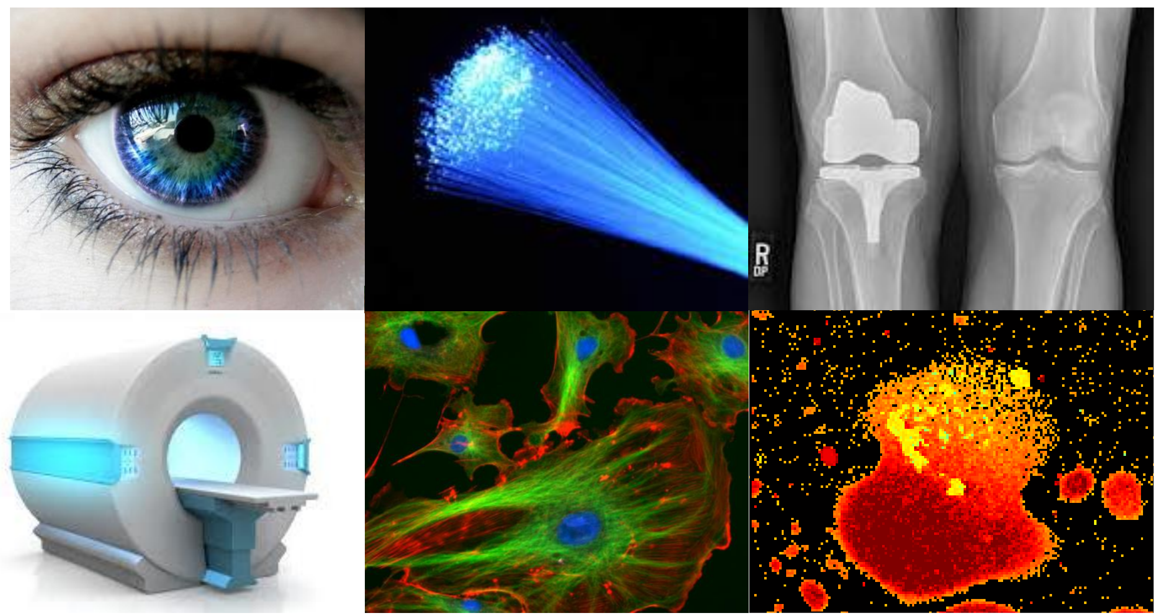
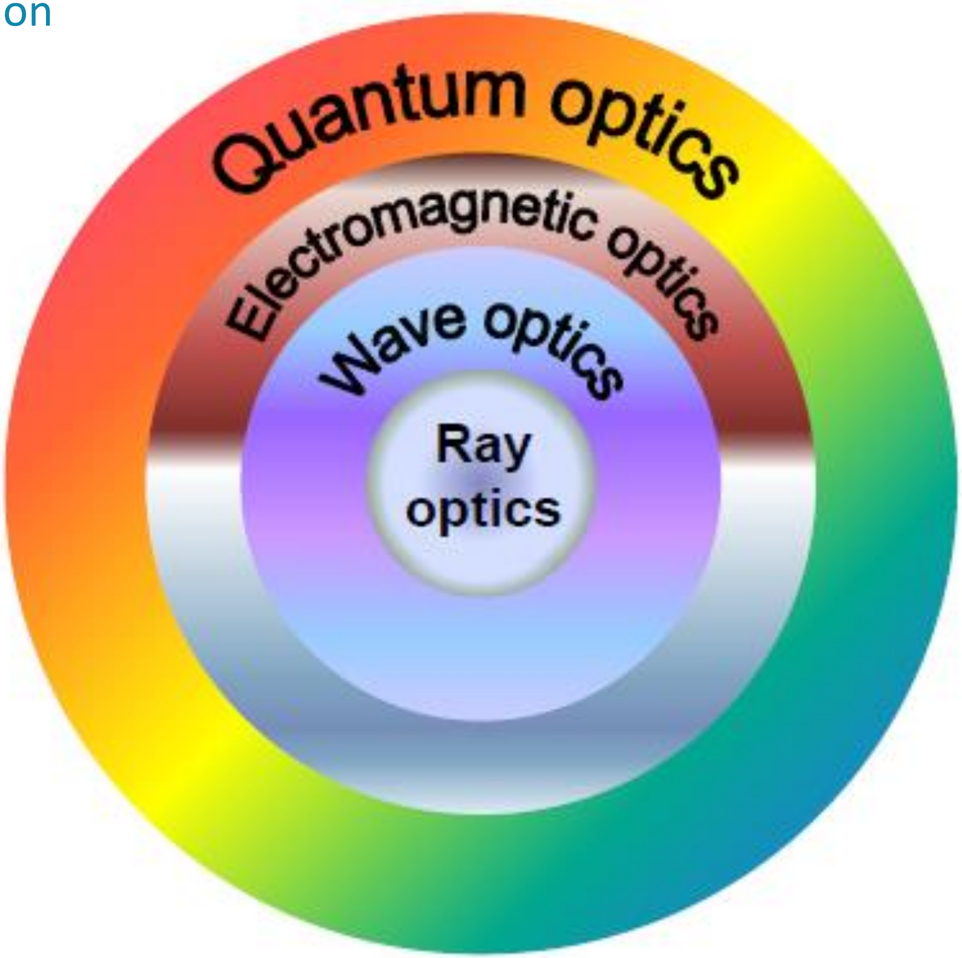
Lasers e Ótica Biomédica



Pedro Jorge

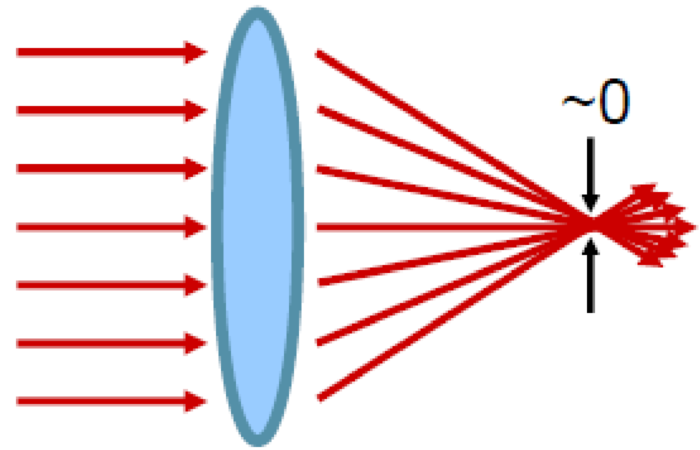
- Geometric Optics

  - Paraxial approximation
  - Thin Lenses
  - Image formation
  - Beam expanders
  - Mirrors



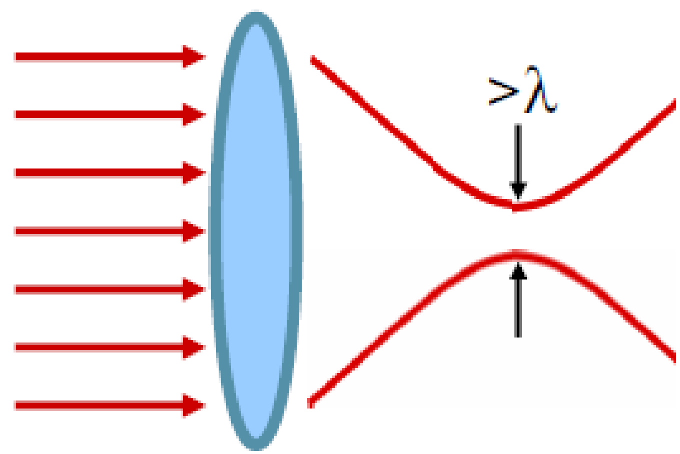
Geometrical Optics

- Geometrical Optics is an approximation!



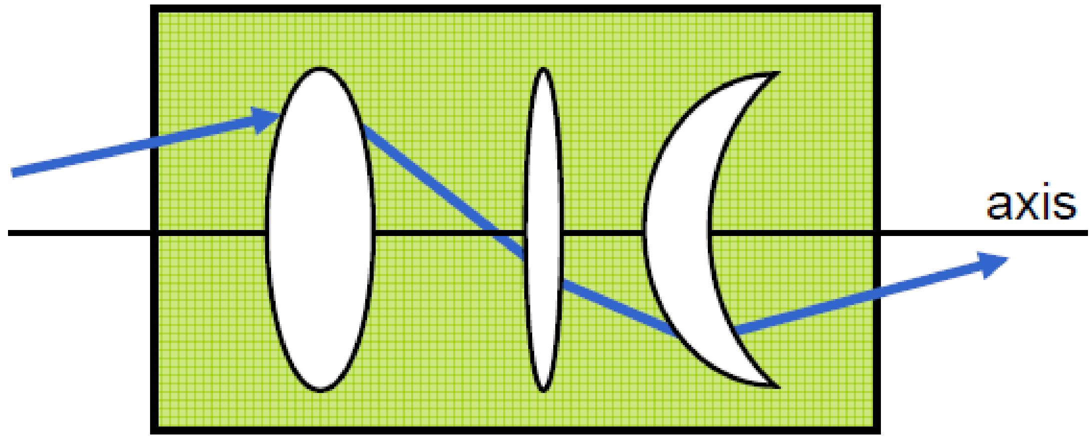
- Phase is not considered

- Ray tracing seems to imply that abeam can be focused in a single point (when aberration is not present, resulting in an infinite resolution. **NOT TRUE**



- Indeed, the smallest focal point that can be achieved as a dimension **~λ**. Therefore, imposing a limit on imaging resolution.
- Such is due to **Diffraction** (which is not considered in Geometric Optics.

Ray Tracing

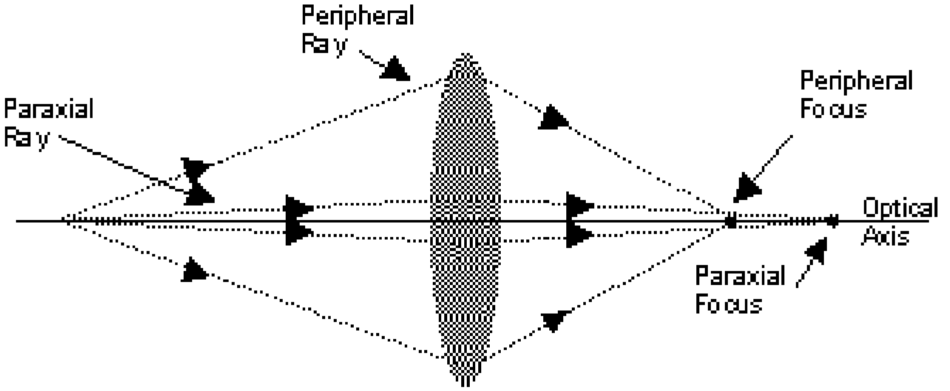


- **Light rays** define directions in space, corresponding to the k vector of electromagnetic waves.

- The phase is not considered

- Each system has an **optical axis**, and all rays propagate with small angles relative to such axis. This is called the **paraxial approximation**

$$\sin\theta \approx \theta$$
$$\tan\theta \approx \theta$$
$$\cos\theta \approx 1$$



Optical devices– Lenses and mirrors

- Terminology and approximation in Geometrical Optics

- **Focus** or **focal point** – point from which the wave converges or to which converges
- **Optical axis** – central axis crossing the optical device.

- **Reversibility principle** – if object (or optical source) switch position with its image, the optical path remains unchanged

- Such is due to **Diffraction** (which is not considered in Geometric Optics.

- An object (in object space) is related to an image (in the image space) as conjugated points (i.e. the object produces a well defined image in any of this points).

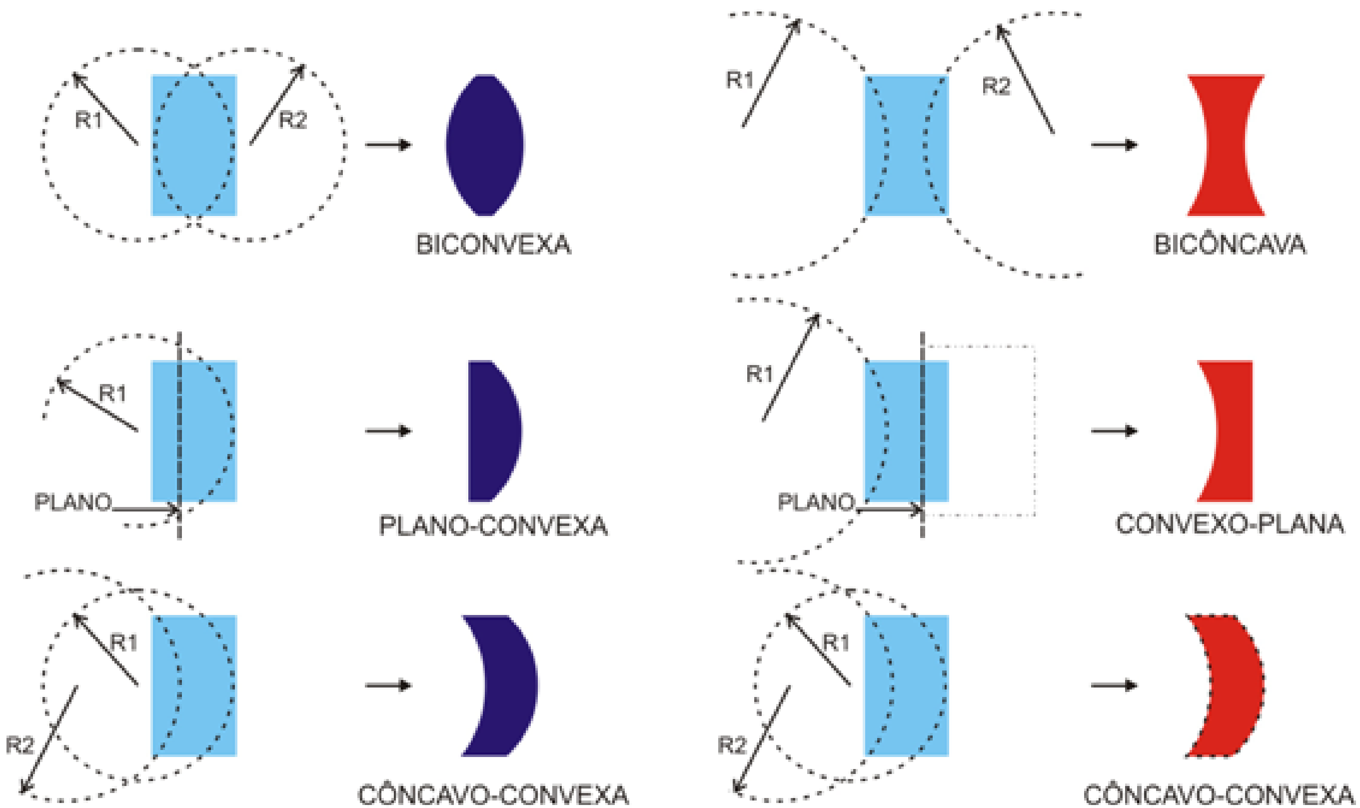
- **Real image** – bright image can be projected into a scree positioned at the focal plane.

