MAT1856/APM466 Assignment 1

Lanlan Yang, Student #: 1003453185

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Fundamental Questions

1.

   (a) A government issues bond to raise fund.Compared to stock, bonds are easier to control and have more security. This is also a way of reducing money supply, by reducing cash in society.

   (b) A yield curve gives information about the whole bond market. It is independent of individual bond, but captures information of all bonds.

   (c) A government can reduce money supply by selling bonds, where people's cash in households will be decreased.

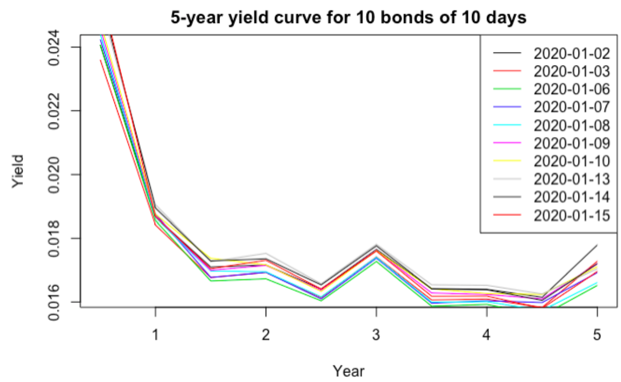
2. 10 bonds I selected are: CAN 1.5 March 01 2020, CAN 0.75 Sept. 01 2020, CAN 0.75 March 01 2021, CAN 0.75 Sept. 01 2021, CAN 0.5 March 01 2022, CAN 2.75 June 01 2022, CAN 1.75 March 01 2023, CAN 1.5 June 01 2023, CAN 2.25 March 01 2024, CAN 1.5 Sept. 01 2024.
   These are the bonds that mature consecutively in 2020 - 2024 five years periods, maturing in every six months, which fit the semi-annual coupon payment. Since for 2022 and 2023, we don't have bonds mature on September 01, I selected bonds mature on June 01 in the same year.
   When calculating yields, spot rate and forward rate, they will take the place of Sept bonds, and we assume they pay coupons in a March-September schedule. This is based on the nearest neighbourhood algorithm.

3. When we have several stochastic curves, there are many information so we want to simplify it, but do not want to lose important ones. Eigenvalues and eigenvectors work in such ways that they tells us the original covariance matrix only change the magnitude of eigenvetors, without changing their directions. It is important because eigenvalues are scalars which we can rank. By ranking the eigenvalues we know which information (eigenvector) the original matrix change the largest, the second largest, etc. Hence we sort out the most important information. The eigenvector with the largest eigenvalue correspond to the largest variance).

Empirical Questions - 75 points

(a)

i. The 5-year yields of 10 selected bonds are calculated using function bond.yield in R, corresponding to each day of data. According to the maturity of 10 bonds, the 5-year yield curve of each day are corresponding to every half year in the next five year.



ii. By the following formula:

$P = \sum_{i=1}^{m-1} p_i \cdot e^{-r(t_i)\cdot t_i} + p_m \cdot e^{-r(t_m)\dot{t}_m}$

We can derive the formula for spot rate:

$r_m = -ln((P - \sum_{i=1}^{m-1} p_i \cdot e^{-r(t_i)\cdot t_i})/p_m)/t_m$

The first bond we chose will mature on March 2020, so it does not have coupon due before maturity, and hence we can solve its spot rate first and use it for solving next one.

pseudo-code:
c1 = coupon rate *100/2
redem = 100
for (i in 1:10)
    date1 = settle day
    spot[i,1] = -log(dirtyprice[i,1]/(redem+c1))/yearFraction(date1, "2020-03-01")

For the second, which we will solve for 1-year spot rate, we can use the above result to solve. The code is largely the same , except that I add one line in the loop :

