ME 132 Dynamic Systems and Feedback
Summer 2025 George Anwar

Extra Credit

Due: Monday, August 11 at 11:59 pm

- This homework is extra credit! It will cover things not in lecture but can be found online and in the Nise textbook AND Oppenheim Signals and Systems!
- If you complete this homework with a score of 90% **or better**, we will **replace your lowest homework score!** This homework will explore the Z-Transforms as well as go into more theory of the Laplace Transform.
- Please write neatly and legibly, because if *we can't read it, we can't evaluate it.* **Box** your final answer.
- In all of the questions, **show your work**, not just the final answer. Unless we explicitly state otherwise, you may expect full credit only if you explain your work succinctly, but clearly and convincingly.
- If you are asked to provide a "sketch," it refers to a *hand-drawn* sketch, well-labeled to indicate all the salient features—not a plot generated by a computing device.
- If you have a confirmed disability that precludes you from complying fully with these instructions or with any other parameter associated with this problem set, please alert us immediately about reasonable accommodations afforded to you by the DSP Office on campus.

Deliverables Submit a PDF of your homework to the Gradescope assignment entitled "{Your Name} HW1". You may typeset your homework in LaTeX or any word-processing application (submit PDF format, not .doc/.docx format) or submit neatly handwritten and scanned solutions.

1 Honor Code

I will adhere to the Berkeley Honor Code: specifically, as a member of the UC Berkeley community, I act with honesty, integrity, and respect for others. Failure to comply with these guidelines can be considered an academic integrity violation. Please email Professor Anwar ganwar@berkeley.edu or post on Ed if you have any questions!

- **List all collaborators. If you worked alone, then you must explicitly state so.**
- **Declare and sign the following statement**:

  "I certify that all solutions in this document are entirely my own and that I have not looked at anyone else's solution. I have given credit to all external sources I consulted."

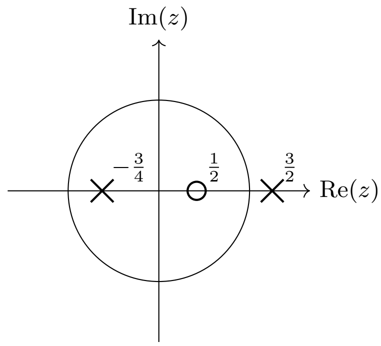
  Signature : ____________________ *Date* : ____________________

  While discussions are encouraged, *everything* in your solution must be your (and only your) creation. Furthermore, all external material (i.e., *anything* outside lectures and assigned readings, including figures and pictures) should be cited properly. We wish to remind you that consequences of academic misconduct are *particularly severe*!
- Violation of the Code of Conduct will result in a **zero** on this assignment and may also result in disciplinary action.

2 Z-Transform Practice

The following pole-zero diagram belongs to a BIBO stable system **H** whose transfer function $\hat{H}$ is rational in z, and whose impulse response h satisfies

$$\sum_{n=-\infty}^{\infty} h[n] = 1$$



(a) Determine $h(n)$, $\forall \in \mathbb{Z}$

(b) Determine whether there exists a stable, causal system whose impulse response h_1 satisfies $(h * h_1)[n] = \delta[n]$. If so, specify the pole-zero diagram and the RoC (Region of Convergence) for $\hat{H}_1$, the Z-transform of h_1. If not, explain briefly why no such system exists. *Note*: The asterisk * denotes the discrete-time convolution of two signals. That is, $(h * h_1)[n] = \sum_{k=-\infty}^{\infty} h[k]h_1[n-k]$.

(c) Determine whether there exists a system whose impulse response h_2 is left-sided and satisfies $(h * h_2)[n] = \delta[n]$. If so, specify the pole-zero diagram and the RoC for $\hat{H}_2$, the Z-transform of h_2. If not, explain briefly why no such system exists.

(d) Consider a complex DT function g related to h according to the relationship

$$g[n] = z_0^n h[n], \forall n$$

where z_0 is a non-zero complex number. Hence, z_0 can be expressed in polar form as $z_0 = r_0 e^{i\omega_0}$, where r_0 and ω_0 are real, $r_0 > 0$, and $-\pi < \omega_0 < \pi$. If g is the impulse response of a discrete-time LTI system $\mathbf{G}$, specify all values of r_0 and ω_0 for which the system $\mathbf{G}$ would be stable.

3 Z-Transform Mystery

You are given the following pieces of information about a real, stable, discrete-time signal x and its Discrete time Fourier Transform (DTFT) X, which can be written in the form

$$X(\omega) = A(\omega)e^{i\theta_x(\omega)}$$

The function A is an even, real-valued *amplitude* function, related to the magnitude $|X|$ by $A(\omega) = |X(\omega)|$. The function θ_x represents the angle.

(a) x is a finite-length signal.

(b) The Z-transform $\hat{X}$ has exactly two poles at $z = 0$ (and no zeros at $z = 0$).

(c) $\theta_x(\omega) = \begin{cases} \omega/2 + \pi/2, & 0 < \omega < \pi, \\ \omega/2 - \pi/2, & -\pi < \omega < 0 \end{cases}$. *Hint: think about if this property tells you something about the evenness/oddness of the original signal about a non-zero location.*

(d) $X(\omega)|_{\omega=\pi} = 2$.