- **Virtual image** – No image can be projected (can only be observed by a secondary optical system, like the eye).

- C

Types of lenses

- A lens is and optical system established by two or more dioptres (interfaces) which transmits and refracts light, changing the convergence or divergence of optical rays, according to the surface curvature and its refractive index



- Different combinations enable different applications such as image formation, magnification or size reduction.

Lenses

Object focus, *F_0*, object focal distance *f_0*,

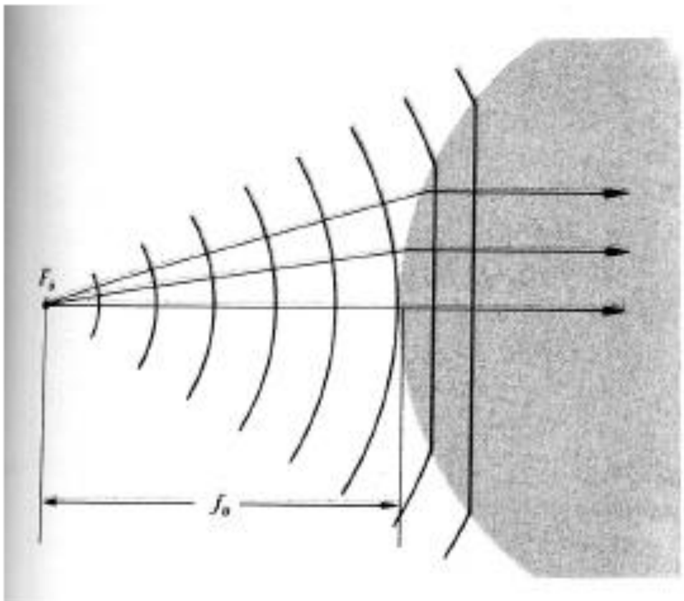
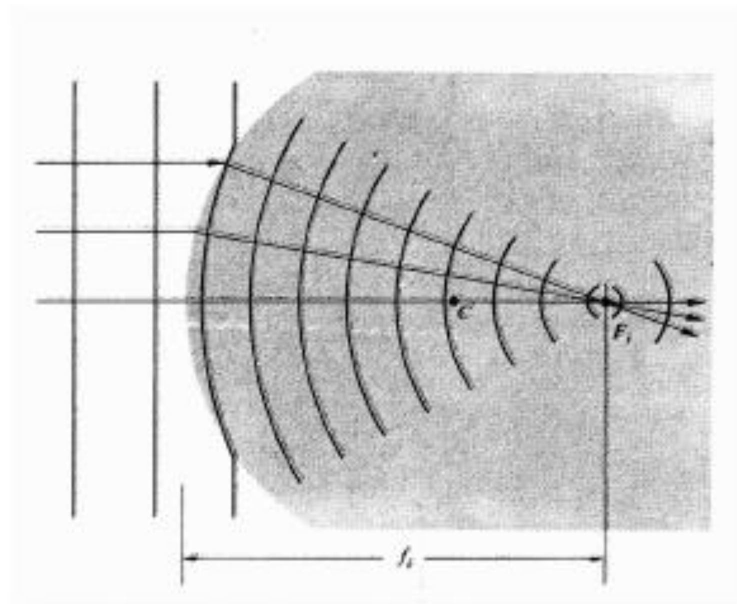
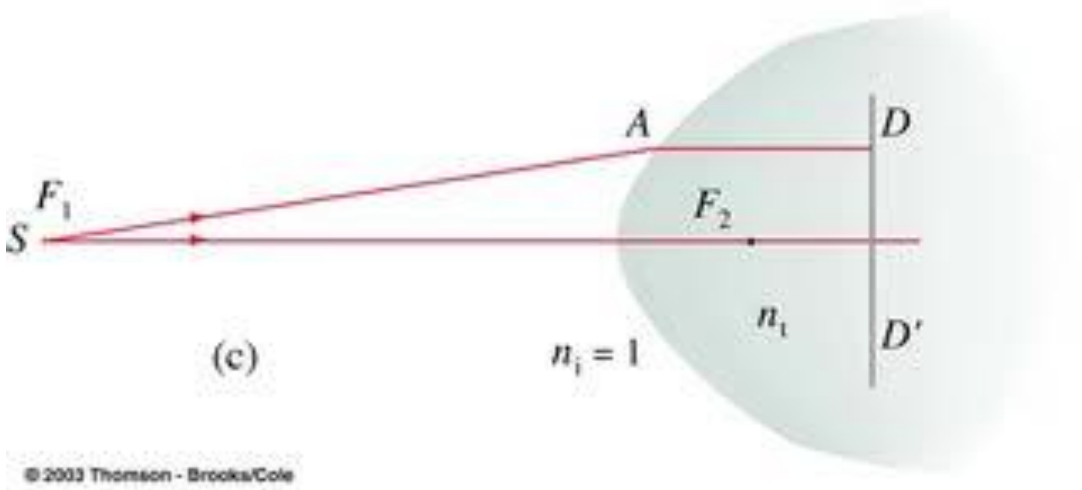
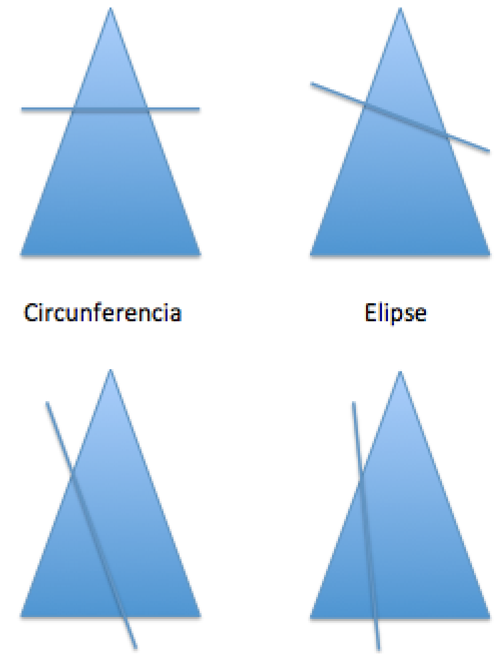


Image Focus *F_i*, image focal distance *f_i*



Refraction in an aspheric surface





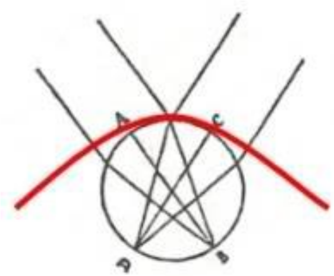
$$n_i\,(\overline{F_1A}) + n_t\,(\overline{AD}) = \text{constant}$$

$$\overline{F_1A} + \left(\frac{n_t}{n_i}\right)\overline{AD} = \text{constant}$$

- Equation describing an **Hyperbole** with eccentricity e=nt/ni >1

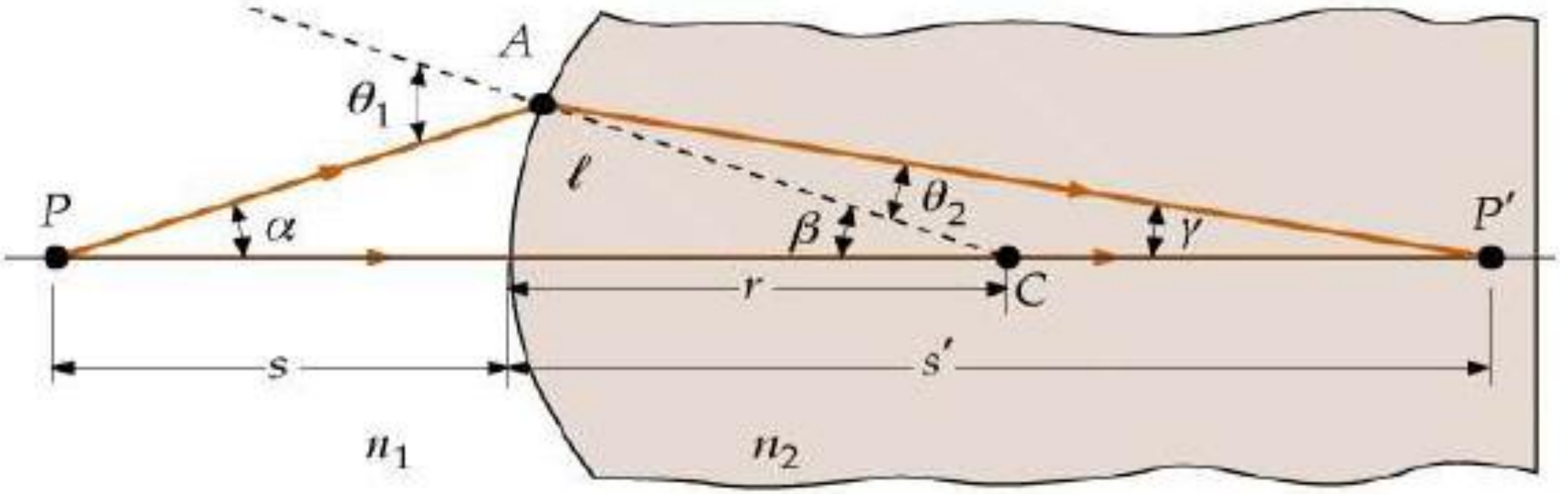
Conical surfaces of revolution resulting from the rotation of an hyperbole, parabola or ellipse.

Provide ideal focusing or collimating devices. Very hard to produce!



Approximation to a sphere

Refraction in a spherical surface

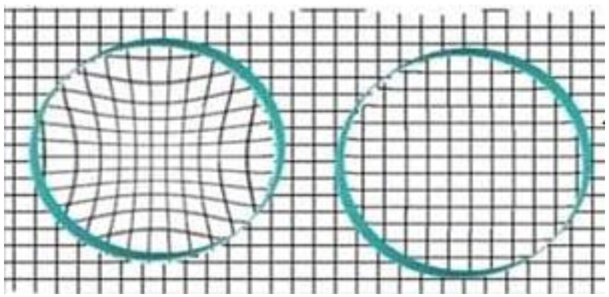


Considering the Fermat principle, the total optical path PO, from P to P', must be the same for any position of A in the surface.

$$(PO) = n_1 \overline{PA} + n_2 \overline{AP'}$$

Considering the paraxial approximation (small angles) it can be derived:

$$\boxed{\frac{n_1}{s} + \frac{n_2}{s'} = \frac{n_2 - n_1}{r}}$$

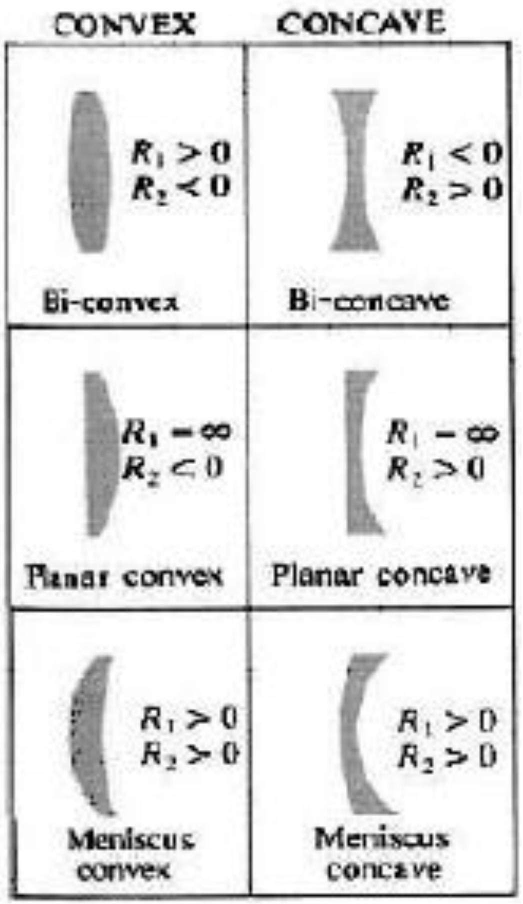


Signal convention for spherical diopters

TABLE 5.2 Meanings Associated with the Signs of Various Thin Lens and Spherical Interface Parameters

Quantity	Sign +	Sign −
s_o	Real object	Virtual object
s_i	Real image	Virtual image
f	Converging lens	Diverging lens
y_o	Erect object	Inverted object
y_i	Erect image	Inverted image
M_T	Erect image	Inverted image

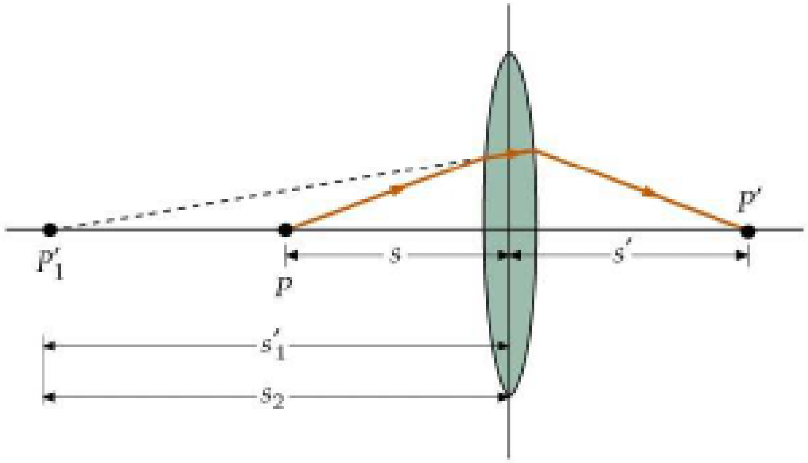
$$f_o = \frac{n_1}{n_2 - n_1} R \qquad f_i = \frac{n_2}{n_2 - n_1} R$$

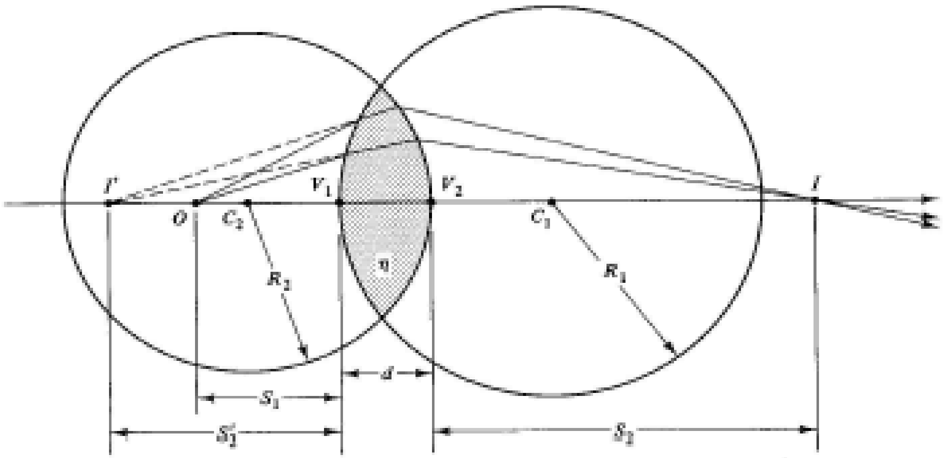


Thin lenses

Lenses can be considered **thin Lenses (d~0)**, when its thickness is small compared to the distances between object and images, and its diameter.

