ANOVA - Analysis of Variance

ANOVA

- A procedure for comparing more than two groups
  - **independent variable**: smoking status
    - *non-smoking*
    - *one pack a day*
    - *> two packs a day*
  - **dependent variable**: number of coughs per day
- Number of conditions = in this case 3.

One-Way ANOVA

- One-Way ANOVA has one independent variable (**_1 factor_**) with > 2 *conditions*
  - conditions = levels = treatments
  - e.g., for a brand of cola factor, the levels are:
    - Coke, Pepsi, RC Cola
- Independent variables = factors

Two-Way ANOVA

- Two-Way ANOVA has **2** independent variables (**factors**)

– each can have multiple conditions

Example

- Two Independent Variables (IV's)

– IV1: Brand; and IV2: Calories

– Three levels of Brand:

- Coke, Pepsi, RC Cola

– Two levels of Calories:

- Regular, Diet

When to use ANOVA

- **One-way ANOVA:** you have more than two levels (conditions) of a single factor.

– EXAMPLE: studying effectiveness of three types of pain reliever

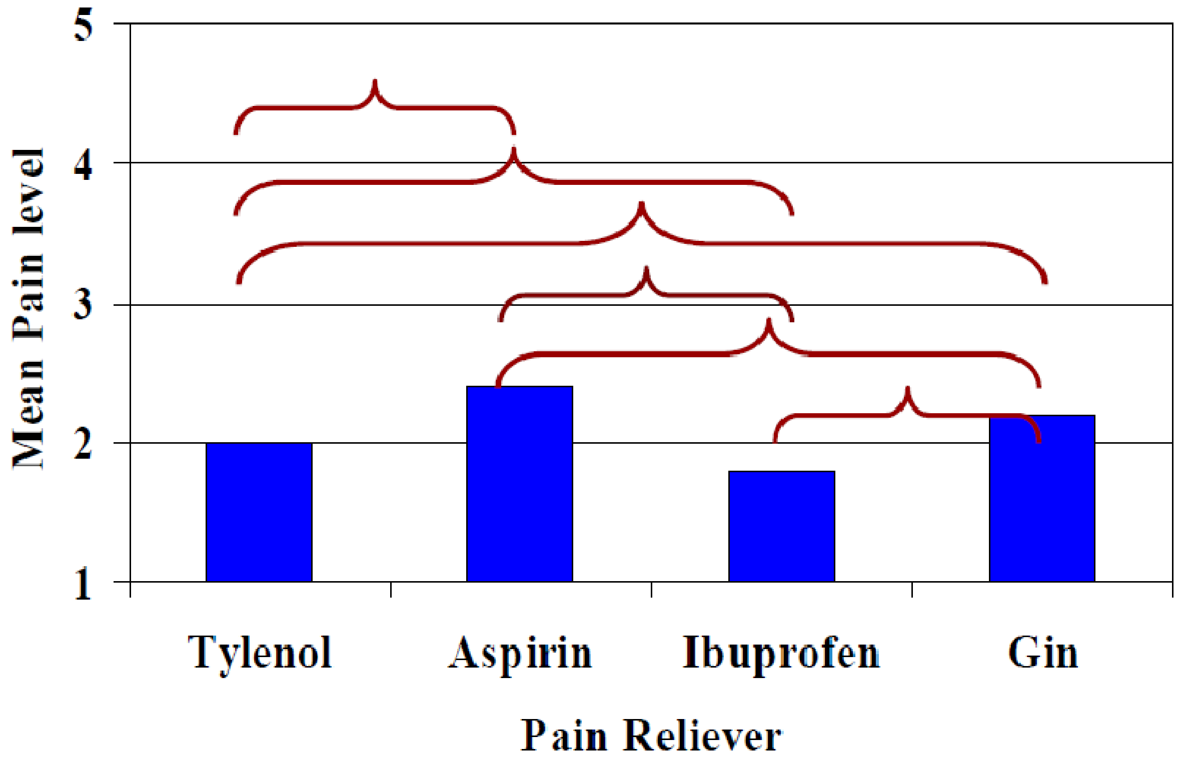
- *aspirin* vs. *tylenol* vs. *ibuprofen*

- **Two-way ANOVA:** you have more than one factor.

– EXAMPLE: studying pain relief based on pain reliever and type of pain

- Factor A: Pain reliever (*aspirin* vs. *tylenol*)
- Factor B: type of pain (*headache* vs. *back pain*)

Why bother with ANOVA?



