ME 132 — Dynamic Systems and Feedback
Summer 2025 — George Anwar

HW2

Due: Monday, June 30 at 11:59 pm

- Homework 2 is a written assignment with 1 coding question; **Please read Nise Chp. 2 and 3**.
- For coding questions, attach a screenshot of the script and output.
- Please write neatly and legibly, because if *we can't read it, we can't evaluate it.* **Box** your final answer.
- In all of the questions, **show your work**, not just the final answer. Unless we explicitly state otherwise, you may expect full credit only if you explain your work succinctly, but clearly and convincingly.
- If you are asked to provide a "sketch," it refers to a *hand-drawn* sketch, well-labeled to indicate all the salient features—not a plot generated by a computing device.
- If you have a confirmed disability that precludes you from complying fully with these instructions or with any other parameter associated with this problem set, please alert us immediately about reasonable accommodations afforded to you by the DSP Office on campus.
- **Start early. Some of the material is prerequisite material not covered in lecture; you are responsible for finding resources to understand it.**

Deliverables Submit a PDF of your homework to the Gradescope assignment entitled "{Your Name} HW1". You may typeset your homework in LaTeX or any word-processing application (submit PDF format, not .doc/.docx format) or submit neatly handwritten and scanned solutions.

1 Honor Code

I will adhere to the Berkeley Honor Code: specifically, as a member of the UC Berkeley community, I act with honesty, integrity, and respect for others. Failure to comply with these guidelines can be considered an academic integrity violation. Please email Professor Anwar ganwar@berkeley.edu or post on Ed if you have any questions!

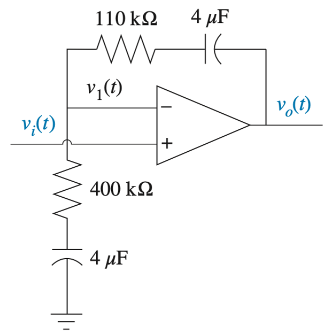
- **List all collaborators. If you worked alone, then you must explicitly state so.**
- **Declare and sign the following statement**:
  "I certify that all solutions in this document are entirely my own and that I have not looked at anyone else's solution. I have given credit to all external sources I consulted."

  Signature : ______________________ *Date* : ______________________

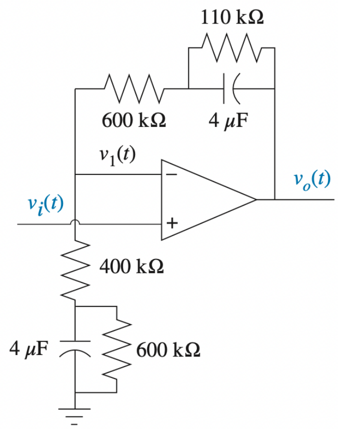
  While discussions are encouraged, *everything* in your solution must be your (and only your) creation. Furthermore, all external material (i.e., *anything* outside lectures and assigned readings, including figures and pictures) should be cited properly. We wish to remind you that consequences of academic misconduct are *particularly severe*!
- Violation of the Code of Conduct will result in a **zero** on this assignment and may also result in disciplinary action.

2 Questions

1. Find the transfer function, $G(s) = V_0(s)/V_i(s)$, for each operational amplifier circuit shown in Figure P2.8 (coninued on next page). [Sec: 2.4]



(a)



(*b*)

2. For the unexcited (no external force applied) system of Figure P2.16, do the following: For the unexcited (no external force applied) system of Figure P2.16, do the following:

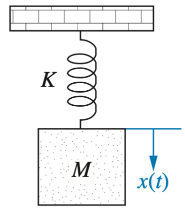


FIGURE P2.16

a. Write the differential equation that describes the system.

b. Assuming initial conditions $x(0) = x_0$ and $\dot{x}(0) = x_1$, write a Laplace transform expression for $X(s)$.

c. Find x(t) by obtaining the inverse Laplace transform from the result in Part c.

d. What will be the oscillation frequency in Hz for this system?