Otherwise they are called Thick lenses (d must be considered)





Refraction in the 1st surface

$$\frac{1}{s}+\frac{n}{s_1'}=\frac{n-1}{r_1}$$

Refraction in the 2nd surface

$$\frac{n}{-s_1'}+\frac{1}{s'}=\frac{1-n}{r_2}$$

$$\boxed{\frac{1}{s}+\frac{1}{s'}=(n-1)\left(\frac{1}{r_1}-\frac{1}{r_2}\right)}$$

Thin Lenses

Like in a mirror, in a thin lens, the focal distance f is defined as the image distance when the object is at infinity $s = \infty \Rightarrow s' = f$, so:

$$\frac{1}{s}+\frac{1}{s'}=(n-1)\left(\frac{1}{r_1}-\frac{1}{r_2}\right) \quad \Rightarrow \quad \boxed{\frac{1}{f}=(n-1)\left(\frac{1}{r_1}-\frac{1}{r_2}\right)}$$

Lens manufacturer equation

$$\boxed{\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}}$$

Thin lens equation, or Gauss formula

Ex:

What is the focal distance of a plano-convex lens with R=50 mm e n=1.5?

- R1=
- R2=

f=

so	600 mm	200 mm	150 mm	100 mm	50 mm
si					

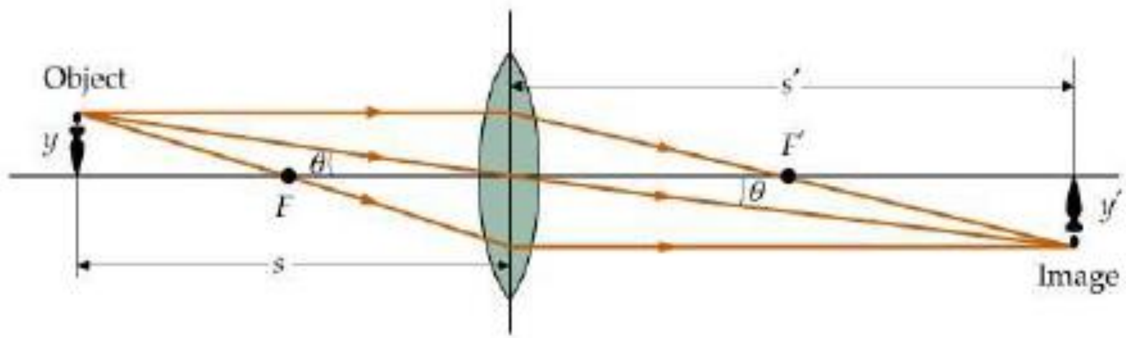
Ray tracing and image formation

For converging (positive) lens, image can be constructed with three **principal rays**:

1. The **parallel ray**
Incoming parallel to the optical axis, it is refracted trough the image focal point.

2. The **focal ray**
Incoming though the object focus it will emerge parallel to the optical axis.

2. The **central ray**
Passing through the center (vertex) of the lens, it does not suffer any deflection.



The intersection of these three rays emerging from a given point in the object localize the corresponding imaging point.

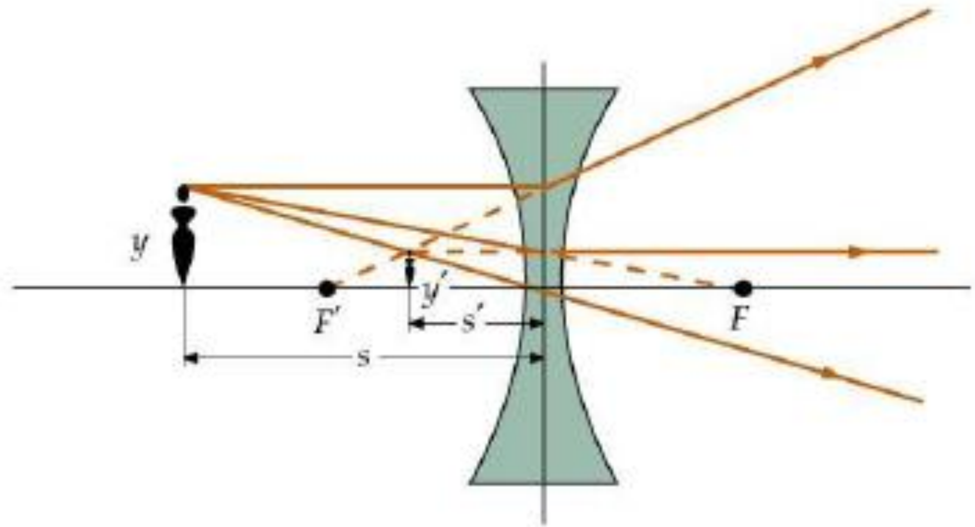
Ray tracing and image formation

For converging (positive) lens, image can be constructed with three **principal rays**:

1. The **parallel ray**
Incoming parallel to the optical axis, it is refracted away from the optical axis, as coming from the image focal point F’

2. The **focal ray**
Drawn towards the object focal point, F) it will emerge parallel to the optical axis.

2. The **central ray**
Passing through the center (vertex) of the lens, it does not suffer any deflection.



The intersection of these three rays emerging from a given point in the object localize the corresponding imaging point.

Magnification

- Lateral or transverse magnification

It is related with the transversal dimensions.

$$\boxed{M_T = \frac{y'}{y} = -\frac{s'}{s}}$$

$M_T > 0 \Rightarrow$ Non inverted image

$M_T < 0 \Rightarrow$ Inverted image

All real images formed by thin lens are inverted

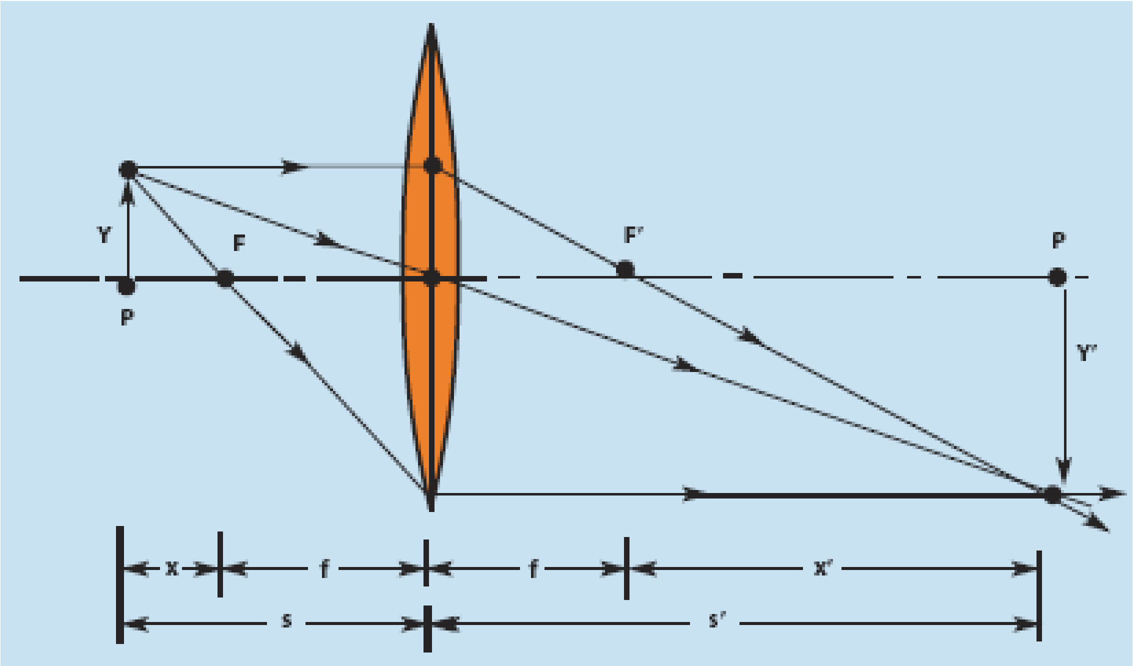
- Longitudinal magnification

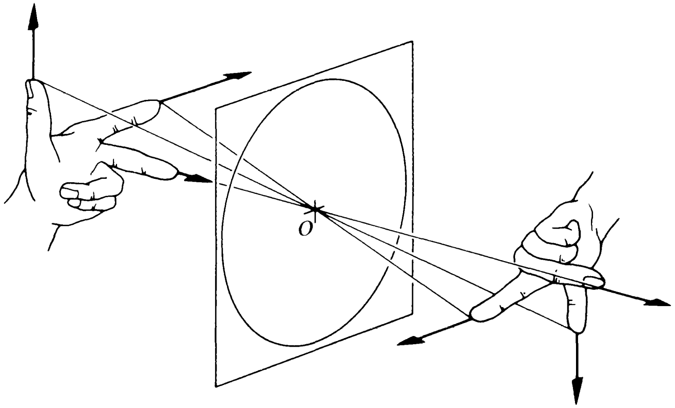
It is related with the axial direction; It represents the ratio between the dimensions a linear (axial) element in the image space, and the corresponding dimensions in the object space .

$$\boxed{M_L = -\frac{f^2}{(s-f)^2} = -M_T^2}$$

This means that, an image of a finger pointing to the lens, will point away from the lens.

Magnification (Gauss and Newton formulas)





$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f} \quad (1)$$

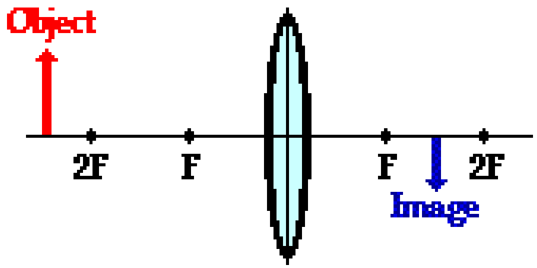
$$xx' = f^2 \quad (2)$$

$$m = -\frac{s'}{s} \quad (3)$$

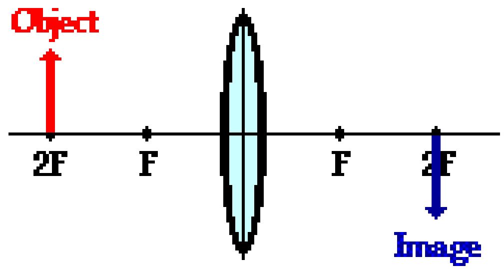
$$m = -\frac{f}{x} = -\frac{x'}{f} \quad (4)$$

Magnification

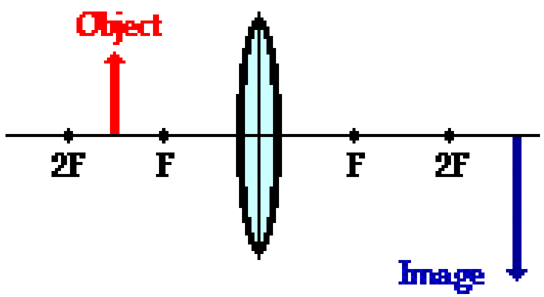
Magnification in positive lenses (convex)



Inverted image; reduced M<1, real



Inverted image; real size M=1, real



Inverted Image; magnified M>1, real

http://www.physicsclassroom.com/class/refrn/Lesson-5/Converging-Lenses-Object-Image-Relations

Magnification in positive lenses (convex)

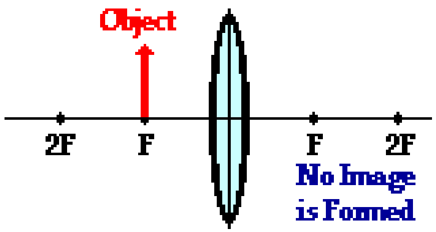
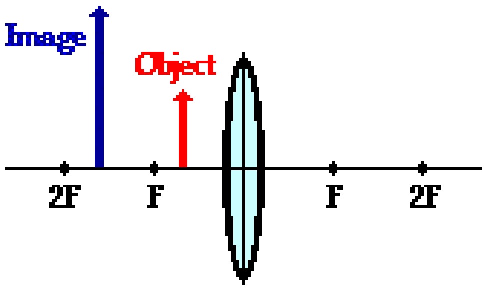
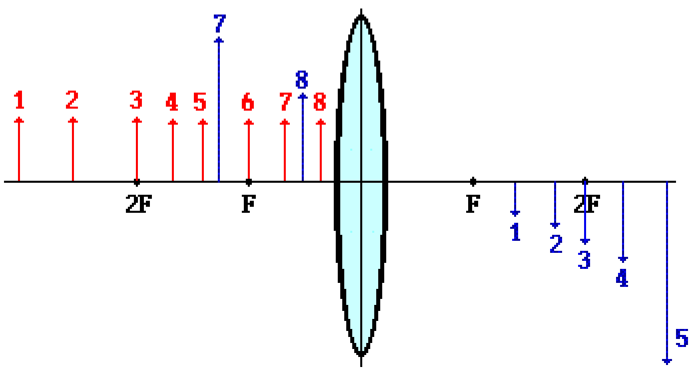


Image focused at infitity. No image formed!

