

### Instructions

- The exam is open book, open notes, and open web. However, you may not consult or communicate with other people (besides course staff).
- When you start, **the first thing you should do is check that you have all 20 pages and all 6 questions** + the extra credit question.
- You have **26.5** hours. (If you are in the Disabled Students' Program and have an allowance of 150% or 200% time, that comes to **39.75 hours** or **53 hours**, respectively.)
- Mark your answers on the exam itself in the space provided. You **may** use the extra sheets at the end of the exam if you run out of space for an answer, write a note that your answer is continued on the bottom of page X. Please **do not attach extra pages** to make grading smoother.
- The total number of points is  $150 + 15(\text{EC})$ . There are 6 multiple choice questions and 3 true or false questions worth 4 points each; 2 short answers worth a total of 38 points; 2 long answers worth a total of 75 points; and the extra credit problem worth 15 additional points.
- For multiple answer questions, fill in the bubbles for **ALL correct choices**: there may be more than one correct choice, but there is always at least one correct choice. **NO Partial credit** will be given on multiple answer questions: the set of all correct answers must be checked.
- For true and false questions, fill in the bubbles for true or false and then provide justification.
- There is an extra credit question at the end. Please do not attempt it until you have finished the rest of the midterm!
- Please write neatly and legibly, because if *we can't read it, we can't evaluate it*. **Box** your final answer.
- In all of the questions, **show your work**, not just the final answer. Unless we explicitly state otherwise, you may expect full credit only if you explain your work succinctly, but clearly and convincingly.
- If you are asked to provide a “sketch,” you are allowed a computer plot or *hand-drawn* sketch. Ensure it is well-labeled to indicate all the salient features.

### Deliverables

Submit a PDF of your exam to the Gradescope assignment entitled “{Your Name} Midterm Exam”. You must neatly handwrite your solutions (submit PDF format, not .doc/.docx format). **Do not** use a text editing software.

- On the next page, please the following statement and sign your signature next to it for **1 point** (Mac Preview, PDF Expert, and FoxIt PDF Reader, among others, have tools to let you sign a PDF file.) We want to make it *extra* clear so that no one inadvertently cheats.
- You've got this! Good Luck! We will award EC points for funny jokes!

First Name	
Last Name	
Student ID	

# 1 Code of Conduct [1 pt]

## Statement of Academic Integrity

- I affirm that this document—which I have produced for the evaluation of my performance—reflects my original, bona fide work, and I have neither provided to, nor received from, anyone **any** assistance that produces unfair advantage for me or for any of my peers.
- I will adhere to the Berkeley Honor Code: specifically, as a member of the UC Berkeley community, I have acted with honesty, integrity, and respect for others, and professional responsibility—and in a manner consistent with the letter and intent of the campus Code of Student Conduct.
- I will complete this assignment entirely on my own, and will not discuss its contents or any related concepts with anyone other than to ask clarification questions directly to the course staff. I will not work on this exam in the physical proximity of any student currently or previously enrolled in this course.
- I will not post any part of this assignment, or related questions, to external websites such as Chegg, CourseHero, StackOverflow, or similar platforms.
- Communication in any other form—oral, written, or electronic; public or private; direct or indirect—with any human being outside the scope permitted above is not allowed.
- I have not violated—nor aided or abetted anyone else to violate—the instructions for this exam given by the course staff, including, but not limited to, those on the cover page of this document.
- More generally, I have not committed any act that violates—nor aided or abetted anyone else to violate—UC Berkeley, state, or Federal regulations, during this exam.
- I hereby acknowledge that I have read and understood the above instructions. I agree that any failure on my part to adhere to these instructions, which may result in a penalty, is solely my responsibility.

Failure to adhere to these guidelines will be considered an academic integrity violation. Please email Professor Anwar [ganwar@berkeley.edu](mailto:ganwar@berkeley.edu) or post on Ed privately if you have any questions!

- **Declare and sign the following statement:**

In the space provided below, **hand-write** the following sentence, verbatim. Then sign and date before uploading your work to Gradescope.

