

```

f[x_] := 4 Cos[ $\sqrt{2} x$ ] - x Sin[ $\frac{1}{x}$ ] -  $\left(x - \frac{1}{2}\right)$  Sin[ $\frac{1}{x - \frac{1}{2}}$ ] -  $\left(x + \frac{1}{2}\right)$  Sin[ $\frac{1}{x + \frac{1}{2}}$ ]

n = 6;
point = Sort[RandomReal[{-1, 1}, n + 1]]
interpoint = point;
b = Table[{f[point[[i]]]}, {i, n + 1}];
a = N[Normal[SparseArray[{i_, 1}  $\rightarrow$  1,
    {i_, j_} /; 2  $\leq$  j  $\leq$  n  $\rightarrow$  Part[point, i]j-1, {i_, n + 1}  $\rightarrow$  (-1)i, {n + 1, n + 1}]], 6 n];
sol = N[Inverse[a].b, 6 n];

ρs = NMaximize[{Abs[f[x] - sol[[1, 1]] -  $\sum_{k=1}^{n-1}$  (sol[[k + 1, 1]] xk)], -1  $\leq$  x  $\leq$  1}, x]

{ρ, x0} = {ρs[[1]], x} /. ρs[[2]]
test = True;

While[test, If[-1  $\leq$  x0 < point[[1]], If[ $\left(f[x0] - \text{sol}[[1, 1]] - \sum_{k=1}^{n-1} (\text{sol}[[k + 1, 1]] x0^k)\right)$ 

     $\left(f[\text{point}[[1]]] - \text{sol}[[1, 1]] - \sum_{k=1}^{n-1} (\text{sol}[[k + 1, 1]] \text{point}[[1]]^k)\right) \geq 0,$ 

    interpoint[[1]] = x0, interpoint = RotateRight[point, 1]; interpoint[[1]] = x0]];

If[point[[-1]] < x0  $\leq$  1, If[ $\left(f[x0] - \text{sol}[[1, 1]] - \sum_{k=1}^{n-1} (\text{sol}[[k + 1, 1]] x0^k)\right)$ 

     $\left(f[\text{point}[[ -1]]] - \text{sol}[[1, 1]] - \sum_{k=1}^{n-1} (\text{sol}[[k + 1, 1]] \text{point}[[ -1]]^k)\right) \geq 0,$ 

    interpoint[[-1]] = x0, interpoint = RotateLeft[point, 1]; interpoint[[-1]] = x0]];

If[Select[Table[i, {i, 1, n}], point[[#]]  $\leq$  x0 &] != {} &&
    Select[Table[i, {i, 2, n + 1}], point[[#]]  $\geq$  x0 &] != {},
    j = Last[Select[Table[i, {i, n}], point[[#]]  $\leq$  x0 &]];

If[ $\left(f[x0] - \text{sol}[[1, 1]] - \sum_{k=1}^{n-1} (\text{sol}[[k + 1, 1]] x0^k)\right)$ 

     $\left(f[\text{point}[[j]]] - \text{sol}[[1, 1]] - \sum_{k=1}^{n-1} (\text{sol}[[k + 1, 1]] \text{point}[[j]]^k)\right) \geq 0,$ 

    interpoint[[j]] = x0, interpoint[[j + 1]] = x0]];

point = interpoint;
b = N[Table[{f[point[[i]]]}, {i, n + 1}], 6 n];
a = N[Normal[SparseArray[{i_, 1}  $\rightarrow$  1,
    {i_, j_} /; 2  $\leq$  j  $\leq$  n  $\rightarrow$  Part[point, i]j-1, {i_, n + 1}  $\rightarrow$  (-1)i, {n + 1, n + 1}]], 6 n];
sol = N[Inverse[a].b, 6 n];

ρs = N[NMaximize[{Abs[f[x] - sol[[1, 1]] -  $\sum_{k=1}^{n-1}$  (sol[[k + 1, 1]] xk)], -1  $\leq$  x  $\leq$  1}, x], 6 n];

If[Abs[Abs[sol[[-1, 1]]] - ρs[[1]]] < 10-n, test = False];
Print[N[sol[[-1]], 6 n]];
{ρ, x0} = {ρs[[1]], x} /. ρs[[2]]
]
point

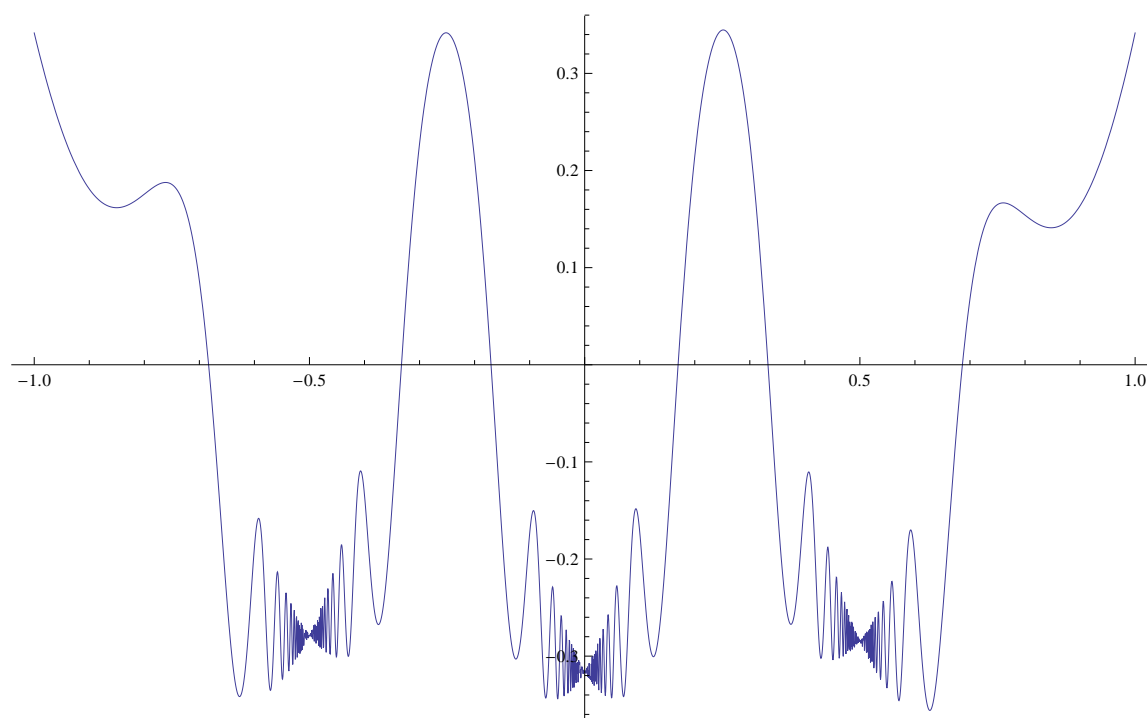
Plot[f[x] - sol[[1, 1]] -  $\sum_{k=1}^{n-1}$  (sol[[k + 1, 1]] xk), {x, -1, 1}]

