Measuring Exploration/Exploitation in Particle Swarms using Swarm Diversity

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***Abstract*— An important factor contributing to the success of particle swarm optimization (PSO) is the balance between exploration and exploitation of the swarm. Exploration is typically preferred at the initial stages of the search but is required to gradually give way to exploitation of promising solutions as the search progresses. The *diversity* of a particle swarm optimization algorithm can be defined, simply, as the degree of dispersion of the particles in the swarm. This dispersion could be defined around some center-point or not. It could also be defined based on the positions of the particles or on their velocities.**

This paper takes a look at some of the different definitions of swarm diversity with the intention of determining their usefulness in quantifying swarm exploration/exploitation. This work is intended to lay the foundations for the development of a suitable means to quantify the rate of change from exploration to exploitation of a PSO, i.e. the rate of change of diversity.

I. Introduction

The diversity of a particle swarm optimization algorithm (PSO) has a very important influence on the performance of the algorithm. Its use in quantifying swarm exploration or exploitation is somewhat intuitive, as a large diversity should directly imply that a large area of the search space is being explored [1]. Similarly, a smaller population diversity should imply that the particles are exploiting a small area of the search space.

The need to quantify the search behavior of a PSO algorithm becomes significant when it is considered that some PSO algorithms make use of swarm diversity to guide their search e.g. the Attractive-Repulsive PSO developed in [2]. In order for such algorithms to demonstrate the correct search behavior, a diversity measure that accurately measures the dispersion of the particles in a swarm at a given time relative to some other time is required.

Several measures of diversity can be found in literature, with different definitions given in [2], [3], [4], [5], [6], [7].

In a few words, there are several ways to measure swarm diversity and even more variations of each of these methods. Not all of these, however can be used to accurately quantify the search behavior of a particle swarm, i.e. whether the swarm explores or exploits.

This paper presents the results of analysis which shows the behavior of these measures over the course of execution of a PSO algorithm.

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Section II provides some background information on the PSO algorithm. Section III gives general definitions for most of the common diversity measures. In Section IV an analysis of the behavior of these diversity measures during a PSO execution is presented. Lastly, in Section V the various conclusions that have been arrived at are given, with details on possible further research efforts.

II. The Particle Swarm Optimization Algorithm

The Particle Swarm Optimization (PSO) algorithm, introduced in [8] and [9], maintains a swarm of particles where each particle in the swarm is a potential solution to the problem being solved.

Let $\tilde{\mathbf{x}}_i(t)$ denote the position of a particle i in the solution search space at a time step t. The position of the particle could be changed by adding a velocity, $\tilde{\mathbf{v}}_i(t)$, to the current position as follows:

$$\tilde{\mathbf{x}}_i(t+1) = \tilde{\mathbf{x}}_i(t) + \tilde{\mathbf{v}}_i(t+1) \tag{1}$$

The particle velocity is the driving force behind the optimization process. It represents the knowledge gained by the particle, both through the experience of the particle itself (cognitive component) and through socially exchanged information acquired from interaction with other particles in its neighborhood (social component).

The first two PSO algorithms that were originally developed are the *gbest* PSO, which uses the *star* social network topology and the *lbest* PSO, based on a ring social network topology [10], [11]. The gbest PSO algorithm is used in this paper and the discussion on PSO will, therefore, be focused on that version of the PSO algorithm.

For a given particle i in a swarm, the velocity is calculated as follows:

$$\begin{aligned}\tilde{\mathbf{v}}_i(t+1) = \omega\tilde{\mathbf{v}}_i(t) + c_1 r_1(t)[\tilde{\mathbf{y}}_i(t) - \tilde{\mathbf{x}}_i(t)] \\ + c_2 r_2(t)[\hat{\mathbf{y}}(t) - \tilde{\mathbf{x}}_i(t)]\end{aligned} \tag{2}$$

where $\tilde{\mathbf{v}}_i(t)$ is the particle i velocity at time step t, $\tilde{\mathbf{x}}_i(t)$ is the particle i position at time step t, c_1 and c_2 are positive constants of acceleration and are used to scale the contributions of the cognitive and social components, respectively, and $r_1(t), r_2(t) \sim U(0,1)$ are random numbers sampled from the uniform distribution $U(0,1)$ used to add a stochastic element to the PSO algorithm. An inertia weight value given by ω is used to control the influence of the previous velocity on the new velocity.