*"I have read the Code of Conduct and certify that all solutions in this document are entirely my own and that I have not looked at anyone else's solution. I have given credit to all external sources I consulted."*

*Signature :* \_\_\_\_\_

*Date :* \_\_\_\_\_

*Everything* in your solution must be your (and only your) creation. Furthermore, all external material (i.e., *anything* outside lectures and assigned readings, including figures and pictures) should be cited properly. We wish to remind you that consequences of academic misconduct are *particularly severe*!

- **Violation of the Code of Conduct will result in a **zero** on this exam and disciplinary referral. However, we reserve the right to give an instant **F** for even one violation.**

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## 2 Multiple Answer [24 pts]

For multiple answer questions, fill in the bubbles for **ALL correct choices**: there may be more than one correct choice, but there is always at least one correct choice. **NO Partial credit** on multiple answer questions is given: the set of all correct answers must be checked

No negative points will be given for incorrect answers. You do not need to justify your answers unless asked; your calculations will not be considered or graded.

- (a) [4 pts] Larry fell asleep in lecture and can't answer the following question. You, a controls expert, want to help him decide which of the following statements pertaining to **feedback** control systems are true.

A: They use the output of the system to **tune** the input.

C: They compare the actual output with the desired output and try to **minimize** the error.

B: They help a system **follow** a desired reference signal.

D: They are **everywhere!** They can be found in everyday systems.

- (b) [4 pts] You just finished your long homework for ME 132. Depressed, you decide to take a shower but you still think about control systems. There is a knob on the wall that can be rotated and pushed/pulled to set a desired temperature  $T$  and water pressure  $p$ . Assuming that **you** are both the **controller** and **sensor** of the system, what are the input(s)  $u$  and output(s)  $y$  of the system?

A: Inputs: Perceived temperature and water pressure; Outputs: Heat and pressure setting of the knob.

C: Inputs: Perceived temperature and heat setting on knob; Outputs: Perceived water pressure and pressure setting on knob.

B: Inputs: Heat and pressure setting of the knob; Outputs: perceived temperature and pressure.

D: Inputs: Perceived temperature and water pressure; Outputs: Rotating, pulling, or pushing of the knob.

- (c) [4 pts] Professor Anwar is tired of driving manually so he wants to design a speed controller for his car. Since you are an expert in controls, he tells you that he knows the current speed  $v$  of the car and is able to accurately input the position of the gas pedal  $p_{pedal}$ . Which type of controller solution would **work** in this scenario.

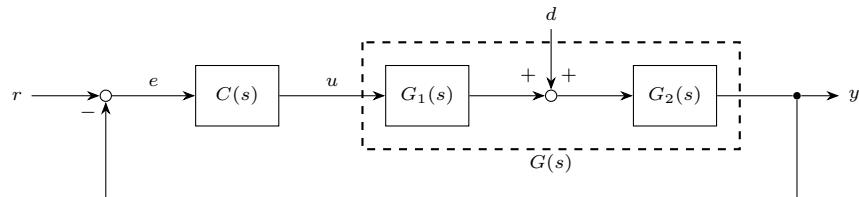
A: Feedforward controller

C: All mentioned options would work.

B: Feedforward and feedback controller.

D: Feedback controller

- (d) [4 pts] Larry and Athul are both unemployed as a MechEs and are working at McDonald's as line cooks. So they made use of their controls knowledge and created a generalized feedback control system to perfect their fries.  $G(s) = G_1(s) \cdot G_2(s)$  is given (fryer equipment),  $d$  as an unknown disturbance (frozen fries dropped in oil), and  $r$  as a reference signal (set temperature):



From the block diagram, what is the correct mapping of  $y$  as a function of  $r$  and  $d$ ? In other words, what is the transfer function(s) of the system?

