

Momentum & Impuls

- momentum (\vec{p}) $\Rightarrow \vec{p} = m \cdot \vec{v}$

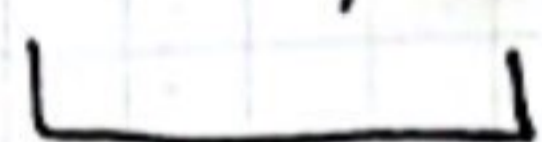
- Impuls (\vec{J}) $\Rightarrow \vec{J} = \vec{F} \cdot \Delta t$

- Teorema Impuls Momentum

$$\vec{J} = \Delta \vec{p}$$

$$\vec{J} = \vec{F} \cdot \Delta t = m \vec{v}_f - m \vec{v}_i$$

$$\vec{p}_i, \vec{v}_i, \vec{J}, \vec{F}$$



vektor

Gambar & arah

① Dik: $m = 0,40 \text{ kg}$

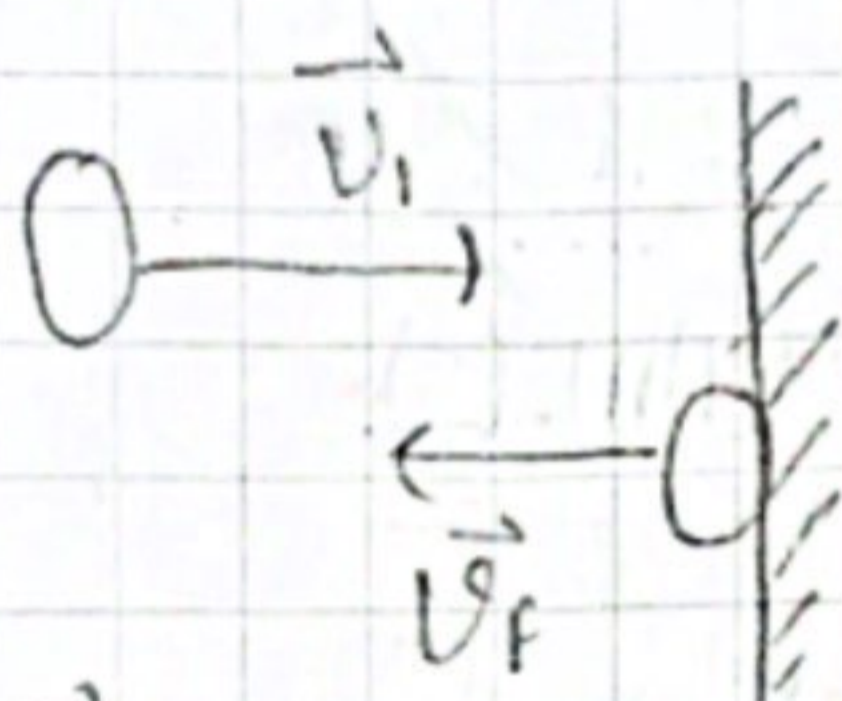
$v_i = 30 \text{ m/s}$

$v_f = 20 \text{ m/s}$

a) $\vec{p}_i = ?$

$\vec{p}_f = ?$

b) $\vec{J} = ?$



$v_i = 30 \text{ m/s}$

$v_f = -20 \text{ m/s}$

a) $\vec{p}_i = m \cdot \vec{v}_i$

$= (0,4 \text{ kg}) \times (30 \text{ m/s})$

$= 12 \text{ kg m/s}$

$\vec{p}_f = m \cdot \vec{v}_f$

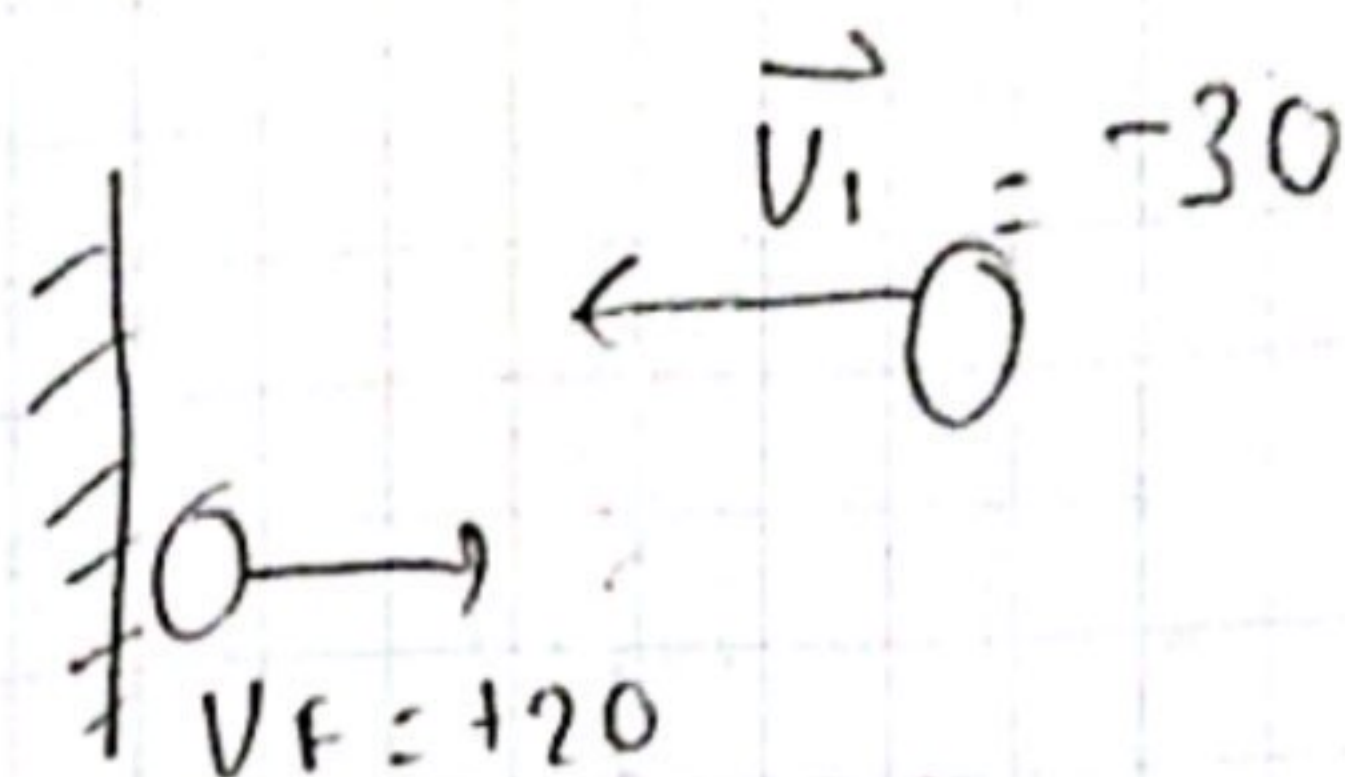
$= (0,4 \text{ kg}) \times (-20 \text{ m/s})$

