Ujian Reval 2023

Bagian A

1.) Himpunan penyelesaian pertaksamaan

$|2u-3| > 7$

adalah $(-\infty, a) \cup (b, \infty)$ dengan

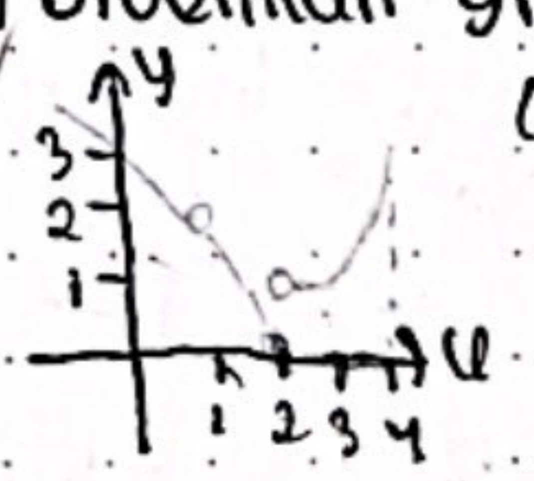
$a = -2$ & $b = 5$

$2u-3=7$	$2u-3=-7$
$2u=7+3$	$2u=-7+3$
$2u=10$	$2u=-4$
$u=\frac{10}{2}=\boxed{5}$	$u=\frac{-4}{2}=\boxed{-2}$
batas	batas bawah

( ) = tidak termasuk

[ ] = termasuk $[5,\infty) \cup (-\infty,-2]$

2.) Diberikan grafik fungsi F sbb:



(a) $\lim\limits_{u\to 1} F(u) = 2$

(b) $\lim\limits_{u\to 4^-} F(u) = 0$

3.) Misalkan $F(c)=3$,
$g(c)=-5$,
$F'(c)=2$,
$g'(c)=-4$

<a> Jika $h(u)=F(u)+g(u)$ maka:

$h'(c) = F'(c)+g'(c)$

$= 2+(-4) = -2$ $\qquad u'.v+u.v'$

<b> Jika $P(u)=F(u)\,g(u)$ maka

$P'(c) = F'(c).g(c)+F(c).g'(c)$

$= 2.-5+3.-4$

$= -10+-12 = -10-12 = -22$

4) Misalkan $F(u)=\sqrt{u}$

(a) $F'(4) =$ $F(u)=\sqrt{u}$

$= u^{1/2}$

$= \frac{1}{2}u^{1/2-1}$

$= \frac{1}{2}u^{-1/2} = \frac{1}{2}(4)^{-1/2}$

$= \frac{1}{2\sqrt{4}} = \frac{1}{2.2} = \frac{1}{4}$

$u^{1/2}=\sqrt{u}$

$u^{-1/2} = \frac{1}{\sqrt{u}}$

$u^{1/2}$ $\quad \frac{1}{2}u^{1/2-1}$ $\quad \frac{1}{2}u^{-1/2}$ $\quad \frac{1}{2\sqrt{u}}$

(b) Dengan menggunakan diferensial atau hampiran linear $\sqrt{4,1} \approx \ldots$ turunan

$\Rightarrow \sqrt{4,1} \approx F(a)+F'(a)\,\Delta u \rightarrow$ aturan perkalian

$\approx 2+\frac{1}{4}\cdot 0,1 = 2+\frac{0,1}{4}$

$= \frac{2}{1}+\frac{0,1}{4}$

$= \frac{8+0,1}{4} = \frac{8,1}{4} = 2,1$

$F(a)=\sqrt{4}=2$

$F'(a)=\frac{1}{4}$

$\Delta u = 4-4,1$
$=0,1$

5) Jika $\int_1^4 F(u)\,du = 3$

$\int_4^6 F(u)\,du = 5$

$\int_1^4 g(u)\,du = 2$

maka:

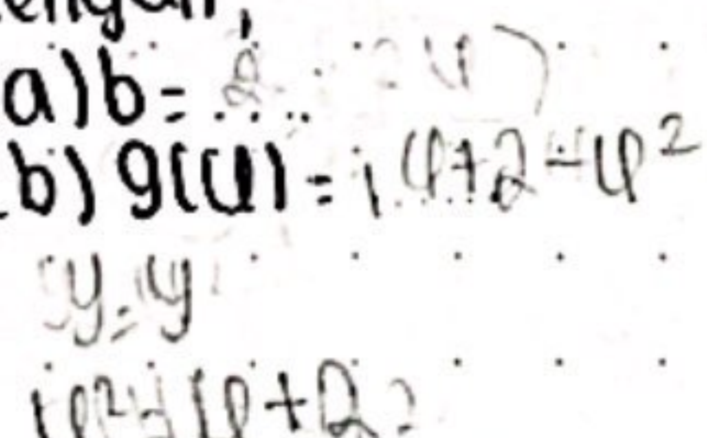
(a) $\int_1^6 F(u)\,du = \int_1^4 F(u)\,du + \int_4^6 F(u)\,du$

$= 3+5 = 8$

(b) $\int_1^4 (F(u)+g(u))\,du = \int_1^4 F(u)\,du + \int_1^4 g(u)\,du$

$= 3+2 = 5$

6) Misalkan R adalah daerah tertutup yang dibatasi oleh parabola $y=u^2$ & garis $y=u+2$ seperti pada gambar:



$\text{luas} = \int (\text{atas} - \text{bawah})\,du$

luas daerah R dapat dinyatakan dalam bentuk integral tentu $\int_{-1}^{b} g(u)\,du$ dengan,

(a) $b = 2$

(b) $g(u) = u+2-u^2$

$y=y$

$u^2 = u+2$

$u^2-u-2=0$

$(u-2)(u+1)=0$

$\boxed{u=2}$ $\quad u=-1$

$P=\int_{-1}^{2}[(u+2)-(u^2)]\,du$

$(u+2)-(u^2)$

$\boxed{=-u^2+u+2}$

$P=\int_{-1}^{2}(-u^2+u+2)\,du$

7) a. $e^{3\ln 2} = e^{a\ln b} = b^a$
$e^{3\ln 2} = 2^3$
$= 8$

6. Jika $f(u) = \ln u$, maka : $f'(2) = \ldots$
$f(u) = \frac{1}{u}$
$f'(2) = \frac{1}{2}$

① $A = e^{4\ln 3}$
A?
$= e^{a\ln b} = b^a = 3^4 = 81$
② $f(u) = \ln(2u^2)$
$= f(u) = \frac{1}{2u^2} \cdot 4u = \frac{4u}{2u^2} = \frac{2}{u}$ //
③ $g(u) = e^{\ln(u^3)}$
$g(u) = u^3$ //

8. Misalkan $F(u) = \int_1^u \sqrt{1+t^3}\,dt$
a. $F(1) = \int_1^1 = 0$ ✓
b. $F'(u) = \int_1^u \sqrt{1+t^3}\,dt$
$= \frac{d}{du}\left[\int_a^u f(t)\,dt\right] = f(u)$ $\boxed{\int_u^1 e^{t^2}\,dt}$
$F'(u) = \sqrt{1+u^3}$

$\int_1^u \frac{1}{1+t^2}\,dt$
$\frac{1}{1+u^2}$
$\cdot 4\, e^{u^2} d$

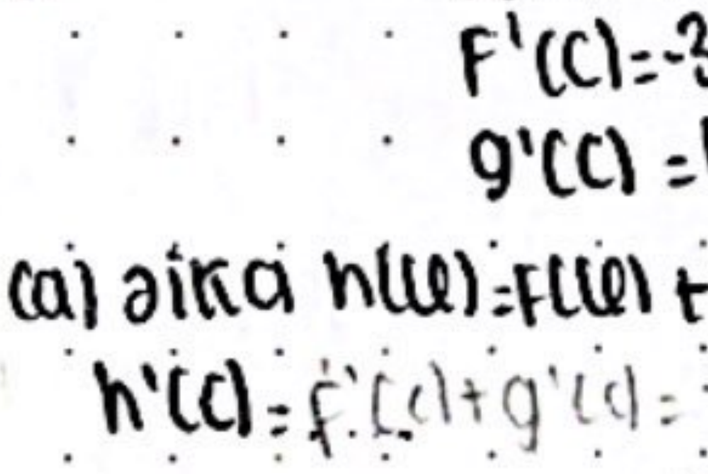
▲ Bagian I

1.) Himpunan penyelesaian pertaksamaan $|2u-3|>5$ adalah $(-\infty, a) \cup (b, \infty)$ dengan $a = -1$ & $b = 4$

$2u-3 > 5$
$2u > 5+3$
$2u > 8$
$u > \frac{8}{2}$
$u > 4$

$2u-3 > -5$
$2u-3 = -5$
$2u = -5+3$
$2u = -2$
$u = \frac{-2}{2} = -1$

2.) Diberikan grafik fungsi sbb:



(a) $\lim_{u \to 1} f(u) = 1$
(b) $\lim_{u \to 4^-} f(u) = -\infty$ (limit tampa batas)

3.) Misalkan $f(c) = 3$,
$g(c) = 4$,
$f'(c) = -3$,
$g'(c) = 1$

(a) Jika $h(u) = f(u) + g(u)$, maka
$h'(c) = f'(c) + g'(c) = -3+1$
$= 2$

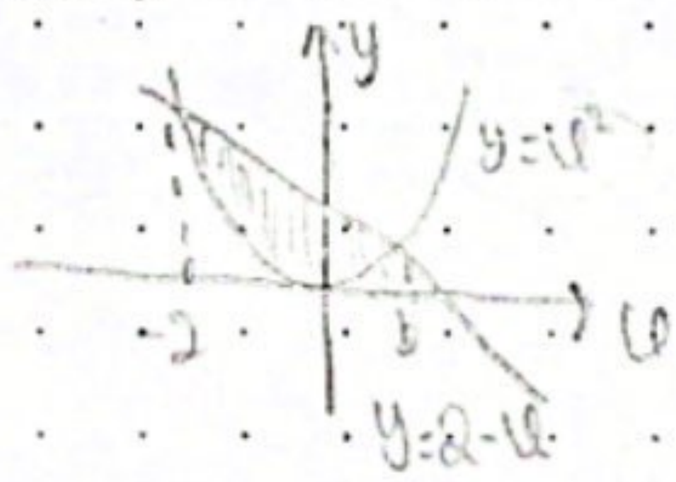
(b) Jika $p(u) = f(u)\,g(u)$, maka
$p'(c) = f'(c) + g'(c)$
$= f'(c) \cdot g(c) + f(c) \cdot g'(c)$
$= -3 \cdot 4 + 3 \cdot 1$
$= -12 + 3 = -9$

4.) Misalkan $f(u) = \sqrt{u}$
(a) $f'(9) = u^{1/2}$
$= \frac{1}{2}u^{-1/2} = \frac{1}{2\sqrt{9}} = \frac{1}{2 \cdot 3} = \frac{1}{6}$ //

(b) Dengan menggunakan diferensial / hampiran linear $\sqrt{9{,}1} \approx \ldots$
$f(a) + f'(u) \cdot \Delta u$
$f(a) = \sqrt{9} = 3$
$f'(u) = \frac{1}{6}$
$\Delta u = 9{,}1 - 9$
$= 0{,}1$
$3 + \frac{1}{6} \cdot 0{,}1$
$3 + \frac{0{,}1}{6}$
$\frac{3}{1} + \frac{0{,}1}{6} = \frac{18+0{,}1}{6} = \frac{18{,}1}{6} = 3{,}1$

5.) Jika $\int_1^4 f(u)\,du = 6$
$\int_4^6 f(u)\,du = 3$
$\int_1^4 g(u)\,du = 1$ maka
(a) $\int_1^6 f(u)\,du = \int_1^4 f(u)\,du + \int_4^6 f(u)\,du$
$= 6+3 = 9$
(b) $\int_1^4 (f(u) + g(u))\,du = \int_1^4 f(u)\,du + \int_1^4 g(u)\,du$
$= 6+1 = 7$

6.) Misalkan R adalah daerah tertutup yang dibatasi oleh parabola $y = u^2$ & garis $y = 2-u$ seperti:



Luas daerah R dapat dinyatakan dalam bentuk integral tentu $\int_{-2}^{b} g(u)\,du$ dengan,
(a) $b = 2$
(b) $g(u) = u^2 - 2 + u$

$y = y$
$u^2 = 2-u$
$2u = 2-u$
$2u + 2 - 2$
$2u = 0$
$u = 2$

$\int_{-2}^{2} g(u)\,du$
$\int_{-2}^{2} (2-u) - (u^2)$
$(-u^2 + 2 - u)$

7) (a) $e^{2\ln 3} = 4\ln b$
$= b^a$
$= 3^2 = 9$
(b) Jika $f(u) = \ln u$ $\frac{1}{u}$
$f'(3) = \frac{1}{3}$

8) Misalkan $f(u) = \int_1^u \sqrt{1+t^4}\,dt$
(a) $f(1) = \ldots$
$\int_1^1 = 0$
(b) $f'(u) = \int_1^u \sqrt{1+t^4}$
$= (1+t^4)^{\frac{1}{2}}$
$= \sqrt{1+t^4} = \sqrt{1+u^4}$

Bagian B
① Diberikan fungsi $f(u) = \sqrt{u^2-4}$. Tentukan asal fungsi (daerah) f. Tentukan $f(-u)$ & simpulkan apakah f fungsi genap/ganjil!
② $f(u) = \sqrt{u^2-4}$
$f(-u) = \sqrt{(-u)^2-4}$
$\boxed{f(-u) = f(u)}$ → genap
③ Fungsi ganjil = $f(-u) = -f(u)$
Fungsi genap = $f(-u) = f(u)$
① Daerah asal fungsi:
$u^2 - 4 \geq 0$ $u^2 < 4$
$u^2 \geq 4$ $u^2 < -2$
$u \geq 2$ $-\infty, -2$
$2, \infty$

② Hitunglah $\lim_{u \to 2} \frac{u^3 - 2u^2}{u^2 - 4}$
$$* \frac{\lim_{u\to 2} u^3 - 2u^2}{\lim_{u \to 2} u^2 - 4} = \frac{2^3 - 2(2)^2}{2^2 - 4}$$
$$= \frac{4-8}{0} = \frac{-4}{0} \times$$

* Substitusi:
$$\lim_{u \to 2} \frac{u^3 - 2u^2}{u^2 - 4} = \frac{2^3 - 2 \cdot 2^2}{2^2 - 4} = \frac{0}{0} \times$$

* Faktorin fungsi:
$(u^2 - 4) = (u-2)(u+2)$
$(u^3 - 2u^2) = u^2(u-2)$
$$\frac{u^2\cancel{(u-2)}}{\cancel{(u-2)}(u+2)} = \frac{u^2}{u+2} = \frac{2^2}{2+2} = 1$$

Mengenal limit
① Substitusi
$\frac{0}{0}$, $\frac{\infty}{\infty}$ (skip)
② Faktorisasi
③ L-Hopital
Syarat: $\lim_{u \to c} \frac{f(u)}{g(u)}$
$= \lim_{u \to c} \frac{f'(u)}{g'(u)}$
$\frac{0}{0}$ atau $\frac{\infty}{\infty}$
$$\lim_{u \to 0} \frac{\sin u}{u} = \lim_{u \to 0} \frac{\cos u}{1} = 1$$
④ Perkalian fungsi:
$$\lim_{u \to 0} \frac{\sqrt{u+1} - 1}{u}$$
$$\frac{\sqrt{u+1} - 1}{u} \cdot \frac{\sqrt{u+1} + 1}{\sqrt{u+1} + 1}$$
$$\lim_{u \to 0} \frac{\frac{1}{u} - \frac{1}{2u}}{u}$$ (kalau bentuk ini maka harus disederhanakan)

Contoh soal
① $\lim_{u \to 0} \frac{\sin u}{u} = \frac{\sin 0}{0} = \frac{0}{0}$ *
L-Hopital.
$f(u) = \frac{d}{du} \sin(u) = \cos u$
$g(u) = \frac{d}{du}(u) = 1$
$$\lim_{u \to 0} \frac{\sin(u)}{u} = \lim_{u \to 0} \frac{\cos u}{1}$$
$$\lim_{u \to 0} \frac{\sin u}{u} = 1$$
② $\lim_{u \to \infty} \frac{u}{e^u}$
$f(u) = \frac{d}{du}(u) = 1$
$g(u) = \frac{d}{du}(e^u) = e^u$
$$\lim_{u \to \infty} \frac{u}{e^u} = \lim_{u \to \infty} \frac{1}{e^u} \to 0$$
$$\lim_{u \to \infty} \frac{u}{e^u} = 0$$

$u^2 - 4 = 0$
$(u-2)(u+2)$ $-\infty, -2$
$u = 2$ $u = (-2)$ $2, \infty$

③ Diketahui:
$F(u) = u^3 - 3u^2$ (batas interval)
Pada interval [1,3]. Tentukan nilai maksimum F pada interval tersebut!

