ME951 - Estatística e Probabilidade I

PROVA 1

FORMULÁRIO

1. $\bar{x} = \dfrac{\sum_{i=1}^{n} x_i}{n}$;

2. $s^2 = \dfrac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n-1} = \dfrac{1}{n-1}(\sum_{i=1}^{n} x_i^2 - n\bar{x}^2)$;

3. $C.V = \dfrac{s}{\bar{x}}$.

Probabilidade

1. $P(A \cup B) = P(A) + P(B) - P(A \cap B)$;

2. $P(A|B) = P(A \cap B)/P(B)$;

3. $P(A) = \sum_{i=1}^{k} P(A|B_i)P(B_i)$;

4. $P(B_i|A) = \frac{P(A|B_i)P(B_i)}{\sum_{i=1}^{k} P(A|B_i)P(B_i)}$.

Distribuição de probabilidade

Seja X uma variável aleatória discreta. Então,

$$\mu = E(X) = \sum_x xP(X = x) \quad \text{e} \quad Var(X) = E(X - \mu)^2 = \sum_x (x - \mu)^2 P(X = x).$$

1. Se $X \sim$ Uniforme Discreta $\quad \Rightarrow \quad P(X = x) = \begin{cases} 1/k, & x = 1, 2, \ldots, k; \\ 0, & \text{caso contrário.} \end{cases}$

2. Se $X \sim b(p) \quad \Rightarrow \quad P(X = x) = \begin{cases} p^x(1-p)^{1-x}, & x = 0, 1; \\ 0, & \text{caso contrário.} \end{cases}$

3. Se $X \sim Bin(n, p) \quad \Rightarrow \quad P(X = x) = \begin{cases} \begin{pmatrix} n \\ x \end{pmatrix} p^x(1-p)^{n-x}, & x = 0, 1, 2, \ldots, n; \\ 0, & \text{caso contrário,} \end{cases}$ onde $\begin{pmatrix} n \\ x \end{pmatrix} = \frac{n!}{x!(n-x)!}$.

4. Se $X \sim G(p) \quad \Rightarrow \quad P(X = x) = p(1-p)^{x-1}, \;\; x = 1, 2, \ldots$.

5. Se $X \sim Hip(N, n, r) \quad \Rightarrow \quad P(X = x) = \begin{cases} \dfrac{\begin{pmatrix} r \\ x \end{pmatrix}\begin{pmatrix} N-r \\ n-x \end{pmatrix}}{\begin{pmatrix} N \\ n \end{pmatrix}}, & x = 0, \ldots, min\{r, n\}; \\ 0, & \text{caso contrário.} \end{cases}$

6. Se $X \sim P(\lambda) \quad \Rightarrow \quad P(x = x) = \begin{cases} \frac{e^{-\lambda}\lambda^x}{x!}, & x = 0, 1, 2, \ldots; \\ 0, & \text{caso contrário.} \end{cases}$

Densidade de probabilidade

Seja X uma variável aleatória contínua. Então,

1. Se $X \sim$ Uniforme(a,b) $\Rightarrow$ $f(x) = \begin{cases} \frac{1}{(b-a)}, & a \leq x \leq b; \\ 0, & \text{caso contrário.} \end{cases}$

2. Se $X \sim \text{Exp}(\lambda)$ $\Rightarrow$ $f(x) = \lambda e^{-\lambda x}, \quad x \geq 0.$

3. Se $X \sim \text{Normal}(\mu, \sigma^2)$ $\Rightarrow$ $f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\{-\frac{(x-\mu)^2}{2\sigma^2}\}, \quad -\infty \leq x \leq \infty.$

4. Se $X \sim \text{Gama}(\alpha, \beta)$ $\Rightarrow$ $f(x) = \begin{cases} \dfrac{\beta^\alpha}{\Gamma(\alpha)} x^{\alpha-1} e^{-\beta x}, \text{ se } x \geq 0 \\ 0, \text{ caso contrário} \end{cases}$

5. Se $X \sim \text{Beta}(\alpha, \beta)$ $\Rightarrow$ $f(x) = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} x^{\alpha-1}(1-x)^{\beta-1}, \quad x \in (0,1) \text{ e } \alpha, \beta > 0.$

Inferência

1. Se $X \sim \text{Normal}(\mu, \sigma^2)$, então $Z = \frac{X-\mu}{\sigma} \sim \text{Normal}(0,1)$;

2. $P(-z_{\alpha/2} < Z <_{\alpha/2}) = 1 - \alpha$, $P(Z > z_\alpha) = \alpha$ e $P(Z < -z_\alpha) = \alpha$;

3. $\hat{p} \sim \text{Normal}(p, \frac{p(1-p)}{n}) \Rightarrow \frac{\hat{p}-p}{\sqrt{p(1-p)/n}} \sim \text{Normal}(0,1)$;

4. $\left[\hat{p} - z_{\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}; \hat{p} + z_{\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}\right]$;

5. $\left[\hat{p} - z_{\alpha/2}\sqrt{\frac{1}{4n}}; \hat{p} + z_{\alpha/2}\sqrt{\frac{1}{4n}}\right]$;

6. $n = \left(\frac{z_{\alpha/2}}{2m}\right)^2$;

7. $\overline{X} \sim \text{Normal}(\mu, \frac{\sigma^2}{n})$, então $\frac{\overline{X}-\mu}{\sigma/\sqrt{n}} \sim \text{Normal}(0,1)$.

8. $\left[\overline{x} - z_{\alpha/2}\sqrt{\frac{\sigma^2}{n}}; \overline{x} + z_{\alpha/2}\sqrt{\frac{\sigma^2}{n}}\right]$;

9. $\left[\overline{x} - t_{n-1,\alpha/2}\sqrt{\frac{s^2}{n}}; \overline{x} + t_{n-1,\alpha/2}\sqrt{\frac{s^2}{n}}\right]$;

10. $n = \left(\frac{z_{\alpha/2}}{m}\right)^2 \sigma^2$;

11. Sob $H_0 : \mu_x = \mu_y$,
$$\frac{\overline{X} - \overline{Y}}{\sqrt{\sigma_x^2/n + \sigma_y^2/m}} \sim \text{Normal}(0,1), \quad \frac{\overline{X} - \overline{Y}}{\sqrt{s_p^2(1/n + 1/m)}} \sim t_{n+m-2},$$
$s_p^2 = \frac{(n-1)s_x^2 + (m-1)s_y^2}{n+m-2}$;

12. Sob $H_0 : p_1 = p_2$,
$$\frac{\hat{p_1} - \hat{p_2}}{\sqrt{p_0(1-p_0)(1/n + 1/m)}} \sim \text{Normal}(0,1),$$
$p_0 = \frac{n\hat{p_1} + m\hat{p_2}}{n+m}$.