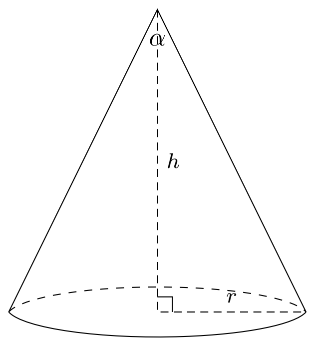

The vertex angle for a cone of radius r and height h is:

$$\alpha = 2 \arctan\left(\frac{r}{h}\right) \tag{1}$$

If the lateral height is constrained then the following must hold for r and h:

$$r^2 + h^2 = C \tag{2}$$

The expression for the volume is:

$$V = \frac{1}{3}\pi r^2 h$$

Using (2), this can be expressed solely in terms of h:

$$V = \frac{1}{3}\pi\left(C - h^2\right) h$$

Taking the first derivative with respect to h gives us:

$$\frac{\mathrm{d}}{\mathrm{d}h} V = \frac{1}{3}\pi\left(C - 3h^2\right)$$

So the critical point must occur when $C = 3h^2$. Returning to (2), this occurs when $r^2 = 2h^2$, or in other words when $\frac{r}{h} = \sqrt{2}$. From (1), we conclude that the vertex angle must be $2\arctan\left(\sqrt{2}\right)$ $\square$