$\Rightarrow F(u) = u^3 - 3u^2$ $\quad \boxed{u \in [1,3]}$
$= 3u^2 - 6u$
$= 3u(u-2)$

$F'(u) = 0$
$\boxed{3u^2 - 6u = 0}$
$3u(u-2) = 0$
$u = 0$ atau $u = 2$
$[1,3] \rightarrow u = 2$
titik stasioner

$\boxed{3u(u-2) = 0}$
$3u = 0 \Rightarrow u = 0$
$u - 2 = 0 \Rightarrow u = 2$
$[1,3]$
→ yg relevan 2

0

1) $F(1) = 1^3 - 3(1)^2 = 1 - 3 = -2$
2) $F(2) = 2^3 - 3(2)^2 = 8 - 3 \cdot 4 = 8 - 12 = -4$
3) $F(3) = 3^3 - 3(3)^2 = 9 - 3 \cdot 9 = 27 - 27 = 0$

④ Tentukan solusi dari persamaan diferensial $\frac{dy}{du} = 5u^4 - 1$ yang memenuhi $y(0) = 1$!

$\Rightarrow \frac{dy}{du} = 5u^4 - 1$
$y(0) = 1$
$y = u^5 - u + 1$

$u^5 - u + c$
$1 = (0)^5 - 0 + c$
$\boxed{c = 1}$

$3u^2 - 2u + 4$
$= \int 3u^2 \, du - \int 2u \, du$
$x = u^3 - u^2 + 4u + 5$

⑤ Hitung $\int_0^{\sqrt{\pi/6}} 2u \cos(u^2) \, du$ dengan menggunakan metode substitusi!

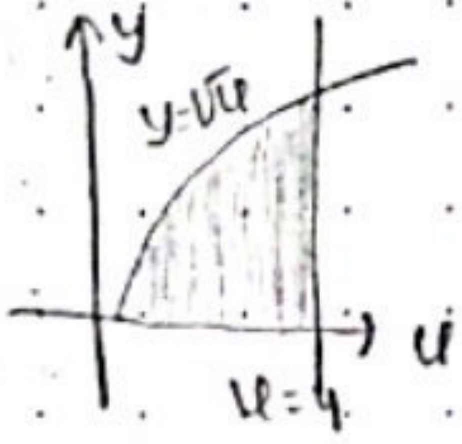
$\hookrightarrow \int_0^{\sqrt{\pi/6}} 2u \cos(u^2)$ $\quad \boxed{du = 2u\,du}$
$\Rightarrow \int \cos u \, du = \sin u + C$
$= \sin(u^2) + C$

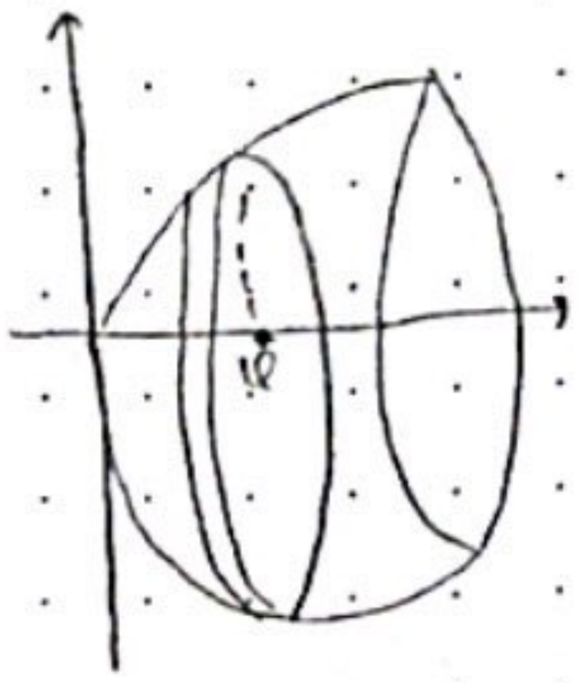
$\int_0^{\sqrt{\pi/6}} 2u \cos(u^2) \, du = [\sin(u^2)]_0^{\sqrt{\pi/6}}$
$= \sin\left(\frac{\pi}{6}\right) - \sin(0)$
$= \sin\left(\frac{180}{6}\right) - \sin 0$
$= \sin 30 - 0$

	0	30	45	60	90
sin	0	1/2	1/2 √2	1/2 √3	1
cos	1	1/2 √3	1/2 √2	1/2	0
tan	0	1/3 √3	1	√3	∞

$\left(\sqrt{\frac{\pi}{4}}\right)^3 = \frac{\pi^{3/2}}{8}$
$= \frac{\sqrt{\pi^3}}{2^3} = \frac{\pi^{3/2}}{8}$

⑥ Taksiran volume irisan vertikal dengan tebal Δu adalah?
Diketahui sebuah benda pejal diperoleh dengan cara memutar daerah yang dibatasi garis $u = 4$, sumbu-u & kurva $y = \sqrt{u}$ terhadap sumbu u. Tentukan volume benda tersebut!





$\hookrightarrow \Delta V \approx \pi(\sqrt{u})^2 \Delta u$
$\approx \pi u \Delta u$

volume benda tersebut =
$V = \pi \int_0^4 u \, du$
$= \pi \left[\frac{u^2}{2}\right]_0^4 = \pi\left(\frac{4^2}{2} - \frac{0^2}{2}\right)$
$= \pi \cdot \frac{16}{2} = 8\pi$ satuan volume

Menentukan fungsi ganjil/genap

① $F(u) = \sqrt{9 - u^2}$
ganjil: $F(-u) = -F(u)$
genap: $F(-u) = F(u)$
$F(-u) = \sqrt{9 - (-u)^2}$
$F(u) = \sqrt{9 + u^2}$
$\boxed{= 3 - u}$ → genap

$9 - u^2 = 0$
$u^2 = 9$
$u = \sqrt{9}$
± 3
$(-\infty, -3) \cup (3, \infty)$

② $g(u) = \frac{1}{\sqrt{u^2 - 1}}$ = genap

ganjil $F(u) = -F(u) = 2$
genap $F(-u) = F(u) = 0$

$F(u) = \frac{1}{\sqrt{u^2 - 1}}$ $\quad (-u)$
$= \frac{1}{\sqrt{(-u)^2 - 1}}$

$g(-u) = g(u) \rightarrow$ genap

$u^2 - 1 = 0$
$u^2 = 1$
$u = \pm 1$

ujian akhir

Tahun : 2025

Bagian 1

1) Misalkan f kontinu pada (a,b). Jika F adalah suatu anti turunan dari f dengan $F(b)=18$ & $F(a)=5$ maka $\int_a^b f(u)\,du = \ldots$

↳ F anti turunan dari f
Berdasarkan TDK II

$\int_a^b f(u)\,du = F(b) - F(a)$
$= 18 - 5$
$= 13$

$-\frac{13}{1} - \frac{21}{4}$

$= \frac{21}{4} - \frac{15}{1} = \frac{81-60}{4} = \frac{21}{4}$

$= \frac{1}{4} - \frac{4}{3} + \frac{1}{1} = \frac{3-16-1}{12}$
$= \frac{-13-1}{12} = \frac{-26}{12} = -\frac{13}{6}$

2) Jika $f(u) = \begin{cases} -1, & \text{untuk } 0 \le u < 2 \\ 4, & \text{untuk } 2 \le u \le 6 \\ 0, & \text{untuk } u \text{ lainnya} \end{cases}$, maka $\int_0^6 f(u)\,du = \ldots$

↳ $\int_0^6 f(u)\,du =$

$\int_0^2 f(u)\,du + \int_2^6 f(u)\,du$

$= \int_0^2 (-1)\,du + \int_2^6 4\,du$

$= (-1)(2-0) + 4(6-2)$

$= -1(2) + 4(4) = -2 + 16 = 14$

$\int_0^6 f(u)\,du$, $0 \le u < 2$, $3 \le u \le 6$

$\int_0^3 (-2)\,du + \int_3^6 (3)\,du$
$= -2(3) + 3.(3)$
$= -6 + 9 = 3$

3) Misalkan f kontinu pada interval $[1,3]$. Nilai fungsi f di beberapa titik pada interval $[1,3]$ diberikan:

u	1	$1\frac{1}{2}$	2	$2\frac{1}{2}$	3
$f(u)$	5	3	2	4	7

Hampiran jumlah rieman kanan u/ nilai $\int_1^3 f(u)\,du$ dengan partisi seragam sebanyak $n = 4$ subinterval adalah...

↳ $\Delta u = \frac{b-a}{n} = \frac{3-1}{n} = \frac{2}{n} = \frac{2}{4} = \frac{1}{2}$

← kanan

1, $\frac{3}{2}$, 2, $\frac{5}{2}$, 3

kiri →

$\int_1^3 f(u)\,du = \Delta u\left[f(1) + f(3/2) + f(2) + f(5/2)\right]$

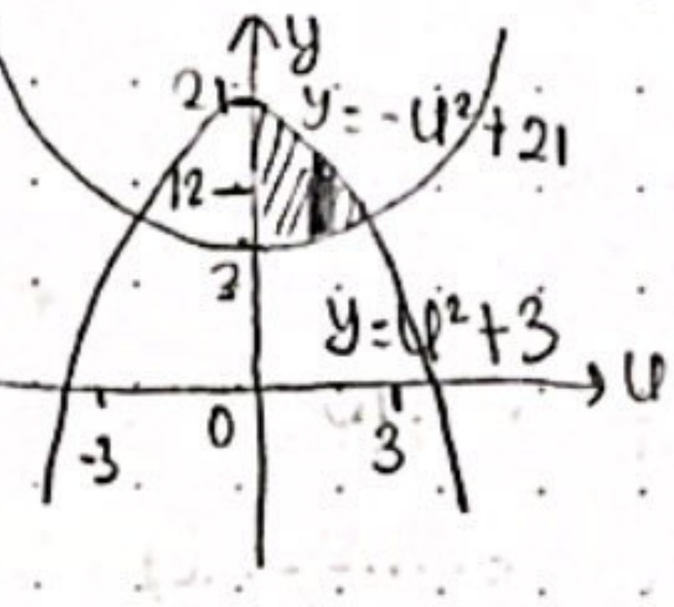
$\int_1^3 f(u)\,du = \Delta u\left[f(\tfrac{3}{2}) + f(2) + f(\tfrac{5}{2}) + f(3)\right]$

$= \frac{1}{2}(3+2+4+7) = \frac{1}{2}\cdot 16 = 8$

4) Luas daerah tertutup di kuadran pertama yang dibatasi oleh parabola $y = u^2+3$ & $y = -u^2+21$ serta sumbu-y adalah $\int_0^b f(u)\,du$

$f(u)\ldots$ & $b\ldots$

↳



$\Delta L \approx [(-u^2+21) - (u^2+3)]\Delta u$
$= [-2u^2 + 18]\Delta u$

$L = \int_0^3 (18 - 2u^2)\,du$

$b = 3$
$f(u) = 18 - 2u^2$

5) Jika $y = \ln(u^2-4)$, maka $\frac{dy}{du} = \ldots$

→ $y = \ln(u^2-4)$

$\boxed{\frac{d}{du}\ln(u) = \frac{1}{u}}$

$\frac{d}{du}\ln(f(u)) = \frac{1}{f(u)} \cdot f'(u)$

$\frac{d}{du}\ln(u^2-4) = \frac{1}{u^2-4} \cdot \frac{d}{du}(u^2-4)$

$= \frac{2u}{u^2-4}$

6) Perhatikan bahwa fungsi $f(u) = \sqrt{u+9}$ mempunyai invers pada daerah asalnya & titiknya $(7,4)$ berada pada grafik f. Nilai $(f^{-1})'(4) = 8$

→ Berdasarkan teorema fungsi invers

$(f^{-1})'(y) = \frac{1}{f'(u)}$

$(f^{-1})'(4) = \frac{1}{f'(7)} = \frac{1}{1/8} = 8$

$f(u) = \sqrt{u+9} = (u+9)^{1/2}$

$f'(u) = \frac{1}{2}(u+9)^{1/2-1}$

$= \frac{1}{2}(u+9)^{-1/2}$

⇒

$F'(7) = \frac{1}{2}(16)^{-1/2}$

$= \frac{1}{2}(4^2)^{-1/2}$

$= \frac{1}{2}4^{-1} = \frac{1}{2}\cdot\frac{1}{4} = \frac{1}{8}$ //

7) $\int_5^{15} \frac{1}{u \ln 2} du = \log_2(b)$ dengan $b = \ldots$

$\Rightarrow \int_5^{15} \frac{1}{u \ln 2} = {}^2\log b$

$\frac{1}{\ln(2)} \int_5^{15} \frac{1}{u} du = \frac{1}{\ln(2)} [\ln(u)]_5^{15}$

$= \frac{1}{\ln(2)} [\ln 15 - \ln 5]$

$= \frac{1}{\ln(2)} \cdot \left(\ln\left(\frac{15}{5}\right)\right)$

$= \frac{1}{\ln(2)} \cdot \ln(3)$

$= \frac{\ln(3)}{\ln(2)}$

ingat: ${}^a\log b = \frac{\log(b)}{\log(a)}$

${}^a\log b = \frac{\log b}{\log a}$

$= {}^2\log 3$

$= {}^2\log 3$

8) Jika $a = \ln(3)$ maka $\cosh a = \frac{p}{q}$, dengan bilangan bulat p & q adalah $p = \ldots$ & $q = \ldots$

$\Rightarrow \cosh(u) = \frac{e^u + e^{-u}}{2}$ // $\sinh(u) = \frac{e^u - e^{-u}}{2}$

$\cosh(\ln(3)) = \frac{e^{\ln(3)} + e^{-\ln(3)}}{2}$

$= \frac{3 + e^{\ln(\frac{1}{3})}}{2} = \frac{3 + \frac{1}{3}}{2}$

$= \frac{\frac{3}{1} + \frac{1}{3}}{2}$

$= \frac{\frac{9+1}{3}}{2} = \frac{\frac{10}{3}}{2}$

$= \frac{\cancel{10}^5}{3} \times \frac{1}{\cancel{2}}$

$= \frac{5}{3}$ //

Tougas 8

$e^{\ln(3)} = 3$

$e^{-\ln(3)} = \frac{1}{e^{\ln(3)}} = \frac{1}{3}$ //

Bagian B

① Misalkan $F(u) = \int_1^{u^2+u} \sqrt{3+t^2}\, dt$

Tentukan interval terbesar sehingga F monoton naik

$F'(u) > 0$ ✓

Berdasarkan TDK & aturan rantai

$F'(u) = \frac{d}{du}\left[\int_1^{u^2+u} \sqrt{3+t^2}\, dt\right]$

$= \sqrt{3+t^2} \cdot \frac{d}{du} 3+t^2$

$= \sqrt{3+(u^2+u)} \cdot \frac{d}{du}(u^2+u)$

$= \sqrt{3+(u^2+u)^2} \cdot (2u+1)$

$F(u)$ monoton naik saat $F'(u) > 0$

$\underbrace{\sqrt{3+(u^2+u)^2}}_{>0} \cdot (2u+1) > 0$

$2u+1 > 0$

$2u > -1$

$\boxed{u > \frac{-1}{2}}$

② Misalkan $I = \left[\frac{-\pi}{3}, \frac{\pi}{3}\right]$ & $F(u) = \pi \cos(3 \cdot u)$ untuk setiap u di I.