Would require <u>six</u> *t*-tests, each with an associated Type I (false alarm) error rate.

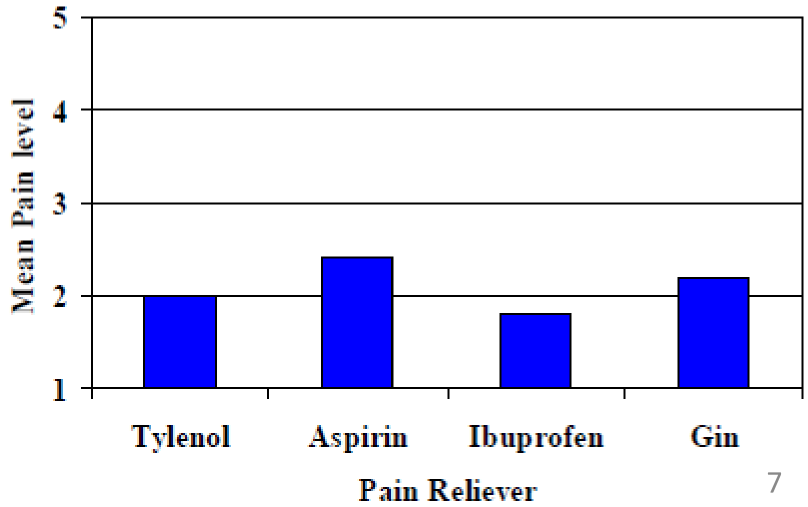
Post-hoc Tests

- If the ANOVA is significant

– *at least* one significant difference between conditions

- In that case, we follow the ANOVA with posthoc tests that compare two conditions at a time

– post-hoc comparisons identify the specific significant differences between each pair



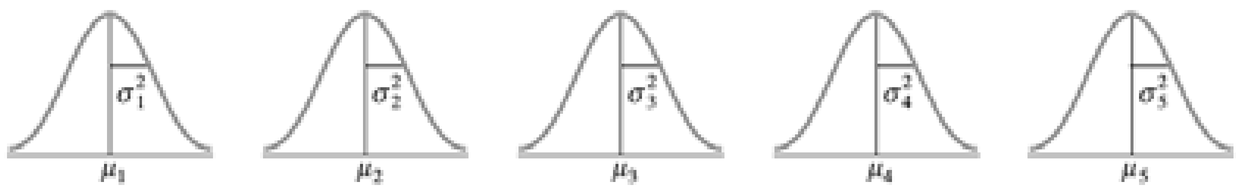
ANOVA Assumptions

- Homogeneity of variance

– $\sigma^2_1 = \sigma^2_2 = \sigma^2_3 = \sigma^2_4 = \sigma^2_5$

- Independence
- Normality

– scores in each population are normally distributed



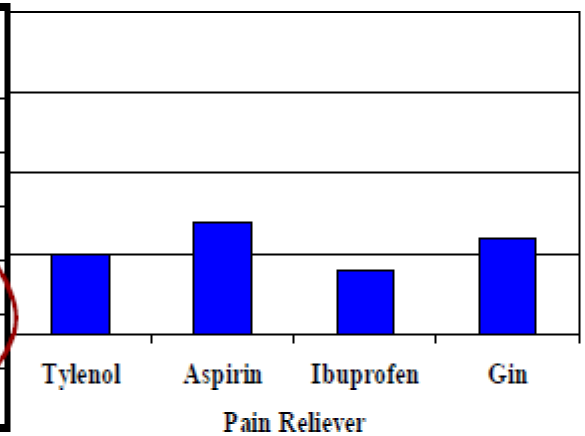
Partitioning Variance

*MS*error (mean square error) is an estimate of the variability as measured by differences within the conditions

– sometimes called the **within group variance** or the errorterm

– chance variance (random error + **individual differe**

	Tylenol	Aspirin	Ibuprofen	Gin
	3	1.6	2.1	1
	2.3	2.7	1.6	3.3
	1.7	2.4	2.4	1.9
	1	2	1.9	2.7
	2	3.3	1	2.1
Mean:	**2**	**2.4**	**1.8**	**2.2**



MS_{error} = error variance (within groups)

- *MS*group is an estimate of the differences in scores
- that occurs between the levels in a factor
- – also called *MS*between
- – Treatment effect (systematic variance)

