Patents Concentration and Productivity Slowdown

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Abstract

The present work aims to understand the role that patents played in determining the observed productivity growth slowdown of the U.S. in the past couple of decades. It documents the salience of patenting activity in determining economic outcomes and the rise of concentration along the patenting dimension. Patents are also shown to be only weakly correlated with a firm's productivity. The paper then develops a simple theoretical model that replicates the observed features of the empirical trends and it introduces extensions that integrate more realistic components.

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1 Introduction

It has long been recognized by economists that productivity is the driving force of long-run economic growth and well-being, Solow (1956). Given its central importance, the slowdown in productivity growth that occurred after the 2000s as shown in Fig. 1 has sparked an intense policy and scientific debate to unravel the causes and consequences of the observed trends, especially but not limited to the U.S. economy. The productivity slowdown is comprised of wider debate that has documented the decline in Business Dynamism, (Akcigit and Ates (2023)) which documents the rise of several empirical trends. A clear and amply considered one is the increasing market concentration, entailed by the reallocation of share of sales to larger firms at the industry level, where larger firms outperform the laggards (Autor, Dorn, et al. (2020)).

Parallel to these trends, Fig. 2 displays the great increase in the use of patenting, in this case, invention patents, exploded especially after the 2000s. Likewise Productivity, patents have received attention in the literature to understand how they shape economic outcomes, specifically as a means of storing intangible capital, which exhibits distinct features from standard physical capital (Crouzet, J. C. Eberly, et al. (2022)).

The present work thus aims to answer the questions: Is there a relationship between the boost in patenting, concentration and productivity slowdown? How are these trends related?

To do that I first develop an empirical section using Compustat data, a large firm-level dataset on publicly listed firms, which although does not show any causal link, documents the dramatic importance of patenting in determining firms' economic outcomes. Moreover, I illustrate how the positive relationship between productivity and patents fades away once Research and Development expenditure is controlled for, suggesting that the increased use of patents is not reflected in a firm's productivity. Empirical trends associated with concentration, such as rising markups among larger firms, seem to revolve around the ability of firms to acquire patents. To make sense of the movements observed in the data, I developed a simple theoretical framework, a partial equilibrium Schumpeterian growth model, where firms compete by investing in innovation to be the sector's leader, that replicates the main features documented in the empirical section. The rest of the work is organized as follows: Section 1.1 discusses related literature, Section 2 introduces and explain the empirical findings, Section 3 presents the model , Section 4 concludes.

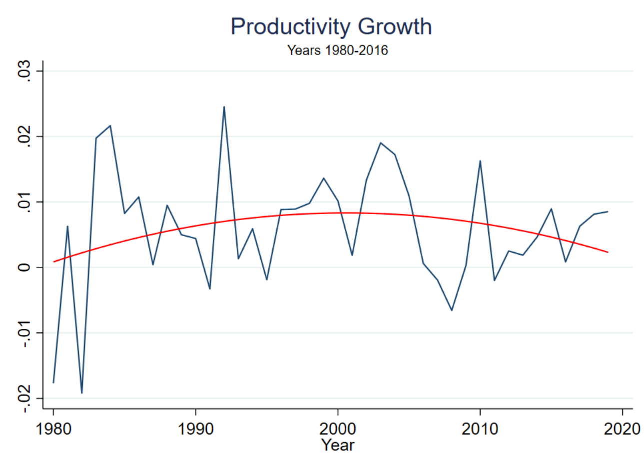

Figure 1: Growth Rate of Productivity measured with Total Factor Productivity. Source: Fred

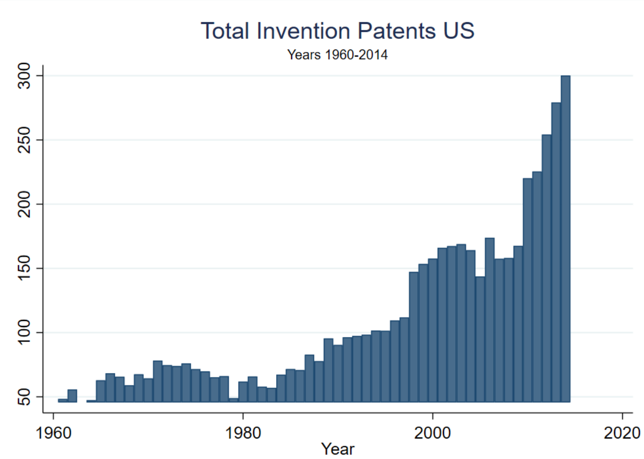

Figure 2: Total Invention Patents Granted in the U.S. Granted from the 1960 to 2014, in thousands. Source:https://ourworldindata.org/research-and-development

1.1 Related Literature

Although most of economic literature has always considered intellectual property as necessary feature to incentives innovation, since ideas are non-rival and can be then appropriated easily by competitors (Romer (1990)), different approaches have tried to challenge this view, arguing that on the contrary intellectual property be detrimental for growth. Boldrin and Levine (2008) sustain that innovation is possible and occurred frequently throughout history in the absence of intellectual property spurred by copying and incremental advancement, while patents prevent competitors from improving upon existing technology, reducing the incentive to innovate of the incumbent by granting them monopolistic power.

Recent years have witnessed increasing attention to patents in the literature in the form of intangible capital which is capital that does not have a physical presence and thus requires a means of storage, which in many cases can be a patent(Crouzet, J. C. Eberly, et al. (2022)). Intangibles, it is argued, share different features from standard physical capital, such as scalability(the ability to be used in several places at the same without virtually any impact on marginal costs), which have made them the object of interest of several studies trying to understand their impact on the real economy(Haskel and Westlake (2017)). Of specific interest for this work are the papers Crouzet and J. C. Eberly (2019) and De Ridder (2024). The first links the the recent rise in intangibles with industry concentration arguing that intangible-intensive firms enjoy higher market power. The former develops a similar point using an endogenous growth model that predicts that intangible rises market power precisely because of scalability, which in turn leads to aggregate decline in productivity growth.

Similar to the latter paper is Aghion, Bergeaud, et al. (2023), which builds a general equilibrium with firm heterogeneity and endogenous markups, where a reduction in overhead costs due to higher efficiency of big firms brings about their expansion in different product lines that eventually causes lower aggregate productivity.

Related to the work mentioned so far, the trend of increasing market concentration to big firms has been also a central topic of recent research, which includes but certainly is not limited to Gutiérrez and Philippon (2016), Bessen (2020) and Autor, Dorn, et al. (2020). Specifically this latter documents the rise of superstar firms, which are getting bigger, more productive and are according to them largely responsible for the observed fall in the labor share. One observation they made is that superstar firms are responsible for a rise in market power as high sales firms are driving up markups. De Loecker, Eeckhout, and Unger (2020) estimate markups with several measures and under different assumptions using Compustat and the Census of Manufacturing concluding that this rise in market power is consistent and driven by reallocation toward large firms.

All of these trends are discussed in Akcigit and Ates (2023) and are listed as overall evidence of a decline in business dynamism of the U.S. economy. They found that the main driver of the decline is to be attributed to a reduction in knowledge diffusion. The present paper expands in that direction offering evidence on the increasing role of patents in determining firms' performances also by means of a reduction in the spread of knowledge. Closely related is the study of Bloom et al. (2020) which remark how ideas are getting harder to find, meaning that the resources devoted to R&D has increased dramatically to maintain constant trend in economic growth, therefore leading to a reduction in research productivity. Investment in research may need to increase if patents are responsible for the reduction in knowledge diffusion by preventing spillovers from occurring, especially if their cost-effectiveness increased, as shown by Bessen and Hunt (2007) and firms invest more in protective patenting, defined ”internal” patenting by Akcigit and Kerr (2018).

Patents and intangibles have also been shown to be important in explaining phenomena related to different economic dimensions, such as Koh, Santaeulàlia-Llopis, and Zheng (2020) Aghion, Akcigit, et al. (2019) Crouzet and J. Eberly (2023) Autor, Salomons, and Seegmiller (2023) among others.

The present work is also related to endogenous growth literature, Romer (1990),Klette and Kortum (2004) and especially to Aghion and Howitt (1990), and builds on the model exposed in Aghion, Akcigit, et al. (2019) expanding on the role of protecting patenting.

2 Empirical Evidence

The empirical section employs firm-level data to document the importance of patents in explaining the declining business dynamism and concentration trends analyzed by the literature in the past years. Specifically, we are going to focus our attention on Compustat which contains financial statements of all publicly traded firms from 1950 to 2016. The dataset has been extensively used for several studies, and although it represents one of the main sources of firm-level data for the US economy, it possesses some potential limitations. Davis et al. (2006) reports that Compustat firms account for 29% of U.S employment albeit being a relatively small sample compared to the overall economy, due to the larger size and high capital intensity on average(De Loecker, Eeckhout, and Unger (2020)).
Patenting Data have been borrowed from the dataset constructed for Kogan et al. (2017) which the authors made publicly available on their GitHub profile, and then merged with Compustat using the perno variable as a mapping between the two datasets. It reports Invention Patents Granted to firms, forward citations, and the private value of each patent computed using stock market response to news about patents. A more detailed description of datasets is reported in Appendix A.1.