Tentukan suatu bilangan real e di interval I yang memenuhi F(e) adalah nilai rata² integral dari F pada interval I!

$f_{avg} = \frac{\int_{-\pi/3}^{\pi/3} \pi \cdot \cos(3u)\, du}{\pi/3 - (-\pi/3)}$ interval, cos → integral sin

$\cos(3u)$ $\frac{1}{3} \cdot \pi$:2 $\frac{2\pi}{3}$

$= \frac{3}{2\pi}\left[\pi \cdot \frac{1}{3} \sin(3u)\right]_{-\pi/3}^{\pi/3}$

$= \frac{3}{2\pi} \cdot \frac{1}{3\pi}\left[\underbrace{\sin(\pi)}_{0} - \underbrace{\sin(-\pi)}_{0}\right]$

$= 0$ //

$F(c) = f_{avg} = 0$

$\pi \cdot \cos(3c) = 0$

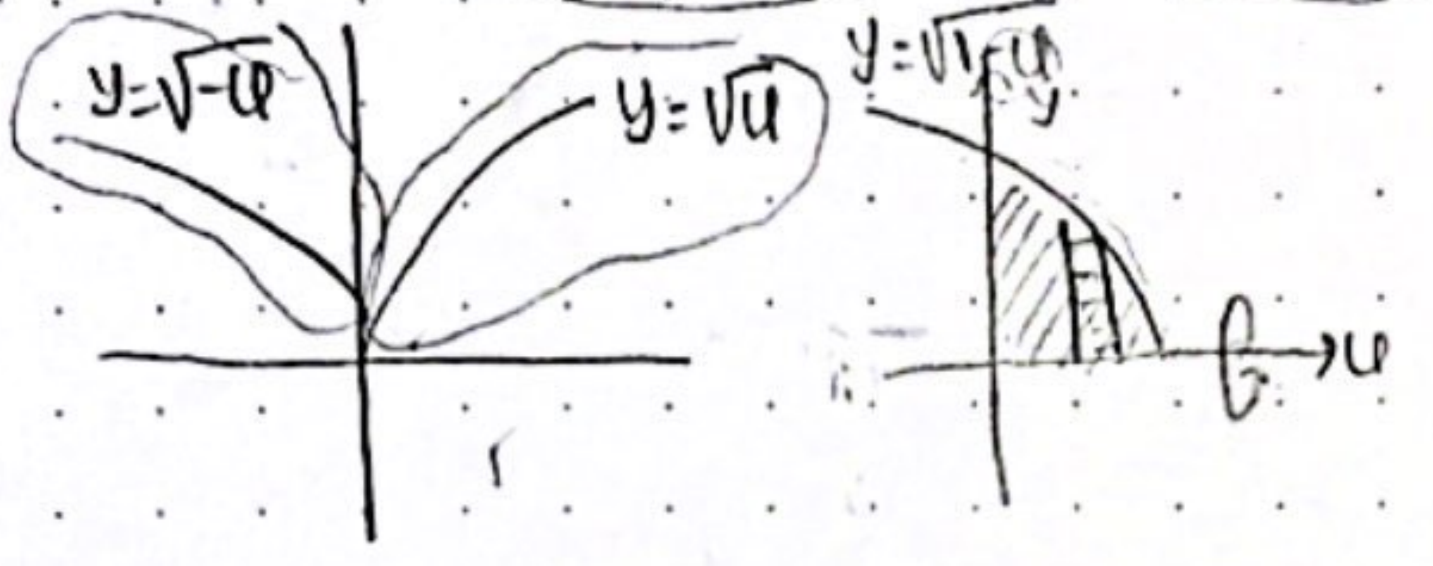
$\cos(3c) = 0$

$3c = \frac{-3\pi}{2}, \frac{-\pi}{2}, \frac{\pi}{2}, \frac{3\pi}{2} \ldots$ =0

$x = -\frac{3\pi}{2}, -\frac{\pi}{2}, \frac{\pi}{2}, \frac{3\pi}{2}$, dst

$c = \frac{\pi}{2}, \boxed{-\frac{\pi}{6}, \frac{\pi}{6}}, \frac{\pi}{2}$

(3) Sketsa daerah tertutup di kuadran pertama yang dibatasi oleh kurva $y = \sqrt{1-x}$, sumbu x & sumbu y kemudian tentukan integral tentu yang menyatakan volume benda yang dibentuk dari memutar daerah tersebut terhadap sumbu x (integral tidak perlu dihitung)



o Partisi ⊥ sb. putar → ga ada lubang
↳ Metode $\boxed{\text{cakram}}$

$\Delta V = \pi R^2 \Delta u$

$= \pi(\sqrt{1-x})^2 \Delta u$ → tebal cakram

$= \pi(1-u)\Delta u$

$V = \pi \int_0^1 (1-u)\,du$

#cincin → lubang ditengah

$\boxed{V = \pi \int_a^b (R_{luar}^2 - R_{dalam}^2)\,du}$

Singgung

(4) Tentukan titik (a,b) pada kurva $y = e^{y/2}$ sehingga gradien garis singgung pada kurva di titik tersebut adalah 2

↳ $y = e^{y/2}$ (turunan implisit)

$\frac{d}{du}(y) = \frac{d}{du}(e^{y/2})$

$m = F'(a)$ → gradien garis singgung ↓ nilai turunan fungsi

$\frac{d}{du}(y) = \frac{dy}{du}$ $\quad y' = e^u$ (pertama turunan) (u → y)

$\frac{d}{du}(e^{y/2})$: $\quad y = e^{F(u)}$

$e^{y/2} \cdot \frac{d}{du}(y/2)$ $\quad y' = e^{F(u)} \cdot F'(u)$

$= e^{y/2} \cdot \frac{1}{2} \frac{dy}{du}$ $\quad y = e^{u/2}$

$\frac{dy}{du} = e^{y/2} \cdot \frac{1}{2} \frac{dy}{du}$ $\quad y' = e^{u/2} \cdot \frac{d}{du}\left(\frac{u}{2}\right) = \frac{1}{2} \cdot e^{u/2}$

$m = F'(a) = \frac{1}{2} e^{a/2} = 2$

$e^{a/2} = 4$

$\ln(e^{a/2}) = \ln(4)$

$\frac{a}{2} = \ln(4)$

$a = 2\ln(4)$

$b = e^{a/2}$

$b = e^{2\ln(4)/2}$

$b = e^{\ln(4)}$

$b = 4$

$[a,b] = [2\ln(4), 4]$

#(2) $F_{avg} = \dfrac{\int_{-\pi/3}^{\pi/3} \pi \cos(3u)\,du}{\pi/3 - (-\pi/3)}$

⇒ panjang interval $= \frac{\pi}{3} - \left(-\frac{\pi}{3}\right) = \frac{2\pi}{3}$

$\frac{3}{2\pi}\left[\left(\pi \cdot \frac{1}{3}\right)\sin 3u\right]_{\pi/3}^{\pi/3}$

$\int \cos(3u)\,du = \frac{\sin(3u)}{3}$

$= \frac{3}{2\pi}\left[\pi \cdot \frac{1}{3} \sin 3u\right]_{-\pi/3}^{\pi/3}$

$= \frac{3}{2\pi} \cdot \frac{1}{3\pi}\left[\underbrace{\sin(\pi)}_{0} - \underbrace{\sin(-\pi)}_{0}\right]$

$\sin(3u)\Big]_{-\pi/3}^{\pi/3}$

$= \sin\left(3 \cdot \frac{\pi}{3}\right) - \sin\left(3 \cdot -\frac{\pi}{3}\right)$

$= \sin(\pi) - \sin(-\pi)$

Sifat fungsi sinus?

$\sin(0) = 0$

$\sin(\pi) = 0$

$\sin(-\pi) = 0$

(5) Diketahui T(t) adalah temperatur setiap waktu t dari suatu benda didalam ruangan yang memenuhi:

$T(t) = R + 60e^{kt}$

dengan R adalah temperatur ruangan yang konstan & k adalah konstanta. Setelah 1 jam, temperatur benda tsb adalah 60°C. Setelah 2 jam sejak awal temp. benda turun 45°C. Tentukan T ruangan R!

$\boxed{T(t) = R + 60e^{kt}}$

$T(1) = 60 \Rightarrow 60 = R + 60e^{k}$

$T(2) = 45 \Rightarrow 45 = R + 60e^{2k}$ −

$15 = 60e^{k} - 60e^{2k}$

$1 = 4e^{k} - 4e^{2k}$ $\quad$ $60 = R + 60e^{-\ln(2)}$

Misal $x = e^{k}$ $\quad$ $60 = R + 60e^{\ln(\frac{1}{2})}$

$1 = 4x - 4x^2$ $\quad$ $60 = R + 60 \cdot \frac{1}{2} = \boxed{R = 30}$

$4x^2 - 4x + 1 = 0$ $\quad$ $\frac{b \pm \sqrt{b^2 - 4ac}}{2a}$

$(2x-1)^2 = 0$ $\quad$ $x_{1,2} = \frac{4 \pm \sqrt{(-4)^2 - 4 \cdot 4 \cdot 1}}{2 \cdot 4}$

$\boxed{x = \frac{1}{2}}$ $\quad$ $= \frac{4 \pm \sqrt{16 - 16}}{8} = \frac{1}{2}$

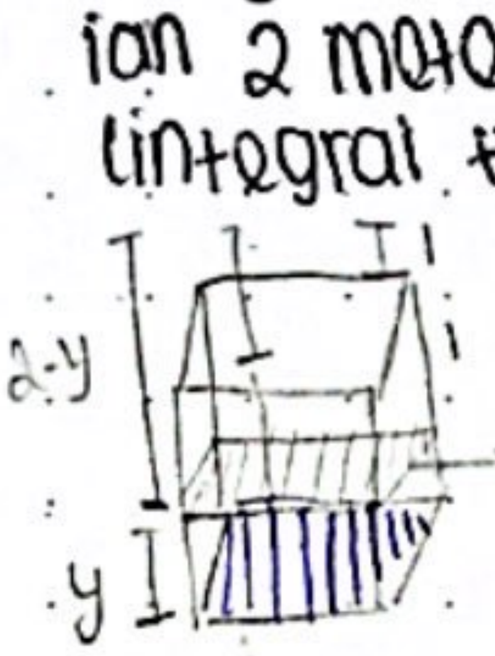
$e^{k} = \frac{1}{2}$

$\ln(e^{k}) = \ln(\frac{1}{2})$

$k = \ln(\frac{1}{2})$

$= -\ln(2)$

6) Suatu tangki terbentuk kubus dengan panjang rusuk 1 meter penuh berisi air dengan berat jenis 10^4 N/m^3. Tuliskan integral yang menyatakan kerja/usaha yang dilakukan w/ memompa seluruh air di dalam tangki hingga ketinggian 2 meter di atas dasar tangki (integral tidak perlu dihitung)!



$\boxed{\Delta V = p.l.t}$

$= 1.1.\Delta y$

$\Delta V = \Delta y$

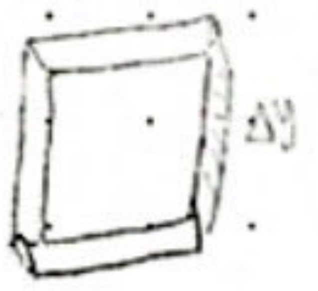
$\rho = \frac{F}{V}$

$F = \rho . V$

$\Delta F = \rho . \Delta V$ → ketetapan

$= \rho . \Delta y$

$= 10^4 . \Delta y$



$W = F.s$

$\Delta W = \Delta F . s$

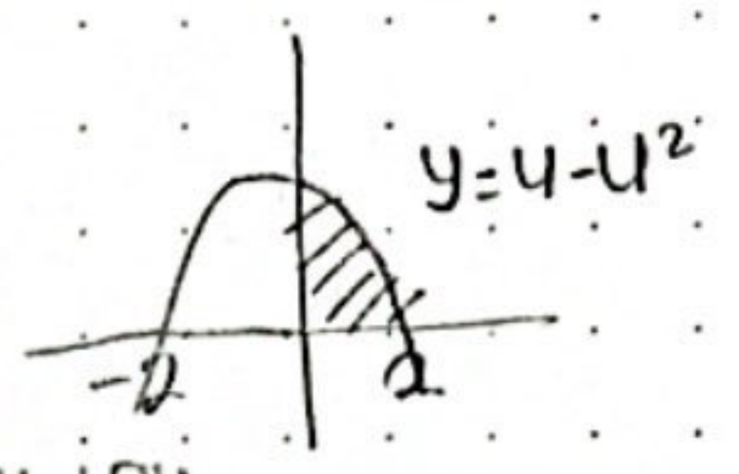
$= 10^4 . \Delta y (2-y)$

$W = 10^4 \int_0^1 (2-y)\,dy$

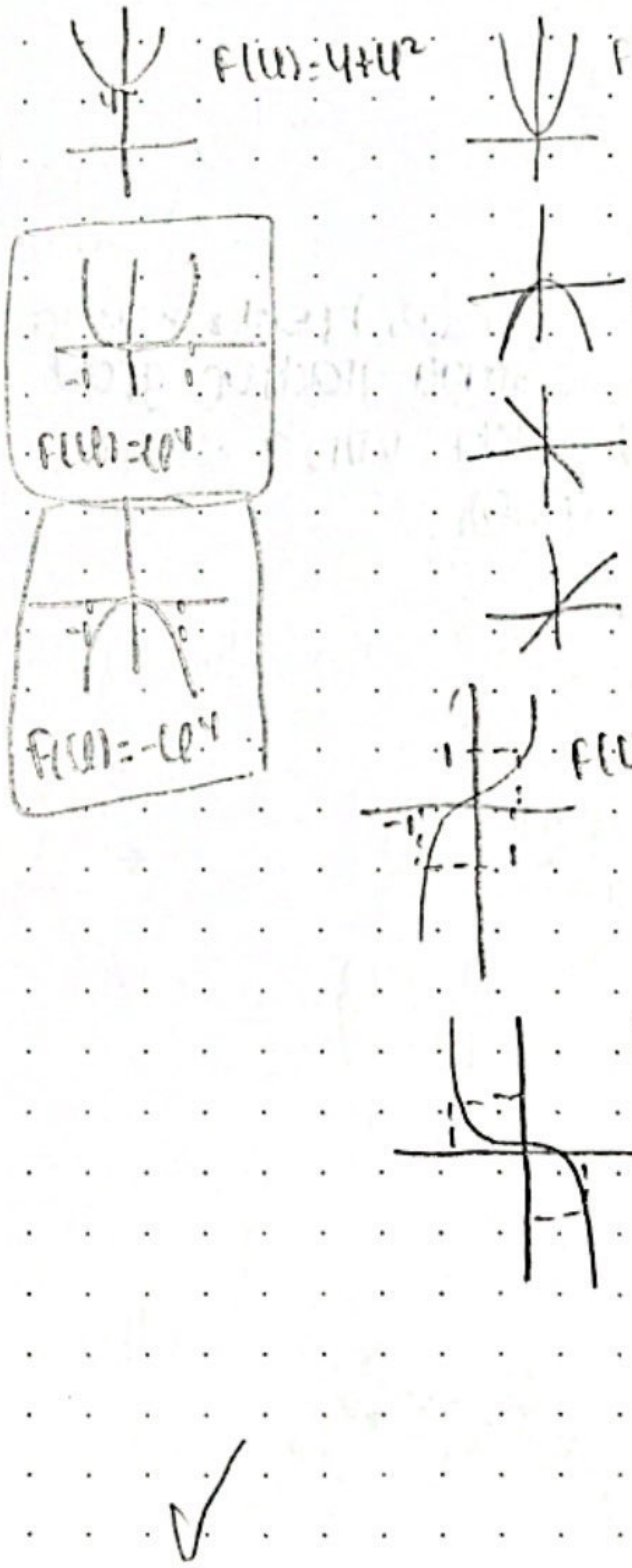
9) Bagian C

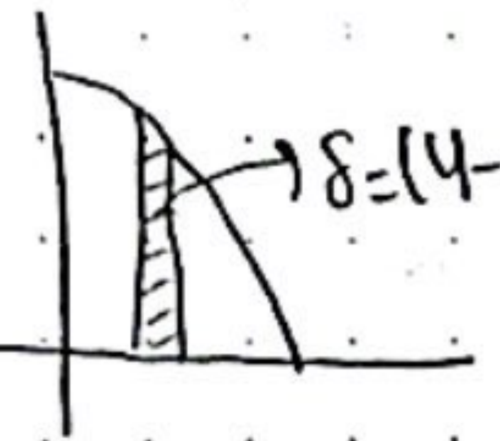
1. Misalkan D adalah daerah tertutup pada kuadran pertama yang dibatasi oleh kurva $y = 4 - x^2$, sumbu x & sumbu y.
(a) Sketsa daerah D!
(b) Tentukan volume benda pejal dengan alas D & irisan penampang yang tegak lurus sumbu-x membentuk persegi
(c) Tentukan nilai positif agar volume benda putar yang dibentuk dari daerah D yang diputar mengelilingi garis $x = b$ adalah 32π satuan volume!