	Tylenol	Aspirin	Ibuprofen	Gin
	3	1.6	2.1	1
	2.3	2.7	1.6	3.3
	1.7	2.4	2.4	1.9
	1	2	1.9	2.7
	2	3.3	1	2.1
$\overline{\mathbf{X}}$:	**2**	**2.4**	**1.8**	**2.2**

Overall $\overline{X} = 2.1$

MS_{group} = variance between groups

Total Variance

(variability among all the scores in the data set)

Between Groups Variance

1. Treatment effect (systematic)
2. Chance *(random error + individual differences)*

Error Variance (within groups)

1. Chance *(random error + individual differences)*

- In ANOVA, variance = Mean Square (*MS*)

$$F\text{-}Ratio = \frac{\text{between group variance}}{\text{error variance (within groups)}} = \frac{MS_{\text{group}}}{MS_{\text{error}}}$$

ANOVA Example: Cell phones

Research Question:

• Is your reaction time when driving slowed by a cell phone? Does it matter if it’s a hands-free phone?

• Twelve participants went into a driving simulator.

1. A random subset of 4 drove while listening to the radio (control group).
2. Another 4 drove while talking on a cell phone.
3. Remaining 4 drove while talking on a hands-free cell phone.

• Every so often, participants would approach a traffic light that was turning red. The time it took for participants to hit the breaks was measured.

1. State your research question

• Is your reaction time when driving influenced by cell-phone usage?

2. Choose a statistical test

• three levels of a single independent variable (cell; hands-free; control)

→ One-Way ANOVA, between subjects

State Hypotheses

referred to as the omnibus null hypothesis

$H0$: μ1 = μ2 = μ3

$H1$: at least one μ is different.

- When rejecting the Null in ANOVA, we can only conclude that there is at least one significant difference among conditions.
- If ANOVA significant

– pinpoint the actual difference(s), with post-hoc comparisons

ANOVA Summary Table

ANOVA Summary Table

Source	Sum of Squares	*df*	Mean Squares	*F*	*p*
Group	.072	2	.0360	6.45	0.001
Error	.050	9	.0056		
Total	.122	11			

Post-hoc Comparisons

- Tukey HSD (**H**onestly **S**ignificant **D**ifference)

– sets the familywise error rate at the error rate for the collection for all pairwise comparisons.

– very common test

- Other post-hoc tests also seen:

– e.g., Newman-Keuls, Duncan, Scheffe'…

Dependent Varia
Tukey HSD

control and cell groups are significantly different

(I) condition	(J) condition	Mean Difference (I-J)	Std. Error	Sig.	95% Confidence Interval Lower Bound	95% Confidence Interval Upper Bound
control	cell	-.17500*	.05270	.022	-.3222	-.0278
	hands	-.15000*	.05270	.046	-.2972	-.0028
cell	control	.17500*	.05270	.022	.0278	.3222
	hands	.02500	.05270	.885	-.1222	.1722
hands	control	.15000*	.05270	.046	.0028	.2972
	cell	-.02500	.05270	.885	-.1722	.1222

*. The mean difference is significant at the .05 level.

hands and cell groups are NOT significantly different

Complete explanation

– Any kind of cell phone conversation can cause a longer reaction time compared to listening to the radio.

– There is no significant difference between reaction times in the normal cell phone and hands-free conditions.

How to report on APA style

A one-way between subjects ANOVA was conducted to compare the effect of sugar on memory for words in sugar, a little sugar and no sugar conditions.

- “There was a significant effect of amount of sugar on words remembered at the $p<.05$ level for the three conditions [$F(2,12) = 4.94$, $p = 0.027$].”

If posthoc was significant:

- Post hoc comparisons using the Tukey HSD test indicated that the mean score for the sugar condition ($M = 4.20$, $SD = 1.30$) was significantly different than the no sugar condition ($M = 2.20$, $SD = 0.84$). However, the a little sugar condition ($M = 3.60$, $SD = 0.89$) did not significantly differ from the sugar and no sugar conditions.”

Exercises

- **Beauty factor!!! Let's perform an one-way ANOVA to examined the effect of alcohol on beauty factor.**
- **Now, using the data set iris, check if there is difference in petal length between iris species.**

- **If we have time…**
- **Let's perform a two one-way ANOVA to examined the effect of alcohol and gender on beauty factor.**