Dr. Sandy Vantika, S.Si   76 mahasiswa   DAY 1

No. 21-08-23
Date senin

Bab 0. Pendahuluan

1. bilangan real
2. pertidaksamaan dan nilai mutlak
3. Sistem koordinat
4. Grafik persamaan
5. fungsi dan grafiknya
6. Operasi pd fungsi
7. Beberapa fungsi khusus

Quiz 4x
Pr tdk hrs dikumpul 2014 → 9

UTS 35% sabtu, 21 oktober 23
UAS 35% kamis, 7 Desember 23
4 Nilai kuis bersama Fakultas KBF 15%
PKH (6 penilaian PR, kuis, dll) 15%

° Kalkulus ° ujian 2x onelsen ° tut. soal (harus aktif) ← zoom ° batas telat (wajar) 10/15 mnt
° 4x kuis

° Permen ✓ ° klo ketoilet langsung aja ° singlet × ° Aksesoris ✓
° sakit → surat kaos berlengan ✓ ° topi klo didlm ×
menyusul
(izin lewat wa ✓) ° sendal ×
📷 difoto

JOYKO® 36 Lines, 6 mm

0.1. Bilangan real, estimasi, dan logika

- imajiner

o Bilangan real = bilangan yang dapat dinyatakan dalam bentuk desimal

$A_n \ldots A_1 A_0, b_1 b_2 b_3 \ldots$ — 2 bilangan bulat

bilangan desimal yg berhenti / berulang menyatakan bilangan rasional

$0,5 = \frac{1}{2}$ (berhenti) $\quad 0,3333\ldots = \frac{1}{3}$ (berulang)

bilangan desimal yg tak henti / tak berulang menyatakan bilangan irasional

$\sqrt{2} = 1,4142135\ldots$

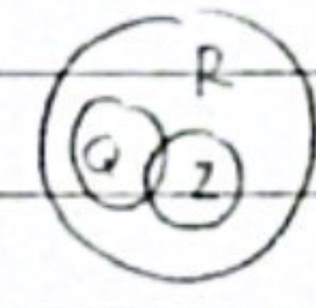
$\pi = 3,14159265\ldots \to \frac{22}{7}$ ✗ $\qquad k = \pi \cdot d$

$\pi = \frac{k}{d}$

- sekumpulan bilangan real

Himpunan bilangan real (R) memuat himpunan bil. rasional (Q), yang memuat himpunan bil. bulat (Z)

$Z = \{\ldots -3, -2, -1, 0, 1, 2, 3 \ldots\}$



himpunan bilangan asli (N) → 0 ✗ $\qquad$ implikasi

$N = \{1, 2, 3 \ldots\}$ $\qquad$ himpunan bagian $\qquad -\frac{1}{2}$

→ merupakan himpunan semesta → semua

dlm hal ini $N \subseteq Z \subseteq Q \subseteq R$

→ sub set

o Estimasi (perkiraan)

Sistem real (R) + dan × memenuhi: $\qquad \pi \simeq 3,14$

o Aljabar (komutatif, asosiatif, distributif)... $\qquad \sqrt{2} \simeq 1,4$

o urutan (trikotomi, transitif ..) → $<, =, >$ $\qquad 2^{10} \simeq 1000$

o kelengkapan R 'merupakan' garis "tak berlubang" $\qquad$ (estimasi / gampang)

klo mau nunjuk dimanapun pasti ada nilainya

$\longleftarrow -2 \;\; -1 \;\; 0 \;\; \frac{1}{2} \;\; 1 \;\; \sqrt{2} \;\; 2 \;\; \pi \longrightarrow$

o Komutatif (pertukaran)

$a + b = b + a = c$

$a \times b = b \times a = c$

o Asosiatif (pengelompokan)

$(a + b) + c = a + (b + c) = d$

o Distributif (penyebaran)

$a \times (b + c) = ab + ac = d$

o Trikotomi

x dan y bil. real

$x < y \mid x = y \mid x > y$

o Transitif

$x < y$ dan $y < z$, mk

$x < z$

o penambahan & pengurangan

$x < y$, mk

$x \pm z < y \pm z$

o Logika - implikasi (gk bs dibalik)

"$P \to Q$" (jika P, maka Q)

→ bernilai benar / slh

P	Q	$P \to Q$
B	B	B
B	S	S
S	B	B
S	S	B

salah tiba² benar ✓

negasi

ekuivalen $\quad P \leftrightarrow Q$

$\neg P \leftrightarrow \neg Q$

jika dan hanya jika

benar jika

P	Q	
B	B	B
S	B	S

o perkalian

$x < y$, z positif $xz < yz$

$x < y$, z negatif $xz > yz$

↳ awalnya salah terus salah ya ✓ (jk ty orang bandung, mk ty orang sabar)

rasional
3,14285714

No ....
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Latihan / mana yg lbh bsr ?

a. $\frac{22}{7}$ | 3,14 berhenti
1,024

b 2^{10} | 1000

Latihan Benar / salah

a. jika $x > 1$, maka $x^2 > 1$ benar · anggap $x > 1$ B, $x^2 > 1^2$ B } B

$1 > 1$ s, $1^2 > 1$ s
$2 > 1$ b, $4 > 1$ b

b. jika $x^2 > 1$, maka $x > 1$ salah → salah satu penyelesaian bukan 2 2 nya

$1 > 1$ s, $1 > 1$ s
$4 > 1$ b, $2 > 1$ b

$x^2 > 1$ B
$(x-1)(x+1)$
$x = 1$ $x = -1$
+ - +
-1 0 1
$x > 1$ $x < -1$

0.2 pertaksamaan dan nilai mutlak
(satu peubah)

0,25 · 0,5 udh pasti

kalimat $1/4 < 1/2$ ketaksamaan yg benar
$> / <$

" $1/x < 1/2$ pertaksamaan / ketaksamaan yg masih "terbuka"
bisa benar, bisa salah tergantung nilai x

menyelesaikan → menentukan himpunan yg memenuhi

Notasi selang (interval) mutlak → hasil x (-) Pe-an

$(a,b) := \{ x \mid a < x < b \}$ dengan x diantara a dan b
$[a,b] := \{ x \mid a \leq x \leq b \}$ x dari a sampe b "cara baca"
$[a,b) := \{ x \mid a \leq x < b \}$ selang a,b didefinisikan x dengan $a < x \leq b$
$(a,b] := \{ x \mid a < x \leq b \}$
$(-\infty,b) := \{ x \mid x < b \}$
$(-\infty,b] := \{ x \mid x \leq b \}$
$(a,\infty) := \{ x \mid x > a \}$
$[a,\infty) := \{ x \mid x \geq a \}$
$(-\infty,\infty) := R$ → seluruh bil real
tak terbatas → didefinisikan

* perhatikan tanda kurung (⇒ ≠
[⇒ =

JOYKO 38 Lines, 6 mm

blh kalikan dgn penyebut kecuali

Contoh pertaksamaan $\frac{2-x}{2} < 0$

$\frac{1}{x} < \frac{1}{2}$

$\frac{1}{x} - \frac{1}{2} < 0$

penyebutnya (x)

$\frac{2-x}{2x} < 0$ gk blh dikali kan

$2-x<0$, $2<x$, $x>2$ | $2x<0$, $x<0$

$(2-x)(2x) < 0$ → krn blm pasti

$2-x \neq 0$ ↳ supaya bernilai 0

$x \neq 0$

$x = 2$

Hp $= x > 2$ atau $x < 0$

$(2,\infty) \cup (-\infty,0)$

gar. ungar. $-$ | $+$ | $-$
$0 \quad 2$

Nilai Mutlak

↳ "Jarak" dari 0 ke x

* ∞ ∞ selalu kurung lengkung ↳ tak bisa dicapai

$|x| := x$ jk $x>0$

$|x| := 0$ $x=0$

$|x| := -x$ $-(-x)$ $x<0$

SIFAT nilai mutlak

$|a\cdot b| = |a|\cdot|b|$

$|a+b| \leq |a|+|b|$ Jarak dari titik

ketaksamaan Δ $|x|<a \leftrightarrow -a<x<a$ 0 ke X gk mungkin (-)

$|x| \geq a \leftrightarrow x>a \,/\, x<-a$

$|x|^2 = x^2$ jk dan hanya jk

$\{|x| = \sqrt{x^2}\}$ → dari akar pasti (+)

$|a| = |-a|$

$|x| \geq 0$

$|x-y| = |y-x|$

$|x|<|y| = x^2<y^2$

$|xy| = |x||y|$

$|x-y| = |x|-|y|$

$\|\cdot\|$ → norm (vektor)

latihan

1. $x+1 < \frac{2}{x}$
2. $|x-3| < |x+1|$
3. $|x-1| \leq x$
4. $|x-2| \leq x^2$

(1) $x+1 < \frac{2}{x}$

$\frac{x}{1} + \frac{1}{1} - \frac{2}{x} < 0$

$\frac{x^2+x-2}{x} < 0$

$(x^2+x-2)(x) < 0$

$x^2+x-2 < 0$ $\quad x<0$ ✓

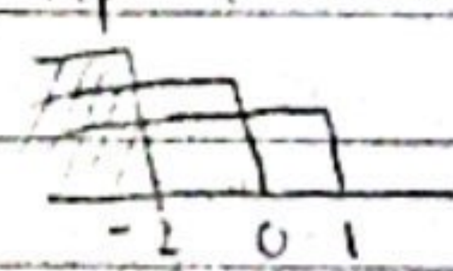
$(x-1)(x+2) < 0$

$x=1$ $\quad x-1<0 \quad x+2<0$

$x=-2$ $\quad x<1 \quad x<-2$

2 -1

Hp $(-\infty,-2)$ ✓ $(-\infty,0)$



(2) $|x-3| < |x+1|$

$(x-3)^2 < (x+1)^2$

$x^2-6x+9 < x^2+2x+1$

$x^2-x^2-6x-2x+9-1<0$

$-8x+8<0$

$-8x<-8$

$x>1$

Hp: $(1,\infty)$

$t = -1$ $\quad 2x^2 - x - 3 = 0$ $\quad 3 \cdot 1$

$x = -3$ $\quad (2x-3)(x+1)$

$2 - 3 = -1$

$|x| \le a$ musalahnya ada x

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3. $|x-1| \le x$

$|x-1| - x \le 0$

$x - 1 - x \le 0$

$-1 \le 0$

$-(x-1) - x \le 0$

$-x + 1 - x \le 0$

$-2x + 1 \le 0$

$-2x \le -1$

$x \ge \frac{1}{2}$

syarat: 1.) $x - 1 \le 0$

$x \le 1$

$- \cdot -$ tanda berubah

$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

dikuadratkan

$1 \le 2x$

$2x \ge 1$

$x \ge \frac{1}{2}$

$\left[\frac{1}{2}, \infty\right)$

$Hp = \left[\frac{1}{2}, \infty\right)$

syarat: 1.) $x - 1 \ge 0$

$x \ge 1$

$[1, \infty)$

$0 \quad \frac{1}{2} \quad 1$

4. $|x-2| \le x^2$

$|x-2| - x^2 \le 0$

$x - 2 - x^2 \le 0$

$-x^2 + x - 2 \le 0$

$t = 1$

$x = -2$

$(-x-2)(x+1)$

3^2

$2x^2 + 7x + 6$

$(2x+3)(x+2)$

$[1, \infty) \cup \left[\frac{1}{2}, 1\right] \quad \left[\frac{1}{2}, \infty\right)$ //

$-(x-2) - x^2 \le 0$

$-x + 2 - x^2 \le 0$

$-x^2 - x + 2 \le 0$

$(-x+1)(x+2)$

$t = -1$

$x = 2$

$(a+b)(a-b)$

$(x-2)^2 \le (x^2)^2$

$x^2 - 4x + 4 - x^4 \le 0$

$-x + 1 \le 0$ $\quad x + 2 \le 0$

$1 \le x$ $\quad x \le -2$

$x \ge 1$

Hp $[1, \infty) \cup (-\infty, -2]$

$x \ge 1 \cup x \le -2$

3 pembenaran

$(x-1)^2 \le x^2$

$x^2 - 2x + 1 \le x^2$

$\cancel{x^2} - \cancel{x^2} - 2x + 1 \le 0$

$-2x \le -1$

$x \ge \frac{1}{2}$

$\left[\frac{1}{2}, \infty\right)$

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Bentuk umum

1. $|f(x)| < k \rightarrow -k < f(x) < k$
2. $|f(x)| > k \rightarrow f(x) > k \cup f(x) < -k$
3. $|f(x)| < |g(x)| \rightarrow (\overset{a+b}{f(x)+g(x)})(\overset{a-b}{f(x)-g(x)}) < 0$ / dikuadratin
4. $a < |f(x)| < b \rightarrow a < f(x) < b \cup -b < f(x) < -a$
5. $\dfrac{|f(x)|}{|g(x)|} < k \rightarrow f(x) < k \cdot g(x)$ terus dikuadratin

* inget tanda kurung

contoh

① $|2x-3| < 7$

$-7 < 2x-3 < 7$ (+3)

$-7+3 < 2x-3+3 < 7+3$

$-4 < 2x < 10$ (:2)

$-2 < x < 5$

② $|2x-1| > 4$

$2x-1 > 4 \quad \cup \quad 2x-1 < -4$

$2x > 5 \quad\quad 2x < -3$

$x > \frac{5}{2} \quad\quad x < -\frac{3}{2}$

③ $|2x-1| > |x+4|$ (dikuadratin)

$(2x-1)^2 > (x+4)^2$

$4x^2-4x+1 > x^2+8x+16$

$4x^2-x^2-4x-8x+1-16 > 0$

$3x^2-12x-15 > 0$

$(3x+3)(x-5)$

$3x+3=0 \quad\quad x-5=0$

$3x=-3 \quad\quad x=5$

$x=\frac{-3}{3}=-1$

+ | − | +

−1 0 5

$x < -1$ atau $x > 5$

$(-\infty, -1)$ $(5, \infty)$

④ $1 < |x-2| < 3$

① $1 < x-2 < 3$ (+2)

$1+2 < x < 3+2$

$3 < x < 5$

② $-3 < x-2 < -1$ (+2)

$-1 < x < 1$

⑤ $\dfrac{|2x+7|}{|x-1|} \leq 1$

$(a+b)(a-b)$

$(+,-)$

$|2x+7| \leq 1 \cdot |x-1|$

$|2x+7| \leq |x-1|$ $-x+1$

$(2x+7+x-1)(2x+7-(x-1)) \leq 0$

* $(3x+6)(x+8) \leq 0$

$\cancel{3x+6 \leq 0}$

$\cancel{3x \leq -6}$

$\cancel{x \leq -2}$

dibuat = 0

$3x+6=0 \quad\quad x+8=0$

$3x=-6 \quad\quad x=-8$

$x=-2$

+ | − | +

−8 −2 0

$-8 \leq x \leq -2$

syarat $x-1 \neq 0$

$x \neq 1$

3.4.5

SISTEM KOORDINAT, GRAFIK PERSAMAAN, FUNGSI dan GRAFIKNYA DAY 2

4umnr

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- koordinat cartesius dan grafik persamaan
- fungsi dan grafiknya

daerah asal (domain) } selang

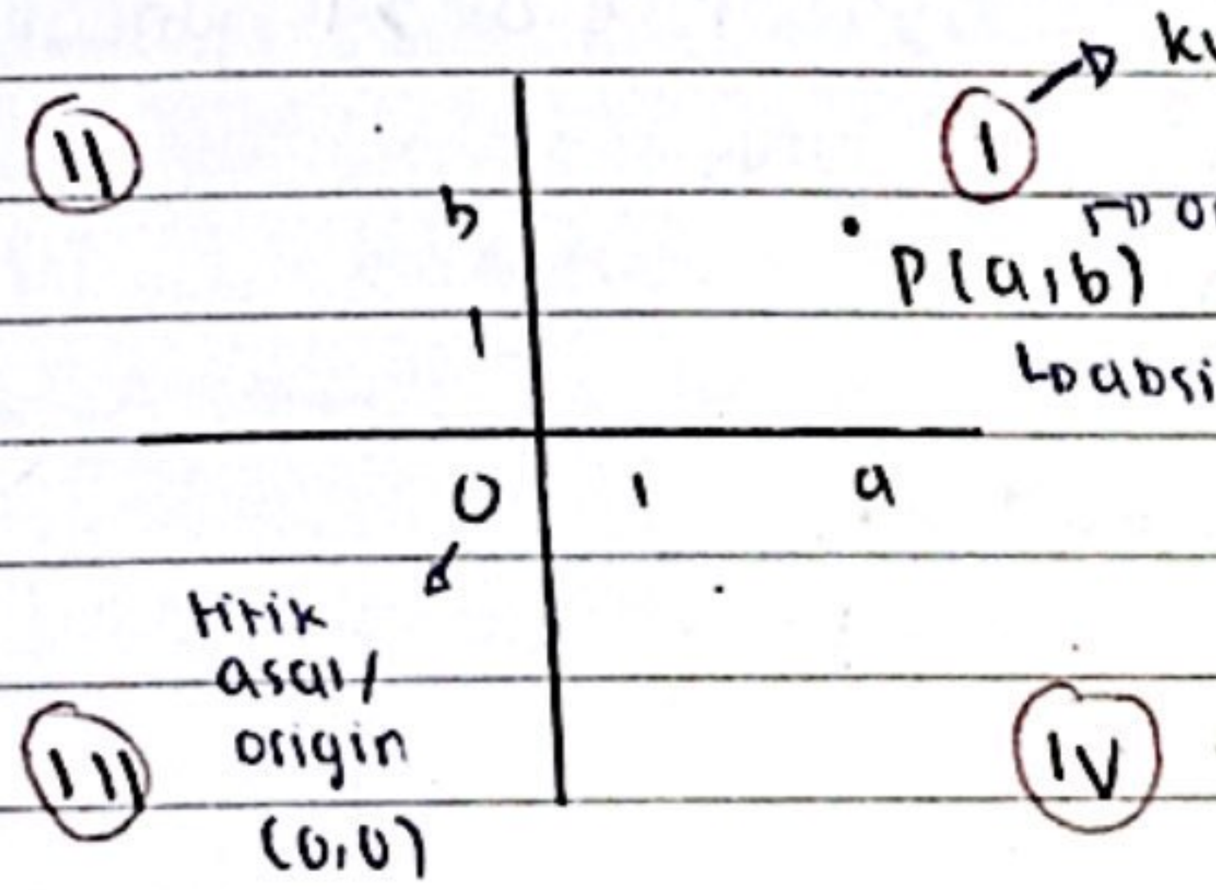
hasil (range) } -> sub himpunan / himpunan bagian dari kodomain; memiliki pasangan dari domain dan kodomain

(kodomain) -> daerah kawan (bisa diperluas), tdk semua kodomain memiliki hasil.

Penemu

Rene descartes -> (1596-1650), filsuf & matematikawan prancis, terkenal karyanya "La geometrie" (1637) dan ucapan "cogito ergo sum."