a) (a) $y = 4 - x^2$

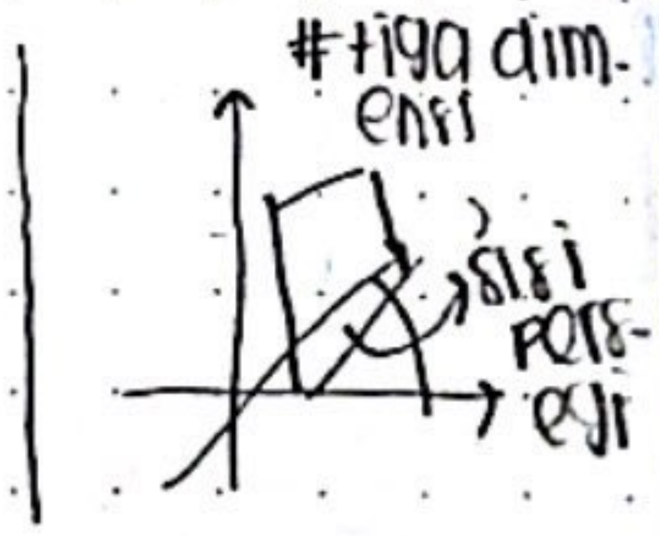


perlu tau



(b) 

$\delta = (4-u^2) - 0$



tiga dimensi

sisi persegi

$\boxed{\Delta V = L.\text{persegi} \cdot \Delta u}$

$= \delta^2 \Delta u$

$= (4-u^2)^2 \cdot \Delta u$

$V = \int_0^2 (4-u^2)^2 du$

$= \int_0^2 (u^4 - 8u^2 + 16) du$

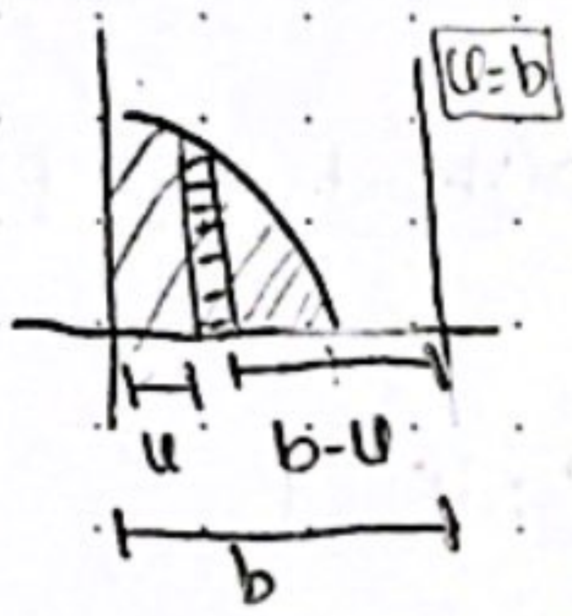
$= \left[\frac{1}{5}u^5 - \frac{8}{3}u^3 + 16u\right]_0^2$

$= \frac{2^5}{5} - \frac{8}{3}(2)^3 + 16(2)$

$= \frac{32}{5} - \frac{8 \cdot 8}{3} + 32$

$= \frac{32}{5} - \frac{64}{3} + \frac{32}{1}$

$= \frac{256}{15}$

(c) 

$\boxed{u=b}$

Partisi // sb. putar

"Metode kulit tabung

$\Delta V = 2\pi r h \Delta u$

$= 2\pi (b-u)(4-u^2)\Delta u$

$V = 2\pi \int_0^2 (b-u)(4-u^2) du$

$V = 2\pi \int_0^2 (u^3 - bu^2 - 4u + 4b) du$

$V = 2\pi \left[\frac{1}{4}u^4 - \frac{b}{3}u^3 - \frac{\cancel{4}^2}{\cancel{2}}u^2 + \frac{\cancel{4}^2}{\cancel{2}}b^2\right]_0^2$

$V = 2\pi \left[\frac{1}{4}u^4 - \frac{b}{3}u^3 - 2u^2 + 4bu\right]_0^2$

$\cancel{32\pi} = \cancel{2\pi}\left[\frac{1}{4}(2)^4 - \frac{b}{3}(2)^3 - 2(2)^2 + 4b(2)\right]$

$16 = \left[\frac{1}{\cancel{4}}(\cancel{16})^4 - \frac{b}{3}(8) - 2 \cdot 4 + 4b2\right]$

$16 = \left[4 - \frac{8b}{3} - 8 + 8b\right]$

$16 = \left[-4 - \frac{8b}{3} + \frac{8b}{1}\right]$

$16 = \left[-4 \; \frac{-8b + 24b}{3}\right]$

$16 = \left[-4 + \frac{16b}{3}\right] \Leftrightarrow 20 = \frac{16}{3}b$

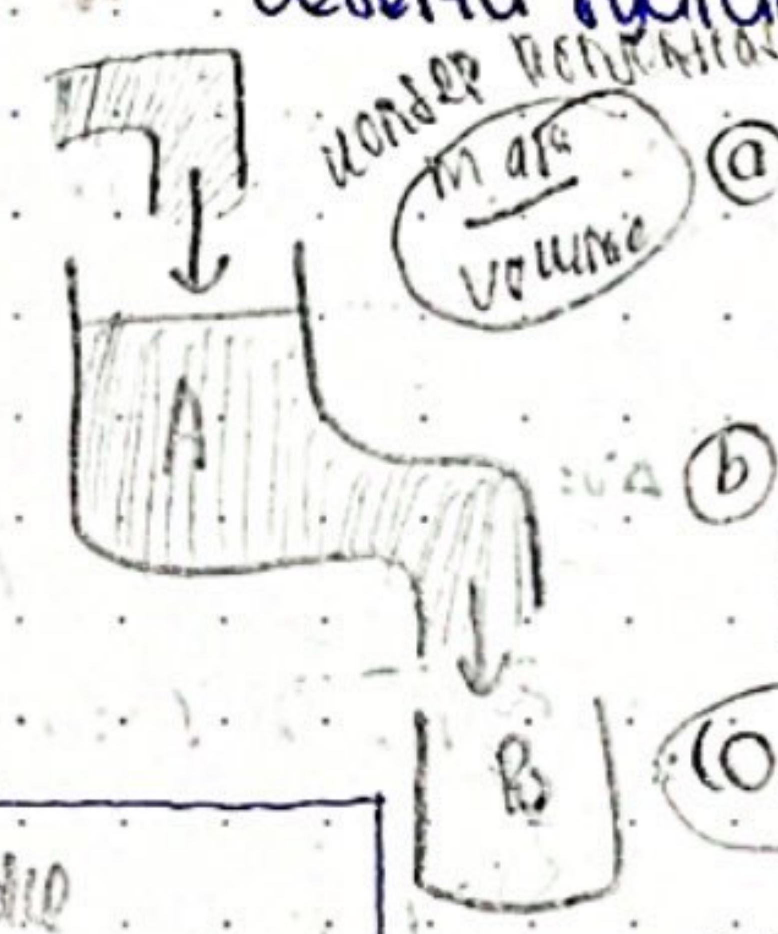
$b = 20 \times \frac{3}{16}$

$= \frac{60}{16} = \frac{15}{4}$ //

2. Tangki A mula-mula berisi 1000 liter larutan garam & tangki B mula² kosong. Larutan garam dengan konsentrasi 0,02 kg / liter mengalir ke dalam tangki A dengan laju 10 liter / menit. Pada saat bersamaan, larutan di dalam tangki A mengalir ke tangki B dengan laju 20 liter / menit tetapi larutan di dalam tangki B tidak mengalir ke luar. Asumsikan bahwa larutan di dalam masing-masing tangki A & B setiap saat teraduk secara sempurna. Diketahui bahwa setelah 20 menit, massa garam terlarut di dalam tangki A adalah 22,4 kg.

<a) Tentukan volume larutan garam setiap saat dalam tangki A!

<b) Misalkan y menyatakan massa garam terlarut di dalam tangki A. Tentukan suatu persamaan diferensial yang dipenuhi oleh y beserta syaratnya!



konsep kekekalan massa volume

(a) $V_A(t) = 1000 + 10t - 20t$

$= 1000 - 10t$

y: massa garam di A

(b) $\boxed{\frac{dy}{dt} = \text{laju masuk} - \text{laju keluar}}$

$(0,\cancel{0}2)\frac{kg}{liter} \cdot 1\cancel{0}\frac{liter}{menit} - \frac{y}{1000-10t} \cdot 20$

$= 0,2\frac{kg}{menit} - \frac{y}{1000-1\cancel{0}t} \cdot 2\cancel{0}$

$0,2\frac{kg}{menit} - \frac{2y}{100-t}$

$\frac{dy}{dt} = 0,2 - \frac{2}{100-t} \cdot y, \; y(20) = 22,4$ //

<c) Tentukan solusi dari persamaan diferensial pd bagian (b)!

$\frac{dy}{dt} + \frac{2}{100-t}y = 0,2$

$FI = e^{\int \frac{2}{100-t} dt} = e^{-2\ln(100-t)}$

$= \frac{1}{(100-t)^2}$

$\frac{1}{(100-t)^2}\frac{dy}{dt} + \frac{2}{(100-t)^3}y = \frac{0,2}{(100-t)^2}$

$\frac{d}{dt}\left[y \cdot \frac{1}{(100-t)^2}\right] = \frac{0,2}{(100-t)^2}$

$y \cdot \frac{1}{(100-t)^2} = \int \frac{0,2}{(100-t)^2} dt$

$y = \frac{1}{(100-t)^2} = \frac{0,2}{(100-t)} + C$

$= 0,2(100-t) + C \cdot [100-t]^2$

$y(20) = 22,4$

$22,4 = 0,2(80) + C \cdot (80)^2$ $\quad y(t) = 0,2(100-t) + \frac{1}{1000}(100-t)^2$

$22,4 = 16 + 6400C$

$6,4 = 6400C$

$C = \frac{6,4}{6400} = \frac{1}{1000}$

nomor 2

$y(t) 22,4 \quad \frac{dy}{dt} = 0,2 - \frac{2y}{100-t}$

$\frac{dy}{0,2 - \frac{2y}{100-t}} = dt$

$0,2 - \frac{2y}{100-t} = \frac{0,2(100-t) - 2y}{100-t}$

$\frac{100-t}{0,2(100-t)-2y} \, dy = dt \quad = \frac{100-20}{0,2(100-20)-2y}$

$\frac{dy}{dt} = 0,2 - \frac{2y}{100-t} \quad = 1,26$

$t(20) = \frac{0,2(100-t)-2y}{100-t}$

$\frac{100-t}{0,2(100-t)-2y} \, dy = dt$

$0,2(100-20) - 2y = 0,2(80) - 2y$

$= 16 - 2y$

$\frac{dy}{16-2y} = \frac{dt}{80}$

$\int \frac{1}{u} \, du = \int \frac{1}{80} \, dt$

$-\ln|16-2y| = \frac{t}{80} + C$

$16 - 2y = Ce^{-t/80}$

$y = 22,4$

$16 - 2(22,4) = Ce^{-20/80}$

(d) Hitung massa garam terlarut di dalam tangki B ketika volume larutan di dalam tangki tersebut 1000 liter!

$\frac{dy}{dt} = \frac{\cancel{0,2(100-t)} + \frac{1}{1000}(100-t)^2}{\cancel{10(100-t)}} \cdot \cancel{20}$

$\frac{dy}{dt} = 0,2 + \frac{1}{1000}(100-t)$

$\frac{dy}{dt} = 0,2 + \frac{1}{1000}(100-t)$

$y = \int 0,2 + \frac{1}{1000}(100-t) \, dt$

$= 0,2t + \frac{1}{1000}\left[100t - \frac{1}{2}t^2\right] + C$

$\boxed{y(t) = 0,2(100-t) + \frac{1}{1000}(100-t)^2}$

$y(0) = 0,2(100) + \frac{1}{1000}(100)^2$

$= 20 + \frac{(100)^2}{1000}$

$= 20 + \frac{10.\cancel{000}}{1\cancel{000}}$

$= 30$

$y(0) = 30$

$30 = 0 + \frac{1}{1000}[0-0] + C$

$C = 30$

$y(t) = 0,2t + \frac{1}{1000}\left[100t - \frac{1}{2}t^2\right] + C$

$\boxed{1000 \text{ liter} = 20t \quad t = \frac{1000}{20} = 50}$

$y(50) = 0,2(50) + \frac{1}{1000}\left[1000(50) - \frac{1}{2}(50)^2\right] + 30$

$= 40 + \frac{5000 - 1250}{1000}$

$= 40 + \frac{3750}{1000} = 43,75$

X

Bagian B

1.) Hitunglah $\lim\limits_{u \to 2} \frac{u^2-u-2}{u^2-4}$

$$= \lim_{u \to 2} \frac{u^2-u-2}{u^2-4} = \lim_{u \to 2} \frac{\cancel{(u-2)}(u+1)}{\cancel{(u-2)}(u+2)}$$

