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An Incompressible Fluid Flow Model with Mutual Information for MR Image Registration

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ABSTRACT

Image registration is one of the fundamental and essential tasks within image processing. It is a process of determining the correspondence between structures in two images, which are called the template image and the reference image, respectively. The challenge of registration is to find an optimal geometric transformation between corresponding image data. This paper develops a new MR image registration algorithm that uses a closed incompressible viscous fluid model associated with mutual information. In our approach, we treat the image pixels as the fluid elements of a viscous fluid flow governed by the nonlinear Navier-Stokes partial differential equation (PDE). We replace the pressure term with the body force mainly used to guide the transformation with a weighting coefficient, which is expressed by the mutual information between the template and reference images. To solve this modified Navier-Stokes PDE, we adopted the fast numerical techniques proposed by Seibold[1]. The registration process of updating the body force, the velocity and deformation fields is repeated until the mutual information weight reaches a prescribed threshold. We applied our approach to the BrainWeb and real MR images. As consistent with the theory of the proposed fluid model, we found that our method accurately transformed the template images into the reference images based on the intensity flow. Experimental results indicate that our method is of potential in a wide variety of medical image registration applications.

Keywords: MRI, image registration, incompressible fluid flow, non-rigid transformation, mutual information

1. INTRODUCTION

Current imaging techniques can be broadly divided into two main categories: anatomical and functional modalities. Anatomical imaging includes computed tomography (CT) and magnetic resonance imaging (MRI). Functional imaging includes positron emission tomography (PET), single photon emission computed tomography (SPECT), and functional MRI (fMRI). Different modalities provide complementary information to interpret the human bodies. In medical image processing applications one is often interested not only in analyzing one image but in comparing or combining the information given by a series of images as well as different subjects and modalities. Accordingly, image registration is one of the fundamental and essential tasks within image processing. Image registration is the process of determining the correspondence between structures in two images, which are respectively called the template image and the reference image. The challenge of registration is to find an optimal geometric transformation between corresponding image data. In other words, given a reference and a template image, the goal is to find a suitable transformation such that the transformed template becomes similar to the reference.

Transformations of maintaining distances between all pixels in images are referred to as rigid-body transformations that are based on coordinate changing by translation and rotation. In contrast, non-rigid transformations map straight lines into curves in such a way to determine the transformation function. Non-rigid methods are often used to handle large scale and complicated deformation such as the registration of MRI scans. In general, non-rigid transformation models can be divided into two main categories: physical based models and function representations. The physical models are generally derived from the theory of continuum mechanics and can be divided into elasticity and fluid flow subcategories. Fluid registration is of particular importance in MRI, as it can be used to localize regions of anatomical change in longitudinal studies.

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Christensen et al.[2] proposed a viscous fluid model to allow large magnitude deformations based on dynamical partial differential equations (PDEs). In their approach, image pixels are regarded as viscous fluid particles that are governed by the PDEs, which constrain the movement during the registration process. To numerically solve the PDEs, they used conventional finite difference techniques associated with the successive over-relaxation (SOR). A regridding procedure is required when the nonlinear transformations evaluated on a finite lattice become singular. Subsequently, Freeborough and Fox[3] adopted a full multi-grid (FMG) framework along with the SOR for the coarse solution to match a set of Alzheimer disease (AD) images. Crum et al.[4] used semi-coarsening and exploited the inherent multi-resolution nature of FMG to implement a multi-scale approach. Xu and Dony[5] proposed to use the least mean square inverse filter to solve the PDEs of the viscous fluid model by Christensen et al.[2]. D'Agostino et al.[6] proposed a multimodal viscous fluid model registration algorithm based on maximization of mutual information. The ambition of this paper is in an attempt to develop a more accurate and robust MR image registration algorithm that makes use of a closed incompressible viscous fluid model associated with mutual information.

2. NONRIGID REGISTRATION

We start by briefly describing the general models of non-rigid registration methods, which can be divided into two main categories: physical based models and function representations as shown in Fig. 1. The physical based models are generally derived from the theory of continuum mechanics and can be divided into three main subcategories: linear elasticity, fluid flow, and optical flow[7].

