A Constrained Ant Colony Algorithm for Image Registration

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Abstract. Ant Colony optimization takes inspiration from the behavior of real ant colony to solve optimization problems. We attach some constraints to ant colony model and present a parallel constrained ant colony model to solve the image registration problem. The problem is represented by a directed graph so that the objective of the original problem becomes to find the shortest closed circuit on the graph under the problem-specific constraints. A number of artificial ants are distributed on the graph and communicate with one another through the pheromone trails which are a form of the long-term memory guiding the future exploration of the graph. The algorithm supports the parallel computation and facilitates quick convergence to the optimal solution. The performance of the proposed method as compared to those of the genetic-based approaches is very promising.

1 Introduction

Swarm intelligence research originates from work into the simulation of the emergence of collective intelligent behaviors of real ants. Ants are able to find good solutions to the shortest path problems between the nest and a food source by laying down, on their way back from the food source, a trail of an attracting substance – a pheromone. Based on the pheromone level communication, the shortest path is considered that with the greatest density of pheromone and the ants will tend to follow the path with more pheromone. Dorigo and his colleagues were the first to apply this idea to the traveling salesman problem [1]. This algorithm is referred to as ant colony algorithm (ACA). ACA has achieved widespread success in solving different optimization problems, such as the vehicle routing problem [2], the machine tool tardiness problem [3] and the multiple objective JIT sequencing problem [4].

Image registration is to find a correspondence function mapping coordinates from a source image to coordinates of homologous points in a target image. In clinical applications, it can be used to match images taken from the same patient over a period of time. Generally, image registration methods can be divided into intensity-based and landmark-based methods. Intensity-based methods [5, 6] find the deformation function by optimizing some criterion function that incorporates a measure of image similarity and can produce more precise registration results because of using the information of the whole images. Landmark-based methods [7, 8, 9, 10] involve extraction of landmarks that need to be matched. By interpolating discrete landmarks, a dense mapping for the whole image can be obtained quickly.

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In this paper, a constrained ant colony model for solving the image registration problem is developed. Our image registration combines intensity and landmark information and exploits the advantages of both classes of information. Ant colony algorithm, which is constrained and modified to deal with image registration, can make algorithm amenable to parallel implementations, compared with other optimal approaches, for its distinct feature -- distributed computation. The proposed parallel algorithm can obtain the optimal solution in a reasonably shorter period of time.

2 The Constrained Ant Colony

2.1 Original Ant Colony

The ant colony algorithms have been introduced with Dorigo's Ph.D thesis. They are based on the principle that by using very simple communication mechanisms, an ant group is able to find the shortest path between any two points. During their trips a chemical trail (pheromone) is left on the ground. The role of this trail is to guide the other ants towards the target point. For one ant, the path is chosen according to the quantity of pheromone. Furthermore, this chemical substance has a decreasing action over time, and the quantity left by one ant depends on the amount of food found and the number of ants using this trail. As illustrated in Fig. 1, when facing an obstacle, there is an equal probability for every ant to choose the left or right path. As the left trail is shorter than the right one and so requires less travel time, it will end up with higher level of pheromone. More ants take the left path, higher pheromone trail is.

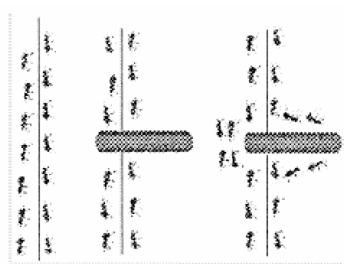


Fig. 1. Ants face an obstacle. When facing an obstacle, there is an equal probability for every ant to choose the left or right path. As the left trail is shorter than the right one and so requires less travel time, it will end up with higher level of pheromone. More ants take the left path, higher pheromone trail is.

The general principles for the ant colony simulation of real ant behavior are as follows.

