Appendix D

Constants

D.1 PHYSICAL CONSTANTS*

The speed of light is an important fundamental constant since it effectively defines the length scale in a range measurement. As described in Chapter 3, the range measurement actually is based on a measurement of the time required for a signal to travel from the transmitter to the receiver. This time interval measurement is converted into a distance measurement (range) using the speed of light, c. The value of c was adopted by the International Astronomical Union (IAU) in 1976, to be a defining constant with the value

$$c = 299,792,458 \ \mathrm{m\,s^{-1}}.$$

The constant of gravitation, G, is experimentally determined. A recent value is (Mohr and Taylor, 2003)

$$G = 6.673 \times 10^{-11} \pm 1.0 \times 10^{-13} \mathrm{m^3\ kg^{-1}\ s^{-2}}.$$

The IAU (1976) System of Astronomical Constants can be found in The Astronomical Almanac for the Year 2000 on pages K6 and K7. Updated constants are given by Seidelmann (1992) and McCarthy (1996).

D.2 EARTH CONSTANTS

For an Earth-orbiting satellite, the normalized gravity coefficients ($\bar{C}_{lm}$ and $\bar{S}_{lm}$), GM, and a_e are required. Recent determinations include the WGS-84 (DMA, 1987), JGM-3 field (Tapley *et al.*, 1996), EGM-96 (Lemoine *et al.*, 1998),

*The physical constants given in this appendix use SI units (Le Systeme International d'Unites (SI), 1991).

and the GRIMS-C1 (Gruber *et al.*, 2000). The degree and order eight subset of JGM-3 is given in the Table D.1 with standard deviations. Conversion of normalized coefficients to conventional coefficients can be accomplished by:

$$
\begin{aligned}
C_{lm} &= N_{lm}\bar{C}_{lm} \\
S_{lm} &= N_{lm}\bar{S}_{lm} \\
N_{lm} &= \sqrt{\frac{(l-m)!(2l+1)(2-\delta_{0m})}{(l+m)!}}
\end{aligned}
$$

where δ_{0m} is the Kronecker delta function, which is zero when m is not zero, otherwise it is one. Furthermore, for zonal harmonics, the commonly used J_ℓ correspond to $m = 0$, and

$$J_l = -C_{l0}\,.$$

The ellipsoidal model of the Earth was described in Chapter 2. Current ellipsoid parameters adopted by the IERS (McCarthy, 1996) are:

$$
\begin{aligned}
a_e &= 6378136.49 \pm 0.1 \text{ m} \\
1/f &= 298.25645 \pm 0.00001\,.
\end{aligned}
$$

D.3 LUNAR, SOLAR, AND PLANETARY MASSES

Additional parameters required for the description of satellite motion include the gravitational parameters for the Sun, Moon, and planets. The values that are used with the planetary ephemerides, such as DE-405 (Standish, *et al.*, 1997), are given in Table D.2. Additional information can be found in McCarthy (1996), Seidelmann (1992), and Standish *et al.* (1997). All mass parameters have been determined from observations; consult the references for uncertainties.

Table D.1

JGM-3 Earth Gravity Model

l m	$\overline{C}$	$\overline{S}$	$\sigma_{\overline{C}}$	$\sigma_{\overline{S}}$
2 0	−0.48416954845647E−03	0.00000000000000E+00	0.4660E−10	0.0000E+00
3 0	0.95717059088800E−06	0.00000000000000E+00	0.3599E−10	0.0000E+00
4 0	0.53977706835730E−06	0.00000000000000E+00	0.1339E−09	0.0000E+00
5 0	0.68658987986543E−07	0.00000000000000E+00	0.8579E−10	0.0000E+00
6 0	−0.14967156178604E−06	0.00000000000000E+00	0.2428E−09	0.0000E+00
7 0	0.90722941643232E−07	0.00000000000000E+00	0.2604E−09	0.0000E+00
8 0	0.49118003174734E−07	0.00000000000000E+00	0.3996E−09	0.0000E+00
2 1	−0.18698764000000E−09	0.11952801000000E−08	0.0000E+00	0.0000E+00
3 1	0.20301372055530E−05	0.24813079825561E−06	0.1153E−09	0.1152E−09
4 1	−0.53624355429851E−06	−0.47377237061597E−06	0.8693E−10	0.8734E−10
5 1	−0.62727369697705E−07	−0.94194632134383E−07	0.2466E−09	0.2465E−09
6 1	−0.76103580407274E−07	0.26899818932629E−07	0.2702E−09	0.2752E−09
7 1	0.28028652203689E−06	0.94777317813313E−07	0.4361E−09	0.4344E−09
8 1	0.23333751687204E−07	0.58499274939368E−07	0.5070E−09	0.5137E−09
2 2	0.24392607486563E−05	−0.14002663975880E−05	0.3655E−10	0.3709E−10
3 2	0.90470634127291E−06	−0.61892284647849E−06	0.9378E−10	0.9375E−10
4 2	0.35067015645938E−06	0.66257134594268E−06	0.1559E−09	0.1560E−09
5 2	0.65245910276353E−06	−0.32333435244435E−06	0.2392E−09	0.2398E−09
6 2	0.48327472124892E−07	−0.37381591944355E−06	0.3145E−09	0.3160E−09
7 2	0.32976022742410E−06	0.93193696831045E−07	0.4635E−09	0.4587E−09
8 2	0.80070663931587E−07	0.65518559097464E−07	0.5185E−09	0.5323E−09
3 3	0.72114493982309E−06	0.14142039847354E−05	0.5755E−10	0.5720E−10
4 3	0.99086890577441E−06	−0.20098735484731E−06	0.7940E−10	0.7942E−10
5 3	−0.45183704808780E−06	−0.21495419346421E−06	0.1599E−09	0.1616E−09
6 3	0.57020965757974E−07	0.88894738008251E−08	0.2598E−09	0.2574E−09
7 3	0.25050152675038E−06	−0.21732010845254E−06	0.3656E−09	0.3736E−09
8 3	−0.19251764331400E−07	−0.86285836534248E−07	0.4947E−09	0.4918E−09
4 4	−0.18848136742527E−06	0.30884803690355E−06	0.7217E−10	0.7228E−10
5 4	−0.29512339302196E−06	0.49741427230934E−07	0.9264E−10	0.9288E−10
6 4	−0.86228032619800E−07	−0.47140511232148E−06	0.1656E−09	0.1663E−09
7 4	−0.27554096307403E−06	−0.12414151248516E−06	0.2665E−09	0.2656E−09
8 4	−0.24435806439297E−06	0.69857074850431E−07	0.4033E−09	0.4063E−09
5 5	0.17483157769990E−06	−0.66939293724911E−06	0.8139E−10	0.8131E−10
6 5	−0.26711227171966E−06	−0.53641016466390E−06	0.8465E−10	0.8510E−10
7 5	0.16440038146411E−08	0.18075335233506E−07	0.1832E−09	0.1835E−09
8 5	−0.25498410010257E−07	0.89090297494640E−07	0.2586E−09	0.2571E−09
6 6	0.95016518338557E−08	−0.23726147889522E−06	0.8021E−10	0.8081E−10
7 6	−0.35884263307918E−06	0.15177808443426E−06	0.5899E−10	0.5913E−10
8 6	−0.65859353864388E−07	0.30892064157956E−06	0.1566E−09	0.1569E−09
7 7	0.13795170564076E−08	0.24128594080773E−07	0.9709E−10	0.9747E−10
8 7	0.67262701848734E−07	0.74813196768710E−07	0.9308E−10	0.9378E−10
8 8	−0.12397061395498E−06	0.12044100668766E−06	0.1379E−09	0.1384E−09