2.1 Linear Elastic Transformations

The basic theory of linear elasticity is based on stress, which is the contact force per unit area acting on orthogonal planes, and strain, which is a measure of the amount of deformation. Considering a linear elastic material in force equilibrium and assuming that the stress components vary linearly across an infinitesimal element, the body forces balance with the surface stress as:

$$\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \sigma_{xy}}{\partial y} + \frac{\partial \sigma_{zx}}{\partial z} + f_x = 0 \tag{1}$$

where $\sigma_{xx}, \sigma_{xy}, \sigma_{zx}$ represent the stress tensor in the three principal axes (x, y, z) and f_x represents the body force in the x component. The force balance equations in the y and z components are similarly defined. Using the Gaussian integral or divergence theorem, it can be shown that the stress tensor is symmetric and the number of independent stress components can be reduced to six[8]. The normal and shear infinitesimal strain $\in$ can then be expressed in terms of the spatial derivatives of the displacement as:

$$\in = \frac{1}{2}\left(\nabla\vec{d} + \left(\nabla\vec{d}\right)^T\right) \tag{2}$$

where $\vec{d}$ represents the displacement and the superscript T represents the transpose operator. For a homogeneous isotropic material the stress-strain relationship can be simplified to the following Piola-Kirchoff form:

$$\boldsymbol{\sigma} = \lambda tr(\in)\boldsymbol{I} + 2\mu \in \tag{3}$$

where λ and μ are the Lamé constants, $\boldsymbol{I}$ is the unit tensor, and tr is the trace operator. By combining Eqs. (1)-(3), we obtain the Navier-Cauchy linear elastic PDE as follows:

$$\mu\nabla^2\vec{d} + (\mu + \lambda)\nabla\left(\nabla \cdot \vec{d}\right) + \vec{F} = 0 \tag{4}$$

where $\nabla^2 = \nabla \cdot \nabla$, ∇ is the gradient operator, and $\vec{F}$ denotes the body force per unit volume, which drives registration. Equation (4) is the governing transformation equation for linear elastic registration[7, 8].

2.2 Fluid Flow Transformations

Continuum mechanics provides the theoretical foundation for fluid flow registration. Fluid flow models are based on physical properties of fluids that follow Newtonian mechanics and must satisfy physical laws such as the conservation of linear momentum that leads to the equation of motion:

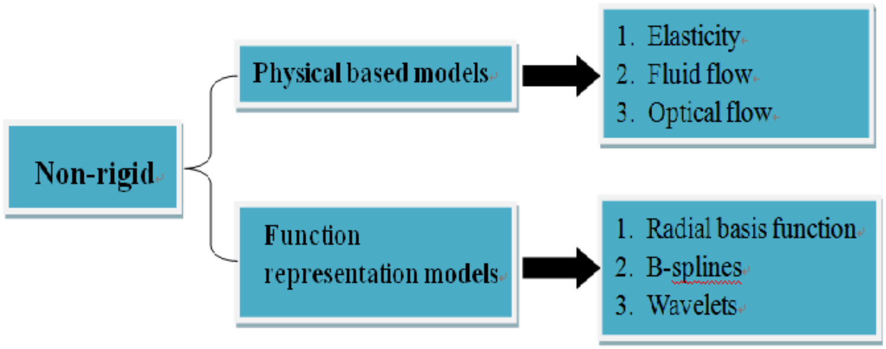

Figure 1. Classification of non-rigid registration methods.

$$\rho \frac{d\vec{V}}{dt} + \vec{V}\eta - \nabla\boldsymbol{\sigma} = \vec{F} \tag{5}$$

where ρ represents the density of the fluid, $\vec{V}$ represents the velocity vector, t represents time, and η represents the mass source term that can be expressed by the continuity equation:

$$\eta = \frac{\partial\rho}{\partial t} + \nabla\cdot\left(\rho\vec{V}\right). \tag{6}$$

For a Newtonian fluid the viscous stress tensor $\boldsymbol{\sigma}$ in Eq. (5) is linearly related to the rate of deformation tensor using:

$$\boldsymbol{\sigma} = -p\boldsymbol{I} + \lambda_f tr(\boldsymbol{D})\boldsymbol{I} + 2\mu_f\boldsymbol{D} \tag{7}$$

where p represents the hydrostatic pressure, λ_f and μ_f are the viscosity coefficients of the fluid, and $\boldsymbol{D}$ is the deformation tensor that is related to the velocity through

$$\boldsymbol{D} = \frac{1}{2}\left(\nabla\vec{V} + (\nabla\vec{V})^{\mathrm{T}}\right). \tag{8}$$