```

```

{-0.888298, -0.784824, -0.595155, 0.0383027, 0.595875, 0.940728, 0.976649}
{0.816829, {x → -0.250006}}
{0.816829, -0.250006}
{0.0923368}
{0.146709}
{0.16377}
{0.19771}
{0.237423}
{0.250585}
{-0.321327}
{-0.331739}
{-0.340522}
{-0.341659}
{-1., -0.627091, -0.250006, 0.0708147, 0.259882, 0.634988, 1.}

```



```
n = 20;
```

```
point = Sort[N[ $\frac{\text{RandomInteger}[-10^{20}, 10^{20}], n+1]}{10^{20}}, 6n]];$ 
```

```
interpoint = point;
```

```
b = N[Table[{f[point[[i]]], {i, n+1}}, {i, n+1}], 6n];
```

```
a = N[Normal[SparseArray[{{i_, 1} → 1,
    {i_, j_} /; 2 ≤ j ≤ n → Part[point, i]^(j-1), {i_, n+1} → (-1)^i, {n+1, n+1}}], 6n];
```

```
sol = N[Inverse[a].b, 6n];
```

```
 $\rho_s = \text{NMaximize}\left[\left\{\text{Abs}\left[f[x] - \text{sol}[[1, 1]] - \sum_{k=1}^{n-1} (\text{sol}[[k+1, 1]] x^k)\right], -1 \leq x \leq 1\right\}, x\right]$ 
```

```
{ρ, x0} = {ρs[[1]], x} /. ρs[[2]]
```