Since a central part of this work is the productivity slowdown of the U.S it is useful to estimate total factor productivity from our dataset. We have opted for estimating the production using the Levinson-Petrin method (LP) developed in Levinsohn and Petrin (2003), which uses intermediate outputs as a proxy for productivity, in this case, the variable COGS. The production function is assumed to be Cobb-Douglas and the factors of production are labor and capital(the variables EMP and PPEGT). Appendix A.1 provides further description of the method.

At first inspection, the estimated productivity reveals a similar trend as the one reported for the U.S economy in Section 1. In Fig. 3 average TFP from 1980 to 2016 whereas Fig. 4 growth rates are shown over the same period. This latter graph reveals an inverted U-shaped trend closely resembling the one in Fig. 1, a first indication of the goodness of our estimate and similarity of our sample with the overall economy. The key message remains that productivity is slowing down in the U.S economy.

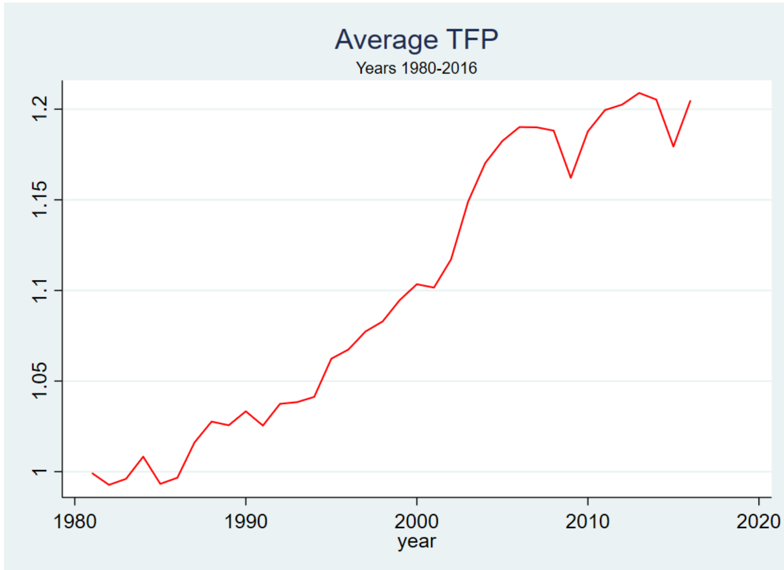

Figure 3: Average TFP from Compustat

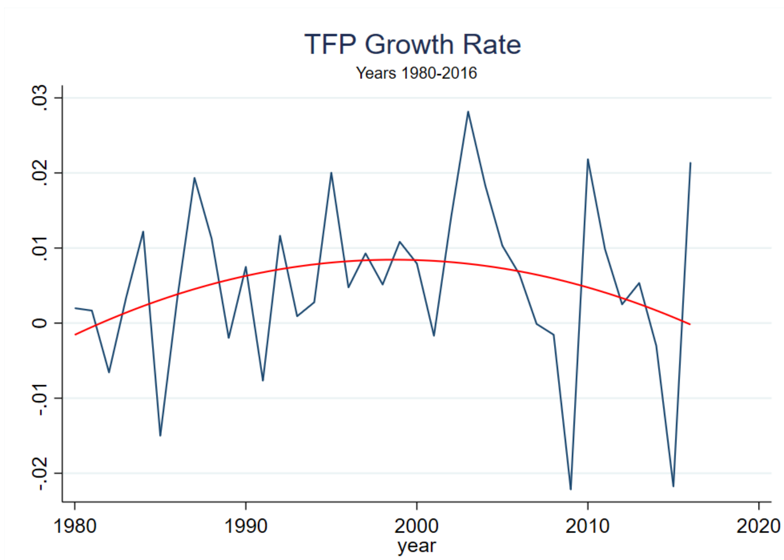

Figure 4: Growth Rate of Productivity measured with Total Factor Productivity, computed from Compustat dataset

Similarly, Patenting activity is a quantitatively salient feature of our dataset, and in particular, the increasing trend displayed in the last decades. Fig. 5 displays the average number of patents issued to a firm while Fig. 6 represents the total amount of patents granted. In both images one can notice a steady increase in patents granted both in mean and in absolute terms from the end of the 90's, probably due to the greater role of software patents (Bessen and Hunt (2007), which also employs Compustat in their empirical analysis), which we were not unfortunately able to distinguish in our dataset. The trends in both figures mirror the tendency shown for the economy in Fig. 2.

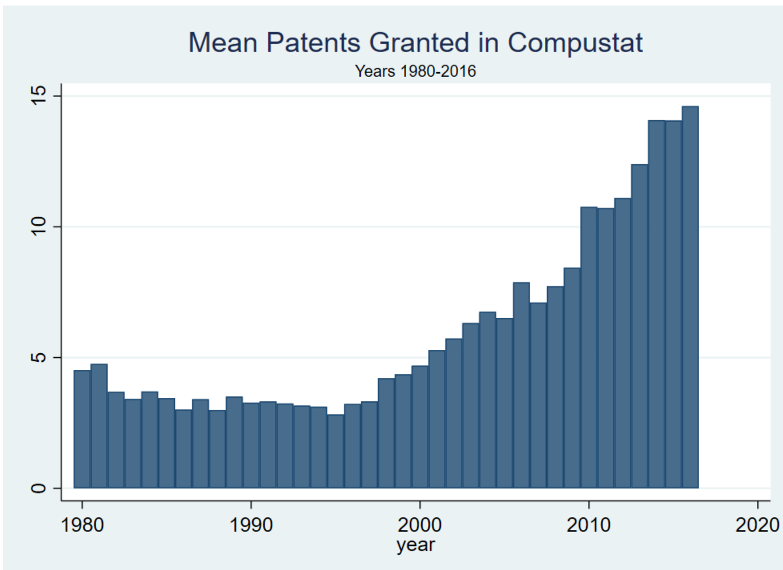

Figure 5: Average Patents Granted by firm in Compustat

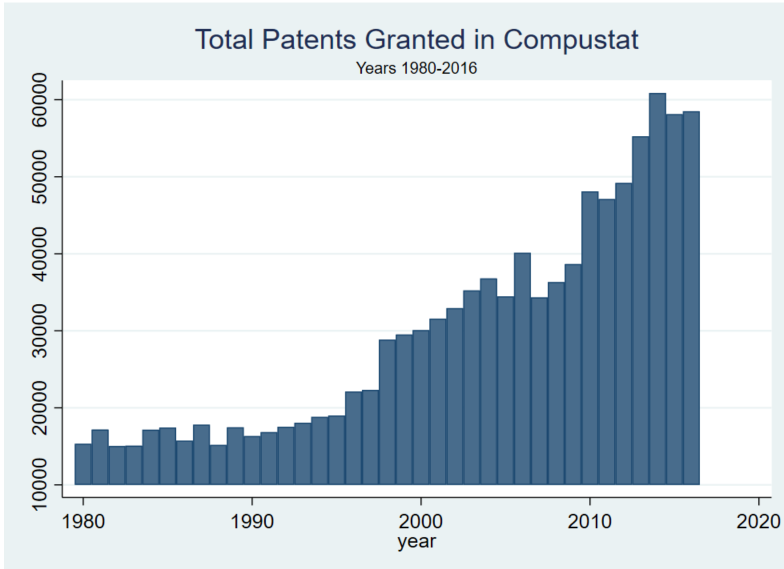

Figure 6: Total Patents Granted in Compustat

Having now verified that our sample matches the our main motivating trends, we can now analyze more deeply our dataset to show the importance that patent have in shaping their performances and why the greater salience of patenting activity should be seen as major determinant in the decline in Productivity Growth.

Below are reported figures that document the striking difference in firms' performance along the line of patenting activity, by dividing the sample in patenting and not patenting firms. To prevent firms from being in both groups at different points in time, we have computed the cumulative stock of patents of each firm. A patenting firm is therefore a firm that was issued a patent at some point in time. Thus, once a firm crosses the extensive margin of patenting remains in the patenting group until it drops out of the sample.

The first figure below, Fig. 7 plots the average firm's productivity between the two groups just mentioned. It is striking to observe the divergence occurred from the mid 90's to the end of the sample. Specifically, the trend of not patenting firms is slowly increasing until the setback of the Great Financial Crisis and remained flat after that, whereas patenting firms perform better every year without any sign of slowing down, in contrast with the general trend of the U.S. economy. On average patenting firms are more productive than not patenting ones and increasingly so in the last years of the sample.

The picture below, Fig. 8 plots average profits for the two groups. Profits have been computed following De Loecker, Eeckhout, and Unger (2020), by subtracting from sales total costs, in our sample obtained by the variable COGS plus capital expenses (as well computed using other variables as described in Appendix A.1). The difference perhaps this time between the groups is less striking. However, patenting firms make on average always more profits, and even though the difference shrinks over the first year of the new century the divergence becomes more prominent after 2010. Profit Rates, calculated as profits over sales, are reported in Fig. 20 of Appendix A.1. The result remains unchanged and if anything it highlights a more pronounced divergence between the groups.