$$= \lim_{u \to 2} \frac{u+1}{u+2}$$

$$= \frac{2+1}{2+2} = \frac{3}{4}$$

2) $F(u) = \sqrt{u^2+5}$

$F'(2) = ?$

↳ $u = u^2+5$

$\frac{du}{du} = 2u$

$F(u) = \sqrt{u} = u^{1/2}$

$\frac{dF(u)}{du} = \frac{1}{2} u^{-1/2} = \frac{1}{2\sqrt{u}}$

$F'(u) = \frac{dF(u)}{du} = \frac{dF(u)}{du} \cdot \frac{du}{du}$

$= \frac{1}{\cancel{2}\sqrt{u}} \cdot \cancel{2}u$

$= \frac{u}{\sqrt{u}} = \frac{u}{\sqrt{u^2+5}}$

$F'(2) = \frac{2}{\sqrt{2^2+5}} = \frac{2}{\sqrt{9}} = \frac{2}{3}$

3) $F(u) = u + \frac{4}{u}$

$[1,5]$

$u = 1$ & $u = 5$

$F(u) = u + 4u^{-1}$

$F'(u) = 1 - 4u^{-2}$

$= 1 - \frac{4}{u^2} = \frac{u^2-4}{u^2} = \frac{(u-2)(u+2)}{u^2}$

$F'(u) = \frac{(u-2)(u+2)}{u^2} = 0$

$u = -2 \vee u = 2$

$\left(u + \frac{4}{u}\right)$

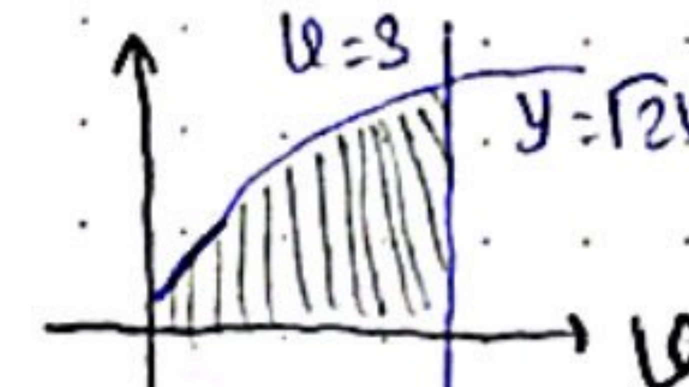
$F(-2) = -2 + \frac{4}{-2} = -4$

$F(1) = 1 + \frac{4}{1} = 5$

$F(2) = 2 + \frac{4}{2} = 4$

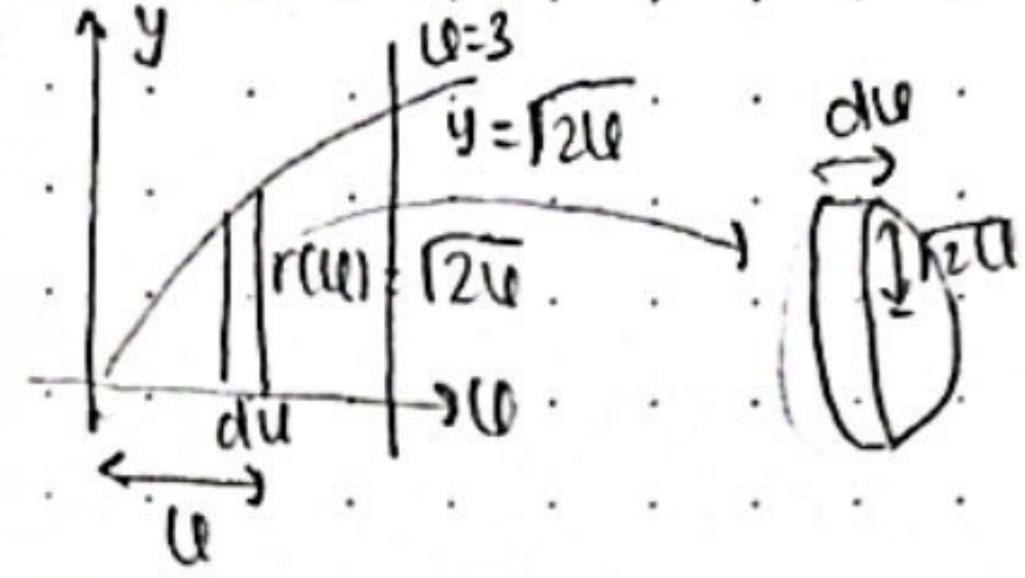
$F(5) = 5 + \frac{4}{5} = \frac{5}{1} + \frac{4}{5} = \frac{25+4}{5} = \frac{29}{5}$

4) Suatu daerah D di kuadran I dibatasi oleh kurva $y = \sqrt{2u}$ garis $u = 3$ & sumbu u



Tentukan volume padat yang diperoleh dengan memutar daerah D mengelilingi sumbu-u!

↳



Taksiran volume satu irisan pada daerah D:

$$dV = \pi\left(\sqrt{2u}\right)^2 du$$

$$= 2\pi u \, du$$

Daerah D memanjang dari $u=0$ hingga $u=3$

$V = \int_0^3 dV$

$= \int_0^3 2\pi u \, du = 2\pi \frac{u^2}{2} \Big|_0^3$

$= \pi u^2 \Big|_0^3$

$= \pi(3^2 - 0^2) = 9\pi$

5) Tentukan solusi umum persamaan diferensial $\frac{dy}{dt} = 3t^2 y$ dengan $y(t) > 0$!

↳ $\frac{dy}{dt} = 3t^2 y$

$\frac{1}{y} dy = 3t^2 dt$

dengan mengintegralkan kedua ruas:

$\int \frac{1}{y} dy = \int 3t^2 dt$ ↳ $\frac{\cancel{3}}{\cancel{3}} t^3$

$\ln|y| = t^3 + C$

$e^{\ln|y|} = e^{t^3+C}$

$|y| = e^{t^3+C}$

$|y| = y$ mengingat $y > 0$

Solusi umum persamaan diferensial =

$y = e^{t^3+C}$

UAS Math

① Diketahui fungsi f dengan turunan pertama:
$f'(u) = u^2 + 8u$
$f''(u) = 2u + 8$

<a> Interval buka terbesar sehingga grafik fungsi f monoton turun adalah ...,

$= f'(u) = u^2 + 8u$
$= u(u+8)$

$\xleftarrow{\;+\quad -\quad +\;}$ f' (titik -8, 0)

$f''(u) = 2u + 8$

$\xleftarrow{\;-\quad +\;}$ f'' (titik -4)

Monoton turun: // naik: $f'(u) \geq 0$

$f'(u) \leq 0$

$(u^2 + 8u) \leq 0$

$u(u+8) \leq 0$

$u(u+8) = 0$
$u = 0 \quad u + 8 = 0$
$u = -8$

$u = 0$ → max
$u = -8$ → min. } lokal

$-\infty, -8 = +$

$-8, 0 = \ominus \leftarrow (-8, 0)$

$0, \infty = +$

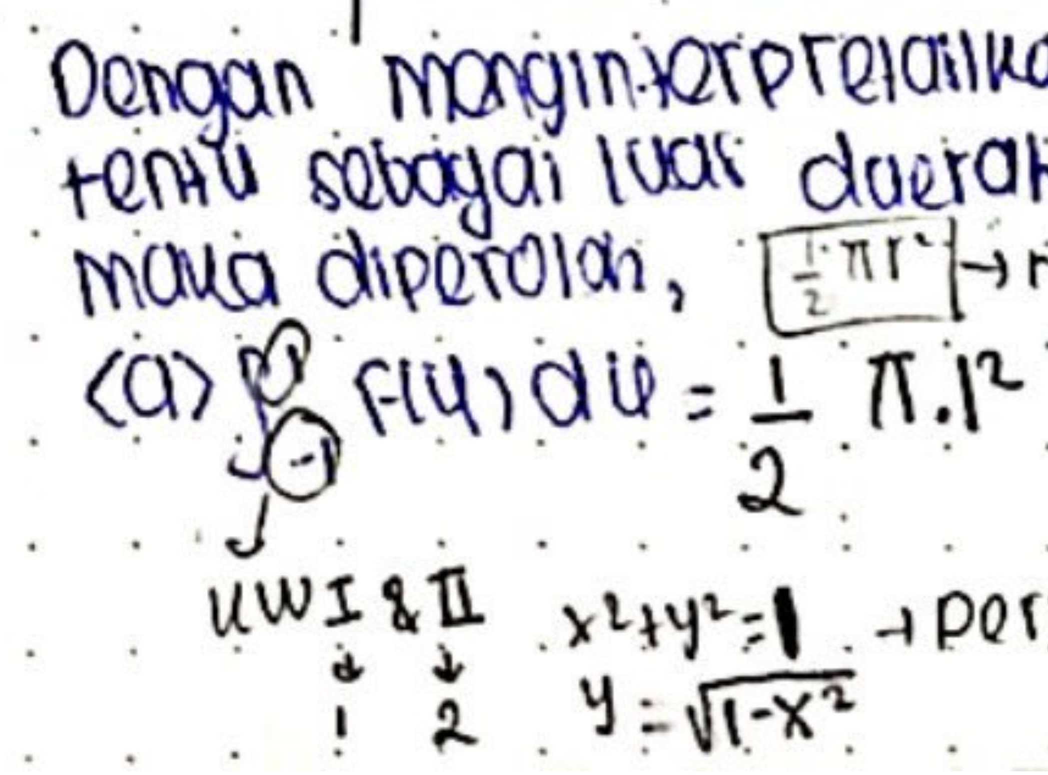
$\circled{0}$ $-9(-9+8)$
$-9 \cdot -1 = 9$
$\circled{-3}$ $-3(-3+8)$
$-3(5) = -$
$\circled{1}$ $1(1+8) = +$

<b> Interval buka terbesar sehingga grafik f cekung ke bawah adalah

$= f''(u) = u^2 + 8u$
$f'(u) \leq 0$
$2u + 8 \leq 0$
$u \leq -4$
$(-\infty, -4)$

cekung keatas
$f''(u) > 0$
cekung kebawah
$f''(u) < 0$
$2u + 8 = 0$
$2u = -8$
$u = -4$

② Diberikan grafik $y = f(u)$



Dengan menginterpretasikan integral tentu sebagai luas daerah bertanda maka diperoleh, $\boxed{\frac{1}{2}\pi r^2}$ → rumus luas lingkaran

<a> $\int_{-1}^{0} f(u)\,du = \frac{1}{2}\pi \cdot 1^2 = \frac{\pi}{2}$

KW I & II
↓ ↓
1 2

$x^2 + y^2 = 1$ → persamaan O
$y = \sqrt{1 - x^2}$

<b> $\int_1^5 f(u)\,du = \int_1^3 f(u)\,du + \int_3^5 f(u)\,du$

$= -\frac{2 \cdot \cancel{2}}{\cancel{2}} + \frac{2+1}{\cancel{2}} \cdot \cancel{2}$

$= -2 + 3 = 1$

③ Jika f ⇒ fungsi kontinu $\int_1^u f(t)\,dt = 2u^3 + C$
C = konstanta
maka $f(u) = \ldots$
$C = \ldots$

$\frac{d}{du}\int_1^u f(t)\,dt = \frac{d}{du}(2u^3 + C)$

$f(u) = 6u^2$

$u = 1$

$\int_1^1 f(t)\,dt = 2 \cdot 1^3 + C$

$0 = 2 + C$

$C = -2$

④ Diberikan data nilai fungsi g di beberapa titik pada $[2, 8]$

u	2	3	4	5	6	7	8
$g(u)$	3	4	5	5	7	3	2

Jika $\int_2^8 g(u)\,du$ dihampiri dengan metode jumlah Rieman kanan dengan $n = 3$ subinterval yang sama panjang, maka....

(a) lebar tiap subinterval adalah
$\Delta u = \frac{b-a}{n} = \frac{8-2}{3} = \frac{\cancel{6}}{\cancel{3}} = 2$

(b) Nilai hampiran:
$\int_2^8 g(u)\,du = \Delta u[g(u_1) + g(u_2) + g(u_3)]$
$= 2[g(4) + g(6) + g(8)]$
$= 2[5 + 7 + 2]$
$= 2[14] = 28$

⑤ Suatu zat radioaktif memiliki waktu paruh 500 tahun. Misalkan $y(t) = Ae^{kt}$ menyatakan massa (dalam gram) zat radioaktif tersebut setelah t tahun. Jika mula-mula massa zat radioaktif ini sebanyak 12 gram, maka

(a) $y(500) = \ldots$ gram
(b) $y(250) = \ldots$ gram

$y(t+500) = \frac{1}{2} y(t)$

$Ae^{k(t+500)} = \frac{1}{2} Ae^{kt}$

$e^{kt} e^{k.500} = \frac{1}{2} e^{kt} \Rightarrow e^{500k} = \frac{1}{2}$

$500k = \ln\left(\frac{1}{2}\right) = \ln(2^{-1}) = -\ln 2$

$k = \frac{-\ln 2}{500}$

$\ln(e^{500k}) = \ln\left(\frac{1}{2}\right)$
$\ln(e^{500k}) = 500k$
$500k = \ln\left(\frac{1}{2}\right)$
$e^{500k} = 500k$
$\ln(e^u) = u$
$\ln(a/b) = \ln|a| - \ln|b|$
$\ln(a^n) = n.\ln(a)$

$y(0) = 12$, maka:
$Ae^{k.0} = 12$
$A.1 = 12$
$A = 12$

$y(t) = 12 \exp\left(-\frac{\ln 2}{500} t\right)$
$= 12 (e^{\ln 2})^{-t/500}$
$= 12 . 2^{-t/500}$

(a) $y(500) = 12 . 2^{-500/500}$
$= \cancel{12} . \frac{1}{\cancel{2}} = 6$

(b) $y(250) = 12 . 2^{-250/500}$
$= 6\sqrt{2}$

>> Bagian a <<

① $F(u) = 6u - u^2$
$F''(u) = 6 - 2u$
(a) monoton naik
$f(u) \geq 0$
$F'(u) = 6u - u^2 \geq 0$
$u(6-u) \geq 0$
$6 - 2u = 0$
$u = 3$
$u = 0.86$
(b) cekung atas
$F''(u) > 0$
$6 - 2u > 0$
$3 - u > 0$
$+u > +3$
$u > 3$
$(-\infty, 3)$

② $\int_{-2}^{2} F(u)\,du$
$= u^2 + y^2 = 2$
$\frac{1}{2}\pi r^2 = \frac{1}{2}\pi 2^2$
$= \frac{4\pi}{2} = 2\pi$

(b) $\int_{2}^{6} F(u)\,du$
$\frac{a \times t}{2} + \frac{(a+b).t}{2}$
$\frac{2 \times 1}{\cancel{2}} + \frac{(2+1).1}{2}$
$-1 + \frac{\cancel{2}}{\cancel{2}}$
$= 1 + 1,5 = 0,5$

③ $\int_{1}^{u} F(t)\,dt = 3u^3 + C$
$F(u) = ?$ → TDK I
$\frac{d}{dt}(3u^3) = F(u) = 9u^2$
$C = 1$
$3(1)^3 = 3$

④ $\Delta u = \frac{b-a}{n} = \frac{8-2}{3} = \frac{6}{\cancel{3}} = 2$

b. $\int_{2}^{8} g(u)\,du = g(u_2) + g(u_4) + g(u_6)$
$= 2(3+5+7)$
$= (8+7).2 = 30$

⑤ $y = Ae^{kt}$
$\boxed{y^{600+t} = Ae^{kt}}$ → waktu paruh
$Ae^{k(t+600)} = \frac{1}{2} Ae^{kt}$
$e^{kt} + e^{k600} = \frac{1}{2} e^{kt}$
$600k = \ln\left(\frac{1}{2}\right)$
$\boxed{\ln 2^{-1} = 2}$
$k = -\frac{\ln 2}{600}$
$y(0) = 10$ gram, maka:
$Ae^{k.0} = 10$
$A.1 = 10$
$A = 10$
$y(t) = 10 \exp\left(-\frac{\ln 2}{600} t\right) = 10 (e^{\ln 2})^{-t/600}$
$= 10 . 2^{-t/600}$
(a) $y(600) = 10 . 2^{-(600)/600}$
$= 10 . 2^{-1}$
$= 10 . \frac{1}{2} = 5$ gr
(b) $y(300) = 10 . 2^{-300/600}$
$= 10 . \frac{1}{2^2}$
$= 10 . \frac{1}{4} = \frac{5}{2} = 2,5$ gr

Pemfaktoran

1) $x^2 - 11x + 24 = 0$ — jumlah

$(x-3)(x-8) = 0$

24	
1	24
2	12
-3	-8
4	6

2) $x^2 - 2x - 15$

$(x+3)(x-5)$

15	
1	15
3	5

3) $2x^2 - 5x - 3 = 0$

$(2x+1)(x-3)$

x

$-6x$ +

$-5x$

$2x+1 = 0$ | $x - 3 = 0$

$x = -\frac{1}{2}$ | $x = 3$

4) pemfaktoran distributif

$3x^2 - 6x = 3x(x-2)$

5) $2x^2 - 5x - 12$

$= \frac{1}{\cancel{2}}(2x+3)(2x-8) = (2x+3)(x-4)$

-24	
-1 . 24	1 . -24
-2 . 12	2 . -12
-3 . 8	3 . -8
-4 . 6	4 . -6

jumlah (-5)

6) Rumus abc:

$x^2 - 6x + 5 = 0$

$a = 1$, $b = -6$, $c = 5$

$$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-6) \pm \sqrt{-6^2 - 4(1).(5)}}{2(1)}$$

$$= \frac{6 \pm \sqrt{36 - 4.5}}{2}$$

$$= \frac{6 \pm 4}{2}$$

$\frac{6+4}{2} = \frac{10}{\cancel{2}} = 5$

$\frac{6-4}{2} = \frac{\cancel{2}}{\cancel{2}} = 1$

① $x^2 + x - 12 = 0$

$(x-3)(x+4)$

$x = 3$ $\quad x = -4$

-12	
1	12
2	6
3	4

= 1

+ − − +

-4 0 3

$F(-5) = x^2 + x - 12 = 0$

$= -5^2 + (-5) - 12 = 0$

$= 25 - 5 - 12 = 0$

$= 20 - 12 = 0$

$= (+)$

$F(-1) = x^2 + x - 12 = 0$

$= -1^2 + (-1) - 12 = 0$

$= 1 - 1 - 12 = 0$

$= (-)$

$F(1) = x^2 + x - 12 = 0$

$= 1^2 + 1 - 12 = 0$

$= (-)$

$F(8) = x^2 + x - 12 = 0$

$= 8^2 + 8 - 12 = 0$

$= 64 + (-4) = 0$

$= 60 (+)$

$HP = \{-4, 3\}$

② $x^2 - 15x + 50 = 0$

$(x-10)(x-5)$

$x = 10$ $\quad x = 5$

$HP = \{5, 10\}$

50	
1	50
2	25
5	10

(15)

③ $2x^2 + 17x + 21 = 0$

$(2x+3)(x+14)$ 7

$2x + 3 = 0$ $\quad x = -7$

$2x = -3$

$x = \frac{-3}{2}$

$HP = \{-7, \frac{-3}{2}\}$

42	
1	42
2	21
3	14

④ $6x^2 - 19x + 15 = 0$

$$\frac{(6x-10)(6x-9)}{\cancel{6}\,2}$$

$$\frac{(6x-10)(2x-3)}{\cancel{2}}$$

$(3x-5)(2x-3)$

$3x - 5 = 0$ $\quad 2x - 3 = 0$

$3x = 5$ $\quad 2x = 3$

$x = \frac{5}{3}$ $\quad x = \frac{3}{2}$

$HP = \{3/2, 5/3\}$

90	
1	90
2	45
3	30
5	18
6	15
9	10

-10 -9

JOYKO

Ujian Perb. 2021

Bagian 1-5

① Daerah asal & daerah hasil dari fungsi $f(u) = 4\sqrt{9-u^2}$ adalah ....