$= -8 \text{ kg m/s}$

b) $\vec{J} = m \cdot \vec{v}_f - m \cdot \vec{v}_i$

$= -8 - 12 = -20 \text{ kg m/s}$

↑
arah

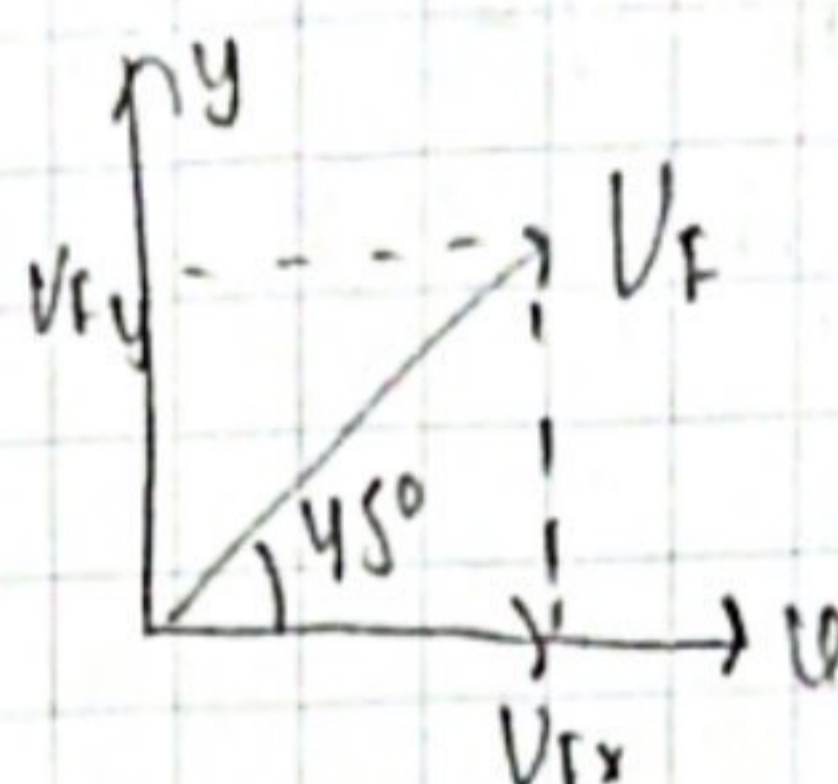


② Dik: $m = 0,40 \text{ kg}$

$\Delta t = 0,018$

$\vec{v}_i = -30 \text{ m/s } (\hat{i})$

$\vec{v}_f = v_{fx} \hat{i} + v_{fy} \hat{j}$



$v_{fx} = v_f \cos 45^\circ = 30 \frac{1}{2} \sqrt{2}$

$= 15 \sqrt{2} \text{ m/s}$

$v_{fy} = v_f \sin 45^\circ = 30 \frac{1}{2} \sqrt{2}$

$= 15 \sqrt{2} \text{ m/s}$

$\vec{v}_f = 15 \sqrt{2} \hat{i} + 15 \sqrt{2} \hat{j} \text{ m/s}$

a) $\vec{J} = ?$

$\vec{J} = m \cdot \vec{v}_f - m \cdot \vec{v}_i$

$15 \sqrt{2} = 21,21$

$= m(v_{fx} - v_i)$

$= 0,40 (15 \sqrt{2} \hat{i} + 15 \sqrt{2} \hat{j} - (-20 \hat{i}))$

$= 0,4 [21,21 \hat{i} + 21,21 \hat{j} - (-20 \hat{i})]$

$= 0,4 [41,2 \hat{i} + 21,21 \hat{j}]$

$= 16,48 \hat{i} + 8,48 \hat{j} \text{ kg m/s}$

b) $F = ?$

$\vec{J} = \vec{F} \cdot \Delta t$

$\vec{F} = \frac{\vec{J}}{\Delta t} = \frac{16,48 \hat{i} + 8,48 \hat{j}}{0,018}$

$= (1648 \hat{i} + 848 \hat{j}) \text{ N}$

① Dik: $m = 0,5 \text{ kg}$

$v_i = (5\hat{i} + 8\hat{j}) \text{ m/s}$

$v_f = (-15\hat{i} - 7\hat{j} + 10\hat{k}) \text{ m/s}$

Dit: a) perubahan kecepatan pada sumbu x, y, z !

$v_f - v_i$

$(-15\hat{i} - 7\hat{j} + 10\hat{k}) - (5\hat{i} + 8\hat{j})$

$= -20\hat{i} - 8\hat{j} + 10\hat{k}$

$= -20(4)\hat{i}$

$= -8(4)\hat{j}$

$= 10(2)\hat{k}$

b) Perubahan momentum!

$p = m(v_f - v_i)$

$= 0,5 \text{ kg} (-20\hat{i} - 8\hat{j} + 10\hat{k})$

$= -10\hat{i} - 4\hat{j} + 5\hat{k}$

c) Hitung impuls bola!

$\Delta = \Delta p$

$\Delta = F \cdot \Delta t$

$= -10\hat{i} - 4\hat{j} + 5\hat{k}$

d) $0,0028$

besar gaya rata-rata pada bola!

