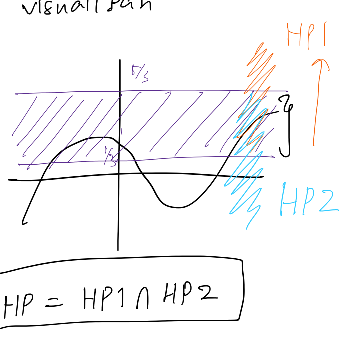
Wednesday, 20 December 2023 18.24

(a) $\left|\frac{1}{x}-1\right| \le \frac{2}{3}$

$-\frac{2}{3} \le \frac{1}{x}-1 \le \frac{2}{3}$ (tambahin 1)

$\frac{1}{3} \le \frac{1}{x} \le \frac{5}{3}$

visualisasi



$\boxed{HP = HP1 \cap HP2}$

tinjau $\frac{1}{3} \le \frac{1}{x}$

$\boxed{\text{untuk } x>0:}$

$\frac{1}{3} \le \frac{1}{x} \Leftrightarrow \frac{x}{3} \le 1$

$\Leftrightarrow x \le 3$

$HP1A = (0,\infty) \cap (-\infty, 3]$

$= (0,3]$

untuk $x<0$:

$\frac{1}{3} \le \frac{1}{x} \Leftrightarrow \frac{x}{3} \ge 1$

$\Leftrightarrow x \ge 3$

$HP1B = (-\infty,0) \cap [3,\infty)$

$= \emptyset$.

$HP1 = HP1A \cup HP1B$

$= (0,3]$

$\frac{1}{x} \le \frac{5}{3}$, $x \ne 0$

$\boxed{\text{untuk } x>0:}$

$\frac{1}{x} \le \frac{5}{3} \Leftrightarrow 1 \le \frac{5x}{3}$ $\times\frac{3}{5}$

$\Leftrightarrow \frac{3}{5} \le x$

$HP2A = (0,\infty) \cap [\frac{3}{5},\infty)$

$= [\frac{3}{5},\infty)$

untuk $\boxed{x<0:}$

$\frac{1}{x} \le \frac{5}{3} \Leftrightarrow 1 \ge \frac{5x}{3}$

$\Leftrightarrow \frac{3}{5} \ge x$

$HP2B = (-\infty,0) \cap (-\infty, \frac{3}{5}]$

$= (-\infty, 0)$

$HP2 = HP2A \cup HP2B$

$= (-\infty,0) \cup [\frac{3}{5},\infty)$

$HP = (0,3] \cap \left((-\infty,0) \cup [\frac{3}{5},\infty)\right)$

$= [\frac{3}{5},3]$

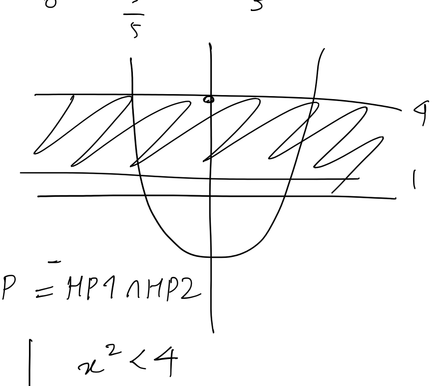
(b) $|2x^2-5| < 3$

$\Leftrightarrow -3 < 2x^2-5 < 3$

$\Leftrightarrow 2 < 2x^2 < 8$

$\Leftrightarrow 1 < x^2 < 4$

HP1 HP2 $HP = HP1 \cap HP2$



$1 < x^2$: $x=1$

$0 < x^2-1 \Leftrightarrow 0 < (x+1)(x-1)$

$HP1 = (-\infty,-1) \cup (1,\infty)$

$x^2 < 4$

$x^2-4 < 0$

$(x+2)(x-2) < 0$

$HP2 = (-2,2)$

$HP = HP1 \cap HP2 = (-2,-1) \cup (1,2)$

(c) $|x+2| < |2x-3|$

$|x+2| = \begin{cases} -x-2, & x \le -2 \\ x+2, & x > -2 \end{cases}$ $x=-2$

$|2x-3| = \begin{cases} 3-2x, & x \le \frac{3}{2} \\ 2x-3 & x > \frac{3}{2} \end{cases}$ $x=\frac{3}{2}$

HP1 HP2 HP3

untuk $(-\infty,-2]$:

$|x+2| < |2x-3| \Leftrightarrow -x-2 < 3-2x$

$\Leftrightarrow x < 5$

$HP1 = (-\infty,-2) \cap (-\infty,5)$

$= (-\infty,-2)$

untuk $-2 < x \le \frac{3}{2}$:

$|x+2| < |2x-3| \Leftrightarrow x+2 < 3-2x$

$\Leftrightarrow 3x < 1$

$\Leftrightarrow x < \frac{1}{3}$

$HP2 = (-\infty,\frac{1}{3}) \cap (-2,\frac{3}{2}]$:

$= (\frac{1}{3},\frac{3}{2}]$

untuk $x > \frac{3}{2}$:

$|x+2| < |2x-3| = x+2 < 2x-3$

$= 5 < x$

$HP3 = (5,\infty) \cap (\frac{3}{2},\infty)$

$= (5,\infty)$

$HP = HP1 \cup HP2 \cup HP3 = (-\infty,-2) \cup (\frac{1}{3},\frac{3}{2}] \cup (5,\infty)$