(1) *Initialization.* The initialization of the AC includes two parts: the problem graph representation and the initial ant distribution. First, the underlying problem should be represented in terms of a graph, G = <N,E>, where N denotes the set of nodes, and E the set of edges. The graph is connected, but not necessarily complete,

such that the feasible solutions to the original problem correspond to paths on the graph which satisfy problem-domain constraints. Second, a number of ants are arbitrarily placed on the nodes chosen randomly. Then each of the distributed ants will perform a tour on the graph by constructing a path according to the node transition rule described next.

(2) *Node transition rule.* The ants move from node to node based on a node transition rule. According to the problem-domain constraints, some nodes could be marked as inaccessible for a walking ant. The node transition rule is probabilistic. For the kth ant on node i, the selection of the next node j to follow is according to the node transition probability:

$$p_{ij}^k = \begin{cases} \dfrac{(\tau_{ij})^\alpha (\eta_{ij})^\beta}{\sum_{h \notin tabu_k} (\tau_{ih})^\alpha (\eta_{ih})^\beta} & if \;\; j \notin tabu_k \\ 0 & otherwise \end{cases} \tag{1}$$

where τ_{ij} is the intensity of pheromone laid on edge (i,j), η_{ij} is the value of visibility of edge (i,j), α and β are control parameters, and $tabu_k$ means the set of currently inaccessible nodes for the *k*th ant according to the problem-domain constraints. The intensity of pheromone laid on edge (i,j) reflecting the previous experience of the ants about this edge is shared memory which provides indirect communication between the ants.

(3) *Pheromone updating rule.* The ant keeps walking through edges to different nodes by iteratively applying the node transition rule until a solution to the original problem is constructed. We define that a cycle of the AC algorithm is completed when every ant has constructed a solution. At the end of each cycle, the intensity of pheromone trails on each edge is updated by the pheromone updating rule:

$$\tau_{ij} \leftarrow \rho \tau_{ij} + \sum_{k=1}^{m} \Delta \tau_{ij}^k \tag{2}$$

where $\rho \in (0,1)$ is the persistence rate of previous trails, $\Delta \tau_{ij}^k$ is the amount of pheromone laid edge (i,j) by the *k*th ant at the current cycle, and *m* is the number of distributed ants. In a real ant system, shorter paths will retain more quantities of pheromone; analogously, in the AC, the paths corresponding to fitter solutions should receive more pheromone quantities and become more attractive in the next cycle. Hence, if we define L_k, the total length of the *k*th ant in a cycle, as the fitness value of the solution, then $\Delta \tau_{ij}^k$ can be given by

$$\Delta \tau_{ij}^k = \begin{cases} \dfrac{Q}{L_k} & if \;\; edge\,(i,j)\, is\, traversed\, by \\ & the\; kth\; ant\; at\; this\; cycle \\ 0 & otherwise \end{cases} \tag{3}$$

where *Q* is a constant.

(4) *Stopping criterion.* The stopping criterion of the AC algorithm could be the maximal number of running cycles or the CPU time limit.

2.2 Constrained Ant Colony

For solving image registration, we modify and attach some constraints to the original ant colony model. In original ant colony model, the underlying problem should be represented in terms of a graph G = <N,E>, which is connected, but not necessarily complete, as shown in Fig. 2 (a). Now, following constraints are added to G for reducing the search space. 1) All the nodes construct a loop. 2) The adjacent node has no less than one edge. 3) The nonadjacent node has no edge. After applying these rules, an example of the graph with constraints is shown in Fig. 2 (b). When ant colony algorithm works on the graph with constraints, we call it constrained ant colony.

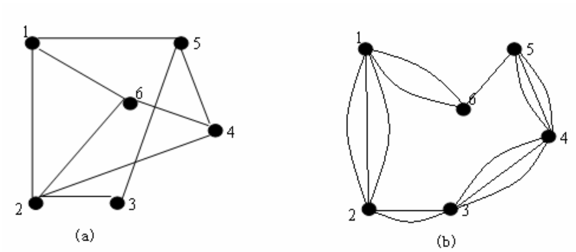


Fig. 2. The original graph and constrained graph. The constrained graph must satisfy following constraints. G. 1) All the nodes Ni construct a loop. 2) The joint node has no less than one edge. 3) The un-joined node has no edge.