$4\sqrt{9-u^2}$

$9-u^2 \geq 0$

$u^2 \leq 9 \rightarrow \sqrt{9} \pm 3$

$-3 \leq u^2 \leq 3$

$D_f = (-3, 3)$

$R_f = f(u) = 4\sqrt{9-u^2}$

$f(0) = 4\sqrt{9-0^2} \rightarrow$

$= 4\sqrt{9} = 4 \times 3 = 12$

$f(-3) = 4\sqrt{9-(-3)^2}$

$= 4\sqrt{9-(9)} = 4\sqrt{0} = 4.0$
$= 0$

$f(3) = 4\sqrt{9-3^2} = 4\sqrt{9-9}$
$= 0$

$R_f = (0, 12)$

[ ] → notasinya ... termasuk dlm himpunan

$\therefore D_f = [-3, 3]$ & $R_f = [0, 12]$

② $\lim_{n \to \infty} \sum_{i=1}^{n} \left(\frac{8i}{n} + 5\right) \frac{2}{n} = \ldots$

↳ $\sum_{i=1} f(u_i) \Delta u$

<Bentuk umum jumlah riemann>

$u_i = a + i\Delta u$ ↳ titik partisi interval

$\Delta u = \frac{b-a}{n} = \frac{2}{n}$

$f(u_i) = \frac{8i}{n} + 5$

$\Delta u = \frac{2}{n}$

$u_i = \boxed{\frac{8i}{n}}$ $(u \rightarrow 0, \boxed{2})$

$\int_0^2 (8u+5)\,du = ?$

$\boxed{\int_0^2 (8u+5)\,du = \int_0^2 8u\,du + \int_0^2 5u\,du}$

$\int_0^2 8u = 8\left[\frac{u^2}{2}\right]_0^2 = 8\left(\left(\frac{2^2}{2} - \frac{0^2}{2}\right)\right)$

$= 8\left(\left(\frac{4}{2} - \frac{0}{2}\right)\right)$

$= 8((2-0))$

$= 8.2 = 16$

$\int_0^2 5u = 5\left[\frac{u^2}{2}\right]_0^2 = 5\left(\left(\frac{2^2}{2} - \frac{0^2}{2}\right)\right)$

$= 5\left(\left(\frac{4}{2} - \frac{0}{2}\right)\right)$

$= 5((2-0)) = 5.2 = 10$

$\boxed{= 26}$

3) Luas daerah di kuadran I yang dibatasi oleh kurva $y = u^2+3$, garis $y = u+9$ & sumbu y adalah ..... satuan luas

↳ $y = u^2+3 \rightarrow y = u+9$

titik puncak: $(0, 3)$

* gambaran umum:

$y = u^2+3$ → parabola membuka keatas $(0, 3)$

$y = u+9$ → garis lurus dan gradien (+) $(0, 9)$

$(u=0)$ → membatasi daerah di kiri

* $y = u^2+3$

$y = u+9$

$u^2+3 = u+9$

$u^2 - u - 6 = 0$

faktorkan: $(u-3)(u+2) = 0$

$u = 3 \vee u = -2$

* $y = u+9$

$y = u^2+3$

$u = 0$

$u = 3$

Bagian B

1) persamaan diferensial

$$\frac{dy}{du} = \frac{4u^3}{y^2}$$

$\boxed{y^2 = 4u^3}$

$\int y^2 dy = \int 4u^3 dy$

$\frac{y^3}{3} = u^4 + C$

$y^3 = 3u^4 + C$

$y = \sqrt[3]{3u^4 + C}$

2) $\int_0^2 3u^2 \sqrt{u^3+1}\, du$

= kompleks jadi

$u = u^3 + 1 \rightarrow u = (0) = 1$

$du = 3u^2\, du \quad u = (2) = 2^3 + 1 = 9$

$\int_0^2 3u^2 \sqrt{u^3+1}\, du$

$= \int_1^9 \sqrt{u}\, du = \left.\frac{u^{3/2}}{3/2}\right]_1^9$

$= \frac{2}{3}(9^{3/2} - 1^{3/2})$

$= \frac{2}{3}(27 - 1) = 26$

$\int \sqrt{u}\, du = \sqrt{u} = u^{1/2}$

$= \frac{u^{1/2+1}}{1/2+1} = \frac{u^{3/2}}{3/2}$

$= \frac{2}{3} u^{3/2} + C$

$= \frac{2}{3}(9^{3/2} - 1^{3/2})$

$= \frac{2}{3}(27-1) = \frac{52}{3}$

3) $F(u) = \cos(2u)$ → ganjil:

$I = \left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$ $\quad F(-u) = -F(u)$

$\sin(-3u) = -\sin(3u)$

→ genap:

$F(-u) = F(u)$

$\cos(-2u) = \cos(2u)$

genap

$$\boxed{\int_{-a}^{a} F(u)\, du = 2\int_0^a F(u)\, du}$$

$\int_{-a}^{0} F(u)\, du = -\int_0^a F(u)\, du$

$\Rightarrow \int_{-a}^{a} F(u)\, du = \int_{-a}^{0} F(u)\, du + \int_0^a F(u)\, du$

$\Rightarrow$ Fave $= \frac{1}{\pi/2 - (-\pi/2)} \int_{-\pi/2}^{\pi/2} \cos(2u)\, du$

$$\boxed{\frac{1}{b-a}\int_a^b F(u)\, du}$$

$\frac{1}{\pi} \cdot \int_{-\pi/2}^{\pi/2} \cos(2u)$

$$\boxed{\int \cos(ku)\, du = \frac{1}{k}\sin(ku) + (C)}$$

$k = 2$

$\int \cos(2u)\, du = \frac{1}{2}\sin(2u)$

integral batas tertentu $(0, \pi/2)$

$\int_0^{\pi/2} \cos(2u)\, du = \left[\frac{1}{2}\sin(2u)\right]_0^{\pi/2}$

$= \frac{1}{2}\sin\left(2 \cdot \frac{\pi}{2}\right) - \frac{1}{2}\sin(2 \cdot 0)$

$= \frac{1}{2}\sin\pi - \frac{1}{2}\sin 0$

$= \frac{1}{2}\sin\pi = \frac{\sin(180)}{2}$

$= 0$

$\cos\theta = 0$

$\theta = \frac{\pi}{2} + k\cdot\pi$

$\theta = 2c$

$2c = \frac{\pi}{2} + k\cdot\pi$

$c = \frac{\pi}{4} + k\cdot\frac{\pi}{2}$

$0 = \frac{\pi}{4} + 0\cdot\frac{\pi}{2} = \frac{\pi}{4}$

$-1 = \frac{\pi}{4} - \frac{\pi}{2} = -\frac{\pi}{4}$

(4) $F(u) = 3u + \ln u$

$u > 0$ ✓ u invers

$(F^{-1})'(3)$?

Cari $(F^{-1})'(3) = 3u + \ln u = 3$

$u = 1$

Turunan fungsi invers:

Jika f punya invers:

$y = F(u)$

$u = F^{-1}(y)$

$\frac{dy}{du} = F'(u)$

$(F^{-1})'(y) = \frac{du}{dy}$

$= \frac{1}{dy/du} = \boxed{\frac{1}{F'(u)}}$

Jadi: $(F^{-1})'(3) = \frac{1}{F'(1)}$

$= \frac{1}{3 + \frac{1}{u}\big|_{u=1}} = \frac{1}{3+\frac{1}{1}} = \frac{1}{4}$

(5) Diketahui $F(u) = e^{2u-4} + \boxed{\log_3 u}$. Akan ditentukan $F'(2)$:

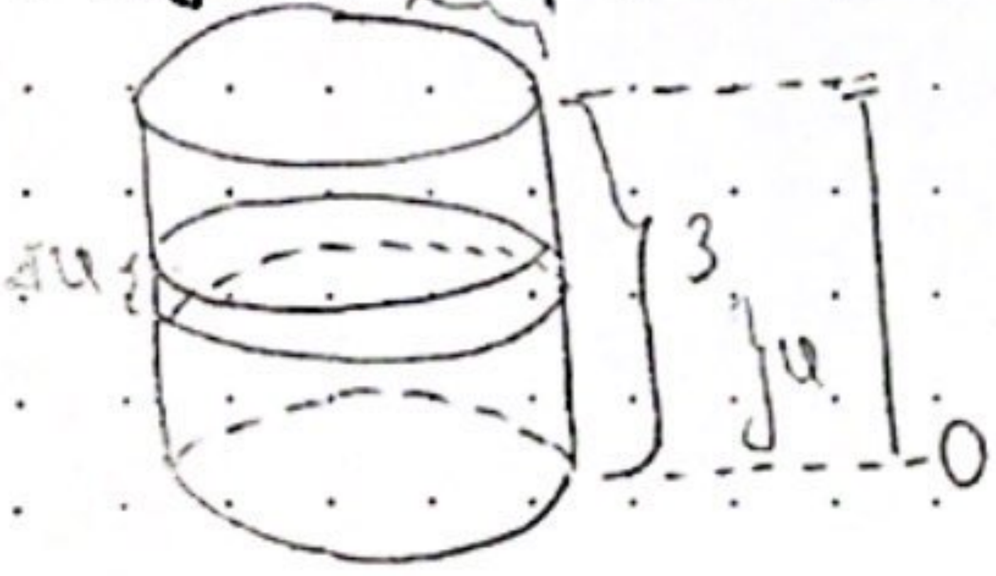
$F'(u) = 2e^{2u-4} + \frac{1}{u \ln 3}$

$F'(2) = 2e^0 + \frac{1}{2\ln 3}$

$= 2 + \frac{1}{2\ln 3}$

(6) Diberikan suatu tangki berbentuk tabung yang berisi air setinggi 2 meter

Misalkan u ketinggian air dari permukaan tanah (meter).
Bayangkan usaha untuk memompa air sehingga tinggi air berkurang sejauh Δu!



Volume irisan airnya:

$\Delta V \approx \pi . 1^2 . \Delta u$

$= \pi . \Delta u$

Berat jenis air adalah: 10000 N/m^3

Berat jenis = $\frac{\text{Berat}}{\text{Volume}}$

Berat = Berat jenis x Volume

$\Delta F \approx 10.000 . \Delta V$

$= 10.000\, \pi\, \Delta u$

$(4-u)$ meter

$\Delta W \approx \Delta F\, (4-u)$

$= 10.000\pi\, (4-u)\, \Delta u$

Jadi, usaha totalnya (joule) adalah:

$W = 10.000\pi \int_0^2 (4-u)\, du$

$= 10.000\pi \left(4x - \frac{u^2}{2}\right)\Big|_0^2$

$= 10.000\pi \left(4.2 - \frac{2^2}{2}\right) =$

$= 60.000\pi$ $8 - \frac{4}{2} = 6 . 10\,000\pi$

Review UR

a) Integral

*** Nilai deret berhingga**

- $\sum_{i=1}^{n} 1 = n$
- $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$
- $\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$
- $\sum_{i=1}^{n} i^3 = \left(\frac{n(n+1)}{2}\right)^2$

*** Menghitung luas dengan limit** dibawah kurva

$\lim_{n\to\infty} \left(\sum_{i=1}^{n} f(u_i).\Delta u\right)$ i=1 sampai n, fungsi, lebar

$\Delta u = \frac{b-a}{n}$, $u_i = a + i\Delta u$ titik² interval, a=awal interval, Δu = lebar interval

$i = 1, 2, \ldots n$

*** Integral tentu**

- Jumlah riemann:

$R_p = \sum_{i=1}^{n} f(u_i)\,\Delta u_i$

Fungsi disebut integral tentu jika f(u) terdefinisi pada [a,b] fungsi kontinu, sehingga bisa diintegralkan pada [a,b]:

$$\boxed{\int_a^b f(u)\,du = \lim_{|P|\to 0} \sum_{i=1}^{n} f(u_i)\Delta u_i}$$

integral tentu, lim. jumlah riemann

Contoh soal

1) Hitung luas daerah dibawah kurva $f(u) = u^2$ dari $u=0$ hingga $u=2$ pakai konsep jumlah riemann & limit!