$|x| > 1$

$x = -2$

$|-2| = 2$

2. $f(x) \to$ ganjil dan periodik dgn $p=8$

$0 \le x \le 4$ $f(x) = 2^x + c$

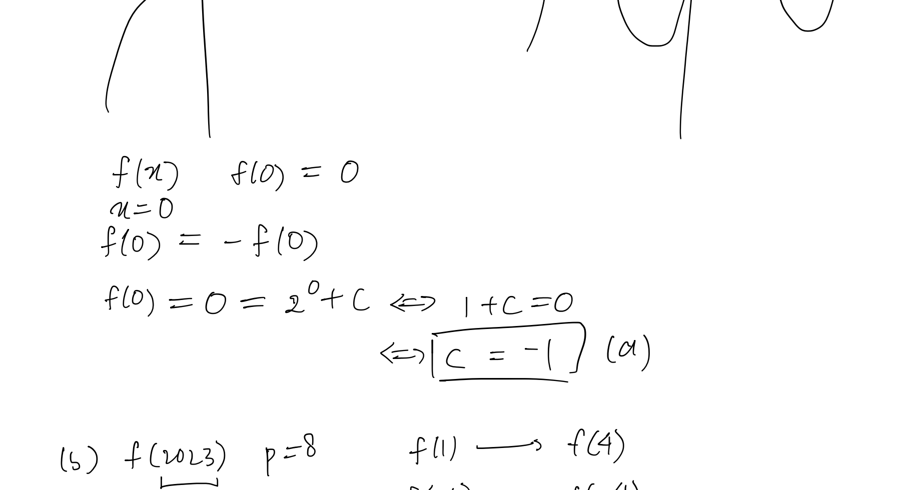
(a) $c =$

(b) $f(2023)$

konsep

$f(x) = f(x+p)$

$= f(x+np)$



$f(x)$ $f(0) = 0$

$x=0$

$f(0) = -f(0)$

$f(0) = 0 = 2^0 + c \Leftrightarrow 1+c=0$

$\Leftrightarrow \boxed{c=-1}$ (a)

(b) $f(2023)$ $p=8$

$f(x+np) = f(x)$

2024 2016

$f(1) \longrightarrow f(4)$

$f(-1) \longrightarrow f(-4)$

$-4 \longrightarrow 4$

$f(x) = 2^x - 1$

$f(2023) = f(7+2016) = f(7)$ ✗

$f(2023) = f(\underbrace{-1+2024}_{2023}) = f(-1) = -f(1)$

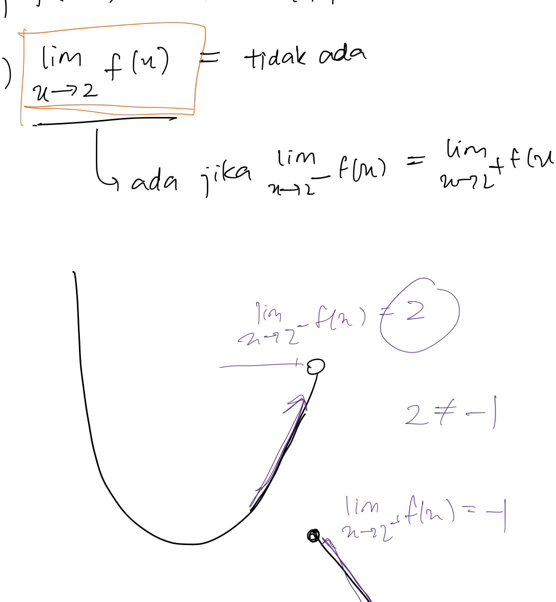
$= -(2^1-1)$

$= -1$ (b)

a) $f(-1) = 1$ $\lim_{x\to -1} f(x) = -1$

(b) $\boxed{\lim_{x\to 2} f(x)}$ = tidak ada

↳ ada jika $\lim_{x\to 2^-} f(x) = \lim_{x\to 2^+} f(x)$



(c) $h(x) = \frac{f(x)}{g(x)}$ aturan pembagian

$h'(x) = \frac{f'(x)g(x) - f(x)g'(x)}{(g(x))^2}$

$h'(0) = \frac{f'(0)g(0) - f(0)g'(0)}{(g(0))^2}$

$f'(0) = 0$

$g'(0) = \frac{\Delta y}{\Delta x} = \frac{1}{1} = 1$

$f(0) = -2$

$g(0) = 1$

$h'(0) = \frac{(0)(1) - (-2)(1)}{1^2}$

$= \frac{2}{1} = 2$ (c)

(d) $\int_{-3}^{5} 2x + 3g(x)\, dx$

$= \int_{-3}^{5} 2x + 3\int_{-3}^{5} g(x)\, dx$

$= \int_{-3}^{5} 2x + 3\left(\int_{-3}^{-1} g(x)\,dx + \int_{-1}^{1} g(x)\,dx + \int_{1}^{5} g(x)\,dx\right)$

$= x^2\Big|_{-3}^{5} + 3(\cancel{-2+2}+4)$

$= (5^2-(-3)^2) + 3(4)$

$= 25 - 9 + 12$

$= 28$

turunan

$\frac{d}{dx} x^2 = 2x$

anti turunan

turunan

$x^n \longrightarrow nx^{n-1}$

anti turunan

$x^n \longrightarrow \frac{x^{n+1}}{n+1}$

$2x \longrightarrow \frac{\cancel{2}x^2}{\cancel{2}}$

$= x^2$

4. turunan f

$\lim_{x\to 3} \frac{x^3+x-30}{x-3}$

definisi

$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$

$\boxed{f'(c) = \lim_{x\to c} \frac{f(x)-f(c)}{x-c}}$

2

$\frac{x^3+x-30}{x-3} = \frac{f(x)-f(3)}{x-3}$

$f(x) \to$ variabel x + konstanta

$f(3) \to$ gaada variable x

$\to$ bil. real

$\boxed{f(x) = x^3+x+a}$ (jawaban a)

$f(3) = 3^3+3+c = 30+c$

$f(x)-f(3) = x^3+x+\cancel{c}-(30+\cancel{c})$

$= \boxed{x^3+x-30}$

$\boxed{(b)\ f'(x) = 3x^2+1}$

(c) $f'(2) = 12+1 = 13$

$f'(c) = \lim_{x\to c} \frac{f(x)-f(c)}{x-c}$

$= \lim_{x\to c} \frac{(x^3+x+\cancel{a})-(c^3+c+\cancel{a})}{x-c}$

$= \lim_{x\to c} \frac{x^3+x-c^3-c}{x-c}$ 0

$= \lim_{x\to c} \frac{(x^3-c^3)+(x-c)}{x-c}$

$= \lim_{x\to c} \frac{\cancel{(x-c)}(x^2+xc+c^2)+\cancel{(x-c)}}{\cancel{(x-c)}}$