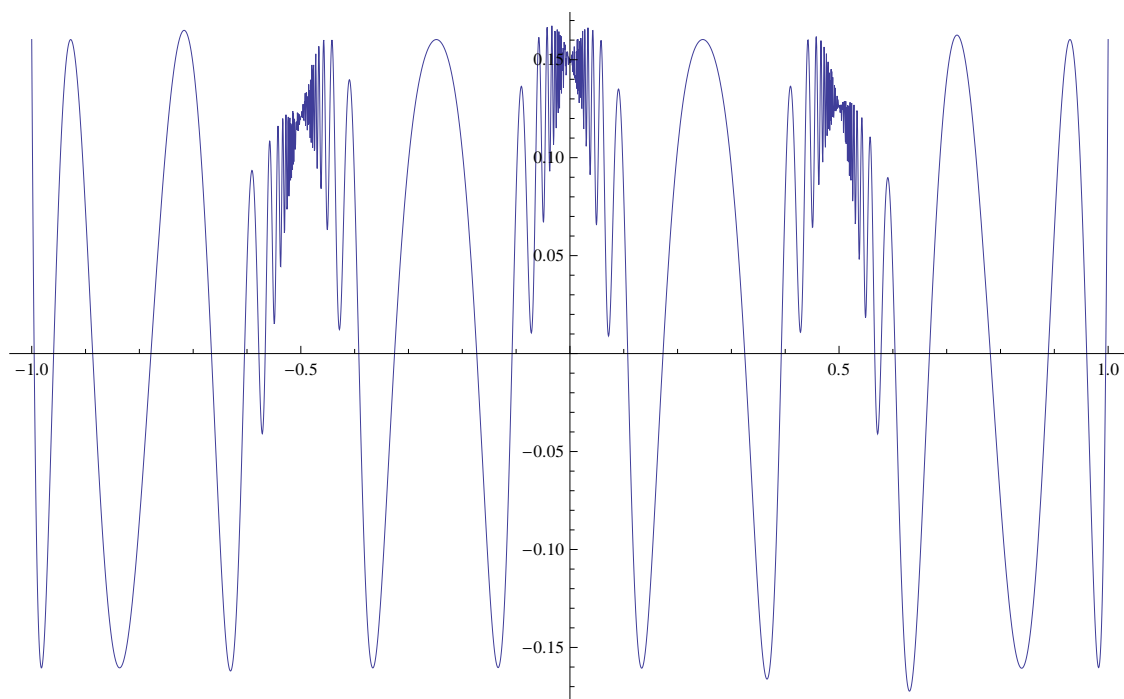
```
test = True;
```

```

While[ $\text{test}$ , If[ $-1 \leq x_0 < \text{point}[[1]]$ , If[ $\left(f[x_0] - \text{sol}[[1, 1]] - \sum_{k=1}^{n-1} (\text{sol}[[k+1, 1]] x_0^k)\right)$ 
 $\left(f[\text{point}[[1]]] - \text{sol}[[1, 1]] - \sum_{k=1}^{n-1} (\text{sol}[[k+1, 1]] \text{point}[[1]]^k)\right) \geq 0$ ,
interpoint[[1]] =  $x_0$ , interpoint = RotateRight[point, 1]; interpoint[[1]] =  $x_0$ ]];
If[point[[-1]] <  $x_0 \leq 1$ , If[ $\left(f[x_0] - \text{sol}[[1, 1]] - \sum_{k=1}^{n-1} (\text{sol}[[k+1, 1]] x_0^k)\right)$ 
 $\left(f[\text{point}[[ -1 ]]] - \text{sol}[[1, 1]] - \sum_{k=1}^{n-1} (\text{sol}[[k+1, 1]] \text{point}[[ -1 ]]]^k)\right) \geq 0$ ,
interpoint[[-1]] =  $x_0$ , interpoint = RotateLeft[point, 1]; interpoint[[-1]] =  $x_0$ ]];
If[Select[Table[i, {i, 1, n}], point[[#]] ≤  $x_0$  &] != {} &&
Select[Table[i, {i, 2, n+1}], point[[#]] ≥  $x_0$  &] != {},
j = Last[Select[Table[i, {i, n}], point[[#]] ≤  $x_0$  &]];
If[ $\left(f[x_0] - \text{sol}[[1, 1]] - \sum_{k=1}^{n-1} (\text{sol}[[k+1, 1]] x_0^k)\right)$ 
 $\left(f[\text{point}[[j]]] - \text{sol}[[1, 1]] - \sum_{k=1}^{n-1} (\text{sol}[[k+1, 1]] \text{point}[[j]]^k)\right) \geq 0$ ,
interpoint[[j]] =  $x_0$ , interpoint[[j+1]] =  $x_0$ ]];
point = interpoint;
b = N[Table[{f[point[[i]]}], {i, n+1}], 6 n];
a = N[Normal[SparseArray[{{i_, 1} → 1,
{i_, j_} /; 2 ≤ j ≤ n → Part[point, i]j-1, {i_, n+1} → (-1)i, {n+1, n+1}]], 6 n];
sol = N[Inverse[a].b, 6 n];
ρs = N[NMaximize[ $\left\{\text{Abs}\left[f[x] - \text{sol}[[1, 1]] - \sum_{k=1}^{n-1} (\text{sol}[[k+1, 1]] x^k)\right], -1 \leq x \leq 1\right\}, x\right], 6 n];
If[Abs[Abs[sol[[-1, 1]]] - ρs[[1]]] < 10-10, test = False];
Print[N[sol[[-1, 1]], 6 n]];
{ρ, x0} = {ρs[[1]], x} /. ρs[[2]]
]
point
g1 = Plot[ $f[x] - \text{sol}[[1, 1]] - \sum_{k=1}^{n-1} (\text{sol}[[k+1, 1]] x^k)$ , {x, -1, 1}]
Print[sol[[1, 1]] +  $\sum_{k=1}^{n-1} (\text{sol}[[k+1, 1]] x^k)$ ]
{2100.66, {x → -1.}}
{2100.66, -1.}
0.0279517
0.0280597
-0.0303653
-0.0303933
-0.0304138
-0.0304766
-0.0306248$ 
```