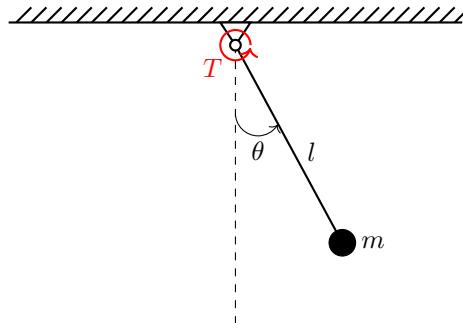
A:  $y(s) = \frac{C(s)G(s)}{1+C(s)G_1(s)}r(s) + \frac{G_1(s)}{1+C(s)G(s)}d(s).$

C:  $y(s) = \frac{C(s)G(s)}{1+C(s)G(s)}r(s) + \frac{1}{1+C(s)G(s)}d(s).$

B:  $y(s) = \frac{C(s)G(s)}{1+C(s)G(s)}r(s) + \frac{G_2(s)}{1+C(s)G(s)}d(s).$

D:  $y(s) = \frac{1}{1+C(s)G(s)}r(s) + \frac{G_2(s)}{1+C(s)G(s)}d(s).$

- (e) [4 pts] Matt, a civil engineer, inspired from his recent trip to Six Flags, wants to develop a swinging display feature for riders on a roller coaster. The mechanism consists of a rigid, massless rod of length  $l$ , with a mass  $m$  attached at the end. The rod itself is connected to a pivot on the roller coaster car, allowing it to swing freely in a vertical plane, similar to a pendulum as modeled below. To move the display and adjust its angle ( $\theta$ ) during the ride, a motor is installed at the pivot. The motor applies a torque  $T$  to the rod. When Matt did his analysis he assumed no friction, drag, or damping. Which of the following govern the motion of the system?



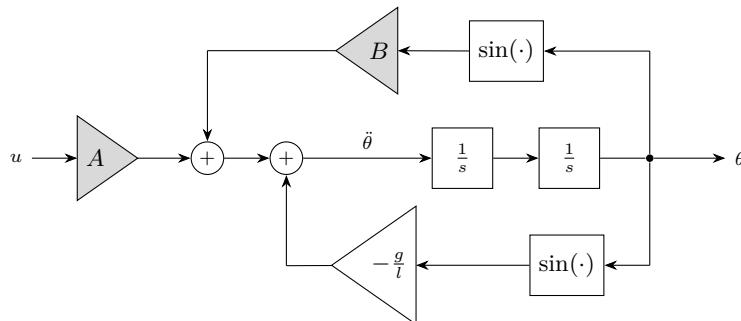
A:  $ml^2\ddot{\theta} = T - mgl \sin(\theta)$

C:  $ml^2\ddot{\theta} = T + mg \sin(\theta)$

B:  $ml\ddot{\theta} = T - mg \cos(\theta).$

D:  $m\ddot{\theta} = T - mgl \cos(\theta)$

- (f) [4 pts] From above, to make sure Matt's design does not kill anyone on the ride, he has hired you and a friend to deal with the non-linear nature of the system. Your friend designed a controller to control the system using the output feedback to linearize it, where  $u$  is the input to a new linearized system as shown below:



Unfortunately, he suddenly passed away on a drop tower that Matt designed. You need to now work backwards and figure out what values your friend set for  $A$  and  $B$  to be in order to linearize the system? Note: All triangles represent a gain block. A gain block multiplies the input into the block by the factor inside of it and outputs that scaled output. Hint: rearrange the EoM for  $\ddot{\theta}$ , then create an expression for  $T$  to remove the  $\sin(\theta)$  term.  $T$  should be of the form  $T = u + \dots$  where  $u$  is a new signal.

A: Block  $A$  is 1; Block  $B$  is  $mgl$ .

C: Block  $A$  is  $\frac{1}{ml^2}$ ; Block  $B$  is  $\frac{g}{l}$ .

B: Block  $A$  is  $\frac{1}{ml^2}$ ; Block  $B$  gain is  $-\frac{g}{l}$ .

D: Block  $A$  is  $\frac{1}{ml^2}$ ; Block  $B$  gain is  $-mgl$ .

### 3 True or False? [12 pts]

Determine whether each of the following statements is **True or False**. Provide a brief explanation or calculation for your answer. Each correct True/False answer will earn 1 point, while a correct explanation will earn 3 points. Please state all assumptions made.

**The correct selection of True/False must be selected in order to get the 3 points for explanation.**  
Partial credit will be given on the explanation.

