

# Ionospheric Time-Delay Algorithm for Single-Frequency GPS Users

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**The goal in designing an ionospheric time-delay correction algorithm for the single-frequency global positioning system user was to include the main features of the complex behavior of the ionosphere, yet require a minimum of coefficients and user computational time, while still yielding an rms correction of at least 50 percent. The algorithm designed for this purpose, and implemented in the GPS satellites, requires only eight coefficients sent as part of the satellite message, contains numerous approximations designed to reduce user computational requirements, yet preserves the essential elements required to obtain group delay values along multiple satellite viewing directions.**

The U.S. Department of Defense has developed an advanced satellite navigation system, called the NAVSTAR global positioning system (GPS), which will allow users to measure range and range rate information simultaneously from four satellites to determine user position and velocity [1–5]. The primary satellite system operating frequency is 1.575 GHz. Even at this relatively high frequency, the Earth's ionosphere can retard radio waves from their velocity in free space by more than 300 ns on a worst case basis, corresponding to range errors of 100 m. Even monthly average diurnal maximum ionospheric time-delay values can be as high as 150 ns, or 50 m of range error, when a GPS satellite is observed at low elevation angles.

In order to nearly eliminate the effects of the Earth's ionosphere on users of the GPS, a secondary frequency, 1.227 GHz, was specifically incorporated into the system to allow users to automatically correct for the effects of both the range and range rate errors induced by the ionosphere. Some users of the GPS, however, may not have a requirement for nearly complete, automatic correction for ionospheric range and range rate errors as the two frequency GPS provides. For these users an algorithm was designed, using eight coefficients, transmitted as part of the satellite message, to provide a correction for approximately 50 percent rms of the ionospheric range error. Corrections for ionospheric range rate errors for a single frequency user are not practical by modeling techniques, due to the impossibility of predicting, except in a statistical manner, the small undulations in the ionosphere which produce range rate errors on time scales of a few seconds to minutes. The goal of a 50 percent rms correction for the ionospheric algorithm was arrived at somewhat arbitrarily as a compromise between number of coefficients required to be sent as part of the satellite message and the realization that even a state of the art ionospheric model, requiring many coefficients, would provide only a 70 to 80 percent rms correction to the ionospheric time delay. In some comparisons of the algorithm against actual ionospheric total electron content (TEC) data presented here, and additional comparisons given by Feess and Stephens [6], it is shown that the goal of a 50 percent rms correction for ionospheric time delay by use of the algorithm described here has been met.

## CHARACTERISTICS OF IONOSPHERIC TIME DELAY

The important parameter responsible for ionospheric time delay is the total number of electrons encountered by the radio wave on its path from each satellite to the GPS system user. This TEC is a function of many variables, including long and short term changes in solar ionizing flux, magnetic activity, season, time of day, user location and viewing direction. An example of the geographic variation of monthly average vertical time delay at the

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primary GPS system operating frequency  $L1$  is shown in Fig. 1 for 2000 UT. At this time of day the maximum values of ionospheric time delay occur in the American longitude sector. As illustrated in Fig. 1, the world-wide average maximum values of TEC do not occur at the geographic equator, but at latitudes approximately plus and minus  $15^\circ$  on either side of the geomagnetic equator. Note that the time delay contours plotted in Fig. 1 are for a satellite at the zenith. At the minimum design elevation angle for the GPS system of  $5^\circ$  the ionospheric time delay values will be approximately three times higher than those shown in Fig. 1.

Many studies have been made of the behavior of TEC, and the characteristics of this ionospheric parameter are now fairly well known, especially in the midlatitude regions of the world, where most single-frequency GPS users are likely to be located. The behavior of TEC in the equatorial and high latitudes is less well known. Basically, the TEC behavior in many parts of the world has a diurnal maximum near 1400 hr local time, with the standard deviation of the TEC about the monthly average value for any given daytime hour generally between 20 and 25 percent of the mean value. The midlatitude ionosphere generally has smoother latitudinal gradients than either the low or high latitude regions; thus, a single-frequency GPS user, in the midlatitudes, has the additional advantage of both a smoother and more well studied ionosphere!

