对于每个固定的 x，对 y 求导知 $f(x, y)$ 是增加的，于是 $y = c - x$ 时，$f(x, y)$ 达到最大$f(x, c-x)$

$$f(x, c-x) = \frac{1}{3}(1 + b(c-x))\alpha + \frac{2}{3}(1 + a x)\alpha + \frac{(1 + b(c-x))\beta}{2(1 + a x)} =$$

$$\frac{\alpha}{3} + \frac{b\alpha}{3a} + \frac{bc\alpha}{3} + (a x + 1)\left(\frac{2\alpha}{3} - \frac{b\alpha}{3a}\right) - \frac{b\beta}{2a} + \frac{\frac{\beta}{2} + \frac{b\beta}{2a} + \frac{bc\beta}{2}}{a x + 1}$$

$x \in (0, c)$，不论 $\left(\frac{2\alpha}{3} - \frac{b\alpha}{3a}\right)$ 的符号如何，$f(x, c-x)$ 都在端点取得最大值，

$$f(0, c) = \frac{1}{3}\alpha(1 + bc) + \frac{\beta(1 + bc)}{2}$$

$$f(c, 0) = \frac{2}{3}\alpha(1 + ac) + \frac{\beta}{2(1 + ac)}$$

$\max f(x, y) = \max\{f(0, c), f(c, 0)\}$

$\alpha = 0, b = 6, a = 9, c = 3 \Longrightarrow f(0, c) > f(c, 0)$

$\alpha = 1, b = 6, a = 9, c = 3 \Longrightarrow f(0, c) < f(c, 0)$