Cvičenia.

V úlohách 1 - 14 vypočítajte pomocou Laplaceovej transformácie riešenie začiatočnej úlohy:

1. $x'''(t) + 2x''(t) + 5x'(t) = 0,\ x(0+) = -1,\ x'(0+) = 2,\ x''(0+) = 0.$
   $\left[x(t) = -\frac{1}{5} - \frac{4}{5}e^{-t}\cos(2t) + \frac{3}{5}e^{-t}\sin(2t)\right]$
2. $x^{(4)}(t) + 2x''(t) + x(t) = 1,\ x(0+) = x'(0+) = x''(0+) = x'''(0+) = 0.$
   $\left[x(t) = 1 - \cos t - \frac{t}{2}\sin t\right]$
3. $x''(t) - 3x'(t) + 2x(t) = e^{3t},\ x(0+) = x'(0+) = 0.$
   $\left[x(t) = \frac{1}{2}e^{t} - e^{2t} + \frac{1}{2}e^{3t}\right]$
4. $x''(t) - 3x'(t) + 2x(t) = 2e^{3t},\ x(0+) = x'(0+) = 0.$
   $[x(t) = e^{t} - 2e^{2t} + e^{3t}]$
5. $x''(t) - x'(t) = te^{t},\ x(0+) = 1,\ x'(0+) = 0.$
   $\left[x(t) = e^{t}\left(\frac{t^2}{2} - t + 1\right)\right]$
6. $x'(t) + x(t) = t^2e^{-t},\ x(0+) = a.$
   $\left[x(t) = ae^{-t} + \frac{t^3}{3}e^{-t}\right]$
7. $x''(t) + 4x'(t) + 4x(t) = t^3e^{-2t},\ x(0+) = 1,\ x'(0+) = 2.$
   $\left[x(t) = e^{-2t}\left(1 + 4t + \frac{t^5}{20}\right)\right]$
8. $x'''(t) - x''(t) = \sin t,\ x(0+) = x'(0+) = x''(0+) = 0.$
   $\left[x(t) = -1 - t + \frac{1}{2}e^{t} + \frac{1}{2}(\cos t + \sin t)\right]$
9. $x'(t) + x(t) = f(t),\ x(0+) = 0,\ f(t) = \begin{cases} 0 & t \notin \langle 0, 2) \\ 1 & t \in \langle 0, 2) \end{cases}.$
   $\left[x(t) = 1 - e^{-t} - \Theta(t-2)\left(1 - e^{-(t-2)}\right)\right]$
10. $x''(t) + 2x'(t) + x(t) = f(t),\ x(0+) = x'(0+) = 0,\ f(t) = \begin{cases} 0 & t < 0 \\ t & t \in \langle 0, 1) \\ 1 & t \geq 1 \end{cases}.$
    $\left[x(t) = -2 + t + 2e^{-t} + te^{-t} - \Theta(t-1)\left[-2 + (t-1) + 2e^{-(t-1)} + (t-1)e^{-(t-1)}\right]\right]$
11. $x''(t) + x(t) = f(t),\ x(0+) = 1,\ x'(0+) = 0,\ f(t) = \begin{cases} 0 & t < 0 \\ b & t \in \langle 0, a) \\ 2b & t \geq a \end{cases}.$
    $[x(t) = \Theta(t)[b + (1-b)\cos t] + \Theta(t-a)[b - b\cos(t-a)]]$
12. $x''(t) + x(t) = |\sin t|,\ x(0+) = x'(0+) = 0.$
    $\left[X(p) = \frac{p}{(p^2+1)^2(1-e^{-\pi p})} + e^{-\pi p}\frac{p}{(p^2+1)^2(1-e^{-\pi p})}, x(t) =\right]$

13. $x'(t) + 3x(t) = e^{-t} + |\sin t|\,,\ x(0+) = 0.$

$\left[X(p) = \frac{1}{(p+3)(p+1)} + \frac{p}{(p+3)(p^2+1)(1-e^{-\pi p})} - e^{-\pi p}\frac{p}{(p+3)(p^2+1)(1-e^{-\pi p})}, x(t) = \right]$

14. $x'(t) + 2x(t) = Ae^{-t} + g(t)\,,\ x(0+) = 0$, kde $A = \frac{1}{1-e}$ a $g(t)$ je periodická s periódou 1, ktorá na intervale $\langle 0, 1)$ je $g(t) = (1-A)\,e^{-t}$.

$$\left[\begin{array}{c} x(t) = \frac{1}{1-e}e^{-t} - \frac{1}{1-e^2}e^{-2t} + h(t)\,, \quad \text{kde } h(t) \text{ je periodická funkcia} \\ \text{s periódou 1, ktorá sa na intervale } \langle 0,1) \text{ rovná} \\ h(t) = \left(1 - \frac{1}{1-e}\right)e^{-t} - \left(1 - \frac{1}{1-e^2}\right)e^{-2t}. \end{array}\right]$$