-0.030831  
-0.0310583  
-0.0312045  
-0.0313969  
-0.0322491  
-0.03226  
-0.0322839  
-0.0323214  
-0.0323722  
-0.0324335  
-0.0325179  
-0.0326586  
-0.0332485  
-0.0350557  
-0.0350846  
0.0362425  
0.0362468  
0.0362535  
0.0362651  
0.0362825  
0.0363067  
0.0363396  
0.036383  
0.0364514  
0.0365798  
0.036794  
0.0378726  
0.0499207  
0.0697973  
0.0748881  
0.0787707  
0.081174  
0.0829542  
0.0842572  
0.0844658  
0.0853194  
0.0856813  
0.0859688  
0.0860777

[illegible]



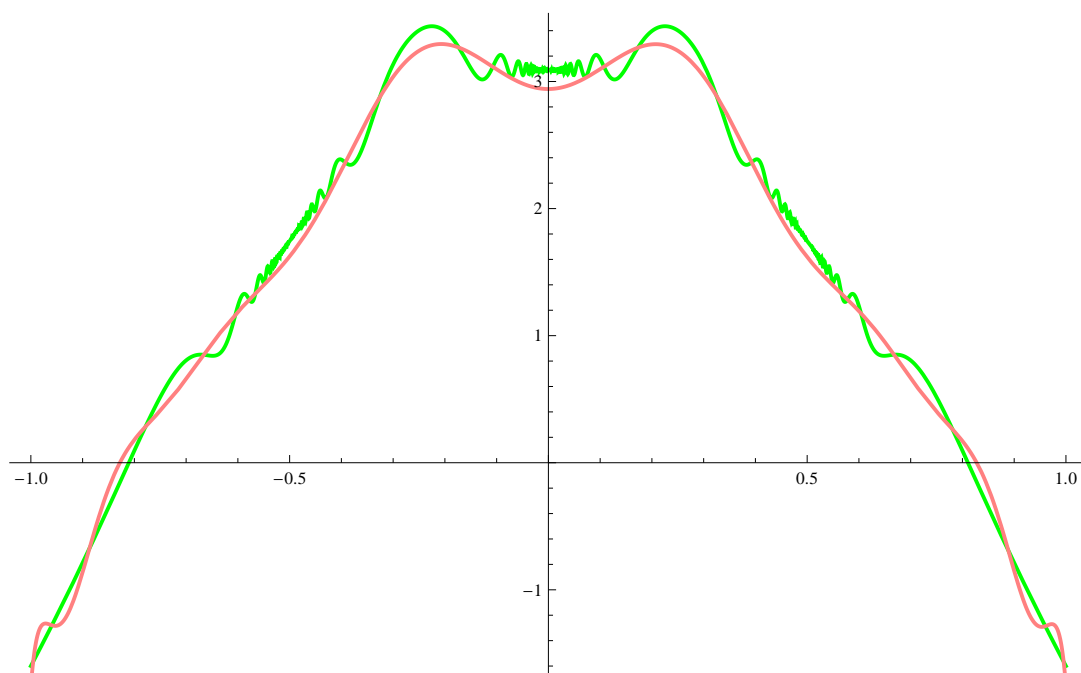
$$2.94096 + 0.0168035 x + 18.2032 x^2 - 1.35426 x^3 - 273.564 x^4 + 31.3196 x^5 + 965.774 x^6 - 299.529 x^7 + 283.195 x^8 + 1464.47 x^9 - 10360. x^{10} - 4052.96 x^{11} + 29099.3 x^{12} + 6612.33 x^{13} - 37683.8 x^{14} - 6300.41 x^{15} + 24052. x^{16} + 3242.17 x^{17} - 6105.86 x^{18} - 696.053 x^{19}$$