With these general facts in mind, an algorithm was designed to give best fit to the large daytime values of monthly average TEC, and to accept any difference from monthly average TEC behavior as part of the residual error for a GPS system single-frequency user. Since the

correlation distance [7] and correlation time [8] of TEC deviations from monthly average conditions are small, and each GPS satellite message is normally updated no more often than once each day, no attempt was made to make corrections for short term, of the order of a few hours, variability of the ionospheric TEC. Also, since the correlation of TEC daily values with day-to-day solar flux values is low, no attempt was made to update the algorithm on a daily basis.

## FORM OF THE IONOSPHERIC TIME-DELAY ALGORITHM

The algorithm developed is a compromise of several factors. These include 1) user computational complexity, 2) present state of knowledge of temporal, diurnal, and geographic variations of TEC, 3) number of coefficients available in the satellite message for the ionospheric correction algorithm, and 4) likely single-frequency system user geographic operational areas.

The algorithm, incorporated in the GPS system for single-frequency users, consists of a cosine representation of the diurnal curve, allowed to vary in amplitude and in period, with user latitude. Fig. 2 shows how a simple, positive half cosine-shaped curve has been made to fit a typical monthly average TEC diurnal variation from the station on Jamaica.

Fig. 2 illustrates the four potential parameters to this simple cosine form of time-delay algorithm for the single-frequency GPS user. They are 1) the night-time constant, or dc, term, 2) the amplitude of the cosine term, 3) the phase of the cosine term, and 4) the period of the cosine term. Note that the period of the rms best fit is not 24 hr,

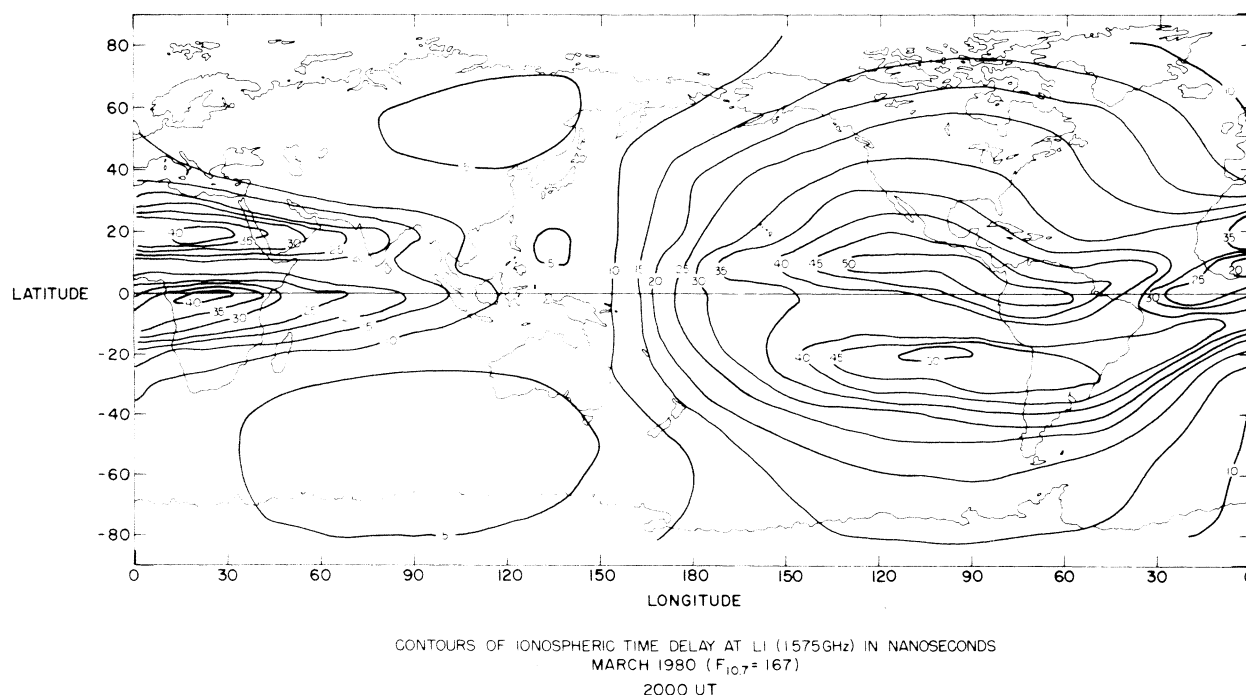


Fig. 1. Contours of monthly average ionospheric time delay, units of nanoseconds at 1.6 GHz.