Lastly, in Fig. 9 average markups are reported for both groups. Markups measures have been obtained again emulating De Loecker, Eeckhout, and Unger (2020), which make firm-level markups estimation the central results of their work (using Compustat as well), arguing that the steady rise in the past decades is evidence of growing market power due to reallocation of economic activity to large firms. Markups are derived using the production function approach as recommended by Loecker and Warzynski (2012) derived from cost minimization problem of the firm, without the need to specify a demand structure. In Appendix A.1 I report the same picture using markups obtained from cost shares. Following the argument De Loecker, Eeckhout, and Unger (2020), one observes that patenting firms enjoy higher markups, most likely pushed by higher productivity observed which in turn makes them able to extract larger profits. Therefore, this last picture add consistency to the point being made with the two previous plots; patenting firms are outperforming not patenting firms over different dimensions.

Appendix A.2 also reports medians of the measures just described in this part, as well as the average profit share between patenting and not-patenting firms, which is obtained by dividing profits by sales. Moreover, it reports the average TFPs and Markups estimated using different techniques and assumptions. The stark division in performances between the groups holds.

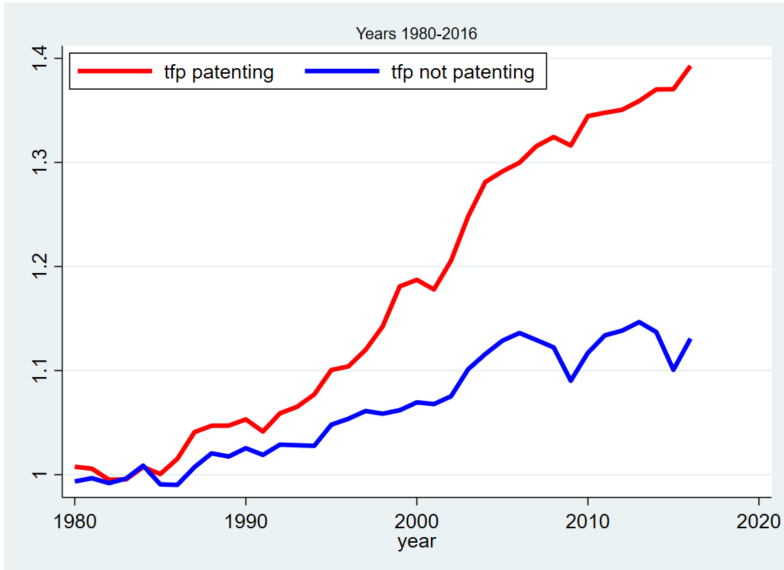

Figure 7: Average TFP of Patenting and not Patenting Firms

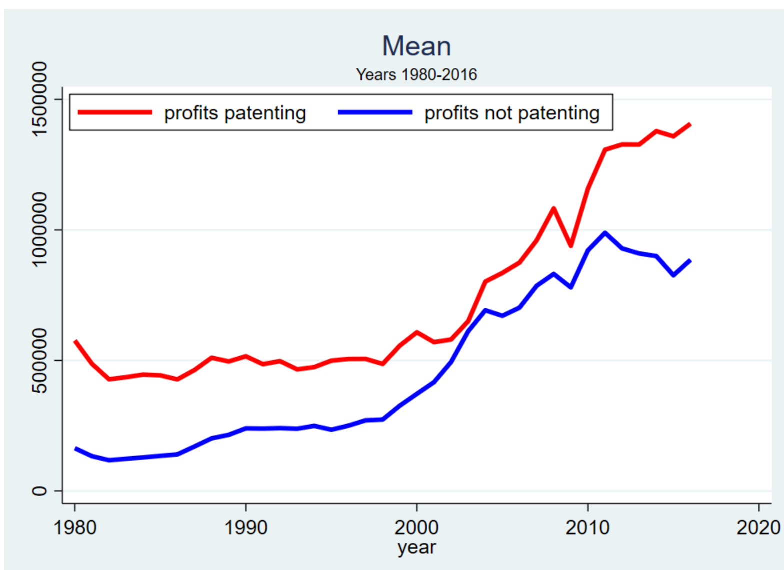

Figure 8: Average Profits of Patenting and not Patenting Firms

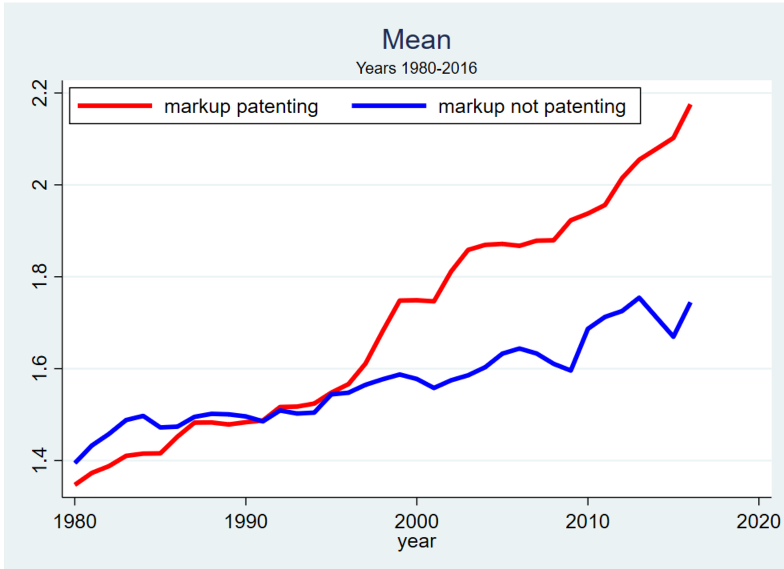

Figure 9: Average Markups of Patenting and not Patenting firms

It may be instructive to also see the distribution of TFP between the two groups and how it evolved over time to get a better sense of the evidence that has been shown so far. We do that in Fig. 10 where we plot the density of the two groups over four years, 1980,1995, 2005, and 2005. A clear shift has occurred throughout these periods with the patenting group passing from having less highly productive firms than not patenting firms, to patents being concentrated only on very highly productive firms.

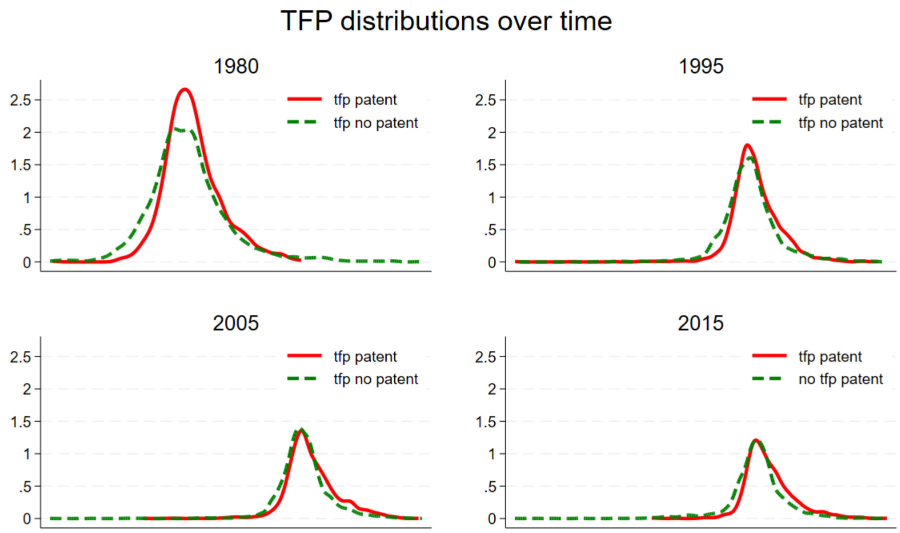

Figure 10: Kernel Density of TFP Patenting vs Not Patenting Firms over time

Although patenting activity and its importance in determining economic outcomes has increased, being a patenting firm seems harder over time. Fig. 11 shows the number of new patenting firms, which here are defined as firms that in year t started to patent but in year t-1 were not patenting. To do that, I employ the measure introduced before of cumulative patents such that once a firm enters the patenting group it remains there until present in the sample.

The plot shows how the number of firms able to start patenting has declined in the new century, contrary to the trend that occurred in the previous two decades. Therefore, with an increasing trend in patenting activity, we found it harder to pass into the group of patenting firms. Fig. 24 in Appendix A.2 shows the new patenting firms over the total firms in the sample. The result is robust as it proves that the trend is not driven by simply fewer firms in the sample. A similar trend is observed when one analyzes forward citations of patents. Forward citations are the references made to that patent when other patents are issued. To account for attrition, namely that younger patents will have fewer citations we employ the metric used in Kogan et al. (2017), which weights forward citations of each patent using the average forward citations in that year. This measure can be viewed as a scientific (Kogan et al. (2017)) or public value(Kogan et al. (2017)) of each patent and they can function as a proxy for knowledge diffusion. To account for the greater patenting activity, we have divided the metric of forward citation by the patents granted at the firm level. Fig. 12 is showing that on average forward citations that a patenting firm receives in a year are declining with respect to the total patents granted to that firm. In other words, the average scientific value of patents is decreasing with respect to the number of patents that a firm is able to get granted. The two observations just commented are line with a decline in knowledge diffusion investigated in Akcigit and Ates (2023) as a central driver of business dynamism and less straightforwardly with the notion that idea are harder to find (Bloom et al. (2020)), as fewer firms can patent and public value of the patent issue declined.