Combining Eqs. (5)-(8), we obtain the Navier-Stokes-Duhem equation as follows:

$$\rho\frac{d\vec{V}}{dt} = \vec{F} - \nabla p + \left(\mu_f + \lambda_f\right)\nabla\cdot\nabla\vec{V} + \mu_f\nabla^2\vec{V} - \eta\vec{V}. \tag{9}$$

If we assume it is Stokes flow (low Reynolds number), the terms $\rho\frac{d\vec{V}}{dt}$ and $\eta\vec{V}$ in Eq. (9) are neglected. Further assume there is only a small spatial variation in the hydrostatic pressure to ignore ∇p, Eq. (9) simplifies to the Navier-Stokes equation for a viscous fluid as[7, 8]

$$\mu_f\nabla^2\vec{V} + \left(\mu_f + \lambda_f\right)\nabla\left(\nabla\cdot\vec{V}\right) + \vec{F} = 0 \tag{10}$$

where the term $\mu_f\nabla^2\vec{V}$ indicates constant volume or incompressible viscous flow and the $\left(\mu_f + \lambda_f\right)\nabla\left(\nabla\cdot\vec{V}\right)$ term controls expansion or contraction of the fluid. Note that the Navier-Stokes PDE (10) is equivalent to the Navier-Cauchy linear elastic PDE (4) with the variable velocity $\vec{V}$ replaced by displacement $\vec{d}$.

2.3 Optical Flow Transformations

Optical flow methods have been used to find small scale deformations in temporal sequences of images. The basic assumption of optical flow is based on the principle of intensity conservation between image frames. For image registration, the motion equation of optical flow is approximated to give a numerically stable expression as[9]

$$\vec{V} = \frac{(T_i - R_i)\nabla R_i}{|\nabla R_i|^2 + (T_i - R_i)^2} \tag{11}$$

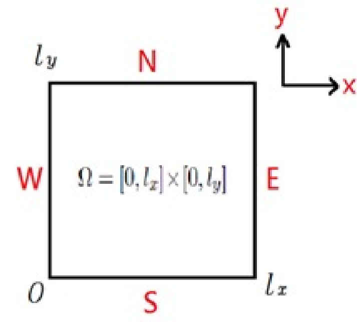

Figure 2. Illustration of flow domain boundaries.

where T_i and R_i respectively represent the intensities of template and reference images, ∇R_i is the internal edge-based force that drives the deformation, $T_i - R_i$ represents the external force, and $(T_i - R_i)^2$ is used to stabilize the velocity field.

3. VISCOUS FLUID REGISTRATION

3.1 Navier-Stokes Equations

In our approach, we consider the Navier-Stokes equations in fluid mechanics as stated in the following formula:

$$\rho\left(\frac{\partial \vec{V}}{\partial t} + \vec{V} \cdot \nabla \vec{V}\right) = -\nabla P + \mu_f \nabla^2 \vec{V} + \rho \vec{g} \tag{12}$$

where P represents pressure and $\vec{g}$ represents the gravity. We further consider a closed 2-D flow on a rectangular domain Ω=[0, l_x]×[0, l_y]. As shown in Fig. 2, we denote the four boundaries as **E**, **S**, **W**, **N** that are time invariant and nonslip on each wall, i.e.,

$$u(x, l_y) = u_N(x),\ v(x, l_y) = 0 \tag{13}$$

$$u(x, 0) = u_S(x),\ v(x, 0) = 0 \tag{14}$$

$$v(0, y) = 0\ ,\ v(0, y) = v_W(y) \tag{15}$$

$$v(l_x, y) = 0,\ v(l_x, y) = v_E(y) \tag{16}$$

where l_x and l_y represent the dimension of the flow domain that corresponds to the image under registration and $u_N(x)$, $u_S(x)$, $v_W(y)$, $v_E(y)$ represent the nonslip velocity on the four walls.