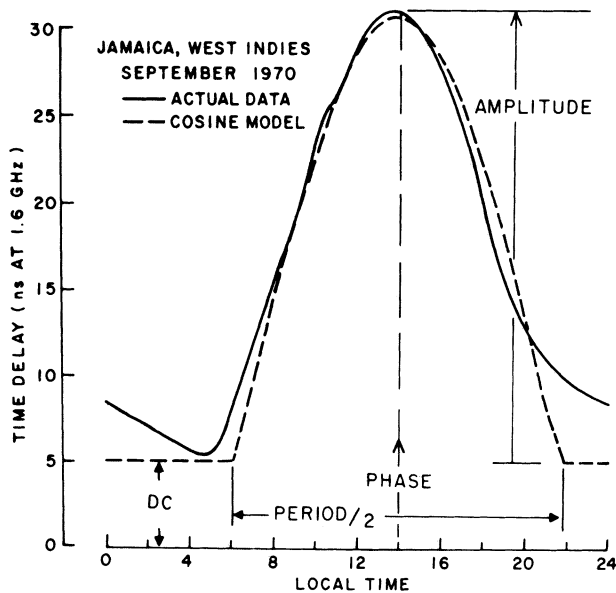


Fig. 2. Example of actual monthly average time-delay data, along with cosine model fit to data.

but is whatever period is required to minimize the rms difference between the actual monthly mean diurnal curve and the cosine fit. This equivalent period is generally significantly longer than 24 hr.

A study of the behavior of the dc and phase over a wide range of latitudes showed that both these terms could be made constant with little overall algorithm rms error increase. Thus, the dc term was made a constant 5 ns and the phase was set at a constant 1400 hr local time.

The remaining two parameters, the amplitude of the cosine term and its period, are functions of latitude, ordered by geomagnetic, rather than geographic, latitude, and are represented in the algorithm by third degree polynomials. The coefficients of these polynomials are transmitted as part of the satellite message. The algorithm coefficients were computed from an empirical model of world-wide ionospheric behavior derived by Bent [9], for each 10 day period of the year and for several values of average solar flux conditions. Coefficients transmitted by the GPS satellites are updated once each ten days, or sometimes more frequently, if the five day running mean solar flux changes by a large amount during that period of time.

Simple cubic fits to both the amplitude and the period terms are not sufficient to adequately represent the actual gradients in the equatorial anomaly region. However, the variability of the equatorial anomaly location and its magnitude are large, not well represented in the original Bent model from which the algorithm coefficients were derived, and, few actual TEC measurements were available in this latitude region, when the Bent model was constructed. Therefore, no attempts were made to use an a priori shape in the algorithm for the TEC behavior in the low latitude ionosphere.

## GEOMETRIC APPROXIMATIONS USED IN THE IONOSPHERIC TIME-DELAY ALGORITHM

The actual cosine representation of ionospheric time delay is much simpler than the geometrical calculations required to find the appropriate geographic location, time, and zenith angle for the required time-delay algorithm calculation. Since the GPS satellites are nearly always viewed at oblique elevation angles, the overhead ionosphere is not the appropriate one for a single-frequency observer to use. The TEC must be found at the geographic point where the ray path from each satellite intersects the mean ionospheric height, rather than at the user location. This point is taken at a mean vertical height of 350 km along the path to each satellite. The obliquity, or slant factor, also must be calculated for the mean ionospheric location, and the vertical TEC obtained from the algorithm at the mean ionospheric location must be multiplied by this slant factor. Finally, since the TEC is best ordered in geomagnetic, rather than geodetic coordinates, a conversion to geomagnetic latitude also is required. If the exact form of the calculations for all the necessary geometry were to be used, the computational assets would be excessive. Therefore, in the derivation of these geometrical functions, simplifying assumptions were made in many cases to reduce the calculation complexity. A discussion of each of the steps required in the geometrical calculations, and the simplifying assumptions made, follows.