$\Delta u = \frac{2-0}{n} = \frac{2}{n}$

$\boxed{u_i = a + i.\Delta u}$

$= 0 + i.\frac{2}{n} = \frac{2i}{n}$

$\text{luas} \approx \sum_{i=1}^{n} f(u_i).\Delta u$

$= \sum_{i=1}^{n} \left(\frac{2i}{n}\right)^2.\frac{2}{n}$

$= \sum_{i=1}^{n} \frac{4i^2}{n^2}.\frac{2}{n} = \sum_{i=1}^{n} \frac{8i^2}{n^3}$

Jumlah Σi^2:

$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$

$= \frac{8}{n^3}.\frac{n(n+1)(2n+1)}{6}$ # jumlah kuadrat bilangan

$= \frac{\cancel{8}}{\cancel{6}n^3}.n(n+1)(2n+1)$ # u/o dibagi & ga sesuai dgn hasil akhir

$= \frac{4}{3n^{\cancel{3}2}}\cancel{n}(n+1)(2n+1)$

$= \frac{4(n+1)(2n+1)}{3n^2}$

$= \lim_{n\to\infty} \frac{4(n+1)(2n+1)}{3n^2}$

$= \lim_{n\to\infty} \frac{4(2n^2+n+2n+1)}{3n^2}$

$= \lim_{n\to\infty} \frac{4(2n^2)}{3n^2}$

$= \frac{8}{3}$

$\frac{1+2}{6} = \frac{3}{6} = \frac{1}{2}$

#Sifat-sifat integral:

1) $\int_a^a f(u)\,du = 0$
2) $\int_a^b f(u)\,du = -\int_b^a f(u)\,du$
3) $\int_a^c f(u)\,du = \int_a^b f(u)\,du + \int_b^c f(u)\,du$

→ TDK 1:

- $F(u) = \int_a^u f(t)\,dt$

Perluasan aturan rantai:

$F(u) = \int_a^{w(u)} f(t)\,dt$, maka:

$F'(u) = f(w(u)).w'(u)$

#Kelinearan integral tentu:

Konstanta keluar integral:

$\int_a^b k.f(u)\,du = k.\int_a^b f(u)\,du$

Penjumlahan & pengurangan:

$\int_a^b (f(u) \pm g(u))\,du = \int_a^b f(u)\,du \pm \int_a^b g(u)\,du$

↳ TDK 2:

$F(u)$ = Fungsi kontinu:

→ $\int_a^b f(u)\,du = F(b) - F(a)$

$F(u)$ = antiturunan dari $f(u)$

→ $\int f(g(u)) . g'(u)\, du$:

$\int f(g(u)) . g'(u)\, du = F(g(u)) + C$

contoh soal:

1) # sifat integral:

$\int_2^2 (3u^2+2)\,du = 0$

2) # TDK I dengan aturan rantai:

$F(u) = \int_0^{u^2} t^2\,dt$

$F'(u) = ?$

→ $F'(u) = f(u^2) . (u^2)' - 0$

$= (u^2)^2 . 2u - 0$

$= u^4 . 2u = 2u^5$

3) # Kelinieran integral:

$\int_0^1 (3u^2 - 2u)\,du$

$= \int_0^1 3u^2\,du - \int_0^1 2u\,du$

$= \frac{3}{3}u^3 \Big]_0^1 - \frac{2}{2}u^2 \Big]_0^1$

$= u^3 \Big]_0^1 - u^2 \Big]_0^1$

$= 1^3 - 1^2 = 0$

Teorema nilai rata-rata untuk integral

→ nilai rata² fungsi:

$\frac{1}{b-a}\int_a^b f(u)\,du$

interval $[a,b]$

→ Teorema nilai rata²:

$f(u)$ kontinu di interval $[a,b]$ maka:

$\int_a^b f(u)\,du = f(c).(b-a)$

→ Teorema simetri u/ genap & ganjil:

a.) Fungsi genap:

$f(-u) = f(u)$:

$\int_{-a}^{a} f(u)\,du = 2\int_0^a f(u)\,du$

b) Fungsi ganjil:

$f(u) = -f(u)$:

$\int_{-a}^{a} f(u)\,du = 0$

kiri & kanan saling menghilangkan

integral numerik (Riemann)

(a) riemann kiri:

$\sum_{i=1}^{n} f(u_{i-1}) . \Delta u$

(b) riemann kanan:

$\sum_{i=1}^{n} f(u_i) . \Delta u$

(c) riemann tengah:

u_i tengah $= \frac{u_{i-1} + u_i}{2}$

$\sum_{i=1}^{n} f\left(\frac{u_{i-1}+u_i}{2}\right) . \Delta u$

contoh soal:

1) # Teorema nilai rata²

Hitung nilai rata² $f(u) = u^2$ di interval $[0,2]$

$\frac{1}{b-a}\int_a^b f(u)\,du$

$\Rightarrow \frac{1}{2-0}\int_0^2 u^2\,du$

$\frac{1}{2}\int_0^2 u^2\,du$

$\frac{1}{2} \cdot \frac{u^3}{3}\Big]_0^2$

$\frac{1}{2}\left[\frac{2^3}{3}\right] = \frac{1}{2}\left[\frac{8}{3}\right]$

$\frac{\cancel{8}}{\cancel{6}} = \frac{4}{3}$

2) # Teorema simetri

Hitung $\int_{-2}^{2} u^3\,du$!

$\Rightarrow \int_{-2}^{2} u^3\,du$

$= \frac{u^4}{4}\Big]_{-2}^{2} = \frac{(2)^4}{4} - \frac{(-2)^4}{4}$

$= \frac{16}{4} - \frac{16}{4} = 0$

3) # riemann tengah!

$f(u) = u^2 \to [0,2]$

riemann tengah

$u_4 \boxed{\sum_{i=1}^{n} f\left(\frac{u_{i-1}+u_i}{2}\right) . \Delta u}$

$\Delta u = \frac{2-0}{2} = 1$

$u_1 = \frac{0+1}{2} = 0,5$

$u_2 = \frac{1+2}{2} = 1,5$

3) # Jumlah riemann:

luas $= \sum_{i=1}^{2} f(u_i \text{ tengah}) \cdot \Delta u$

$= [f(0,5) + f(1,5)] \cdot 1$

$f(0,5) = (0,5)^2 = 0,25$

$f(1,5) = (1,5)^2 = 2,25$

$= (0,25 + 2,25) \cdot 1$

$= 2,50 \times 1$

$= 2,50$

(d) Aturan trapesium

$\boxed{\frac{(a+b) \cdot t}{2}}$ → rumus trapesium (1 interval):

$$\int_a^b f(u)\,du \approx \frac{b-a}{2}[f(a)+f(b)]$$

→ aturan trapesium dgn banyak subinterval:

$\Delta u = \frac{b-a}{n}$

f_1, f_2, f_3

luas $= \frac{\Delta u}{2}[f(u_{i-1}) + f(u_i)]$

$$\int_a^b f(u)\,du \approx \frac{\Delta u}{2}\left[f(u_0) + 2\sum_{i=1}^{n-1} f(u_i) + f(u_n)\right]$$

$\frac{\Delta u}{2}\left[f(u_0) + 2\sum_{i=1}^{n-1} f(u_i) + f(u_n)\right]$

- batas bawah: $u_0 = a$
  batas atas: $u_n = b$
- $2\sum_{i=1}^{n-1} f(u_i)$ = nilai fungsi di titik tengah setiap subinterval dikalikan 2
- $\frac{\Delta u}{2}$ = pembagi luas trapesium

rumus eror: E_n

$E_n = -\frac{(b-a)^3}{12n^2} \cdot f''(c)$ → $f(u)$

eror semakin kecil

$\frac{3}{12n^2}$

Contoh soal

1) Hitung aproksimasi integral $\int_0^2 u^2\,du$ dengan menggunakan aturan trapesium dengan $n=2$!

(a) $\Delta u = \frac{b-a}{n} = \frac{2-0}{2} = 1$

$u_0 = 0$

$u_1 = 1$

$u_2 = 2$

$f(u_0) = (0)^2 = 0$

$f(u_1) = (1)^2 = 1$

$f(u_2) = (2)^2 = 4$

(fn)

Rumus Trapesium:

$$\int_0^2 u^2\,du = \frac{\Delta u}{2}[f(u_0) + 2f(u_1) + f(u_2)]$$

→ $2f(u_2)$ → tanpa trapesium akhir

Substitusi:

$\int_0^2 u^2\,du \approx \frac{1}{2}[0 + 2(1) + 4]$

$\int_0^2 u^2\,du \approx \frac{1}{2} \cdot 6 = 3$

(e) Aturan parabola (genap) pakai pola koef: 1, 4, 2, 4, ...

$$\int_a^b f(u)\,du \approx \frac{\Delta u}{3}[f(u_0) + 4f(u_1) + 2f(u_2) + 4f(u_3) + \dots + 2f(u_{n-2}) + 4f(u_{n-1}) + f(u_n)]$$

$\frac{\Delta u}{3}$ ≈ $\boxed{\Delta u = \frac{b-a}{n}}$

koef: 1, 4, 2, 4, 2, 4, 1 → kurva parabola

erorgalat: $E_n = -\frac{(b-a)^5}{180n^4} \cdot f^{(4)}(c)$ → selisih parabola

→ kelengkungan fungsi $f(u)$ → $c \in [a,b]$

hasil turunan matematis aproksimasi parabola | semakin besar n (jumlah subinterval) eror semakin kecil pembagiannya >

$\frac{5}{180n^4} \cdot f^{(4)}(c)$

contoh soal

1) Hitung $\int_0^2 u^2\,du$ dengan aturan parabola dengan $n=2$!

$\Delta u = \frac{b-a}{n} = \frac{2-0}{2} = 1$

$u_0 = 0$

$u_1 = 1$

$u_2 = 2$

$f(u_0) = 0^2 = 0$

$f(u_1) = 1^2 = 1$

$f(u_2) = 2^2 = 4$

$\int_0^2 u^2\,du \approx \frac{\Delta u}{3}[f(u_0) + 4f(u_1) + f(u_2)]$

$\int_0^2 u^2\,du \approx \frac{1}{3}[0 + 4(1) + 4]$

$\approx \frac{1}{3} \cdot 8 = \frac{8}{3}$

2) Hitung galat

$E_t = -\frac{(b-a)^3}{12n^2} F''$

$F(x) = x^2$
$F'(x) = 2x$
$F''(x) = 2$

$= -\frac{(b-a)^3}{12n^2} \cdot F''$

$= -\frac{(3-1)^3}{12(2)^2} \cdot 2$

$= -\frac{(2)^3}{12 \cdot 4} \cdot 2 = \frac{-\cancel{8}^{4}}{\cancel{48}_{24\,12\,3}}$

$= -\frac{1}{3}$ //

Parabola

Dik: $F(x) = \sin(x)$
$[0, \pi]$
$n = 6$

$\rightarrow -\frac{(b-a)^5}{180 \cdot n^4} \cdot F^4$

$F(x)$
$\sin(x) \rightarrow \cos(x)$
$\downarrow F''(x)$
$-\sin(x)$
$\downarrow F^3(x)$
$-\cos(x)$
$\downarrow F^4(x)$
$\sin(x)$

$= -\frac{(\pi-0)^5}{180(\sin 6)^4} \cdot \sin(x)$

B) Aplikasi Integral

1.) Daerah atas sb. x:
$A = \int_a^b F(x)\,dx$

2) Daerah antara 2 kurva:
$A = \int_a^b [\underbrace{F(x)}_{\text{atas}} - \underbrace{g(x)}_{\text{bawah}}]\,dx$

3) Volume sebagai limit jumlah riemann:
$V = \lim_{\Delta x \to 0} \sum_{i=1}^{n} A(x_i)\Delta x_i$

$= \int_a^b A(x)\,dx$

4) Volume benda padat hasil perputaran:
$V = \pi \int_a^b [F(x)]^2\,dx$

$F(x) \rightarrow$ diputar terhadap sumbu x $\Rightarrow$ hasilnya: Benda padat

5) Metode cincin: (sumbu x)
$V = \pi \int_a^b [\underbrace{F(x)}_{\text{luar}}{}^2 - \underbrace{g(x)}_{\text{dalam}}{}^2]\,dx$

6) Kulit Tabung:
$V = 2\pi \int_a^b x\,F(x)\,dx$

$\sqrt{x} = x^{1/2}$

Konsep Dasar Kerja

$\boxed{W = F \cdot s}$

$W \rightarrow$ usaha, $F \rightarrow$ gaya, $s \rightarrow$ perpindahan

$\boxed{W = \int_a^b F(x)\,dx}$

Kerja dalam Fluida

$\Delta W = F(x_i) \cdot \Delta x$

$W = \int_a^b F(x)\,dx$

Gaya Fluida:

$F = \rho \cdot g \cdot A \cdot h$

$\rho \rightarrow$ massa jenis cairan

Dik: ① Hitung kerja yang diperlukan untuk memompa air ke atas dari tangki berbentuk prisma tegak dengan alas $2\,m^2$ & tinggi 3 m. Massa jenis air $\rho = 1000\ kg/m^3$ & $g = 10\ m/s^2$!

$\Rightarrow A = 2\,m^2$
$t = 3\,m \quad = 0,3$
$\rho = 1000\ kg/m^3$
$g = 10\ m/s^2$

Dit: kerja?

$\boxed{F = \rho \cdot g \cdot A \cdot h}$

$= 1000 \cdot 10 \cdot 2 \cdot h$

$= 20.000\,h$

$W = \int$

uas math 22

① Misal: $\int_1^4 F(u)\,du = 7$

$\int_4^6 F(u)\,du = 10$

$\int_4^6 g(u)\,du = 3$

a) $\int_1^6 F(u)\,du = \int_1^4 F(u)\,du + \int_4^6 F(u)\,du$
$= 7 + 10 = 17$

b) $\int_4^6 (F(u)+g(u))\,du = \int_4^6 (F(u)+g(u))\,du$
$= 10 + 3$
$= 13$

② $F(u) = \log_6(3u)$

a) $F(a) = 1$
$a = {}^6\log 3u$

$F(a) = 1$
$1 = {}^6\log 3a$
$3a = 6^1 = 6$
$a = 2$

$\boxed{*\ {}^a\log b = c \quad b = a^c}$

$\boxed{*\ \log_a(b)^c = c.\log_a b}$

$a^c = b$
$6^1 = 3a$
$a = 2$

b) $F(b) = 5$
$F(3b^2) = ?$
$F(b) = 5$
$5 = {}^6\log 3^b \rightarrow {}^6\log 3 + {}^6\log b = 5$
$F(3b^2) = {}^6\log (3)(3b^2)\ 3.3.b^2$
$= {}^6\log 3 + {}^6\log 3b^2$
$= {}^6\log (3b)(3b)$
$= {}^6\log 3b + {}^6\log 3b$
$= 5 + 5 = 10$

④ Jika $F(u) = \int_u^1 \sqrt{t^4+3}\,dt$

a) $F(1) = \int_1^1 \sqrt{t^4+3}\,dt = 0$ $\quad \boxed{\int_a^a F(u)\,du = 0}$

b) $F''(1) = \int_x^1 \sqrt{t^4+3}\,dt$
$= -\int_1^x \sqrt{t^4+1}$

$F'(u) = \frac{d}{du}\left[-\int_1^u \sqrt{t^4+1}\,dt\right]$

$= -\sqrt{x^4+1} \cdot \frac{d}{du}(u) = -\sqrt{u^4+1}$

$F'(u) = -\sqrt{x^4+1}$
$F'(1) = -\sqrt{2}$

⑤ Pada tabel dibawah ini disajikan data kecepatan sesaat itu kendaraan pada beberapa waktu tertentu $[0, 6]$ (dalam jam)

t(jam)	V(t) (km/jam)
0	10
1	40
2	50
3	55
4	35
5	30
6	25

Jika jumlah riemann kiri dengan $n=3$ untuk menaksir jarak yang ditempuh dalam selang waktu tsb, maka:

(a) Jumlah lebar subinterval adalah
$\Delta t = ?$

0 2 4 6

$\Delta t = \frac{b-a}{n} = \frac{6-0}{3}$
$= \frac{6}{3} = 2$

(b) $\int_0^6 v(t)\,dt = \Delta t\,[F(0) + F(2) + F(4)]$
$= 2[10 + 50 + 35] = 190$

jarak

⑦ $\int_0^3 6u\sqrt{u^2+16}\,du \quad du = \int_{16}^{25} 3\sqrt{u}\,du$
$3 \cdot 2u$

$\Rightarrow$ Metode subs

$U = u^2 + 16$ $\quad U(3) = 25,\ U(0) = 16$
$dU = 2u\,du$

$\int u^n\,du = \frac{1}{n+1} u^{n+1} \rightarrow \int U^{1/2}\,dU = \frac{1}{\frac{1}{2}+1} U^{\frac{1}{2}+1}$

JOYKO

$\frac{1}{\frac{1}{2}+1} u^{1/2+1} = \frac{2}{3} u^{3/2}$

$\int_0^3 6u\sqrt{u^2+16}\, du$ → perkalian harus dicari gitu

$= \int_{16}^{25} 3\sqrt{v}\, dv = \left[\frac{2}{3} . 3 v^{3/2}\right]_{16}^{25}$

$= 2 . [u^{3/2}]_{16}^{25}$

$= 2 . [125 - 64]$

$= 2 . 61 = 122$

misal: $\int x^2 dx = \frac{1}{3}x^3$

- $\int x^2 x^3 dx = \int x^5 dx$

- $\int x\sqrt{x}\, dx = \int x^{3/2} dx$ ($1/2+1 = 3/2$)

- $\int x\sqrt{x^2+1}\, dx =$

② $y(u) = e^{2u} + \ln(u+1)$ tentukan $y'(1)$

↳ $\frac{d}{du} e^u = e^u$ → $\frac{d}{du} \ln(u) = \frac{1}{u}$

→ $\frac{d}{du} e^{f(u)} = e^{f(u)} . f'(u)$ → $\frac{d}{du} \ln(f(u)) = \frac{1}{f(u)} . f'(u)$