$= \lim_{x\to c} x^2+xc+c^2+1$

$= c^2+c^2+c^2+1$

$= \boxed{3c^2+1}$

$\lim_{x\to 3} \frac{x^3+x-30}{x-3}$

turunan

$\lim_{x\to c} \frac{f(x)-f(c)}{x-c}$

g mungkin ada ✓ $c=3$

$f(x) - \boxed{f(3)} = \boxed{x^3+x} - 30$

$f(x) = x^3+x+c$

Mockup Ujian 3

Wednesday, 20 December 2023 19:17

5. Diketahui fungsi

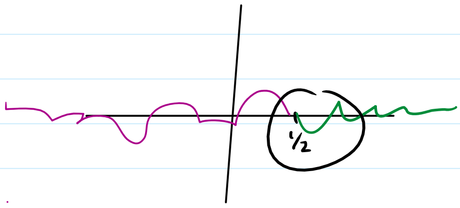
$$f(x) = \begin{cases} x \cdot \cos(\pi x) & x < \frac{1}{2} \\ ax + b & x \geq \frac{1}{2} \end{cases}$$

⇒ trigonometri (sin / cos) ⇒ polinom / sukubanyak } diferensiabel

memiliki turunan (diferensiabel) di semua titik.

(a) $a = \ldots$ $-\frac{1}{2}\pi$ dan $b = \ldots$ $\frac{1}{4}\pi$

(b) $f(1) = \ldots$ $x = 1$ > $\frac{1}{2}$



※ f(x) diferensiabel di semua titik ⇒ a.

$\Rightarrow \lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x)$ ✓ (kontinu)

$\Rightarrow \lim_{x \to a^-} f'(x) = \lim_{x \to a^+} f'(x)$ ✓

Cek untuk $a = \frac{1}{2}$

① $f(x) = \underbrace{x}_{u} \cdot \underbrace{\cos(\pi x)}_{v}$

$u' = 1$ || $v' = -\sin(\pi x) \cdot \pi$.

$f'(x) = u' v + u v'$

$= 1 \cdot \cos(\pi x) + x \cdot (-\sin \pi x) \cdot \pi$

$= \boxed{\cos(\pi x) - \pi \cdot x \cdot \sin(\pi x)}$ //

② $f(x) = ax + b$

$\boxed{f'(x) = a}$ // ✓

$\lim_{x \to \frac{1}{2}^-} f'(x) = \lim_{x \to \frac{1}{2}^+} f'(x)$

$\pi = 180°$

$\frac{1}{2}\pi = 90°$

$\underbrace{\cos(\frac{1}{2}\pi)}_{0} - \pi \cdot \frac{1}{2} \cdot \underbrace{\sin(\frac{1}{2}\pi)}_{1} = a.$

$\boxed{-\frac{1}{2}\pi = a}$ ✓

Diferensiabel ⇒ kontinu

Karena kontinu . $\lim_{x \to \frac{1}{2}^-} f(x) = \lim_{x \to \frac{1}{2}^+} f(x)$

$\lim_{x \to \frac{1}{2}^-} [x \cdot \cos(\pi x)] = \lim_{x \to \frac{1}{2}} [ax + b]$

$\frac{1}{2} \cdot \underbrace{\cos(\pi \frac{1}{2})}_{0} = (-\frac{1}{2}\pi) \cdot \frac{1}{2} + b$

$0 = -\frac{1}{4}\pi + b$

$\Leftrightarrow \boxed{b = \frac{1}{4}\pi.}$

b. $f(1) = \ldots ?$

$f(x) = a \cdot 1 + b$

$= -\frac{1}{2}\pi \cdot 1 + \frac{1}{4}\pi$

$f(1) = -\frac{1}{4}\pi$ //

Type equation here.

$f(1) = -\frac{1}{4}\pi$

Type equation here.

$\sqrt{4,6} = \sqrt{x}$ dengan $x = 4,6$

$f(x) = \sqrt{x}$

$\sqrt{4,6} = f(4,6)$

$\ln 3,2 = \ldots ?$

$\hookrightarrow f(x) = \ln x$

$\cos(0,5) = \ldots ?$

$\hookrightarrow f(x) = \cos(x)$

6. Gunakan diferensial atau hampiran linear untuk menaksir nilai $\sqrt{4.6}$.

mendekati benar

↳ gar singgung.

$$\boxed{f(x_0 + \Delta x) \approx f(x_0) + f'(x_0) \cdot \Delta x}$$

turunan dari $f(x)$

$\sqrt{4,6}$

$\hookrightarrow f(x) = \sqrt{x}$

$x_0 + \Delta x = 4,6 \longrightarrow$ dekat dengan 4

Cari x_0 sehingga $f(x_0)$ mudah cari

$\sqrt{\ldots}$ mudah dicari

$\sqrt{4} = 2 \Rightarrow x_0 = 4$

$\Delta x = 4,6 - 4 = 0,6$

Cari yang lebih kecil

↳ observasi dekat x_0

$\Rightarrow$ perhatikan $(2,1)^2 = 4,41 \longrightarrow \sqrt{4,41} = 2,1$

✓ $x_0 = 4,61$ ~ lebih dekat.

✓ $\Delta x = 4,6 - 4,41 = 0,19$

$f(x) = \sqrt{x} = x^{1/2}$

$f'(x) = \frac{1}{2} \cdot x^{-1/2} = \frac{1}{2 \cdot x^{1/2}} = \frac{1}{2\sqrt{x}} \Rightarrow \frac{1}{2\sqrt{4,41}} = \frac{1}{2 \cdot 2,1} = \frac{1}{4,2}$

$f(x_0 + \Delta x) = f(x_0) + f'(x_0) \cdot \Delta x$

$= f(4,41) + f'(4,41) \cdot [0,19]$

$$= \boxed{2,1 + \frac{1}{4,2} \cdot 0,19}$$

7. Dengan substitusi $u = x + 1$, diperoleh

$$\int_0^2 f(x)dx = \int_0^2 \frac{1}{(x+1)^2}dx = \int_1^b g(u)du$$

(a) $g(b) = \ldots$

(b) Tentukan nilai rata-rata dari fungsi $f(x)$ pada interval $[0, 2]$.

