Gabarito 2ª Lista

① $\mathbb{P}(A|B) = \frac{\mathbb{P}(A \cap B)}{\mathbb{P}(B)} = \mathbb{P}(A)$

$\Rightarrow \mathbb{P}(A \cap B) = \mathbb{P}(A)\,\mathbb{P}(B)$

$\mathbb{P}(A \cap B) = \mathbb{P}(A).\mathbb{P}(B) \Rightarrow A$ e B são independentes

$\mathbb{P}(A/B) = \frac{\mathbb{P}(A).\mathbb{P}(B)}{\mathbb{P}(B)} = \mathbb{P}(A)$

② $\mathbb{P}(A \cup B) \leq \mathbb{P}(A) + \mathbb{P}(B)$

$\mathbb{P}(A \cup B) = \underbrace{\mathbb{P}(A)}_{\geq 0} + \underbrace{\mathbb{P}(B)}_{\geq 0} - \underbrace{\mathbb{P}(A \cap B)}_{\geq 0}$

$\Rightarrow \mathbb{P}(A \cup B) \leq \mathbb{P}(A) + \mathbb{P}(B)$

③ $\Omega = \{(1,1), (1,2), (1,3), (1,4), (1,5), (1,6),$
$(2,1), (2,2), (2,3), (2,4), (2,5), (2,6),$
$(3,1), (3,2), (3,3), (3,4), (3,5), (3,6),$
$(4,1), (4,2), (4,3), (4,4), (4,5), (4,6),$
$(5,1), (5,2), (5,3), (5,4), (5,5), (5,6),$
$(6,1), (6,2), (5,3), (6,4), (6,5), (6,6)\}$

A = Soma dos pontos é par $\quad \mathbb{P}(A) = 18/36 = 1/2$ (i).

B = Soma é ímpar $\quad \mathbb{P}(B) = 18/36 = 1/2$ (ii).

C = o primeiro lançamento é menor que o segundo $\quad \mathbb{P}(C) = 15/36$ (iii)

D = a soma é diferente de dois $\quad \mathbb{P}(D) = 35/36$. (v)

E = a soma ≤ 4 $\quad \mathbb{P}(E) = 6/36 = 1/6$

F = a soma > 2 $\quad \mathbb{P}(F) = 35/36$

$\mathbb{P}(E \cup F) = \frac{6}{36} + \frac{35}{36} - \frac{5}{36} = 1$ (vi).

G = iguais resultados nos dois lanç. $\quad \mathbb{P}(G) = 6/36 = 1/6$

H = a soma é igual a 7. $\quad \mathbb{P}(H) = 6/36 = 1/6$ (iv)

$\mathbb{P}(C \cap A) = 6/36 = 1/6$ (vii).

$\mathbb{P}(B \cap G) = 0$

④ 5 bolas pretas.
10 bolas Vermelhas. $\Big\rangle$ 15 bolas

PP $\quad \mathbb{P}(PP) = \frac{5}{15} \cdot \frac{4}{14} = 20/210 = \frac{2}{21}$ $\quad \mathbb{P}(VV) = \frac{10}{15} \cdot \frac{9}{14} = \frac{9}{21} = 9/$

PV $\quad \mathbb{P}(PV) = \frac{5}{15} \cdot \frac{10}{14} = \frac{50}{210} = 5/21$

VP $\quad \mathbb{P}(VP) = \frac{10}{15} \cdot \frac{5}{14} = 50/210 = 5/21$

VV

i) A = 1° bola preta $\quad \mathbb{P}(A \cap B) = 2/21$

B = 2° bola preta $\quad \mathbb{P}(B) = 7/21$

C = 1° bola vermelha $\quad \mathbb{P}(C) = 14/21$

⑤ A: Viver 70 ou mais $\quad \mathbb{P}(A) = 0.6$

B: Viver 80 ou mais $\quad \mathbb{P}(B) = 0.2$

$$\mathbb{P}(B/A) = \frac{\mathbb{P}(B \cap A)}{\mathbb{P}(A)} = \frac{\mathbb{P}(A/B) \cdot \mathbb{P}(B)}{\mathbb{P}(A)} = \frac{\mathbb{P}(B)}{\mathbb{P}(A)} = \frac{0.2}{0.6}$$

⑥ Restaurante < salada / carne

Homens 20% salada $\quad$ B: prefere carne

mulheres 30% carne $\quad$ A: prefere salada

M o freguês é mulher $\quad \mathbb{P}(H) = 0.25 \quad \mathbb{P}(A/H) = 0.20$

H o freguês é homem $\quad \mathbb{P}(M) = 0.75$

$\mathbb{P}(A/H) = 0,20$

$$\mathbb{P}(M/A) = \frac{\mathbb{P}(M \cap A)}{\mathbb{P}(A)} = \mathbb{P}(A/M) \cdot \mathbb{P}(M)$$

$\mathbb{P}(B/M) = 0,30$

$$\mathbb{P}(A) = \mathbb{P}(A/M)\,\mathbb{P}(M) + \mathbb{P}(A/H)\,\mathbb{P}(H)$$
$$= 0.70 \cdot 0.25 + 0.20 \cdot 0.75$$