$$\text{sol}[[1, 1]] + \sum_{k=1}^{n-1} (\text{sol}[[k+1, 1]] x^k)$$

$$2.94096 + 0.0168035 x + 18.2032 x^2 - 1.35426 x^3 - 273.564 x^4 + 31.3196 x^5 + 965.774 x^6 - 299.529 x^7 + 283.195 x^8 + 1464.47 x^9 - 10360. x^{10} - 4052.96 x^{11} + 29099.3 x^{12} + 6612.33 x^{13} - 37683.8 x^{14} - 6300.41 x^{15} + 24052. x^{16} + 3242.17 x^{17} - 6105.86 x^{18} - 696.053 x^{19}$$

Show[Plot[f[x], {x, -1, 1}, PlotStyle -> {Thick, Green}],

Plot[sol[[1, 1]] +  $\sum_{k=1}^{n-1} (\text{sol}[[k+1, 1]] x^k)$ , {x, -1, 1}, PlotStyle -> {Thick, Pink}]]



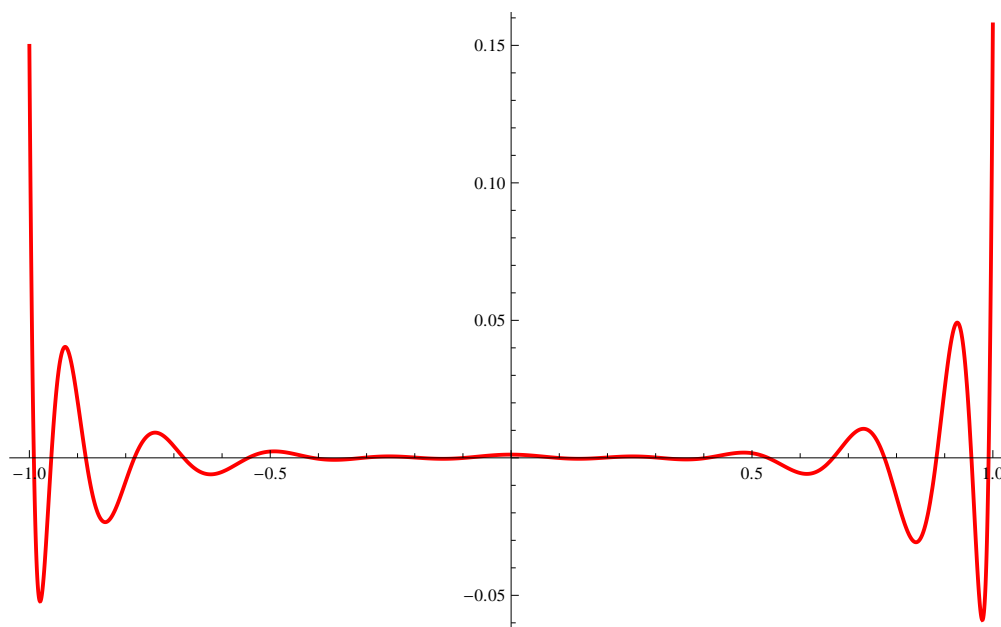
[illegible]

$$S[x_] := \prod_{k=1}^n (\alpha[k] - x)$$

```
NMaximize[Abs[4000 S[x]], -1 ≤ x ≤ 1, x]
```