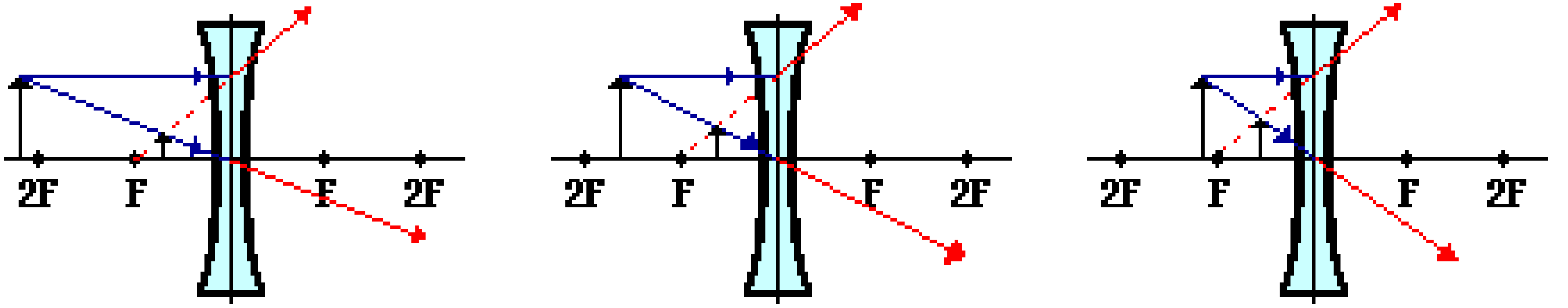


Upritgh image (not inverted); Magnified M>1, virtual

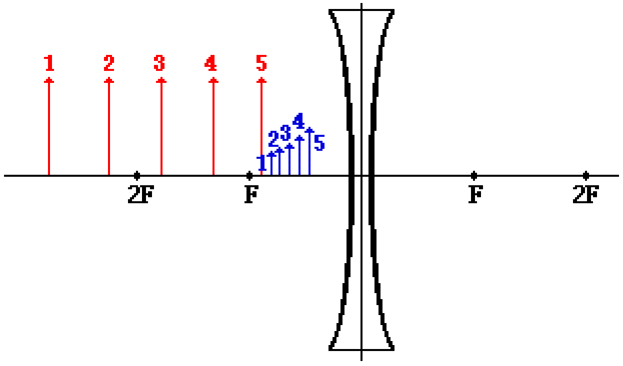


Each of the numbered objects (except #6) has an image with the corresponding number; its relative location, size, and orientation are shown.

Magnification with negative lenses (concave)



In all cases:
Virtual image, upright (not inverted) <and reduced M<1



Lenses – sign convention

- Sign convention for thin lenses and spherical diopters

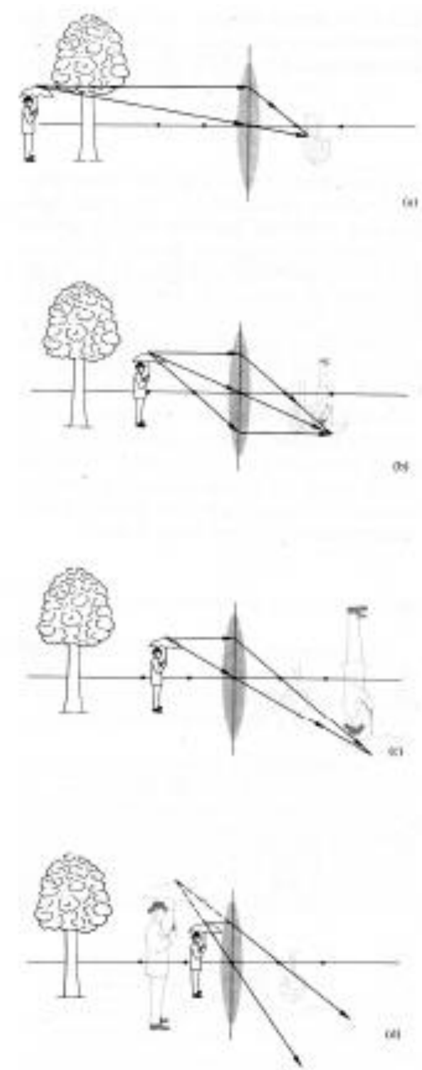
GRANDEZA	SINAL	
	+	-
s	objecto real	objecto virtual
s'	imagem real	imagem virtual
f	lente convergente	lente divergente
y	objecto não invertido	objecto invertido
y'	imagem não invertida	imagem invertida
M_T	imagem não invertida	imagem invertida

Lenses –image formation

- Image for real objects formed by thin lenses

CONVEXA				
OBJECTO	IMAGEM			
Localização	Tipo	Localização	Orientação	Tamanho Relativo
$\infty > s > 2f$	Real	$f < s' < 2f$	Invertida	Reduzida
$s = 2f$	Real	$s = 2f$	Invertida	Igual
$f < s < 2f$	Real	$\infty > s' > 2f$	Invertida	Aumentada
$s = f$		$\pm \infty$		
$s < f$	Virtual	$\lvert s' \rvert > s$	Direita	Aumentada

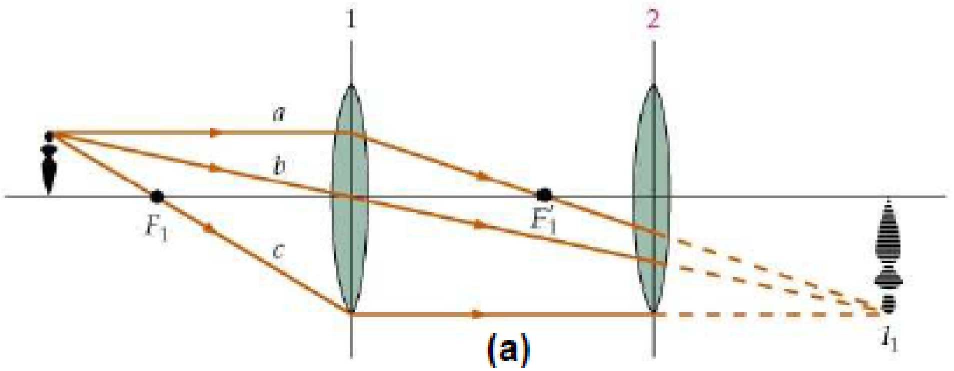
CÔNCAVA				
OBJECTO	IMAGEM			
Localização	Tipo	Localização	Orientação	Tamanho Relativo
Qualquer	Virtual	$\lvert s' \rvert < \lvert f \rvert$ $s > \lvert s' \rvert$	Direita	Reduzida



Combination of lenses: example

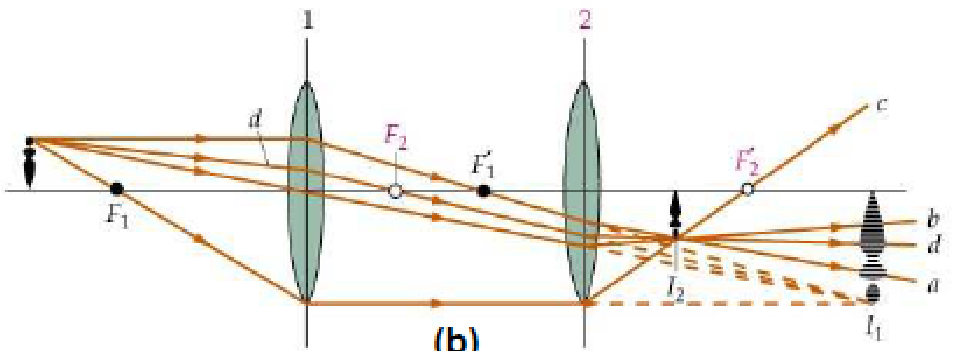
Two lenses, each having a focal distance of 10 cm, are separated by 15 cm. Localize the image position of an object standing 15 cm from the first lens.

1. Draw the parallel (a), central (b) and focal (c) rays, of the first lens. If lens 2 is ignored, image is formed in I1 (fig (a))



(a)

2. The focal ray (c) strikes lens 2, parallel to the optical axis, refracting towards the image focus F_2'. Tracing an additional ray (d) going trough F_2 (like any other ray it would go towards I1) it should exit parallel to the optical axis. Intersection of (c) and (d) localize the final image (fig b).



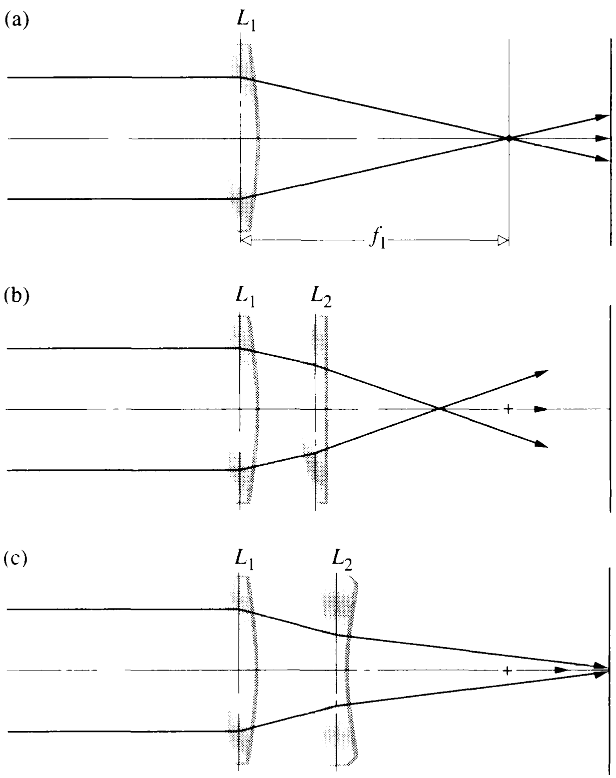
(b)

3. Using the thin lens equation, we can obtain the distance of the image of lens 1, which is S_1'=30 cm

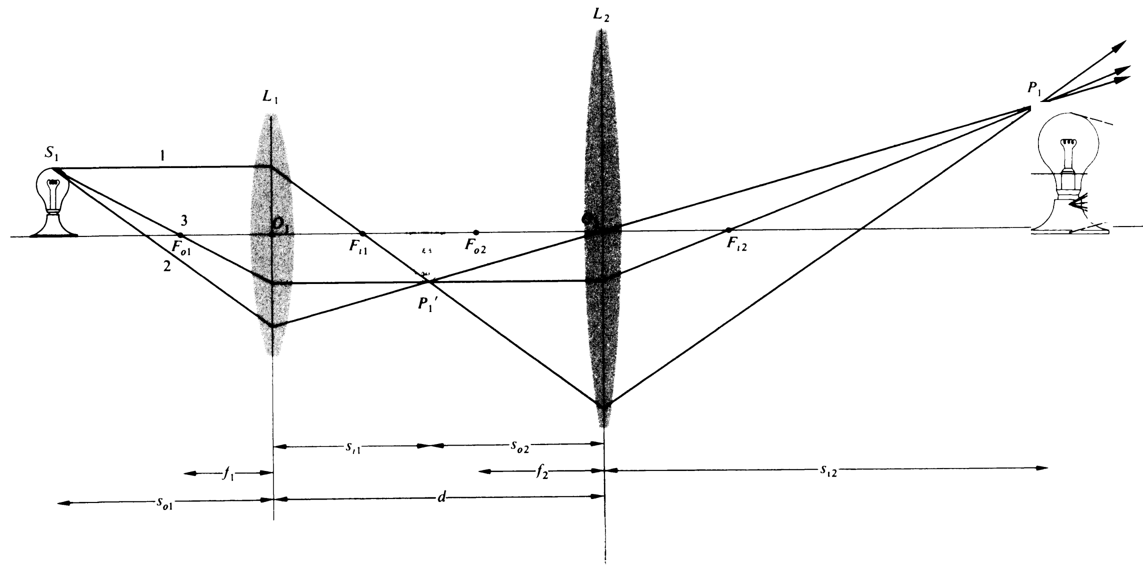
4. For lens 2, the image I1, is 15 cm away from the lens (transmission side) so S2=-15 cm. Using the thin lens equation we can obtain the final image distance which is S_2'=6 cm

Combination of Lenses

Combination of Lenses at distances smaller than f. Reinforcement or reduction of the convergence or divergence.