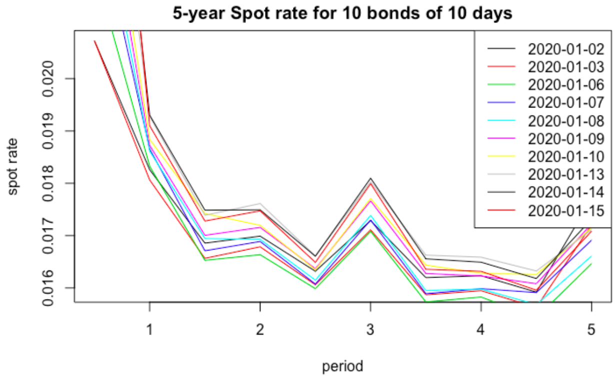
    cf2 = c2*exp(-spot[i,1]*t1)

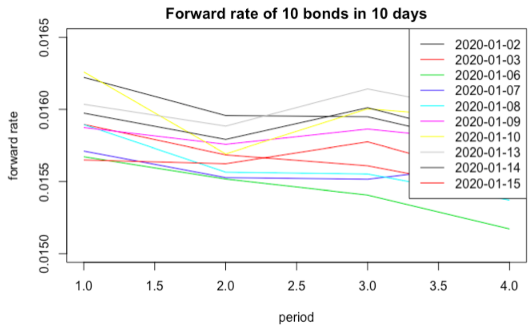
iii. According to the formula:

$r_1, j = (r_j t_j - r_1 \dot{t}_1)/t_j - t_1$

r_i: spot rate, t_i: settle day to maturity t
We set up pseudo code as follow:
for (i in 1:10)
    day1 = settle day

tt1 = yearFraction(day1, "2020-03-01")
tt2 = yearFraction(day1, "2021-03-01")
forwardrate[i,1] = (spot[i,3]*tt2 - spot[i,1]*tt1)/(tt2-tt1)





(b) Covariance matrix of log return of yields =

1.456076e-04	8.871111e-05	3.922183e-05	6.801543e-05	7.473699e-05
8.871111e-05	1.146489e-04	4.758666e-05	8.596835e-05	7.383219e-05
3.922183e-05	4.758666e-05	1.656424e-04	4.531644e-05	5.327988e-05
6.801543e-05	8.596835e-05	4.531644e-05	1.419230e-04	1.046521e-04
7.473699e-05	7.383219e-05	5.327988e-05	1.046521e-04	9.812230e-05

Covariance of log return of forward rates =

2.272317e-04	5.045522e-05	0.0001285449	0.0001272191
5.045522e-05	1.042927e-04	0.0001266534	0.0001227353
1.285449e-04	1.266534e-04	0.0001885356	0.0002154810
1.272191e-04	1.227353e-04	0.0002154810	0.0003671712

(c) The eigenvalues for log return of yield covariance are as follows:
4.090391e-04 1.321614e-04 8.176811e-05 3.383887e-05 9.136676e-06
Eigenvectors as follows:
1. -0.4628869 -0.27416544 0.72200854 0.38577412 -0.20112436
2. -0.4556333 -0.15331294 0.14053749 -0.85754631 0.11729013
3. -0.3551199 0.92615790 0.08558833 0.02475754 -0.09046043
4. -0.5005897 -0.19322443 -0.61656445 0.12247653 -0.56295827
5. -0.4488137 -0.07889438 -0.26734958 0.31651101 0.78783686

The eigenvalues for log return of forward covariance are as follows:
6.546766e-04 1.552793e-04 7.575451e-05 1.520704e-06
Eigenvectors as follows:
1. -0.3977378 0.88728176 0.1341188 0.1911749
2. -0.3095590 -0.16895565 -0.6943810 0.6272656
3. -0.5144855 -0.00562931 -0.4361724 -0.7382592
4. -0.6937453 -0.42913095 0.5564178 0.1579980

References

Markets Insider, bond finder: issuer: Government of Canada, maturity: 0-10 years.
https://markets.businessinsider.com/bonds/finder?borrower=71maturity=shorttermyield=bondtype=2.
Accessed January 29 2020.

PCA: Eigenvectors and Eigenvalues, Towards Data science
URL:https://towardsdatascience.com/pca-eigenvectors-and-eigenvalues-1f968bc6777a

GitHub Link to Code

https://github.com/larissayang/apm466-assignment1