(c) Tentukan semua nilai c yang memenuhi Teorema Nilai Rata-rata untuk $f(x)$ pada interval $[0, 2]$.

$$\int_{x_1 \to 0}^{x_f \to 2} \frac{1}{(x+1)^2}\,dx = \int_1^3 \frac{1}{u^2}\,du = \int_1^b g(u)\,du$$

sub $u = x + 1 \rightarrow x = u - 1$

$du = dx$

$u_T = x_T + 1 = 0 + 1 = 1$

$u_f = x_f + 1 = 2 + 1 = 3$

$\Rightarrow \quad b = 3$

$g(u) = \frac{1}{u^2}$ $\qquad\qquad u = x+1$

a.) $g(b) = g(3) = \frac{1}{3^2} = \frac{1}{9}$

b.) $\int f(x)\,dx = \int g(u)\,du = \int \frac{1}{u^2}\,du = \int u^{-2}\,du$

$= \frac{u^{-1}}{-1} + C = -\frac{1}{u} + C = -\frac{1}{x+1} + C$

$\Rightarrow$ Nilai Rata-Rata $f(x) : [a,b]$

$$\langle f(x) \rangle = \frac{\int_a^b f(x)\,dx}{b-a}$$

Rata-rata $f(x) : [0,2]$

$$= \frac{\int_0^2 \frac{1}{(x+1)^2}\,dx}{2-0} = \frac{\left[-\frac{1}{x+1}\right]_0^2}{2} = \frac{\left(-\frac{1}{3} + \frac{1}{1}\right)}{2}$$

$$= \frac{\left(1-\frac{1}{3}\right)}{2} = \frac{2/3}{2} = \frac{1}{3}$$

b.) $\langle f(x) \rangle_{[0,2]} = \frac{1}{3}.$

a) Nilai c untuk TNR integral di interval [0,2]

$$f(c) = \boxed{\frac{\int_a^b f(x)\,dx}{b-a}}$$

$f(x) = \frac{1}{(x+1)^2}$

$$f(c) = \frac{1}{3}$$

$$\frac{1}{(c+1)^2} = \frac{1}{3}$$

$3 = (c+1)^2 \iff (c+1) = \sqrt{3}$ atau $-\sqrt{3}$

$\boxed{c = \sqrt{3} - 1}$ atau $-\sqrt{3} - 1$ //.

8. Diketahui

$$\lim_{n\to\infty} \sum_{i=1}^{n} \frac{4}{4i+n} = \int_1^5 f(x)\, dx$$

(a) $\int_1^5 f(x)\, dx = \ldots$

(b) Dengan menggunakan aproksimasi numerik metode trapesium dengan 4 partisi, $\int_1^5 f(x)\, dx \approx \ldots$

※ Integral Riemann.

$$\int_a^b f(x)\, dx = \underbrace{\lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\, \Delta x}$$

$$\Delta x = \boxed{\frac{b-a}{n}} \qquad x_i = a + \Delta x \cdot i = a + \frac{(b-a)\cdot i}{n}$$

$$\underbrace{\lim_{n\to\infty} \sum_{i=1}^{n} \frac{4}{4i+n}} \cdot \frac{n}{n} = \int_1^5 f(x)\, dx = \lim_{n\to\infty} \sum_{i=1}^{n} f\left(\frac{n}{n} + \frac{4i}{n}\right) \cdot \frac{4}{n}$$

✓ $f\left(\frac{n+4i}{n}\right)$

$\Rightarrow b = 5 \quad a = 1$

$$\Delta x = \frac{5-1}{n} = \frac{4}{n} \qquad x_i = 1 + \frac{4}{n} \cdot i$$

$$\lim_{n\to\infty} \sum_{i=1}^{n} \frac{n}{4i+n} \cdot \frac{4}{n} = \lim_{n\to\infty} \sum_{i=1}^{n} f\left(\frac{n+4i}{n}\right) \cdot \frac{4}{n}$$

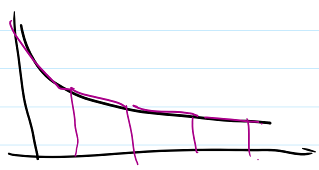
$$\frac{n}{4i+n} = f\left(\frac{n+4i}{n}\right) \qquad x = \frac{n+4i}{n}$$

$$\boxed{\frac{1}{x} = f(x)} \qquad \frac{1}{x} = \frac{n}{4i+n}$$

$$\boxed{= \int_1^5 \frac{1}{x}\, dx} \Rightarrow \int_1^x \frac{1}{x}\, dx = \ln(x)$$

a. $\int_1^5 f(x)\, dx = \int_1^5 \frac{1}{x}\, dx = \ln(5)$ //

b. ※ Metode Trapesium.



$$\int_a^b f(x)\, dx \approx \left(f(x_1) + 2f(x_2) + 2f(x_3) \cdots + 2f(x_{n-1}) + f(x_n)\right) \cdot \frac{b-a}{2n}$$

1 2 2 2 2 ⋯ 2 2 2 1

※ Metode ~~Pe...~~ Parabola.