If we further consider a viscous incompressible fluid and introduce the Reynolds number to Eq. (12), the governing equation can be decomposed into the principal directions in 2-D as

$$\begin{cases} u_t + P_x = -(u^2)_x - (uv)_y + \frac{1}{RE}(u_{xx} + u_{yy}) \\ v_t + P_y = -(v^2)_y - (uv)_x + \frac{1}{RE}(v_{xx} + v_{yy}) \end{cases} \tag{17}$$

where u and v respectively represent the velocity in the x- and y-axis, RE represents the Reynolds number. In Eq. (17), $-(u^2)_x - (uv)_y$ is the inertia or convection term and $\frac{1}{RE}(u_{xx} + u_{yy})$ is the diffusion or dissipation term. For image registration, the pressure term in Eq. (17) is replaced by a body force as

$$\begin{cases} u_t + \alpha f_x = -(u^2)_x - (uv)_y + \frac{1}{RE}(u_{xx} + u_{yy}) \\ v_t + \alpha f_y = -(v^2)_y - (uv)_x + \frac{1}{RE}(v_{xx} + v_{yy}) \end{cases} \tag{18}$$

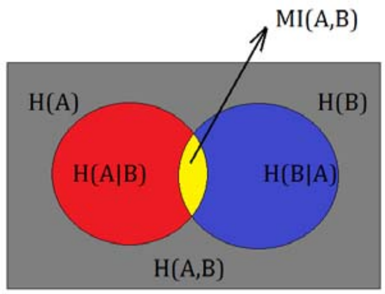

Figure 3. Illustration of mutual information.

where f_x and f_y represent the body force in the *x*- and *y*-axis, respectively, and α is a weighting function for controlling the contribution of the body force, which will be described in detail later.

3.2 Mutual Information

In information theory, entropy is a measure of the uncertainty associated with a random variable given by the following equation:

$$H(X) = E\big(I(X)\big) = \sum pi(X)\log(\frac{1}{pi(X)}) \tag{19}$$

where *X* represents a discrete random variable, *I*(*X*) represents the information content of *X*, *E* represents the expectation, *pi* represents the probability mass function in terms of *X*, and *H*(*X*) represents the entropy of *X*. We can compute the entropy of an image by treating the image pixel as a random variable, say, images *A* and *B* using

$$H(A) = \sum pi(A)\log\left(\frac{1}{pi(A)}\right) \tag{20}$$

$$H(B) = \sum pi(B)\log\left(\frac{1}{pi(B)}\right) \tag{21}$$

where *H(A)* and *H(B*) represent the image intensity entropy of *A* and *B*, respectively. In addition, the joint entropy $H(A, B)$ of images *A* and *B* can be defined as

$$H(A, B) = \sum\sum pi(A, B)log\, pi(A, B) \tag{22}$$

where $pi(A, B)$ represents the joint intensity probability.

As illustrated in Fig. 3, the mutual information between *A* and *B* is given by

$$MI(A, B) = H(A) + H(B) - H(A, B) \tag{23}$$

where *MI* represents the mutual information, *H(A)*, *H(B*), and $H(A, B)$ are defined in Eqs. (20), (21), and (22), respectively. The mutual information is a quantity that measures the mutual dependence of two random variables, i.e., the intensity distribution correlation between images *A* and *B* in our case. When the intensity distributions between two images are greatly intersected, the mutual information is larger, and vice versa. On the other hand, when the intensities of two images are completely independent, the mutual information is zero. As such, we can introduce the mutual information to the weighting function α in Eq. (18) using

$$\alpha = log\frac{MI(R,R)}{MI(T,R)} \tag{24}$$

where $MI(R, R)$ represents the self-mutual information of the reference image R and $MI(T, R)$ represents the mutual information between the template image *T* and the reference image *R*. At the beginning of registration, $MI(T, R)$ is quite small and α is large that gives a strong body force to drive the flow with large deformations. However, when the template image is getting similar with the reference image, the value of α becomes small (close to zero) that results in small deformations and finally stops the registration process.