1. [4 pts] The step-response of the transfer function  $G(s)$  approaches the value of  $\frac{1}{3}$  as time goes to infinity.

$$G(s) = \frac{s+4}{(s-6)(s-2)}$$

**True**      Explanation: \_\_\_\_\_

**False**      \_\_\_\_\_

2. [4 pts] If two systems have the same transfer function, they must have the same state-space realization.

**True**      Explanation: \_\_\_\_\_

**False**      \_\_\_\_\_

3. [4 pts] The input-output relationship is given by  $y(t) = x(t) \cdot (\text{sinc}(t^2 - 3t) + e^{-t})$  where  $\text{sinc}(t) = \frac{\sin(\pi t)}{\pi t}$  and  $\text{sinc}(0) = 1$ . The system is linear, time-variant, and SISO.

**True**      Explanation: \_\_\_\_\_

**False**      \_\_\_\_\_

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Extra space: if you need extra space for your answer to the above true and false, you may write here. **Be sure to write the corresponding question number under the unfinished answer!** (If needed, there is more extra space on page 18 to 20.)

## 4 Short Answers [38 pts]

### 1. ReLU? Is That You? (8 pts)

While writing the ME132 midterm, Larry decided to take a break but stumbles across Peter Griffin explaining deep learning—specifically, on training neural networks—on reels. He keeps saying something called “ReLU” like it is the best thing since open-book exams. Peter defined ReLU as a system **H** that zeros out all inputs if our input signal is negative, and only preserves the non-negative input values as the output as shown below:

$$x(t) \longrightarrow \boxed{\mathbf{H}} \longrightarrow y(t) = \max(x(t), 0) = \begin{cases} 0 & x(t) < 0 \\ x(t) & x(t) \geq 0 \end{cases}$$

However, as a loyal controls student, you aren’t convinced. Is ReLU even a system? Is it linear? Is it time-invariant? Would Professor Anwar approve of this? Now it’s your turn to help Larry out. Analyze this mysterious ”ReLU” system using your understanding of linear systems theory. Don’t let the AI/ML crowd fool you—show them what real systems analysis looks like!

- (i) [4 pts] Select the strongest assertion from the list below **and** provide a clear and detailed explanation.

- The system must be *linear*. (If you choose this, give a short proof.)
- This system could be *linear*, but does not have to be. (If you choose this, explain why and specify additional conditions needed to determine linearity.)
- The system cannot be *linear*. (If you choose this, give a counterexample.)

- (ii) [4 pts] Select the strongest true assertion from the list below **and** provide a clear and detailed explanation.

- The system must be *time invariant*. (If you choose this, give a short proof.)
- This system could be *time invariant*, but does not have to be. (If you choose this, explain why and specify additional conditions needed to determine time invariance.)
- The system cannot be *time invariant*. (If you choose this, give a counterexample.)

## 2. Block To The Future! (30 pts)

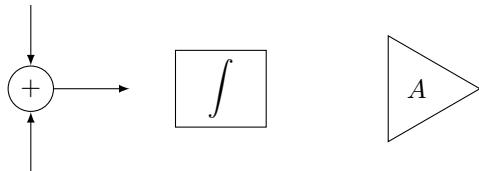
While reviewing an old set of engineering notes left behind in Kresge, Athul and Larry find a page labeled as "DeLorean Flux Stabilization System" with a handwritten equation—clearly the work of the eccentric physicist named Dr. E. Brown. In his engineering notes, Doc Brown states:

A continuous-time LTI system  $\mathbf{F}$  whose input and output signals  $x(t)$  and  $y(t)$ , respectively, are related by the following linear, constant coefficient differential equation (LCCDE):

$$\ddot{y}(t) - 3\dot{y}(t) + y(t) + 5 \int_{0-}^t y(\tau) d\tau = \ddot{x}(t) + 2\dot{x}(t) + 7x(t)$$

It is not immediately clear what the system does so Athul and Larry decide to analyze it. Your goal is to help them understand the system's internal structure by answering the following questions to determine how the DeLorean will behave.

- (i) [10 pts] Draw a block diagram using only integrators, summing junctions, and gain blocks to represent this system in the **time domain**. **You must have 4 integrators**. The symbols for the blocks, respectively, are as follows:



Clearly label all intermediate signals. You may assume initial conditions are zero.