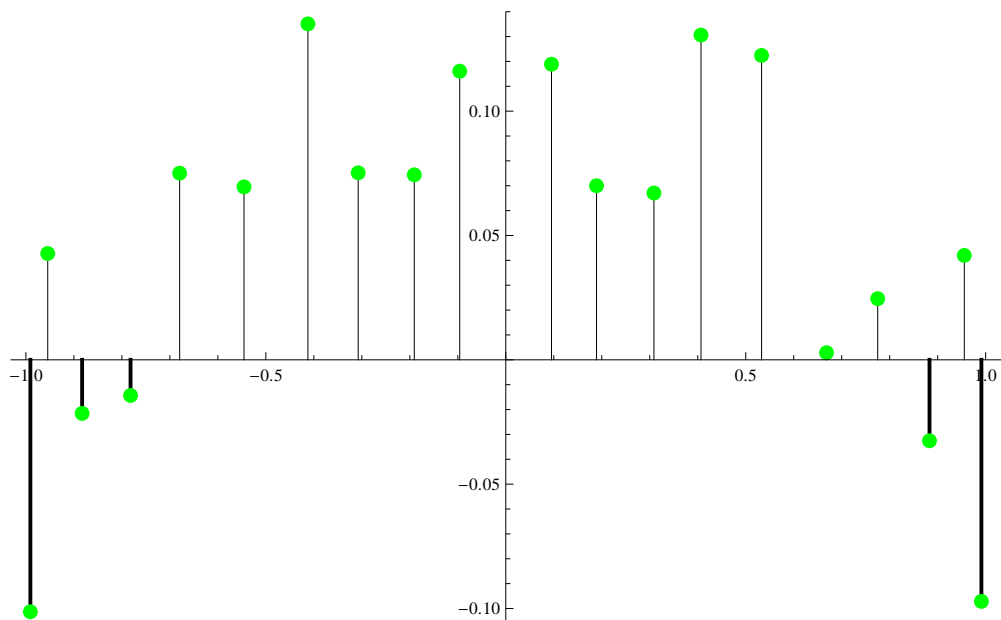
```
{0.157618, {x → 1.}}
```

```
g2 = Plot[4000 S[x], {x, -1, 1}, PlotRange → Full, PlotStyle → {Red, Thick}]
```



```
g3 =
```

```
ListPlot[Table[{α[k], f[α[k]] - sol[[1, 1]] - ∑_{j=1}^{n-1} (sol[[j+1, 1]] α[k]^j)}, {k, 1, n}],  
PlotStyle → {PointSize[0.015], Green}, Filling → Axis, FillingStyle → {Thick, Black}]
```