$\tilde{\mathbf{y}}_i(t)$ is the *personal best* position, for a particle i, and is the best position that the particle has visited up until time

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step t. $\hat{\mathbf{y}}(t)$ represents the *global best* position, at time step t. It denotes the best position (and consequently, solution) that has been found so far by any of the particles in the swarm.

A summary of the gbest PSO algorithm is given below:

1) Create and initialize an n_x-dimensional swarm, S.
2) **for each** particle $i = 1, \ldots, S.n_s$ set the personal and global best positions; update the particle's velocity and position
3) Repeat step 2 until a stopping condition is reached.

III. Measures of Swarm Diversity

A. On Variations

As was mentioned in Section I, there are many different diversity measures and different variations of these. Many of these variations are due to differences in the distance metric used, or differences in the normalization parameters. Another reason for the variations in some of the measures is the choice of the swarm center, which can either be spatial or gbest.

Due to space constraints it is not possible to consider every single variation of the different diversity measures. Consequently, only the most *general form* is used, which makes use of the Euclidean distance metric wherever a distance metric is required. Also, a spatial swarm center is used as opposed to a gbest swarm center. This is due to the fact that a gbest centered diversity can be seen as equivalent to a spatially centered diversity where the former makes use of an unfixed, "not necessarily centered" position as the swarm center and hence both measures should, on average (over a number of algorithm iterations), return the same value for diversity. Also, where normalization is required, the diameter of the swarm is used, as opposed to the swarm radius.

The diversity measures considered in this paper are given in the next sub-sections.

B. The Swarm Diameter and Swarm Radius

The diameter of a swarm is defined as the maximum distance between any 2 particles in the swarm [2] and the radius can be calculated as the distance between the swarm center and the particle in the swarm which is furthest away from it. The diameter is calculated as follows:

$$|D| = \max_{(i \neq j) \in [1, |S|]} \left(\sqrt{\sum_{k=1}^{I} (x_{ik} - x_{jk})^2} \right) \tag{3}$$

where $|S|$ is the swarm size, I is the dimensionality of the problem (and its solution) and x_{ik} is the k-th dimension of the i-th particle position. The radius is calculated as:

$$|R| = \max_{i \in [1, |S|]} \left(\sqrt{\sum_{k=1}^{I} (x_{ik} - \bar{\mathbf{x}}_k)^2} \right) \tag{4}$$

where the variables are as defined for the swarm diameter and $\bar{x}_k$ is the k-th dimension of the swarm center position $\bar{\mathbf{x}}$.

Both the swarm diameter and radius could be used as diversity measures, where a large value signifies high particle dispersion and smaller values signify convergence.

C. The Average Distance around the Swarm Center

This measure is given in [5] by:

$$\mathcal{D} = \frac{1}{|S|} \sum_{i=1}^{|S|} \sqrt{\sum_{k=1}^{I} (x_{ik} - \bar{\mathbf{x}}_k)^2} \tag{5}$$

where the variables are as defined in sub-section B.

The diversity of the swarm as calculated using this measure is the average distance value returned by equation 5. A small value indicates swarm convergence around the swarm center while a large value indicates a higher dispersion of particles away from the center.

D. The Normalized Average Distance around the Swarm Center

This is exactly the same as the above, except for the further normalization that is done with regards to the swarm diameter. The normalized average distance around the swarm center is given in [2] by:

$$\mathcal{D}^N = \frac{1}{|S|.|D|} \sum_{i=1}^{|S|} \sqrt{\sum_{k=1}^{I} (x_{ik} - \bar{\mathbf{x}}_k)^2} \tag{6}$$

The radius of the swarm could also be used as a normalizing variable.

E. The Average of the Average Distance around all Particles in the Swarm

This extends the concept of distance around the swarm center by evaluating the average distance around each particle in the swarm, i.e. each swarm particle is used as a center, and then calculating the average over all these distances. It is calculated as follows:

$$\mathcal{D}_{all} = \frac{1}{|S|} \sum_{i=1}^{|S|} \left(\frac{1}{|S|} \sum_{j=1}^{|S|} \sqrt{\sum_{k=1}^{I} (x_{ik} - x_{jk})^2} \right) \tag{7}$$

where

$$\frac{1}{|S|} \sum_{j=1}^{|S|} \sqrt{\sum_{k=1}^{I} (x_{ik} - x_{jk})^2}$$

is the average distance around the particle x_i.