$\Delta = F \cdot \Delta t$

$-10\hat{i} - 4\hat{j} + 5\hat{k} = F \cdot 0,0028$

$F = -5000\hat{i} - 2000\hat{j} + 2500\hat{k}$

$F = \frac{-10\hat{i} - 4\hat{j} + 5\hat{k}}{0,0028}$

Besar: $|\vec{F}| = \sqrt{F_x^2 + F_y^2 + F_z^2}$

$= \sqrt{(-5000)^2 + (-2000)^2 + (2500)^2}$

$= 500\sqrt{141}$

$\approx 5937,17104$

Kekekalan momentum

$\vec{p}_i = \vec{p}_f$

Sistem pusat massa: ↑ acceleration
↓ acceleration

$X_{pm} = \frac{X_1 m_1 + X_2 m_2 + \dots + X_n m_n}{m_1 + m_2 + \dots + m_n}$

③ kondisi awal (i)

$M = 4 \text{ kg}$

$m = 0,01 \text{ kg}$

↓ sistem
 $v_i = 0$

kondisi akhir (f)

$M = 4 \text{ kg} \rightarrow v_{sf} = ?$

$m = 0,01 \text{ kg}$

$v_{pf} = 300 \text{ m/s}$

↑ tekanan
kekanan

Hukum kekekalan momentum

$\vec{p}_i = \vec{p}_f$

$m \vec{v}_{si} + M \cdot \vec{v}_{pi} = m \cdot \vec{v}_{sf} + M \cdot \vec{v}_{pf}$

$\therefore 0 + 0 = (4) \vec{v}_{sf} + (0,01)(300)$

$0 = 4 \vec{v}_{sf} + 3$

$-3 = 4 \vec{v}_{sf}$

$-\frac{3}{4} = \vec{v}_{sf} //$

④ Dik: diam $\therefore v_i = 0$

$M_{\text{proton}} = 1836$ kali

M_{elektron}

Dit: perbandingan dari energi total yang dilepaskan proton \rightarrow e. kinetik?

$$\Rightarrow \boxed{\vec{p}_i = \vec{p}_f}$$

$$M_e \vec{v}_{ef} + M_p \vec{v}_{pf} = M_e \vec{v}_{ef} + M_p \vec{v}_{pf}$$

Kondisi awal Kondisi akhir

M_{elektron}

$$v_i = 0$$

$$\text{Dit: } \frac{K_{\text{total}}}{K_p} = ?$$

$$M_{\text{proton}} = M_p \quad \text{dan} \quad M_p = 1836 M_e$$
$$M_{\text{elektron}} = M_e$$

$$K = \frac{1}{2} M v^2$$

$$\Downarrow$$
$$M_e = \frac{M_p}{1836}$$

$$\vec{p}_i = \vec{p}_f$$

$$0 = \frac{M_p}{2} v_{pf}^2 + \frac{M_e}{2} v_{ef}^2$$

$$0 = M_p v_{pf}^2 + \frac{M_e}{1836} v_{ef}^2$$

$$\frac{M_e}{1836} v_{ef}^2 = M_p v_{pf}^2$$

$$\frac{1}{1836} v_{ef}^2 = v_{pf}^2$$

$$v_{ef} = -1836 v_{pf}$$

$$\frac{K_{\text{total}}}{K_p} = \frac{K_{pf} + K_{ef}}{K_{pf}}$$

$$= \frac{\frac{1}{2} M_p v_{pf}^2 + \frac{1}{2} M_e v_{ef}^2}{\frac{1}{2} M_p v_{pf}^2}$$

$$= \frac{\frac{1}{2} M_p v_{pf}^2 + \frac{1}{2} M_e v_{ef}^2}{\frac{1}{2} M_p v_{pf}^2}$$

$$= \frac{M_p v_{pf}^2 + \frac{M_e}{1836} (-1836 v_{pf})^2}{M_p v_{pf}^2} = 1 + \frac{(1836)^2}{1836} = 1 + 1836 = 1837$$

5) Dik: $M = 75 \text{ kg} \rightarrow \text{Orang}$
 $m = 125 \text{ kg} \rightarrow \text{Perahu}$
 $m = 15 \text{ kg} \rightarrow \text{paket}$
horizontal ke kanan

$$v_1 = 4,5 \text{ m/s}$$

abaikan hambatan udara,
perahu tidak bergerak
sebelum paket dilempar.

Dit: Tentukan kecepatan perahu!

by Kondisi awal (i) Kondisi akhir (f)

$$M_{\text{orang}} = 75 \text{ kg}$$

$$\textcircled{1} \text{ Momentum awal} = 0$$

$$\textcircled{2} \text{ kondisi akhir (setelah paket dilempar):}$$

= Misalkan kecepatan perahu (beserta orang di dalamnya) setelah paket dilempar $= v$.

