Time Series Forecasting of Chaotic Dynamical Systems

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Interest

I've always really enjoyed problem solving, and in college, I was drawn towards mathematics. After a few modeling and statistics courses, I learned to use mathematics to model and solve problems. I am interested in researching is chaotic forecasting problems because they occur frequently in nature and daily life. It's important to be able to understand what might come next, and it has many practical applications: weather patterns, orbiting bodies in space, fluid flow, and the dynamics of ecosystems. I want to build complex models stemming from deep learning (such as recurrent neural networks) and research how well they can solve problems like these. Being chaotic means that there are very complex patterns occurring. These models may capture those patterns, so I am very interested in building models to simulate the chaotic phenomenon and then implementing recurrent neural nets to perform a time-series forecasting analysis on the simulations. It will involve data analytics with neural networks, solving systems of PDEs numerically, and designing models to simulate physical phenomenon. With software development as another passion of mine, programming these models is very rewarding. I am interested in building a small python software module to be available and useful to anyone hoping to solve forecasting problems.

Project Description

The central idea of my research will evaluate the effectiveness of deep learning models in time-series forecasting of chaotic (and deterministic) dynamical systems. Chaotic systems have the property of having extremely high variance given slight perturbation of input data. Typically, these systems seem to have random properties, which can be difficult to model. However, deep neural networks may be strong enough to capture the complexity. The three chaotic systems I will study are the 3-body problem, the Lorenz butterfly, and the double pendulum. All three are classic examples of chaotic systems.

My project will consist of creating a Python package to model and analyze the chaotic systems, and I will evaluate how well deep learning techniques perform in comparison to more traditional methods. First, I will need to collect data to analyze. I will model the chaotic systems and collect necessary data from these models. Since chaotic models are very sensitive to perturbations, I will have to pay special attention to accuracy and correctness when using numerical methods to simulate the problems. Second, I will make visualization tools to view simulations generated by the models. Third, I will implement and apply forecasting models to the generated data and evaluate performance. Models will include classical forecasting approaches such as ARIMA, and it will include deep learning techniques such as recurrent neural networks, echo state networks, and long short-term memory networks. Finally, I will compare the performance of classical and deep learning approaches and conclude with how well these chaotic problems can be modeled.

Major Tasks

- Models for chaotic dynamical systems—such as 3-body problem, Lorenz butterfly problem, and double pendulum problem—will be implemented with Python.

- Tools to generate and visualize data for these models will be created using available python packages such as NumPy, SciPy and Matplotlib.
- Time-series forecasting models—such as ARIMA, Echo State Neural Network, Recurrent Neural Network, and LSTM—will be implemented and applied to the generated data.
- Results from the analysis will be interpreted to conclude how effectively deep learning techniques model chaotic dynamical systems.
- A Python module will be created containing all the models of chaotic systems, data analytic tools, and generated datasets.

Key Resources

I. Goodfellow, Y. Bengio, and A. Courville, *Deep Learning*. MIT Press, 2016. http://www.deeplearningbook.org.

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S. Ho, M. Xie, and T. Goh, "A comparative study of neural network and box-jenkins arima modeling in time series prediction," *Computers & Industrial Engineering*, vol. 42, no. 2-4, pp. 371–375, 2002.

G. E. Box, G. M. Jenkins, G. C. Reinsel, and G. M. Ljung, *Time series analysis: forecasting and control*. John Wiley & Sons, 2015.

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E. N. Lorenz, "Deterministic nonperiodic flow," *Journal of the atmospheric sciences*, vol. 20, no. 2, pp. 130–141, 1963.

Timeline of Completion

- March 5th: Python analytic tool finished. Data generated and analyzed.
- March 28th: First draft of scientific paper completed.
- April 4th: Penultimate draft submitted
- April 19th: Oral thesis presentation to Dr. Chuck Anderson and Dr. Gerhard Dangelmayr
- May 13th: All thesis components submitted

Final Product Description

The final product will be a Python module for analyzing chaotic dynamical systems and a written report evaluating how well deep learning techniques can model chaotic dynamical systems. The Python module will simulate three chaotic systems: the 3-body problem, the Lorenz butterfly, and the double pendulum. It will be capable of visualizing the simulations. The core of it will be capable of creating, training, using and evaluating time series forecasting models—ARIMA, recurrent neural network, echo state network, and long short-term memory networks.