Sistem koordinat cartesius dibatasi sumbu x (+) dan y (+)



-> kuadran

-> ordinat

-> absis (jarak suatu titik kesumbu y) (+)

-> ta

-> tapi klo (-) bkn jarak

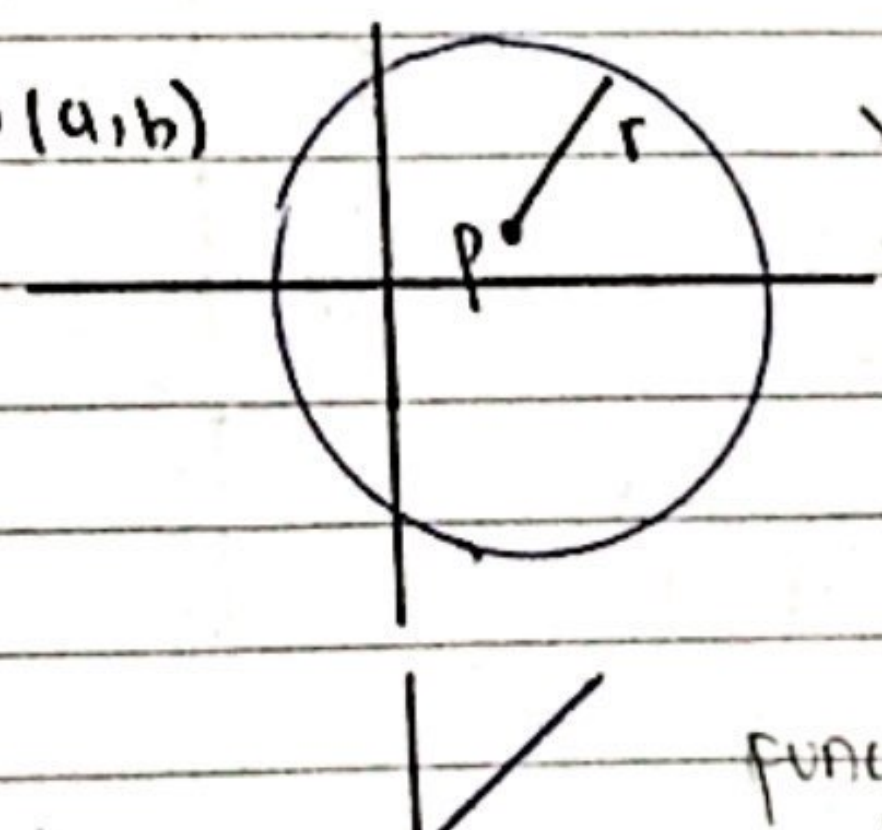
berlawanan arah

Jarak antara 2 titik P(a,b) dan Q(c,d)

$$\boxed{d(P,Q) = \sqrt{(c-a)^2 + (d-b)^2}}$$

persamaan lingkaran yg berpusat di P(a,b)

$$\boxed{(x-a)^2 + (y-b)^2 = r^2}$$



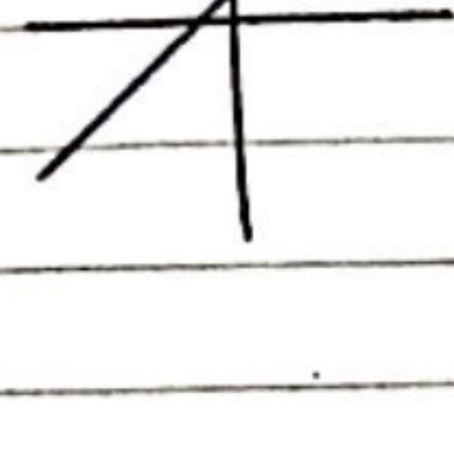
X fungsi

Garis lurus -> punya gradien

$$\boxed{Ax + By + C = 0}$$

-> konstanta

syarat A dan B tak keduanya 0, $B \neq 0$



fungsi

$$\boxed{y = mx + n}$$

-> intercept (titik potong sumbu y)

-> gradien (slope)

- sejajar : $m_1 = m_2$
- tegak lurus = $m_1 \cdot m_2 = -1$

geogebra

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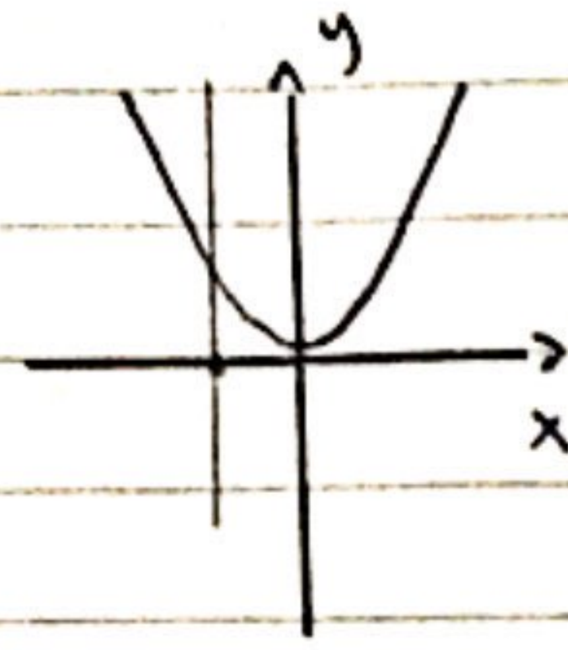
Grafik persamaan

grafik fungsi -> bloh disebut persamaan

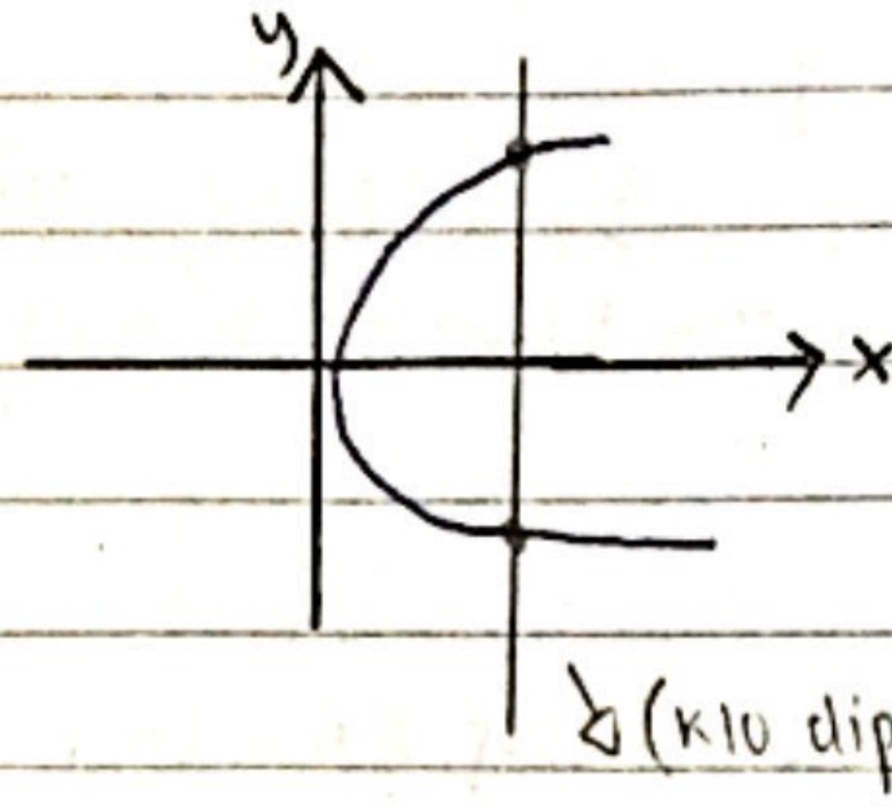
contoh 1. $\boxed{y = x^2}$



setiap nilai x itu hanya punyai pasangan dng y (klo dipotong sejajar sumbu y memperoleh 1 titik) -> fungsi

contoh 2. $\boxed{x = y^2}$

grafik persamaan -> belum tentu fungsi



(klo dipotong sejajar sumbu y memperoleh 2 titik) -> bukan fungsi

* klo pangkat > 1 jangan lurus

Bantuan dalam menggambar grafik $y = x^2$

x	-2	-1	0	1	2
y	4	1	0	1	4

-> plot (tandai)

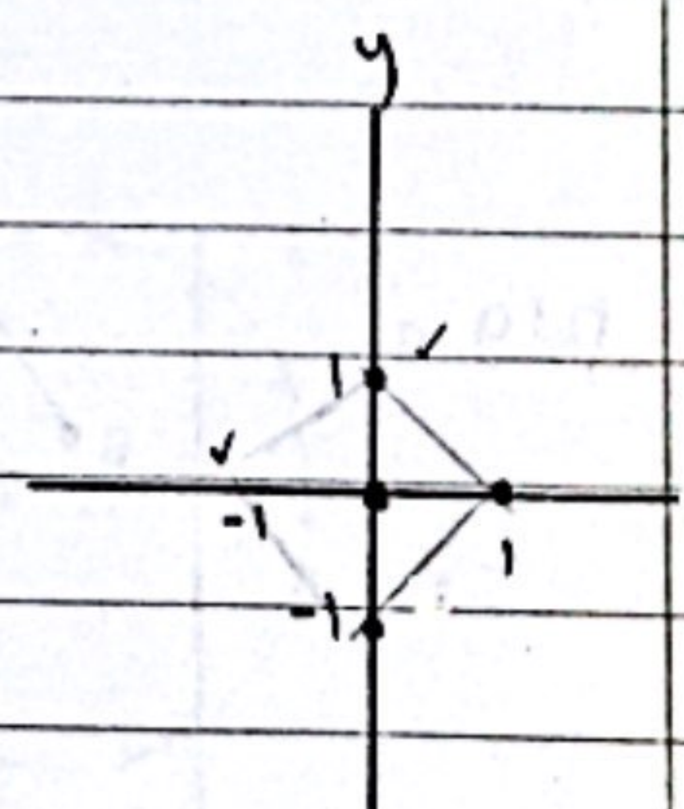
Latihan gambar grafik pers.

(2) $|x| + |y| = 1$

$|y| = 1 - |x|$

misal ~~x = 0~~ ~~misal y = 0~~

- misal $x = 0$
  $y = 1 - |0|$
  $y = 1$
- $x = 1$
  $y = 1 - |1|$
  $y = 0$
- $x = -1$
  $y = 1 - |-1|$
  $y = 1 - 1$
  $= 0$



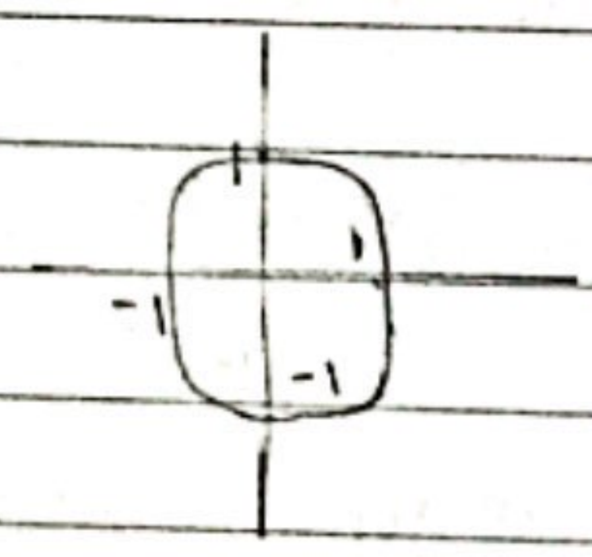
x	0	1	-1
y	1	0	0
	0,1	1,0	-1,0

(3) $x^4 + y^4 = 1$

$y^4 = 1 - x^4$

~~$x^4 = 1 - y^4$~~

$x = 0$
$y^4 = 1 - 0^4$
$y^4 = 1$
$y = 1$ 0,1



$x = 1$
$y^4 = 1 - 1^4$
$y^4 = 1 - 1$
$y^4 = 0$
$y = 0$ 1,0

$x > 1$ tdk memenuhi

$x = -1$
$y^4 = 1 - (-1)^4$
$y^4 = 1 - 1$
$y^4 = 0$
$y = 0$ -1,0

$x = 2$
$y^4 = 1 - (2)^4$
$y^4 = 1 - 16$
$y^4 = -15$
$y = \sqrt{-15}$

Fungsi & Grafik

• dalam kehidupan sehari-sahari besaran/ nilai bergantung pada nilai lain

ex. berat badan fungsi dari waktu

$t = t_1$

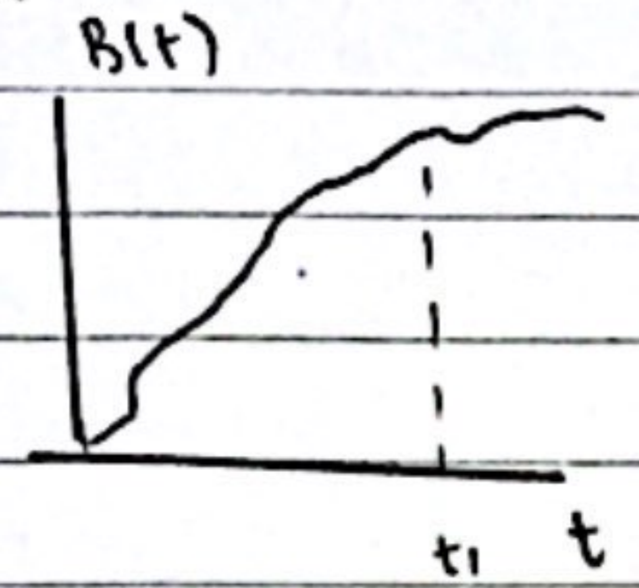
Berat badan $B_1 = B(t_1)$

$t = t_2$

$B_2 = B(t_2)$

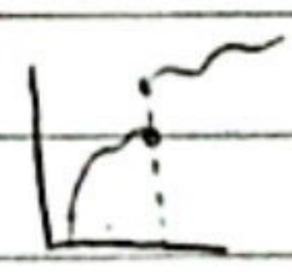
umum: pd saat $t \to B(t)$

grafik fungsi berat badan



* fungsi kontinu (tdk pernah putus)

fungsi x kontinu



- fungsi

fungsi f (dari R ke R) bil real, dngn range bil real

domain (daerah asal)

$D \subset R$, range → daerah asal

himpunan — karena fungsi (gk boleh berpasangan >1)

nilai tunggal $f(x) \in R$

$f: x \to f(x)$, $x \in D$ berada

x berada di daerah asal

$y = f(x)$, $x \in D$

$\{y = f(x) \mid x \in D\}$

→ daerah hasil fungsi f

(x nya berapapun pasti akan selalu bernilai)

contoh fungsi

$f(x) = x^2$ → hasil nya gk mungkin (-), pasti $x \geq 0$

daerah asal fungsi adalah R (bil real),

daerah hasil adalah $[0, \infty)$

$g(x) = \frac{1}{x}$ dimana $x \neq 0$

daerah asal fungsi adalah $R - \{0\}$ (kecuali)

daerah hasil fungsi adalah $R - \{0\}$

yg

Catatan

bila daerah asal tdk disebutkan secara detail berarti ~~bil real~~ himpunan terbesar dari R

ex: daerah asal fungsi $f(x) = \sqrt{1-x}$ adalah .... $\sqrt{x} \to x \geq 0$

klo didalam kurung → $1 - x \geq 0$

$x \leq 1$

$(-\infty, 1)$

GRAFIK FUNGSI

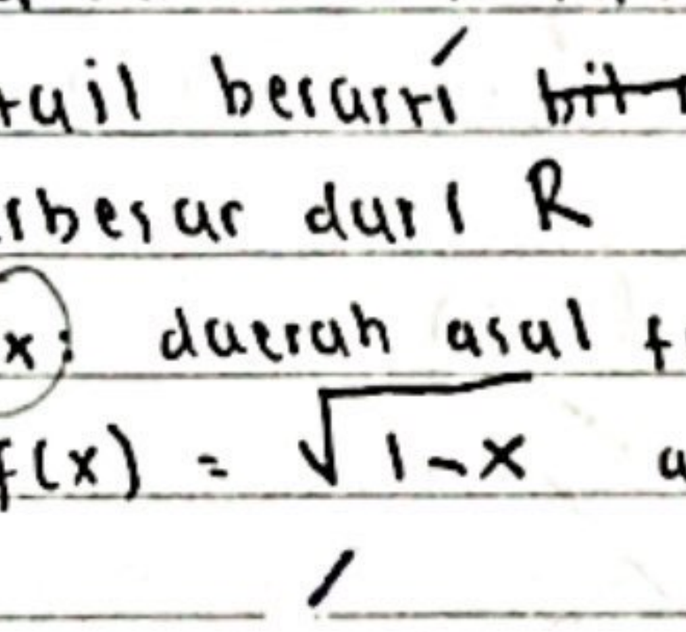
$y = f(x)$

$f(x) = x^2$

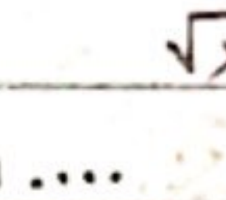
$y = x^2$

karena nilai fungsi tidak mungkin (-)

fungsi kuadrat & simetris



fungsi nilai mutlak



simetris

$f(x) = |x|$

sama² sah

sumbu x berpotongan di (0,0) dan simetri terhadap sumbu -y mri

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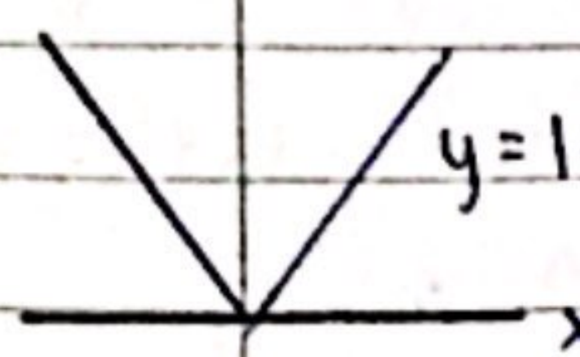
grafik fungi nilai mutlak →



grafik fungsi $f(x) = |x|$

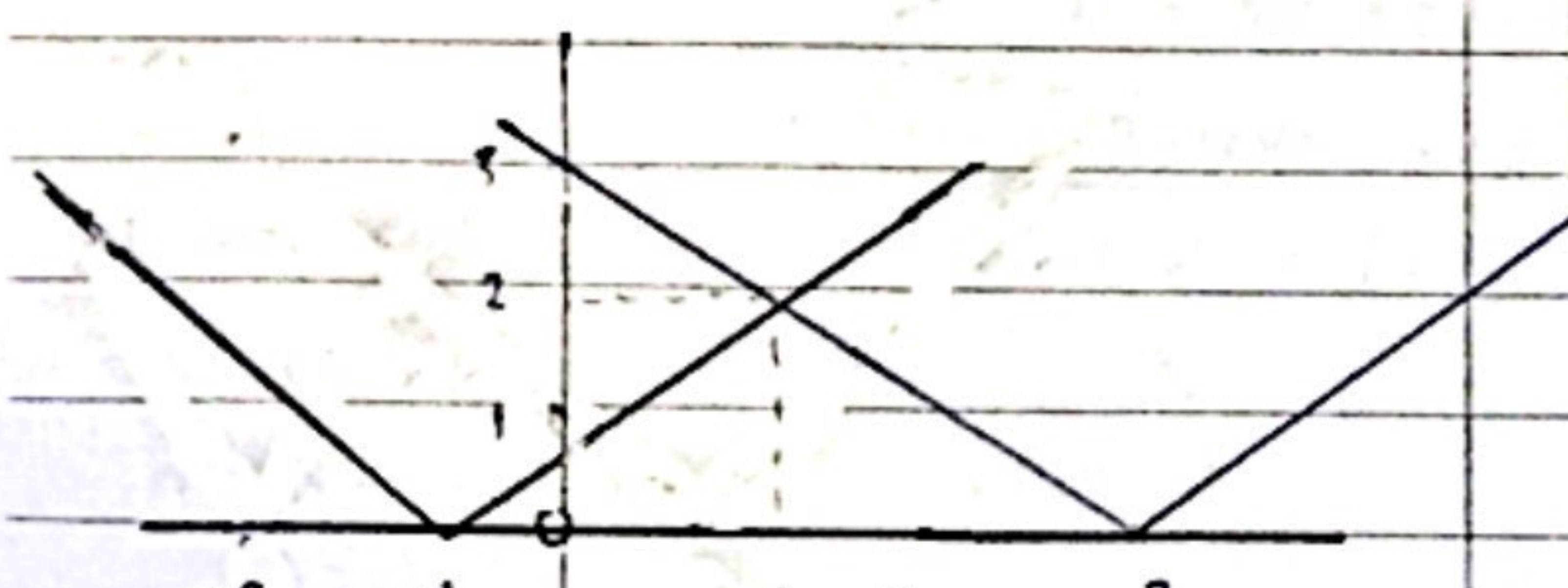
pertaksamaan

$|x-3| < |x+1|$

↓

dapt diperoleh dari ~~dngn~~
menggambar grafik fungsi
$y = |x-3|$ dan $y = |x+1|$

$x = 0$	$x-3=0$	$x = 0$	$x+1=0$
$y = \|0-3\|$	$x = 3$	$y = \|0+1\|$	$x = -1$
$\cancel{y = \|-3\|}$	↓	$\cancel{y = \|1\|}$	↓
$y = 3$	$(3,0)$	$y = 1$	$(-1,0)$



fungsi kuadrat $f(x) = y$ fungsi nilai mutlak $f(x) = |x|$

$\frac{f(x)}{g(x)} \to g(x) \neq 0$ — $R - \{0\}$

$x^2 = x \geq 0$? real

$\sqrt{x} \to x \geq 0$ ~ Real

$\exists$ = terdapat $\forall$ = untuk setiap

$\ni$ = sehingga

$\in$ = anggota

$x < 0 \to$ imajiner
✗ real

06. OPERASI PADA FUNGSI dan BEBERAPA FUNGSI KHUSUS

operasi al-jabar didefinisikan

irisan

$(f+g)(x) := f(x) + g(x) \quad x \in D_f \cap D_g$

$(f-g)(x) := f(x) - g(x) \quad x \in D_f \cap D_g$

$(fg)(x) := f(x) \cdot g(x) \quad x \in D_f \cap D_g$

$\left(\frac{f}{g}\right)(x) := \frac{f(x)}{g(x)} \quad x \in D_f \cap D_g$

lbh sempit

syarat $g(x) \neq 0$ / memastikan $\neq 0$

penyebut

Contoh

$f(x) := x^2$ ($x \in \mathbb{R}$) $\quad g(x) := \sqrt{x}$ ($x \geq 0$)