3. For each of the rotational mechanical systems shown in Figure P2.17, write, but do not solve, the equations of motion. [Sec: 2.6]

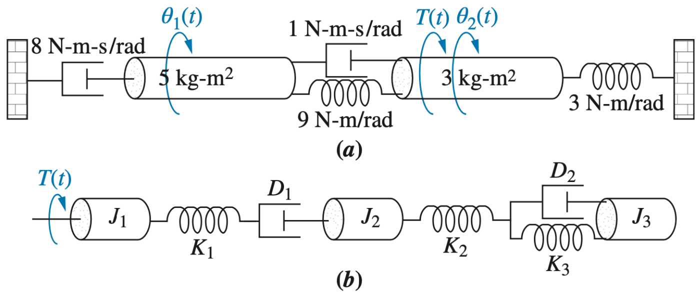


FIGURE P2.17

4. In the system shown in Figure P2.27, the inertia, J, of radius, r, is constrained to move only about the stationary axis A. A viscous damping force of translational value f_v exists between the bodies J and M. If an external force, $f(t)$, is applied to the mass, find the transfer function $G(s) = \theta(s)/F(s)$. [Sec: 2.5; 2.6]

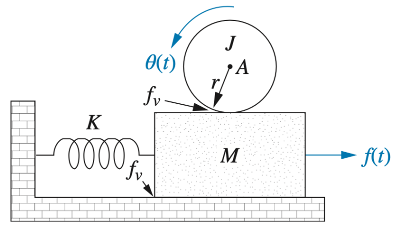


FIGURE P2.27

5. Figure P2.36 shows a crane hoisting a load. Although the actual system's model is highly nonlinear, if the rope is considered to be stiff with a fixed length L, the system can be modelled using the following equations:

$$m_L \ddot{x_{La}} = m_L g\phi \tag{1}$$
$$m_T \ddot{x_T} = f_T - m_L g\phi \tag{2}$$
$$x_{La} = x_T - x_L \tag{3}$$
$$x_L = L\phi \tag{4}$$

where m_L is the mass of the load, m_T is the mass of the cart, x_T and x_L are displacements as defined in the figure, ϕ is the rope angle with respect to the vertical, and f_T is the force applied to the cart $(Marttinen, 1990)$

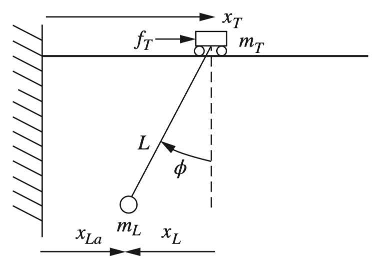


FIGURE P2.36[16]

a. Obtain the transfer function from cart velocity to rope angle $\frac{\Phi(s)}{V_T(s)}$

b. Assume that the cart is driven at a constant velocity V_0 and obtain an expression for the resulting $\phi(t)$. Show that under this condition, the load will sway with a frequency $\omega_0 = \sqrt{\frac{g}{L}}$.

c. Find the transfer function from the applied force to the cart's position, $\frac{X_T(s)}{F_T(s)}$

d. Show that if a constant force is applied to the cart, its velocity will increase without bound as $t \to \infty$

6. Find the state-space representation of the network shown in Figure P3.3 if the output is $\nu_0(t)$. [Sec: 3.4]

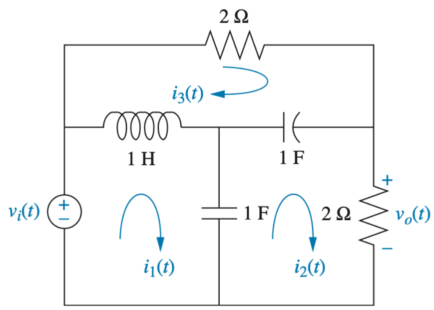


FIGURE P3.3

7. Represent the translational mechanical system shown in Figure P3.5 in state space, where $x_1(t)$ is the output. [Sec: 3.4]

