

ME414 - ESTATÍSTICA PARA EXPERIMENTALISTAS

PROVA II - 22/11/2018

FORMULÁRIO

1. $\bar{x} = \frac{\sum_{i=1}^n x_i}{n}$;
2. $s^2 = \frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n - 1} = \frac{1}{n - 1} \left(\sum_{i=1}^n x_i^2 - n\bar{x}^2 \right)$;
3. $C.V = \frac{s}{\bar{x}}$.

Probabilidade

1. $P(A \cup B) = P(A) + P(B) - P(A \cap B)$;
2. $P(A|B) = P(A \cap B)/P(B)$;
3. $P(A) = \sum_{i=1}^k P(A|B_i)P(B_i)$;
4. $P(B_i|A) = \frac{P(A|B_i)P(B_i)}{\sum_{i=1}^k P(A|B_i)P(B_i)}$.

Distribuição de probabilidade

Seja X uma variável aleatória discreta. Então,

$$\mu = E(X) = \sum_x xP(X=x) \quad \text{e} \quad Var(X) = E(X - \mu)^2 = \sum_x (x - \mu)^2 P(X=x).$$

1. Se $X \sim \text{Uniforme Discreta} \Rightarrow P(X=x) = \begin{cases} 1/k, & x = 1, 2, \dots, k; \\ 0, & \text{caso contrário.} \end{cases}$
2. Se $X \sim b(p) \Rightarrow P(X=x) = \begin{cases} p^x(1-p)^{1-x}, & x = 0, 1; \\ 0, & \text{caso contrário.} \end{cases}$
3. Se $X \sim Bin(n, p) \Rightarrow P(X=x) = \begin{cases} \binom{n}{x} p^x (1-p)^{n-x}, & x = 0, 1, 2, \dots, n; \\ 0, & \text{caso contrário,} \end{cases} \quad \text{onde } \binom{n}{x} = \frac{n!}{x!(n-x)!}.$
4. Se $X \sim G(p) \Rightarrow P(X=x) = p(1-p)^{x-1}, \quad x = 1, 2, \dots$
5. Se $X \sim Hip(N, n, r) \Rightarrow P(X=x) = \begin{cases} \frac{\binom{r}{x} \binom{N-r}{n-x}}{\binom{N}{n}}, & x = 0, \dots, \min\{r, n\}; \\ 0, & \text{caso contrário.} \end{cases}$
6. Se $X \sim P(\lambda) \Rightarrow P(X=x) = \begin{cases} \frac{e^{-\lambda} \lambda^x}{x!}, & x = 0, 1, 2, \dots; \\ 0, & \text{caso contrário.} \end{cases}$

Densidade de probabilidade

Seja X uma variável aleatória contínua. Então,

1. Se $X \sim \text{Uniforme}(a, b) \Rightarrow f(x) = \begin{cases} \frac{1}{(b-a)}, & a \leq x \leq b; \\ 0, & \text{caso contrário.} \end{cases}$

2. Se $X \sim \text{Exp}(\lambda) \Rightarrow f(x) = \lambda e^{-\lambda x}, \quad x \geq 0.$

3. Se $X \sim \text{Normal}(\mu, \sigma^2) \Rightarrow f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left\{-\frac{(x-\mu)^2}{2\sigma^2}\right\}, \quad -\infty \leq x \leq \infty.$

Inferência

1. Se $X \sim \text{Normal}(\mu, \sigma^2)$, então $Z = \frac{X-\mu}{\sigma} \sim \text{Normal}(0, 1)$;

2. $P(-z_{\alpha/2} < Z < z_{\alpha/2}) = 1 - \alpha$, $P(Z > z_{\alpha}) = \alpha$ e $P(Z < -z_{\alpha}) = \alpha$;

3. $\hat{p} \sim \text{Normal}(p, \frac{p(1-p)}{n}) \Rightarrow \frac{\hat{p}-p}{\sqrt{p(1-p)/n}} \sim \text{Normal}(0, 1)$;

4. $\left[\hat{p} - z_{\alpha/2} \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}; \hat{p} + z_{\alpha/2} \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \right]$;

5. $\left[\hat{p} - z_{\alpha/2} \sqrt{\frac{1}{4n}}; \hat{p} + z_{\alpha/2} \sqrt{\frac{1}{4n}} \right]$;

6. $n = \left(\frac{z_{\alpha/2}}{2m} \right)^2$;

7. $\bar{X} \sim \text{Normal}(\mu, \frac{\sigma^2}{n})$ então $\frac{\bar{X}-\mu}{\sigma/\sqrt{n}} \sim \text{Normal}(0, 1)$, $\frac{\bar{X}-\mu}{s/\sqrt{n}} \sim t_{n-1}$;

8. $\left[\bar{x} - z_{\alpha/2} \sqrt{\frac{\sigma^2}{n}}; \bar{x} + z_{\alpha/2} \sqrt{\frac{\sigma^2}{n}} \right]$;

9. $\left[\bar{x} - t_{n-1, \alpha/2} \sqrt{\frac{s^2}{n}}; \bar{x} + t_{n-1, \alpha/2} \sqrt{\frac{s^2}{n}} \right]$;

10. $n = \left(\frac{z_{\alpha/2}}{m} \right)^2 \sigma^2$;

11. Sob $H_0 : \mu_x = \mu_y$,

$$\frac{\bar{X} - \bar{Y}}{\sqrt{\sigma_x^2/n + \sigma_y^2/m}} \sim \text{Normal}(0, 1), \quad \frac{\bar{X} - \bar{Y}}{\sqrt{s_p^2(1/n + 1/m)}} \sim t_{n+m-2},$$

$$s_p^2 = \frac{(n-1)s_x^2 + (m-1)s_y^2}{n+m-2};$$

12. Sob $H_0 : p_1 = p_2$,

$$\frac{\hat{p}_1 - \hat{p}_2}{\sqrt{p_0(1-p_0)(1/n + 1/m)}} \sim \text{Normal}(0, 1),$$

$$p_0 = \frac{n\hat{p}_1 + m\hat{p}_2}{n+m}.$$