Swarm diversity, as calculated by this measure, gives an indication of the average dispersion of all particles in the swarm relative to every other particle in the swarm.

F. Swarm Coherence

Swarm coherence is defined in [4] as follows:

$$S_c = \frac{\mathbf{v}_s}{\bar{\mathbf{v}}} \tag{8}$$

where $\mathbf{v}_s$ represents the speed of the swarm center given by:

$$\mathbf{v}_s = \frac{1}{|S|} \left\| \sum_{i=1}^{|S|} \tilde{\mathbf{v}}_i \right\|_2 \tag{9}$$

and $\bar{\mathbf{v}}$ represents the average particle speed of the swarm and is given by:

$$\bar{\mathbf{v}} = \frac{1}{|S|}\sum_{i=1}^{|S|}\|\tilde{\mathbf{v}}_i\|_2 \qquad (10)$$

Equation 8 could be used as a measure of diversity which is based on the average speed of the particles in a swarm relative to that of the swarm center.

The diversity measures presented in this section all quantify, in some way, the dispersion of the particles in a PSO. The usefulness, however, of these measures in showing the trend of a swarm from exploration to exploitation is analyzed in the next section.

IV. Analysis and Results

A. On Graphical Illustrations of the Different Diversity Measures

Graphs of some diversity measures are provided where required as an illustration of certain behavior in those measures.

The results on which the graphs are based are from PSO simulations run on the following benchmark functions as defined in [12]:

- The **Spherical function**:

$$f(x) = \sum_{i=1}^{n} x_i^2 \qquad (11)$$

  with $-5.12 \leq x_i \leq 5.12$ and global minimum given by $f(0,\ldots,0) = 0$
- The **Griewank function**:

$$f(x) = \left(\sum_{i=1}^{n}\frac{x_i^2}{4000}\right) - \left(\prod_{i=1}^{n}\cos\left(\frac{x_i}{\sqrt{i}}\right)\right) + 1 \qquad (12)$$

  with $-600.0 \leq x_i \leq 600.0$ and global minimum given by $f(0,\ldots,0) = 0$
- The **Ackley function**:

$$f(x) = -20.\exp(-0.2.\sqrt{\frac{1}{n}\sum_{i=1}^{n}x_i^2}) - \exp(\frac{1}{n}\sum_{i=1}^{n}\cos(2\pi x_i)) + 20 + e \qquad (13)$$

  with $-30.0 \leq x_i \leq 30.0$ and global minimum given by $f(0,\ldots,0) = 0$
- The **Rosenbrock function**:

$$f(x) = \sum_{i=1}^{\frac{n}{2}}(100(x_{2i} - x_{2i-1}^2)^2 + (1 - x_{2i-1}^2)) \qquad (14)$$

  with $-2.048 \leq x_i \leq 2.048$ and global minimum given by $f(1,\ldots,1) = 0$

All the simulations were run for 30 samples with random starting conditions and the graphs were plotted using the mean of each run. The following scaling process is applied, where necessary, to each plotted mean value, $\bar{\mathbf{z}}$, in order to constrict the graph outputs to the interval $[0,1]$:

$$\bar{\mathbf{z}} = \frac{\bar{\mathbf{z}} - \min(\bar{\mathbf{z}})}{\max(\bar{\mathbf{z}}) - \min(\bar{\mathbf{z}})} \qquad (15)$$

This is done so that any comparisons of the measures occurs in the same range.

The diversity measures are plotted relative to the fitness of the particles in the swarm in order to observe trends in the measures as the swarm converges. A global best PSO was used for all simulations, with asynchronous updates [13] and with 30 particles all of which were initialized with random velocities and positions. The particle positions and velocities were updated using equations 1 and 2, respectively and each PSO algorithm was executed for 75000 fitness evaluations.

Code implementations of the particle swarm optimization algorithm and of the different diversity measures can be found in CILib (**C**omputational **I**ntelligence **Lib**rary), an open source library and framework for computational intelligence algorithms, implemented in Java (see http://cilib.sourceforge.net for more information).

The next sub-section details the criteria based on which the different diversity measures are evaluated and demonstrates the performance of the different measures based on these criteria.