3 Image Registration Problem

In this section, we introduce the structure of image registration problem and give some definitions of the notations. The goal of image registration algorithm is to determine a transformation function based on the landmarks from the source image $f_S(\mathbf{i})$ to the target image $f_T(\mathbf{i})$, where $\mathbf{i} \in \mathbf{I} \subset \mathbb{R}^2$, and $\mathbf{I}$ is a 2D discrete interval representing the set of all pixel coordinates in the image. We suppose that the source image is a geometrically deformed version of the target image. That is to say that the points with the same coordinate $\mathbf{x}$ in the target image $f_T(\mathbf{x})$ and in the warped source image $f_w(\mathbf{x})=f_S^c(\mathbf{g}(\mathbf{x}))$ should correspond. Here, f^c is a continuous version of the image and $\mathbf{g}(\mathbf{x}) : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ is a deformation function to be identified.

3.1 Image Representation

To interpolate the images accurately, the image is represented as a continuous version using uniform B-splines [11]:

$$f^{c}(\mathbf{x}) = \sum_{\mathbf{i}\in\mathbf{I}\subset\mathbb{Z}^2} \mathbf{b}_{\mathbf{i}}\beta_3(\mathbf{x}-\mathbf{i}) \tag{4}$$

where β_3 is a tensor product of B-splines of degree 3, that is $\beta_3(\mathbf{x}) = \prod_{\mathbf{k}=1}^{2}\beta_3(\mathrm{x}_{\mathrm{k}})$, with $\mathbf{x} = (\mathrm{x}_1, \mathrm{x}_2)$.

3.2 Transformation Function

We choose the Locally Constrained Deformation (LCD) [12], shown in Eq. (5), as transformation function

$$\begin{aligned} \mathbf{g}(\mathbf{x}) &= \mathbf{x} + \sum_{\mathbf{i}=1}^{\mathbf{N}} \alpha_{\mathrm{i}}\rho_{\mathbf{LCD}}\left(\frac{\|\mathbf{x}-\mathbf{p}_{\mathbf{i}}\|}{\mathbf{r}_{\mathbf{i}}}\right) \\ \rho_{\mathbf{LCD}}(\mathbf{r}) &= \begin{cases} (1-\mathbf{r}^2)^3 & 0 \le \mathbf{r} \le 1 \\ 0 & 1 < \mathbf{r} \end{cases} \end{aligned} \tag{5}$$

and it must fulfill the following constraints:

$$\mathbf{g}(\mathbf{p}_{\mathbf{i}}) = \mathbf{q}_{\mathbf{i}} \qquad \mathbf{i} = 1\cdots\mathbf{N} \tag{6}$$

where α_{i} is the coefficients, $\mathrm{p}_{\mathrm{i}} \in \mathbb{R}^2$ $\mathrm{i} = 1...\mathrm{N}$ constitute a given set of landmark points in the source image, $\mathrm{q}_{\mathrm{i}} \in \mathbb{R}^2$ $\mathrm{i} = 1...\mathrm{N}$ are the corresponding landmark points in the target image and **N** is the number of the landmark points.

3.3 Cost Function

The two images f_T and f_W will not be identical because the assumption that there is a geometrical mapping between the two images is not necessarily correct. Therefore, we define the solution to our registration problem as the result of the minimization $\mathbf{g} = \arg\min_{\mathbf{g}\in\mathbf{G}} \mathbf{E}(\mathbf{g})$, where **G** is the space of all admissible deformation functions **g**. We choose the sum of squared difference (SSD) criterion as cost function because it is fast to evaluate and yields a smooth criterion surface. The form of SSD is

$$\boldsymbol{E} = \sum_{\boldsymbol{i}\in\boldsymbol{I}} (\boldsymbol{f}_{\mathrm{S}}^{\,c}(\boldsymbol{g}(\boldsymbol{i})) - \boldsymbol{f}_{\mathrm{T}}^{\,c}(\boldsymbol{i}))^2 \tag{7}$$

3.4 Optimization Strategy

To minimize the criterion **E** in (7), some local iterative algorithms have been cast into the framework [13], but the computation of the derivate is required. Therefore, the constrained ant colony algorithm is introduced to minimize the criterion and the details are shown in section 4.