Combination of lenses at distances higher than f. Real magnified images



$$s_{i2} = \frac{f_2 d - f_2 s_{o1} f_1/(s_{o1} - f_1)}{d - f_2 - s_{o1} f_1/(s_{o1} - f_1)}$$

$$M_T = M_{T1} M_{T2}$$

Front or anterior focal length

$$\text{f.f.l.} = \frac{f_1(d - f_2)}{d - (f_1 + f_2)}$$

Back or posterior focal length

$$\text{b.f.l.} = \frac{f_2(d - f_1)}{d - (f_1 + f_2)}$$

For d equal to zero $\text{b.f.l.} = \text{f.f.l.} = \dfrac{f_2 f_1}{f_2 + f_1}$

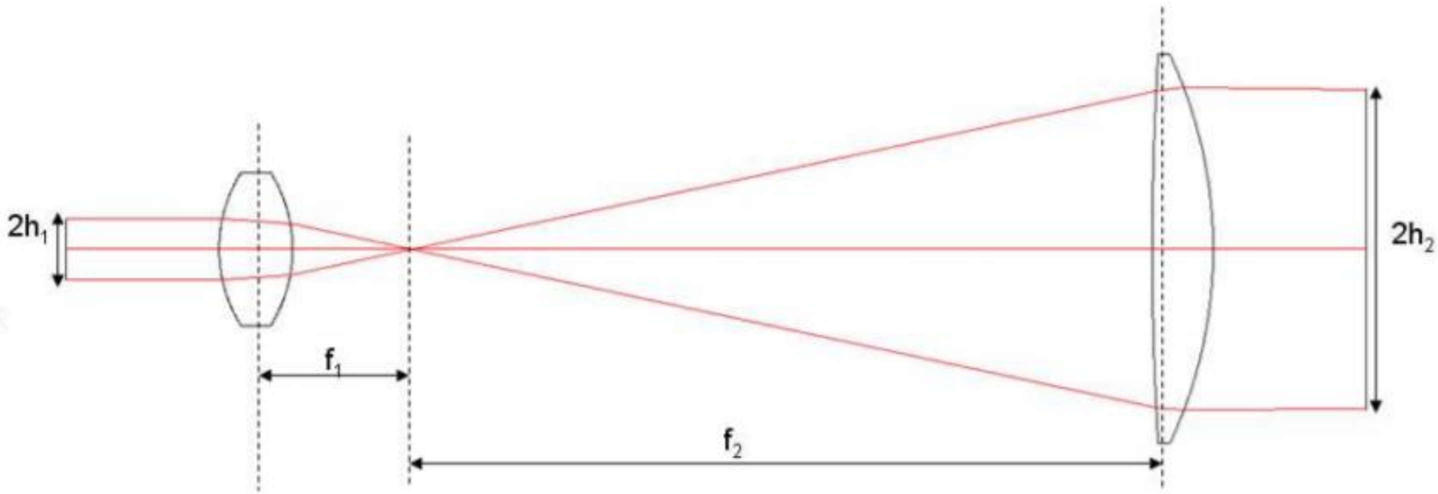
$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}$$

Beam Expander

Click for Details
The 2X magnfication beam expanders have an externally SM1-threaded output.

Click for Details
The 3X and 5X magnfication beam expanders share the same housing dimensions. Both versions have an M43 x 0.5-threaded output.

$$t = f_1 + f_2$$

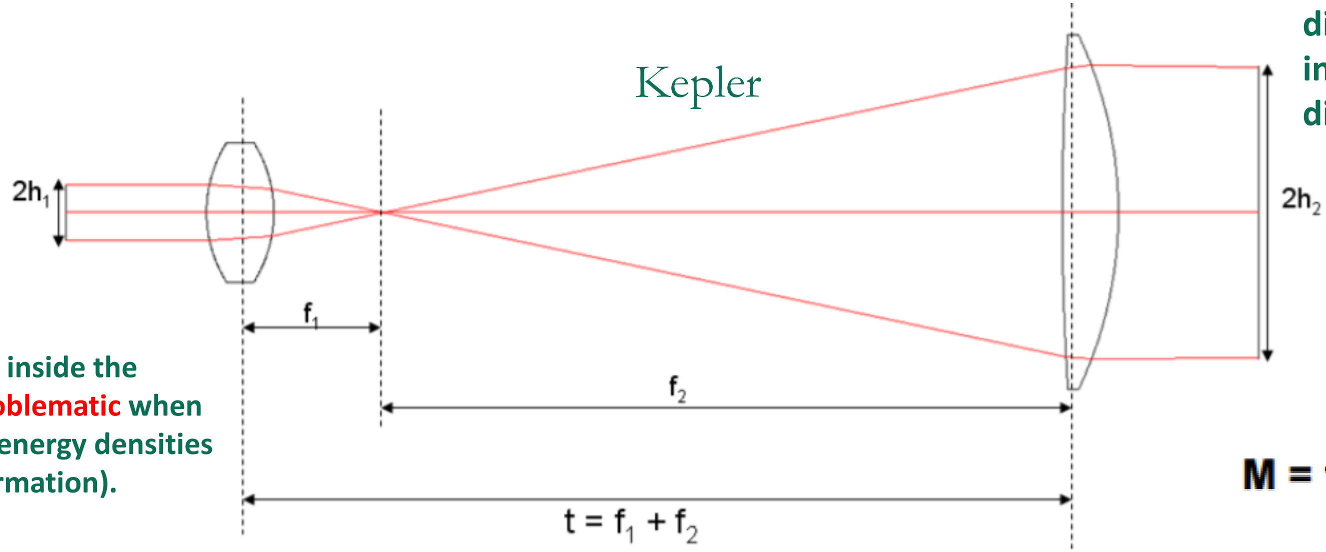


$$M = f_2/f_1 = R_2/R_1 = h_2/h_1$$

https://www.thorlabs.com/newgrouppage9.cfm?objectgroup_id=2980

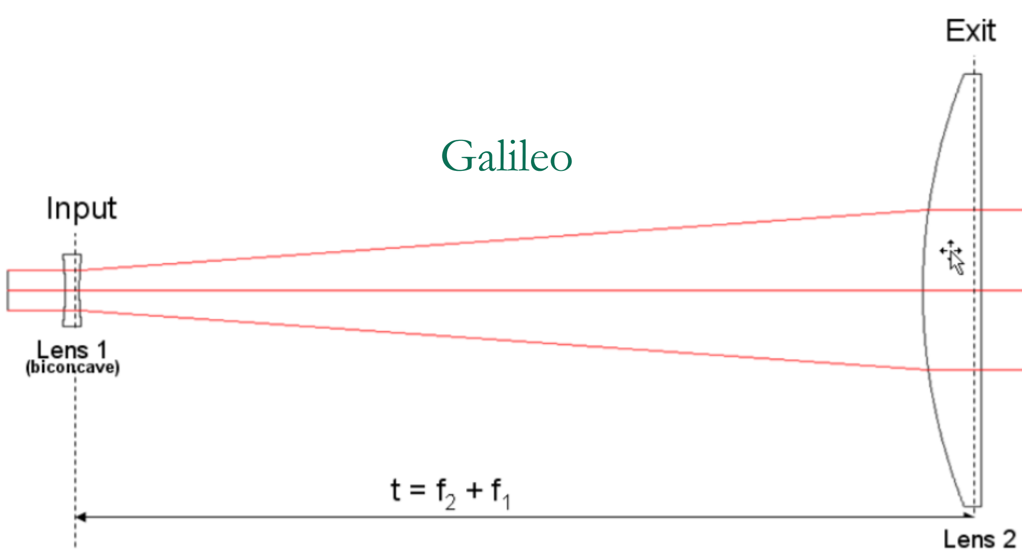
Kepler and Galileo configurations

Choice of lenses should consider the useful diameter (80%) and initial and final beam diameters.



Focal point inside the system. Problematic when using high energy densities (plasma formation).

$$M = f_2/f_1 = R_2/R_1 = h_2/h_1$$



No focal point inside system!

$$M = -f_2/f_1$$

Bibliography and multimedia

2002, Óptica, Eugene Hecht, Fundação Calouste Gulbenkian. **Capítulos 5**

Websites additional information and examples:

http://hyperphysics.phy-astr.gsu.edu/hbase/phyopt/reflex.html

http://research.vuse.vanderbilt.edu/bmeoptics/bme285/mainframes/module3/module3sec3.htm

http://demonstrations.wolfram.com/FresnelEquations/

http://www.physicsclassroom.com/class/refln

http://www.lessonpaths.com/learn/i/o-estudo-de-espelhos-esfricos/geogebratube

http://www.tutorvista.com/content/science/science-ii/reflection-light/formation-plane-mirror.php

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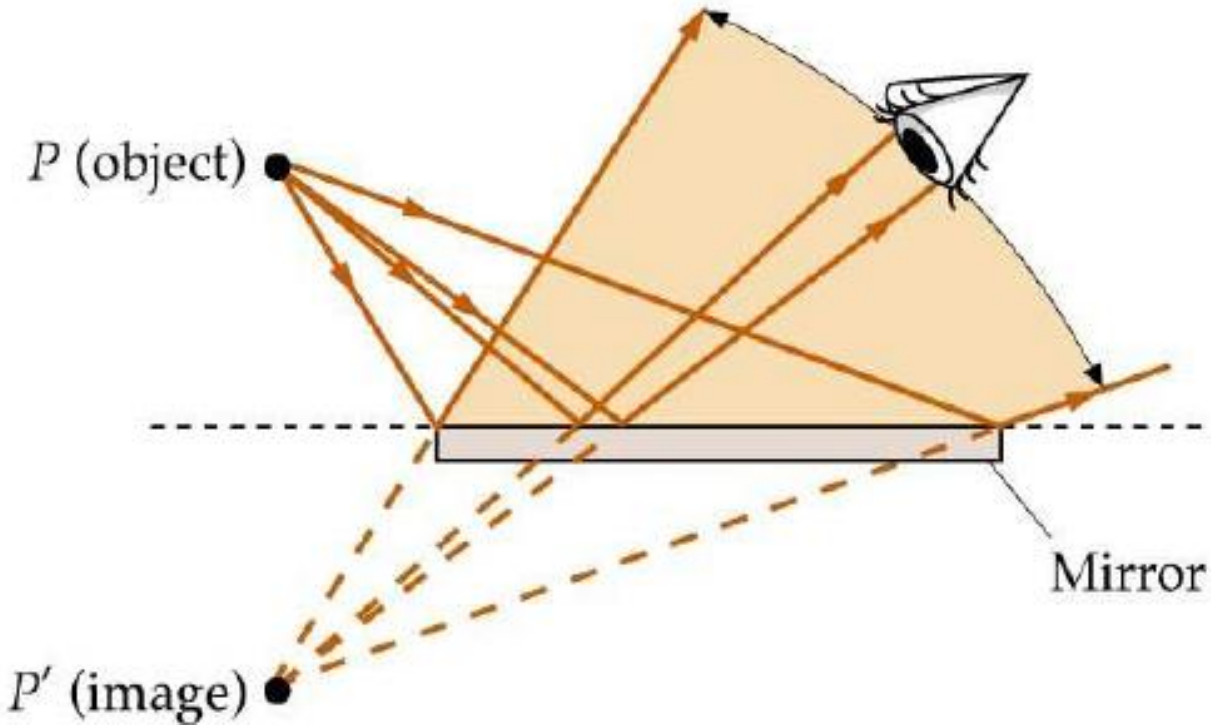
https://www.newport.com/n/gaussian-beam-optics

			Tsia chapts	Niemz	Hecht
18/2	Semana 2	Gaussian optics: laser beam manipulation and calculations. Optical systems for beam maniipulation. **Geometric optics basics.image formation, beam expander;**	1.8,1.2.1 .3 1.9, 7		5.1-2, 5.2.3, 5.4.2, 5.6, 5.7.1-2

Aditional Information (Mirrors)

Optical devices– Plane Mirrors

- Plane Mirrors

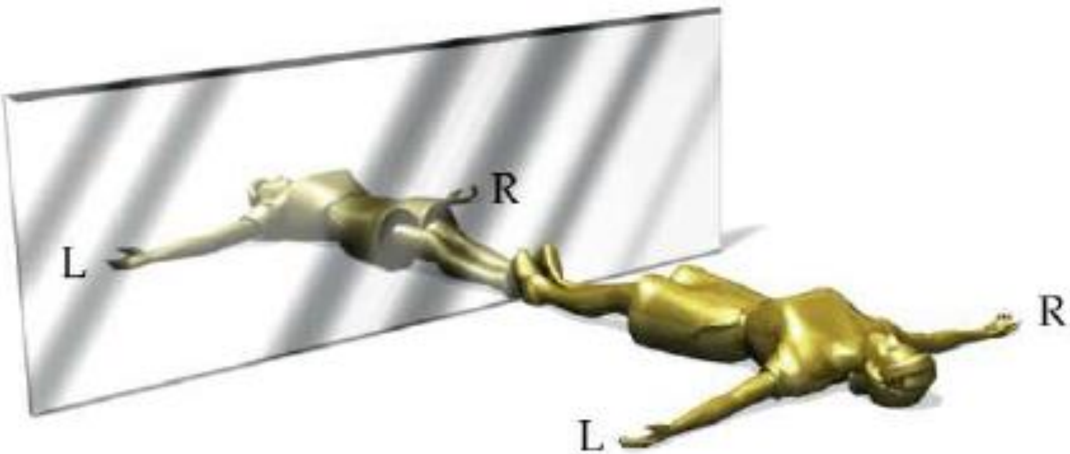


The image is said to be virtual, because light only appears to be coming from P'.

Surfaces with high reflectivity. Polished Metals, thin metallic film (aluminum, silver…)

Optical devices– Plane Mirrors

- Depth Inversion



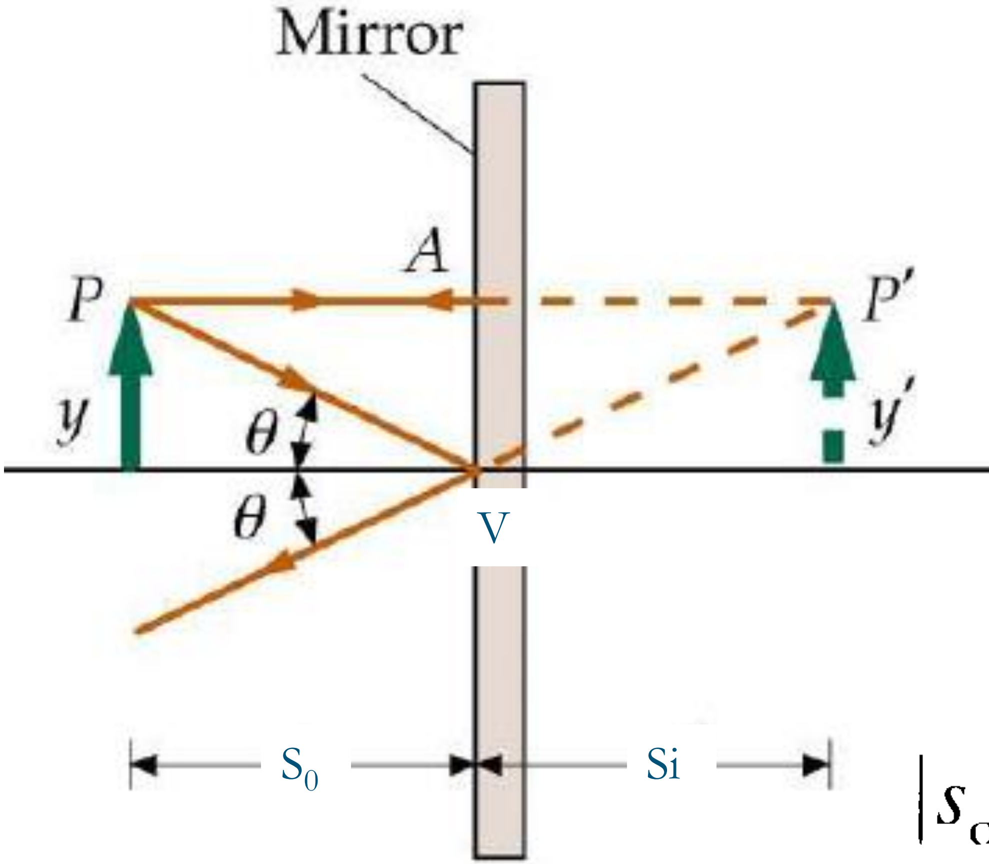
The right-left inversion of the hand, results from depth inversion..