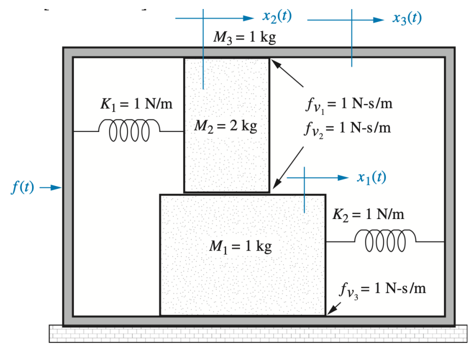


FIGURE P3.5

8. Given the DC servomotor and load shown in Figure P3.11, represent the system in state space, where the state variables are the armature current, i_a, load displacement, θ_L, and load angular velocity, ω_L. Assume that the output is the angular displacement of the armature. Do not neglect armature inductance. [Sec: 3.4]

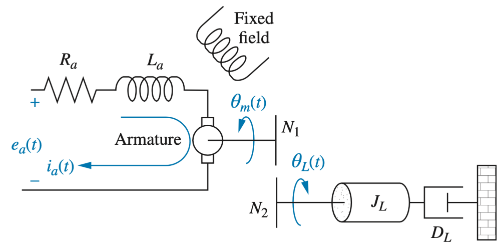


FIGURE P3.11 Motor and load

9. Experiments to identify precision grip dynamics between the index finger and thumb have been performed using a ball-drop experiment. A subject holds a device with a small receptacle into which an object is dropped, and the response is measured (*Fagergren, 2000*). Assuming a step input, it has been found that the response of the motor subsystem together with the sensory system is of the form

$$G(s) = \frac{Y(s)}{R(s)} = \frac{s+c}{(s^2+as+b)(s+d)} \tag{5}$$

Convert this transfer function to a state-space representation.

10. Figure P3.17 shows a free-body diagram of an inverted pendulum, mounted on a cart with a mass, M. The pendulum has a point mass, m, concentrated at the upper end of a rod with zero mass, a length, l, and a frictionless hinge. A motor drives the cart, applying a horizontal force, $u(t)$. A gravity force, mg, acts on m at all times. The pendulum angle relative to the y-axis, θ, its angular speed, $\dot{\theta}'$, the horizontal position of the cart, x, and its speed, x', were selected to be the state variables. The state space equations derived were heavily nonlinear. They were then linearized around the stationary point, $\mathbf{x_0} = \mathbf{0}$ and $u_0 = 0$, and manipulated to yield the following open-loop model written in perturbation form:

$$\frac{d}{dt}\delta\mathbf{x} = \mathbf{A}\delta\mathbf{x} + \mathbf{B}\delta u \tag{6}$$

However, since $\mathbf{x_0} = \mathbf{0}$ and $u_0 = 0$, then let: $\mathbf{x} = \mathbf{x_0} + \delta\mathbf{x} = \delta\mathbf{x}$ and $u = u_0 + \delta u$. Thus the state equation may be rewritten as (*Prasad, 2012*):

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}u \tag{7}$$

$$\text{where } \mathbf{A} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ \frac{(M+m)g}{Ml} & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ -\frac{mg}{M} & 0 & 0 & 0 \end{bmatrix} \text{ and } \mathbf{B} = \begin{bmatrix} 0 \\ \frac{-1}{Ml} \\ 0 \\ \frac{1}{M} \end{bmatrix}$$

Assuming the output to be the horizontal position of $m = x_m = x + l\sin\theta = x + l\theta$ for a small angle, θ, the output equation becomes:

$$y = l\theta + x = \mathbf{C}\mathbf{x} = \begin{bmatrix} l & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \theta \\ \dot{\theta} \\ x \\ \dot{x} \end{bmatrix} \tag{8}$$

Given that: $M = 2.4$ kg, $m = 0.23$ kg, $l = 0.36$ m, $g = 9.81$ m/s^2, use MATLAB to find the transfer function, $G(s) = Y(s)/U(s) = X_m(s)/U(s)$.