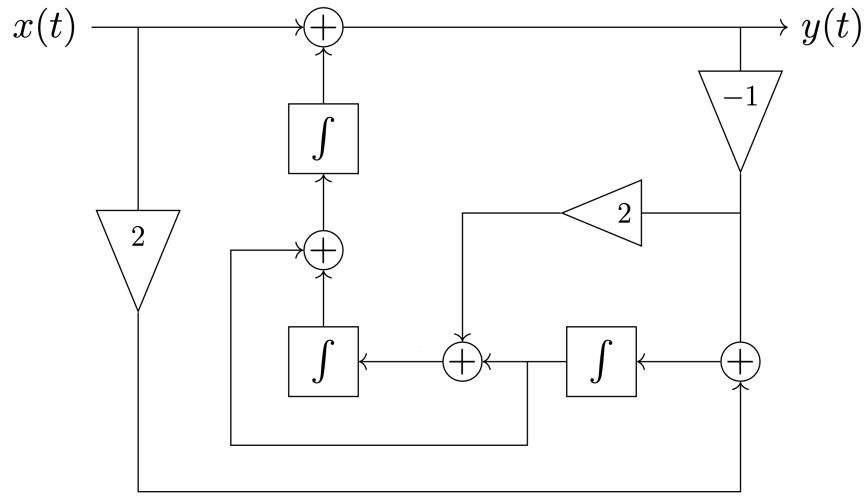
- (ii) [4 pts] Using the Laplace Transform (with zero initial conditions), convert the differential equation into the  $s$ -domain and solve for the transfer function  $Y(s)/X(s)$ .

- (iii) [10 pts] Find the state-space representation of this system where  $\mathbf{q}(t) = [q_1(t) \quad q_2(t) \quad q_3(t) \quad q_4(t)]^\top$  is the state vector. Explicitly write the entries for  $A$ ,  $B$ ,  $C$ , and  $D$ , where

$$\text{SEE : } \dot{\mathbf{q}}(t) = A\mathbf{q}(t) + Bx(t)$$

$$\text{OE : } y(t) = C\mathbf{q}(t) + Dx(t)$$

- (iv) [6 pts] Larry and Athul also stumble across the 1st edition of Doc Brown's “Original Delorean Flux Stabilization System” and this time, instead of an equation, there is a block diagram with a note saying: “consider this to be the block diagram of a continuous-time LTI system  $\mathbf{G}$ ”. Out of curiosity, they want to see if it matches with the original LCCDE they found. Find the LCCDE of the system  $\mathbf{G}$ . State if it is the same!

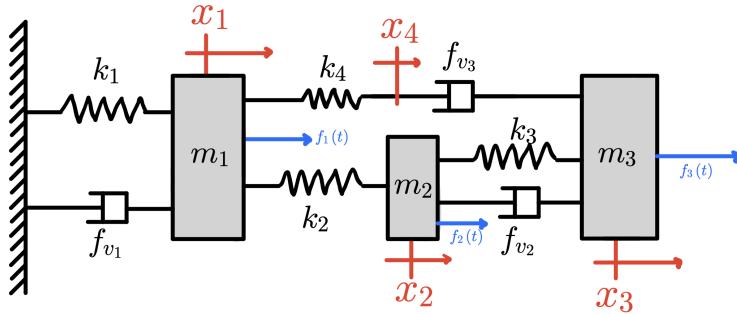


## 5 Long Answer [75 pts]

### 1. Hop to it! [32 pts]

Owen, after a rough spring semester, has turned to beer for support. He is a passionate beer lover and has always wanted to open up his own restaurant called *OP's Beer Garden*—a fast-paced IPA brewery and pub. He has created a system where bottles are transported along a conveyor and placed in sliding bottle cradles during the filling and capping process. Each cradle holds one bottle, is mounted on a precision linear guide rail and is mechanically coupled to adjacent cradles using spring-damper mechanisms to reduce the vibration ensuring consistent alignment under motion.