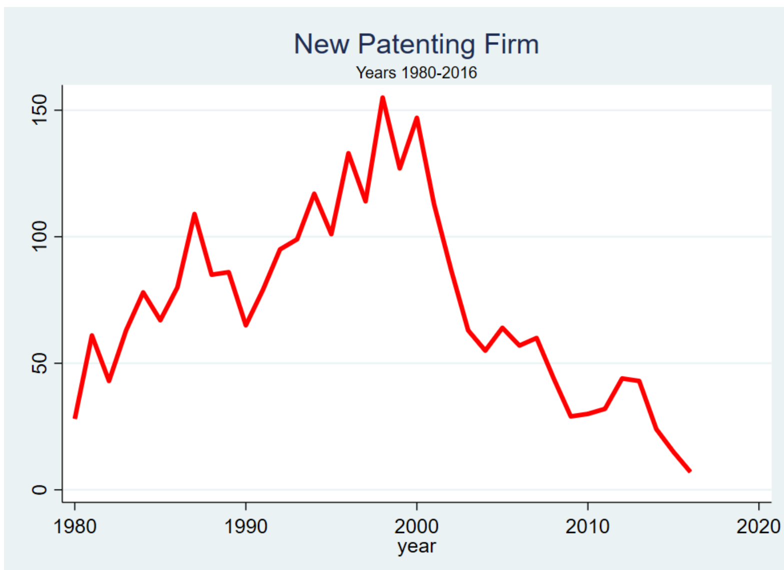

Figure 11: Extensive margin Patenting firm

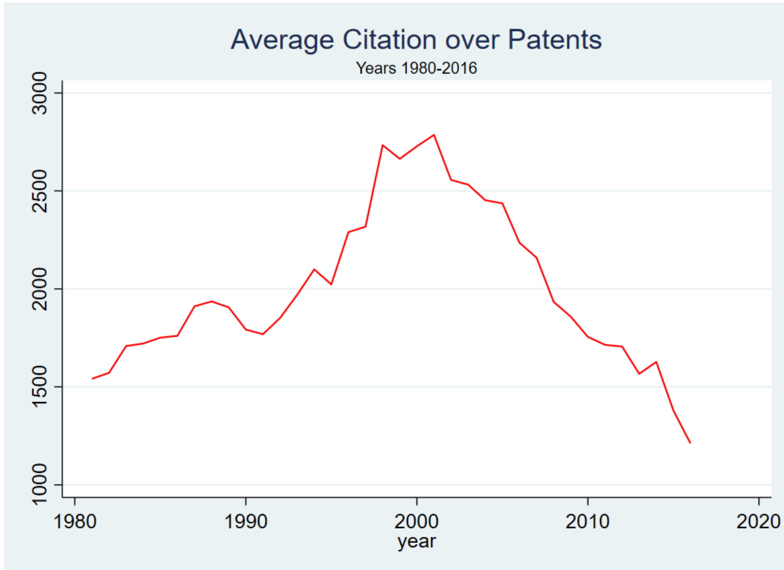

Figure 12: Forward Citation over Patents Granted

Whether all this patenting activity is eventually beneficial to help advance firms' productivity is a question that has been largely debated. As we saw in Section 1.1 Standard growth theories such as the one presented in Romer (1990) argue that this should be the case due to the non-rivalry of ideas and the need to create an incentive to innovate, whereas Boldrin and Levine (2008) argue that patents may be detrimental for growth as it prevents a potential competitor from innovating and give incumbent producers harmful and unnecessary monopoly powers. The results we present in the following figures are far from being causal and even less conclusive on the debate. It does nonetheless offer evidence in support of the latter theory and it connects with existing research on business dynamism. Fig. 13 and Fig. 14 are added value graphs, plotting log TFP on log of value of patents, when R&D spending is controlled for. The value of patents was preferred as it is more instructive on how that patent may contribute to a firm's performance. We observe a negative relation between productivity and the value of patents using both metrics. The public value of a patent we have already encountered while the Private value of a patent is computed from market returns by Kogan et al. (2017), as described in Appendix A.1. For the sake of completeness, I report in Appendix A.2 the same exercise done with patents granted and the trend obtained is along the same lines.[1] The plots tell us that, at the same level of R&D spending a percentage increase in the value of a patent is not correlated with a percentage increase in Productivity. This evidence is consistent with Akcigit and Ates (2023) which documents a greater use of strategic patenting after the 2000's, finding a larger adoption of patents that build on existing firm's technology with the aim of preventing competitors from accessing that knowledge (defined as "internal patents" by Akcigit and Kerr (2018)). Therefore, the evidence presented in this work, along with other contributions, such as the one we have just mentioned, seems to point to an impacting role of patents in the decline of productivity growth and the rise in concentration, in line with the argument made by Boldrin and Levine (2008). The specification remains robust when additional variables are controlled for and a different estimate of TFP is employed as shown in the regressions at the end of Appendix A.2.

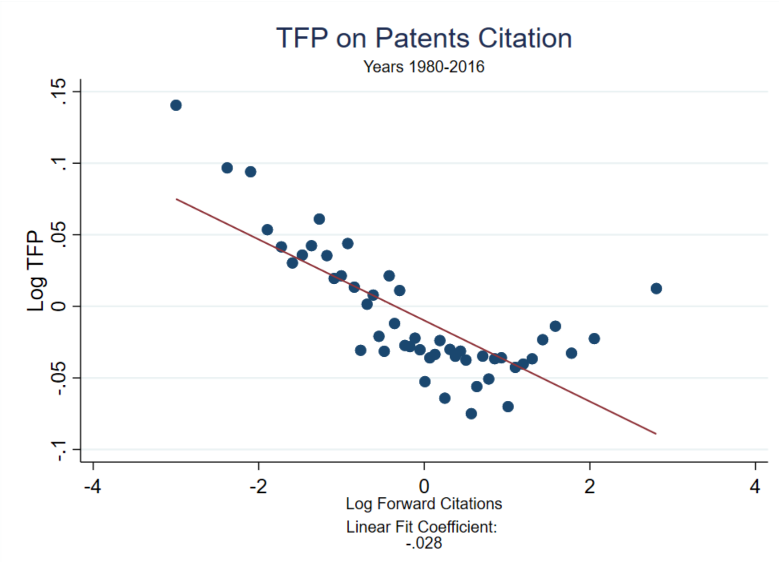

Figure 13: TFP and Public Value of Patents when R&D is controlled for

[1] Fig. 13 Fig. 14 Fig. 25 were created using the "binscatter" function in Stata that creates , in this case 50, equally sized bins out of all observations to improve the visual appearance of the table

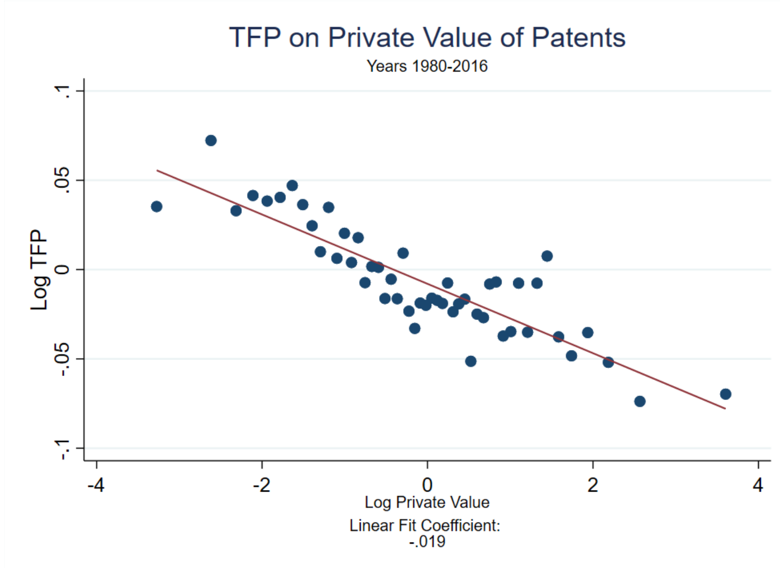

Figure 14: TFP and Private Value of Patents when R&D is controlled for

3 The Model

We will consider the Shumpeterian growth model that only focuses on the production side of the economy. As pointed out more precisely later, the setting borrows from and extends the model presented in Aghion, Akcigit, et al. (2019), particularly the distinction between productive and defensive innovation. There is one final good firm and a mass of unit 1 of intermediate producers. In each sector, firms compete on prices to be the monopolist leader by investing in R&D and Patents to be the technological leader in that sector.

3.1 Final good

The Final good is produced by combining each intermediate good y_i produced by the firm i according to a Cobb-Douglas technology such that:

$$Y_t = exp(\int_0^1 ln(y_{it})\, di) \tag{3.1}$$

From the cost minimization program, one easily derives the inverse demand of intermediate good product y_i, which is given by:

$$p_{it} = Y_t P_t y_{it}^{-1} \tag{3.2}$$

where the price of the final good P given by:

$$P_t = exp(\int_0^1 ln(p_{it})\, di) \tag{3.3}$$

which, for simplicity, can be normalized to 1.