B. Proportionality to Particle Dispersion

Consider the full PSO model with a global best topology [10], [11], in which the particle positions and velocities have been randomly initialized in the search space, i.e. a PSO with an initially high diversity. Such a swarm can be said to converge [14], [15] when:

$$\lim_{t\to+\infty}\hat{\mathbf{y}}_t = p \qquad (16)$$

where $\hat{\mathbf{y}}_t$ is a term from the sequence, $\{\hat{\mathbf{y}}_t\}_{t=0}^{+\infty}$, of global best solutions in the swarm, p is a point in the search space which is a weighted average between the personal best and neighborhood best positions of $\hat{\mathbf{y}}_t$ and all other particles are in "close" vicinity of p. At this stage, the following should hold for the swarm diversity:

$$\lim_{t\to+\infty}\mathcal{D}_t = 0 \qquad (17)$$

where $\mathcal{D}_t$ is a term from the sequence, $\{\mathcal{D}_t\}_{t=0}^{+\infty}$, of swarm diversities.

In order to accurately reflect the transition from exploration to exploitation in a swarm, a measure of diversity should, under the conditions above show an average decrease proportional with time to a value that approaches zero. Based on this, the following observations have been made:

- The **diameter of the swarm** shows the required decrease with time, as $(x_{ik} - x_{jk})^2$ will approach zero for all particles as the swarm converges to a point in the search space. This is the same for the **swarm radius** as the distance between the center of the swarm and the furthest particle also approaches zero as the swarm converges.

- Considering the **average distance around the swarm center**, we have that as the swarm converges, $(x_{ik} - \bar{\mathbf{x}}_k)^2 \to 0$ for all dimensions k in all particles i and hence the measure is also proportional to particle dispersion. This is also the case for the **average distance around all particles in the swarm** as the component $(x_{ik} - x_{jk})^2$ will also approach zero.
- Consider that the **normalized average distance around the swarm center** is a ratio of average distance around the swarm center and swarm diameter, and that the average distance around the swarm center can be defined as a uniform random variable in the interval $[0, |D|]$, where $|D|$ is the diameter of the swarm. The ratio is thus a standard uniform random variable from the interval $[0, 1]$ and consequently conforms to a uniform probability density function, given by:

$$f(x) = \begin{cases} 1 & \text{if } 0 \leq x \leq 1; \\ 0 & \text{otherwise.} \end{cases} \tag{18}$$

  . This implies that, over the course of algorithm execution, the ratio of average distance around the swarm center to swarm diameter takes on, with equal probability, any real value in the interval $[0, 1]$. This does not conform with the required proportional decrease with time.

  Going further, when considering the fact that both the average distance around the swarm center and the swarm diameter approach zero proportionally with time, we have that their quotient will approximate a constant graph.

  Figures $1 - 4$ illustrate the swarm diversity, as returned by the normalized average distance around the swarm center relative to the best fitness of the swarm particles and plotted against the number of algorithm iterations. The component parts of the measure, i.e. swarm diameter and the average distance around the swarm center are also plotted for comparison.

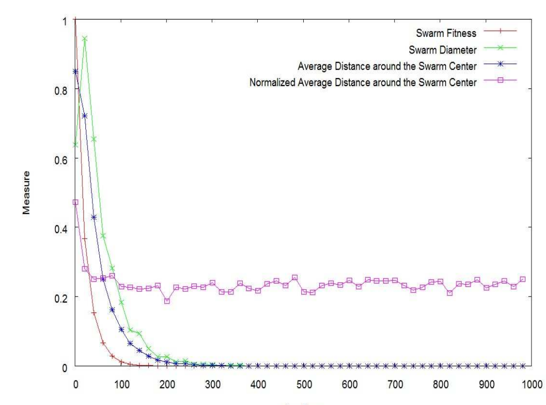

Fig. 1. Normalized Average Distance with respect to Fitness, Diameter and Average Distance on the Spherical Function