The cradles are modeled as masses  $m_1, m_2$ , and  $m_3$ . Springs  $k_1, k_2, k_3$ , and  $k_4$  represent the stiffness of the elastic linkages and supports. Dampers  $f_{v1}$  and  $f_{v2}$  account for the sliding friction and energy dissipation in the linkage and rail system. The system is shown below:



Owen's brewery wants to ensure that any external disturbances like nozzle impacts or sudden conveyor motions (represented by external forces  $f_1(t)$ ,  $f_2(t)$ , and  $f_3(t)$ ) do not lead to misalignment or spillage of that precious golden IPA.

- (i) [12 pts] Draw the Free Body Diagrams (FBDs) for each of the three bottle cradle masses. Label the directions of all spring and damping forces clearly, as well as the applied forces  $f_1(t)$ ,  $f_2(t)$ , and  $f_3(t)$ . Derive the equations of motion for the system, one for each mass. Express your answer in terms of  $x_1(t)$ ,  $x_2(t)$ ,  $x_3(t)$ ,  $x_4(t)$  and their derivatives. *Leave symbolic answers!*

- (ii) [4 pts] Write your equations from part ii) in matrix form. That is,

$$\mathbf{M}\ddot{\mathbf{x}} + \mathbf{C}\dot{\mathbf{x}} + \mathbf{K}\mathbf{x} = \mathbf{F}(t)$$

where  $\mathbf{M}, \mathbf{C}, \mathbf{K} \in \mathbb{R}^{n \times n}$  and  $\ddot{\mathbf{x}}, \dot{\mathbf{x}}, \mathbf{x} \in \mathbb{R}^n$ . Leave symbolic answers!

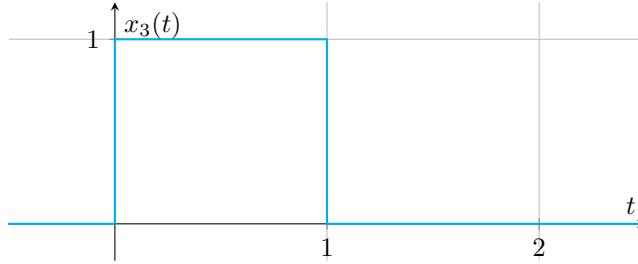
- (iii) [6 pts] Owen, in an attempt to save the beer, has somehow stabilized cradle 2 cradle 3, so now the only vibration present (external force) is  $f_1(t)$ . Assuming zero initial conditions, use Cramer's Rule or MATLAB to find the transfer function

$$G(s) = \frac{X_3(s)}{F_1(s)}$$

i.e. the displacement of the third bottle cradle in response to the nozzle impact on the first. If you are using MATLAB, please include the script and output **fully**. Owen is cheap, so he bought dampers and spring with constants  $1 \text{ N/m}$  and  $\text{Ns/m}$  respectively. You may assume that  $f_1(t) = 100N$  and all masses are  $1 \text{ kg}$ .

- (iv) [10 pts] For this subpart, assume that the system has changed entirely and that **there is a new unknown transfer function  $H(s)$  relating  $X_3$  and  $F_1$** , formally defined as  $H(s) = \frac{X_3(s)}{F_1(s)}$ .

Owen wants to create a system resistant to parts getting jammed in the system, exerting a constant force. More formally, we need to design  $H(s)$ , such that for a **constant applied force**  $f_1(t) = 1$ ,  $t \geq 0$ , the displacement  $x_3(t)$  is **temporary**, and the last cradle can recover its position! For the purposes of this subpart, we will look at a reasonable case; the graph  $x_3(t)$ , given the input  $f_1(t)$  defined above, is shown below.



**Find the transfer function  $H(s)$ .**

**Hints:**

- Multiplying a signal  $R(s)$  by  $e^{-sT}$  shifts the output to the right by  $T$ . This is known as a **delay operator**.
- Think of  $x_3(t)$  as the sum of shifted and scaled step functions! How would you express these shifts, scales, and sum within a transfer function?

## 2. Perpetual Motion Machine...? [43 pts]

It's 2025 and Athul begins his lucrative career conning the public. He begins by proposing a **perpetual motion machine** to venture capitalists! As expected, he gets millions of dollars in funding, but after release day, customer reviews are overwhelmingly negative!