$(f+g)(x) = x^2 + \sqrt{x}, \quad x \geq 0$

$(f-g)(x) = x^2 - \sqrt{x}, \quad x \geq 0$

$(fg)(x) = x^2\sqrt{x}, \quad x \geq 0$

$\left(\frac{f}{g}\right)(x) = \frac{x^2}{\sqrt{x}} = x\sqrt{x}, \quad x > 0$

$\frac{x^2}{\sqrt{x}} \cdot \frac{\sqrt{x}}{\sqrt{x}} = \frac{x^2\sqrt{x}}{x} = x\sqrt{x}$

catatan

daerah usal f/g tidak mencungkup $x=0$ mka sekalipun $x\sqrt{x}$ terdefinisi $x=0$

Pangkat akar

real

asli

$(f^n)(x) := (f(x))^n, \quad x \in D_f$

$(f^{-n})(x) := \frac{1}{f(x)^n}, \quad x \in D_f \quad f(x) \neq 0$

$(f^{\frac{1}{n}})(x) := \sqrt[n]{fx}, \quad x \in D_f \quad f(x) \geq 0$ u/ n = genap

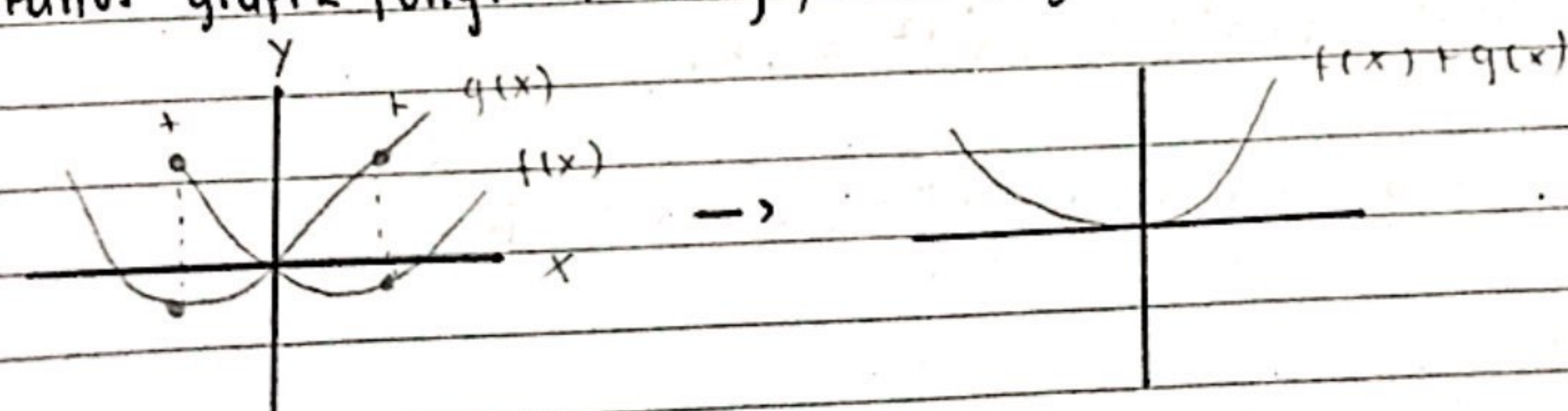
x(-)

$f(x)^{\frac{1}{n}}$

catatan

* $f^{-1}(x) = \frac{1}{f(x)}$ $\quad$ * $f^{\frac{1}{2}}(x) = f(x)^{\frac{1}{2}} = \sqrt{f(x}, \quad f(x) \geq 0$

diketahui grafik fungsi f dan g, klo f+g?



$\in$ akan berada

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Fungsi komposisi

bila x dipetakan ke $y = f(x)$ oleh f
y " ke $z = g(y)$ oleh g diperoleh $z = g(f(x))$

$x \to y = f(x) \to z = g(f(x))$
$g(y)$

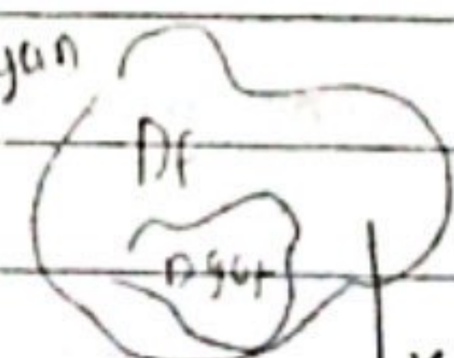
komposisi f dan g dilambangkan $g \circ f$, merupakan fungsi yg memetakan x ke $z = g(f(x))$, yakni
$(g \circ f)(x) = g(f(x))$

Daerah Asal fungsi komposisi $g \circ f$ adalah himpunan semua $x \in D_f$ (anggota), sehingga $\underbrace{f(x)}_{y} \in D_g$

$h(x) = x$
daerah asal
$D_h = \{x \in \mathbb{R}\}$

f - seluruh bilangan real

$D_{g \circ f} = \{ x \in D_f \mid f(x) \in D_g \}$



$(g \circ f)(x) = x$

x semua

$\boxed{ex}$ $f(x) = \sqrt{x}$ $\to x \geq 0$ $g(x) = x^2$ hasil $x \geq 0$ $[0, \infty)$ * apapun dikuadratkan pasti ada nilainya makanya REAL
mk,
$(g \circ f)(x) = g(f(x)) = g(\sqrt{x}) = x^2 = (\sqrt{x})^2 = x$ $\to x \geq 0$

$D_f = g$ $D_g = f$

daerah asalnya $\to$ $\boxed{D_{g \circ f}} = \{ x \in [0, \infty) \mid \sqrt{x} \in \mathbb{R} \} = [0, \infty)$ irisan

sekalipun $g \circ f$ memetakan x, daerah asalnya hanya $[0, \infty)$ yg bs dua²nya

* Komposisi dua fungsi tidak bersifat Komutatif

$\{x \in D_f \mid f(x) \in D_g\}$

$(f \circ g)(x) = f(g(x)) = f(x^2) = (\sqrt{x^2}) = |x|$

$D_g = f$ $D_f = g$

daerah asalnya $\to$ $\boxed{D_{f \circ g}} = \{ x \in \mathbb{R} \mid x^2 \in [0, \infty) \} = \mathbb{R}$?

$f \circ g \neq g \circ f$ $D_{g \circ f} = \{ x \in D_f \mid f(x) \in D_g \}$

tidak komutatif

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Latihan $\{x \in D_f \mid f(x) \in D_g\}$

2. $f(x) := \sqrt{x}$ dan $g(x) := \frac{1}{x}$ tentukan $g \circ f$ dan $f \circ g$, beserta daerah asalnya

$x \geq 0 : [0, \infty)$ $x \to x \in R - \{0\}$

$g \circ f = g(f(x)) = g(\sqrt{x}) = \frac{1}{\sqrt{x}}$

df dg

$\{x \in [0,\infty) \mid \sqrt{x} \in R - \{0\}\} = (0,\infty)$

$f \circ g = f(g(x)) = f\left(\frac{1}{x}\right) = \sqrt{\frac{1}{x}}$

dg df

$\{x \in R - \{0\} \mid \frac{1}{x} \in [0,\infty)\} = \cancel{R - \{0\}}\ (0,\infty)$

Fungsi Polinom

fungsi konstan	$f(x) = k$ (konstan)
fungsi identitas	$f(x) = x$
fungsi linear	$f(x) = mx + n$
fungsi kuadrat	$f(x) = ax^2 + bx + c$
fungsi polinom	$f(x) = a_n x^n + a_{n-1} x^{n-1} + \dots a_1 x + a_0$, dngn $a_0, a_1, \dots a_n$ konstan dan $a_n \neq 0$, dngn $n \in N$

~~catatan~~

daerah asal fungsi polinom adalah R

Fungsi Rasional & Fungsi Aljabar

$\boxed{f(x) = \frac{P(x)}{q(x)}}$, dengan p dan q polinom, disebut fungsi rasional

ex:

$f(x) = \frac{x}{x^2 + 1} \to$

fungsi Aljabar

ex: $g(x) = \sqrt{x}$ dan $h(x) = x^{\frac{1}{3}} + x - 10$

DV EDIpT

Catatan

fungsi nilai mutlak $f(x) = |x|$ → termasuk Aljabar, mengingat $\sqrt{x^2} = |x|$ dlm hal ini $y = |x|$, mk y yang memenuhi persamaan $y^2 = x^2$ $(y \geq 0)$, fungsi Al-jabar yg memenuhi persamaan aljabar $y^2 = x^2$ / $y^3 = 3x^2 + 5x$

θ	0	30	45	60	90	120	135	150	180
	0	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$	$2\pi/3$	$3\pi/4$	$5\pi/6$	π
$\sin\theta$	0	$1/2$	$1/2\sqrt{2}$	$1/2\sqrt{3}$	1	$1/2\sqrt{3}$	$1/2\sqrt{2}$	$\frac{1}{2}$	0
$\cos\theta$	1	$1/2\sqrt{3}$	$1/2\sqrt{2}$	$1/2$	0	$-1/2$	$-\frac{1}{2}\sqrt{2}$	$-\frac{1}{2}\sqrt{3}$	-1
$\cot\theta$	∞	$\sqrt{3}$	1	$1/3\sqrt{3}$	0				
$\tan\theta$	0	$1/3\sqrt{3}$	1	$\sqrt{3}$	∞				
$\sec\theta$	1	$\frac{2}{3}\sqrt{3}$	$\sqrt{2}$	2	∞	-2	$-\sqrt{2}$	$-\frac{2}{3}\sqrt{3}$	-1
$\csc\theta$	∞	2	$\sqrt{2}$	$\frac{2}{3}\sqrt{3}$	1	$\frac{2}{3}\sqrt{3}$	$\sqrt{2}$	2	∞

$\sin\alpha + \sin\beta = 2\sin\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$

$\sin\alpha - \sin\beta = 2\cos\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right)$

$\cos\alpha + \cos\beta = 2\cos\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$

$\cos\alpha - \cos\beta = -2\sin\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right)$

$\sin\left(\frac{\pi}{2}-\alpha\right) = \cos\alpha$

$\cos\left(\frac{\pi}{2}-\alpha\right) = \sin\alpha$

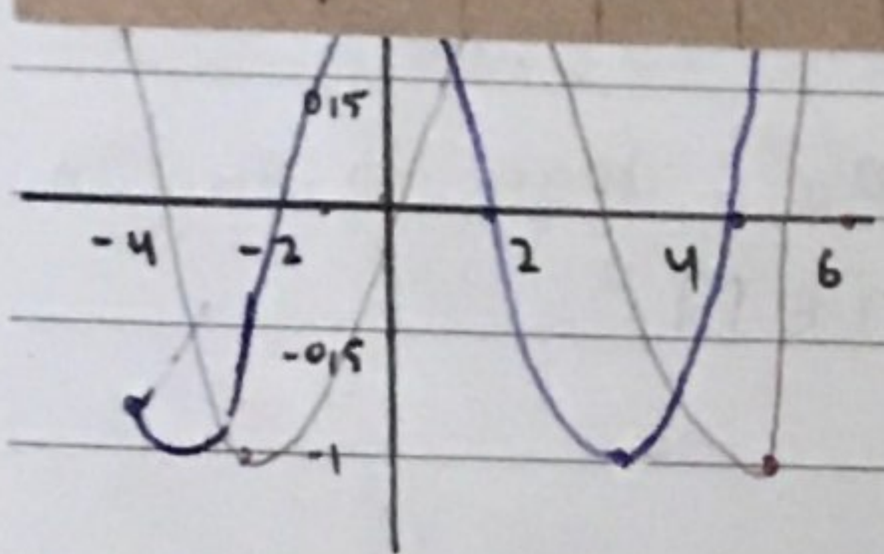
$\tan\left(\frac{\pi}{2}-\alpha\right) = \cot\alpha$

$\sin\frac{1}{2}\alpha = \pm\sqrt{\frac{1-\cos\alpha}{2}}$

$\cos\frac{1}{2}\alpha = \pm\sqrt{\frac{1+\cos\alpha}{2}}$

$\tan\frac{1}{2}\alpha = \pm\sqrt{\frac{1-\cos\alpha}{1+\cos\alpha}} = \frac{\sin\alpha}{1+\cos\alpha} = \sqrt{\frac{1-\cos\alpha}{\sin\alpha}}$

2	$\sqrt{2}$	$2/3\sqrt{3}$	1



$\cos(-x) = \cos x$
$\sin(-x) = -\sin x$

- $\sin(x) = \cos x$
- $\cos(x) = -\sin x$
- $\tan(x) = \sec^2 x$
- $\sec(x) = \sec x \cdot \tan x$
- $\cot(x) = -\csc^2 x$
- $\csc(x) = -\csc x \cdot \cot x$

$\sin 2x = 2\sin x\cos x$

$\cos 2x = \cos^2 x - \sin^2 x$
$= 1 - 2\sin^2 x$
$= 2\cos^2 x - 1$

$\tan 2x = \frac{2\tan x}{1-\tan^2 x}$

$\cot 2x = \frac{\cot^2 x - 1}{2\cot x}$

$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$
$\sin(\alpha-\beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$
$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$
$\cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$

$\tan(\alpha+\beta) = \frac{\tan\alpha + \tan\beta}{1-\tan\alpha\tan\beta}$

$\tan(\alpha-\beta) = \frac{\tan\alpha - \tan\beta}{1+\tan\alpha\tan\beta}$

Fungsi Tan, cot, sec, dan csc

$\tan t = \frac{\sin t}{\cos t}$ $\sec = \frac{1}{\cos t}$

$\cot t = \frac{\cos t}{\sin t}$ $\csc = \frac{1}{\sin t}$

Sifat fungsi trigonometri

$\cos(-x) = \cos x$ → genap
$\sin(-x) = -\sin x$ → ganjil
$\tan(-x) = -\tan x$

$\cos^2 x + \sin^2 x = 1$
$\tan^2 x + 1 = \sec^2 x$
$\cot^2 x + 1 = \csc^2 x$

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TUTORIAL 1

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2d

3a. $(3y+2)^3 = (3y+2)(3y+2)$ $\qquad (a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$

$(9y^2 + 6y + 6y + 4)$ $\qquad$ ↳ binomial

$(9y^2 + 12y + 4)(3y+2)$

$27y^3 + 18y^2 + 36y^2 + 24y + 12y + 8$

$= 27y^3 + 54y^2 + 36y + 8$

4c. bilangan real antara

$-\frac{3}{4}$ dan $-\frac{4}{5}$ bilangan real akan selalu ada

$\boxed{\text{sembarang}}$ fokus y (mau x nya berapapun akan ada)

5d. Untuk setiap bilangan real x, terdapat bilangan real y sehingga

$x^2 + 1 < y$ (Benar)

y supaya benar (cukup 1 → ada)

↳ misal pilih y misal $y = x^2 + 2$ $\qquad x^2 + -$

↳ jangan langsung pilih angka harus ada x nya

fokus x

E. Terdapat bilangan real y, sehingga untuk setiap bilangan bilangan real x

berlaku $x^2 + 1 < y$ ↳ cari dulu y nya ↳ tdk semua x masuk

(salah)

$x^2 < 4 \to -4 < x^2 < 4 \to \sqrt{-4} < x < \sqrt{4} \to -2 < x < 2$

$x^2 + 1 < y$ $\qquad -y+1$ $\qquad$ pasti

$x^2 < y-1 \to -(y-1) < x^2 < y-1$ $\qquad x^2 \geq 0$ $\qquad$ pasti

$-\sqrt{y-1} < x^2 < y-1$ $\qquad x^2 + 1 \geq 1$

$-\sqrt{y-1} < x < \sqrt{y-1}$ $\qquad$ misal $y \in (-\infty, 1)$

↳ $x^2 + 1 \geq 1 \geq y$

fokus x

f. Terdapat bilangan real y, sehingga untuk setiap bilangan real x $\qquad x^2 + 1 \geq y$

berlaku $x^2 + 1 \geq y$ (Benar)

$x^2 + 1 \geq y$

$x^2 \geq y - 1 \to$ $\quad x^2 \geq y-1 \;\cup\; x^2 \leq -(y-1)$ $\quad$ kuantor $y \leq 1$ ada

$x \geq \sqrt{y-1} \;\cup\; x^2 \leq -\sqrt{y-1}$ $\qquad \exists$: terdapat

$(\sqrt{y-1}, \infty) \cup (-\infty, \sqrt{y-1})$ $\qquad \exists$: sehingga

 misal $y = 0 \to (0, \infty) \cup (-\infty, 0) \to (-\infty, \infty)$ $\qquad \in$: anggota

$\forall$: untuk setiap

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6. hitung jarak dan titik tengah

a) (7,2) dan (3,2)

Jarak

$\sqrt{(3-7)^2 + (2-2)^2}$

$= \sqrt{(-4)^2 + 0^2}$

$= \sqrt{16}$

$= 4$

titik tengah

$\left(\frac{7+3}{2}, \frac{2+2}{2}\right)$

$\left(\frac{10}{2}, \frac{4}{2}\right)$

$(5,2)$

7. tentukan persamaan ○ yg memenuhi

(b) memiliki pusat (5,-4) dan melalui (1,-1)

$(x-a)^2 + (y-b)^2 = r^2$

$\cancel{(5-a)^2 + (-4-b)^2}$

$(1-5)^2 + (-1-(-4))^2 = r^2$

$(-4)^2 + (-1+4)^2 = r^2$

$(-4)^2 + (3)^2$

$16 + 9 = \sqrt{25}$

$= 5$ r

$(x-5)^2 (y-1-4) = 5^2$

$(x-5)^2 (y+4) = 25$

8. Tentukan pusat dan jari-jari ○

(c) $x^2 + y^2 + 10y = 56$

melengkapi kuadrat sempurna

$\frac{5}{9}(F - 32)$

$\frac{5}{9} \cdot 100$

$\frac{5}{9}(100 - 32)$

$\frac{5}{9} \cdot 68$

100

$55 - 32$

$= 23$

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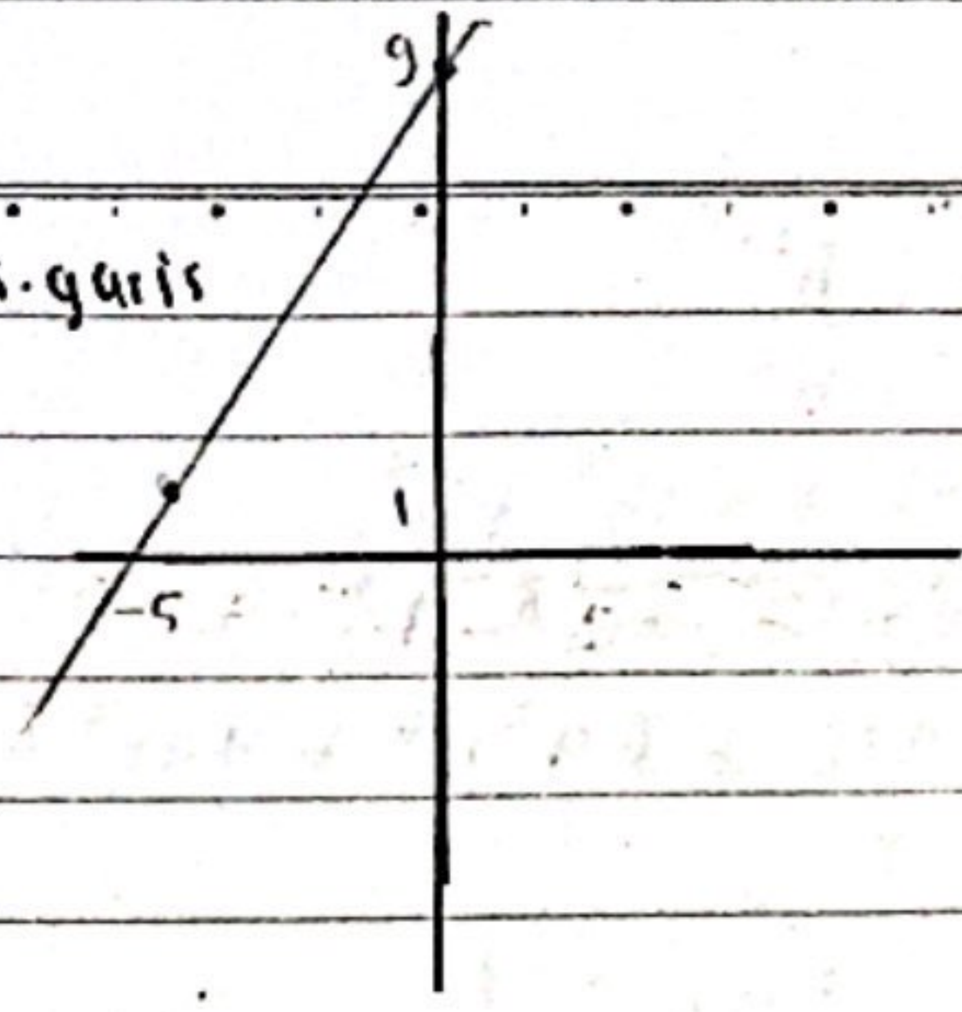
a b