### Earth Angle

The TEC is computed at the point where the ray from the satellite intersects the ionosphere, rather than at the observer. For an observer located near the earth's surface, looking at a satellite at 5° elevation, the ionospheric intersection location is approximately 14° of earth-entered angle from the observer. Since the gradient in TEC can be large over Earth angles much less than 14° we must find the actual point where the satellite intersects the ionosphere, rather than simply using the observer's location. The exact Earth angle representation, as well as the approximate form, are shown in Fig. 3, versus satellite elevation angle. The approximate form is less than 0.2° in error for all elevation angles greater than 10°, and is only 0.4° and 0.3° from the exact form at 5° and 0° elevation, respectively. Now that we have a useful form of the Earth angle we can compute the coordinates of the ionospheric intersection.

### Computing the Mean Ionospheric Location

The actual form of the equations to obtain the ionospheric intersection coordinates, as well as the flat-Earth approximation used, are shown in Fig. 4. Fortunately, at the low and mid-latitudes, any errors in using the approximate form to calculate the ionospheric intersection coordinates is small. To keep away from the

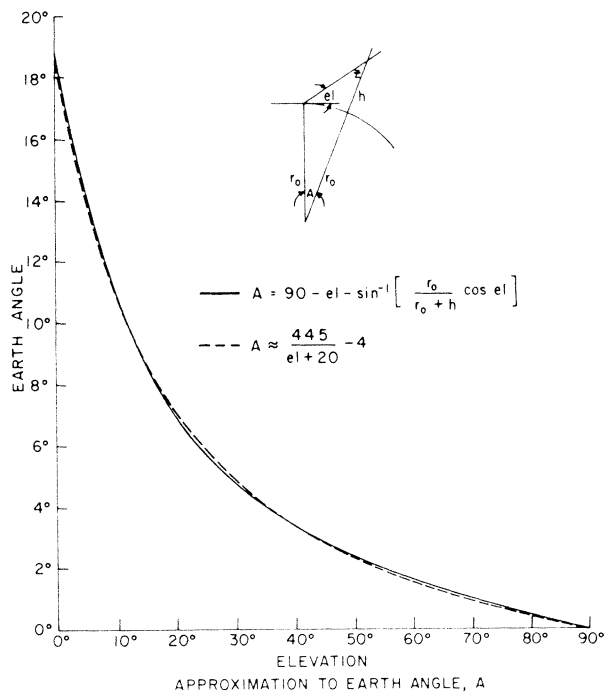


Fig. 3. Approximation to computing Earth angle, A.

large errors this approximation would yield at the extremely high latitudes, if the ionospheric latitude is calculated to be greater than 75°, the latitude is defined as 75°. This approximation is reasonable because little single-frequency user activity is expected above 75° latitude, and the TEC is generally low at those very high latitudes. The few actual TEC measurements available in the polar cap region show high variability, which is not well represented by currently available ionospheric models.

### Conversion from Geodetic to Geomagnetic Latitude

Ionospheric time delay is best ordered by geomagnetic, rather than geographic, latitude. Thus, it is necessary to perform a transformation from geodetic to

geomagnetic latitude. This transformation from geodetic to geomagnetic latitude, assuming that the earth's magnetic field can be represented by an Earth-centered dipole, is given by

$$\sin \Phi = \sin \phi \sin \phi_p + \cos \phi \cos \phi_p \cos(\lambda - \lambda_p)$$

where

$$\phi_p = 78.3^\circ \text{ N}$$

$$\lambda_p = 291.0^\circ \text{ E.}$$

The approximation

$$\Phi = \phi + 11.6^\circ \cos(\lambda - 291)$$

represents the exact form to within 1° at all geomagnetic latitudes equatorward of 40°, and is within 2° up to 65° either side of the geomagnetic equator. Over the entire CONUS region the error in the approximate form is less than 1°.

### Finding Local Time

Given the approximate universal, or Greenwich, time (UT) and the approximate longitude of the ionospheric point, the local time at the ionospheric point is simply:  $t = \lambda_i/15 + \text{UT}$ . If  $t$  comes out greater than 24 hr, simply subtract 24 hr to keep the resultant time between 0 and 24 hr.

