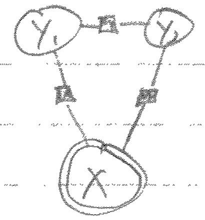
Démo mean-field



y_1 et y_2 sont binaires

$$p(y_1, y_2 \mid x) = \exp\left(y_1 a_1(x) + y_2 a_2(x) + y_1 y_2 v\right) / Z(x)$$

$$q(y_1, y_2) = q(y_1)\, q(y_2)$$

$$q(y_i) = y_i \mu_i + (1-y_i)(1-\mu_i) \qquad \begin{pmatrix} y_i = 1 \to \mu_i \\ y_i = 0 \to 1-\mu_i \end{pmatrix}$$

$$\begin{aligned}
KL\left(q(y_1,y_2) \,\|\, p(y_1,y_2 \mid x)\right) &= \sum_{y_1}\sum_{y_2} q(y_1,y_2) \log \frac{q(y_1,y_2)}{p(y_1,y_2 \mid x)} \\
&= \sum_{y_1}\sum_{y_2} q(y_1)\,q(y_2) \log\left(q(y_1)\,q(y_2)\right) \\
&\quad - \sum_{y_1}\sum_{y_2} q(y_1)\,q(y_2) \log p(y_1,y_2 \mid x) \\
&= \sum_{y_1}\sum_{y_2} q(y_1)\,\cancel{q(y_2)} \log q(y_1) \\
&\quad + \sum_{y_2}\sum_{y_1} \cancel{q(y_1)}\,q(y_2) \log q(y_2) \\
&\quad - \sum_{y_1}\sum_{y_2} q(y_1)\,q(y_2) \left[y_1 a_1(x) + y_2 a_2(x) + y_1 y_2 v\right] \\
&\quad + \sum_{y_1}\sum_{y_2} q(y_1)\,q(y_2) \log Z(x) \\
&= \underbrace{\begin{aligned}&\mu_1 \log \mu_1 + (1-\mu_1)\log(1-\mu_1) \\ &+ \mu_2 \log \mu_2 + (1-\mu_2)\log(1-\mu_2) \\ &- \mu_1 a_1(x) - \mu_2 a_2(x) - \mu_1 \mu_2 v \\ &+ \log Z(x)\end{aligned}}
\end{aligned}$$

On souhaite minimiser ça avec la contrainte $\mu_1 \in [0,1]$ et $\mu_2 \in [0,1]$

$$\frac{\partial KL(\text{—})}{\partial \mu_1} = \log \mu_1 + 1 - \log(1-\mu_1) - 1 - a_1(x) - \mu_2 v$$

Fixe à 0 :

$$0 = \log \mu_1 - \log(1-\mu_1) - a_1(x) - \mu_2 v$$

$$\Leftrightarrow \log\left(\frac{1-\mu_1}{\mu_1}\right) = -a_1(x) - \mu_2 v$$

$$\Leftrightarrow \frac{1-\mu_1}{\mu_1} = \exp(-a_1(x) - \mu_2 v)$$

$$\Leftrightarrow 1 = \mu_1(1 + \exp(-a_1(x) - \mu_2 v))$$

$$\Leftrightarrow \frac{1}{1+\exp(-a_1(x) - \mu_2 v)} = \mu_1$$

Peut démontrer que, pour μ_2 fixe, cette valeur de μ_1 est optimale.

Si on fixe μ_1 : $\mu_2 = \dfrac{1}{1+\exp(-a_2(x) - \mu_1 v)}$

Algorithme naive mean-field = coordinate descent

tant que pas convergé

$$\mu_1 \leftarrow \frac{1}{1+\exp(-a_1(x) - \mu_2 v)}$$

$$\mu_2 \leftarrow \frac{1}{1+\exp(-a_2(x) - \mu_1 v)}$$