9. Hitung gradien garis dan pers. garis

b. $(\underset{x}{-5},\underset{y}{1})$ dan $(\underset{x}{0},\underset{y}{9})$



$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{9-1}{0-(-5)} = \frac{8}{5}$$

$$y - b = m(x-a)$$

$$y - 1 = \frac{8}{5}(x - (-5))$$

$$y - 1 = \frac{8}{5}(x+5)$$

$$y - 1 = \frac{8}{5}x + \frac{8}{\cancel{5}} \cdot \cancel{5}$$

$$y - 1 = \frac{8}{5}x + 8$$

$$y - 1 = \frac{8}{5}x + 8 + 1$$

$$y = \frac{8}{5}x + 9$$

11 12 - 129

10 b. Tentukan persamaan garis yg memenuhi melalui (1,-2) dan sejajar dengan garis $2y = 3x + 10$

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$(a+b)^2 = a^2 + 2ab + b^2$
$(a-b)^2 = a^2 - 2ab + b^2$
$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$
$(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$
$(a+b)^4 = a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$
$a^2 - b^2 = (a+b)(a-b)$
$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$
$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$

$f(-x) = f(x) \rightarrow$ genap
$f(-x) = -f(x) \rightarrow$ ganjil

fungsi f disebut periodik, jk
terdapat bilangan real p
sehingga untuk setiap $x \in D_f$
berlaku $f(x+p) = f(x)$

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menyinggung titik — Mgg-6

BAB 2. Turunan

No. 25-09-23
Date Senin

Kecepatan sesaat

↳ hukum mekanika klasik

kec sesaat

$x = x(t)$ ⇒ posisi benda → kec hanya saat suatu t

kec. rata² dari $t = a$ s/d $t = b$

$$v[a,b] = \frac{[x(b) - x(a)]}{b - a}$$

kec. sesaat

$$v(a) = \lim_{b \to a} \frac{(x(b) - x(a))}{b - a}$$

Garis singgung - gradien

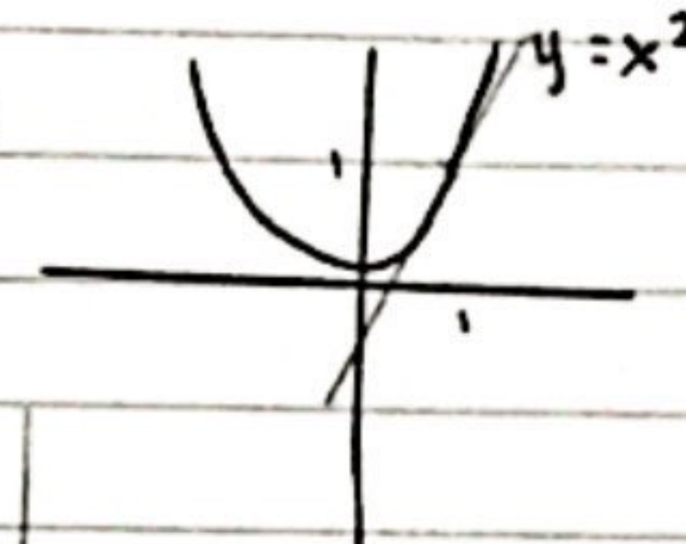
$y = f(x)$ $x = a$, sehingga punya garis singgung $P(a, f(a))$

$P(a, f(a))$

$Q(b, f(b))$ garis singgung pd $y = f(x)$ di $P((a, f(a)) =$

$$\boxed{m_a = \lim_{b \to a} \frac{f(b) - f(a)}{b - a}} \qquad \boxed{y - y_1 = m(x - x_1)}$$

contoh y (0)



$y = x^2$ ditiri k (1,1)

x y

$y = 1^2$ $y = 1$

100 - 4,9

dik $h = 100$ m

$h(t) = 100 - 4{,}9t^2$. kec. sesaat saat $t = 1$

$$v(1) = \lim_{t \to 1} \frac{h(t) - h(1)}{t - 1}$$

$$= \lim_{t \to 1} \frac{100 - 4{,}9t^2 - (100 - 4{,}9)}{t - 1}$$

$$= \lim_{t \to 0} \frac{\cancel{100} - 4{,}9t^2 - \cancel{100} + 4{,}9 \cdot 1}{t - 1}$$

$$= \lim_{t \to 0} \frac{-4{,}9(t^2 - 1)}{(t - 1)}$$

$$= \lim_{t \to 1} \frac{-4{,}9\,\cancel{(t-1)}(t+1)}{\cancel{(t-1)}}$$

$$= \lim_{t \to 1} (-4{,}9)(t+1)$$

$$= \lim_{t \to 1} (-4{,}9)(t+1)$$

$$= (-4{,}9)(2)$$

$$= -9{,}8 \text{ m/s}$$

$$m = \lim_{x \to 1} \frac{f(x) - f(1)}{x - 1} = \frac{x^2 - 1}{x - 1} = \frac{\cancel{(x-1)}(x+1)}{\cancel{(x-1)}}$$

$$\lim_{x \to 1} (x+1) = 2$$

$y - y_1 = m(x - x_1)$

$y - 1 = 2(x - 1)$

latihan

1. $h = 50$

$h_t = 50 - 4{,}9t^2$

$t = 2$

$$v(2) = \lim_{t \to 2} \frac{h(t) - h(2)}{t - 2}$$

$$= \lim_{t \to 2} \frac{50 - 4{,}9t^2 - (50 - 4{,}9t^2)}{t - 2}$$

$$= \lim_{t \to 2} \frac{\cancel{50} - 4{,}9t^2 - \cancel{50} + 4{,}9 \cdot 4}{t - 2}$$

$$\frac{-4{,}9(t^2 - 4)}{t - 2}$$

$$\frac{-4{,}9\,\cancel{(t-2)}(t+2)}{\cancel{(t-2)}}$$

$(-4{,}9)(2+2)$

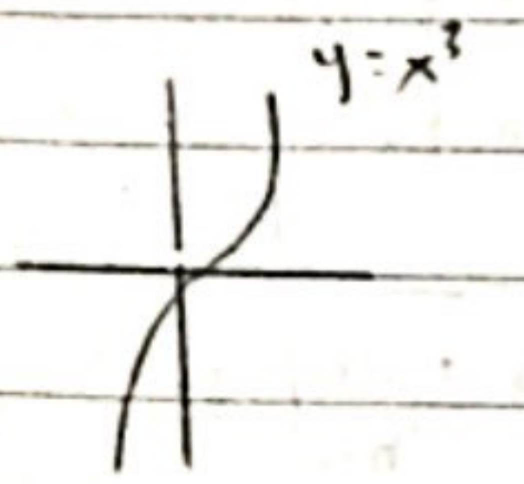
$(-4{,}9)(4) = -19{,}6$ m/s

② tentukan pers garis singgung pd kurva $y = x^3$ di titik (2, 8)

$y = x^3$ $\quad y - y_1 = m(x - x_1)$

$x_1 \quad y_1$

$m = \lim_{x \to 2} \frac{f(x) - f(2)}{x - 2}$



$= \lim_{x \to 2} \frac{x^3 - 2^3}{x - 2}$

$= \lim_{x \to 2} \frac{x^3 - 8}{x - 2}$

$= \lim_{x \to 2} \frac{\cancel{(x-2)}(x^2 + 2x + 4)}{\cancel{(x-2)}}$ $\quad (a-b)(a^2 + ab + b^2)$

$= \lim_{x \to 2} x^2 + 2x + 4$ $\quad (x-2)(x^2 + 2x + 4)$

$2^2 + 2(2) + 4$

$4 + 4 + 4 = 12$

$y - y_1 = m(x - x_1)$

$y - 8 = 12(x - 2)$

$b = a + h$

$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$

$f(x) = x^2 \quad a = 1$

$\lim_{h \to 0} \frac{f(1+h) - f(1)}{h} = \lim_{h \to 0} \frac{(1+h)^2 - 1}{h} =$

$\frac{h^2 + 2h + \cancel{1} - \cancel{1}}{h}$

$\lim_{h \to 0} (2 + h) = 2$

$f'(1) = 2 \quad a^2 \to 2a$

$f'(a) = 2a$

~ Aturan dasar

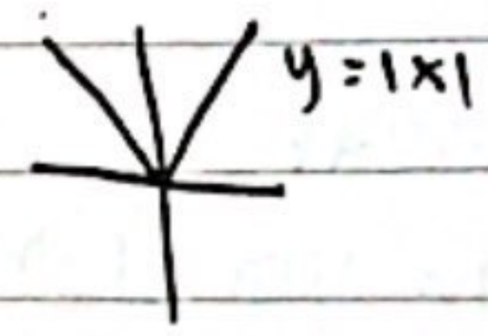
① $f(x) = k \to f'(x) = 0$

② $f(x) = x \to f'(x) = 1$

③ $f(x) = x^n \to f'(x) = n x^{n-1}$

Hub antara turunan & kekontinuan

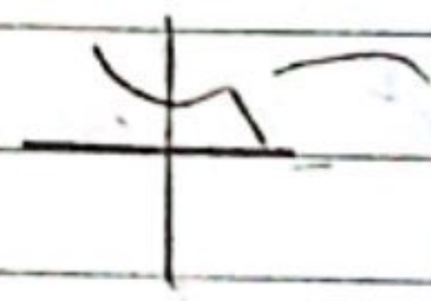
$f(x) = |x|$



↓

kontinu di 0

tapi tak pny turunan 0



yg gk punya turunan hanya yg dilancip nya saja

COBA

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Latihan (15)

1. $f(x) = \sqrt{x}$

$a > 0$ sembarang

3. $f(x) = \frac{1}{x}$ di $a \neq 0$ sembarang

$h = a + h$

$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$

$f(x) = \frac{1}{x} \quad a = a$

$\lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$

$\lim_{h \to 0} \frac{\frac{1}{a+h} - \frac{1}{a}}{h}$

$\lim_{h \to 0} \frac{\frac{a - (a+h)}{a(a+h)}}{h}$ $-a-h$

$\lim_{h \to 0} \frac{-\cancel{h}}{(a^2+ah)\cancel{h}}$

$\lim_{h \to 0} \frac{-1}{a^2+ah}$

$= \frac{1}{a^2+0} = \frac{-1}{a^2}$

2.3 Aturan Turunan

④ $(kf)'(x) \to k \cdot f'(x)$

⑤ $(f+g)'(x) \to f'(x) + g'(x)$

⑥ $(f \cdot g)'(x) \to f'(x) \cdot g(x) + f(x) \cdot g'(x)$

$(u \cdot v) \qquad u'v + v'u$

⑦ $\left(\frac{f}{g}\right)'(x) \to \frac{f'(x)\, g(x) + f(x) \cdot g'(x)}{(g(x))^2}$

$\frac{u}{v} \qquad \frac{u'v + v'u}{v^2}$

Latihan (6) $\quad u \quad v$

a. $f(x) = (x^3+1)\sqrt{x}$ $\qquad \frac{1}{2} - \frac{2}{2} = \frac{-1}{2}$

$u = x^3+1 \qquad v = \sqrt{x}$

$u' = 3x^2 \qquad v' = \frac{1}{2} x^{\frac{1}{2}-1}$

$= \frac{1}{2} x^{-\frac{1}{2}}$

$= \frac{1}{2} \cdot \frac{1}{\sqrt{x}} = \frac{1}{2\sqrt{x}}$

$u'v + v'u$

$3x^2 \cdot \sqrt{x} + \frac{1}{2\sqrt{x}} \cdot x^3 + 1$

$\frac{3x^2 \cdot \sqrt{x}}{1} + \frac{x^3+1}{2\sqrt{x}}$

$\frac{2\sqrt{x}(3x^2\sqrt{x}) + x^3 + 1}{2\sqrt{x}}$

$\frac{6x^2 \cdot x + x^3 + 1}{2\sqrt{x}}$

$\frac{6x^3 + x^3 + 1}{2\sqrt{x}} = \frac{7x^3+1}{2\sqrt{x}}$

minggu-3
DAY 6

BAB 1 Limit dan kekontinuan

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penghantar limit (1.1)

$\hookrightarrow$ pendekatan nilai fungsi

fungsi $f(x) = \dfrac{(x^3-1)}{(x-1)}$ $\quad x-1 \neq 0$

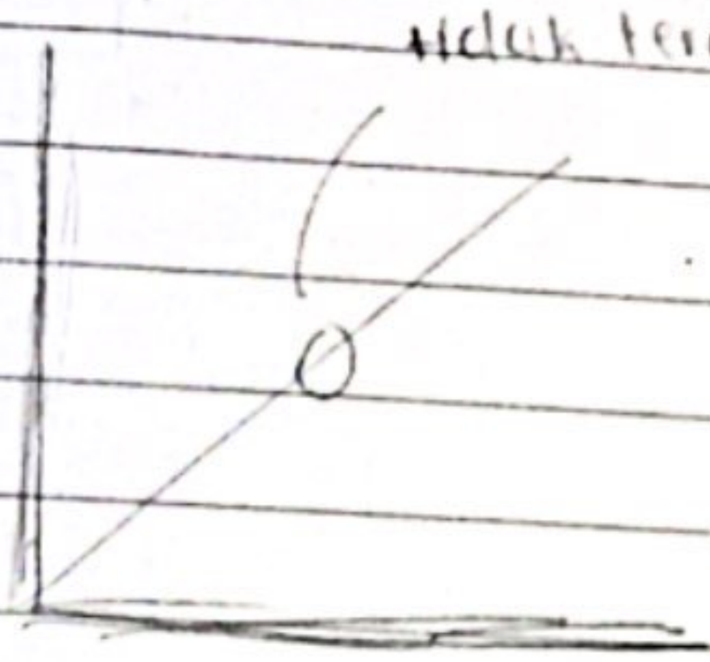
$x \in R - \{1\}$ $\quad x \neq 1$

kanan

x	f(x)
1,1	3,31
1,01	3,0301
1,001	3,003001
1	?
0,999	2,997001
0,99	2,9701
0,9	2,71

kiri

mendekati 3 (secara intuisi)



tidak terdefinisi

bth beda limit dng

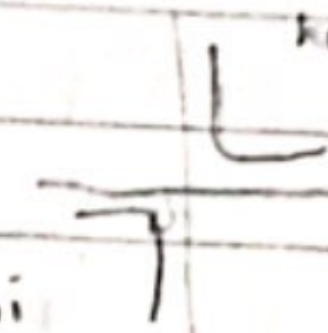
fungsi c tdk sm → diskontinue

fungsi c sama → kontinu

$f(x) = f(c)$

→ c ←

"perilaku dari fungsi" $\downarrow$ u/apa?



kanan

kiri

limit → x ada

bs tetap mencari walau tdk tau

nilai limit
nilai fungsi } sama → kontinu

Makna limit secara intuitif

$\boxed{\lim\limits_{x \to c} f(x) = L}$ disebut "limit f di c"

$\hookrightarrow$ nilai limit

* nilai x didekat c tdk mencangkup $x = c$

jika x dekat ke c, mk f(x) dekat ke L

$\hookrightarrow$ dkt dari kiri/kanan

ex $\lim\limits_{x \to 1} \dfrac{x^3-1}{x-1} = \dfrac{3x^2-1}{x-1} = \dfrac{3x^2}{x} = 3x = 3 \cdot 1 = 3$

Limit kanan dan limit kiri

limit kanan

$\boxed{\lim\limits_{x \to c^+} f(x) = L}$

$x > c$ dan dekat ke c, mk f(x) dekat ke L

limit kiri

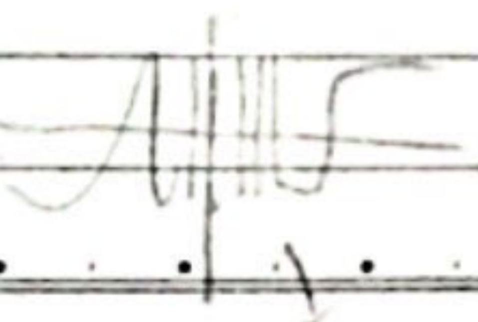
$\boxed{\lim\limits_{x \to c^-} f(x) = L}$

$x < c$ dan dekat ke c, mk f(x) dekat ke L

Bisakah limit ada/tdk ada?

- Limit fungsi di c $\boxed{\text{ada}}$ jika ~~dan hanya jk~~ limit kanan dan limit kiri f di c harus ada dan nilainya sama
- limit f di c $\boxed{\text{tidak ada}}$ jk
  1. limit kanan dan limit kiri f di c nilainya tidak sama
  2. limit kanan dan limit kiri f di c tidak ada, karena
     a. nilai f didekat c menuju tak terhingga
     b. nilai f didekat c berosilasi

nilainya tdk bisa menentukan → bergerak



$\sin\left(\frac{1}{x}\right)$

naik turun

limit kanan tdk selalu sm dengan limit kiri

JOYKO 36 Lines, 6 mm

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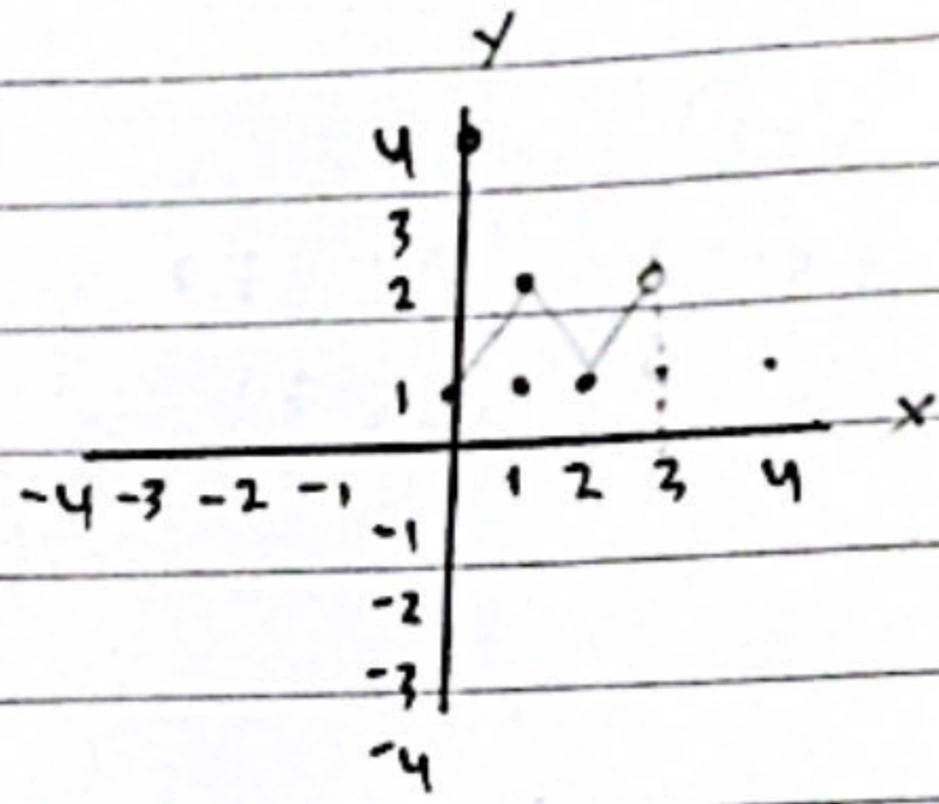
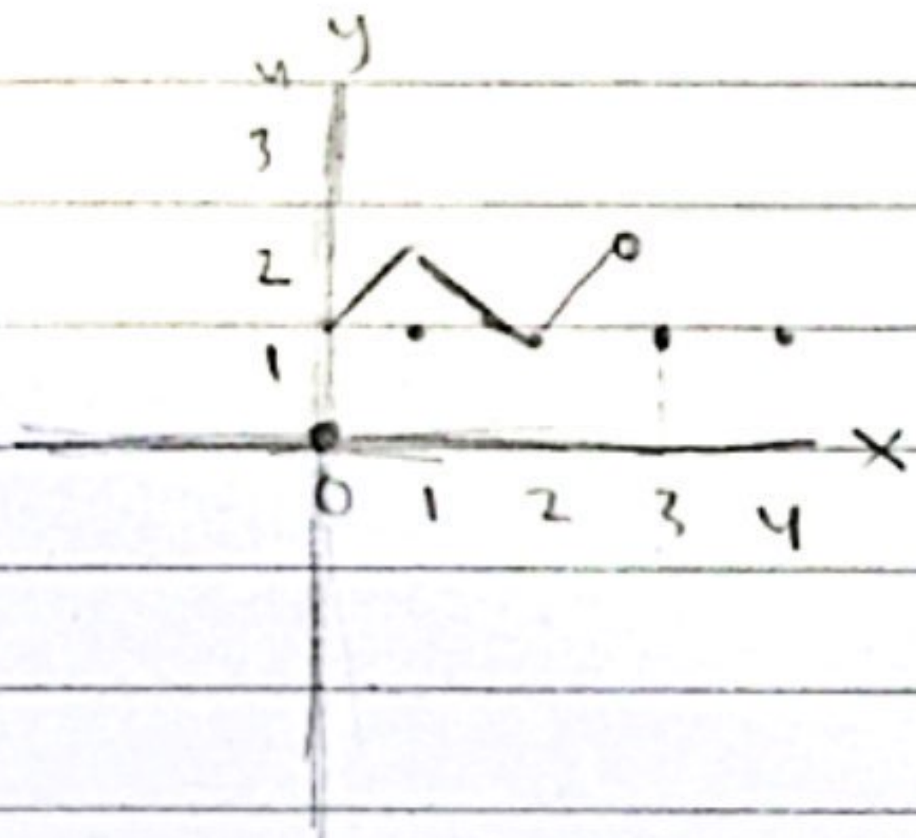
Latihan