### The Obliquity Factor

The vertical time delay at the subionospheric point must be multiplied by an obliquity, or slant, factor defined as the secant of the zenith angle at the mean ionospheric height. An average ionospheric height of 350 km is assumed. Fig. 5 shows both the exact form of the slant factor calculation as well as the approximate form used in the algorithm. This approximate form is within 2 percent of the exact value for all elevation angles greater than 5°.

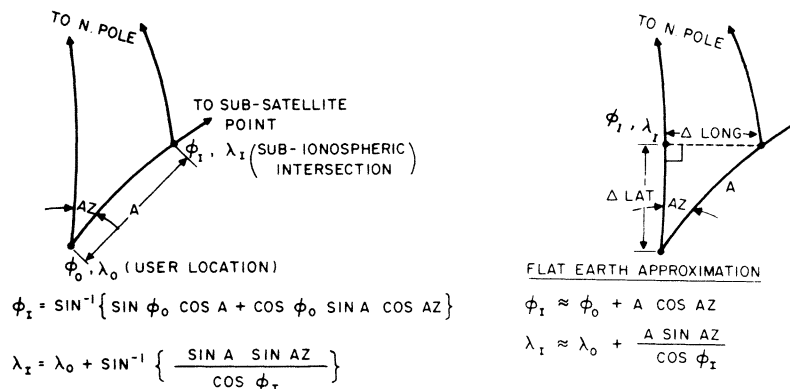


Fig. 4. Approximations to geometrical calculations for finding subionospheric coordinates.

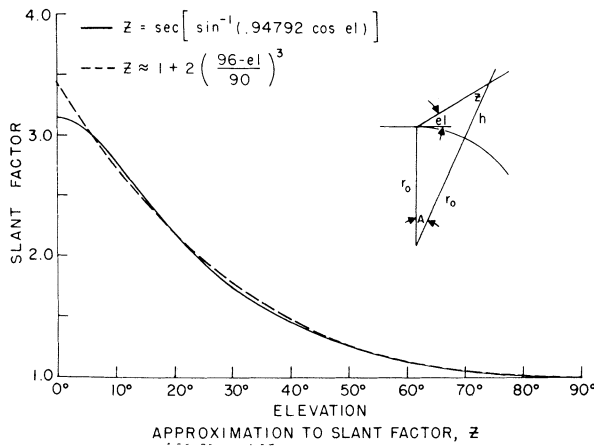


Fig. 5. Approximation to computing the slant factor, SF.

## SUMMARY OF ALGORITHM EQUATIONS

The form of the ionospheric time-delay correction algorithm as given in the GPS user "Interface Control Document" [10] is summarized in the following equations. In the actual implementation of the ionospheric correction algorithm the units used in angular measure are semicircles, and time is expressed in seconds.

*Given.* User approximate geodetic latitude  $\Phi_u$ , longitude  $\lambda_u$ , elevation angle  $E$ , and azimuth  $A$ , to the particular GPS satellite for which you wish to calculate the ionospheric time delay. Also given are the coefficients  $\alpha_n$  and  $\beta_n$  which are transmitted as part of the satellite message.

*Note.* All angles are in units of semi-circle; time is in seconds.

- 1) Calculate the Earth-centered angle:

$$\psi = \frac{0.0137}{E + 0.11} - 0.022 \quad (\text{semicircles}).$$

- 2) Compute the subionospheric latitude:

$$\Phi_I = \Phi_u + \psi \cos A.$$

If  $\Phi_I > +0.416$ , then  $\Phi_I = +0.416$ . If  $\Phi_I < -0.416$ , then  $\Phi_I = -0.416$ .

- 3) Compute the subionospheric longitude:

$$\lambda_I = \lambda_u + \frac{\psi \sin A}{\cos \Phi_I}.$$

- 4) Find the geomagnetic latitude:

$$\Phi_m = \Phi_I + 0.064 \cos(\lambda_I - 1.617)$$

- 5) Find the local time:

$$t = 4.32 \times 10^4 \lambda_I + \text{GPS time (sec)}.$$

If  $t > 86\,400$ , use  $t = t - 86\,400$ . If  $t < 0$ , add 86 400.