(e) $\int_{-\pi}^{\pi} e^{2i\omega} \left[\frac{dX(\omega)}{d\omega}\right] d\omega = 4\pi i$

(f) The sequence v whose DTFT is $V(\omega) = \text{Re}\{X(\omega)\}$ satisfies $v(2) = 3/2$.

By interpreting the pieces of information and putting them together, determine and sketch x. You may continue to use the space on the following page to show your work. For those of you who are unfamiliar with Fourier Analysis, the Discrete Time Fourier Transform (DTFT) comes in a pair of equations, the Fourier Transform (Analysis Equation) and the Inverse Fourier Transform (Synthesis), much like the Laplace Transform! The formulas are given as follows

$$\text{Analysis (FT):} \quad X(\omega) = \sum_{n=-\infty}^{\infty} x[n]e^{i\omega n}$$

$$\text{Synthesis (IFT):} \quad x[n] = \frac{1}{2\pi}\int_{\langle 2\pi \rangle} X(\omega)e^{i\omega n} d\omega$$

More working space here for the above question!

4 Impulse Response from Z-Transforms

Consider a discrete-time LTI system with system function that has the following algebraic form (for all z in an appropriately defined RoC):

$$\hat{H}(z) = \frac{1}{\left(1 - \frac{1}{2}z^{-1}\right)\left(1 + \frac{1}{3}z^{-1}\right)\left(1 - 2z^{-1}\right)}$$

(a) Draw the pole-zero diagram for $\hat{H}$.

(b) Suppose we are told that the system is stable. Could it also be causal? Briefly justify your answer. *Hint: A system is causal if the RoC includes* ∞

(c) Suppose instead that we are given the following candidate forms for the impulse response of the system. The exact values of the non-zero constants $A, B, \ldots$ are not important for this problem, and need not be determined.For each candidate impulse response, determine if it could or could not be the impulse response of the system, and if it could, determine the RoC for $\hat{H}$ which is consistent with that particular form for the impulse response. If the candidate response could not be the impulse response, briefly explain why.

$$h_1(n) = A\left(\tfrac{1}{2}\right)^n u(n) + B\left(\tfrac{1}{4}\right)^n u(n) + C(2)^n u(n)$$
$$h_2(n) = D\left(\tfrac{1}{2}\right)^n u(n) + E\left(-\tfrac{1}{3}\right)^n u(n) + F(2)^n u(n)$$
$$h_3(n) = G\left(\tfrac{1}{2}\right)^n u(-n-1) + H\left(-\tfrac{1}{3}\right)^n u(n) + I(2)^n u(n-1)$$
$$h_4(n) = J\left(\tfrac{1}{2}\right)^n u(n) + K\left(-\tfrac{1}{3}\right)^n u(n-1) + L(2)^n u(n-1)$$

(d) What is the constant-coefficient difference equation governing the discrete time LTI system whose transfer function is characterized by $\hat{H}(z)$?

5 Pole-Zero Properties of Finite-Length Discrete-Time Signals

In this problem, we examine the locations of the poles and zeroes of prototypical finite-length DT signals. A signal x is *finite-length* if $x[n] = 0$ outside a finite set of samples n.

For each of the following real-valued, finite-length discrete-time signals r, v, and w—partially characterized by the constant parameters α, β, and γ—determine a reasonably simple expression for the corresponding Z-transform, and determine all the pole locations. Furthermore, explain why finite-length signals—such as r, v, and w—are somewhat justifiably referred to as *all-zero* signals. $\hat{R}(z), \hat{V}(z)$, and $\hat{W}(z)$.

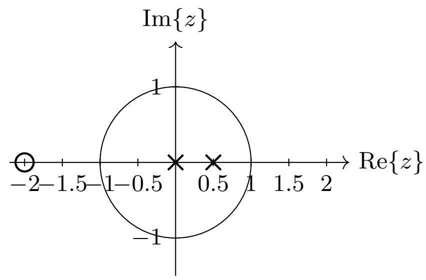
(a) $r[n] = \alpha\delta[n] + \beta\delta[n-1] + \gamma\delta[n-2]$

(b) $v[n] = \alpha\delta[n+1] + \beta\delta[n] + \gamma\delta[n-1]$

(c) $w[n] = \alpha\delta[n+2] + \beta\delta[n+1] + \gamma\delta[n]$

6 The South Pole

The following pole-zero diagram belongs to a BIBO stable DT-LTI system **H** whose transfer function $\hat{H}$ is rational in z. Additionally, when an input of $x[n] = \delta[n+1]$ is fed into the system, the output $y[n]$ has Z-transform $\hat{Y}(z)$ with $\hat{Y}(1) = 18$.



1. Determine an expression for $\hat{H}(z)$.

2. Consider a different BIBO stable DT-LTI system **F**, whose transfer function is

$$\hat{F}(z) = \frac{z - \frac{3}{2}}{z(z + \frac{1}{4})}$$

Determine an expression for the impulse response $f[n]$. Justify your answer.

3. Determine whether there exists a BIBO stable DT-LTI system $\mathbf{F}_1$ that inverts system $\mathbf{F}$ from part (b). In other words, the impulse response $f_1[n]$ of $\mathbf{F}_1$ satisfies $(f * f_1)[n] = \delta[n]$. If so, specify the pole-zero diagram and the RoC for $\hat{F}_1$, the Z-transform of $f_1[n]$. If not, briefly explain why no such system exists.

