ME414 2S2019

FORMULÁRIO

1.
$$\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n}$$
;

2.
$$s^2 = \frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n-1} = \frac{1}{n-1} (\sum_{i=1}^n x_i^2 - n\bar{x}^2);$$

$$3. C.V = \frac{s}{\bar{x}}.$$

Probabilidade

1.
$$P(A \cup B) = P(A) + P(B) - P(A \cap B);$$

3.
$$P(A) = \sum_{i=1}^{k} P(A|B_i)P(B_i);$$

2.
$$P(A|B) = P(A \cap B)/P(B)$$
;

4.
$$P(B_i|A) = \frac{P(A|B_i)P(B_i)}{\sum_{i=1}^k P(A|B_i)P(B_i)}$$
.

Distribuição de probabilidade

Seja X uma variável aleatória discreta. Então,

$$\mu = \mathbb{E}(X) = \sum_{x} x P(X = x)$$
 e $Var(X) = \mathbb{E}(X - \mu)^2 = \sum_{x} (x - \mu)^2 P(X = x)$.

1. Se
$$X \sim$$
 Uniforme Discreta $\Rightarrow P(X = x) = \begin{cases} 1/k, & x = 1, 2, ..., k; \\ 0, & \text{caso contrário.} \end{cases}$

2. Se
$$X \sim b(p)$$
 \Rightarrow $P(X = x) = \begin{cases} p^x (1-p)^{1-x}, & x = 0, 1; \\ 0, & \text{caso contrário.} \end{cases}$

3. Se
$$X \sim Bin(n,p)$$
 \Rightarrow $P(X=x) = \begin{cases} \binom{n}{x} p^x (1-p)^{n-x}, & x=0,1,2,\ldots,n; \\ 0, & \text{caso contrário,} \end{cases}$ onde $\binom{n}{x} = \frac{n!}{x!(n-x)!}$.

4. Se
$$X \sim G(p)$$
 \Rightarrow $P(X = x) = p(1 - p)^{x-1}$, $x = 1, 2, ...$

5. Se
$$X \sim Hip(N, n, r)$$
 \Rightarrow $P(X = x) = \begin{cases} \frac{\binom{r}{x} \binom{N-r}{n-x}}{\binom{N}{n}}, & x = 0, \dots, min\{r, n\}; \\ \binom{N}{n}, & 0, & \text{caso contrário.} \end{cases}$

6. Se
$$X \sim P(\lambda)$$
 \Rightarrow $P(x = x) = \begin{cases} \frac{e^{-\lambda}\lambda^x}{x!}, & x = 0, 1, 2, ...; \\ 0, & \text{caso contrário.} \end{cases}$

Densidade de probabilidade

Seja X uma variável aleatória contínua. Então,

$$\mu = \mathbb{E}(X) = \int_{-\infty}^{\infty} x f_X(x) dx$$
 e $\operatorname{Var}(X) = \mathbb{E}(X - \mu)^2 = \int_{-\infty}^{\infty} (x - \mu)^2 f_X(x) dx$.

1. Se
$$X \sim \text{Uniforme}(\mathbf{a}, \mathbf{b}) \implies f(x) = \begin{cases} \frac{1}{(b-a)}, & a \le x \le b; \\ 0, & \text{caso contrário.} \end{cases}$$

2. Se
$$X \sim \text{Exp}(\lambda) \implies f(x) = \lambda e^{-\lambda x}, \quad x \ge 0.$$

3. Se
$$X \sim \text{Normal}(\mu, \sigma^2)$$
 \Rightarrow $f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\{-\frac{(x-\mu)^2}{2\sigma^2}\}, -\infty \le x \le \infty.$

4. Se
$$X \sim \text{Gama}(\alpha, \beta)$$
 \Rightarrow $f(x) = \begin{cases} \frac{\beta^{\alpha}}{\Gamma(\alpha)} x^{\alpha - 1} e^{-\beta x}, \text{ se } x \geq 0 \\ 0, \text{ caso contrário.} \end{cases}$

5. Se
$$X \sim \text{Beta}(\alpha, \beta) \implies f(x) = \frac{\Gamma(\alpha + \beta)}{\Gamma(\alpha)\Gamma(\beta)} x^{\alpha - 1} (1 - x)^{\beta - 1}, \quad x \in (0, 1) \text{ e } \alpha, \beta > 0.$$