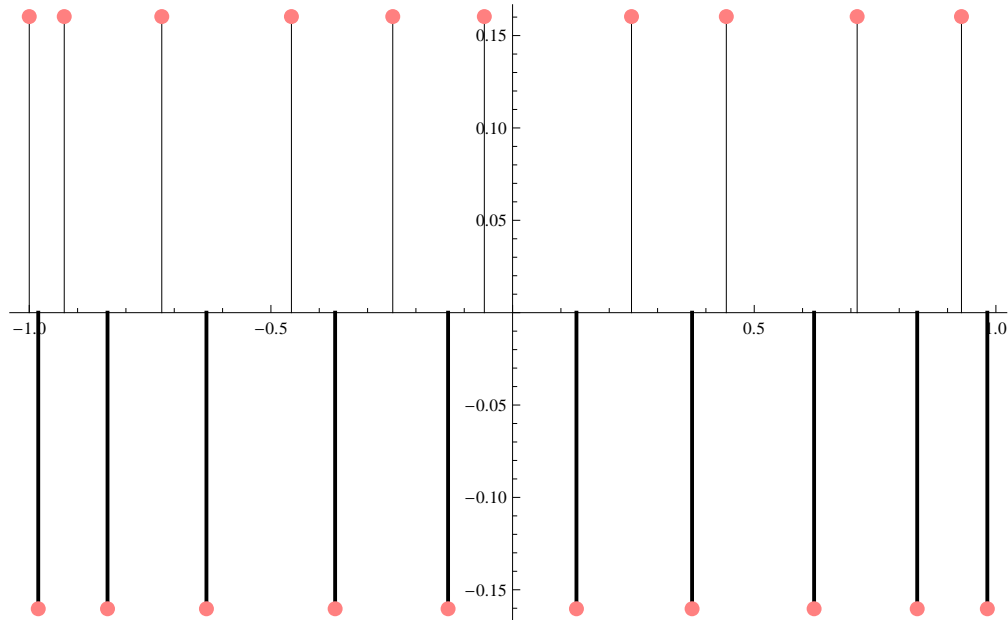




```

g4 = ListPlot[Table[
  {point[[k]], f[point[[k]]] - sol[[1, 1]] -  $\sum_{j=1}^{n-1} (\text{sol}[[j+1, 1]] \text{point}[[k]]^j)$ }, {k, 1, n}],
  PlotStyle -> {PointSize[0.015], Pink}, Filling -> Axis, FillingStyle -> {Thick, Black}]

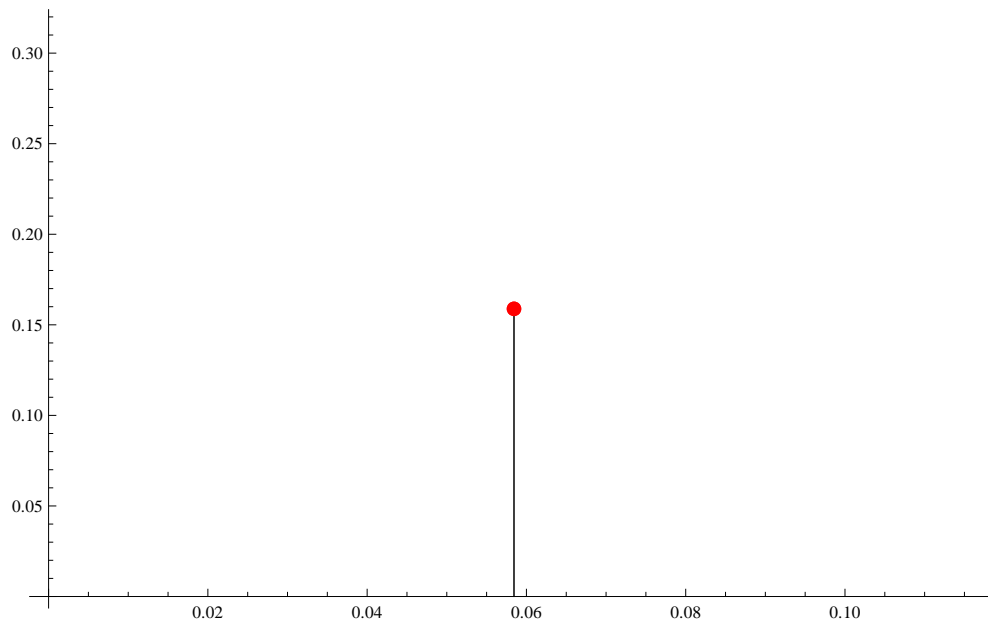
```