Momentum akhir sistem terdiri =

- Momentum paket ke kanan =

$$m' \times v_1 = 15 \text{ kg} \times 4,5 \text{ m/s}$$

$$= 67,5 \text{ kg m/s}$$

- Momentum perahu + Orang ke kiri =

$$(M + m) \times v = (75 \text{ kg} + 125 \text{ kg}) \cdot v$$

$$= 200 \text{ kg} \times v$$

- Berdasarkan huk. Kekekalan =

$$0 = (200 \text{ kg}) \cdot v - 67,5 \text{ kg m/s}$$

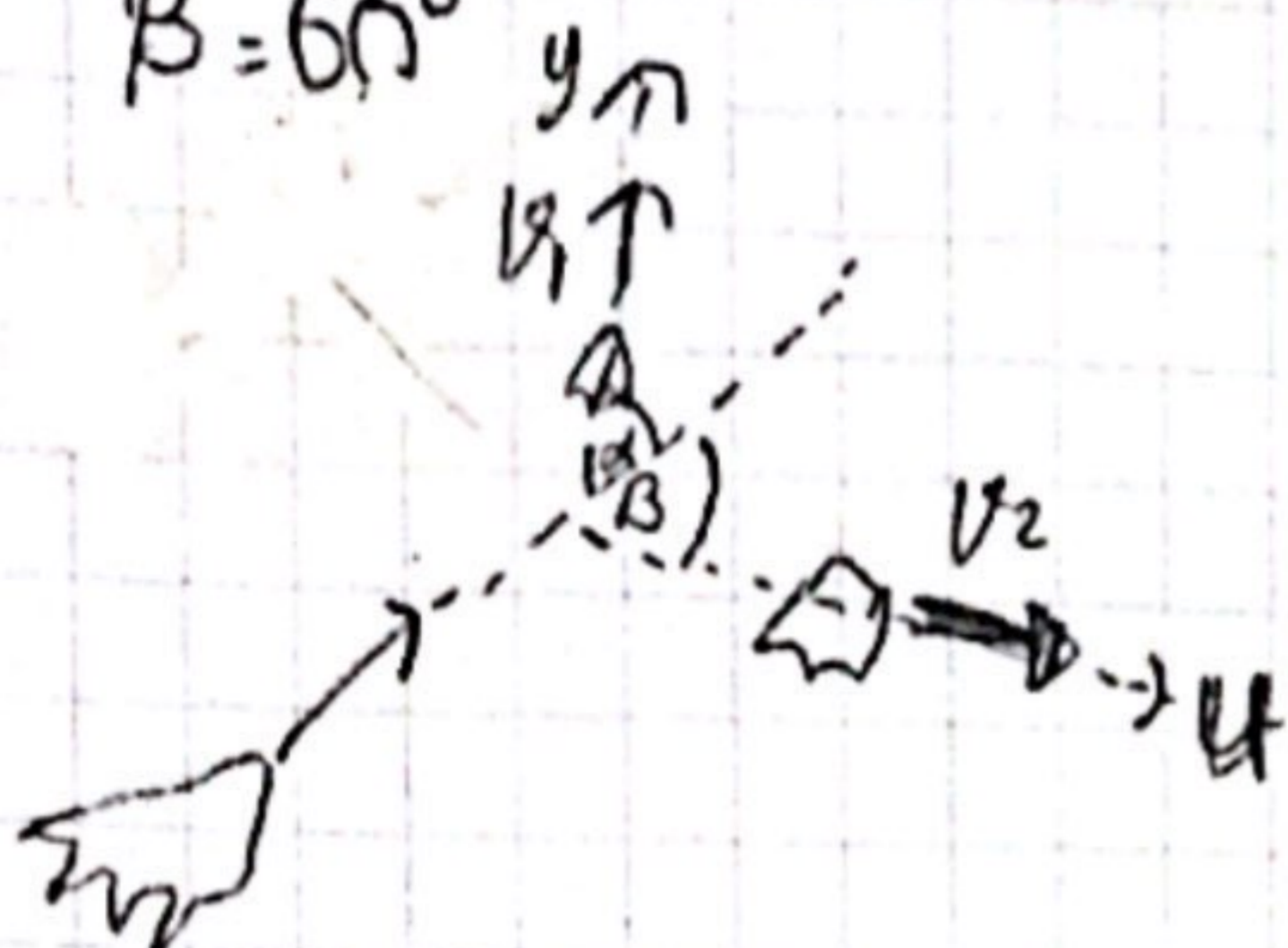
$$200 \text{ kg} \cdot v = 67,5 \text{ kg m/s}$$

$$v = \frac{67,5}{200} = 0,3375 \text{ m/s}$$

6) Sebuah roket kembang api, bermassa m , $v_0: 48 \text{ m/s}$ terpecah menjadi 2 bagian massanya sama, masing-masing bergerak v_1 & v_2

Dik: $\alpha: 30^\circ$

$\beta: 60^\circ$



Dit: a) tulis persamaan momentum linear!

b) tentukan laju v_1 & v_2 !

c) tentukan v pm selokh pecah!

Jawab: a) $\vec{p}_i = \vec{p}_f$

$$m \cdot v_0 = m_1 v_1 + m_2 v_2$$

$$m(v_0 \cos \alpha \hat{i} + v_0 \sin \alpha \hat{j}) = \frac{1}{2} m v_1 \hat{j} + \frac{1}{2} m v_2 \hat{i}$$

$$(24 \hat{i} + 24\sqrt{3} \hat{j}) = \frac{1}{2} v_1 \hat{j} + \frac{1}{2} v_2 \hat{i}$$

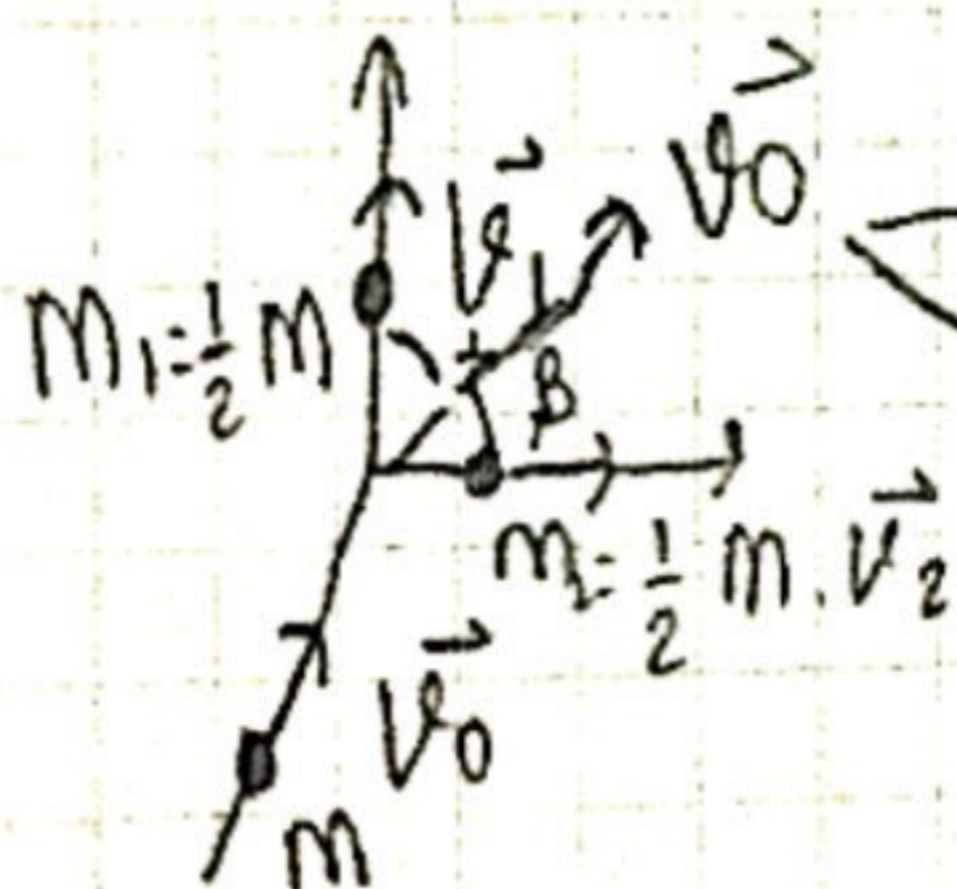
b) v_1 & $v_2 = ?$

$$\text{dari } \hat{i} \Rightarrow 24 \hat{i} = \frac{1}{2} v_2 \hat{i}$$

$$v_2 = 48 \text{ m/s}$$

$$\text{dari } \hat{j} \Rightarrow 24\sqrt{3} \hat{j} = \frac{1}{2} v_1 \hat{j}$$