Larry wants to demonstrably prove that the perpetual motion machine doesn't work at all, but he doesn't know how to do so **rigorously**. We'll help him out in this problem, by showing that the machine does not work **regardless of its initial condition**.

After some research online, Larry finds out that he can linearly approximate the dynamics of the system over **small time intervals**. Formally, the dynamics of the system at time  $i$  are given as:

$$x[i] = \begin{bmatrix} v_x[i] \\ v_y[i] \end{bmatrix}, \dot{x}[i] = \mathbf{A}x[i] + \mathbf{B}u[i]$$

Here, the vector  $x$  has the components of the measured velocity of the machine. Additionally, Larry can estimate the state at the following timestep  $x + \delta t$  as:

$$x[i + \delta t] = x[i] + \dot{x}[i]\delta t$$

Larry begins by attempting to run some experiments on the machine. He *wants* to run an experiment where he provides an initial input  $u[0] = 1$ , and all following inputs  $u[i] = 0, \forall i > 0$ . The machine will start at an initial state  $x[0]$  and all future states will be noted down.

After several tries and multiple electrocutions, he finally gets an experiment with consistent readings! He notes his measurements down below:

$$x[0] = \begin{bmatrix} 2 \\ 1 \end{bmatrix}, x[0.5] = \begin{bmatrix} 3 \\ 2.25 \end{bmatrix}, x[1] = \begin{bmatrix} 3.75 \\ 3.1875 \end{bmatrix}$$

However, Larry noticed he was accidentally sitting on the motor input controller, leaving it on for the whole experiment! This means that  $u[0] = u[0.5] = u[1] = 1$ . Even with this faulty experiment, we can still prove that the machine doesn't actually work as intended.

- (i) [12 pts] **Assume that  $\mathbf{A}$  is a diagonal matrix.** Write, **but do not solve**, a set of equations to solve for  $\mathbf{A}$  and  $\mathbf{B}$ , given the measurements for  $x$  and  $u$  above. You will have 4 unknown variables, and 4 equations.

- (ii) [9 pts] Solve the system of equations using MATLAB or by hand. If using MATLAB, include your code and output below. Your answer should be the matrices **A** and **B**.

- (iii) [7 pts] It is possible to rewrite the dynamics of the system in the form  $\mathbf{C}x[i] + \mathbf{D}u[i] = x[i + \delta t]$ . What are **C** and **D**, **in terms of the variables A, B, and  $\delta t$** ? You may also have constant values as part of your answer.

(iv) [15 pts] For the perpetual motion machine to work, it takes in an initial input  $u[0] \neq 0$ , with all following inputs  $u[i > 0] = 0$ , and the machine runs forever. In this final subpart, we will construct a proof for why the machine will never work, regardless of the initial conditions  $x[0], u[0]$ .

i. If the machine is *not* working, then, given this input sequence, it will **consistently slow down**. Knowing this, as  $t \rightarrow \infty$ , what vector  $x_f$  should  $x$  converge to?

ii. Now, to show that the machine does not work, **we want to prove  $x[t] \rightarrow x_f$  as  $t \rightarrow \infty$  using the dynamics we calculated** in previous subparts. **Write an expression for  $x[n]$  in terms of  $\mathbf{A}, \mathbf{B}, \mathbf{C}, \mathbf{D}, \delta t, u[0], x[0], n$  and/or any other necessary constants.** You may not use  $x[i]$  or  $u[i]$  for any  $i \neq 0$  in your expression. As a way to see if you are on the right path, your answer will have a matrix exponential!