→ $\frac{d}{du} e^{2u} = e^{2u} . 2$

→ $\ln(u+1) = \frac{1}{u+1} . 1$

$y'(u) = e^{2u} . 2 + \frac{1}{u+1}$

$y'(1) = e^2 . 2 + \frac{1}{2}$

③ Tentukan solusi persamaan diferensial $\frac{dy}{dx} = \frac{10x^4}{3y^2}$ dengan $y(1) = 1$

→ $\frac{dy}{dx} = \frac{10x^4}{3y^2}$

$\int 3y^2 dy = \int 10u^4 du$

$= \int 3y^2 dy = \int 10x^4 dx$

$\frac{\cancel{3}y^3}{\cancel{3}} = \frac{\cancel{10}^{2} x^5}{\cancel{5}} + C$

$y^3 = 2x^5 + C$

$y(1) = 1$ ⇒ Ketika $u = 1$ nilai $y = 1$

$1^3 = 2 . 1^3 + C$

$\boxed{C = -1}$

$y^3 + C = 2x^5$ ⇒ $1 + C = 2$

$C = 1$

⇒ $y^3 = 2x^5 - 1$ ⇒ $y^3 + 1 = 2x^5$

$y = \sqrt[3]{2x^5 - 1}$ $y = \sqrt{2u^5 - 1}$

④ Misalkan $f(u) = a + b \tan^{-1}(u)$. Tentukan nilai a & b agar grafik fungsi f melalui titik $P(0, 5)$ & persamaan garis singgung grafik f di P adalah $u - 2y + 10 = 0$

→ $f(0) = 5$

$f(0) = a + b \tan^{-1}(0)$

$5 = a + b . 0$

$\boxed{a = 5}$

⇒ $m = f'(0)$

$f(u) = a + b \tan^{-1}(u)$

$f'(u) = 0 + b . \frac{1}{1+x^2}$

$m = f'(0) = 0 + b . \frac{1}{1+0^2} = b$

$\boxed{m = b}$

$\frac{d}{dx} \tan^{-1}(x) = \frac{1}{1+x^2}$

$\frac{d}{dx} \tan^{-1}(f(u)) = \frac{1}{1+f^2(u)} f'(u)$

$x - 2y + 10 = 0$

$2y = x + 10$

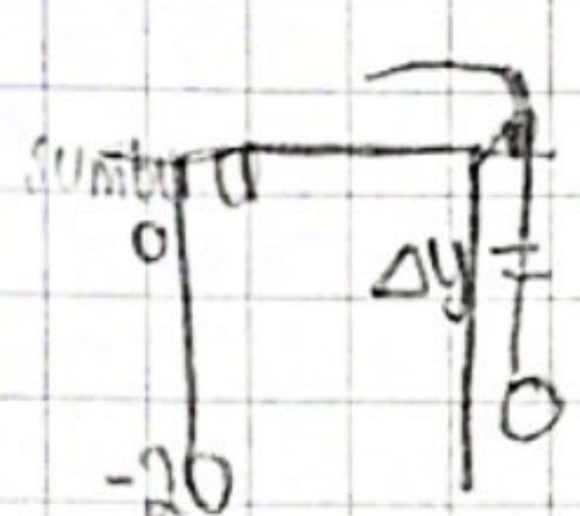
$\boxed{y = \frac{1}{2}x + 5}$

$m = \frac{1}{2}$

(5) Suatu tali tambang dengan berat jenis 15 Newton/meter digunakan untuk menarik benda dengan berat 1000 Newton dari dasar sumur dengan kedalaman 20 meter dari permukaan tanah. Tentukan integral yang menyatakan kerja yang dilakukan u/ menarik benda tsb sampai kedalaman 10 meter dari permukaan tanah

Jawab: Dik: $\rho = \frac{F}{\ell}$

$15 = \frac{F}{\ell}$



$F = 15\ell$

$\Delta F = 15\Delta \ell$

$= 15\Delta y$

$\Delta W = \Delta F . s, \ 15 . \Delta y \, (-y)$

↳ perpindahan

$W_{tali} = \int_{-20}^{-10} -15 \, y . dy$

Beban: $\underbrace{1000 \text{ N}}_{F}$

$\Delta W = F . \Delta s$

$= 1500 \Delta y$

$W_{beban} = \int_{-20}^{-10} 1500 \, dy$

$\boxed{W_{total} = W_{tali} + W_{beban}}$

$= \int_{-20}^{-10} (1500 - 15y) \, dy$

(6) Tentukan nilai rata² fungsi $f(x) = 3x + \sin^7(2x+1)$ pada selang $[-2, 1]$!

$\Rightarrow \bar{x} = \frac{\sum x}{n}$, $f_{avg} = \frac{\int_{-2}^{1} f(x)}{1-(-2)}$

$= \frac{1}{3}\int_{-2}^{1} (3x + \sin^7(2x+1)) \, dx$

$\int_{-2}^{1} [3x + \sin^7(2x+1)] \, dx = \int_{-2}^{1} 3x \, dx + \int_{-2}^{1} \sin^7(2x+1) \, dx$

$= \left[\frac{3x^2}{2}\right]_{-2}^{1}$

misal $u = 2x + 1$ → $u(1) = 3$, $u(-2) = -3$

$du = 2\,dx$

$dx = \frac{1}{2} du$

$\boxed{\int_{-3}^{3} \frac{1}{2} \underbrace{\sin^7(u)}_{\text{ganjil}} \, du}$

$\boxed{\int_{-a}^{a} f(u) \, du = 0}$ jika $f(x)$ ganjil

↓

$f(-u) = -f(u)$

$f(u) = \frac{1}{2}\sin^7 \ldots$

$f(-1) = -\frac{1}{2}(\sin \ldots$

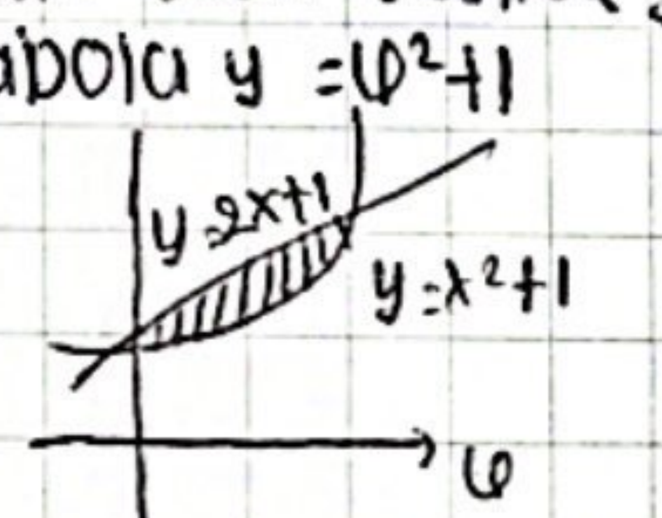
$= \left[\frac{3x^2}{2}\right]_{-2}^{1} + 0$

$= \frac{3}{2} - 6$

$= 1{,}5 - 6$

$= -\frac{9}{2} = -4{,}5$

(1) Diketahui bahwa R adalah daerah yang dibatasi oleh sumbu y, garis $y = 2x + 1$ & parabola $y = x^2 + 1$



(a) Tentukan dua titik potong garis & parabola tsb!

(b) Hitunglah luas daerah R!

(c) Hitunglah volume benda pejal yang didapat dengan memutar daerah R mengelilingi garis $x = -1$!

$\Rightarrow$ a) $x^2 + 1 = 2x + 1$

$x^2 - 2x = 0$

$x(x - 2) = 0$ → $x = 0$

$x - 2 = 0$ → $x = 2$

$x(x-2)=0 \rightarrow \boxed{x=0}$

$y=1$

$\rightarrow x-2=0$

$\boxed{x=2} \rightarrow y=5$

$(0,1)$ & $(2,5)$

b) luas $= \int [F_{atas} - F_{bawah}]\, du$

$= \int_0^2 [(2x+1)-(x^2+1)]\, du$

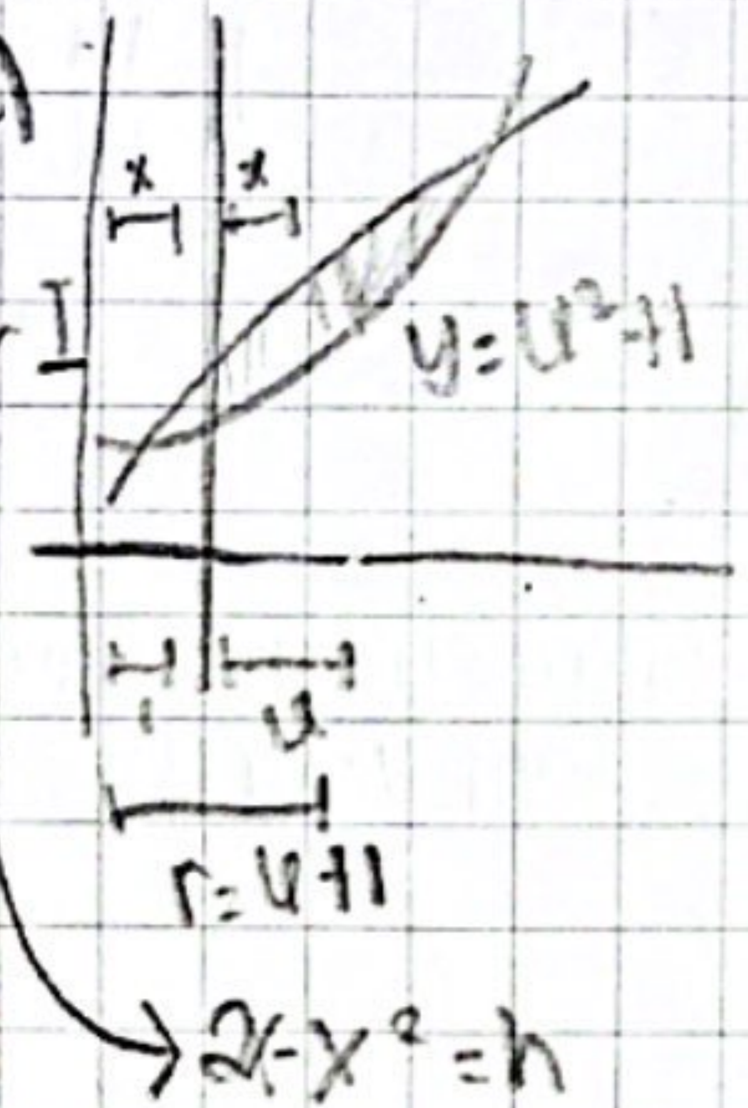
$= \int_0^2 [(2u-u^2)]\, du$

$= \left[\frac{2u^2}{2} - \frac{u^3}{3}\right]_0^2 = \left[u^2 - \frac{u^3}{3}\right]_0^2$

$= \left[\frac{8}{2} - \frac{8}{3}\right]$

$= \frac{24-16}{6} = \frac{8}{6}$

c)



partisi // sb. putar

→ Metode kulit tabung

$\Delta V = 2\pi r h\, \Delta u$

$= 2\pi (x+1)(2x-x^2)\Delta x$

$V = 2\pi \int_0^2 (x+1)(2x-x^2)\, dx$

$= 2\pi \int_0^2 (2x^2-x^3+2x-x^2)\, dx$

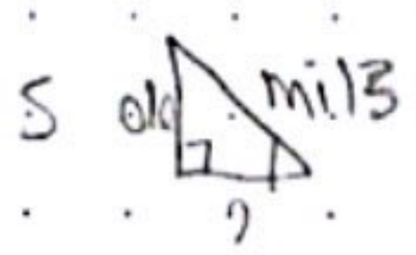
$= 2\pi \int_0^2 ($

7) Hitunglah nilai $\cos\left(\sin^{-1}\left(\frac{5}{13}\right)\right)$!

$\Rightarrow \cos\left(\sin^{-1}\left(\frac{5}{13}\right)\right)$

$\sin^{-1} // \cos$

$\sin = \frac{de}{mi} = \frac{5}{13}$



$\sqrt{13^2 - 5^2}$

$\sqrt{169 - 25}$

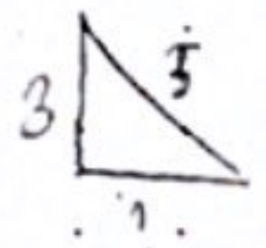
$\sqrt{144} = 12$

$\cos = \frac{sa}{mi} = \frac{12}{13}$ //

#soal serupa:

↳ $\tan\left(\sin^{-1}\left(\frac{3}{5}\right)\right)$!

↳ $\sin \frac{de}{mi} \; \frac{3}{5} = (4) \to$ samping



$\sqrt{5^2 - 3^2}$

$\sqrt{25 - 9} = \sqrt{16}$

$\tan = \frac{de}{sa} = \frac{3}{4}$ //

Bagian B

1) Diketahui sebuah kurva dengan persamaan $x^2 - xy + y^2 = 4$

(a) Tentukan semua titik potong kurva tersebut dengan garis $y = x$!

(b) Tentukan persamaan garis singgung kurva tersebut pada titik² tersebut!

(a) $x^2 - xy + y^2 = 4$

$\frac{d}{dx}(x^2) - \frac{d}{dx}(xy) + \frac{d}{dx}(y^2) = \frac{d}{dx}(4)$

$(2x) - \left(x\frac{dy}{dx} + y\right) + 2y\frac{dy}{dx} = 0$

$2x - \left(x\frac{dy}{dx} + y\right) + 2y\frac{dy}{dx} = 0$

$2x - x\frac{dy}{dx} - y + 2y\frac{dy}{dx} = 0$

$\frac{dy}{dx}$:

$2x - y + \frac{dy}{dx}(-x + 2y) = 0$

$\frac{dy}{dx}(-x + 2y) = y - 2x$

$\frac{dy}{dx} = \frac{y - 2x}{-x + 2y}$

Substitusi titik:

a) titik $(2,2)$:

$x = 2$

$y = 2$

ke $\frac{dy}{dx} =$

$\frac{dy}{dx} = \frac{2 - 2(2)}{-2 + 2(2)}$

$\frac{dy}{dx} = \frac{2-4}{-2+4} = \frac{-\cancel{2}}{\cancel{2}} = -1$

$y - 2 = -1(x - 2)$

$y - 2 = -x + 2$

$y = -x + 2 + 2$

$= -x + 4$

b) titik $(-1,-1)$:

$x = -1$

$y = -1$

ke $\frac{dy}{dx} = \frac{-1 - 2(-1)}{-(-1) + 2(-1)}$

$= \frac{-1+2}{1-2} = \frac{1}{-1} = -1$

persamaan garis singgung:

$\boxed{y - y_1 = m(x - x_1)}$

$y - (-1) = -1(x - (-1))$

$y + 1 = -1(x + 1) \Leftrightarrow y + 1 = -x - 1$

$y = -x - 2$

persamaan garis singgung:

$(2,2): y = -x + 4$

$(-1,-1) = y = -x - 2$ //

$f(x) = 2x$

$g(x) = x$

maka

$\pi \int_1^3 (f(x)^2 - g(x)^2)\, dx$

$\pi \int_1^3 ((2x)^2 - (x)^2)\, dx$

$\pi \int_1^3 [4x^2 - x^2]$

$\pi \int_1^3 3x^2\, dx$

$\Rightarrow 3x^2 \qquad 3x^2$

↳ $\int x^2\, dx = \frac{x^3}{3} \qquad \frac{\cancel{3}}{\cancel{3}} x^2 = \frac{x^3}{3}$

$\int 3x^2\, dx = x^3 \to 3 \int x^2\, dx$

$\int 3x^2\, dx = \cancel{3} \cdot \frac{x^3}{\cancel{3}}$

$V = \pi \left[x^3\right]_1^3$

$= \pi (3^3 - 1^3)$

$= \pi (27 - 1) = \pi \cdot 26 = 26\pi$ //