Aerial robotics challenge

1 Mathematical prerequisites

1.1 Model of the quadrotor

In this section we describe the model of the quadrotor [3],[2] used in the Gazebo simulator. You will use this model in the design of quadrotor control algorithms.

First, note the coordinate system assigned to the quadrotor body (Fig. 1.1). As a 3D rigid body, quadrotor pose is described by 6 parameters - x,y,z position and 3 Euler angles. Throughout this seminar, when we refer to the quadrotor position, we assume the position relative to the inertial coordinate frame (i.e. global coordinates w.r.t to the Gazebo origin). For the Euler angles, we use notation yaw-pitch-roll, which means that we assume that the quadrotor is first rotated around body z-axis (this is also a global z-axis), followed by a rotation around body y-axis, and finally, a rotation around body x-axis.

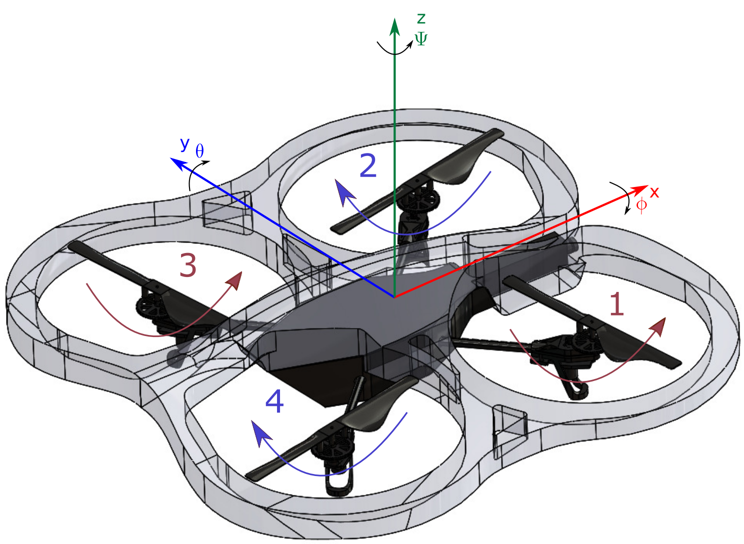

Figure 1: Coordinate frame assigned to the quadrotor body (body frame). The origin of the frame coincides with tdhe center of the mass. Note the numbering of the motors and their direction of rotation.

A quadrotor motor in the simulator is modelled as:

$$T_m \cdot \dot{\Omega}_i + \Omega_i = \Omega_{i,ref} \quad \text{i=1,2,3,4,} \tag{1}$$

where Ω_i is the rotational velocity of the i-th motor (in rad/sec), T_m is the motor time constant and $\Omega_{i,ref}$ is a motor reference velocity. We assume that time constants of all motors are equal.

The thrust force F_i that each motor produces is:

$$F_i = b_f \cdot {\Omega_i}^2 \cdot \hat{k}, \tag{2}$$

b_f is a motor thrust constant, $\hat{k}$ is a unit vector in the direction of the body z-axis.

Each motor also produces moment M_i (due to induced drag):

$$M_i = \zeta_i b_f b_m \cdot {\Omega_i}^2 \cdot \hat{k}, \ \ \zeta_i = 1 \ \ \text{(i=2,4) or} - 1 \ \ \text{(i=1,3)}, \tag{3}$$

b_m is a motor moment constant and ζ_i indicates whether the propeller rotates clockwise ($\zeta = 1$) or counter-clockwise ($\zeta = -1$).

The change of the quadrotor attitude is described by the following equation:

$$\frac{d}{dt}\left(\mathbf{I} \cdot \omega\right) = -\omega \times \left(\mathbf{I} \times \omega\right) + \mathbf{M}_{sum}, \tag{4}$$

where $\mathbf{I}$ is the quadrotor tensor matrix w.r.t. the center of the mass, ω is the quadrotor angular velocity in the body frame (see Fig. 1.1), $\mathbf{M}_{sum}$ is the sum of all external moments acting on the vehicle.