$$v_1 = 48\sqrt{3} \text{ m/s}$$



$$v_{0y} = v_0 \sin \alpha = v_0 \sin \beta (\hat{j}) = 48 \cos 30^\circ$$

$$= 48 \sin 60^\circ$$

$$= 24\sqrt{3} \hat{j} \text{ m/s}$$

$$v_{0x} = v_0 \cos \alpha = v_0 \sin \alpha (\hat{i})$$

$$= 48 \cos 60^\circ$$

$$= 48 \sin 30^\circ$$

$$= 24 \hat{i} \text{ m/s}$$

c) Kecepatan pusat massa =

$$V_{pm} = \frac{m_1 \cdot v_1 + m_2 \cdot v_2}{m_1 + m_2}$$

$$= \frac{\frac{1}{2} m \cdot 40\sqrt{3} \hat{i} + \frac{1}{2} m \cdot 40\hat{j}}{\frac{1}{2} m + \frac{1}{2} m}$$

$$= \frac{24\sqrt{3} m \hat{i} + 24 m \hat{j}}{1 m}$$

$$= 24\sqrt{3} \hat{i} + 24 \hat{j} \text{ m/s}$$

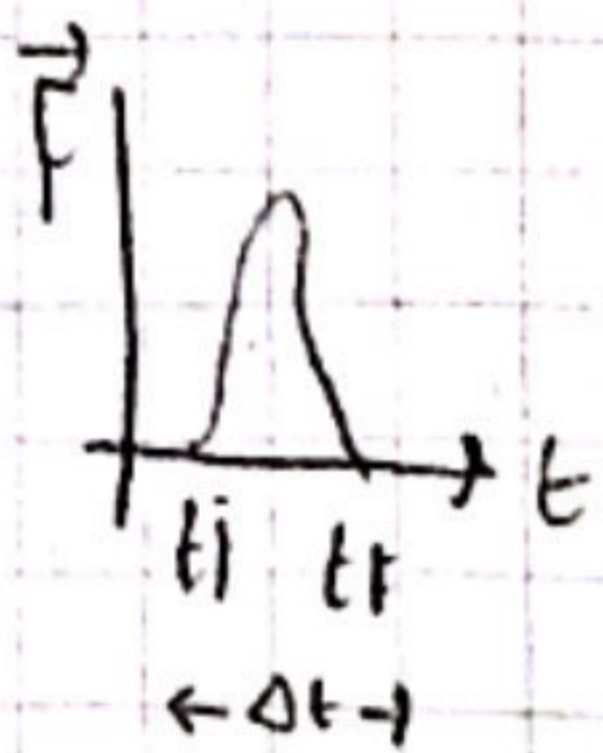
Momentum dan Impuls

9-Nov-2024

kinematika, dinamika, usaha & energi, momentum

Impuls = perkalian antara gaya rata-rata yang bekerja

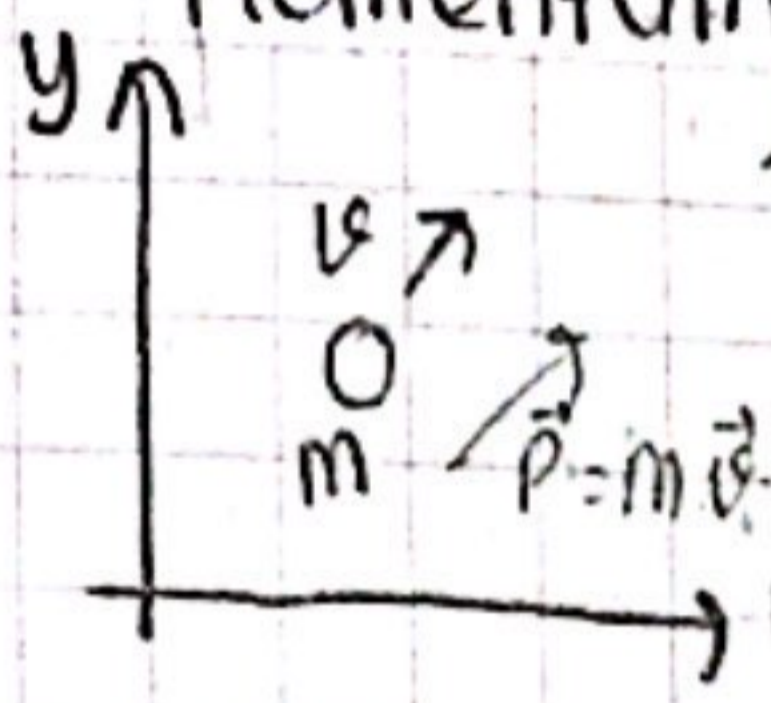
$$\vec{J} = \vec{F} \cdot \Delta t$$



$$\vec{J} = \int_{t_i}^{t_f} \vec{F} dt$$

(satuan = Nm)

Momentum linear:



$$\vec{p} = m \cdot \vec{v}$$

Satuan = kg, m/s

$$\begin{aligned} \vec{p} &= p_x \hat{i} + p_y \hat{j} + p_z \hat{k} \\ &= m v_x \hat{i} + m v_y \hat{j} + m v_z \hat{k} \end{aligned}$$

