AI 512 Part 1: Assignment -2 (10 Marks)

Q1 Eigenvalues and eigenvectors play an important part in the applications of linear algebra. The naive method of finding the eigenvalues of a matrix involves finding the roots of the characteristic polynomial of the matrix. In industrial sized matrices, however, this method is not feasible, and the eigenvalues must be obtained by other means. Fortunately, there exist several other techniques for finding eigenvalues and eigenvectors of a matrix, some of which fall under the realm of iterative methods. These methods work by repeatedly refining approximations to the eigenvectors or eigenvalues, and can be terminated whenever the approximations reach a suitable degree of accuracy. Iterative methods form the basis of much of modern day eigenvalue computation.

- Read and Understand the iterative methods discussed in the attached document **Power_Iteration_Methods.pdf**

- Create a 10x10 random matrix and convert it to a symmetric matrix (use python programming). Find the eigen values and eigenvectors of the matrix with simultaneous iteration method discussed in Section 3.4 of the document **Power_Iteration_Methods.pdf** . Obtain the eigen values using standard library packages and plot the eigenvalue convergence for top -5 eigenvalues using simultaneous iteration method .

(X-axis to capture iterations and Y-axis to show absolute difference between eigenvalues obtained using standard library and iteration method.)

Attach the python file along with the report.

5 Marks

Q2

Laplacian matrix of a graph. Let A be the incidence matrix of a directed graph with n nodes and m edge . The *Laplacian matrix* associated with the graph is defined as $L = AA^T$, which is the Gram matrix of A^T. It is named after the mathematician Pierre-Simon Laplace.

(a) Show that $\mathcal{D}(v) = v^T Lv$, where $\mathcal{D}(v)$ is the Dirichlet energy .

(b) Describe the entries of L. *Hint.* The following two quantities might be useful: The *degree* of a node, which is the number of edges that connect to the node (in either direction), and the number of edges that connect a pair of distinct nodes (in either direction).

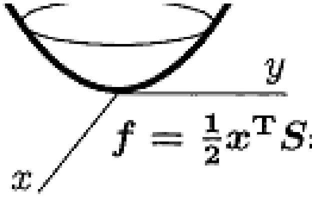
Refer to **Assign2Q2.png** for definitions of directed graph and Dirichlet Energy

3 Marks

Q3

Second derivatives $S = \begin{bmatrix} a & b \\ b & c \end{bmatrix}$

$a > 0$ and $ac > b^2$



$$f = \tfrac{1}{2}\boldsymbol{x}^{\mathrm{T}} S \boldsymbol{x} > 0$$

The graph of $2f = ax^2 + 2bxy + cy^2$ is a bowl when S is positive definite.

If S has a negative eigenvalue $\lambda < 0$, the graph goes below zero. There is a *maximum* if S is negative definite (all $\lambda < 0$, upside down bowl). Or a *saddle point* when S has both positive and negative eigenvalues. A saddle point matrix is "*indefinite*".

The energy $\boldsymbol{x}^{\mathrm{T}} S \boldsymbol{x} = 2x_1x_2$ certainly has a saddle point and not a minimum at $(0,0)$. What symmetric matrix S produces this energy? What are its eigenvalues?

2 Marks