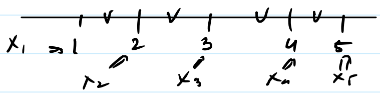
$$\approx \left(f(x_1) + 4f(x_2) + 2f(x_3) + 4f(x_4) + \cdots + 2f(x_{n-2}) + 4f(x_{n-1}) + f(x_n)\right) \cdot \frac{b-a}{3n}$$

1 4 2 4 2 4 2 4 2 ⋯ 2 4 1

$$\approx \left(f(x_1) + 4f(x_2) + 2f(x_3) + 4f(x_4) + \cdots + 2f(x_{n-2}) + 4f(x_{n-1}) + f(x_n) \right) \cdot \frac{b-a}{3n}$$

1 4 2 4 2 4 2 4 2 ⋯ 2 4 1

$f(x) = 1/x \qquad \int_1^5 f(x)\,dx \to 4$ partisi



$$\int_1^5 \frac{1}{x}\,dx = \left(f(x_1) + 2f(x_2) + 2f(x_3) + 2f(x_4) + f(x_5) \right) \cdot \frac{5-1}{(4\cdot 2)}$$

$$= \left(\frac{1}{1} + 2\cdot\frac{1}{2} + 2\cdot\frac{1}{3} + 2\cdot\frac{1}{4} + \frac{1}{5} \right) \cdot \frac{4}{8}$$

$$= \frac{60 + 60 + 40 + 30 + 12}{60} \cdot \frac{4}{8}$$

$=$

$$\Rightarrow \left[\underbrace{\frac{f(x_1) + f(x_2)}{2}} + \underbrace{\frac{f(x_2) + f(x_3)}{2}} + \cdots + \underbrace{\frac{f(x_{n-1}) + f(x_n)}{2}} \right] \cdot \Delta x \qquad \Delta x = \frac{b-a}{n}$$

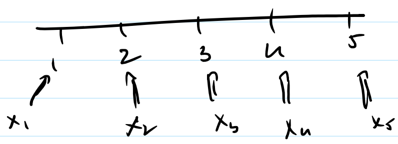
$$\Rightarrow \frac{f(x_1) + 2f(x_2) + 2f(x_3) + \cdots + 2f(x_{n-1}) + f(x_n)}{2} \quad \frac{(b-a)}{n}$$

※ Parabola → Partisinya genap

$$\approx \left[\underbrace{\frac{f(x_1) + 4f(x_2) + f(x_3)}{3}} + \underbrace{\frac{f(x_3) + 4f(x_4) + f(x_5)}{3}} + \underbrace{\cdots} + \frac{f(x_{n-2}) + 4f(x_{n-1}) + f(x_n)}{3} \right] \Delta x$$

$$= \frac{f(x_1) + 4f(x_2) + 2f(x_3) + 4f(x_4) \cdots + 2f(x_{n-2}) + 4f(x_{n-1}) + f(x_n)}{3} \cdot \frac{(b-a)}{n}$$

$$f(x)_{b} = \int_1^5 \frac{1}{x}\,dx$$



$$\frac{f(1) + 2f(2) + 2f(3) + 2f(4) + f(5)}{2} \cdot \frac{4}{4}$$

$$\frac{1}{2}\left[\frac{1}{1} + 2\cdot\frac{1}{2} + 2\cdot\frac{1}{3} + 2\cdot\frac{1}{4} + \frac{1}{5} \right] = \frac{1}{2}\left[\frac{30 + 30 + 20 + 15 + 6}{30} \right]$$

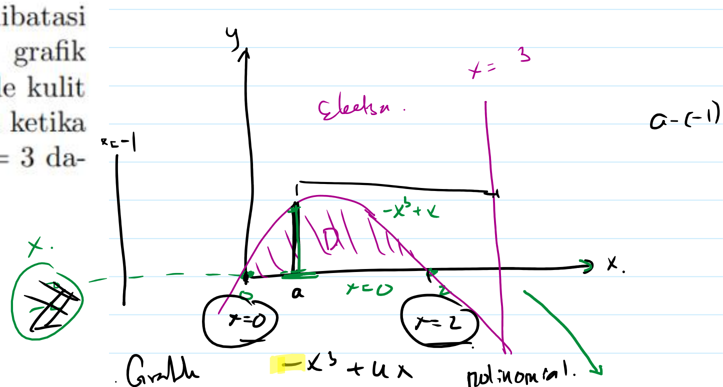
(lupa penyebut)

9. Misalkan daerah D adalah daerah yang dibatasi oleh sumbu-x positif, sumbu-y positif dan grafik $y = -x^3 + 4x$. Dengan menggunakan metode kulit tabung, volume benda pejal yang terbentuk ketika memutar daerah D dengan sumbu putar $x = 3$ dapat dituliskan sebagai

$$\int_0^b g(x)\,dx$$

(a) $b = \ldots$

(b) $g(1) = \ldots$



Grafik $-x^3 + 4x$ polinomial.

→ Tentukan titik potong →

$$0 = -x^3 + 4x$$
$$= x\,(-x^2 + 4)$$
$$= x\,(4 - x^2)$$
$$0 = x\,(2 + x)(2 - x)$$

$\Rightarrow$ $x = 0$ atau $x = -2$ atau $x = 2$

$\Uparrow$ $x = 0$

$y = 0$

※ **Metode Kulit Tabung** ~

$L = 2\pi \cdot r \cdot t$

$V = \sum L.$

r = jarak x ke $x = 3$

$= 3 - x.$

$t = (-x^3 + 4x) - 0$

$t =$

$$V = \int_{x_0}^{x_f} 2\pi \cdot r \cdot t \; dx.$$

$$= \int_0^2 2\pi\,(3 - x)\cdot(-x^3 + x)\; dx$$

$$= \int_a^b g(x)\; dx$$ — titik potong daerah

a.) $b = 2.$

b) $g(x) = 2\pi\,(3 - x)\,(-x^3 + 4x)$

$g(1) = 2\pi\,\underbrace{(3 - 1)}_{2}\,\underbrace{(-1^3 + 4 \cdot 1)}_{3}$

$=$

$= 12\pi$ //.

4. Diketahui

$$F(x) = \int_4^{x^2} (t-1)\sqrt{2t+1}\, dt$$

Jika $F^{-1}(x)$ adalah invers dari $F(x)$ pada interval (a, ∞), tentukan selang kemonotonan $F(x)$ untuk mencari nilai a kemudian tentukan turunan pertama dari $F^{-1}(x)$ di $x = 0$.

