= -100 + 300 = 200 \text{ m}$$

$$x + m = x_0 + \Delta x$$

$$= -10 + 200$$

$$x + m = 190 \text{ m}$$

- (a) Sebuah partikel bergerak dalam bidang xy . Pada saat $t=0$, partikel berada pada koordinat asal. Pada sumbu x , kecepatan partikel terhadap waktu dituliskan dengan persamaan: $\vec{v}_x = l - 10t \hat{i} \text{ ms}^{-1}$ sedangkan pada sumbu y , posisi partikel tiap waktu di deskripsikan oleh grafik berikut.



(a) Tuliskan vektor persamaan posisi, kecepatan & percepatan partikel sebagai fungsi waktu.

(b) Berapakah kecepatan rata-rata partikel dalam selang waktu 0-2s?

(c) Berapa besar kecepatan partikel saat $t=1s$?

(d) Hitung vektor posisi partikel saat ketinggian benda 100 m

$$a) \vec{r}(t) = x(t)\hat{i} + y(t)\hat{j} \text{ m} \quad 23/08/2024$$

$$\vec{v}(t) = v_x(t)\hat{i} + v_y(t)\hat{j} \text{ ms}^{-1}$$

$$\vec{a}(t) = a_x(t)\hat{i} + a_y(t)\hat{j} \text{ ms}^{-2}$$

Sumbu x

$$\vec{v}_x = (-10t)\hat{i} \quad \rightarrow x(t) = x(0) + \int v_x dt$$

$$= 0 + \int (-10t) dt$$

$$x(t) = l - 5t^2/2 \text{ m}$$

$$\rightarrow a_x(t) = \frac{dv_x}{dt} = \frac{d}{dt} (-10t)$$

$$a_x(t) = -10 \hat{i} \text{ ms}^{-2}$$

→ kecepatan rata-rata $\langle \vec{v} \rangle = \frac{\vec{r}}{t} = \frac{\vec{r}_f - \vec{r}_i}{t} = \frac{\vec{r}_f - \vec{r}_i}{\Delta t}$

→ kecepatan sejati $\vec{v} = \frac{dr}{dt}$

$$\vec{r} = x\hat{i} + y\hat{j}$$

$$|\vec{r}| = \sqrt{x^2 + y^2}$$

Sumbu y

$$y(t) = ?$$

$$\text{grafik linear: } y = mt + c \quad (\text{konstanta } = y(t=0) = 0)$$

$$19 \text{ radian } \frac{20}{10} = 20$$

$$4 \text{ hari } y(t=0) = 0$$

$$y(t) = 20t + 0$$

$$\vec{y}(t) = (20t)\hat{j} \text{ m}$$

$$\rightarrow v_y(t) = \frac{dy}{dt} = \frac{d}{dt} (20t) = 0$$

$$a_y(t) = \frac{dv_y}{dt} = \frac{d}{dt} (0) = 0$$

$$a_y(t) = \frac{d v_y}{dt}$$

$$= 0 \frac{1}{dt} (20) = 0$$

$$= 0$$

$$b) \langle \vec{v}_{0-2} \rangle = \frac{\vec{r}(t=2) - \vec{r}(t=0)}{2-0}$$

$$= \frac{l - 5.4\hat{i} - [20.2]\hat{j}}{2} = \frac{-20\hat{i} + 40\hat{j}}{2}$$

$$\langle \vec{v}_{0-2} \rangle = -10\hat{i} + 20\hat{j} \text{ m/s}$$

$$c) \vec{v}(t=1) = (-10)\hat{i} + 20\hat{j} = -10\hat{i} + 20\hat{j} \text{ m/s}$$

$$\text{besar } |\vec{V}_{0,2}| = \sqrt{(10)^2 + (20)^2}$$

$$= \sqrt{100 + 400} = \sqrt{500}$$

$$= \sqrt{25 \cdot 20}$$

$$= 5\sqrt{20}$$

$$|\vec{v}(t+1)| = \sqrt{(1-10)^2 + (20)^2}$$

$$= \sqrt{500} = 5\sqrt{20}$$

⑤ sebuah balon bergaya naik dengan kelecekan konstan $2,50 \text{ m/s}$. ketika balon mencapai ketinggian $3,00 \text{ m}$ diatas permukaan tanah, seorang penumpang tidak sengaja melepasikan kompas ke arah luar balon. gunakan nilai percepatan gravitasi ($g = 9,8 \text{ m/s}^2$)

a) Berapa lama waktu yg dibutuhkan yg mencapai permukaan tanah?

b) Berapa kec. kompas saat mencapai tanah?

Jawab: Dik: $v_0 = 2,50 \text{ m/s}$
 $h = 3 \text{ m}$
 $g = 9,8 \text{ m/s}^2$

Dit: Berapa lama?

a) $\boxed{h = v_0 \cdot t + \frac{1}{2} \cdot g \cdot t^2}$

$$3,00 = 2,50 \cdot t + \frac{1}{2} \cdot 9,8 \cdot t^2$$

$$3 = 2,5 \cdot t + 4,90 \cdot t^2$$

$$4,90t^2 + 2,5 \cdot t - 3 = 0$$

$$t \approx 0,57 \text{ detik}$$

b) $\boxed{v = v_0 + g \cdot t}$

$$= 2,50 + 9,80 \cdot 0,57$$

$$v = 8,00 \text{ m/s}$$

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