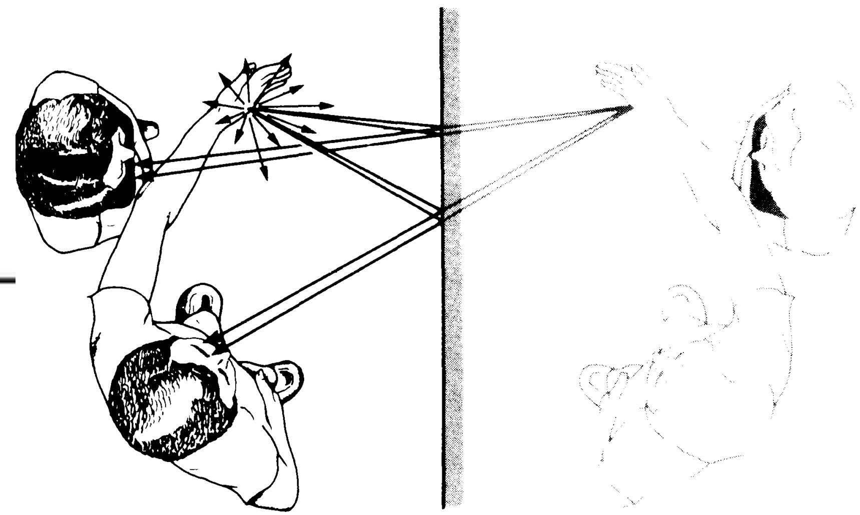
The plane mirror transforms a right coordinate axis $(\hat{i}\times\hat{j}=\hat{k})$ Into a left coordinate axis $(\hat{i}\times\hat{j}=-\hat{k})$

Optical devices– Plane Mirrors

- Ray tracing for image location

$$\theta_i = \theta_r$$





$$|s_o| = |s_1| \qquad M_T \equiv \frac{y_i}{y_o} = 1$$

From the scheme: i) the image is at the same distance behind the mirrors, as the object is from its surface; ii) the image is upright (same as the object) and has the same size has the object.

Optical devices– Plane Mirrors

- Ray tracing for image location

$\mathbf{P_1'}$ é a imagem do objecto **P** no espelho 1;

$\mathbf{P_2'}$ é a imagem do objecto **P** no espelho 2;

$\mathbf{P_{1,2}''}$ é a imagem de $\mathbf{P_1'}$ no espelho 2, vista quando os raios vindos do objecto são reflectidos primeiro no espelho 1 e depois no espelho 2;

A imagem $\mathbf{P_2'}$ não tem imagem no espelho 1 porque está atrás desse espelho.

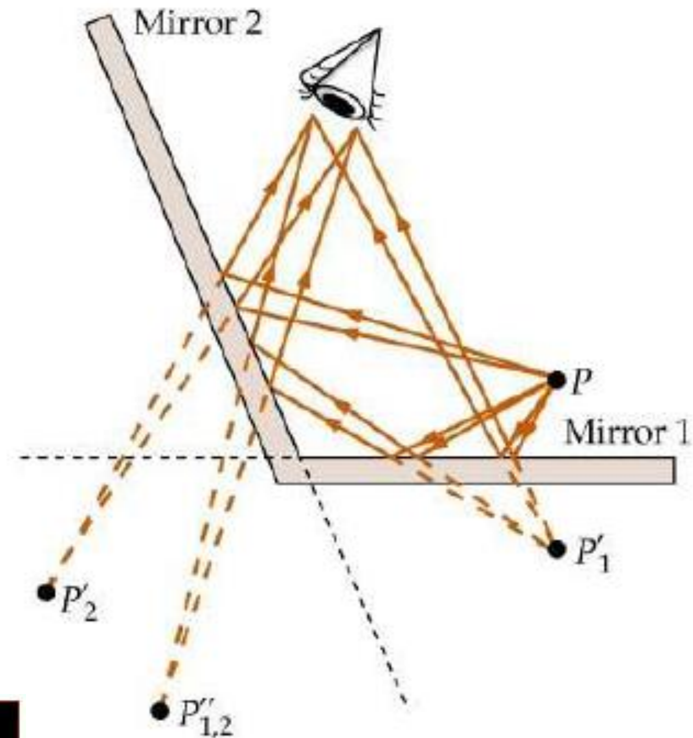
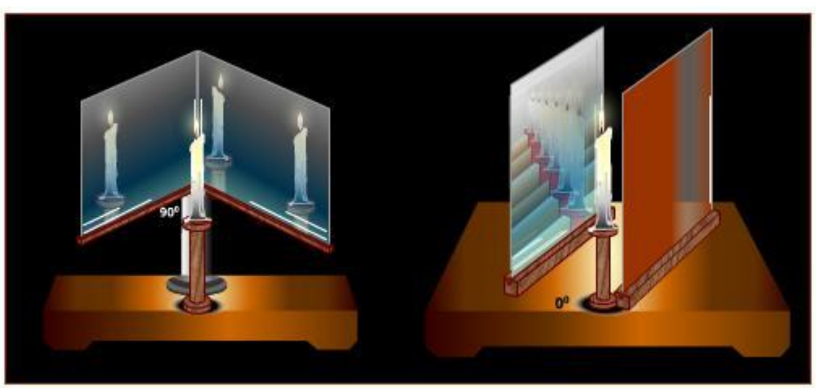


Imagem formada por 2 espelhos planos



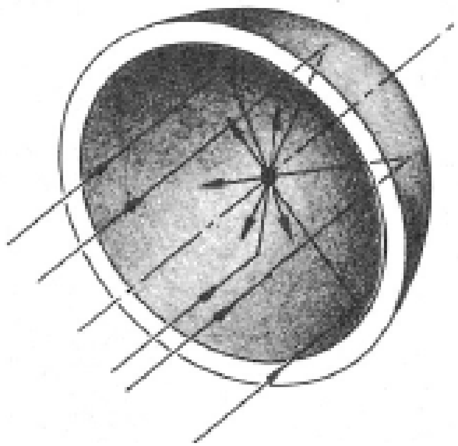
$$n_{\,images} = \frac{360}{\theta} - 1$$

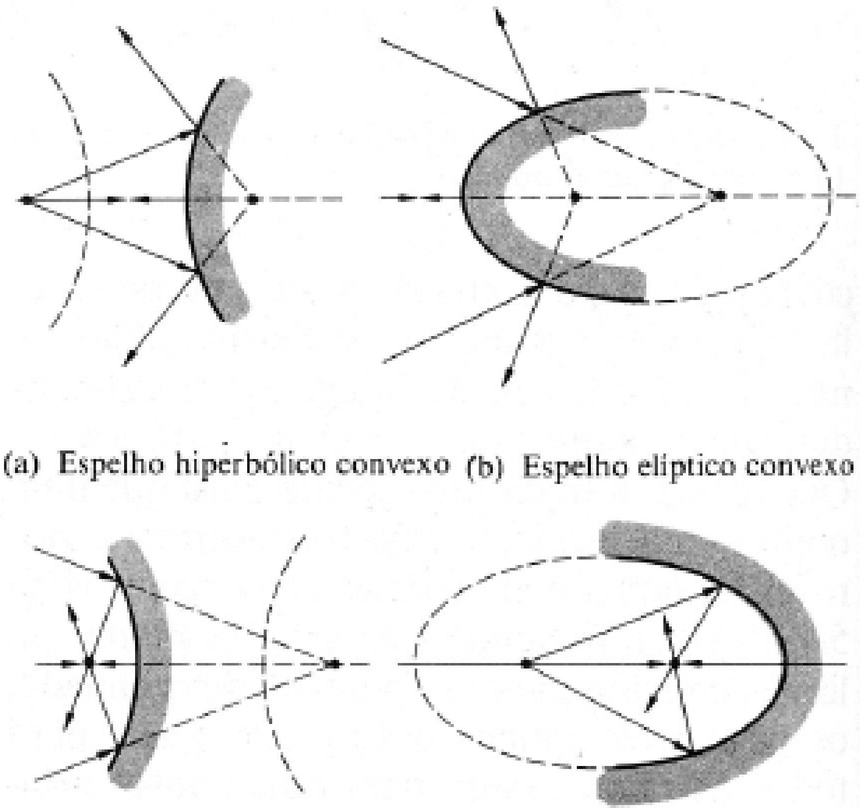
Optical devices – curved mirrors

- Aspherical mirrors

Espelhos baseados em cónicas também focam raios paralelos para um determinado ponto: parabolóides, hiperbólicos e elípticos. Designam-se por **espelhos asféricos**.

Espelho paraboloidal

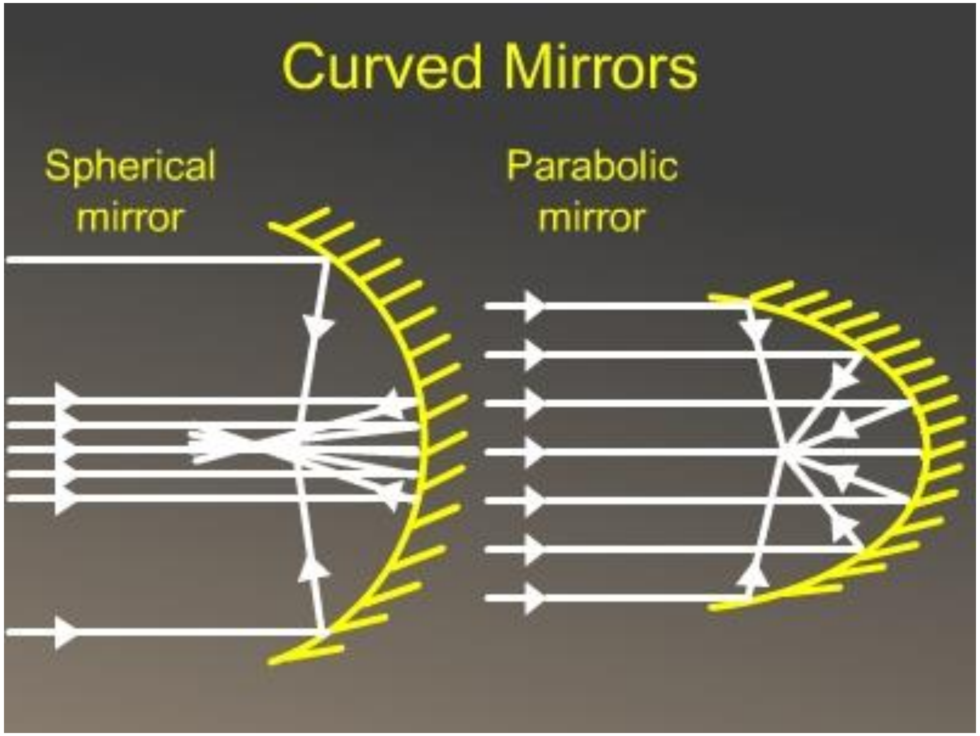


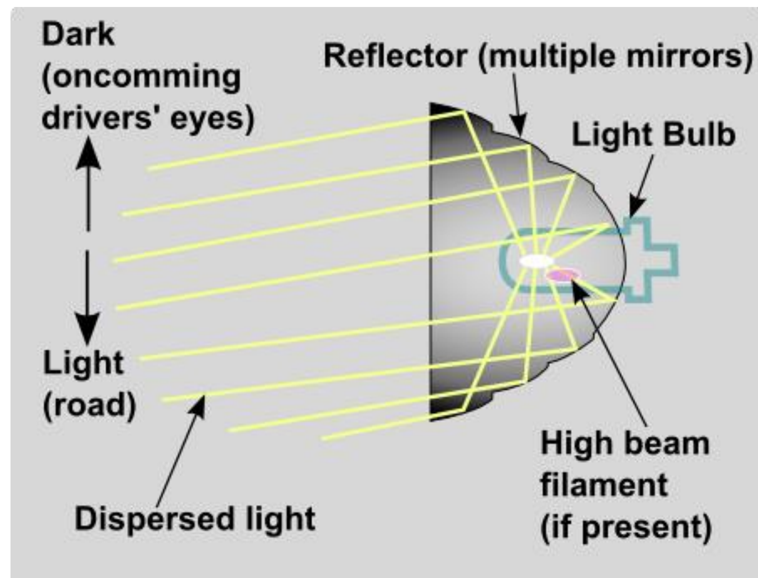


(a) Espelho hiperbólico convexo (b) Espelho elíptico convexo

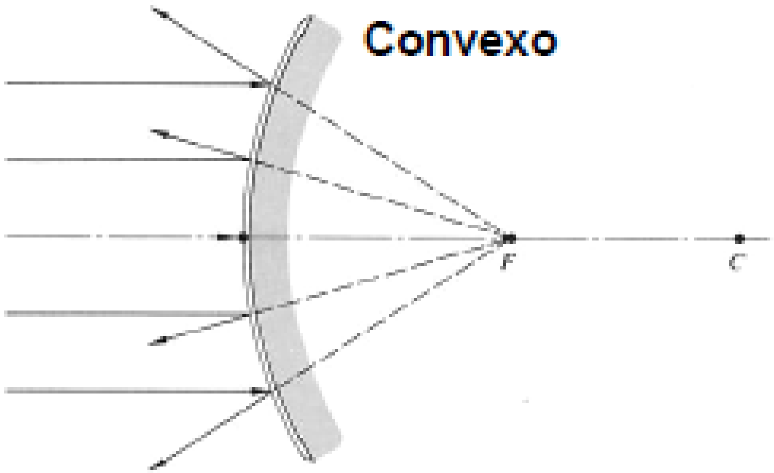
(c) Espelho hiperbólico côncavo (d) Espelho elíptico côncavo