3.2 Intermediate goods

The intermediate good in each sector is produced by a firm using as only factor labor, which is supplied inelastically, according to the linear technology:

$$y_{it} = q_{it} l \tag{3.4}$$

where l is labor and q_i can be seen as the productivity parameter of firm i.The value of q_{it} is determined by the following law of motion.

$$q_{it} = v_{it} q_{it-1} \tag{3.5}$$

where $v_{it} \in \{v_H, v_L\}$ with $v_H > v_L > 1$.

The term v_H occurs when a new innovation materializes, either to the incumbent monopolist or a competitor, giving the innovator the technological lead in that sector. This is the result of a successful innovation resulting from investment in productive research. On the other hand, when there is no technological advancement, v_L gives an edge to the incumbent, which can therefore remain the monopolist. This can result from the failure to innovate of the competitor, which, as it will be shown below, can be influenced by the investment in protective innovation or patenting by the incumbent. In this setting, we are assuming that each technology at the end of each period becomes publicly available and that the competitor is perfectly able to replicate it.

3.2.1 Innovation

At each point in time, in each sector, I assume that a competing firm invests in productive research to get the technological lead in the sector. In particular, the competitor to innovate with probability x_C needs to invest:

$$\frac{\theta_C x_C^2}{2}$$

The incumbent, on the other, has the option not only to invest in productive research but in defensive patenting. By investing:

$$\frac{\theta_P x_P^2}{2} + \frac{\sigma d^2}{2}$$

she can innovate with probability x_P and prevents the competitor from innovating with probability d. Intuitively, the incumbent by being the leader in that sector can invest in patenting that do not enhance the productivity of the firm but can be very valuable as it prevents the competitor from catching up. On the other can invest in more groundbreaking research that would make it more vulnerable to competition. On can think as these two investments as the internal patenting and external patenting, two terms already discussed and introduced by Akcigit and Kerr (2018) to identify patents that aim at preventing competitors to copy , the former, while the latter refer to patents of newer and disruptive technology. In our model one should think d as an internal patent and x_P (and x_C) as external patents.

This simple cost structure will be later be modified to allow for more complex strategic interaction between investments. Indeed, one can anticipate that additive separability will lead the two investments of the incumbent to be independent of one another. Costs need to be convex given the linearity of the production function to create boundaries to the firm, that otherwise would scale up production to keep making profits indefinitely.

3.2.2 Solution of the model

Limiting price in any sector at any t, leads the producer set the price of the intermediate equal to the marginal cost times, here given by the wage over the productivity parameter, multiplied by the technological lead of the incumbent producer:

$$p_{it} = \nu_{it}\frac{w}{q_{it}} \tag{3.6}$$

In this first part, where prices are set and incorporated into profits, it is important to notice that each agent treats the probability of innovation as endogenously set and firms treat them as given. Thus, profits net of the innovation investments are therefore given by:

$$\Pi_{it} = (p_{it} - \frac{w}{q_{it}})y_{it} = (1 - \frac{w}{q_{it}p_{it}})Y_t = (\frac{\nu_{it}-1}{\nu_{it}})Y_t \tag{3.7}$$

where the first step uses the optimal demand for the intermediate good derived in Section 3.1, while the last step incorporates the optimal pricing behavior of the incumbent producer.
In this section, it will present the case for the simultaneous game, in which investments in innovation from both competitor and incumbent monopolist are decided at the same time. However, in the Appendix A.3 it is presented an extension where the incumbent monopolist has a first mover advantage and can therefore observe the best reply of the competitor and maximize her profits accordingly.

In the simple setting, both competitor and incumbent maximizes their expected profits by simultaneously optimally investing innovation. The former solves then the following program:

$$\max_{x_C} \quad \{(1-d)x_C\pi_H - \frac{\theta_C x_C^2}{2}\}Y_{it} \tag{3.8}$$

whereas the incumbent:

$$\max_{x_I,d} \quad x_I\{\pi_H + (1-x_I-(1-d)x_C)\pi_L - (\frac{\theta_P x_I^2 + \sigma d^2}{2})\}Y_{it} \tag{3.9}$$

The competitor can only take the lead by being the one getting the best technology and patenting it. Therefore, she maximizes her expected profits by investing in research, treating as given the choice of the incumbent. However, the incumbent will optimally split the investment between productive and defensive innovation. Investing in defensive, she reduce the chances of the competitor to improve her technology, however, by doing so it will take away resources from productive research, in turn reducing her expected profits. We should point here at one of the main differences with the model proposed in Aghion, Akcigit, et al. (2019). Productive and defensive innovation have two potentially different marginal costs that depend on exogenous values of θ_P and σ respectively. The intuition is that these two different investment may entail different actions in reality that may therefore have different impacts for the firm at the margin. These parameters therefore play a crucial role both for the individual firms as well as for social welfare consideration, as we will see below, they impact the probability of a successful innovation in a given sector.

In order for these quantities to be meaningful within the setting of these model, we need that the overall probability of a technological improvement to be bounded below 1. To do so, first define $\Delta \equiv \pi_H - \pi_L$, then we should take the following assumption:

Assumption 1. *The cost parameters associated with productive innovation, θ_P and θ_C, need to be high enough.*

We can now characterize the optimal investments in innovation by the incumbent and competitor in a given sector.

Proposition 1. *The optimal investments are given by:*

$$x_P^* = \frac{\pi_H - \pi_l}{\theta_P}; \quad d^* = \frac{\pi_H \pi_L}{\pi_H \pi_L + \theta_C \sigma}; \quad x_C* = \frac{\pi_H \sigma}{\pi_H \pi_L + \theta_C \sigma}$$

Proof. see Appendix A.3 □

As we have already mentioned earlier, the optimal productive innovation by the incumbent only depends on profits and the cost parameter associated with that investment, θ_P. On the other hand, both defensive innovation and productive innovation by the competitor are dependent on each other cost parameters. Indeed, exogenous variations of one of these two parameters, possibly due to a change in legislation, will influence simultaneously both investments. Specifically, we are going to focus on the impact of a change in σ on the overall probability of innovation, as it may inform us about the trends that have been documented in Section 2. In equilibrium, the probability is given by:

$$P(v_{it} = v_H) = x_P^* + (1 - d^*) x_C^* = \frac{\pi_H - \pi_l}{\theta_P} + \frac{\pi_H \theta_C \sigma^2}{(\pi_H \pi_L + \sigma \theta_C)^2} \tag{3.10}$$

It is therefore easy to see that:

Proposition 2. *The probability of technological advancement in a given sector is increasing in σ*

Proof. see Appendix A.3 □

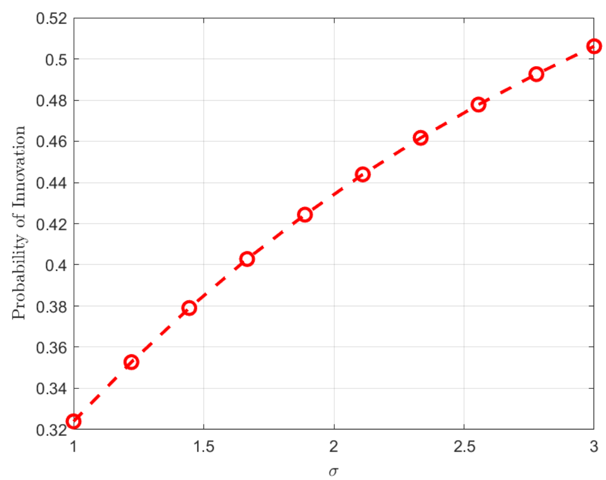

Figure 15: Probability of innovation over values of σ, Simple Model

The result establishes that when for the incumbent is costly to invest in defensive technology the overall probability of technological advancement in a given sector increases. Hence, conversely, the cheaper it is for an incumbent to protect her monopoly position, the lower the probability of a productive innovation. In this simple model, a lower probability implies that v_L occurs more often, therefore reducing the overall productivity growth of the economy. The result is in line with the stylized facts shown in Section 2, in particular with decreasing knowledge diffusion measured as citation over patents, the lower number of firms able to patent, and an overall slowdown in productivity. Moreover, a lower value of σ can be viewed as a change in legislation in patenting between the '80s and the '90s documented by Bessen and Hunt (2007) Boldrin and Levine (2008) among others[2].

This simple just illustrated should thus serve as a basic conceptual framework to understand the role that patents have played in shaping the U.S. economy in the last decades.

3.2.3 Relaxing additive separability

We shall advance to analyze what happens to our predictions when the cost structure for the incumbent may be different, specifically when additive separability is relaxed. Suppose that now to get the technological lead with probability x_P and prevent the competitor from successfully innovating with probability d the incumbent has to invest:

$$\frac{(\theta_P x_P + \sigma d)^2}{2} \tag{3.11}$$

Notice that the problem of the maximization program carried out by the competitor is unchanged, even though the optimal investment will differ as it depends on the value of d. The new cost structure relaxes additive separability maintaining a convex cost which ensures boundary to the firm to keep the problem well defined.

However, the incumbent now solves:

$$\max_{x_P, d} \quad x_I \pi_H + (1 - x_P - (1-d)x_C)\pi_L - \frac{(\theta_P x_P + \sigma d)^2}{2} \tag{3.12}$$

From the first order condition of the two profit maximizations, one derives the optimal investments.