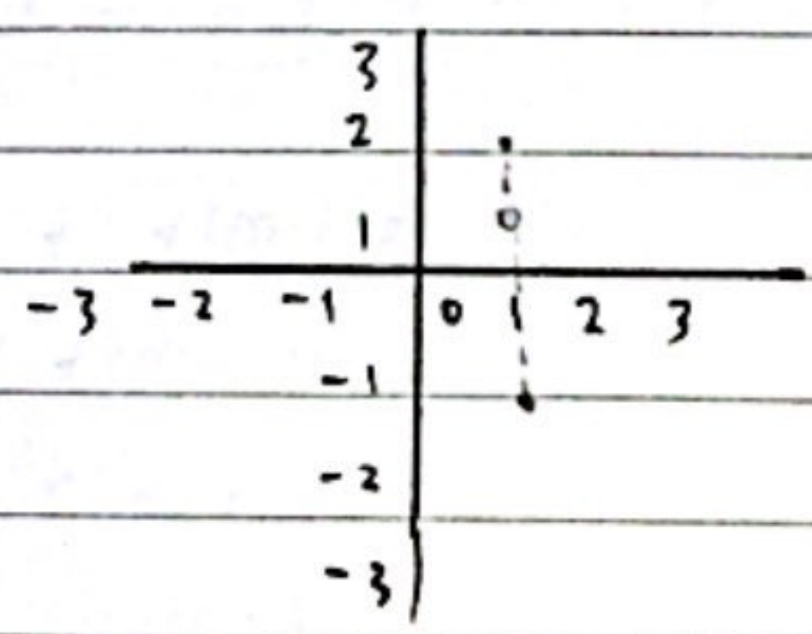
2. Sketsalah grafik suatu fungsi f memenuhi semua syarat berikut



a. daerah asalnya [0,4]
b. f(0) = f(1) = f(2) = f(3) = f(4) = 1
c. limit f di 1 = 2
d. limit f di 2 = 1
e. limit kanan f di 3 = 1
f. limit kiri f di 3 = 2



1. sketsalah
$f(x) = -x$ jk $x < 0$ -1
$f(x) = x$ $0 \leq x < 1$ kiri 1
$f(x) = 1 + x$ $x \geq 1$ kanan 2
tentukan f(1) dan nilai limit f di 1
f(1) = -1 1,-1
f(1) = 1 1,1
f(1) = 2 1,2



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1.2 Limit fungsi :

↳ definisi normal → nilai mullah / definisi presisi Limit

lim mendekati c dari suatu fungsi

menuju / akan menuju

definisi : $\lim_{x \to c} f(x) = L$ jika dan hanya jika "untuk setiap $\varepsilon > 0$ terdapat $\delta > 0$

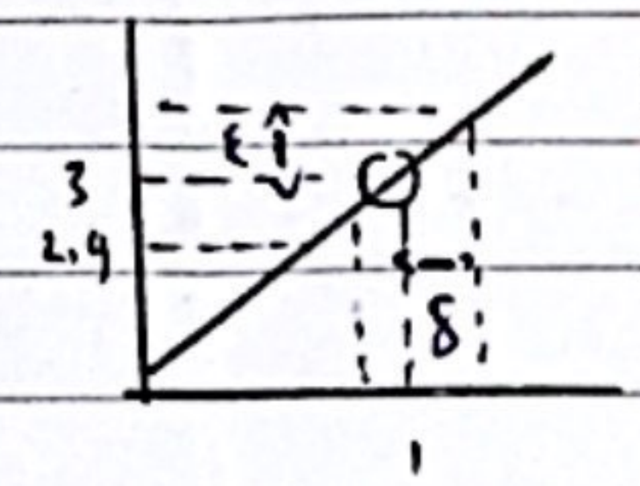
epsilon delta

sehingga : $\boxed{0 < |x - c| < \delta \text{, maka } |f(x) - L| < \varepsilon}$

jarak jarak

+ yg nilainya kecil

lim lit (mendekati 0)



$\frac{(x^3 - 1)}{(x - 1)}$ $0 < |x - c| < \delta \Rightarrow |f(x) - L| < \varepsilon$

Benar/salah $\lim_{x \to c} f(x) = L$

1. Jika $0 < |x - 1| < 0{,}1$, maka $|5x - 5| < 0{,}5$. B $\lim_{x \to 1} 5x = 5$

tinggal di kali 5

dikali 5

3. Jika $0 < |x - 1| < 0{,}005$, maka $|5x - 5| < 0{,}050$ B

$\frac{5 \times}{0{,}025}$ $0{,}025 < 0{,}050$ ✓

dikali 5

4. Jika $0 < |x - 1| < 0{,}005$, maka $|5x - 5| < 0{,}010$ S

$0{,}025$ $0{,}025 > 0{,}010$ ×

5. terdapat $\boxed{\delta = 0{,}002}$ sehingga :
Jika $0 < |x - 1| < \delta$, maka $|5x - 5| < 0{,}010$ B

$0 < |x - 1| < 0{,}002$

$\frac{5 \times}{0{,}010}$

6. terdapat $\boxed{\delta > 0}$, ~~mk~~ sehingga : B
Jika $0 < |x - 1| < \delta$, mk $|5x - 5| < 0{,}001$

kita anggap aja δ $5x = 0{,}001$

$x = \frac{0{,}001}{5} = 0{,}0002$

dikali 5

7. Untuk setiap $\varepsilon > 0$ terdapat $\boxed{\delta = \frac{\varepsilon}{5}}$, sehingga
Jika $0 < |x - 1| < \delta$, maka $|5x - 5| < \varepsilon$

$0 < |x - 1| < \frac{\varepsilon}{5}$ $\times 5$

B

$0 < 5x - 5 < \varepsilon$.

2. Untuk setiap $\varepsilon > 0$ terdapat $\delta > 0$ sehingga:

~~Jk $0 < x < \delta$, mk ...~~

Jk $0 < |x-4| < \delta$, mk $|\sqrt{x}-2| < \varepsilon$

$$\frac{|\sqrt{x}-2||\sqrt{x}+2|}{|\sqrt{x}+2|} = \frac{|x-4|}{\sqrt{x}+2}$$

$|x-2|\,|x+2| < \varepsilon$

$0 < |x-2| < \delta$

$|x+2| = |(x-2)+4| \leq |x-2| + 4 < \delta + 4$

Pilih $\sqrt{x}+2 \geq 2$

artinya: $\cancel{\frac{|x-4|}{\sqrt{x}+2} < |x-4| < \varepsilon}$

$$\frac{|x-4|}{|\sqrt{x}+2|} < \frac{|x-4|}{2} < \frac{\delta}{2} = \varepsilon$$

$$\varepsilon = \frac{\delta}{2} \qquad \frac{1}{2}|x-4| < \varepsilon$$

$$\delta = 2\varepsilon \qquad |x-4| < 2\varepsilon$$

JOYKO® 36 Lines, 6 mm

Lan δ

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Date

ε Benar / salah

fokus ke situ

1. Untuk setiap $\varepsilon > 0$ terdapat $\delta > 0$ sehingga : B
jk $\varepsilon < \boxed{x < \delta}$, maka $|\sqrt{x}| < \varepsilon$

$\delta = \varepsilon^2$ → mengikuti pola — idk bth variabel lain

$\lim_{x \to 0} (\sqrt{x}) = 0$

$\varepsilon < \sqrt{x} < \varepsilon^2$ → diakarin

$\sqrt{x}^2 < \varepsilon^2$

$\sqrt{x} < \varepsilon$

$x < \varepsilon^2$

2. Untuk setiap $\varepsilon > 0$ terdapat $\delta > 0$ sehingga : B
jk $0 < |x-1| < \delta$ maka $|\sqrt{x}-1| < \varepsilon$

$\delta = \varepsilon^2$

$\lim_{x \to 1} \sqrt{x} = 1$

$|x-1| < \varepsilon^2$

$|\sqrt{x-1}| < \sqrt{\varepsilon^2}$

$|\sqrt{x}-1| < \varepsilon$ benar

$|\sqrt{x}-1| < \varepsilon$

$(\sqrt{x}-1)^2 < \varepsilon^2$

$x - 1 < \varepsilon^2$

3. Untuk setiap $\varepsilon > 0$ terdapat $\delta > 0$ sehingga B
jika $0 < |x-2|^2 < \delta$, mka $|x^2-4| < \varepsilon$

$|x-c| < \delta$ → $\delta = ?$ $|f(x) - L| < \varepsilon$

membuktikan bahwa:

$\lim_{x \to 2} x^2 = 4$

$\boxed{\lim_{x \to c} f(x) = L}$ $\boxed{0 < |x-c| < \delta \text{, mk } |f(x) - L| < \varepsilon}$

Latihan

buktikan

1. $\lim_{x \to 3} (2x-5) = 1$ → $0 < |x-3| < \delta$, mk $|(2x-5)-1| < \varepsilon$

$2(3) - 5 = 1$

$2x-5-1$

$|2x-6| < \varepsilon$ → $2|x-3| < \varepsilon$

2. $\lim_{x \to 4} \sqrt{x} = 2$ → $0 < |x-4| < \delta$, mk $|\sqrt{x}-2| < \varepsilon$

$\sqrt{x}^2 - 2^2$

$\sqrt{4} = 2$

$|x-4| < \varepsilon$

3. $\lim_{x \to 2} x^2 = 4$ → $0 < |x-2| < \delta$ mk $|x^2-4| < \varepsilon$

$(x-2)(x+2) < \varepsilon$

$2^2 = 4$

$|x-2| < \varepsilon$

petable

$\forall$ -JK $\exists$ -mk

→ Trigonometri

Teorema apit

$f(x) \le g(x) \le h(x)$

- $\lim_{x \to c} f(x) = \lim_{x \to c} h(x) = L$
- $\lim_{x \to c} g(x) = L$

teorema apit berlaku limit sepihak

sin diantara $-1 < x < 1$

$\boxed{\text{contoh}}$

$x > 0$, dikekat 0

$\boxed{-1 \le \sin\frac{1}{2} \le 1}$ dikalikan x

~~karena lim (-x) = lim~~

$-x \le x\sin\frac{1}{2} \le x$

- $\lim_{x \to 0} (-x) = \lim_{x \to 0} x = 0$
- $\lim_{x \to 0} x \sin\frac{1}{2} = 0$

Latihan

2. Buktikan bahwa $\lim_{x \to 0} x^2 \cos\frac{1}{x} = 0$

$\boxed{-1 \le \cos\frac{1}{x} \le 1}$ dikali x^2

$-x^2 < x^2 \cos\frac{1}{x} \le x^2$

- $\lim_{x \to 0} -x^2 = 0 \quad \lim_{x \to 0} x^2 = 0$
- $\lim_{x \to 0} x^2 \cos\frac{1}{x} = 0$

$\lim_{x \to 0^-} x^2 \cos\frac{1}{x} = 0$

$\lim_{x \to c} \sqrt{x} = \sqrt{c}$

$0 < |x - c| < \delta \Rightarrow |f(x) - L| < \varepsilon$

$0 < |x - c| < \delta \Rightarrow |\sqrt{x} - \sqrt{c}| < \varepsilon$

$\sqrt{x} + \sqrt{c} \ge 0$

$\sqrt{x} + \sqrt{c} \ge \sqrt{c}$

$\frac{1}{\sqrt{x}+\sqrt{c}} \le \frac{1}{\sqrt{c}}$

① $\dfrac{\sqrt{x} - \sqrt{c} \cdot \sqrt{x} + \sqrt{c}}{\sqrt{x} + \sqrt{c}}$

$\dfrac{x + \sqrt{x}\sqrt{c} - \sqrt{x}\sqrt{c} - c}{\sqrt{x} + \sqrt{c}}$

x-c

$\dfrac{|x - c|}{|\sqrt{x} + \sqrt{c}|} < \varepsilon$

~~$\frac{1}{\sqrt{x}+\sqrt{c}} < \frac{1}{\sqrt{c}}$~~

* $\frac{1}{\sqrt{x}+\sqrt{c}} < \frac{1}{\sqrt{c}}$

$\dfrac{|x - c|}{|\sqrt{x} + \sqrt{c}|} < \dfrac{|x - c|}{\sqrt{c}} < \dfrac{\delta}{\sqrt{c}}$ ─ ε

$\varepsilon = \dfrac{\delta}{\sqrt{c}} \qquad \dfrac{1}{\sqrt{c}} |x - c| < \varepsilon$

$\delta = \varepsilon\sqrt{c} \qquad |x - c| < \varepsilon\sqrt{c}$

$\delta = \varepsilon\sqrt{c}$

② $\dfrac{2}{\sqrt{2x+6}+4} < \dfrac{2}{4}$

$\dfrac{2}{\sqrt{2x+6}+4} < \dfrac{1}{2} < \dfrac{\delta}{2}$ ─ ε

$\varepsilon = \dfrac{\delta}{2}$

$\delta = 2\varepsilon$

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$$\lim_{x \to 5} \sqrt{2x+6} = 4$$

$0 < |x - c| < \delta \Rightarrow |f(x) - L| < \varepsilon$

$0 < |x - 5| < \delta \Rightarrow |\sqrt{2x+6} - 4| < \varepsilon$

$$\frac{(\sqrt{2x+6} - 4) \cdot (\sqrt{2x+6} + 4)}{\sqrt{2x+6} + 4} < \varepsilon$$

$$\frac{2x+6 - 16}{\sqrt{2x+6} + 4} < \varepsilon$$

$\sqrt{2x+6} \geq 0$ $^{+4}$

$\sqrt{2x+6} \geq 4$

$\frac{1}{\sqrt{2x+6}} < \frac{1}{4}$

$$\frac{2x - 10}{\sqrt{2x+6} + 4} < \varepsilon$$

* klo bs diperkecil perkecil dl

$$\frac{2|x-5|}{\sqrt{2x+6} + 4} < \frac{2|x-5|}{4} < \varepsilon$$

$$< \frac{1}{2}|x-5| < \varepsilon$$

$$|x-5| < 2\varepsilon$$

$$\delta = 2\varepsilon$$

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TUTORIAL 2

KUIS 1

No. rabu
Date 8-09-23

1. $|2x^2-5| < 3$

2. persamaan O memiliki pusat $(5,-4)$ dan melalui $(1,-1)$

3. sketsa grafik
$y = 4x - x^2$

4. Jelaskan mengapa implikasi berikut benar
$1 < c < 2 \Rightarrow \frac{24}{(1+c)^5} < \frac{3}{4}$

Jawab.

1. $-3 < 2x^2-5 < 3$ (+5)
$2 < 2x^2 < 8$
$1 < x^2 < 4$
$\sqrt{1} < x < \sqrt{4}$
$1 < x < 2 \cup -1 < x < -2$
Hp: $\{(1,2) \cup (-1,-2)\}$

$x^2 < 4$	$x^2 > 1$
$x^2 - 4 < 0$	$x^2 - 1 > 0$
$(x-2)(x+2) < 0$	$(x-1)(x+1)$
$x = 2 \quad x = -2$	$x = 1 \quad x = -1$
$+ \; - \; +$	$+ \; - \; +$
$-2 \quad 0 \quad 2$	$-1 \quad 0 \quad 1$
$-2 < x < 2 \quad (-2,2)$	$x < -1 \overset{\text{atau}}{\cup} x > 1$
	$(-\infty,-1) \cup (1,\infty)$

mk $Hp_1 \overset{\text{gabungan (irisan)}}{\cap} Hp_2$

$-2 \; -1 \; 0 \; 1 \; 2$

$Hp = \{(-2,-1), (1,2)\}$

2. $P(5,-4)$ melalui $(1,-1)$

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3. $y = 4x - x^2$ $a = -1$

$y = -x^2 + 4x$ $b = 4$

$x = 0$ $y = 0$

$y = 0$ $0 = -x^2 + 4x$

$(0,0)$ $(-x\ \ )(\ \ )$

$x(-x+4)$

$x = 0$ $-x + 4 = 0$

$x = 4$

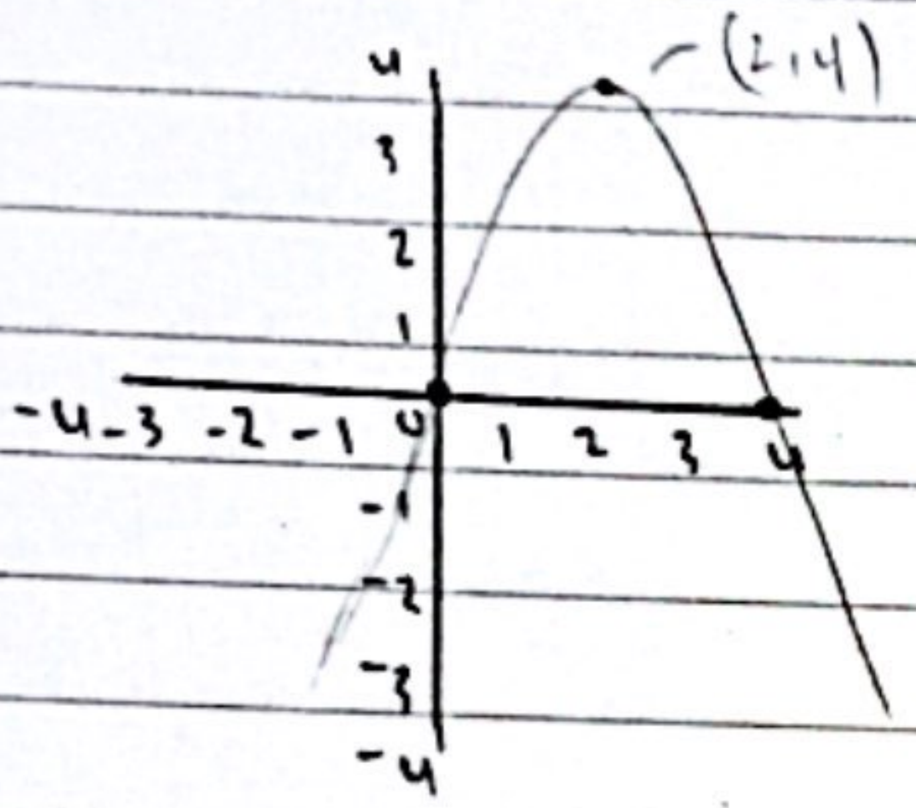
$(0,0)$ $(4,0)$

titik balik

$\left(\frac{-b}{2a}, \frac{-D}{4a}\right)$ $b^2 - 4ac$

$\frac{-4}{2.(-1)}$, $\frac{-4^2 - 4.-1.0}{4.-1}$

$\left(\frac{-4}{-2}, \frac{-16}{-4}\right) = (2, 4)$



$y = 4x - x^2$

4. $1 < c < 2 \Rightarrow \frac{24}{(1+c)^5} < \frac{3}{4}$ +1

ditambah 1

$(=) 2 < c+1 < 3$

~~jadiin 1 per~~

$\cancel{\frac{1}{2} > \frac{1}{c+1} > \frac{1}{3}}$

0 Jadiin 1 per $\boxed{\text{dibalik}}$

$\frac{1}{3} < \frac{1}{c+1} < \frac{1}{2}$ krna C gk akan (−)

dipangkati 5

$\frac{1}{(3)^5} < \frac{1}{(c+1)^5} < \frac{1}{(2)^5}$

$\frac{1}{243} < \frac{1}{(c+1)^5} < \frac{1}{32}$ $\times 24$

$\frac{24}{243} < \frac{24}{(1+c)^5} < \frac{24}{32}$ $: 8$

$\frac{24}{243} < \boxed{\frac{24}{(1+c)^5} < \frac{3}{4}}$

JOYKO 36 Lines, 6 mm

Mgg 1

No. 11 September
Date 2023

LIMIT FUNGSI

δ bilangan dari 1

$0 < |x-2| \delta \Rightarrow |2x-4| < \varepsilon$
$2|x-2| < \varepsilon$
$x-2 \le \frac{\varepsilon}{2}$