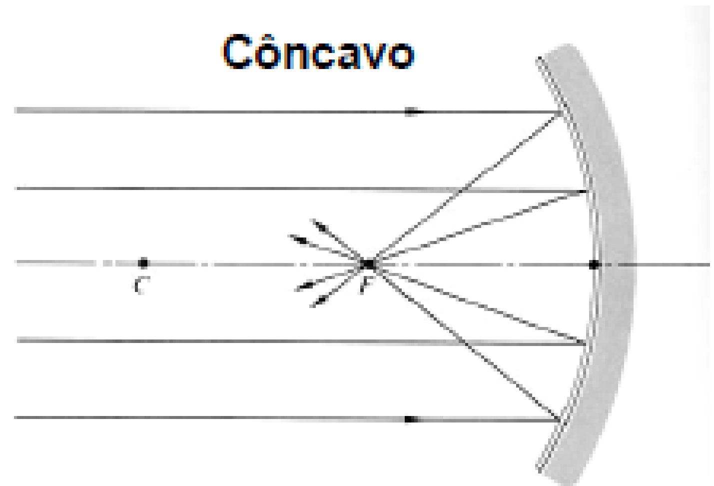
Optical devices– Aspherical Mirrors





Optical devices– spherical mirrors





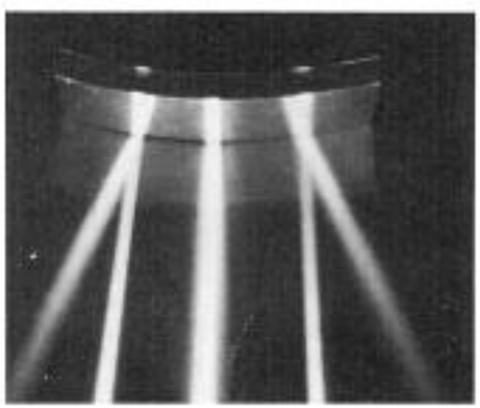


Imagem Virtual

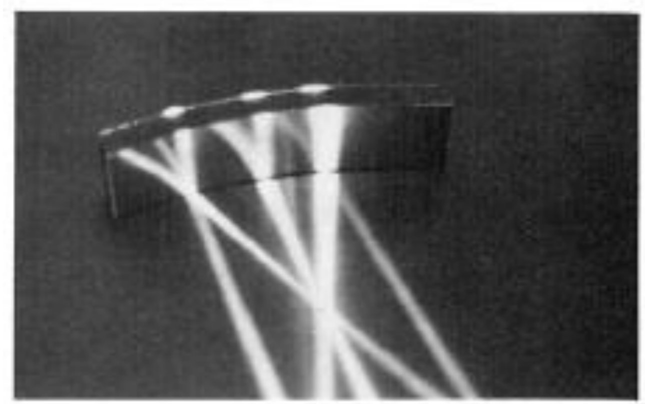
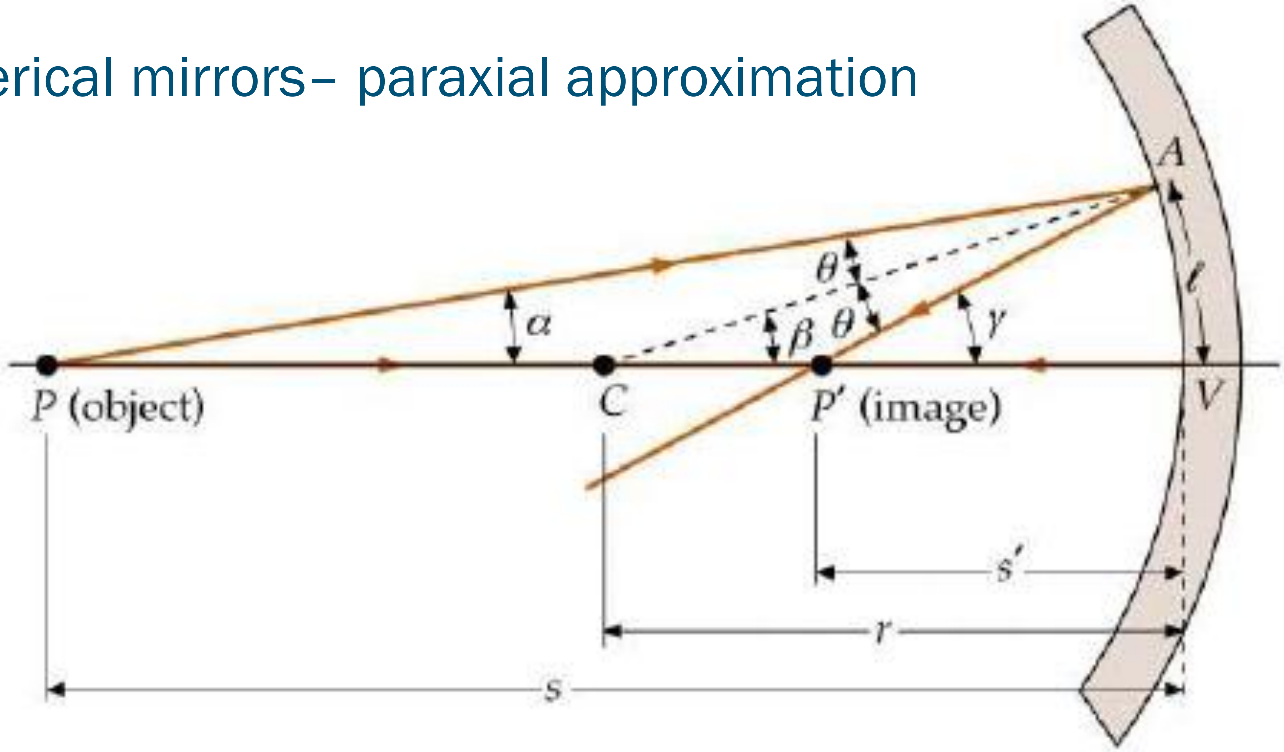


Imagem Real

Optical devices– spherical mirrors

- Spherical mirrors– paraxial approximation



$$\boxed{\frac{1}{s}+\frac{1}{s'}=\frac{2}{r}}$$

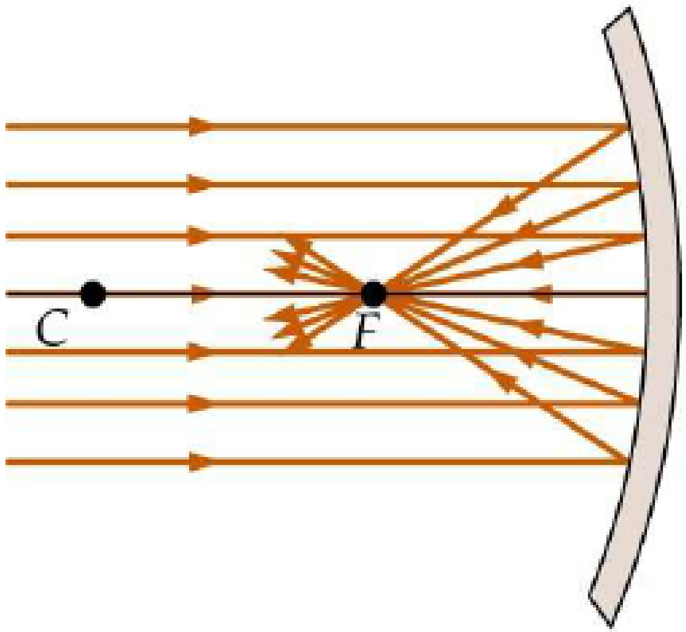
r < 0 ⇒ espelho côncavo

r > 0 ⇒ espelho convexo

Optical devices– spherical mirrors

Se a distância objecto $s = \infty$ $\Rightarrow$ distância imagem $\underbrace{s' = r/2}$

Esta distância designa-se por ***distância focal f***



f < 0 ⇒ espelho côncavo

f > 0 ⇒ espelho convexo

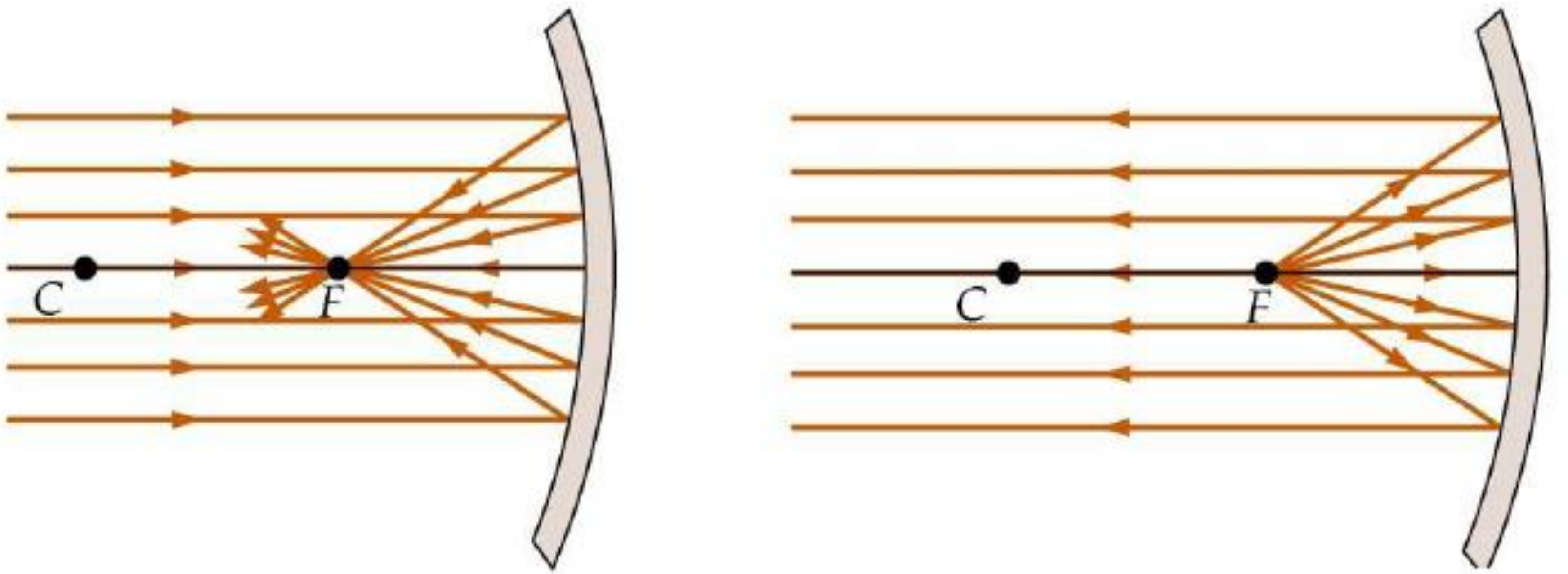
$$\boxed{\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}}$$

Equação dos espelhos esféricos

O ponto no qual os raios paralelos incidentes no espelho convergem designa-se por ***foco F***.

Dispositivos Óticos - Espelhos

- Princípio da Reversibilidade



Raios paralelos incidentes no espelho após reflexão convergem num ponto – foco F.

Raios divergindo de uma fonte pontual no foco F são reflectidos no espelho como raios paralelos.

O percurso dos raios luminosos é o mesmo mas com direcções invertidas.

Optical devices–mirrors

- Ray tracing

Para a construção das imagens usam-se 3 **raios principais:**

1. O **raio paralelo**

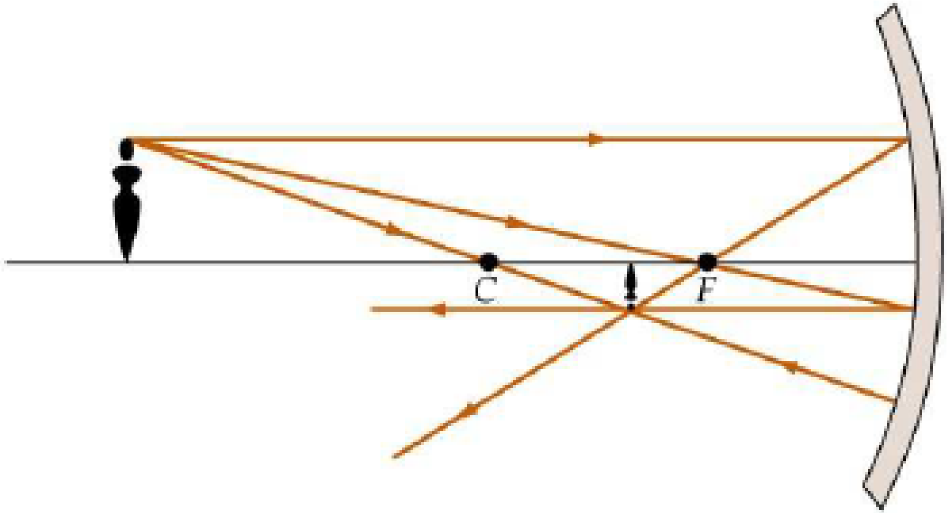
   Desenhado paralelo ao eixo óptico, é reflectido através do foco.

2. O **raio focal**

   Desenhado através do foco, é reflectido paralelamente ao eixo.