Proposition 3. *The optimal investments when additive separability is relaxed are given by :*

$$x_P^* = \frac{\Delta(\sigma^2 \theta_C + \pi_H \pi_L)}{\theta_P^2 \pi_H \pi_L} - \frac{\sigma}{\theta_P}; \quad d^* = 1 - \frac{\theta_C \sigma \Delta}{\theta_P \pi_H \pi_L}; \quad x_C* = \frac{\sigma \Delta}{\theta_P \pi_L} \tag{3.13}$$

Proof. see Appendix A.3 □

[2] from Boldrin and Levine (2008) "The long-standing tradition of free competition and lack of intellectual monopoly began to crumble in 1981, with the Supreme Court decision in Diamond v. Diehr, and collapsed completely with the publication of new examination guidelines by the U.S. Patent and Trademark Office in 1996, which made computer programs fully and clearly patentable."

Interestingly, all optimal investments in this setting depend on each other's cost parameters, that is, the cost associated with investments in innovation influence simultaneously all optimal decisions. Perhaps, the new result may be more realistic, as one should expect the incumbent to also tailor strategically her productive investments when an exogenous variation in the cost parameter of other investments occurs. For example, when productive innovation gets cheaper for the competitor, maybe due to spillovers from other sectors, the monopolist internalizes that both investments need to be adjusted strategically.

As before, we are going to focus on the role played by σ on the overall probability of a technological improvement in a sector, to see if we can relate the results of our model with the empirical evidence shown in Section 2.

Before doing that, let us impose parameter restrictions that ensure that the probability of an invention is bounded below 1.

Assumption 2.

$$\frac{\Delta}{\theta_P \sigma} < \frac{\pi_H \pi_L + \theta_C \Delta}{\theta_C \sigma^2 + \pi_H \pi_L + \sigma \theta_P \pi_H} \tag{3.14}$$

In equilibrium, the probability of a technological advancement is given by:

$$P(v_{it} = v_H) = x_I^* + (1 - d^*) x_C^* = \frac{\Delta(\sigma^2 \theta_C + \pi_H \pi_L)}{\theta_P^2 \pi_H \pi_L} - \frac{\sigma}{\theta_P} + \frac{\theta_C \Delta 2 \sigma^2}{\theta_P \pi_H \pi_L^2} \tag{3.15}$$

We can now establish the following result:

Proposition 4. *The probability of a technological advancement is increasing in σ if $\pi_H > 2$ and $\theta_C > 2$*

Proof. see Appendix A.3 □

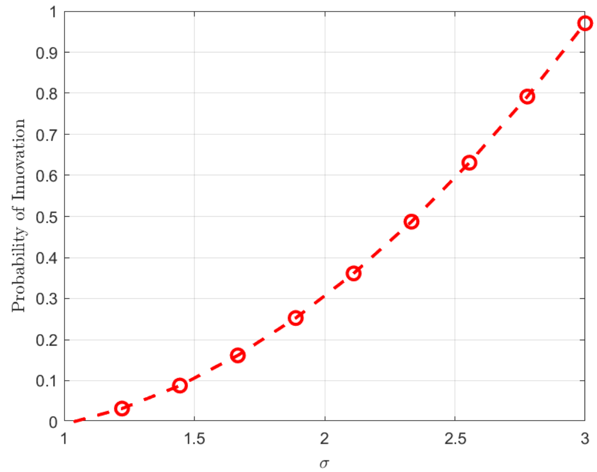

Figure 16: Probability of innovation over values of σ, Simultaneous Move Model

The above proposition tells us that, when additive separability is relaxed in the way we have proposed, to get the same result as in the simple case, the profits from technological advancement and cost of productive innovation of competitor need to be high enough. The result stems from the consideration that we made above about the strategic interaction across investments. The intuition is that when the cost of a defensive innovation increases, a reduction in defensive investment spurs higher productive research if the returns are sufficiently high and at the same time, the incumbent knows that being replaced by the competitor is costly. The inverse process we are observing in the empirical section laid out in Section 2. Indeed, the same considerations proposed when we analyzed Section 3.2.2 remain valid here, with the addition that certain conditions need to be met. Specifically, for the same result to hold we need profits from technology to be high and the cost for competitors to enter the market to be high enough. These features resemble the greater differential in profits between patenting and not patenting firms more clearly and the greater cost of producing innovation from competitors is reflected in the lower firms able to patent during the last of the sample we have considered.

Appendix A.3 introduces a further extension of the model here presented, which assumes a different timing in the competition, allowing the incumbent to have a First Mover Advantage. The intuition is that the incumbent from its privileged position and greater capabilities may possess an edge in the competition with rivals.

4 Conclusion

The present work has highlighted the importance of firms' economic outcomes along the dimension of patenting activity by linking it with empirical trends such as market power and productivity differential between leaders and laggards related to the rise in market concentration.
The evidence presented also pointed out the growing difficulty in entering the patenting group and the reduced spillovers observed measured using patent citations. Moreover, although patenting firms are on average more productive, patent values display a negative correlation with productivity once Research and Development expenditures are accounted for, thus suggesting a weak contribution of patents in determining actual TFP. All of this evidence displays of picture of an economy where productivity growth is slowing down and favorable economic performances are concentrated on firms that are able to patent.
The third section has developed a simple Schumpeterian model of firms' competition in innovation that is able to predict the trends underlined in the empirical section. The model is then extended to allow for greater strategic interaction between investments, therefore describing a more realistic environment, while a third version, placed in the appendix, changes the timing of the game, allowing the incumbent producer to enjoy a First Mover Advantage. The prediction beyond the technicalities of parameter restrictions remains the same: when the cost of investing in protective patenting diminishes the overall productivity of the economy decreases as a result of the reduced competition from laggard competitors.

The work calls for several potential extensions and improvements. On the empirical side, being able to recognize between different kinds of patents, in particular between internal and external patents, and how they relate to the relate with the economic measures we have shown may shed further light on the role of patents in shaping firms' performances and determining productivity, as well as determining their relationship with measures patents value we have employed.
Further work should also extend the theoretical framework by introducing a general equilibrium dimension and a more dynamic framework, where elements can be determined endogenously, as markups in Aghion, Bergeaud, et al. (2023), and it should eventually be brought to the data for more structural analysis. Such an exercise would give a quantitative effect of the channels we have

tried to highlight with our simple theoretical framework.

To conclude, the analysis has tried to point to a clear relationship between some sluggish economic trends of the U.S. economy and patenting activity, hopefully contributing to the debate on business dynamism. Future research will be needed to shed more light on the phenomena and mechanisms behind these aggregate trends.

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A Appendix

A.1 Data Description

Kogan et al. (2017) download the entire history of patents from Google patents and make them publicly available on their GitHub web page [3]. In their website they extend the original dataset used in their initial paper, to allow for data to run until 2022, starting in 2016. Specifically, they made available the patent number identification, the issue date of each patent, the filling date of the the patent application, the private value estimated, forward citations that each patent received, and the CRSP permno variable, which is a unique numeric code assigned to each stock traded in the U.S. equity markets. They estimate private value of patents from the stock market return of the firm when the patent is issued assuming that the return is composed by the value of the patent and an additional unrelated component that follow a given distribution. The final private value of a patent is the estimated using the expected value of the patent and function of a firm's market capitalization. On the other hand, what is defined public or scientific value of a patent is obtained as a weighted sum of forward citation of a firm in given year. Let P be the public Value, j a patent belonging the set of all the patents issued to a firm in a given year J_t , C_j forward citation of patent j and $\bar{C}_t$, the public value of firm i in year t is obtained then :

$$P_{it} = \sum_{j}^{J_i}(1 + \frac{C_j}{\bar{C}_t}) \tag{A.1}$$

To match the patent dataset I merge it using the permno and year such that for each firm in each year I know how many patents were issued and the overall value these patents. The total number of firms in the sample from 1956 to 2016 to which at least one patent in a year is 40,689 , while the overall observations in the sample contains 226,552 observations. The Compustat dataset have been initially eliminating observations with negative sales(SALE), overhead costs(XSGA) and total costs directly allocated to production (COGS) and the first and last percentile of the variable sales over COGS as in De Loecker, Eeckhout, and Unger (2020) to rule out outliers. All the variables used in Compustat have been deflated using industry-level input price (PIRIC from FRED).

[3] available here https://github.com/KPSS2017/Technological-Innovation-Resource-Allocation-and-Growth-Extended-Data?tab=readme-ov-file

Total Factor Productivity was estimated following production function estimation using Levinson-Petrin method. A key assumption is that intermediate input is a proxy variable for productivity. Moreover it assumes that state variables used in the production function evolve according to a policy function whereas free variables are static as their current choice do not impact future and are decided after productivity shock is realized. For intermediate inputs I have used the variable COGS, for state variable, which is understood as capital, PPEGT (the gross value of Property, Plant, and Equipment) and EMP, the number of employees as a free variable. For robustness I also estimate TFP from production function estimation using Olley-Pakes method(from Olley and Pakes (1992))

Markups were estimated following De Loecker, Eeckhout, and Unger (2020), which employs the production function approach, which derives first order conditions of firm's cost minimization. Markups in their model are given by:

$$\mu_{it} = \theta_{it}^{V} \frac{P_{it}^{V} Q_{it}}{P_{it}^{V} V_{it}} \tag{A.2}$$

The first term in the equation represents the output elasticity of inputs while the nominator in the fraction is the total value of inputs (here COGS) and the denominator the total value of sales (here SALE). In Fig. 9 we plot the benchmark markups used in De Loecker, Eeckhout, and Unger (2020) which derive output elasticity estimating the production function from the method proposed by Ackerberg, Caves, and Frazer (2015).In Appendix A.2average markups for patenting and not patenting firms where the output elasticity is derived from cost shares as indicated in the figures, as in De Loecker, Eeckhout, and Unger (2020).