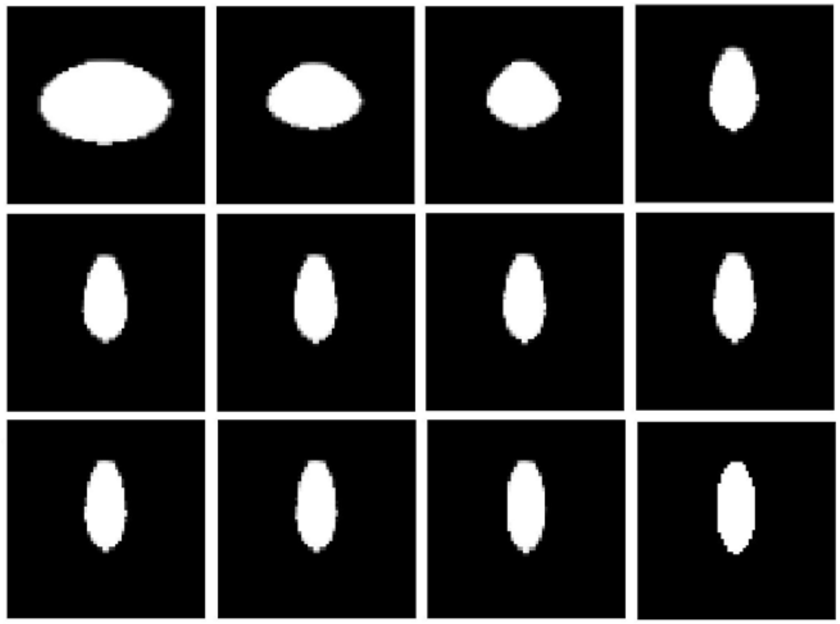

Figure 4. Evolution of the registration process from top left to bottom right.

4. RESULTS

A fast and simple numerical technique proposed by Seibold[1] was adopted to solve the governing PDEs of the proposed method. We have applied our approach to a variety of phantom, BrainWeb[10], and real MR images. We randomly shrank or enlarged the images on different anatomical structures for the experiments. To analyze the performance of the proposed algorithm, the sum of squared difference (SSD) and correlation coefficients (CC) between the deformed template image and the reference image are used as given in the following equations:

$$SSD = \sum\sum \frac{\|A(x,y) - B(x,y)\|^2}{N} \tag{25}$$

$$CC = \frac{\sum\sum\big(A(x,y) - \bar{A}(x,y)\big)\sum\sum\big(B(x,y) - \bar{B}(x,y)\big)}{\sqrt{\sum\sum\big(A(x,y) - \bar{A}(x,y)\big)^2 \sum\sum\big(B(x,y) - \bar{B}(x,y)\big)^2}} \tag{26}$$

where *A(x, y)* and *B(x, y)* represent the intensities of images *A* and *B* at *(x, y)*, respectively, $\|\cdot\|$ represents the Euclidean norm, *N* represents the total pixel number, $\bar{A}(x,y)$ and $\bar{B}(x,y)$ represent the average intensities of images *A* and *B*, respectively.

Figure 4 shows the evolution of the registration process of a template image from a horizontal, big oval transforming to a vertical, small oval that corresponds to the reference image. The corresponding displacement field and deformation grid maps are shown in Fig. 5. In Fig. 6, we show the template, reference, and registered brain MR images of using our method. Finally, Table 1 summarizes the SSD and CC scores of the experiments in Figs. 4 and 6. Clearly, significantly smaller SSD values and much higher CC values were obtained after the registration.