Hukum II Newton:

$$\begin{aligned} \sum \vec{F} &= m \cdot \vec{a} \\ &= m \cdot \frac{d\vec{v}}{dt} \end{aligned}$$

massa partikel konstan =

$$\sum \vec{F} = \frac{d(m\vec{v})}{dt}$$

$$\sum \vec{F} = \frac{d\vec{p}}{dt}$$

Teorema Impuls & Momentum:

$$\vec{F} = m \cdot \vec{a}$$

$$= m \cdot \frac{\vec{v}_f - \vec{v}_i}{\Delta t}$$

$$\Leftrightarrow \vec{a} = \frac{\Delta \vec{v}}{\Delta t} = \frac{\vec{v}_f - \vec{v}_i}{\Delta t}$$

$$\vec{F} \Delta t = m(\vec{v}_f - \vec{v}_i)$$

$$\vec{F} \cdot \Delta t = m \vec{v}_f - m \vec{v}_i$$

Impuls

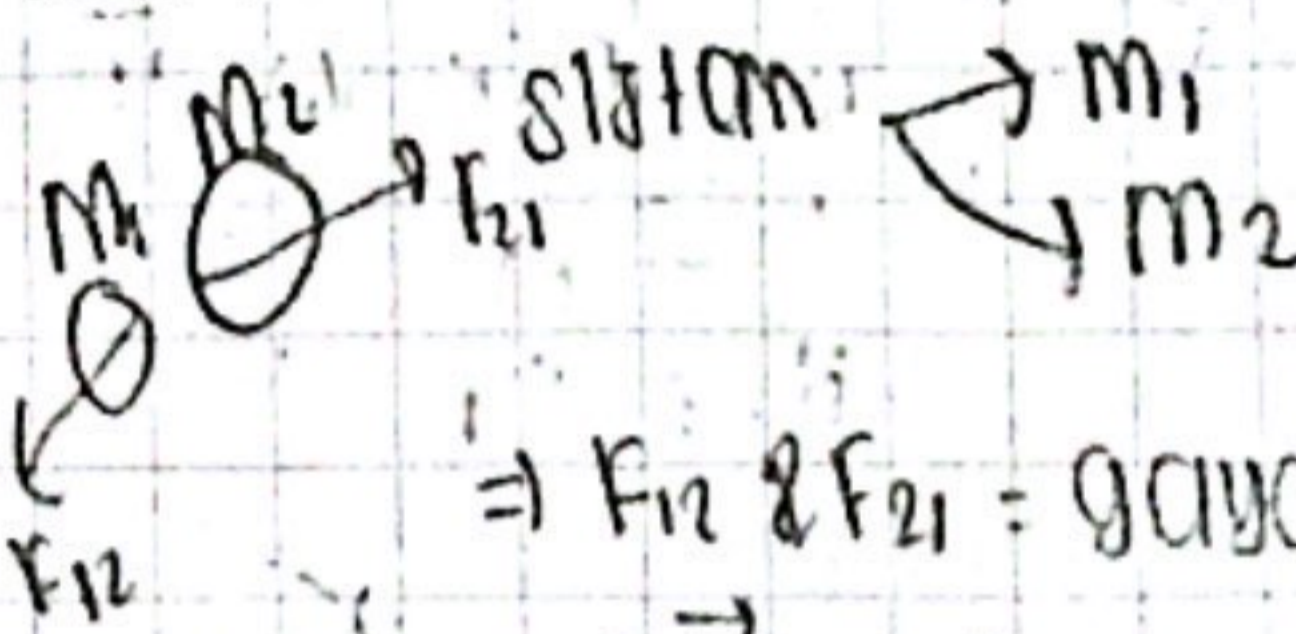
$$\vec{J} = \vec{p}_f - \vec{p}_i$$

Perubahan momentum

Teorema Impuls - Momentum:

$$\text{"Impuls" = "Δmomentum"}$$

kekawalan Momentum linear:



$\Rightarrow F_{12} \text{ \& } F_{21}$ = gaya aksi & reaksi

$$\sum \vec{F}_{out} = 0$$

$$\frac{\Delta \vec{p}}{\Delta t} = 0$$

$$\Delta \vec{p} = 0$$

$$\vec{p}_f - \vec{p}_i = 0$$

$$\vec{p}_i = \vec{p}_f$$

$$m \vec{v}_i = m_1 \cdot \vec{v}_{1f} + m_2 \vec{v}_{2f}$$

Sistem Partikel

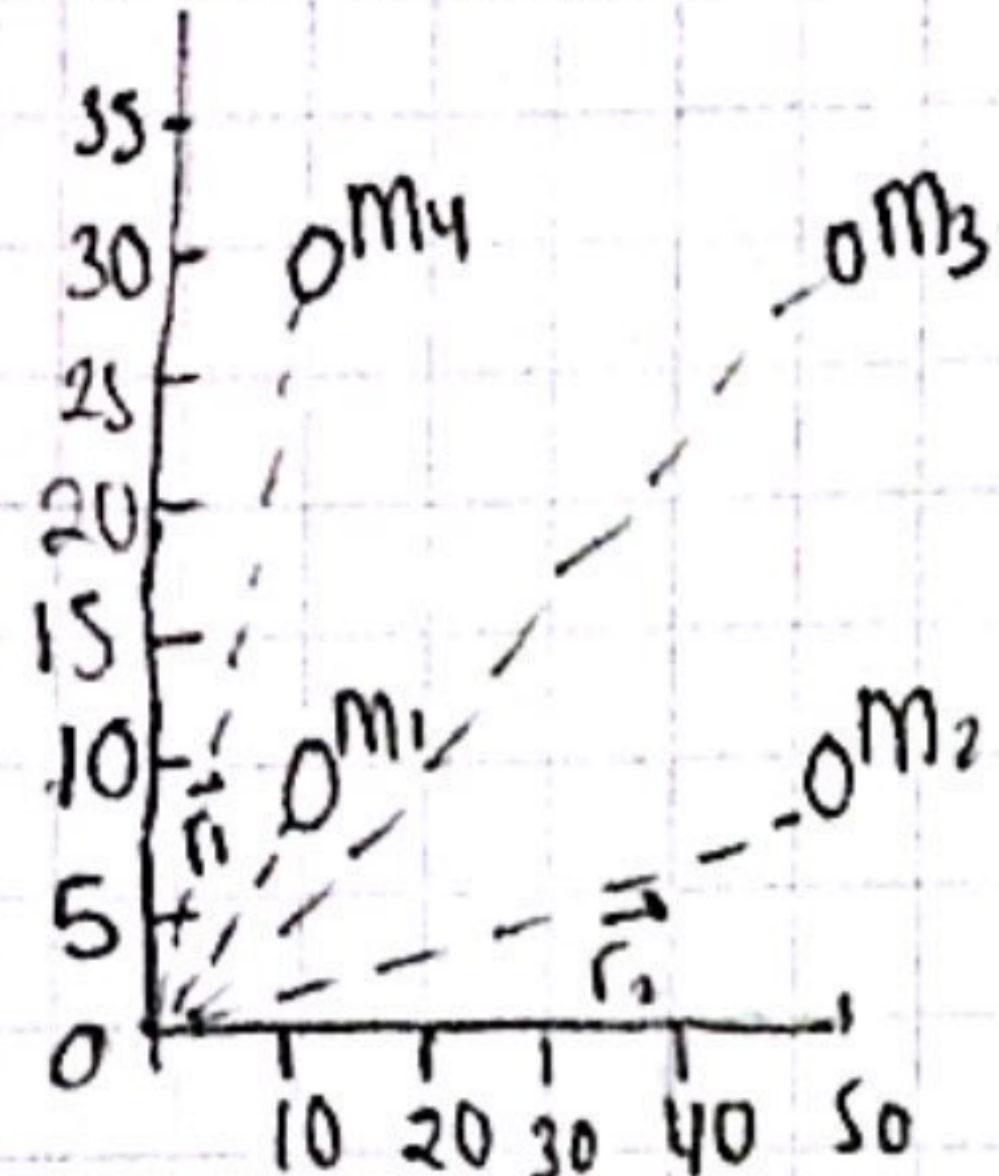
Analisis gerak benda:

↳ gerak benda runtuh

↳ bentuk & ukuran benda diperhitungkan

↳ objek → banyak partikel

Sistem Benda Diskrit:



posisi pusat massa
< 4 partikel >

$$\vec{r}_{pm} = x_{pm}\hat{i} + y_{pm}\hat{j}$$

$$x_{pm} = \frac{m_1 \cdot x_1 + m_2 \cdot x_2 + m_3 \cdot x_3 + m_4 \cdot x_4}{m_1 + m_2 + m_3 + m_4}$$

$$y_{pm} = \frac{m_1 \cdot y_1 + m_2 \cdot y_2 + m_3 \cdot y_3 + m_4 \cdot y_4}{m_1 + m_2 + m_3 + m_4}$$

sistem benda kontinu:

$$\vec{r}_{pm} = \frac{1}{M} \int \vec{r} dm$$

1-D λ
<Rapat massa per satuan panjang>

$$\lambda = \frac{dm}{du}$$

$$\vec{r}_{pm} = x_{pm}\hat{i}$$

$$x_{pm} = \frac{\int x \lambda du}{\int \lambda du}$$

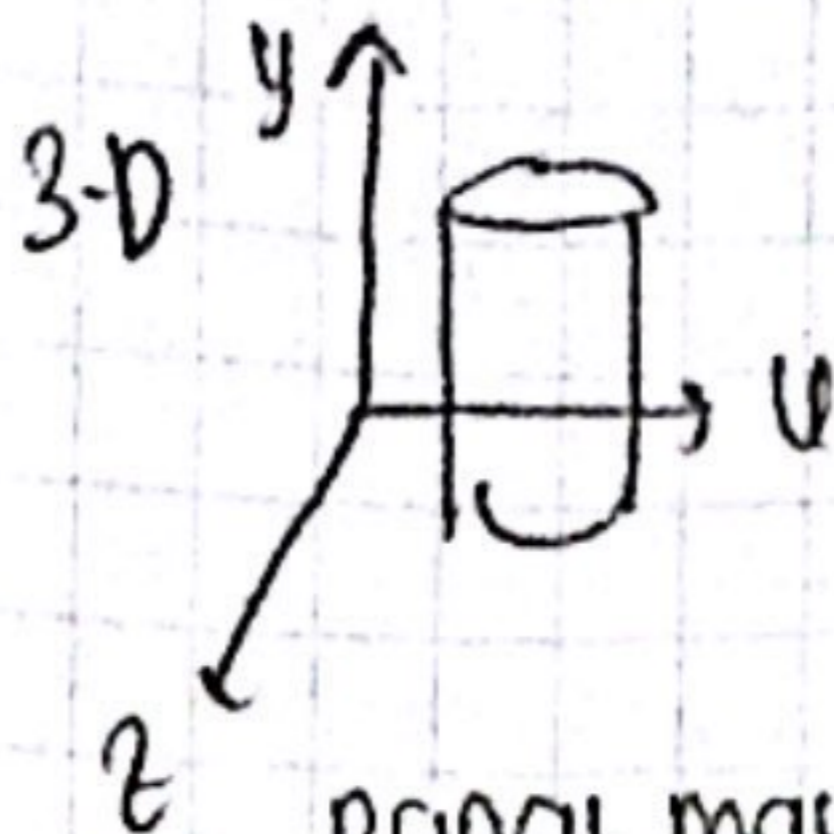
2-D σ
<Rapat massa per satuan luas>

$$\sigma = \frac{dm}{da}$$

$$\vec{r}_{pm} = x_{pm}\hat{i} + y_{pm}\hat{j}$$

$$x_{pm} = \frac{\int x \sigma da}{\int \sigma da}$$

$$y_{pm} = \frac{\int y \sigma da}{\int \sigma da}$$



Rapat massa per volume:

