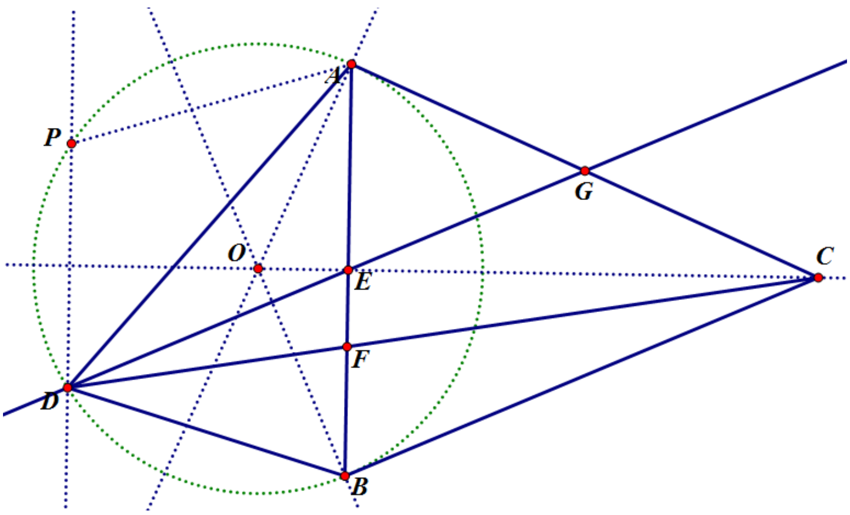

设$\angle ABC = \alpha$, $\angle ABD = \beta$, O是$\triangle ADB$的外心,

则由$\angle AOB = 2\angle ADC$且OC一定为$\triangle ABC$的对称轴$\Longrightarrow AO \perp AC$, $BO \perp BC$

$\angle ADB = \angle BAC = \angle AEG \Longrightarrow \triangle BDE$■$\triangle BAD \Longrightarrow BD = \sqrt{2}\,BE$ (1)

正弦定理：$\dfrac{DE}{\sin\beta} = \dfrac{\sqrt{2}\,BE}{\sin\alpha}$ (2)

将$AD = 2\,OD\sin\beta$与$BE = OB\sin\alpha$和（1）代入余弦定理：$AB^2 = AD^2 - 2\,AD\,BD\cos\alpha$

$\Longrightarrow \sin^2\alpha = 2\sin^2\beta - 2\sqrt{2}\sin\alpha\cos\alpha\sin\beta$ (3)

$DP \perp OC \Longrightarrow PD = 2\,DE\cos\alpha$ ($\angle PDE = \alpha$) (4)

由余弦定理 :$CD = \sqrt{DE^2 + CE^2 + 2\,DE\,CE\sin\alpha}$ (5)

将（2）代入（5）并联立（3），（4）得到 $PD \times CD = \left(2\sqrt{2}\dfrac{BE\sin\beta\cos\alpha}{\sin\alpha}\right)\left(BE\dfrac{\sqrt{2}\sin\beta}{\sin\alpha\cos\alpha}\right)$

$= 4\,BE^2\dfrac{\sin^2\beta}{\sin^2\alpha}$ (注意$\triangle BOE$) $= (2\,OD\sin\beta)^2 = AD^2$ (6)

另外由于$\angle DPA = \angle ADB + \angle DAB = \angle DAB + \angle BAC = \angle DAC$ (联合6) $\Longrightarrow \triangle PDA$■$\triangle ADC$

故$\angle DAF = \angle PDA$ (PD平行AB) $= \angle ADC$