※ Invers.

$y = f(x) \Leftrightarrow f^{-1}(y) = x.$

✓ $\boxed{f(f^{-1}(x)) = x.}$

$f'(f^{-1}(x)) \cdot (f^{-1})'(x) = 1$

turunan $f^{-1}(x)$ $\boxed{(f^{-1})'(x) = \dfrac{1}{f'(f^{-1}(x))}}$ $x = 0$

$\boxed{(f^{-1})'(0) = \dfrac{1}{f'(f^{-1}(0))}}$

$f^{-1}(0) = \dots ?$ ✓

$f'(x) = \dots ?$

$F^{-1}(0) = c\ ? \Leftrightarrow F(c) = 0$

$F(c) = \int_4^{c^2} (t-1)\sqrt{2t+1}\, dt = 0 \Rightarrow$ Ingat bahwa $\int_a^a f(x)\, dx = 0$

$\Rightarrow c^2 = 4 \Rightarrow \boxed{c = 2 \text{ or } -2.}$

※ Selang kemonotonan $\rightarrow f'(x) > 0$: monoton naik

$\hookrightarrow f'(x) < 0$: monoton turun

$\hookrightarrow f'(x) = 0$: stationer.

} tanda

$F(x) = \int_4^{x^2} (t-1)\sqrt{2t+1}\, dt \Rightarrow$ TDK

$F(x) = \int_c^x f(x)\, dx$

$F'(x) = f(x)$

$t = x^2$

$\boxed{F'(x) = (x^2-1)\sqrt{2x^2+1} \cdot 2x}$

$= 2x\,(x+1)(x-1)\sqrt{2x^2+1} \Rightarrow$ pembuat nol.

$0 \quad -1 \quad 1$

$- \quad + \quad - \quad + \Rightarrow (a, \infty)$

turun -1 naik 0 turun 1 monoton naik

$\therefore (a, \infty) = (1, \infty) \Rightarrow a = 1$

$c = 2$ or $\cancel{-2}$. $\Rightarrow c = 2.$

$c = F^{-1}(0) \Rightarrow \boxed{F^{-1}(0) = 2.}$

$$(F^{-1})'(0) = \frac{1}{F'(\underbrace{F^{-1}(0)})} = \frac{1}{F'(2)}$$

$$= \frac{1}{2x(x^2-1)\sqrt{2x^2+1}} = \frac{1}{2 \cdot 2(4-1)\sqrt{2 \cdot 4+1}}$$

$$= \frac{1}{4 \cdot 3 \cdot \sqrt{9}} = \frac{1}{36}$$

$\therefore$ Turunan $F^{-1}(x)$ pada $x = 0$ adalah $\frac{1}{36}$

Mockup UP Bagian 2

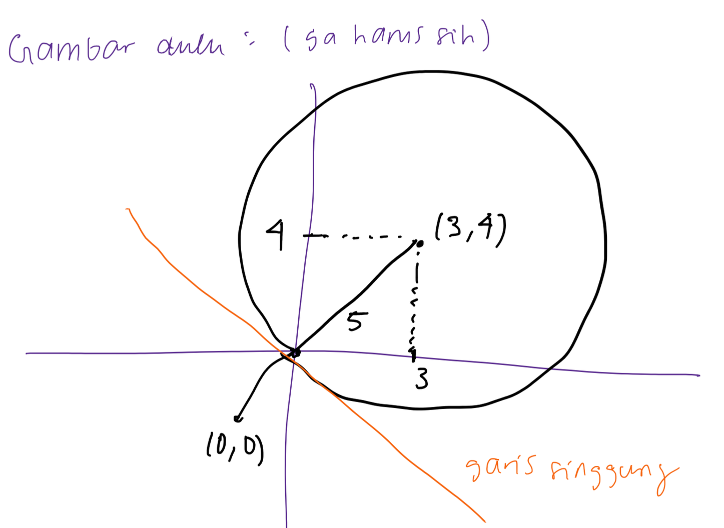
Wednesday, 20 December 2023 20.18

1. jari-jari = 5 = r

titik pusat = (3,4)

? gradien garis singgung di (0,0)

Gambar dulu : (ga harus sih)



persamaan lingkaran :

mengikuti bentuk $(x-a)^2+(y-b)^2=r^2$ dengan (a,b) sbg titik pusat.

TP : (3,4) r = 5 $\longrightarrow$ $(x-3)^2+(y-4)^2=25$

untuk mencari gradien, maka kita cari $\frac{dy}{dx}$ dr persamaan lingkaran.

$$(x-3)^2+(y-4)^2=25$$

$$\Leftrightarrow x^2-6x+\cancel{9}+y^2-8y+\cancel{16}=\cancel{25}$$

$$\Leftrightarrow x^2-6x+y^2-8y=0$$

mencari $\frac{dy}{dx}$:

$$D_x[x^2-6x+y^2-8y]=D_x[0]$$

$$\Leftrightarrow 2x-6+2y\cdot\frac{dy}{dx}-8\frac{dy}{dx}=0 \longrightarrow$$ turunan implicit jika y adalah fungsi x,

$D_x[y]=\frac{dy}{dx}=y'$

$D_x[y^2] \rightarrow$ berlaku aturan rantai

$=2y\cdot\frac{dy}{dx}$

$$\Leftrightarrow \frac{dy}{dx}(2y-8)=6-2x$$

$$\Leftrightarrow \frac{dy}{dx}=\frac{6-2x}{2y-8}$$

di (0,0), gradien garis singgung atau $\frac{dy}{dx}$-nya adalah :

$$\frac{dy}{dx}=\frac{6-2x}{2y-8}=\frac{6-2(0)}{2(0)-8}=\frac{6}{-8}=-\frac{3}{4}$$ (jawab)