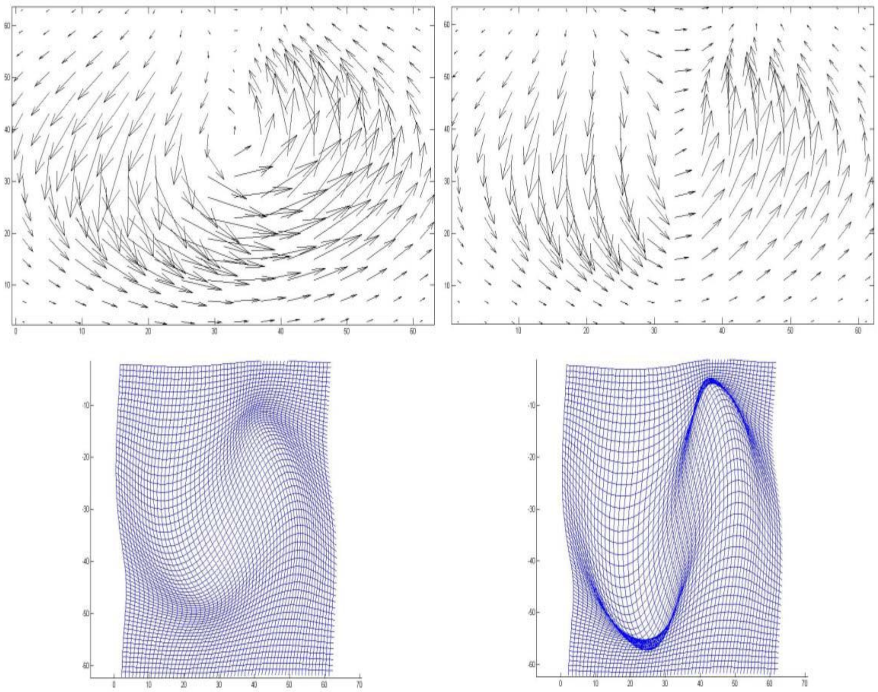

Figure 5. Analysis of Fig. 4. Top: displacement field maps at iteration 10 (left) and 700 (right). Bottom: deformation grid maps at iteration 10 (left) and 700 (right).

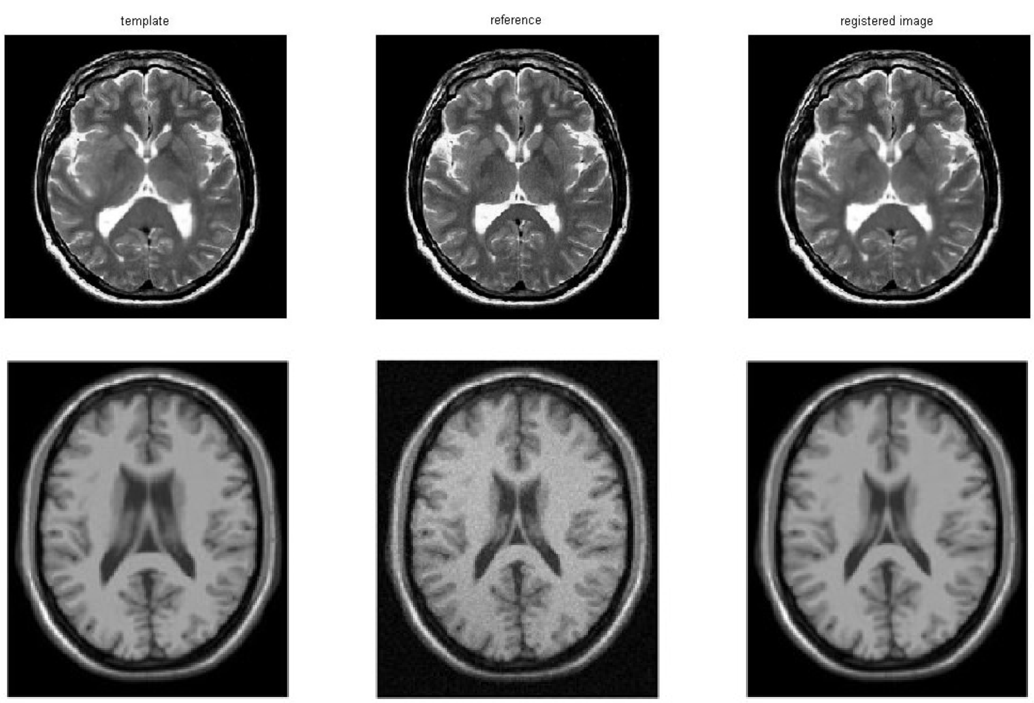

Figure 6. Registration results of brain MR images. Top: real MR data. Bottom: BrainWeb data.

Table 1. Quantitative analysis of the registration results.

Experiment	SSD		CC	
	Initial	Final	Initial	Final
Fig. 4	0.1568	2.3707×10^{-4}	0.4145	0.9937
Fig. 6: Top	0.0013	1.2972×10^{-4}	0.9377	0.9915
Fig. 6: Bottom	0.0012	4.7975×10^{-5}	0.9703	0.9894

5. CONCLUSIONS

Stimulated by the success of the existing fluid flow models, we proposed a new registration algorithm based on the incompressible viscous fluid model associated with mutual information. We replaced the pressure term with the body force to guide the transformation with a weighting coefficient, which is expressed by the mutual information between the template and reference images. A fast and simple numerical technique is adopted to solve the PDE while maintaining high registration accuracy. A variety of phantom, BrainWeb, and real MR images were used to evaluate our algorithm. Experimental results indicate that the proposed method is of potential in a wide variety of medical image registration applications.

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