⑦ A: saber a resposta $\quad \mathbb{P}(A) = p$

B: acertar a resposta $\quad \mathbb{P}(B/A) = 1$

$$\mathbb{P}(A/B) = \frac{\mathbb{P}(A \cap B)}{\mathbb{P}(B)} = \frac{\mathbb{P}(B/A)\,\mathbb{P}(A)}{\mathbb{P}(B/A)\,\mathbb{P}(A) + \mathbb{P}(B/A^c)\,\mathbb{P}(A^c)} = \frac{p}{p + \frac{1}{n}(1-p)}$$

$$= \frac{p}{\frac{np + 1 - p}{n}} = \frac{np}{np + 1 - p} = \frac{np}{p(n-1) + 1}$$

$$\mathbb{P}(B^c) = 1 - \mathbb{P}(B) = 1 - \left(p + \frac{1}{n}(1-p)\right)$$

$$= 1 - (0.2 + 0.8/5) = 1 - 0{,}36 = 0{,}64$$

(8) A: o circuito é da fabrica A (produzido) $\mathbb{P}(A) = 0.50$

B: o circuito é da fabrica B. $\mathbb{P}(B) = 0.25$

C: o circuito é da fabrica C $\mathbb{P}(C) = 0.25$

F: circuito não funcionar

$\mathbb{P}(F/A) = 0{,}01$

$\mathbb{P}(F/B) = 0{,}04$

$\mathbb{P}(F/C) = 0{,}03$

$$\mathbb{P}(F) = \mathbb{P}(F/A)\cdot\mathbb{P}(A) + \mathbb{P}(F/B)\cdot\mathbb{P}(B) + \mathbb{P}(F/C)\cdot\mathbb{P}(C)$$

(9) X = # de navios que chegam a uma certa refinaria / por dia

$X \sim$ Poisson (λ) $\lambda = 2$

a) $\mathbb{P}(X > 3)$

b) $\mathbb{E}(X) = 2$

(10) λ = # médio de carros abandonados semanalmente em uma rodovia

$= 3$

$X \sim$ Poisson (λ)

(a) $\mathbb{P}(X = 0)$

(b) $\mathbb{P}(X \geq 2)$

(11) $X = \begin{cases} 1 & \text{prob } 1/3 \\ 2 & \text{prob } 1/2 \\ 25 & \text{prob } 1/6 \end{cases}$

$$\mathbb{E}(X) = \frac{1}{3} + 1 + \frac{25}{6} = \frac{2+6+25}{6} = \frac{33}{6} = 5{,}5$$

$$\mathbb{E}(X^2) = \frac{1}{3} + 2 + \frac{625}{6} = \frac{2+12+625}{6} = \frac{639}{6} = 106{,}5$$

$$\text{Var}(X) = 106{,}5 - \underbrace{5{,}5^2}_{30{,}25} = 76{,}25$$

(12) $\mathrm{Var}(X) = \mathbb{E}(X^2) - (\mathbb{E}(X))^2$

$\mathbb{E}(X^2) = \mathrm{Var}(X) + (\mathbb{E}(X))^2$

$\mathbb{E}(X) = np = 5/3$

$\mathrm{Var}(X) = np(1-p)$

$X \sim \mathrm{Bin}(5, 1/3)$

$$\mathbb{E}(X^2) = np(1-p) + n^2p^2 = np(1-p+np)$$
$$= \frac{5}{3}\left(\frac{2}{3} + \frac{5}{3}\right) = \frac{5}{3}(7/3) = 35/9$$

(13) X = soma dos resultados

$\mathbb{E}(X) = 7$

$X = 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12$

$\mathbb{P}(X = x)$

(14) $X = 3, 4, 5$

- 3: 1,2 — $1/3 \cdot 1/2 = 1/6$; 2,1 — $1/3 \cdot 1/2 = 1/6$ } 2/6
- 4: 2,3 — $1/3 \cdot 1/2 = 1/6$; 3,2 — $1/3 \cdot 1/2 = 1/6$ } 2/6
- 5: 3,1 — $1/3 \cdot 1/2 = 1/6$; 1 3 — $1/3 \cdot 1/2 = 1/6$ } 2/6

$\Rightarrow$

$X =$
- 3 com prob. 2/6
- 4 com prob. 2/6
- 5 com prob 2/6

$Y =$
- 1 $\leftarrow$ 1/2: 2
- 1 $\leftarrow$ 1/2: 3
- 2 $\leftarrow$ 1/2: 3
- 2 $\leftarrow$ 1/2: 4
- 3 $\leftarrow$ 1/2: 4
- 3 $\leftarrow$ 1/2: 5

$\Rightarrow$

$Y =$
- 2 com prob 1/9
- 3 com prob 2/9
- 4 com prob 3/9
- 5 com prob 2/9
- 6 com prob 1/9

(13)

$\mathbb{P}(X=2) = \frac{1}{36}$

$\mathbb{P}(X=3) = \frac{2}{36}$

$\mathbb{P}(X=4) = \frac{3}{36}$

$\mathbb{P}(X=5) = \frac{4}{36}$

$\mathbb{P}(X=6) = \frac{5}{36}$

$\mathbb{P}(X=7) = \frac{6}{36}$

$\mathbb{P}(X=8) = 5/36$

$\mathbb{P}(X=9) = 4/36$

$\mathbb{P}(X=10) = 3/36$

$\mathbb{P}(X=11) = 2/36$

$\mathbb{P}(X=12) = 1/36$