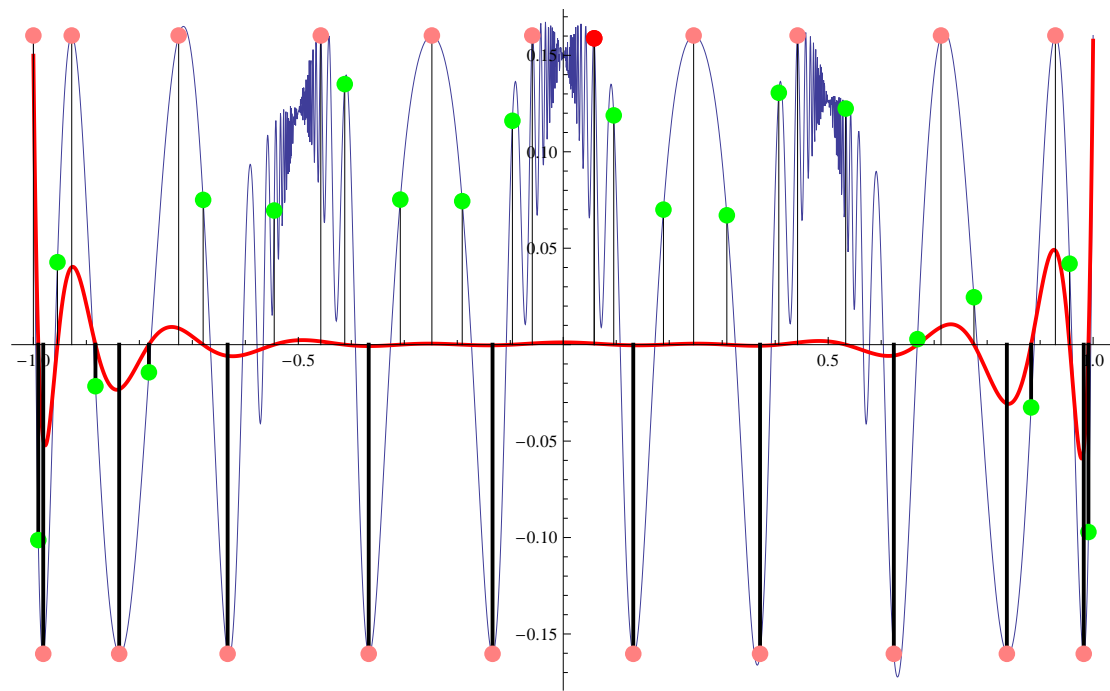
```

g5 = ListPlot[
  Table[{pnt[[k]], f[pnt[[k]]] - sol[[1, 1]] -  $\sum_{j=1}^{n-1} (\text{sol}[[j+1, 1]] \text{pnt}[[k]]^j)$ }, {k, 12, 12}],
  PlotStyle -> {PointSize[0.015], Red}, Filling -> Axis, FillingStyle -> {Thick, Black}]

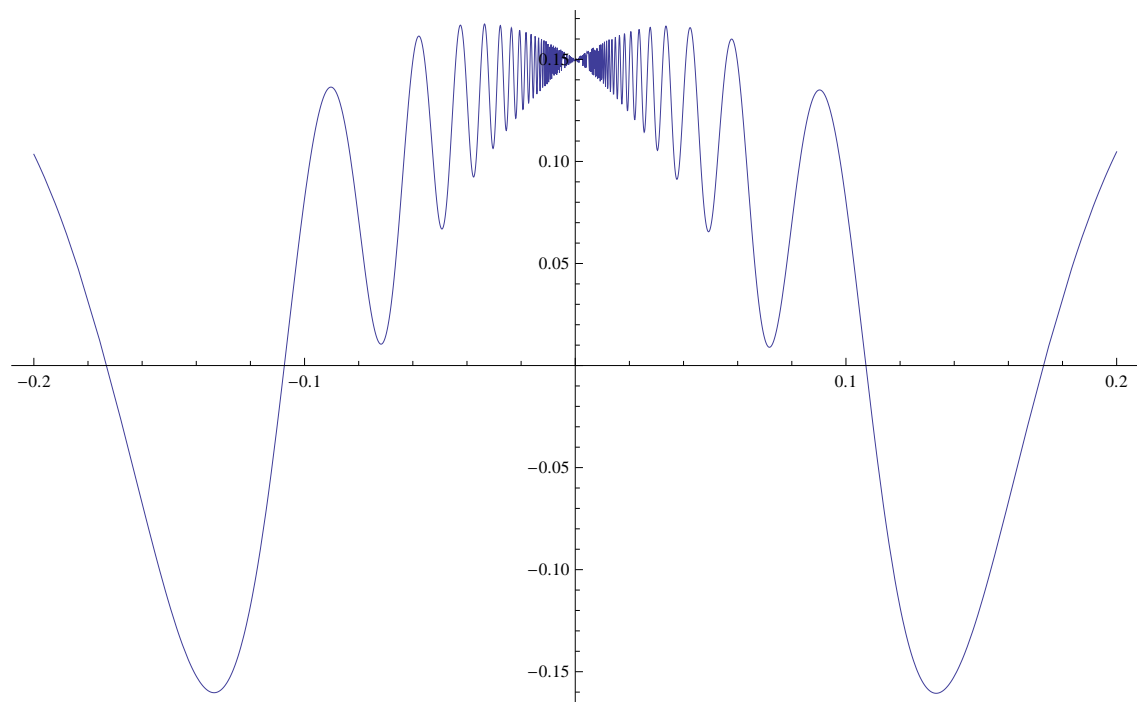
```



Show[g1, g2, g3, g4, g5]



$$g11 = \text{Plot}\left[f[x] - \text{sol}[[1, 1]] - \sum_{k=1}^{n-1} (\text{sol}[[k+1, 1]] x^k), \{x, -0.2, 0.2\}\right]$$



```
Show[g11, g2, g3, g4, g5]
```