$\boxed{UTS}$

$\delta = \frac{\varepsilon}{2}$

Sekecil mungkin

pilih yg minimal dekat ke 0

Buktikan $\lim_{x \to 2} x^2 = 4$

(1) $0 < |x-c| \delta$ $|f(x) - L| < \varepsilon$

jika $0 < |x-2| < \delta$, mk $|x^2-4| < \varepsilon$ #

$|x^2-4| < \varepsilon$ $x^2-4 = |x+2||x-2|$ # $< \varepsilon$

$\cancel{|x-2||x+2| < \varepsilon}$ $\delta \le 1$

dimisalkan

Pilih $(x-2) \to$ minimum | x disekitar 2 maka asumsikan yg mendekati 0

1 (2) 3 $|x+2||x-2| <$

$|x-2| < 1$ $-1 < x-2 < 1$

$-1 < |x-2| < 1$

$-1+2 < x-2+2 < 1+2$

$1 < x < 3$

$3 < x+2 < 5$

$|x+2| < 5$

nilai terbesar 3 sehingga $|x+2|$ terbesar adalah 5

δ $0 < |x-2| < \delta \Rightarrow |x^2-4| < \varepsilon$

$|x+2||x-2| < 5|x-2| < \varepsilon$

$|x-2||x+2| < \varepsilon$

$x-2 < \frac{\varepsilon}{5}$

Kembali ke #

$|x^2-4| < 5|x-2| < \varepsilon$

$|x-2| < \frac{\varepsilon}{5}$

misal $\delta \le 1$ maka nilai terbesar adalah 3 sehingga $|x+2|$ adalah 5

$|x-2| 5 < \varepsilon$

$|x-2| < \frac{\varepsilon}{5}$

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$\lim_{x \to \frac{1}{2}} \frac{1}{x} = 2$

$a < x < b$

$\frac{1}{b} < \frac{1}{x} < \frac{1}{a}$

(1) $0 < |x - c| < \delta$, mk $|f(x) - 2| < \varepsilon$

$0 < |x - \frac{1}{2}| < \delta \Rightarrow |\frac{1}{x} - \frac{2}{x}| < \varepsilon$

$\left|\frac{1}{x} - 2\right| < \varepsilon$

$\left|\frac{1 - 2x}{x}\right| < \varepsilon$

$\frac{2x-1}{\frac{1}{4}} < \varepsilon$

$|1 - 2x| < \varepsilon$

$-\frac{1}{1} + \frac{1}{2} = \frac{-2+1}{2}$

*dibalik $-(|2x| - 1) < \varepsilon$

$4|2x - 1| < \varepsilon$

$\frac{|2x-1|}{|x|} < \varepsilon$

$\frac{2x \cdot 2 - 1}{\frac{1}{2}} < \varepsilon$

$\frac{2x-1}{\frac{1}{4}} < \varepsilon$

$|2x - 1| < \varepsilon$

misal $|x - \frac{1}{2}| < 1$

$4(2x - 1) < \varepsilon$

$2x - 1 < \frac{\varepsilon}{2}$

$-1 + \frac{1}{2} < x < 1 + \frac{1}{2}$

$2|x - \frac{1}{2}| < \frac{\varepsilon}{2}$

$(x - \frac{1}{2}) < \frac{\varepsilon}{4}$

$-\frac{1}{2} < x < \frac{3}{2}$

$|x - \frac{1}{2}| < \frac{\varepsilon}{4}$

$-\frac{1}{2} < |x| < \frac{3}{2}$

$\delta = \frac{\varepsilon}{4}$

$x - \frac{1}{2} < \frac{\varepsilon}{4}$

*jadikan 1per

$\frac{2}{3} < \boxed{\frac{1}{|x|}} < 2$

$-\frac{1}{4} < x < \frac{3}{4}$

$|x| < \frac{1}{2}$

$\frac{4}{3} < \left|\frac{1}{|x|}\right| < 4$

$x - \frac{1}{2} < \frac{\varepsilon}{8}$

$1 < 4|x|$

$|x| < \frac{1}{4}$

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$\lim_{x \to \frac{1}{2}} \frac{1}{x} = 2$

$0 < |x - \frac{1}{2}| < \delta \Rightarrow |\frac{1}{x} - 2| < \varepsilon$

$\left|\frac{1}{x} - \frac{2}{1}\right| < \varepsilon$

$\left|\frac{1 - 2x}{x}\right| < \varepsilon$

$\left|\frac{2x - 1}{x}\right| < \varepsilon$

$\boxed{\frac{|2x-1|}{|x|} < \varepsilon}$

misal $\left|x - \frac{1}{2}\right| < \frac{1}{4}$

$+\frac{1}{2}$ $\quad$ $\frac{1}{4}$

$-\frac{1}{4} < x - \frac{1}{2} < \frac{1}{4}$

$-\frac{1}{4} < |x| < \frac{3}{4}$

* jadiin
1 per

$\frac{4}{3} = \boxed{\frac{1}{|x|} = 4}$

$|x| < \frac{1}{4}$

δ min $\left(\frac{1}{4}, \frac{\varepsilon}{8}\right)$

$\frac{1}{4} + \frac{1}{2}$

$\frac{|2x-1|}{\frac{1}{4}} < \varepsilon$

$4|2x-1| < \varepsilon$

$|2x-1| < \frac{\varepsilon}{4}$

$2\left|x - \frac{1}{2}\right| < \varepsilon \to 4$

$\left|x - \frac{1}{2}\right| < \frac{\varepsilon}{8}$

$\lim_{x \to 2} \frac{1}{x} = \frac{1}{2}$

$0 < |x - c| < \delta \Rightarrow |f(x) - L| < \varepsilon$

$0 < |x - 2| < \delta \Rightarrow \left|\frac{1}{x} - \frac{1}{2}\right| < \varepsilon$

$\left|\frac{1}{x} - \frac{1}{2}\right| < \varepsilon$

$\left|\frac{2 - x}{2x}\right| < \varepsilon$

$\frac{|2-x|}{|2x|} < \varepsilon$

$\boxed{\frac{|x-2|}{|2||x|} < \varepsilon}$

misal $|x - 2| < 1$

$-1 < x - 2 < 1$

$1 < x < 3$

$\frac{1}{3} < \boxed{\frac{1}{|x|} < \frac{1}{1}}$

$|x| < 1$

$\frac{|x-2|}{|2||1|} < \varepsilon$

$\frac{|x-2|}{2} < \varepsilon$

$|x - 2| < 2\varepsilon$

$\delta = \min(1, 2\varepsilon)$

$\frac{}{3})$

$\sin 2a = 2 \sin a \cos a$

Limit fungsi Trigono

(1)

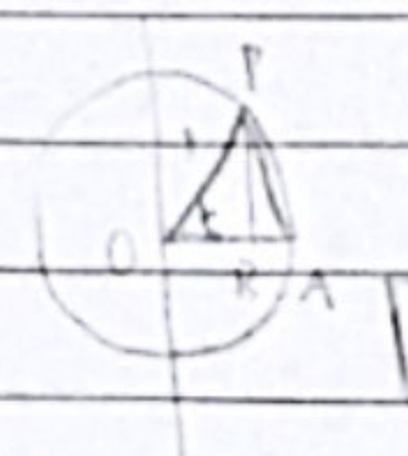
No. 13-09-23
Date rabu

* bukan ° tapi radian

$\lim_{x \to c} \sin x = \sin c \qquad 0 < |BP| < |AP|$

$|AP| < t$

$|BP| = \sin t$



$\boxed{0 < \sin t < t}$

OPA = juring

APO = tembereng

$\frac{1}{2} t r^2$

$\lim_{t \to 0^+} \sin t = 0$

$\lim_{t \to 0^-} \sin t = 0$

$\lim_{x \to c} \sin x = \sin c$

misalkan

$\lim_{t \to 0} \sin t \cdot \cos 2t \rightarrow \frac{\sin t \cdot \cos 2t}{\sin 2t}$

$\frac{\sin t \cdot \cos 2t}{2 \sin t \cos t}$

$\frac{\cos 2t}{2 \cos t} = \frac{\cos 2(0)}{2 \cos 0}$

$= \frac{\cos(0)}{2 \cos(0)}$

$= \frac{1}{2}$

b. $\lim_{x \to 0} \frac{\sin(x^2 + 2x)}{2x^2 + x}$

$\lim_{x \to 0} \frac{\sin(x^2+2x)}{2x^2+x} \cdot \frac{2x^2-x}{2x^2-x}$

$\lim_{x \to 0} \frac{\sin(x^2+2x)}{x^2+2x} \quad \lim_{x \to 0} \frac{x^2+2x}{2x^2+x}$

gampang aja

misal $t = x^2 + 2x$ karena $x \to 0 \quad t \to 0$

$L = \lim_{t \to 0} \frac{\sin(t)}{t} \quad \lim_{x \to 0} \frac{x(x+2)}{x(2x+1)}$

$1. \lim_{x \to 0} \frac{0+2}{0+1} = \frac{2}{1} = 2$

2.

$\lim_{t \to 0} \frac{1 - \cos t}{t} \cdot \frac{1 + \cos t}{1 + \cos t}$

$\lim_{t \to 0} \frac{1 - \cos^2 t}{t(1 + \cos t)} \qquad \sin^2\theta + \cos^2\theta = 1$

$\lim_{t \to 0} \frac{\sin^2 t}{t(1+\cos t)} = \frac{\sin t}{t} \cdot \frac{\sin t}{1 + \cos t}$

$\lim_{t \to 0} = 1 \cdot \frac{\sin(0)}{1 + \cos(0)}$

$= 1 \cdot \frac{0}{1+1}$

$= 1 \cdot \frac{0}{2}$

$= 0$

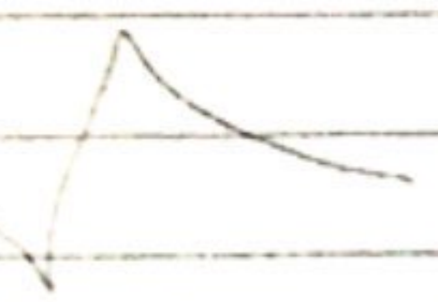
Limit Trigonometri khusus

1. $\lim_{t \to 0} \frac{\sin t}{t} = 1 \qquad \left\{ \cos t \le \frac{\sin t}{t} \le 1 \right\}$

2. $\lim_{t \to 0} \frac{1 - \cos t}{t} = 0$

5. Limit di Tak Hingga dan Limit Tak Hingga

$x \to \infty$ / $x \to -\infty$



Limit di tak hingga $[<, \infty)$

$\lim f(x) = L$

$\lim\limits_{x \to \infty} f(x) = L$ dicari M ???   sebesar mungkin

$\varepsilon > 0$ $M > 0$ sekecil mungkin

$\boxed{\text{Jika } x > M, \ |f(x) - L| < \varepsilon}$

ex: $\lim\limits_{x \to \infty} \frac{1}{x} = 0$

[jika $x > ??$, maka $\left|\frac{1}{x} - 0\right| < \varepsilon$

$\left|\frac{1}{x} - 0\right| < \varepsilon$

$\frac{1}{x} < \varepsilon \Rightarrow x > \frac{1}{\varepsilon}$

kap mutlak bisa dilepas

$x > M$ $M = \frac{1}{\varepsilon}$

Buktikan

$\lim\limits_{x \to \infty} \frac{x}{x^2+1} = 0$ $\varepsilon > M$ $M \geq \frac{1}{\varepsilon}$

Jika $x > \ldots$, maka $\left|\frac{x}{x^2+1} - 0\right| < \varepsilon$

bil terbesar

$\frac{x}{x^2+1} < \frac{1}{x} < \varepsilon$ bil terkecil

ide

$\frac{x}{x^2+1}$ $x > \frac{1}{\varepsilon}$ $M = \frac{1}{\varepsilon}$

$\frac{1}{x+1}$ $x > M$

mgg-6 → quiz (rabu)

Mgg-5

No. 18-09-23
Date senin

sangat bsr

limit ditakhingga → terdapat M besar $x > M$

limit tak hingga → terdapat N besar
diminus

$|f(x) - L| < \varepsilon \Leftarrow x < N$ sangat (-)

$\boxed{\text{Jika } x < N \text{, maka } |f(x) - L| < \varepsilon}$

kerjaan mundur

contoh $\lim\limits_{x \to -\infty} \frac{1}{x} = 0$ jika $x < \ldots$, mak $\left|\frac{1}{x} - 0\right| < \varepsilon$

mundur

maju

$f(x) - L$

$\left|\frac{1}{x} - 0\right|$ jangn ditulis dulu ε nya

bisa sama

$\left|\frac{1}{x}\right| < \varepsilon$

harus dibalik tandanya kna

$-\frac{1}{x} < \varepsilon$

$-\frac{1}{x} < \varepsilon$

dikali (-) → tanda berubah

$\frac{1}{x} > -\varepsilon$

$\frac{1}{x} > -\varepsilon$ dikali (-)

$x < -\frac{1}{\varepsilon}$

$x < \frac{1}{-\varepsilon}$ dijadiin 1 per

$\frac{1}{\text{mundur}}$

contoh

$\lim\limits_{x \to -\infty} \frac{x}{x^2+1} = 0$

sangat kecil (+)

Jika $x < \ldots$, maka $\left|\frac{x}{x^2+1} - 0\right| < \varepsilon$

$\frac{x}{x^2+1} < \frac{1}{x}$ berlaku karena kw dikali

$\left|\frac{x}{x^2+1}\right| < \varepsilon$

$\frac{-x}{x^2} \quad \frac{-1}{x}$

$\left|\frac{x}{x^2+1} < \frac{-1}{x}\right|$ $x < 0$

$\frac{-x}{x^2+1} < -\frac{1}{x} < \varepsilon$

$x(-1)$

• mk tanda berubah

$-\frac{1}{2} < -1$

$\frac{x}{x^2+1} > \frac{1}{x} > -\varepsilon$

$-\ldots > -1$

disatu perkan
• tanda berubah

$\frac{x}{x^2+1} < x < \frac{-1}{\varepsilon}$

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$N \leq -\frac{1}{\varepsilon}$

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dibagi pangkat tertinggi atau dibagi x

nilai limitnya sendiri yg tak hingga

Limit tak hingga

limit kanan x mendekati c

o Limit kanan

limit ada jika limit kanan = kiri

limit kanan x mendekati c

$\lim_{x \to c^+} f(x) = +\infty$

besar bgt   kecil bgt

untuk setiap $M > 0$ terdapat $\delta > 0$,
sehingga $\boxed{0 < x - c < \delta, \text{ mk } f(x) > M}$

jarak x-c dekat

mendekati c dari kanan

c x

cari δ

o Limit kiri

$\lim_{x \to c^-} f(x) = +\infty$

$N < 0$

untuk setiap $M > 0$ terdapat $\delta > 0$
sehingga $\boxed{0 < c - x < \delta, \text{ mk } f(x) > M}$

& $f(x) < -M$
$f(x) < N$

contoh

$\lim_{x \to 0^+} \frac{1}{x} = +\infty$ Jika $0 < x < \dots$ maka $\frac{1}{x} > M$

Jika $0 < x - 0 < \delta$..., maka $\frac{1}{x} > M$

$0 < x < \delta \Rightarrow \frac{1}{x} > M$

$0 < x < \frac{1}{M}$ — δ

2. $\lim_{x \to 0^-} \frac{1}{x} = -\infty$

Jika $0 < 0 - x < \delta$, maka $f(x) < N$
$0 < -x < \delta$, maka $\frac{1}{x} < N$

x (-)

$-\frac{1}{x} > -N$

dibalik

$-x < -\frac{1}{N}$ — δ

bukti

$0 < 0 - x < \delta$

misal $\delta = -\frac{1}{N}$

$0 < -x < -\frac{1}{N}$

$-x < -\frac{1}{N}$

$x > \frac{1}{N}$

$\frac{1}{x} < N$

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Latihan

1\. a. $\lim\limits_{x\to\infty} \frac{x^3-1}{x^3-1} = 1$

c. $\lim\limits_{x\to\infty} \sqrt{x+1} - \sqrt{x} = \frac{(\sqrt{x+1}-\sqrt{x})}{1}\frac{(\sqrt{x+1}+\sqrt{x})}{(\sqrt{x+1}+\sqrt{x})} = \frac{x+1-x}{\sqrt{x+1}+\sqrt{x}} = \frac{1}{\sqrt{x+1}+\sqrt{x}}$

$= \frac{1}{\infty} = 0$

b. $\lim\limits_{x\to\infty} \frac{\sqrt{x^4-1}}{x^2+1} = \frac{x^2}{x^2} = 1$

d. $\lim\limits_{x\to\infty} \frac{\sqrt{x^2+1}}{x} = \frac{x}{x} = 1$

2\. Hitunglah

$\lim\limits_{x\to 2^+} \frac{x}{4-x^2} = -\infty$ $\qquad$ $\lim\limits_{x\to 2^-} \frac{x}{4-x^2} = \infty$

mendekati 2 dari kanan

limit

x dari kanan $-\infty$

menuju 2

1.6 KEKONTINUAN

Fungsi kontinu disuatu titik

$\lim\limits_{x\to c} f(x) = \overset{=L}{f(c)}$ → Fungsi f dikatakan kontinu di c

untuk setiap $\varepsilon > 0$ terdapat $\delta > 0$ sehingga:

$\boxed{\text{Jika } |x-c| < \delta, \text{ maka } |f(x)-f(c)| < \varepsilon}$

<u>keluarga fungsi kontinue</u>

1\. fungsi polinom (° drajat).. disetiap R (bil·real) · fungsi rasional kontinu pd daerah asal (tdk menyebabkan $\frac{}{0}$)

2\. Fungsi nilai mutlak $f(x) = |x|$ kontinu pd $(0,\infty)$. Fungsi ini f· kontinu kanann di $c = 0$

3\. Fungsi akar $f(x) = \sqrt{x}$ $\qquad y = \sqrt{x}$

$y^2 = x$

4\. Fungsi $f(x) = \sin x$ $g(x) = \cos x$ kontinu pd bil real (R)

No ......
Date ......