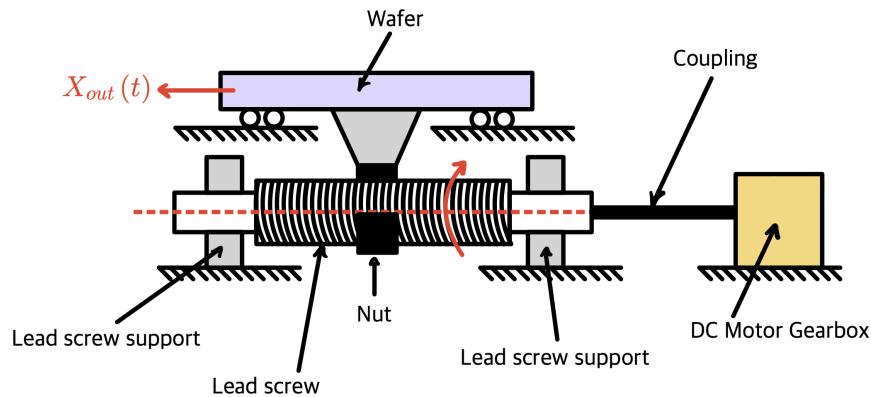
iii. Now, find the numerical value of the matrix  $\mathbf{C}$ . Based on this matrix, and your expression for  $x[n]$  in the previous subpart, explain why  $x[t] \rightarrow x_f$  as  $t \rightarrow \infty$ , irrespective of the values of  $x[0]$  and  $u[0]$ .

### 3. I say Screw It! (EC) [15 pts]

*Note: The following question is for extra credit only. It is designed to be more challenging and exploratory in nature. Please make sure to attempt all required questions on the midterm before working on this one!*

Athul, despite having no love for chip design, is coming to fame as a part of both the ASIC (Application-Specific Integrated Circuit) and FPGA (Field-Programmable Gate Arrays) design team at Intel. He has just finished taping out a new mixed-signal SoC. But before bring-up begins, he has volunteered to help Larry on the motion control team fix a flaky wafer inspection system on the lithography line. The linear stage used to align the wafers has been jittery at best—suspected to be due to Larry’s poor dynamics modeling knowledge (I don’t know how he got the job!).

The system uses a lead-screw driven stage, where the motion is actuated by a DC motor and transformed into linear motion through a torsional mechanical coupling. You and Athul are brought in to help salvage Larry’s catastrophic mistake to model the system so that Athul can go back to debugging clock trees.



The system is shown above and described as follows. You know that the motor torque is proportional to the current input  $I_{in}$  with a proportionality constant  $k_t$ . The motor connects to the lead screw via a coupling which is modeled as a torsional spring and damper in parallel with stiffness  $k_c$  and viscous damping coefficient  $b_c$  respectively. The lead screw has pitch  $l_p$  in  $[mm/rad]$ , a rotational inertia  $J_l$ , and drives a stage with a wafer mass of  $M$ . The leadscrew support is lubricated with a viscous coefficient of friction  $f_v$ . We assume that backlash and cogging effects are neglected.

- (i) [3 pts] Draw the complete block diagram in the  $s$ -domain **with no simplifications**. Treat the motor current  $I_{in}(s)$  as the input and the the linear stage position,  $X_{out}(s)$  as the output. Clearly label all physical parameters and intermediate signals.

- (ii) [4 pts] Derive the state-space representation of the system in the form. Be sure to define all your state variables clearly.

$$\text{SEE: } \dot{\mathbf{x}}(t) = A\mathbf{x}(t) + BI_{in}(t)$$

$$\text{OE: } X_{out}(t) = C\mathbf{x}(t) + DI_{in}(t)$$

- (iii) [2 pts] Athul wants to run a quick parametric study on design sensitivity and limits before bringing up resumes, as he has a reputation to maintain at Intel! Obviously, he is too important to be doing this work, so he assigned it to you! Using your state-space model, find the **steady-state displacement**  $X_{ss}$  (i.e.  $\dot{\mathbf{x}} = 0$ ) when the motor input is held at  $I_{in}(t) = I_0$ . Assume all transients have died out. We are only looking for a symbolic answer.

- (iv) [3 pts] Compute the following derivative.

$$\frac{\partial X_{ss}}{\partial l_p}$$

and **interpret its meaning**. What does this tell Athul about choosing the pitch  $l_p$ ? Does a larger pitch always mean better positioning?

- (v) [3 pts] Evaluate the limit of  $X_{ss}$  as the **coupling stiffness**  $k_c \rightarrow \infty$ . What physical behavior does this model reduction imply?

You may use this page to show extra work. Clearly mark your work with the problem number here, and also mention in the problem-specific box that your work is continued here.

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