We assume that the quadrotor is symmetrical, which results in a diagonal tensor matrix (i.e. $I_{xy} = I_{xz} = I_{yz} = 0$):

$$\mathbf{I} = \begin{bmatrix} I_{xx} & 0 & 0 \\ 0 & I_{yy} & 0 \\ 0 & 0 & I_{zz} \end{bmatrix} \tag{5}$$

The equations describing the angular velocity change in x,y and z direction can be now written as:

$$I_{xx}\dot{\omega}_x = (I_{yy} - I_{zz})\omega_y\omega_z + (F_2 + F_3 - F_1 - F_4) \cdot l\cos(45) \tag{6}$$

$$I_{yy}\dot{\omega}_y = (I_{zz} - I_{xx})\omega_x\omega_z + (F_3 + F_4 - F_1 - F_2) \cdot l\cos(45) \tag{7}$$

$$I_{zz}\dot{\omega}_z = M_1 + M_2 + M_3 + M_4 \tag{8}$$

Note that moments of inertia in x and y axis are equal (Table 1). Therefore, gyroscopic term in Equ. 8 ($(I_{xx} - I_{yy})\omega_x\omega_y$) equals zero and it is omitted.

As we derive the model around the hovering state, the friction in Equ. 6, 7, 8 is neglected, which is a common approach when designing a control law for a quadrotor. The change of the Euler angles (yaw - ψ, pitch - θ, roll - ϕ) is described as:

$$\begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} 1 & \sin(\phi)\tan(\theta) & \cos(\phi)\tan(\theta) \\ 0 & \cos(\phi) & -\sin(\phi) \\ 0 & \frac{\sin(\phi)}{\cos(\theta)} & \frac{\cos(\phi)}{\cos(\theta)} \end{bmatrix} \begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix}, \tag{9}$$

where $\dot{\phi}, \dot{\theta}, \dot{\psi}$ are roll rate, pitch rate and yaw rate, respectively.

We derive the quadrotor position model in the inertial coordinate frame (corresponds to the Gazebo static frame):

$$m\dot{v_x} = F_z^b\left(\cos(\phi)\sin(\theta)\cos(\psi) + \sin(\phi)\sin(\psi)\right) \tag{10}$$

$$m\dot{v_y} = F_z^b\left(\cos(\phi)\sin(\theta)\sin(\psi) - \sin(\phi)\cos(\psi)\right) \tag{11}$$

$$m\dot{v_z} = F_z^b\cos(\phi)\cos(\theta) - mg \tag{12}$$

Table 1: Model parameters

Symbol	Value and unit	Desctiption
m	1.477 kg	Total quadrotor mass
I_{xx}	0.01152 kg·m^2	Quadrotor moment inertia in body x direction
I_{yy}	0.01152 kg·m^2	Quadrotor moment inertia in body y direction
I_{zz}	0.0218 kg·m^2	Quadrotor moment inertia in body z direction
T_m	0.0125 s	Time constant of a motor
b_f	8.54858e-06 kg ·m	Thrust constant of a motor
b_m	0.016 m	Moment constant of a motor
l	0.18 m	The distance of a motor from the center of mass

where m is the quadrotor mass, F_z^b is total thrust force in body frame (z-direction):

$$F_z^b = F_1 + F_2 + F_3 + F_4, \tag{13}$$

To get the position in the inertial coordinate frame, simply integrate the velocity:

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{bmatrix} = \begin{bmatrix} v_x \\ v_y \\ v_z \end{bmatrix} \tag{14}$$

Parameters used in the Gazebo model can be found in Table 1.

References

[1] Catkin tools cheat sheet, January 2018.

[2] Robert C Leishman, John C. MacDonald, Randal W Beard, and Timothy W. McLain. Quadrotors and Accelerometers: State Estimation with an Improved Dynamic Model. *IEEE Control Systems*, 34(1):28–41, 2014.

[3] Philippe Martin, Erwan Salaün, and E Salaun. The true role of accelerometer feedback in quadrotor control. *Robotics and Automation (ICRA), 2010 . . .* , pages 1623–1629, 2010.

[4] D. Miklic, M. Orsag, and I. Markovic. Programming for the robot operating system: Lectures, January 2016.

[5] W. Woodall. Ros kinetic installation instructions, January 2018.