Jika f dan g kontinu di c, mk

$kf, f+g, f-g, fg, \frac{f}{g}, f^n$, dan $\sqrt[n]{f}$ kontinu di c

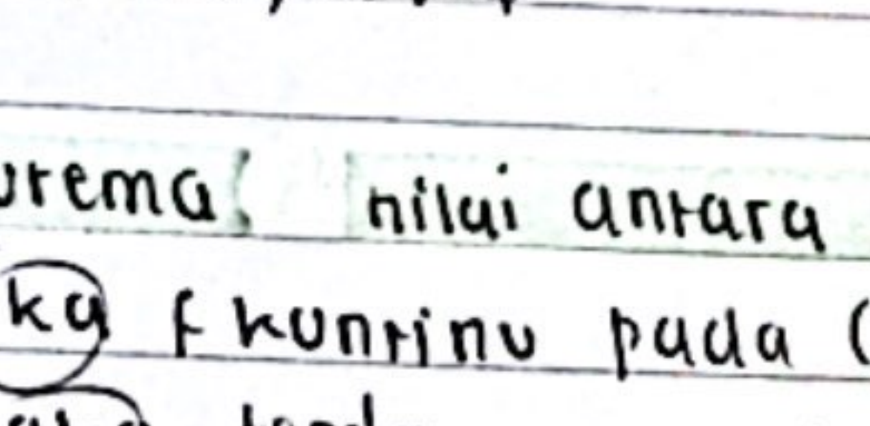
Jika $\lim_{x \to c} g(x) = L$ dan f kontinu di L, mk $\lim_{x \to c} f(g(x)) = f(L)$

| Jika g kontinu di c dan f ~~kontinu~~ kontinu di g(c), mk $f \circ g$ kontinu di c |

Fungsi kontinu pd Selang Tutup!

murni = kiri & kanan

Fungsi f dikatakan kontinu pd [a,b] apabila f kontinu pada (a,b), kontinu kanan di a dan kiri di b



kontinu di antara a dan b

kontinu di kiri dari batas kanan

kontinu di kanan dari

premis = ~~syarat cukup~~

Jika ..., maka

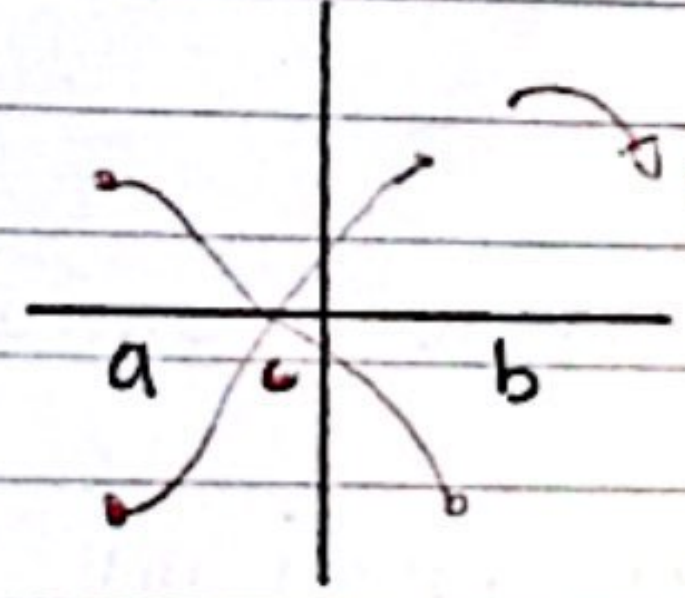
Teorema nilai antara (TNA)

harus berlawanan

Jika f kontinu pada (a,b), f(a)<0 dan f(b)>0 atau sebaliknya, maka terdapat $c \in (a,b)$, sehingga f(c)=0

f(a) (-) F(b) +

$\lim_{x \to c} f(x) = f(c)$



tanpa mengangkat O

ada yg sm dngn O karena

pasti memotong sumbu x

$\frac{\sqrt{0}}{0} = 0$

Latihan (21)

$\lim_{x \to c} f(x) = f(c)$ No. Date

1. Tentukan nilai L agar f dikontinu di 1

$f(x) = \frac{\sqrt[3]{x} - 1}{x - 1}$, $x \neq 1$

$= L$, $x = 1$

$\lim_{x \to 1^-} f(x) = \lim_{x \to 1^+} f(x) = f(1)$

$1 = \lim_{x \to 1^+} \frac{\sqrt[3]{x} - 1}{x - 1} = L$

2. Tentukan a dan b agar f kontinu disetiap titik

$f(x) = -1$, $x < -1$ mendekati -1 dari kiri

$= ax^3 + b$, $-1 \leq x \leq 1$

-1 0 1

$= 2$, $x > 1$ mendekati dari dari kanan

$\boxed{\lim_{x \to c} f(x) = f(c)}$

$\lim_{x \to -1^-} f(x) = \lim_{x \to -1^+} f(x) = f(-1)$ (kiri, kanan, hasil)

$-1 = \lim ax^3 + b = -a + b$

$f(x) = ax^3 + b$

$f(-1) = a(-1)^3 + b = -1$

$f(-1) = -a + b = -1$ $b = -1 + a$

$\lim_{x \to 1^-} f(x) = \lim_{x \to 1^+} = f(1)$

$2 = \lim ax^3 + b = a + b$

$f(1) = a + b = 2$

$a + (-1 + a) = 2$

$a - 1 + a = 2$

$2a = 2 + 1$

$2a = 3$

$a = \frac{3}{2}$

$-a + b = -1$

$a + b = 2$ $-$

$-2a = -3$

$a = +\frac{3}{2}$

$b = -1 + a$

$= \frac{-1}{1} + \frac{3}{2} = \frac{-2+3}{2} = \frac{1}{2}$

$-a + b = -1$

$-\frac{3}{2} + b = -1$

$b = \frac{-1}{1} + \frac{3}{2}$

$= \frac{-2+3}{2} = \frac{1}{2}$

3. buktikan bahwa $p(x) = x^5 - x - 1$ (polinom), mempunyai akar positif

p(x) adalah polinom

p(x) terdefinisi di $\mathbb{R}$

maka p(x) juga $\boxed{\text{terkontinu di } [0,2]}$ pilih selang

tujuan → supaya ada yg berlawanan

$\boxed{\text{Jika } p(0) = -1 < 0 \text{ dan } p(2) = 29 > 0}$, berdasarkan TNA c sebagai akar p(x) sehingga $c \in [0,2]$ dan $p(c) = 0$, karena $c \in (0,2)$

c positif, sehingga terbukti p(x) mempunyai positif

fungsi. hrs berlawanan.

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$$f(x)\begin{cases}\dfrac{\sqrt[3]{x}-1}{x-1} ; & x \neq 1 \\ L , & x = 1\end{cases}$$

supaya f kontinu di L harus $\lim\limits_{x\to 1} f(x) = f(1)$

$\lim\limits_{x\to 1^-} f(x) = \lim\limits_{x\to 1^+} f(x) = f(1)$

$\lim\limits_{x\to 1^-} \left(\dfrac{\sqrt[3]{x}-1}{x-1}\right) \cdot \left(\dfrac{\sqrt[3]{x}+1}{\sqrt[3]{x}+1}\right) = \quad " \quad = f(1)$

$\lim\limits_{x\to 1^-} \quad x$

MANDIRI

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- Kalkulus → limit
- mendekati tapi tdk menyentuh

blh dicoret karena ada limitnya
/ blh

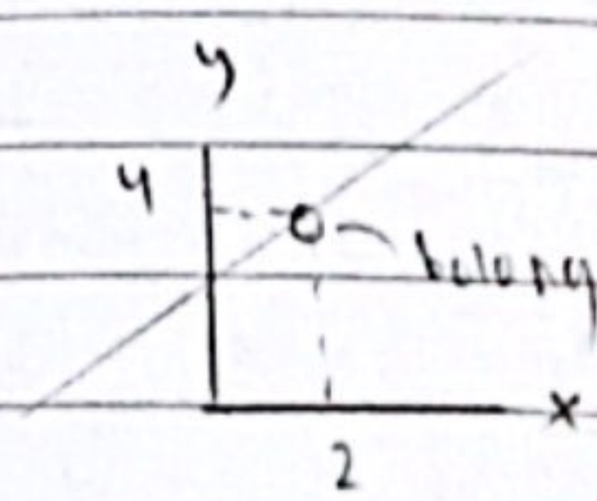
$$\lim_{x \to 2} f(x) = \lim_{x \to 2} \left(\frac{x^2 - 4}{x - 2} \right) = \lim_{x \to 2} \frac{(x+2)(\cancel{x-2})}{(\cancel{x-2})} = \lim_{x \to 2} (x+2) = 4$$

$(x+2) = 4$

$x + 2 = 4$

$x = 2$

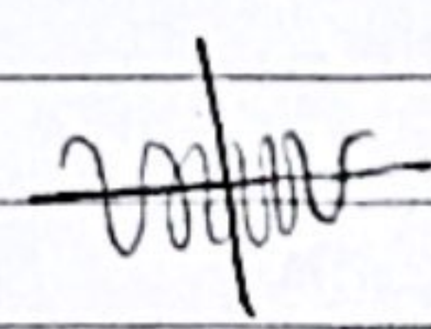
$(2, 4)$



Apakah limit selalu ada?

- $\lim_{x \to 0} \frac{1}{x} = \infty$, limit tdk ada

- berosilasi parah, fungsi ini ketika mendekati 0, nilainya tdk menuju ke 1 nilai tertentu



- grafik fungsi ada loncatan, sehingga limit kanan & kiri tdk ada nilainya

tdk sm

- $\lim_{x \to c} f(x) = L$, jk dan hanya jika $\lim_{x \to c^+} f(x) = L$ (kanan) dan $\lim_{x \to c^-} f(x) = L$ (kiri) → lim kanan = lim kiri

Teorema Limit

1. $\lim_{x \to c} k = k$

2. $\lim_{x \to c} x = c$

3. $\lim_{x \to c} k f(x) = k \lim_{x \to c} f(x)$

4. $\lim_{x \to c} [f(x) \pm g(x)] = \lim_{x \to c} f(x) \pm \lim_{x \to c} g(x)$

5. $\lim_{x \to c} [f(x) \cdot g(x)] = \lim_{x \to c} f(x) \cdot \lim_{x \to c} g(x)$

6. $\lim_{x \to c} \frac{f(x)}{g(x)} = \frac{\lim_{x \to c} f(x)}{\lim_{x \to c} g(x)}$, dngn syarat $\neq 0$

7. $\lim_{x \to c} [f(x)]^n = \left[\lim_{x \to c} f(x) \right]^n$ limitin dl baru dipangkatin / dikaliin

8. $\lim_{x \to c} \sqrt[n]{f(x)} = \sqrt[n]{\lim_{x \to c} f(x)}$, jk n bilangan genap syarat ≥ 0

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Teorema subtitusi

u/ fungsi polinom dan fungsi rasional $\frac{f(x)}{g(x)}$

$$\boxed{\lim_{x\to c} f(x) = f(c)}$$

↳ berlaku asal penyebut $\neq 0$

$\lim_{x\to 2} \frac{(x+2)(x-2)}{(x-2)}$ → blh dicoret kina ada limitnya
→ blh disubstitusi kalo tidak $\frac{0}{0}$

Teorema Apit

$f(x)$, $g(x)$, $h(x)$

$f(x) \le g(x) \le h(x)$

$\lim_{x\to c} f(x) = \lim_{x\to c} h(x) = L$

kesimpulan $\lim_{x\to c} g(x) = L$

LIMIT FUNGSI TRIGONOMETRI

$$\boxed{\lim_{x\to c} \sin x = \sin c}$$ → teorema subtitusi

$$\boxed{\lim_{t\to 0} \frac{\sin t}{t} = 1 \qquad \lim_{t\to 0} \frac{1-\cos t}{t} = 0}$$

→ limit trigonometri khusus

$L\bigcirc = \pi r^2$

sudut $= 2\pi$

rumus juring $\boxed{\frac{1}{2} t r^2}$

op = r

$\cos t = \frac{OB}{r}$

$OB = r \cdot \cos t$

$|OB| = r \cos t = 1 \cos t = \cos t$

$|BP| = r \sin t = 1 \sin t = \sin t$

$|OA| = r = 1$

(2) Bukti 2

$\lim_{t\to 0} \frac{1-\cos t}{t} = \lim_{t\to 0} \left(\frac{1-\cos t}{t} \cdot \frac{1+\cos t}{1+\cos t} \right)$

$\lim_{t\to 0} \frac{(1-\cos^2 t)}{t(1+\cos t)} = \lim_{t\to 0} \frac{\sin^2 t}{t(1+\cos t)} =$

$\lim_{t\to 0} \frac{\sin t}{t} \times \frac{\sin t}{1+\cos t}$

$\lim_{t\to 0} 1 \times \frac{0}{1+1} = 0$

(3) $\lim_{t\to 0} \frac{\tan t}{t} = \lim_{t\to 0} \frac{\frac{\sin t}{\cos t}}{t} = \lim_{t\to 0} \frac{\sin t}{t(\cos t)}$

$\lim_{t\to 0} \frac{\sin t}{t} \cdot \frac{1}{\cos t}$

$1 \cdot \frac{1}{1} = 1$

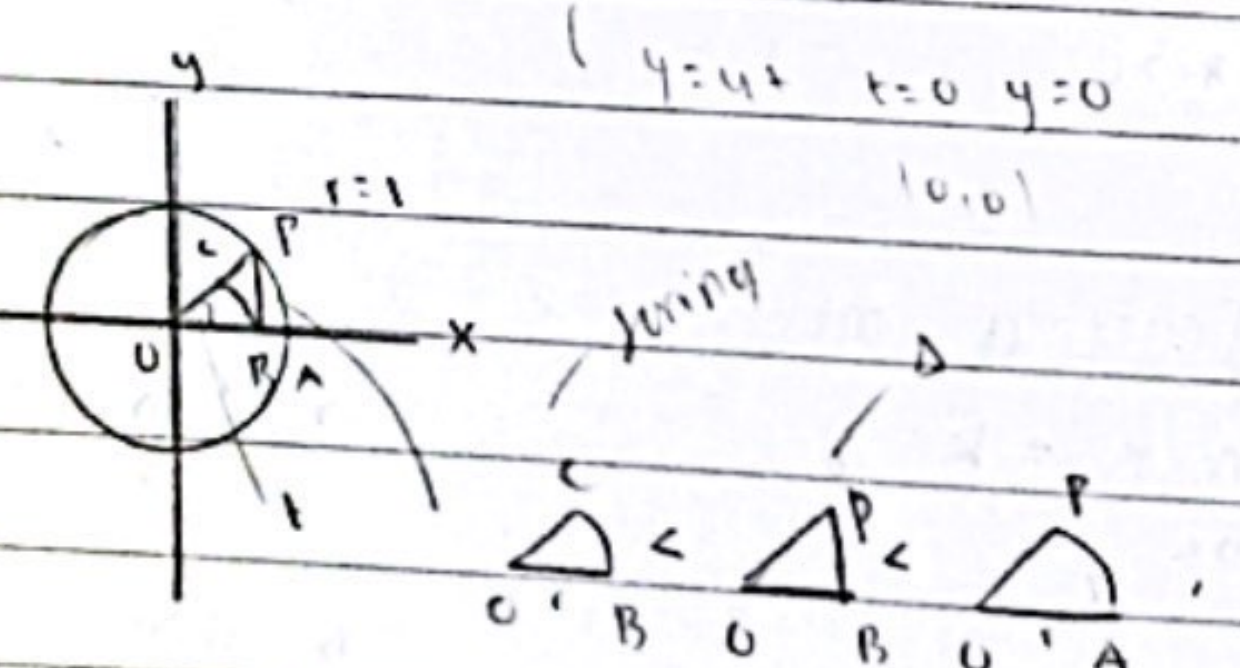
$$\boxed{\lim_{t\to 0} \frac{\tan t}{t} = 1}$$

$\lim_{t\to 0} \frac{\sin 4t}{t} = \frac{\sin 4t}{t} \cdot \frac{4}{4}$

$= \frac{\sin 4t}{4t} \cdot \frac{4}{1} = 1 \cdot 4 = 4$

$y = 4t \quad t=0 \quad y=0$

(1) Bukti 1



luas OBC < luas OBP < luas OAP

$\frac{1}{2} t r^2 < \frac{1}{2} \cdot a \cdot t < \frac{1}{2} t r^2$

$\frac{1}{2} t(\cos t)^2 < \frac{1}{2} \cdot \cos t \cdot \sin t < \frac{1}{2} t (1)^2$

biarsm x2: $t(\cos t)^2 < \cos t \sin t < t$

$t \cos t < \sin t < \frac{t}{\cos t}$ — bagi $\cos t$

$\cos t < \frac{\sin t}{t} < \frac{1}{\cos t}$ — bagi t

tambahin limit: $\lim_{t\to 0} \cos t < \lim_{t\to 0} \frac{\sin t}{t} < \lim_{t\to 0} \frac{1}{\cos t}$

$1 < 1 < 1$

$\frac{1}{0} = \infty$

$\frac{\quad}{\infty} = 0$ $\quad$ $\frac{0}{1} = 0$

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Limit//∞

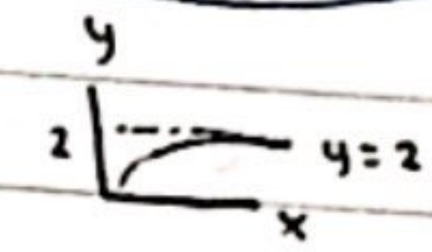
Limit tidak hingga

dibagi pangkat tertinggi

$$\lim_{x \to \infty} \frac{2x^2}{x^2+3} = \frac{\infty}{\infty}$$

$$= \frac{\frac{2x^2}{x^2}}{\frac{x^2+3}{x^2}} = \frac{2}{1+\frac{3}{x^2}} = \frac{2}{1+\frac{3}{\infty}} = \frac{2}{1+0} = \frac{2}{1} = 2$$

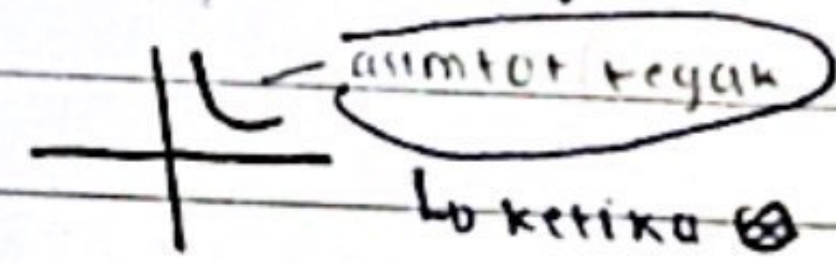
asimtot datar y



Ketakhinggaan limit

$\lim_{x \to 0^+} \frac{1}{x} = \infty$, limit sebenernya tdk ada

asimtot tegak

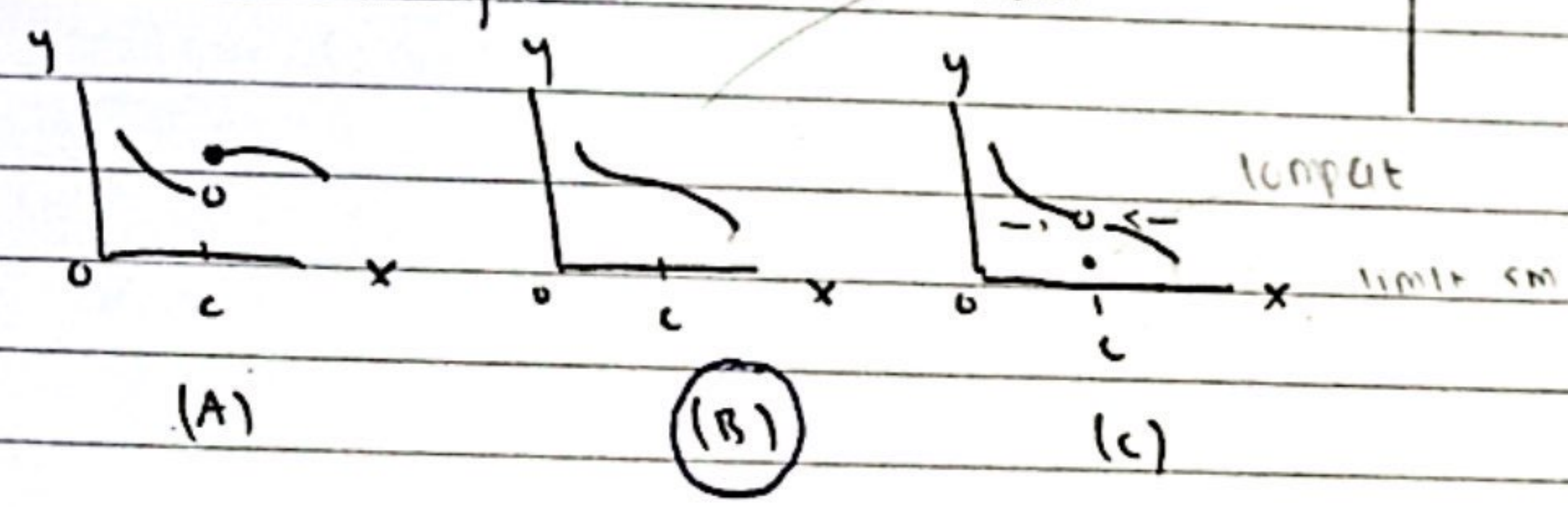


$\lim_{x \to 0^-} \frac{1}{x} = -\infty$

Kekontinuan fungsi

berlanjut (tdk putus)

harus



✓ Fungsi f(x) dinyatakan kontinu pada titik x = c jk

$\boxed{\lim_{x \to c} f(x) = f(c)}$

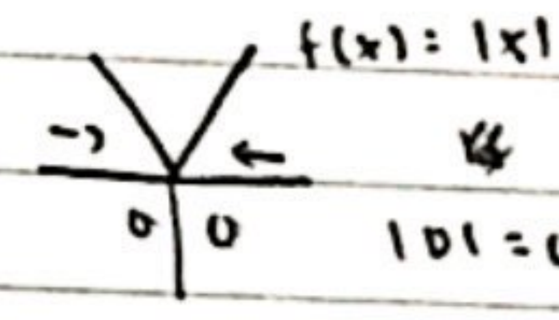
limit $\quad$ fungsi

- ada jika limit kanan = limit kiri
- pada titik x = c , nilai fungsi ada
- $\lim_{x \to c} f(x) = f(c)$
- Harus digambar tanpa pensil lepas