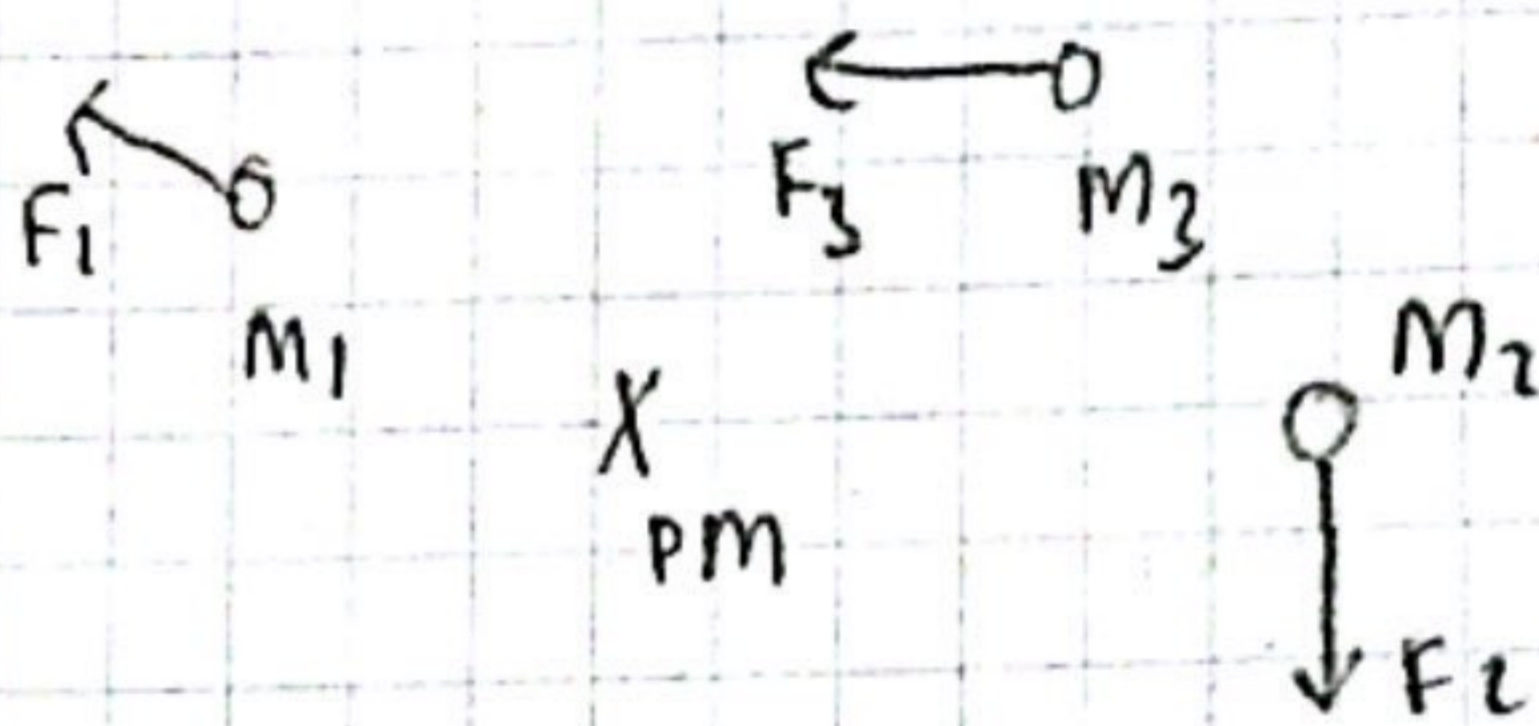
$$\rho = \frac{dm}{dv}$$

$$\vec{r}_{pm} = x_{pm}\hat{i} + y_{pm}\hat{j} + z_{pm}\hat{k}$$

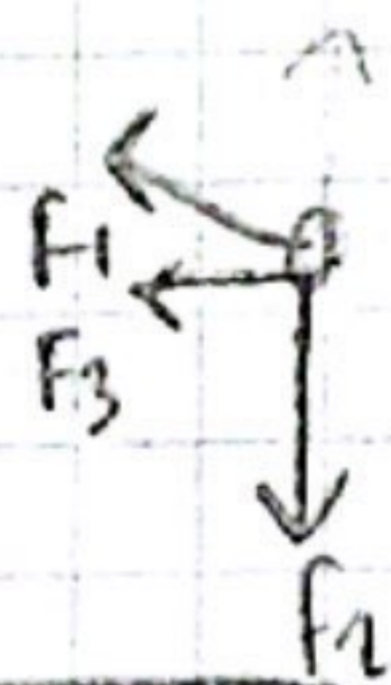
$$x_{pm} = \frac{\int x \rho dv}{\int \rho dv} \quad y_{pm} = \frac{\int y \rho dv}{\int \rho dv}$$

$$z_{pm} = \frac{\int z \rho dv}{\int \rho dv}$$

Hukum II Newton / Sistem Partikel



#ga perlu ninau 1-1
<pusat massa>



$$\sum \vec{F} = M \cdot \vec{a}_{pm}$$

#Ruler = $\sum \vec{F} = 0$ jika tidak

M = total massa sistem kontinu

a_{pm} = percepatan pusat massa

tumbukan :

tumbukan

elastis

atau elastis

elastis = momentum linear kekal:

$$P_{1i} + P_{2i} = P_{1f} + P_{2f}$$

- Energi sistem kekal

$$K_{1i} + K_{2i} = K_{1f} + K_{2f}$$

atau elastis = momentum linear kekal:

$$P_{1i} + P_{2i} = P_{1f} + P_{2f}$$

- Energi sistem tidak kekal:

$$K_{1i} + K_{2i} = K_{1f} + K_{2f} + K_{loss}$$

tumbukan tak elastis (1-D)

$K_{total\ akhir} < K_{total\ awal}$

$$\frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2 < \frac{1}{2} m_1 v_{1i}^2 + \frac{1}{2} m_2 v_{2i}^2$$

momentum sistem tetap kekal:

$$m_1 \vec{v}_{1i} + m_2 \vec{v}_{2i} = m_1 \vec{v}_{1f} + m_2 \vec{v}_{2f}$$

tumbukan tak elastis sama sekali:

$$v_f = \frac{m_1 \vec{v}_{1i} + m_2 \vec{v}_{2i}}{m_1 + m_2}$$

arah

(+) / (-)

$$(+v_i) \quad \leftarrow \quad (-v_i)$$

→

sumbu u+

tumbukan

$$ID$$

$$K_{loss} = 0$$

Tumbukan Dalam 2-D

sumbu u:

$$m_1 v_{1iu} + m_2 v_{2iu} = m_1 v_{1fu} + m_2 v_{2fu}$$

$$m_1 v_{1i} \cos \alpha + m_2 v_{2i} \cos \beta =$$

$$m_1 v_{1f} \cos \theta + m_2 v_{2f} \cos \varphi$$

sumbu y:

$$m_1 v_{1iy} + m_2 v_{2iy} =$$

$$m_1 v_{1fy} + m_2 v_{2fy}$$