对于每个固定的 x，对 y 求导知 $f(x,y)$ 是增加的，于是 $y=c-x$ 时，$f(x,y)$ 达到最大$f(x,c-x)$

$$f(x,c-x)=\frac{1}{3}(1+b(c-x))\alpha+\frac{2}{3}(1+a x)\alpha+\frac{(1+b(c-x))\beta}{2(1+a x)}=$$

$$\frac{\alpha}{3}+\frac{b\alpha}{3a}+\frac{bc\alpha}{3}+(a x+1)\left(\frac{2\alpha}{3}-\frac{b\alpha}{3a}\right)-\frac{b\beta}{2a}+\frac{\frac{\beta}{2}+\frac{b\beta}{2a}+\frac{bc\beta}{2}}{a x+1}$$

$x\in(0,c)$，不论 $\left(\frac{2\alpha}{3}-\frac{b\alpha}{3a}\right)$ 的符号如何，$f(x,c-x)$ 都在端点取得最大值，

$$f(0,c)=\frac{1}{3}\alpha(1+bc)+\frac{\beta(1+bc)}{2}$$

$$f(c,0)=\frac{2}{3}\alpha(1+ac)+\frac{\beta}{2(1+ac)}$$

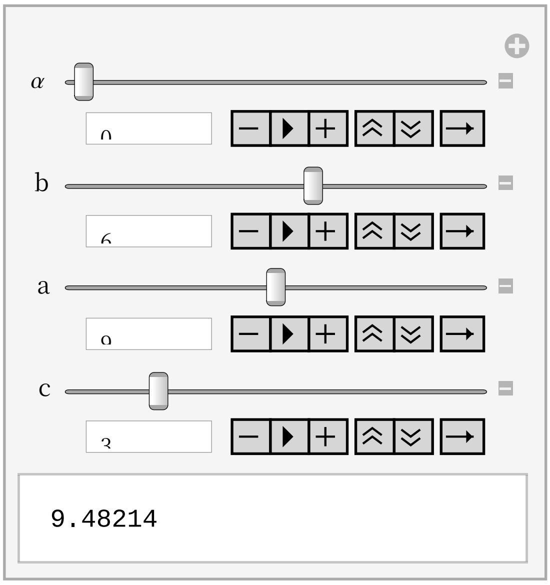
$\max f(x,y)=\max\{f(0,c),f(c,0)\}$

$\alpha=0,\ b=6,\ a=9,\ c=3\Longrightarrow f(0,c)>f(c,0)$

$\alpha=1,\ b=6,\ a=9,\ c=3\Longrightarrow f(0,c)<f(c,0)$

In[39]:= $$\text{Manipulate}\left[\frac{1}{3}\alpha(1+bc)+\frac{(1-\alpha)(1+bc)}{2}-\left(\frac{2}{3}\alpha(1+ac)+\frac{1-\alpha}{2(1+ac)}\right),\right.$$
$$\left.\{\alpha,0,1\},\{b,0,10\},\{a,b,12\},\{c,0,15\}\right]$$

Out[39]=



In[19]:= $$\text{Collect}\left[\frac{1}{3}(1+b(c-x))\alpha+\frac{2}{3}(1+a x)\alpha+\frac{(1+b(c-x))\beta}{2(1+a x)}\ /.\ x\to(y-1)/a,\ y\right]$$

Out[19]= $$\frac{\alpha}{3}+\frac{b\alpha}{3a}+\frac{bc\alpha}{3}+y\left(\frac{2\alpha}{3}-\frac{b\alpha}{3a}\right)-\frac{b\beta}{2a}+\frac{\frac{\beta}{2}+\frac{b\beta}{2a}+\frac{bc\beta}{2}}{y}$$

In[9]:= $$\text{Collect}\left[\frac{1}{3}(1+b(c-x))\alpha+\frac{2}{3}(1+a x)\alpha+\frac{(1+b(c-x))\beta}{2(1+a x)}\ /.\ \beta\to 1-\alpha,\ x\right]$$

Out[9]= $$\frac{(1+b(c-x))(1-\alpha)}{2(1+a x)}+\frac{1}{3}(1+b(c-x))\alpha+\frac{2}{3}(1+a x)\alpha$$

In[10]:=
```
D[1/3 (1 + b (c - x)) α + 2/3 (1 + a x) α + ((1 + b (c - x)) β)/(2 (1 + a x)), x]
```

Out[10]= $$\frac{2\,a\,\alpha}{3} - \frac{b\,\alpha}{3} - \frac{a\,(1 + b\,(c - x))\,\beta}{2\,(1 + a\,x)^2} - \frac{b\,\beta}{2\,(1 + a\,x)}$$

In[15]:=
```
2 a α/3 - b α/3 - (a (1 + b (c - x)) β)/(2 (1 + a x)^2) - (b β)/(2 (1 + a x)) /. x → (y - 1) / a // Simplify
```

Out[15]= $$\frac{4\,a\,y^2\,\alpha - 2\,b\,y^2\,\alpha - 3\,a\,\beta - 3\,b\,\beta - 3\,a\,b\,c\,\beta}{6\,y^2}$$

In[29]:=
```
t = Solve[2 a α/3 - b α/3 - (a (1 + b (c - x)) β)/(2 (1 + a x)^2) - (b β)/(2 (1 + a x)) == 0, x]
a x + 1 /. t // Simplify
```

Out[29]= $$\left\{\left\{x \to \left(-4\,a^2\,\alpha + 2\,a\,b\,\alpha - \sqrt{6}\,\sqrt{\left(2\,a^4\,\alpha\,\beta + a^3\,b\,\alpha\,\beta - a^2\,b^2\,\alpha\,\beta + 2\,a^4\,b\,c\,\alpha\,\beta - a^3\,b^2\,c\,\alpha\,\beta\right)}\right) \Big/ \left(2\,\left(2\,a^3\,\alpha - a^2\,b\,\alpha\right)\right)\right\},\right.$$
$$\left.\left\{x \to \left(-4\,a^2\,\alpha + 2\,a\,b\,\alpha + \sqrt{6}\,\sqrt{\left(2\,a^4\,\alpha\,\beta + a^3\,b\,\alpha\,\beta - a^2\,b^2\,\alpha\,\beta + 2\,a^4\,b\,c\,\alpha\,\beta - a^3\,b^2\,c\,\alpha\,\beta\right)}\right) \Big/ \left(2\,\left(2\,a^3\,\alpha - a^2\,b\,\alpha\right)\right)\right\}\right\}$$

Out[30]= $$\left\{-\frac{\sqrt{\frac{3}{2}}\,a\,(a + b + a\,b\,c)\,\beta}{\sqrt{a^2\,(2\,a - b)\,(a + b + a\,b\,c)\,\alpha\,\beta}},\ \frac{\sqrt{\frac{3}{2}}\,a\,(a + b + a\,b\,c)\,\beta}{\sqrt{a^2\,(2\,a - b)\,(a + b + a\,b\,c)\,\alpha\,\beta}}\right\}$$