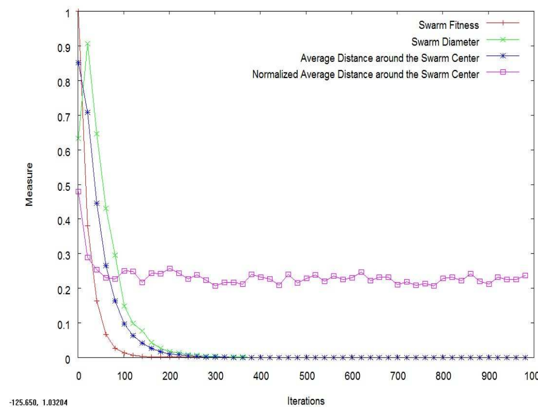

Fig. 2. Normalized Average Distance with respect to Fitness, Diameter and Average Distance on the Griewank Function

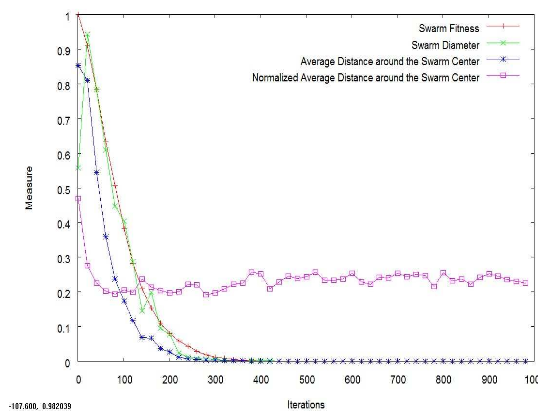

Fig. 3. Normalized Average Distance with respect to Fitness, Diameter and Average Distance on the Ackley Function

- The **swarm coherence** measure is a ratio of the magnitude of the swarm center velocity, i.e. the speed of the swarm center, to the average of the swarm particle speeds.

  A large value for the swarm center speed implies a high swarm center velocity and this implies that a high percentage of the swarm particles have velocity vector components with the same sign (not moving in opposite directions), which do not cancel out one another upon vector addition. A low value could either imply that the particles are moving in opposite directions (towards or away from each other) and canceling out one another's velocity components (large or small) or that the particles are moving in the same direction but with small velocities.

  Large values for the average particle speed imply that the swarm particles are, on average, making large changes to their current positions with respect to their personal best and neighborhood best positions. These large changes would result in an increased exploration

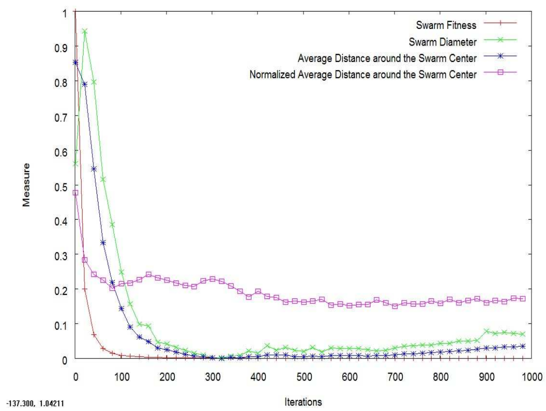

Fig. 4. Normalized Average Distance with respect to Fitness, Diameter and Average Distance on the Rosenbrock Function

of the problem space. Smaller values imply that the swarm particles are conducting a search in relative proximity to their personal and neighborhood bests, hence, exploiting around those solutions.

We thus have that a low value for swarm coherence could either mean that the particles are highly dispersed in the search space (large denominator dominates in the equation), or that they are simply moving so that the resultant of their directions is relatively small (small numerator dominates). On the other hand, a high value for swarm coherence could mean that the swarm is converging (small denominator dominates) or simply that the particles are moving in the same resultant direction (large numerator dominates).

Figure 5 illustrates a situation where swarm diversity is high and the value returned by swarm coherence is relatively small — large average speed and small swarm center speed.

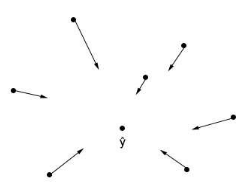

Fig. 5. High Particle Diversity and Small Swarm Coherence

Figure 6 illustrates a situation in which the swarm diversity is also high but swarm coherence returns a relatively large value — large average speed and large swarm center speed.

Considering Figures 7 and 8, we have that the swarm diversity is low but the swarm coherence returns relatively low and relatively high values, respectively.