✓ kekontinuan dari fungsi familiar

- polinom $x^2 + 2x + x^3$ kontinu diseluruh bilangan real.
- fungsi rasional

  $\frac{dx^2+ex+fx^3}{ax^2+bx+cx^3}$, kontinu pd bil real, kecuali penyebut = 0
- fungsi mutlak f(x) = |x| , kontinu pd seluruh bil real



$|0| = 0$

- fungsi akar

  $\sqrt[n]{f(x)}$, kontinu pd bilangan real jk n bilangan ganjil dan jk n bil. genap $f(x) > 0$
- fungsi trigonometri , kontinu pd seluruh bil real.

  tan, cot, cosec, sec → kontinu pd domain

✓ perkalian, penjumlahan, pengurangan, pembagian kontinu jk f(x) || g(x) b kontinu

$\sqrt{7}$)

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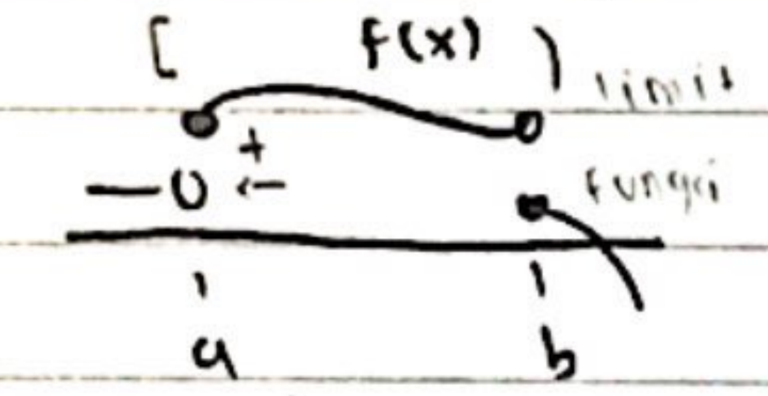
✓ kekontinuan pd interval



[a,b]

(a,b)

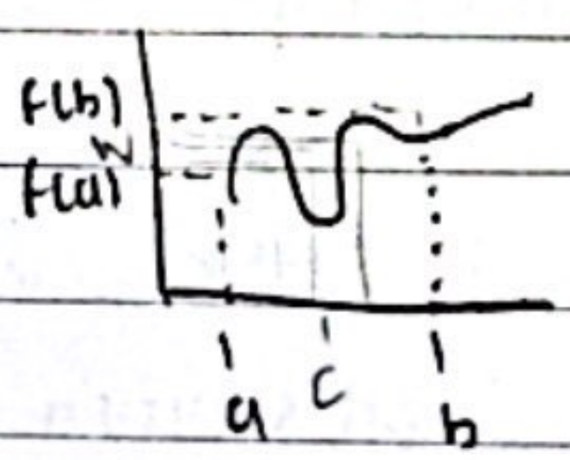
[a,b)

$$\lim_{x \to c^+} f(x) = f(c)$$

$$\lim_{x \to c^-} f(x) = f(c)$$

✓ Teorema nilai antara

fungsi f kontinu [a,b], mk ada c diantara a dan B memiliki nilai fungsi = W
W adalah nilai diantara f(a) dan f(b)



f(c) = W

Part 1

asimtot tegak = ∞

! cari δ

penyebab 0 $x=2$ $x=0$

Buktikan

$\lim\limits_{x \to 2} 2x+8 = 12$

$0 < |x-c| < \delta \Rightarrow |f(x)-L| < \varepsilon$

$0 < |x-2| < \delta \Rightarrow |2x+8-12| < \varepsilon$

$|2x+8-12| < \varepsilon$

$|2x-4| < \varepsilon$

$|2(x-2)| < \varepsilon$

$|x-2| < \frac{\varepsilon}{2}$ maka $\delta = \frac{\varepsilon}{2}$

Problem

1. Hitung limit brk

$\lim\limits_{x \to 0} \left(\frac{1}{x} + \frac{2}{x(x-2)}\right) = \lim\limits_{x \to 0} \left(\frac{(x-2)+2}{x(x-2)}\right) = \lim\limits_{x \to 0} \frac{x}{x(x-2)} = \frac{1}{x-2} = \frac{1}{0-2} = \frac{-1}{2}$

2. Tentukan nilai dari konstanta a agar f kontinu di $x=1$, jk f didefinisikan

$f(x) = \begin{cases} ax^2-1, & x<1 \text{ kiri} \\ x, & x \geq 1 \text{ kanan} \end{cases}$

$1=1$

$f(x)$ $\boxed{\lim\limits_{x \to 1^-} f(x) = \lim\limits_{x \to 1^+} f(x) = f(1)}$

$\lim\limits_{x \to 1^-} ax^2-1 = \lim\limits_{x \to 1^+} x = 1$

$\lim\limits_{x \to 1^-} a(1)^2-1 = \lim\limits_{x \to 1^+} 1 = 1$

$a-1 = 1 = 1$

$a-1=1$

$a=2$

Ln $\lim\limits_{x \to 1^-} 2x^2-1 = 1$

limit kiri

$a-1$

3. Tentukan semua asimtot dari fungsi brk

$f(x) = \frac{x-2}{|x|}$

- Asimtot tgk $(x-2)|x|$ $\lim\limits_{x \to 0^{+-}}$ X

cek $x=2$ X

$\lim\limits_{x \to 2} \frac{(x-2)(x-2)}{(\cancel{x-2})(x)} = \lim\limits_{x \to 2} \frac{x-2}{|x|} = \frac{2-2}{2} = \frac{0}{2} = 0$

cek $x=0$

$\lim\limits_{x \to 0^+} \frac{x-2}{|x|} = -\infty$

$\lim\limits_{x \to 0^-} \frac{x-2}{|x|} = -\infty$

$x=0$ merupakan asimtot tegak

- Asimtot datar $\lim\limits_{x \to \infty}$ Y

$\lim\limits_{x \to \infty} \frac{x-2}{|x|} = \frac{x-2}{x} = \frac{x}{x} - \frac{2}{x} = 1 - \frac{2}{\infty}$

pangkat tertinggi

$= 1-0$

$= 1$

$y=1$ merupakan asimtot datar

$\lim\limits_{x \to -\infty} \frac{x-2}{|x|} = \frac{x-2}{-x} = \frac{x}{-x} - \frac{2}{-x} = -1 + \frac{2}{x}$

$= -1 + \frac{2}{\infty}$

$= -1 - 0$

$= -1$

$y=-1$ adalah asimtot datar

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part 2

1. $$\lim_{x\to 0} \frac{\tan 4x + \tan 2x}{\sin 3x - \tan x} = \frac{4+2}{3-1} = \frac{6}{2} = 3$$

$\frac{\tan 4x}{4x} \cdot 4$

$$\frac{\frac{\tan 4x}{x} \cdot \frac{4}{4} + \frac{\tan 2x}{x} \cdot \frac{2}{2}}{\frac{\sin 3x}{x} \cdot \frac{3}{3} - \frac{\tan x}{x} \cdot \frac{1}{1}} = \frac{1 \cdot 4 + 1 \cdot 2}{1 \cdot 3 - 1} = \frac{4+2}{3-1} = 3$$

2. $$\lim_{x\to \frac{\pi}{3}} \frac{\sin\left(x - \frac{\pi}{3}\right)}{3x - \pi} =$$ misal $x - \frac{\pi}{3} = p$

$\lim_{p\to 0}$

$$\lim_{x\to \frac{\pi}{3}} \frac{\sin\left(x-\frac{\pi}{3}\right)}{3\left(x-\frac{\pi}{3}\right)} = \lim_{p\to 0} \frac{\sin p}{3p} = \frac{1}{3}$$

3. $$\lim_{x\to 0} \frac{x \sin x}{1 - \cos x} = \frac{0}{0} \times$$

$$\lim_{x\to 0} \frac{x \sin x}{1-\cos x} \cdot \frac{1+\cos x}{1+\cos x} = \lim_{x\to 0} \frac{x \sin x (1+\cos x)}{1-\cos^2 x} = \lim_{x\to 0} \frac{x \sin x (1+\cos x)}{\sin^2 x}$$

buh $(a-b)^2 = (a+b)(a-b)$ $\sin x$

$$\lim_{x\to 0} \frac{x(1+\cos x)}{\sin x} = \frac{1+\cos x}{\frac{\sin x}{x}} = \frac{1+\cos(0)}{1} = \frac{1+1}{1} = 2$$

part 3

1. Buktikan bahwa

$$\lim_{x\to 2} (x^2 + x - 3) = 3$$

$x=1$ $x=-6$ -2 3

terkecil

$0 < |x-c| < \delta \Rightarrow |f(x) - L| < \varepsilon$

$0 < |x-2| < \delta \Rightarrow |x^2 + x - 3 - 3| < \varepsilon$

$|x^2 + x - 6| < \varepsilon$

$|(x+3)(x-2)| < \varepsilon$

$|x+3||x-2| < \varepsilon$

$|x+3||x-2| < 6|x-2| < \varepsilon$

$|x-2| < \frac{\varepsilon}{6}$

jd pilih $\delta = \frac{\varepsilon}{6}$

$\min\left(\frac{\varepsilon}{6}, 1\right)$

nilai x tak jauh dari 2
misal $\delta \leq 1$, maka x terbesar adalah 3
sehingga $|x+3|$ terbesar adalah 6

Part 4

1. $\lim_{x\to 2} \frac{x^2-4}{x-\sqrt{3x-2}}$

$\lim_{x\to 2} \frac{x^2-4}{x-\sqrt{3x-2}} \cdot \frac{x+\sqrt{3x-2}}{x+\sqrt{3x-2}}$

$\lim_{x\to 2} \frac{(x^2-4)(x+\sqrt{3x-2})}{x^2-(\sqrt{3x-2})^2}$

$\lim_{x\to 2} \frac{(x-2)(x+2)(x+\sqrt{3x-2})}{x^2-(3x-2)}$

$\lim_{x\to 2} \frac{(x-2)(x+2)(x+\sqrt{3x-2})}{x^2-3x+2}$ $\quad x=2 \quad -2 \cdot 1$, $x=1$

$\frac{\cancel{(x-2)}(x+2)(x+\sqrt{3x-2})}{\cancel{(x-2)}(x-1)}$

$\lim_{x\to 2} \frac{(x+2)(x+\sqrt{3x-2})}{(x-1)}$

$\frac{(4)(2+\sqrt{6-2})}{1}$

$\frac{(4)(2+2)}{1} = 16$ //

$\lim_{x\to 0} x^2 \cos\frac{1}{x} = 0$

• $-1 \le \cos\frac{1}{x} \le 1$

$-x^2 \le x^2\cos\frac{1}{x} \le x^2$

$\lim_{x\to 0^+} -x^2 \le \lim_{x\to 0^+} x^2\cos\frac{1}{x} \le \lim_{x\to 0^+} x^2$

$0 \le \lim_{x\to 0^+} x^2\cos\frac{1}{x} \le 0$

$\lim_{x\to 0^+} x^2\cos\frac{1}{x} = 0$

• $-1 \ge \cos\frac{1}{x} \ge 1$

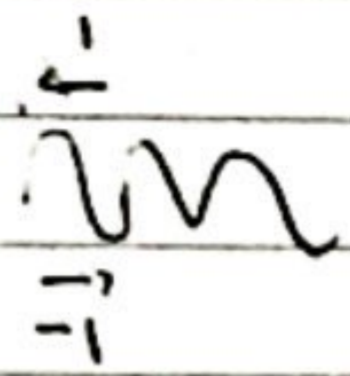
$-x^2 \ge x^2\cos\frac{1}{x} \ge x^2$

$\lim_{x\to 0^-} x^2\cos\frac{1}{x} = 0$

PART 5

$\lim_{x\to 0} x\sin\left(\frac{1}{x^2}\right)$ berhub. dngn teorema apit

$x \begin{cases} (+) \text{— tanda tetap} \\ (-) \text{— tanda berubah} \end{cases}$

sin 

$-1 \le \sin\frac{1}{x^2} \le 1$

$\cancel{\le \lim x \frac{1}{x^2}}$ $\quad x \begin{cases} 0^+ \\ 0^- \end{cases}$

• $x > 0$, $\lim_{x\to 0^+}$ kanan $<$ • $x < 0$, $\lim_{x\to 0^-}$ kiri $>$

$\left(\lim_{x\to 0^+} \sin\frac{1}{x^2} = \lim_{x\to 0^-} \sin\frac{1}{x^2} = 0\right)$

$\boxed{-x \le x\sin\frac{1}{x^2} \le x}$ $\quad$ $\boxed{-x \ge x\sin\frac{1}{x^2} \ge \text{kiri } x}$

$\lim_{x\to 0^+} -x \le \lim_{x\to 0^+} x\sin\frac{1}{x^2} \le \lim_{x\to 0^+} x$ $\quad$ $\lim_{x\to 0^-} -x \ge \lim_{x\to 0^-} \sin\frac{1}{x^2} \ge \lim_{x\to 0^-} x$

$0 \le \lim_{x\to 0^+} x\sin\frac{1}{x^2} \le 0$ $\quad$ $0 \ge \lim_{x\to 0^-} \sin\frac{1}{x^2} \ge -0$

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$\boxed{\lim_{x\to 0^+} \sin\frac{1}{x^2} = 0}$ $\quad$ $\boxed{\lim_{x\to 0^-} \sin\frac{1}{x^2} = 0}$

Pekan-7
Senin
No Date 2-10-23

Notasi leibniz

$\Delta y = f(x+\Delta x) - f(x)$

bagi dngn Δx

$\boxed{\dfrac{\Delta y}{\Delta x} = \dfrac{f(x+\Delta x) - f(x)}{\Delta x}}$

$\Delta x \to 0$ ≈ benambah sedikit bngt

$\lim\limits_{\Delta x \to 0} \dfrac{\Delta y}{\Delta x} = \lim\limits_{\Delta x \to 0} \dfrac{f(x+\Delta x) - f(x)}{\Delta x} = f'(x)$ misal $\Delta x = h$

h

↓ menggunakan lambang $\dfrac{dy}{dx}$ u/ menyatakan $\lim\limits_{\Delta x \to 0} \dfrac{\Delta y}{\Delta x}$, jika $y = f(x)$

$\boxed{\dfrac{dy}{dx} = f'(x)}$ / $\dfrac{dy}{dx} = y'$

contoh

$y = x^3 + x$ maka $\dfrac{dy}{dx} = 3x^2 + 1$

dy dx

Aturan rantai dalam notasi leibniz

$y = f(u) \quad u = g(x)$

$\dfrac{dy}{dx} = \dfrac{dy}{du} \cdot \dfrac{du}{dx}$

contoh

$y = (\underbrace{x^3 + x}_{u})^{10} = u^{10}$

$u = x^3 + x$

$\dfrac{dy}{dx} = 10\, x^3 + x^{10-1} \cdot 3x^2 + 1$

$= 10(x^3 + x)^9 \cdot 3x^2 + 1$

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f(x)	f'(x)
1. $\sin(x)$	$\to \cos(x)$
2. $\cos(x)$	$\to -\sin(x)$
3. $\tan(x)$	$\to \sec^2(x)$
4. $\cot(x)$	$\to -\csc^2 x$
5. $\sec(x)$	$\to \sec(x)\cdot\tan(x)$
6. $\csc(x)$	$\to -\csc(x)\cdot\cot(x)$

① buktikan $\tan(x) \to \sec^2(x)$

$\tan(x) = \dfrac{\sin(x)}{\cos(x)}$ $-u$ $\quad$ $u = \sin(x)$ $\quad$ $v = \cos(x)$

$\qquad\qquad\quad\; v$ $\quad$ $u' = \cos(x)$ $\quad$ $v' = -\sin(x)$

$$\frac{u'v - v'u}{v^2}$$

$$\frac{\cos(x)\cos(x) - (-\sin(x))\sin(x))}{\cos^2(x)}$$

$\sin^2(x)$

$$\frac{\cos^2(x) + \sin^2 x}{\cos^2(x)} = \frac{1}{\cos^2(x)} = \sec^2(x)$$

② buktikan $\sec(x) \to \sec(x)\cdot\tan(x)$

$\sec(x) = \dfrac{1}{\cos(x)}$ $-u$ $\quad$ $u = 1$ $\quad$ $v = \cos(x)$

$\qquad\qquad\quad\; v$ $\quad$ $u' = 0$ $\quad$ $v' = -\sin(x)$

$$\frac{u'v - v'u}{v^2}$$

~~v' = sin(x)~~

$$= \frac{0\cdot\cos(x) - (-\sin(x)\cdot 1)}{\cos^2 x}$$

$$= \frac{\sin(x)}{\cos^2(x)} = \frac{\sin(x)}{\cos(x)}\cdot\frac{1}{\cos(x)} \pm$$

$$\frac{1}{\cos(x)}\cdot\frac{\sin(x)}{\cos(x)}$$

$$\sec(x)\cdot\tan(x)$$

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2.5 Aturan rantai

g mempunyai turunan di x dan f mempunyai turunan di $U = g(x)$

$\boxed{(f \circ g)'(x) = f'(g(x)) \cdot g'(x)}$

$g(x) = U$

contoh : $h(x) = (1 + 0{,}5x)^{12}$

U

$g(x) = U = 1 + 0{,}5x$ $\quad g'(x) = 0{,}5$

$f(U) = U^{12}$ $\quad f'(U) = 12U^{11}$

$(f \circ g)'(x) = f'(g(x)) \cdot g'(x)$

$= 12\,U^{11} \cdot 0{,}5$

$= 12(1 + 0{,}5x)^{11} \cdot 0{,}5$

$= 6(1 + 0{,}5x)^{11}$

Latihan

$f(x) = U = x^2 + 1$ $\quad f'(x) = 2x$

a. $f(x) = \sqrt{x^2+1}$

U

$h(U) = \sqrt{U}$ $\quad h'(U) = U^{\frac{1}{2}}$ $= \frac{1}{2}U^{-\frac{1}{2}}$

$f'(x) : (x^2+1)^{\frac{1}{2}} = \frac{1}{2}(x^2+1)^{-\frac{1}{2}} \cdot 2x = x(x^2+1)^{-\frac{1}{2}}$

$\frac{1}{2}U^{-\frac{1}{2}}$

$x \cdot \frac{1}{\sqrt{x^2+1}}$

$= \frac{x}{\sqrt{x^2+1}}$

$(h \circ f)(x) = h'f(x) \cdot f'(x)$

$= \frac{1}{2}(x^2+1)^{-\frac{1}{2}} \cdot 2x$

$x(x^2+1)^{-\frac{1}{2}}$

$x \cdot \frac{1}{\sqrt{x^2+1}}$

$2\sqrt{}$

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Limit

↳ dekat tapi tidak jadi

$$\lim_{x \to c} f(x) \iff \lim_{x \to c^-} f(x) = \lim_{x \to c^+} f(x)$$

limit tdk hrs sm dngn fungsi

$\varepsilon > 0, \delta > 0$

$\forall x \quad 0 < |x - c| < \varepsilon \Rightarrow 0 < |f(x) - L| < \delta$

Teorema Apit

$f(x) \leq g(x) \leq h(x)$

$\lim_{x \to c} f(x) = \lim_{x \to c} h(x) = L$

Latihan

(9) $\lim_{x \to a} g(x)^5 = 32$, mk $\lim_{x \to a} \frac{3}{\sqrt[3]{g(x)}}$

$$\left(\lim_{x \to a} g(x)\right)^5 = 32$$

$$\lim_{x \to a} g(x) = \sqrt[5]{32}$$

$$\lim_{x \to a} g(x) = 2$$

Sifat khusus

* $\lim_{x \to a} f(x) = f(a)$ → terdefinisi

substitusi ✗

↓

* pemfaktoran

$3x - 12 = 3(x - 4)$

$x^2 - y^2 = (x + y)(x - y)$

$x - 3 = (\sqrt{x} - \sqrt{3})(\sqrt{x} + \sqrt{3})$

contoh

$$\lim_{x \to 9} \frac{\sqrt{x} - 3}{x - 9}$$