- 6) Compute the slant factor:

$$F = 1.0 + 16.0 \times (0.53 - E)^3.$$

- 7) Then compute the ionospheric time delay:

$$T_{\text{IONO}} = F \times \left[ 5 \times 10^{-9} + \sum_{n=0}^3 \alpha_n \Phi_m^n \times \left( 1 - \frac{x^2}{2} + \frac{x^4}{24} \right) \right]$$

where

$$x = \frac{2\pi(t - 50\,400)}{\sum_{n=0}^3 \beta_n \Phi_m^n}.$$

Note:  $T_{\text{IONO}}$  is referred to the  $L1$  frequency. If the user requires the ionospheric time-delay correction on the  $L2$  frequency, the correction term must be multiplied by the constant 1.65.

## EXAMPLE CALCULATION

As an example, assume a station located at 40.0° North, 100.0° West, viewing a GPS satellite at 20° elevation and 210° azimuth. The ionospheric algorithm coefficients transmitted at this time from the GPS satellites are  $\alpha_n = 3.82 \times 10^{-8}, 1.49 \times 10^{-8}, -1.79 \times 10^{-7}, 0$  and  $\beta_n = 1.43 \times 10^5, 0, -3.28 \times 10^5, 1.13 \times 10^5$ . The time is 2045 UT.

The results for this example are

- 1)  $\psi = 0.03996$  semicircles (7.2°)
- 2)  $\Phi_I = 0.215$  semicircles (38.7° N)
- 3)  $\lambda_I = -0.6399$  semicircles (124.795° W)
- 4)  $\Phi_m = 0.2509$  semicircles (45.16° N)
- 5)  $t = 50,700$  s
- 6)  $F = 2.176$
- 7)  $T_{\text{IONO}} = 77.6$  ns (23.3 m).

This example was chosen to represent an actual coefficient set during a spring equinox, high solar flux period.

## SUMMARY OF ALGORITHM CHARACTERISTICS

The ionospheric time-delay algorithm for the single-frequency GPS user was designed to minimize user computational complexity, yet still retain the goal of a 50 percent rms ionospheric range error correction. A major part of the reduction in user computational complexity was accomplished by using approximations to the geometrical calculations involved to reduce the number of trigonometric calculations which can be computationally time consuming. In fact, the algorithm itself, the simple cosine representation of a diurnal curve, was truncated to the first two terms of the cosine expansion in order to save computational time, and incidentally, slightly improve the algorithm fit to the actual data.

## INITIAL TESTS OF IONOSPHERIC ALGORITHM ACCURACY

The ionospheric time-delay algorithm was tested against actual TEC from 18 different stations for a total of 490 station-months of data covering a representative portion of the globe and for both solar maximum and minimum conditions. Fig. 6 shows the distribution of stations and times from which data were used for model tests. Notice that a significant portion of the data is from regions other than the midlatitudes, and that many parts of the world are represented in the available data. Also, much of the data was from a period of high solar activity when absolute values of TEC are high.

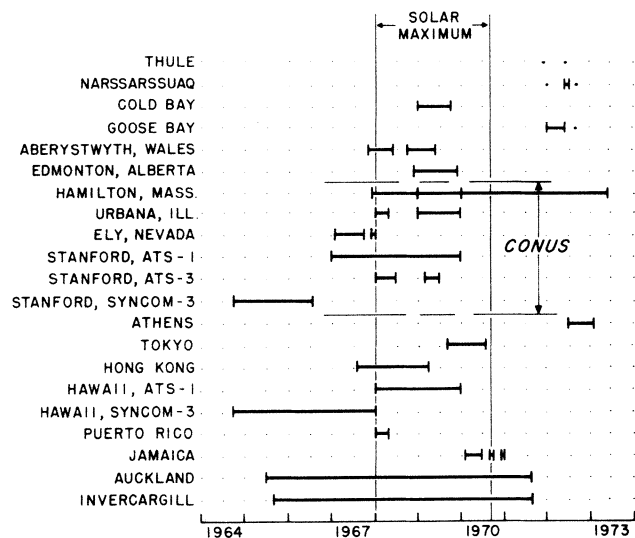


Fig. 6. Distribution of times and locations when monthly average TEC data were available for algorithm testing.