Capital expenses were computed by multiplying the variable USERCOST, referring to cost associated with capital assets to the variable PPEGT.

A.2 Robustness

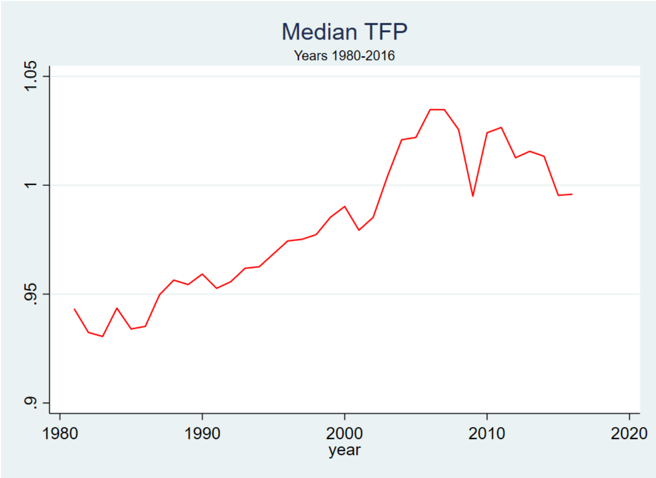

Figure 17: Median of TFP

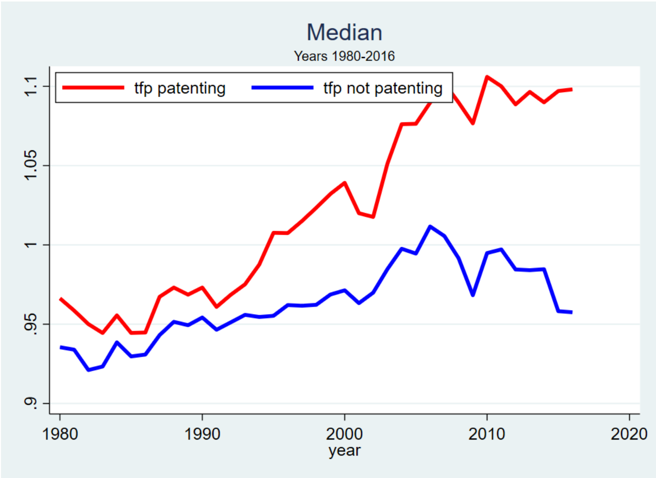

Figure 18: Median of TFP patenting and not patenting firms

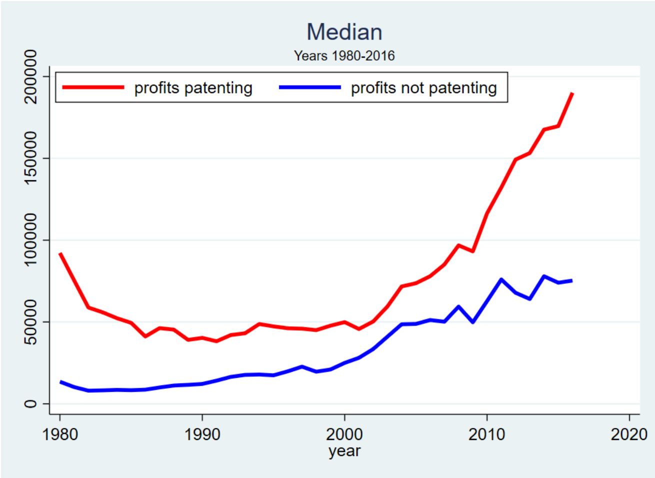

Figure 19: Median Profits patenting vs not patenting firms

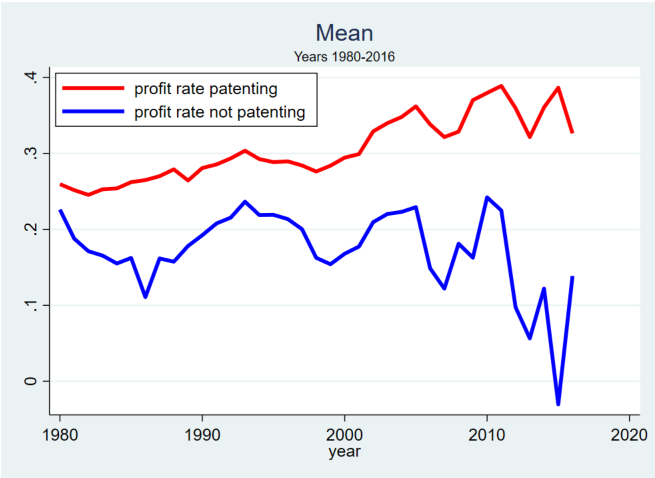

Figure 20: Average Profit Rate of Patenting and not Patenting Firms

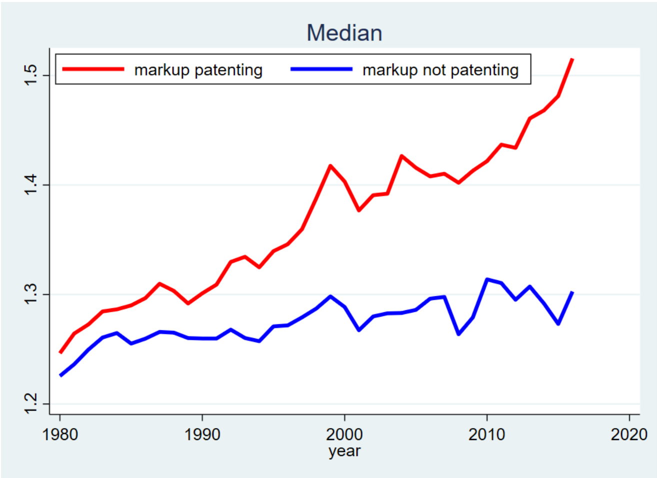

Figure 21: Median Markups of patenting vs not patenting firms

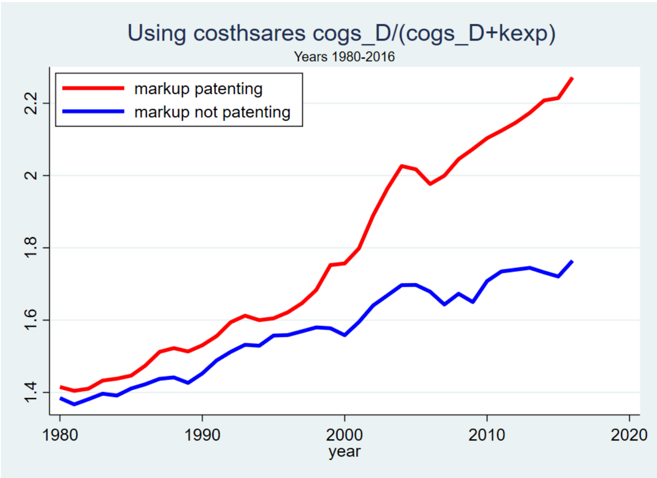

Figure 22: Average Markups of patenting vs not patenting firms using cost shares as output elasticity

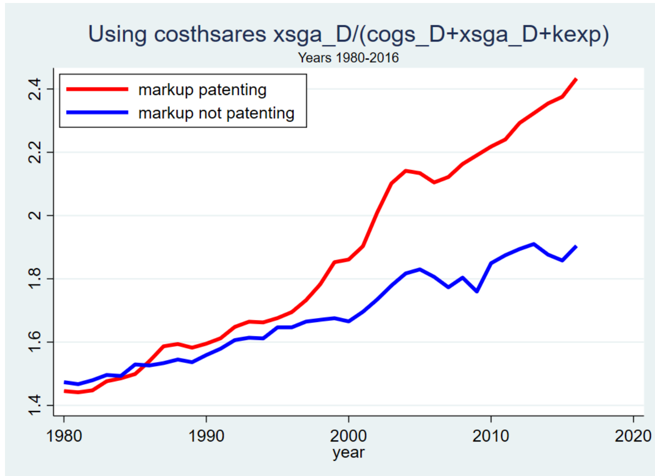

Figure 23: Average Markups of patenting vs not patenting firms using cost shares as output elasticity , where XSGA represents Selling, General, and Administrative Expenses.

