Acceleration discontinuities in dry-friction oscillations

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I recently noticed the paper by Hauko *et al.*[1] in which an interesting pedagogical exposition of oscillatory motion in the presence of dry (Coulomb) friction is provided. It appeared exactly one month after our own paper on the same subject.[2] The timing of publication appears to have precluded the possibility of their authors situating their work with respect to ours, and so I am taking the opportunity to provide this service here, as there are points that will bring valuable insight to readers of either article.

Central to both papers is the discontinuous shift in equilibrium position that occurs at the extremes of motion as the velocity changes sign. The pedagogical analogy that Hauko's paper proposes is based on the equivalence between a horizontally and vertically oriented spring-mass system, an approach that emphasizes the preservation of (quasi-) harmonic behaviour under equilibrium shifts. From this perspective, the shift is a subtle one, a view supported by the fact that the position graph (as well as its first derivative) remains continuous in the presence of Coulomb friction: The deviations from friction-free harmonic motion can only be deduced by a careful examination of the amplitude decay over many oscillations, an examination which Hauko *et al.* carry out based on measurements of position as a function of time.

Our paper, in contrast, begins with a theory that depicts graphs of the *second* derivative in the presence of Coulomb friction. Such graphs of acceleration versus time differ sharply from their friction-free counterparts, exhibiting a dramatic discontinuity in direct (proportional) response to the discontinuous sign-change of the frictional force. One can immediately recognise the presence of friction in even a casual inspection of such curves; and parameters characterising it are in principle obtainable from data over a single oscillation.

Historically, the absence of the latter approach in investigations of oscillatory motion can be attributed to the fact that a sequence of position measurements, when differentiated twice, produces a signal with so much noise as to make the discontinuities undetectable. The present-day broad availability of affordable MEMS-based accelerometer devices, however, is opening up a whole new vista for the observation of classical systems. Our paper, based on the analysis of such accelerometer data, reports the first direct observation of these discontinuities and presents a variety of related plots, such as phase diagrams, that reveal clear discontinuity signatures due to Coulomb friction.

Additional work since the time of our publication[3–6] continues to explore and celebrate the untapped potential of accelerometer technology, and I encourage readers to consider adopting it as part of their arsenal of experimental tools. Being familiar with both position-based and accelerometer-based approaches is valuable: Knowledge of both leads to a richer appreciation of the underlying phenomenon and of the range of data-analysis methods that can be employed in quantifying it.

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