**Time Series Forecasting of Chaotic Dynamical Systems**

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**Interest**

I am interested in forecasting problems because they occur frequently in nature and daily life. I want to evaluate potential tools for solving such problems. I focus specifically on forecasting problems with chaotic properties. These types of problems can be difficult to model and therefore require stronger techniques. As such, I want to investigate the effectiveness of more modern techniques rooted in deep learning and compare them to more traditional methods. Examples of chaotic systems include weather patterns, orbiting bodies, fluid flow, and the dynamics of ecosystems. Research in this area can be very useful.

**Project Description**

The central idea of my research will evaluate the effectiveness of recurrent neural networks in time-series forecasting of chaotic (and deterministic) dynamical systems. Chaotic systems have the property of having extremely high variance given slight perturbation of input data. This typically results in the results having seemingly random properties, which can be hard to model. However, under deterministic dynamical systems, they are not random. In fact, lack of perturbation will have the exact same results every time. I will investigate whether deep networks are capable of learning the intricacies of chaotic systems and whether they are useful for forecasting.

There are several types of problems I can investigate to get a ranged census of how well my techniques are performing: the 3-body problem, Lorenz butterfly, and the double pendulum. With such classic problems, I hope to show the effectiveness of deep networks on these problems as representative of the effectiveness of deep learning on chaos in general. Contributions by Henri Poincaré towards solving the 3-body problem were instrumental in creating chaos theory. The problem models three bodies affected by each other’s gravity in open space. Edward Lorenz is considered the father of chaos, and one of his classic examples, probably the epitome of chaos, is the Lorenz Butterfly (related to the butterfly effect). This model is related to convection currents in the atmosphere. Additionally, one of his major papers outlines a fluid system with dynamical properties. Third, I will investigate the double pendulum problem.

First, I will need to collect data to analyze. Most of the problems are modeled by a system of PDEs, which I can solve with numerical methods. I will need to research what numerical methods apply to the chaotic problems because with the nature of chaos, it is likely the problems are considered non-stiff, meaning some common numerical solving methods don’t apply or require special alterations. By definition, perturbations are amplified greatly, so the numerical solutions I generate have to have as little error as possible. Further, I will write programs implementing the numerical methods to approximate the problems. Also, I will need to do research and find public datasets in order to additionally analyze real, recorded data. Such an analysis could have interesting results.

Second, I will analyze the data to gain a better understanding of the data. One method I will use is dimensionality reduction. A technique called t-sne allows us to have a meaningful visualization of the data by projecting high dimensional space into 2 or 3 dimensions. This is useful for larger chaotic problems (e.g., n-body problem as opposed to 3-body problem).

Third, I will research what standard methods for forecasting problems are. Then, I will research what performance evaluation techniques there are. I will implement and apply these to the data to analyze the results. One forecasting technique I will use is called ARIMA.

Finally, I will analyze data with modern methods. This involves designing a good RNN to fit the generated data. I can supplement the analysis with dimensionality reduction to help the model converge more quickly to good solutions. I will then compare those results to those of the classical approaches.

**Major Tasks**

* **Collect or generate data for 3-body problem, Lorenz butterfly problem, and double pendulum problem.** This will involve programming and simulating these systems to generate data. Simulating these systems will require research in numerical methods to ensure my data is correct and accurate.
* **Apply visualization and dimensionality reduction techniques to better understand the dataset.** This will involve programming the techniques (if not already part of an accepted math package).
* **Research and apply standard time series analysis techniques.** This involves researching strong forecasting techniques and evaluating them.
* **Research and apply deep learning time series analysis techniques.** This involves researching what models have been designed and then building them to apply to my problems.

**Key Resources**

**Timeline of Completion**

Week 1-3: Write the formal proposal, collect resources and begin researching.

Week 4-5: Build the chaotic models and analytic toolsand collect data

Week 6-8: Build the time series forecasting models

Week 9: Perform preliminary data analysis

Week 10: Apply models to data to collect results.

Week 11: Evaluate effectiveness of deep learning models

Week 12-13: Finalize report and submit Penultimate Draft (April 24th)

Week 14: Prepare Oral Presentation

Week 15-Finals Week:

May 10th: Oral Presentation to Dr. Chuck Anderson and TODO

May 14th: All thesis components submitted

**Final Product Description**