2. $g(x)=x^{\frac{2}{3}}-x^{\frac{5}{3}}$, interval $[-1,1]$

titik kritis → titik ujung → $x=-1, x=1$

→ titik stasioner : x dimana $g'(x)=0$

→ titik singuler : x dimana $g'(x)$ tidak terdefinisi

titik stasioner :

$$g'(x)=\frac{2}{3}x^{-\frac{1}{3}}-\frac{5}{3}x^{\frac{2}{3}}=0,\ x\neq 0$$

$$=x^{-\frac{1}{3}}\left(\frac{2}{3}-\frac{5}{3}x\right)=0,\ x\neq 0$$

$$\Leftrightarrow x\neq 0,\ \frac{2}{3}-\frac{5}{3}x=0 \Leftrightarrow 2-5x=0$$

$$\Leftrightarrow 5x=2$$

$$\Leftrightarrow \boxed{x=\frac{2}{5}}$$

titik singuler : $x=0$ → di 0, $g'(x)$ tidak terdefinisi karena di $\frac{2}{3}x^{-\frac{1}{3}}=\frac{2}{3\sqrt[3]{x}}$, x tidak boleh 0.

maksimum dan minimum

$g(-1)=(-1)^{\frac{2}{3}}-(-1)^{\frac{5}{3}}=1-(-1)=2$

$g(0)=0-0=0$

$g\left(\frac{2}{5}\right)=\left(\frac{2}{5}\right)^{\frac{2}{3}}-\left(\frac{2}{5}\right)^{\frac{5}{3}} \longrightarrow \sqrt[3]{\frac{2}{5}^2}-\sqrt[3]{\left(\frac{2}{5}\right)^5} \longrightarrow \left(\frac{2}{5}\right)^2 > \left(\frac{2}{5}\right)^5 \Leftrightarrow \sqrt[3]{\left(\frac{2}{5}\right)^2} > \sqrt[3]{\left(\frac{2}{5}\right)^5}$ (pasti)

$g(1)=1^{\frac{2}{3}}-1^{\frac{5}{3}}=1-1=0$

jadi $g\left(\frac{2}{5}\right)$ pasti >0.

$\left(\frac{2}{5}\right)^{\frac{2}{3}}<2$ dan $\left(\frac{2}{5}\right)^{\frac{5}{3}}>0$,

maka $g\left(\frac{2}{5}\right)$ pasti <2.

maka, $g\left(\frac{2}{5}\right)$ bukan maksimum, maupun minimum

nilai minimum = 0

nilai maksimum = 2

interval kemonotonan

$g'(x)=\frac{2}{3}x^{-\frac{1}{3}}-\frac{5}{3}x^{\frac{2}{3}}$

periksa tanda :

$g'\left(\frac{1}{8}\right)=\frac{2}{3}\left(\left(\frac{1}{2}\right)^{\cancel{3}}\right)^{-\frac{1}{\cancel{3}}}-\frac{5}{3}\left(\left(\frac{1}{2}\right)^{\cancel{3}}\right)^{\frac{2}{\cancel{3}}}=\frac{2}{3}\left(\frac{1}{2}\right)^{-1}-\frac{5}{3}\left(\frac{1}{4}\right)$

$=\frac{4}{3}-\frac{5}{12}$ → positif

(−) $\frac{1}{8}$ (+)   (−)

−1   0 $\frac{2}{5}$ 1

monoton naik : $\left[0,\frac{2}{5}\right]$

monoton turun : $[-1,0]\cup\left[\frac{2}{5},1\right]$

interval kecekungan

$$g''(x)=-\frac{2}{9}x^{-\frac{4}{3}}-\frac{10}{9}x^{-\frac{1}{3}},\ x\neq 0$$

$$=x^{-\frac{4}{3}}\left(-\frac{2}{9}-\frac{10}{9}x\right)=0$$ (cari pembuat 0 buat periksa tanda.)