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Inferência

1. Se $X \sim \text{Normal}(\mu, \sigma^2)$, então $Z = \frac{X - \mu}{\sigma} \sim \text{Normal}(0, 1)$;

2.
$$P(-z_{\alpha/2} < Z < z_{\alpha/2}) = 1 - \alpha$$
, $P(Z > z_{\alpha}) = \alpha$ e $P(Z < -z_{\alpha}) = \alpha$;

3.
$$\hat{p} \sim \text{Normal}(p, \frac{p(1-p)}{n}) \Rightarrow \frac{\hat{p}-p}{\sqrt{p(1-p)/n}} \sim \text{Normal}(0, 1);$$

4.
$$\left[\hat{p} - z_{\alpha/2} \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}; \ \hat{p} + z_{\alpha/2} \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \right];$$

5.
$$\left[\hat{p} - z_{\alpha/2}\sqrt{\frac{1}{4n}}; \ \hat{p} + z_{\alpha/2}\sqrt{\frac{1}{4n}}\right];$$

6.
$$n = \left(\frac{z_{\alpha/2}}{2m}\right)^2$$
;

7.
$$\bar{X} \sim \text{Normal}(\mu, \frac{\sigma^2}{n})$$
, então $\frac{\bar{X} - \mu}{\sigma / \sqrt{n}} \sim \text{Normal}(0, 1)$;

8.
$$\left[\bar{x} - z_{\alpha/2}\sqrt{\frac{\sigma^2}{n}}; \ \bar{x} + z_{\alpha/2}\sqrt{\frac{\sigma^2}{n}}\right];$$

9.
$$\left[\bar{x} - t_{n-1,\alpha/2} \sqrt{\frac{s^2}{n}}; \ \bar{x} + t_{n-1,\alpha/2} \sqrt{\frac{s^2}{n}} \right];$$

10.
$$n = \left(\frac{z_{\alpha/2}}{m}\right)^2 \sigma^2$$
;

11. Sob
$$H_0: \mu_x = \mu_y$$
,

$$\frac{\bar{X} - \bar{Y}}{\sqrt{\sigma_x^2/n + \sigma_y^2/m}} \sim \text{Normal}(0, 1), \quad \frac{\bar{X} - \bar{Y}}{\sqrt{s_p^2(1/n + 1/m)}} \sim t_{n+m-2},$$

onde
$$s_p^2 = \frac{(n-1)s_x^2 + (m-1)s_y^2}{n+m-2}$$

12. Sob
$$H_0: p_1 = p_2$$
,

$$\frac{\hat{p_1} - \hat{p_2}}{\sqrt{p_0(1 - p_0)(1/n + 1/m)}} \sim \text{Normal}(0, 1),$$

onde
$$p_0 = \frac{n\hat{p_1} + m\hat{p_2}}{n+m};$$

13. Sob H_0 ,

$$\chi^2 = \sum_{i=1}^k \frac{(O_i - E_i)^2}{E_i} \sim \chi_{k-1}^2.$$

Regressão

O modelo de regressão é dado por $Y_i = \alpha + \beta X_i + \epsilon_i$, i = 1, ..., n, com $\mathbb{E}(\epsilon_i) = 0$ e $Var(\epsilon_i) = \sigma^2$. A reta estimada é dada por $\hat{Y}_i = \hat{\alpha} + \hat{\beta} X_i$, onde

$$\hat{\alpha} = \bar{Y} - \hat{\beta}\bar{X}; \quad \hat{\beta} = \frac{\sum_{i=1}^{n} (X_i - \bar{X})(Y_i - \bar{Y})}{\sum_{i=1}^{n} (X_i - \bar{X})^2} = \frac{\sum_{i=1}^{n} X_i Y_i - n\bar{X}\bar{Y}}{\sum_{i=1}^{n} X_i^2 - n\bar{X}^2} = \frac{S_{XY}}{S_{XX}}.$$

Coeficiente de Correlação:

$$R = \frac{\sum_{i=1}^{n} (X_i - \bar{X})(Y_i - \bar{Y})}{\sqrt{\sum_{i=1}^{n} (X_i - \bar{X})^2} \sqrt{\sum_{i=1}^{n} (Y_i - \bar{Y})^2}}.$$