Based on these results, the swarm coherence measure is not proportional to the dispersion of particles in a PSO. Figures 9 – 12 illustrate swarm diversity, as returned by

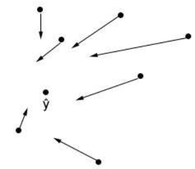

Fig. 6. High Particle Diversity and Large Swarm Coherence

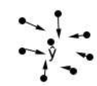

Fig. 7. Low Particle Diversity and Low Swarm Coherence

the swarm coherence measure.

The next sub-section analyzes the sensitivity of the different diversity measures to outliers.

C. Sensitivity to Outliers

A particle is considered as an outlier when its position deviates significantly relative to the other particle positions in the swarm. Figure 13 illustrates an example of an outlier.

Outliers in a PSO are a characteristic of the algorithm and diversity measures should be able to return values which are not affected by the presence of these outliers. Looking at the measures under consideration, the following observations can be made:

- Based on the way it is calculated, the **diameter of the swarm** is extremely sensitive to outliers. Consider Figures 14 and 15 (assume both figures to be in the same scale).
  The swarm diameter would return almost equal values of diversity for both examples where Figure 14 is obviously more diverse than Figure 15.
  The **radius of the swarm** is also very sensitive to outliers, also due to the way in which it is calculated.
- The **average distance around the swarm center** is a more robust measure than **swarm diameter** or **radius** with regards to outliers as it makes use of the average of the particle distances from the center of the swarm. An extreme outlier, however, may skew the average significantly enough to affect its value, especially when there are too few non-outlying particles in the swarm.
- The **average distance around all particles in the swarm** will be slightly less affected by outliers than the average distance around the swarm center. This is because the effect of outliers introduced by the calculation of the average distance around each particle will be reduced slightly in the calculation of the average over all the average distances.
- The **normalized average distance around the swarm center** will be affected as much by outliers as the

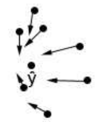

Fig. 8. Low Particle Diversity and Large Swarm Coherence

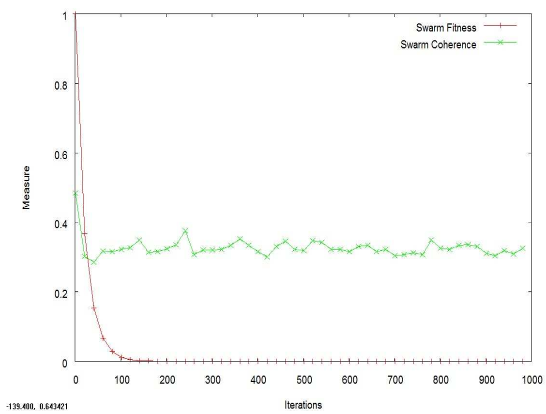

Fig. 9. Swarm Coherence with respect to Fitness on the Spherical Function

diameter of the swarm. This is due to the fact that the diameter is used as the normalizing variable.

- **Swarm coherence** makes use of average velocities (for the swarm center speed) and average speeds of particles in the swarm. Since the velocity of a particle is dependent on its direction, the swarm coherence measure will be affected by outliers in the form of particles going in opposite directions to the rest of the swarm particles. The effects of this have been discussed in the previous sub-section and it was shown that the values returned by this measure are significantly affected by such particle behavior.

It is not possible to eliminate outliers from a particle swarm. Also, it is not possible for a diversity measure to be completely immune to the effects of outliers. Some, however, are more sensitive than others and this section served to outline this.

V. Conclusion

This paper investigated some of the definitions of swarm diversity from various literature. The aim was to determine the usefulness of these measures in quantifying the exploration/exploitation of a particle swarm algorithm.

The behaviors of the different measures were analyzed over the course of execution of a particle swarm algorithm in order to determine whether they showed an average decrease proportional to the dispersion of the particles in the swarm. The sensitivity to outliers of the various measures was also taken into consideration and based on these criteria, the following observations were made:

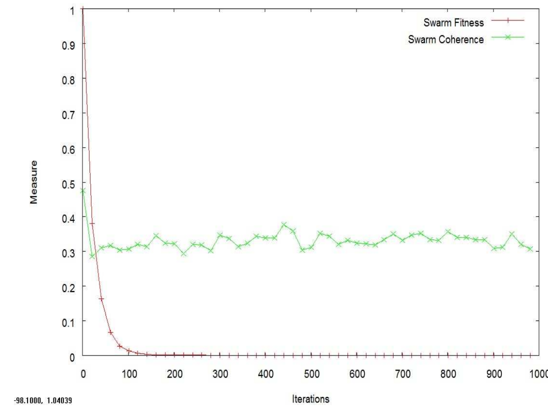

Fig. 10. Swarm Coherence with respect to Fitness on the Griewank Function

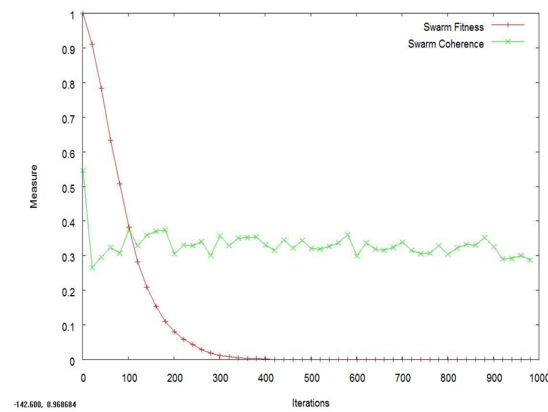

Fig. 11. Swarm Coherence with respect to Fitness on the Ackley Function

- The **normalized average distance around the swarm center** was shown to follow a uniform distribution defined between $[0, 1]$ rendering it unsuitable for quantifying the dispersion of swarm particles at a given time.
- Also, the **swarm coherence** measure was shown to give ambiguous results for swarm diversity based on the different possible values of its components also making it unsuitable for measuring swarm diversity.
- The **diameter** and **radius** of the swarm show the required proportionality to particle dispersion, but are significantly sensitive to the presence of outlying particle positions in the swarm. Hence they are not the best possible means to measure swarm diversity.

Based on the above, we are left with the **average distance around the swarm center** and the **average distance around all particles in the swarm**. The latter measure is based on the former and is a more accurate measure of the dispersion of particles in the swarm. It is also much more computationally complex, requiring $|S|$-times more calculations, where $|S|$ is the swarm size. This could pose a problem for significantly large swarms.

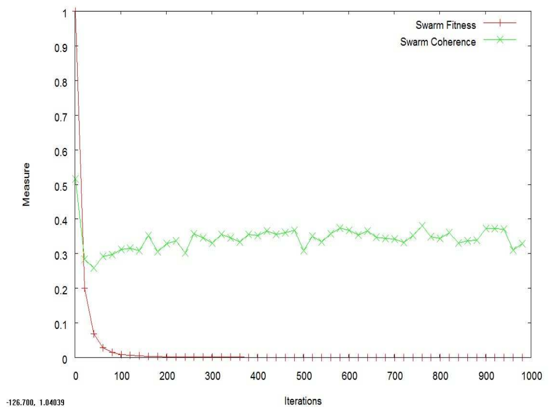

Fig. 12. Swarm Coherence with respect to Fitness on the Rosenbrock Function

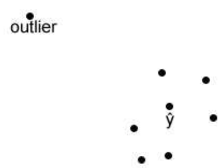

Fig. 13. Illustration of a Particle Outlier

This leaves the **average distance around the swarm center**. This measure shows the required decrease, proportional to particle dispersion and is affected by outliers only in situations where the outlying positions deviate sufficiently from the rest of the population to skew the average distance calculation.

This work is intended as a foundation for the development of a diversity rate of change measure, which could be used to quantify the rate at which a particle swarm switches from exploration to exploitation. Based on the results in this paper the **average distance around the swarm center** will be used

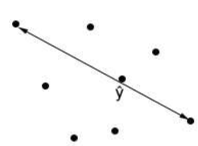

Fig. 14. Swarm Diameter and Particle Outliers I

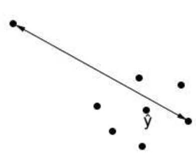

Fig. 15. Swarm Diameter and Particle Outliers II

as the basis for the rate of change measure.

Further research will involve investigating the effects of different distance measures e.g. Manhattan distance, Chebyshev distance, Quadratic distance, etc, on the accuracy and robustness of the different measures. Also, the development of different means for normalization that do not result in a diversity measure becoming a random variable will be investigated. Lastly, the behavior of the different diversity measures on different particle swarm algorithms — different information sharing strategies, charged swarms, etc, — could be investigated.

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