Summary statistics of ionospheric time delay for each station for all available hours and seasons were computed in units of rms error in nanoseconds at 1.6 GHz. Also, the residual rms error for each station was computed after correction by use of the time delay algorithm. These summary results are shown in Fig. 7, with the dashed line indicating the uncorrected ionospheric time delay, and the solid line indicating the residual time delay after correction. Two stations are shown in more than one column in Fig. 7 because the actual ionospheric TEC data available from those stations came from more than one satellite, and from different time periods.

The comparison of the algorithm against actual monthly mean TEC data, for the entire 490 station months of data, gave an overall rms residual error of 38 percent. In order to find the rms error of the model against actual individual data, rather than against monthly average TEC data, the 38 percent algorithm error in representing the monthly mean TEC, was combined in a root of the sum of the square error (rss) sense with the 25 percent rms deviation of individual data from the monthly

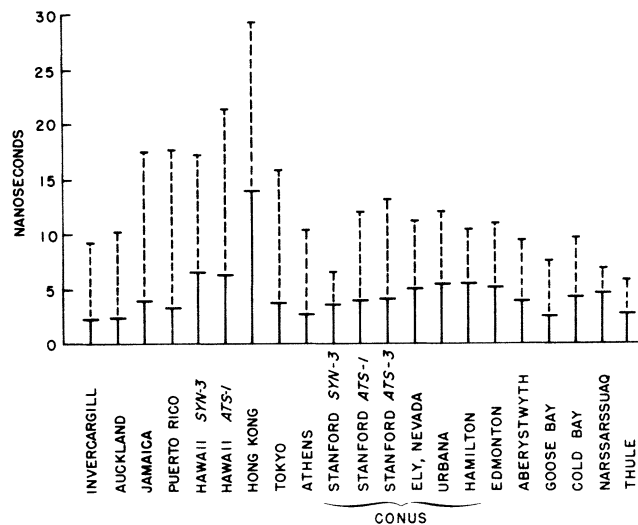


Fig. 7. Results of ionospheric time-delay algorithm by station.

mean. Obviously, any errors in the algorithm are independent of the day-to-day variability of the ionosphere, and thus these two errors can be added in root of the sum of the square errors fashion. The overall algorithm improvement obtained in this manner, from the 490 station months of data, was approximately 55 percent. Thus, these initial algorithm tests showed that our goal of a 50 percent rms ionospheric correction has been met.

## ALGORITHM LIMITATIONS

The algorithm described here is a severely truncated version of a much larger empirical model of TEC developed by Bent [9], over 13 years ago. While significantly more actual TEC data, along with additional bottomside and topside ionospheric profile measurements, are available than at the time of the development of the Bent model, no attempts have been made to improve the present state of world-wide empirical TEC models. This is because the use of even a state of the art ionospheric model would not get around the approximate 20 to 25 percent day-to-day variability of the ionospheric TEC which the single-frequency user must live with.

The high latitude regions deserve some additional mention. Firstly, the auroral regions are characterized by high variability, with little chance for adequately modeling departures in TEC from monthly average behavior, except in a statistical sense. Secondly, most active auroral energy inputs occur during nighttime periods when the background absolute TEC values are considerably lower than during the daytime periods when TEC is higher. Auroral disturbances have been seen which have produced TEC values which rival, and sometimes, even exceed midlatitude midday values, but these are likely infrequent in occurrence, only predictable in a statistical sense, and cannot be used to improve single-frequency user ionospheric range errors.

## CONCLUSIONS

This algorithm has been implemented in a slightly different form for use with the GPS satellites. Details of its implementation in a single-frequency user set are described in detail in the NAVSTAR GPS "Interface Control Document" [10]. It is relatively easy to obtain a 50 percent rms ionospheric error reduction with this algorithm. The trick is in the single-frequency user being able to withstand those infrequent times when a large deviation from average ionospheric behavior will be encountered. If a single-frequency user cannot tolerate those rare, but real, times when the ionosphere differs by substantial amounts from its average behavior, then that user must seriously consider opting for a dual-frequency GPS receiver which has been demonstrated to remove ionospheric errors to a level comparable with other sources of GPS system errors [11].

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