$$-\frac{2}{9}-\frac{10}{9}x=0$$

$$-2-10x=0$$

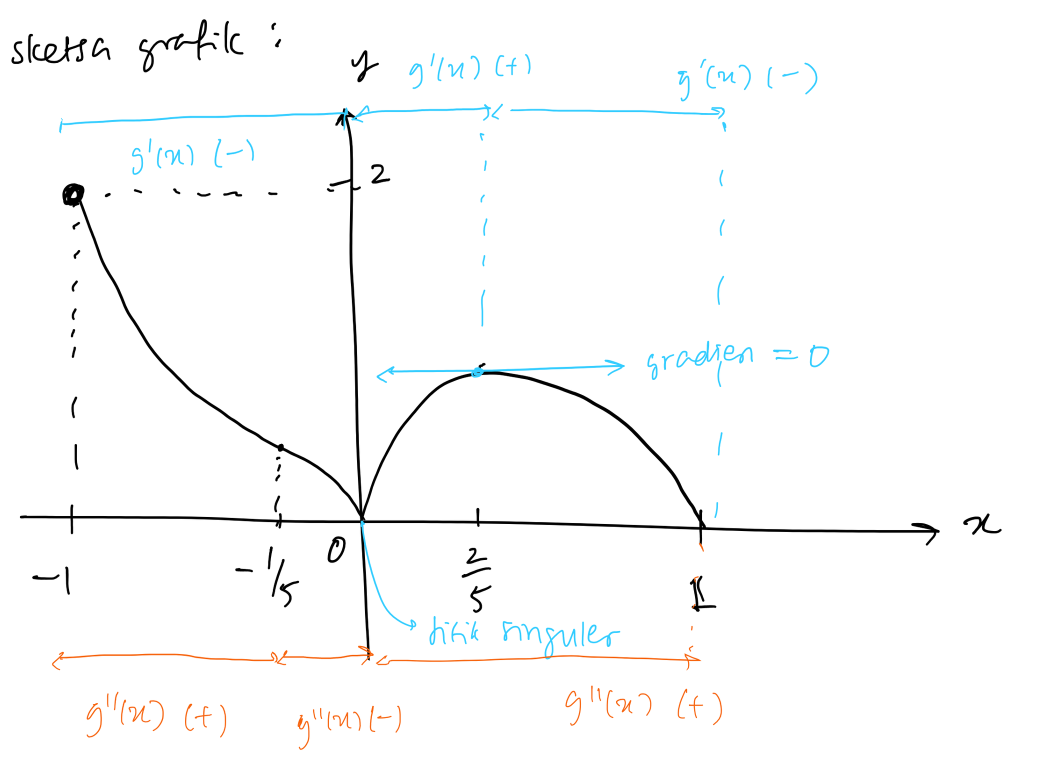
$$10x=-2$$

$$\boxed{x=-0.2}$$

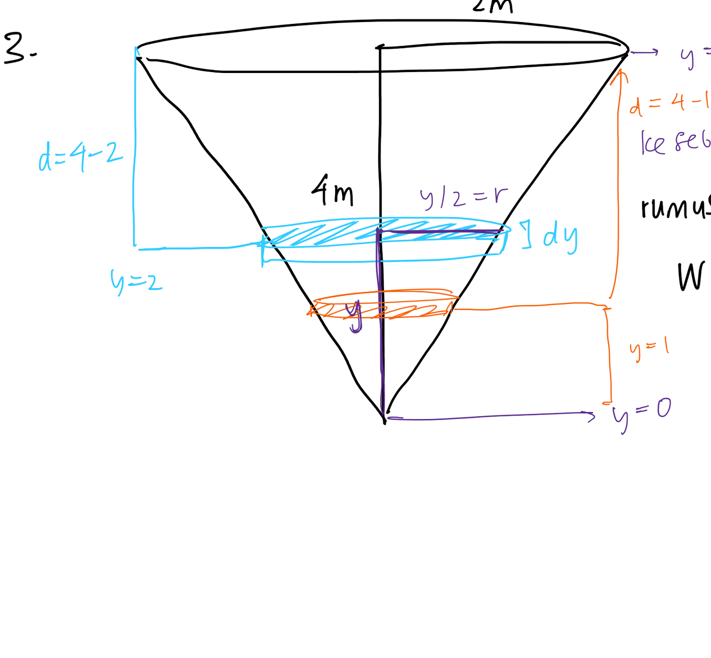
(+)   (−)   (+)

−1   −0,2   periksa $-\frac{1}{8}$ 0   1 → cek tanda

sketsa grafik :



3.



kesebangunan : $\frac{2m}{4m}=\frac{r}{y} \Leftrightarrow r=y/2$

rumus usaha :

$W=F\cdot d$ → gaya × perpindahan

d bergantung pada y :

di $y=1$, $d=4-1$

$y=2$, $d=4-2$

⋮

$y=4$, $d=4-4$

$\boxed{\text{umum : } d=4-y}$

untuk F → gaya berat lempengan

$F_g=(10000\ N/m^3)\times$ alas lempengan × tinggi

$=(10\,000\ N/m^3)\times \pi r^2 \times dy$

$=(10000\ N/m^3)\times \frac{\pi y^2}{4}\cdot dy$

(a) $$W=\int_0^4 F_g\cdot d=\int_0^4 (10\,000\ N/m^3)\frac{\pi y^2}{4}(4-y)\,dy$$

$$=\int_0^4 (10\,000\pi y^2-2500\pi y^3)\,dy$$

$$=\frac{10000}{3}y^3-625\pi y^4\Big|_0^4$$

$$=\boxed{\frac{64(10\,000)}{3}\pi-(625)(256)\pi}$$ jawab (a)

(b) sama kayak (a), tapi $d=5-y$ bukan $4-y$, ς dari $4m+1m=5m$

$$W=\int_0^4 F_g\cdot d=\int_0^4 (10\,000\ N/m^3)\frac{\pi y^2}{4}(5-y)\,dy$$

(tinggal perhitungan)

5. Suatu bakteri berkembang secara eksponensial sehingga banyak bakteri pada hari ke-t mengikuti $f(t) = a \cdot e^{kt}$. Pada suatu laboratorium, terdapat 100.000 bakteri. Kemudian, setelah 20 hari terdapat 400.000 bakteri. Prediksikan pada hari keberapa terdapat 1.000.000 bakteri.

$f(t) = ae^{kt}$

misalkan, Pada $t = t_0$, $f(t_0) = 100.000$ bakteri

Pada $t = t_0 + 20$, $f(t_0+20) = 400.000$ bakteri

maka :

$$\frac{f(t_0+20) = ae^{(t_0+20)k} = 400.000}{f(t_0) = ae^{t_0 k} = 100.000} \Rightarrow e^{((t_0+20)-t_0)k} = 4$$

$$e^{20k} = 4$$

$$\ln(e^{20k}) = \ln(4)$$

$$20k \cdot \ln e = \ln 4$$

$$k = \frac{1}{20} \cdot \ln 4$$

$$\Rightarrow f(t_0) = ae^{\ln 4 \cdot \frac{1}{20} \cdot t_0} = 100.000$$

$$a(4)^{\frac{t_0}{20}} = 100.000$$

$$a = \frac{100.000}{4^{t_0/20}}$$

$$\Rightarrow f(t) = ae^{k \cdot t}$$

$$= \frac{100.000}{4^{t_0/20}} \left(e^{\ln 4 \cdot \frac{1}{20}}\right)^t$$

$$= 10^5 \, 4^{(t/20 - t_0/20)}$$

$$f(t) = 10^5 \cdot 4^{\left(\frac{t-t_0}{20}\right)}$$

Kapan dia mencapai 1 jt?

$$10^6 = 10^5 \cdot 4^{\left(\frac{t-t_0}{20}\right)}$$

$$10 = 4^{\left(\frac{t-t_0}{20}\right)}$$

$$\ln(10) = \left(\frac{t-t_0}{20}\right) \cdot \ln 4$$

$$\frac{\ln(10)}{\ln(4)} \cdot 20 = t - t_0$$

$$t = t_0 + \frac{\ln(10)}{\ln(4)} \cdot 20$$

maka, bakteri mencapai 1 jt pada hari ke - $\frac{\ln(10)}{\ln(4)} \cdot 20$