4 Constrained Ant Colony for Image Registration

Through the analysis of Eq. (5) and Eq. (6), the transformation function $\mathbf{g}(\mathbf{x})$ can be adjusted by varying the value of $\mathbf{r_i}$. Thus the optimal solution of $\mathbf{r_i}$ can be obtained to minimize the SSD of two images using constrained ant colony.

4.1 Graph Representation

To apply ant colony, the underlying problem should be represented in terms of a directed graph, G=<N,E>, which can avoid the ants walking backward. Apparently, for image registration problem, each landmark point p_i should be represented as a node of the graph, N=P. We represent the edge E as $\overrightarrow{(p_i, p_j, r_k)}$, where r is the action radius of landmark point p_j, as shown in Eq. (5). $ssd\overrightarrow{(p_i, p_j, r_k)}$ is the SSD of the edge $\overrightarrow{(p_i, p_j, r_k)}$ and is defined as E in Eq. (7) with only one landmark point p_j and action radius is r_k. In addition, there are many edges between two nodes in our graph, which differs from AC. Now, the problem of image registration is equivalent to finding a closed circuit, which satisfies the minimal SSD, on the directed graph.

4.2 Node Transition Rule

The node transition rule is a probabilistic one determined by the pheromone intensity τ_{ijk} and the visibility value η_{ijk} of the corresponding edge. In the proposed method, τ_{ijk} is equally initialized to any small constant positive value, and is gradually updated at the end of each cycle according to the average quality of the solution that involves this edge. On the other hand, the value of η_{ijk} is determined by a greedy heuristic method, which encourages the ants to walk to the minimal SSD edge. This can be accomplished by setting $\eta_{ijk} = 1/ssd\overrightarrow{(p_i, p_j, r_k)}$.

We now define the transition probability from node i to node j through directed edge $\overrightarrow{(p_i, p_j, r_k)}$ as

$$p_{ijk}(t) = \begin{cases} \dfrac{[\tau_{ijk}(t)]^{\alpha}[\eta_{ijk}]^{\beta}}{\sum\limits_{allowed\_list} [\tau_{ijk}(t)]^{\alpha}[\eta_{ijk}]^{\beta}} & if \;\; j \in allowed\_list \\ 0 & otherwise \end{cases} \tag{8}$$

where *allowed_list* are the accessible nodes by walking ants, and the means of other symbols are same to the Eq. (1).

4.3 Pheromone Updating Rule

The intensity of pheromone trails of an edge is updated at the end of each cycle by the average quality of the solutions that traverse along this edge. We simply apply and modify Eqs. (2) and (3) to update pheromone intensity.

$$\tau_{ijk} \leftarrow \rho.\tau_{ijk} + \sum_{s=1}^{m} \Delta\tau_{ijk}^{s} \tag{9}$$

$$\Delta\tau_{ijk}^{s} = \begin{cases} \dfrac{Q}{SSD_s} & \text{if the sth ant walks } \overrightarrow{(p_i, p_j, r_k)} \\ 0 & \text{otherwise} \end{cases} \tag{10}$$

where *SSD*s is the SSD of the *sth* ant's registration result at current cycle.

4.4 Constrained Ant Colony for Image Registration

According to the given definitions as above, the image registration based on constrained ant colony is described as following.

Input:
m: the number of ants.
MAX_CYCLE: the maximal number of running cycles.

1: *Construct the directed graph G=<N,E> as described in Section 4.1. Set NC=1 (NC is the cycles counter),* $\tau_{ijk}(t) = c$ *(* c *is constant),* $SSD_{global\_best} = \infty$ *(* SSD_{global_best} *saves the minimal SSD of the strip). Compute* η_{ijk} *on every edge* $\overrightarrow{(p_i, p_j, r_k)}$.