$GM_{\text{Earth}} = 3.986004415 \times 10^{14} \pm 8 \times 10^{5}\ \text{m}^3\ \text{s}^{-2}$
$a_e = 6378136.3\ \text{m}$

Table D.2

Lunar, Solar and Planetary Masses

Planet	GM ($m^3\ s^{-2}$)
Mercury	2.203208×10^{13}
Venus	3.248586×10^{14}
Mars	4.282831×10^{13}
Jupiter	1.267128×10^{17}
Saturn	3.794063×10^{16}
Uranus	5.794549×10^{15}
Neptune	6.836534×10^{15}
Pluto	9.816009×10^{11}
Moon	4.902801×10^{12}
Sun	1.327124×10^{20}

The Earth-Moon mass ratio is 81.30056 and
1 Astronomical Unit = 149, 597, 870, 691 m,
as provided with DE-405.

Note that using the Earth-Moon mass ratio in Table D.2 yields $GM_{\text{Moon}} = 4.902804 \times 10^{12} m^3 s^{-2}$ based on the GM_{Earth} from JGM-3. The difference with GM_{Moon} in Table D.2 is caused by the GM_{Earth} adopted in DE-405.

D.4 REFERENCES

Defense Mapping Agency, *Department of Defense World Geodetic System 1984*, DMA Technical Report 8350.2, Washington, DC, September, 1987.

Gruber, T., A. Bode, C. Reigber, P. Schwintzer, G. Balmino, R. Biancale, J. Lemoine, "GRIM5-C1: Combination solution of the global gravity field to degree and order 120," *Geophys. Res. Ltrs.*, Vol. 27, No. 24, pp. 4005–4008, December 2000.

Lemoine, F., S. Kenyon, J. Factor, R. Trimmer, N. Pavlis, D. Chinn, C. Cox, S. Klosko, S. Luthcke, M. Torrence, Y. Wang, R. Williamson, E. Pavlis, R. Rapp, and T. Olson, *The development of the Joint NASA GSFC and the National Imagery and Mapping Agency (NIMA) Geopotential Model EGM96*, NASA/TP–1998–206861, Greenbelt, MD, July 1998.

Le Systeme International d'Unites (SI), Bureau International des Poids et Mesures, Sevres, France, 1991.

McCarthy, D. (ed.), *IERS Conventions (1996)*, IERS Technical Note 21, International Earth Rotation Service, Observatoire de Paris, July 1996.

Mohr, P. J., and B. Taylor, "The fundamental physical constants," *Physics Today*, Vol. 56, No. 8, Supplement, pp. BG6–BG13, August 2003.

Seidelmann, P. (ed.), *Explanatory Supplement to the Astronomical Almanac*, University Science Books, Mill Valley, CA, 1992.

Standish, E. M., X. Newhall, J. Williams, and W. Folkner, *JPL Planetary and Lunar Ephemerides* (CD-ROM), Willmann-Bell, Inc., Richmond, VA 1997.

Tapley, B. D., M. Watkins, J. Ries, G. Davis, R. Eanes, S. Poole, H. Rim, B. Schutz, C. Shum, R. Nerem, F. Lerch, J. Marshall, S. Klosko, N. Pavlis, and R. Williamson, "The Joint Gravity Model 3," *J. Geophys. Res.*, Vol. 101, No. B12, pp. 28,029–28,049, December 10, 1996.