3. O **raio radial**

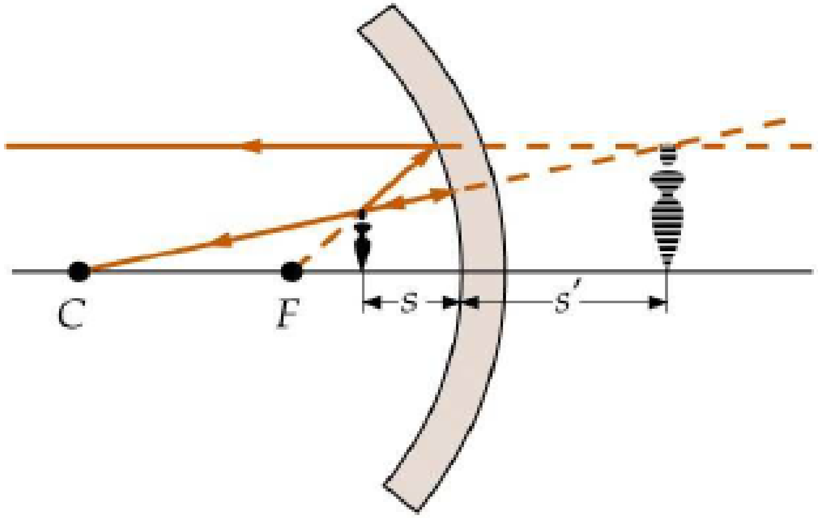
   Desenhado através do centro de curvatura C, incide perpendicularmente ao espelho e é reflectido na mesma direcção.

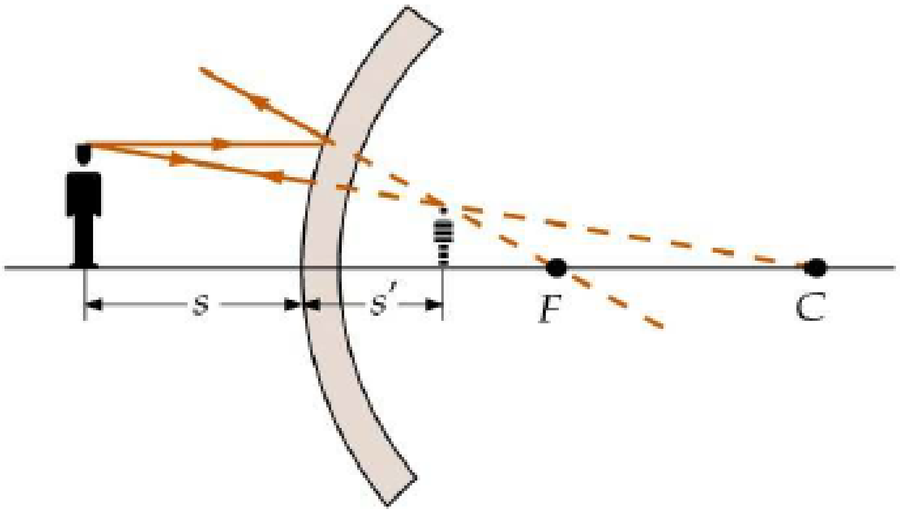


A intersecção de qualquer destes dois raios localiza o ponto imagem do extremo superior do objecto.

Optical devices- mirrors

- Ray tracing

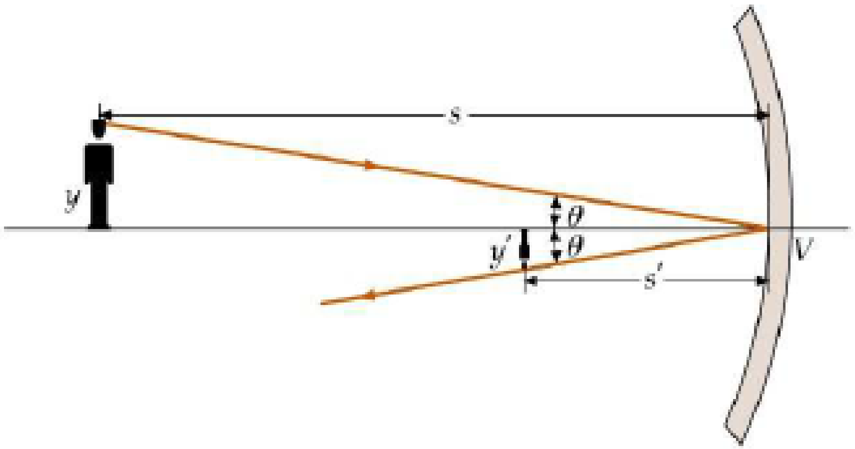




Ampliação lateral ou transversa

A razão entre as dimensões transversas da imagem e do objecto é a **ampliação lateral** ou **transversa M_T** :

$$M_T = \frac{y'}{y} = -\frac{s'}{s}$$



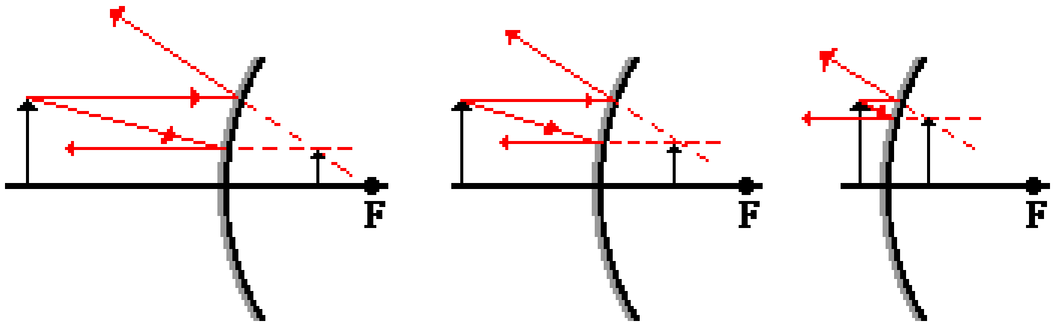
Optical devices– spherical mirrors

- Sign convention for mirrors

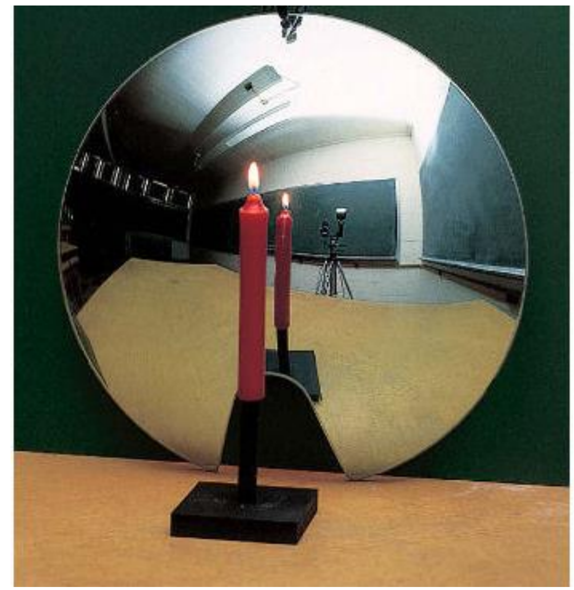
GRANDEZA	SINAL	
	+	-
s	à esquerda de V, objecto real	à direita de V, objecto virtual
s'	à esquerda de V, imagem real	à direita de V, imagem virtual
f	espelho côncavo	espelho convexo
R	C à direita de V, convexo	C à esquerda de V, côncavo
y	acima do eixo, objecto não invertido	abaixo do eixo, objecto invertido
y'	acima do eixo, imagem não invertida	abaixo do eixo, imagem invertida

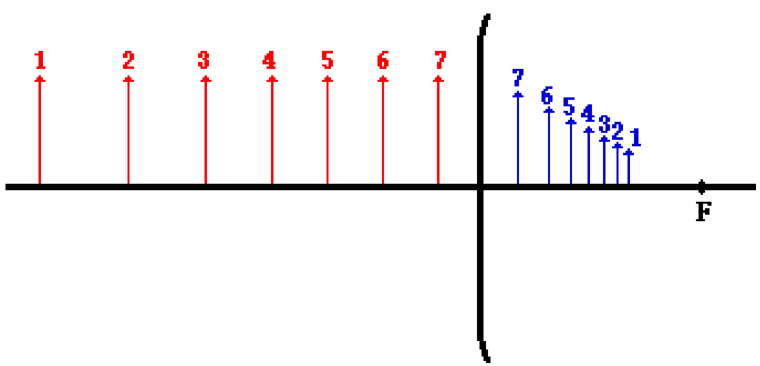
Optical devices– convex mirrors

- Images of real objects formed by spherical mirrors



Upright image (not inverted) ; reduced M<1, virtual



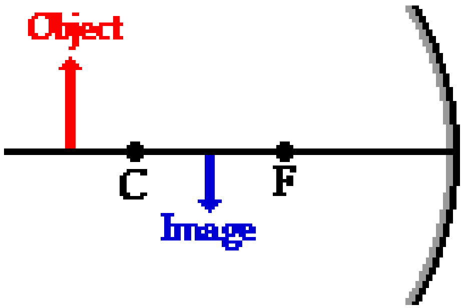


$$\frac{1}{s_o} + \frac{1}{s_i} = \frac{1}{f}$$ f negative

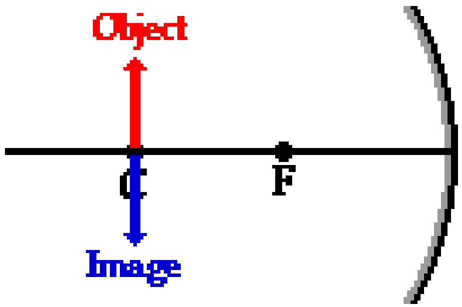
$$M_T \equiv \frac{y_i}{y_o} \qquad M_T = -\frac{s_i}{s_o}$$

Optical devices– concave mirrors

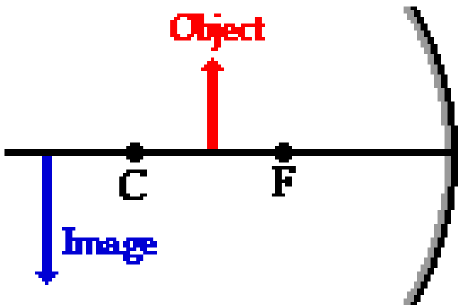
- Images of real objects formed by spherical mirrors



Inverted image; reduced M<1, real



Inverted image; original size M=1, real



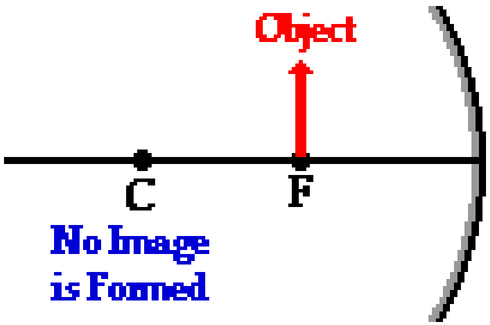
Inverted image; magnified M>1, real

$$\frac{1}{s_o} + \frac{1}{s_i} = \frac{1}{f}$$ f positive

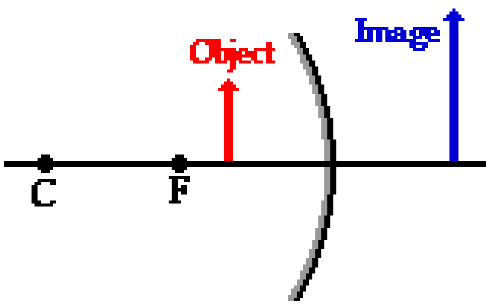
$$M_T \equiv \frac{y_i}{y_o} \qquad M_T = -\frac{s_i}{s_o}$$

Optical devices– concave mirrors

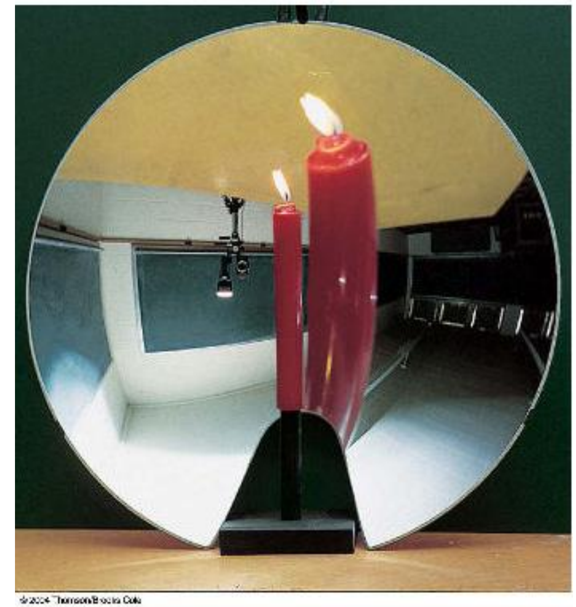
- Images of real objects formed by spherical mirrors

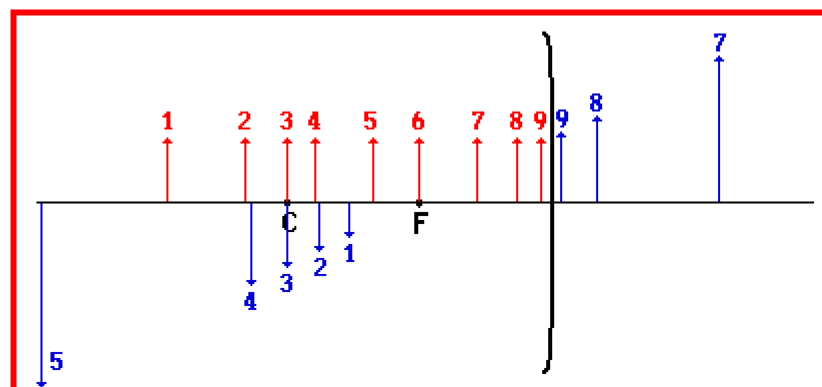


No image; colimated beam converges at ∞



Uprigth imagem (not inverted); magnified M>1, virtual





http://www.physicsclassroom.com/class/refln/Lesson-3/Image-Characteristics-for-Concave-Mirrors

Optical devices - mirrors

- Image of real objects formed by spherical mirrors

CÔNCAVO				
OBJECTO	IMAGEM			
Localização	Tipo	Localização	Orientação	Tamanho Relativo
∞ > s > 2f	Real	f < s' < 2f	Invertida	Reduzida
s = 2f	Real	s = 2f	Invertida	Igual
f < s < 2f	Real	∞ > s' > 2f	Invertida	Aumentada
s = f	Real	± ∞		
s < f	Virtual	| s' | > s	Direita	Aumentada

CONVEXO				
OBJECTO	IMAGEM			
Localização	Tipo	Localização	Orientação	Tamanho Relativo
Qualquer	Virtual	| s' | < | f | s > | s' |	Direita	Reduzida