2: *For every ant do*
Select a starting node.
Repeat
Move to next node according to the node transition rule using Eq. (8).
until a closed tour is completed.
//a closed tours is completed when the ant arrives at the starting node again.

3: *Compute registration results and find out the minimal SSD among the m tours obtained at step2, say SSDcurrent_best.*

4: *If SSDcurrent_best < SSDglobal_best, then SSDglobal_best = SSDcurrent_best.*

5: *For every directed edge* $\overrightarrow{(p_i, p_j, r_k)}$ *do*
Update the pheromone intensity using Eqs. (9) and (10).

6: *If (NC = MAX_CYCLE), then output SSDglobal_best and strop;*
otherwise, NC = NC + 1 and goto step2.

5 Parallel Ant Colony

To solve efficiently large optimization problems, a parallel model of ant colony has been developed. The programming style used is a synchronous master/workers paradigm. The master initializes all kinds of data as stated in section 4.4 step1, and then sends the graph information including the trail density τ_{ijk} and the visibility η_{ijk} to workers. With the graph information, the worker takes charge of searching for a tour composed of the edge and completing image registration. The parallel algorithm works as follows Fig. 3. Each worker returns the SSD of registration result and the tour visited to the master, which later updates the intensity of trail on the edge and controls the flow of the algorithm. By this way all the workers can implement parallel packing by sharing the information from the master.

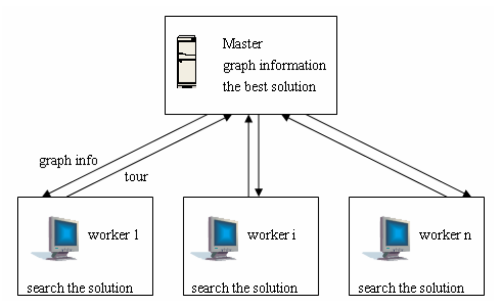


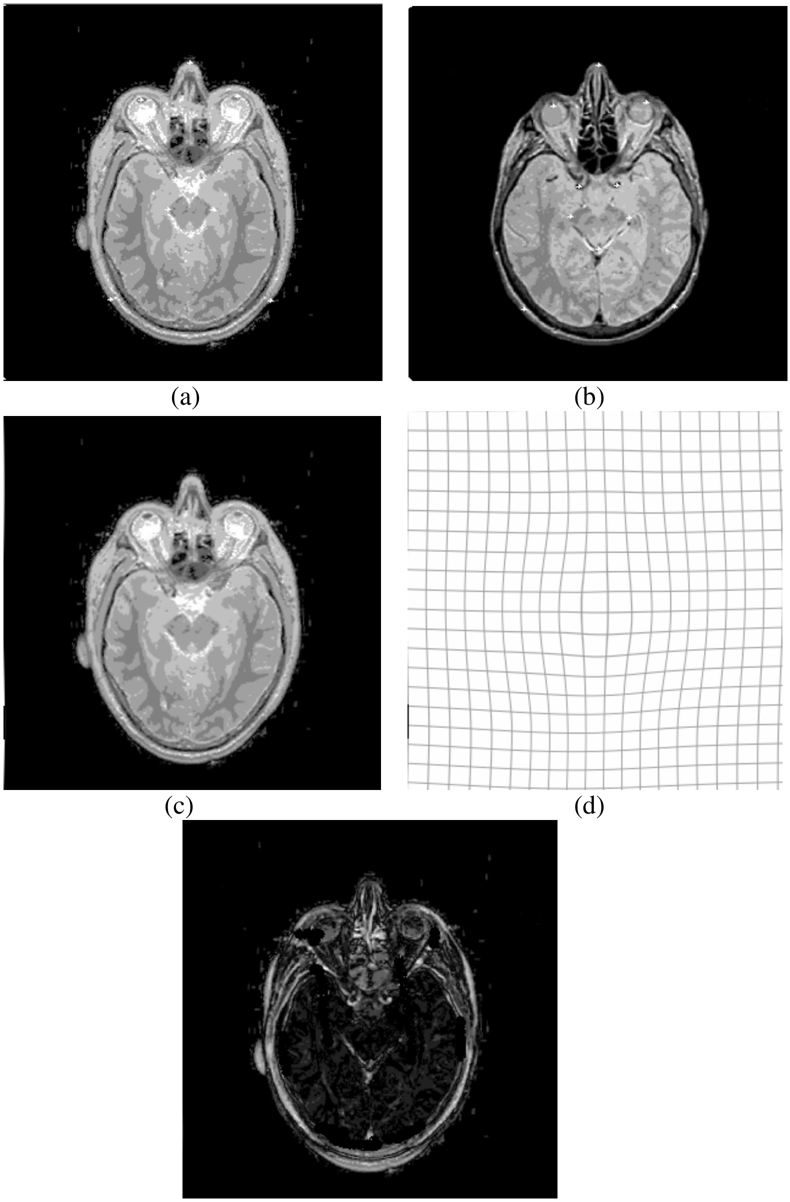
Fig. 3. Synchronous master/workers model for parallel ant colony. The master initializes all kinds of data and the worker takes charge of searching for a tour composed of the edge and completing image registration.

6 Experimental Results

The proposed algorithm has been programmed in VC++ language, and run in Windows 2000 Professional. According to Ref. [12], the action radius of the landmark point is constrained as:

$$\mathrm{r}_\mathrm{i} \geq 1.71 \| \mathrm{q}_\mathrm{i} - \mathrm{p}_\mathrm{i} \| \tag{11}$$

which should be satisfied in the constrained graph. Furthermore, every landmark point has limited action scope and it is constrained as $\mathrm{r}_\mathrm{i} \leq 15 \| \mathrm{q}_\mathrm{i} - \mathrm{p}_\mathrm{i} \|$. Thus between the node pi and pj, there are $\mathrm{NUM}_\mathrm{j} = \lfloor (15 - 1.71) * \| \mathrm{q}_\mathrm{j} - \mathrm{p}_\mathrm{j} \| / \Delta \rfloor$ edges, that is to say $\overrightarrow{(p_i, p_j, r_k)}$ $k = 1, 2, ..., NUM_j$, where Δ is used to control the accuracy of the algorithm and set $\Delta = 0.5$. In our implementation, $\alpha = 1.0$, $\beta = 2.0$, $\rho = 0.9$ and *MAX_CYCLE=200*.



(a) (b) (c) (d) (e)

Fig. 4. Registration results. (a) source image; (b) target image; (c) registration result; (d) the underlying grid of (c); (e) difference between (b) and (c).

Fig. 4 shows an experimental result based on the presented algorithm. In the source image Fig. 4(a) and the target image Fig. 4(b), we interactively selected 10 pairs of point-landmarks. The registration result is shown in Fig. 4(c). To assess the form of the deformation, we have applied the algorithm to a regular grid and the result can be seen in Fig. 4(d). The obvious influence of the deformation is demonstrated in Fig. 4(e) where the reference image is subtracted from the registration result.

In addition, we compare the efficiency of our algorithm to the genetic algorithm. With variable number of the landmark points, we complete four examples whose results are listed in Table 1. In our algorithm, 10 workers are used to search the solutions. It is indicated from the results that our algorithm can converge more quickly to the better solutions because of the parallel implementation.

Table 1. Results from the different algorithms

Examples		Genetic algorithm	Ant colony algorithm
Example 1: 10 points	Time (minute)	10.0	2.9
Example 2: 15 points	Time (minute)	14.3	4.1
Example 3: 20 points	Time (minute)	17.6	6.0
Example 4: 25 points	Time (minute)	19.1	7.7

7 Conclusion

In this paper we have developed a powerful and robust algorithm for image registration, which bases on the constrained ant colony model. The proposed algorithm supports parallel computation and facilitates quick convergence to the optimal solution. The experimental result demonstrates that our algorithm can search the solution space more effectively and obtain the optimal solution in a reasonably shorter period of time.

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