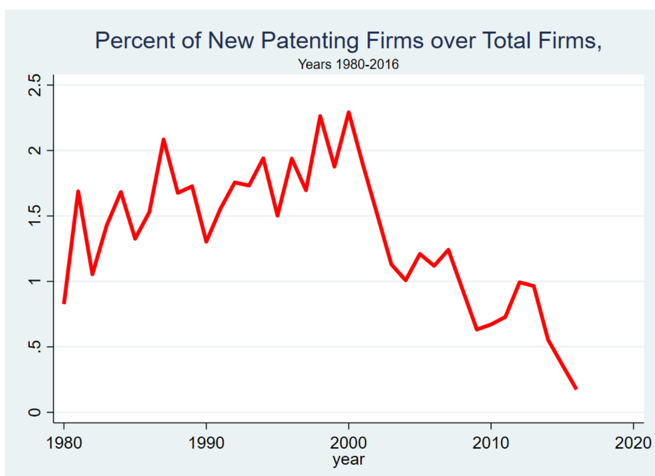

Figure 24

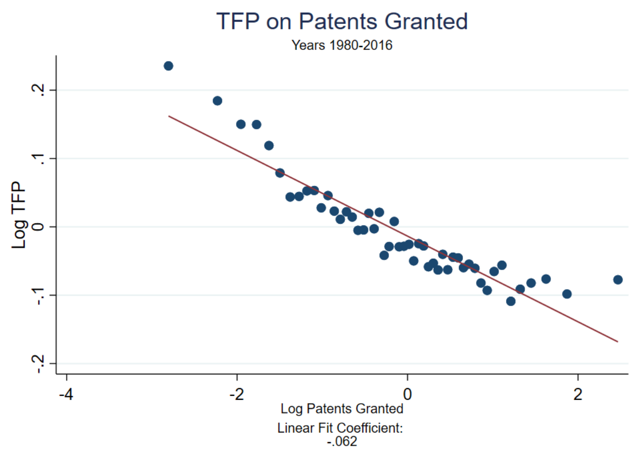

Figure 25: TFP and Patents Granted when R&D is controlled for

Below are presented the regressions related with the last part in Appendix A.1. It shows how the positive relations between patents and productivity fades away once R&D is controlled for. Fig. 14 Fig. 13 Fig. 25 relates with the second column of the respective tables. The third columns add time and sectors fixed effects and the fourth column adds further controls. The last 3 tables repeat the same exercises with TFP estimated using Olley-Pakes method. All the regressions have standard errors clustered at the firm level.

	(1)	(2)	(3)	(4)
	TFP	TFP	TFP(FE)	TFP
Private Value of Patent	0.029***	-0.018***	-0.009*	-0.009**
	(0.002)	(0.003)	(0.003)	(0.004)
R&D		0.076***	0.054***	0.038***
		(0.005)	(0.005)	(0.005)
Sales				-0.131***
				(0.011)
OverHead Costs				0.172***
				(0.013)
Observations	40141	34702	34702	34702

Standard errors in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

	(1)	(2)	(3)	(4)
	TFP	TFP	TFP(FE)	TFP
social value patent	0.029***	-0.032***	-0.016***	-0.028***
	(0.004)	(0.004)	(0.004)	(0.004)
R&D		0.074***	0.053***	0.044***
		(0.004)	(0.004)	(0.005)
Sales				-0.131***
				(0.011)
OverHead Costs				0.171***
				(0.013)
Observations	40141	34702	34702	34702

Standard errors in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

	(1)	(2)	(3)	(4)
	TFP	TFP	TFP(FE)	TFP
Patents Granted	0.021***	-0.061***	-0.042***	-0.051***
	(0.004)	(0.004)	(0.004)	(0.004)
R&D		0.092***	0.069***	0.054***
		(0.004)	(0.004)	(0.005)
Sales				-0.123***
				(0.011)
OverHead Costs				0.169***
				(0.013)
Observations	40141	34702	34702	34702

Standard errors in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

	(1)	(2)	(3)	(4)
	TFP	TFP	TFP(FE)	TFP
Private Value of Patent	0.029***	-0.018***	-0.009*	-0.009**
	(0.002)	(0.003)	(0.003)	(0.004)
R&D		0.076***	0.054***	0.038***
		(0.005)	(0.005)	(0.005)
Sales				-0.131***
				(0.011)
OverHead Costs				0.172***
				(0.013)
Observations	40141	34702	34702	34702

Standard errors in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

	(1)	(2)	(3)	(4)
	TFP	TFP	TFP(FE)	TFP
social value patent	0.029***	-0.032***	-0.016***	-0.028***
	(0.004)	(0.004)	(0.004)	(0.004)
R&D		0.074***	0.053***	0.044***
		(0.004)	(0.004)	(0.005)
Sales				-0.131***
				(0.011)
OverHead Costs				0.171***
				(0.013)
Observations	40141	34702	34702	34702

Standard errors in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

	(1)	(2)	(3)	(4)
	TFP	TFP	TFP(FE)	TFP
Patents Granted	0.021***	-0.061***	-0.042***	-0.051***
	(0.004)	(0.004)	(0.004)	(0.004)
R&D		0.092***	0.069***	0.054***
		(0.004)	(0.004)	(0.005)
Sales				-0.123***
				(0.011)
OverHead Costs				0.169***
				(0.013)
Observations	40141	34702	34702	34702

Standard errors in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

A.3 Proofs and Sequential Move Model Extension

A.3.1 Simultaneous Move

Proposition. *The optimal investments are given by:*

$$x_P^* = \frac{\pi_H - \pi_l}{\theta_P}; \quad d^* = \frac{\pi_H \pi_L}{\pi_H \pi_L + \theta_C \sigma}; \quad x_C* = \frac{\pi_H \sigma}{\pi_H \pi_L + \theta_C \sigma}$$

Proof. x_P^* immediately follows from the F.O.C. of Eq. (3.9). From the F.O.C. of with respect to d, one gets:

$$d = \frac{x_C \pi_L}{\sigma}$$

while from F.O.C. of Eq. (3.8) one gets:

$$x_C = \frac{(1-d)\pi_H}{\theta_C}$$

Solving the system of two equation in two unknowns leads to the optimal quantities. □

Proposition. *The probability of technological advancement in a given sector is increasing in σ*

Proof.

$$\frac{\delta P}{\delta \sigma} = \frac{2\pi_H \theta_C \sigma(\sigma\theta_C + \pi_H \pi_L) - 2\pi_H \theta_C^2 \sigma^2}{(\sigma\theta_C + p_H \pi_L)^3} = \frac{2\pi_H^2 \pi_L \theta_C \sigma}{(\sigma\theta_C + \pi_H \pi_L)^3} > 0$$

□

Proposition. *The optimal investments when additive separability is relaxed are given by :*

$$x_P^* = \frac{\Delta(\sigma^2\theta_C + \pi_H \pi_L)}{\theta_P^2 \pi_H \pi_L} - \frac{\sigma}{\theta_P}; \quad d^* = 1 - \frac{\theta_C \sigma \Delta}{\theta_P \pi_H \pi_L}; \quad x_C* = \frac{\sigma\Delta}{\theta_P \pi_L}$$

Proof. From the F.O.C. of Eq. (3.12)

$$d = \frac{x_C \pi_L - \sigma\theta_P X_P}{\sigma^2} \quad x_P = \frac{\Delta - \sigma\theta_P d}{\theta_P^2}$$

while the F.O.C. of the competitor remain invariant. Substituting x_C into d one gets:

$$d = \frac{\pi_H \pi_L - \theta_P \theta_C \sigma x_P}{\sigma^2 \theta_C + \pi_H \pi_L}$$

Then d into x_P to get x_P^*. d^* follows from inserting x_P^* into d and x_C^* from entering d^* into x_C.

□

Proposition. *The probability of a technological advancement is increasing in σ if $\pi_H > 2$ and $\theta_C > 2$*

Proof.

$$\frac{\delta P}{\delta \sigma} = \frac{2\sigma\Delta\theta_C\pi_L - \theta_P\pi_H\pi_L^2 + 2\theta_C\sigma\Delta^2}{\theta_P^2\pi_H\pi_L^2} > 0 \to 2\sigma\Delta\theta_C(\pi_L + \Delta) > \theta_P\pi_H\pi_L^2 \to \frac{\pi_L\theta_P}{\Delta\sigma} < \frac{\theta_C}{2}$$

From Eq. (3.14) it follows that the probability of technological advancement is bounded below 1. It entails that:

$$x_c < 1 \to \frac{\Delta}{\pi_L} < \frac{\theta_P}{\sigma} \to \frac{\theta_P\pi_L}{\sigma\Delta} > 1$$

from which one deduces that: $\frac{\delta P}{\delta \sigma} > 0$ if $\theta_C > 2$. Likewise:

$$(1-d) < 1 \to \frac{\Delta}{\pi_L} < \frac{\theta_P}{\sigma} \to \pi_H > \frac{\Delta\theta_C\sigma}{\pi_L\theta_P}$$

From $\frac{\delta P}{\delta \sigma} > 0$ one can rewrite:

$$\frac{\Delta\theta_C\sigma}{\pi_L\theta_P} > 2$$

from which follows :

$\frac{\delta P}{\delta \sigma} > 0$ if π_H .

□

A.3.2 Sequential Move

In this last part we are going to present a further modification of the models presented in Section 3.2.2. The only difference is the timing of the investment. We are going to assume that the incumbent has a First Mover Advantage with respect to the competitor and we will eventually analyze the probability of a technological advancement in a sector and compare it with the two results already obtained in Section 3. We are going to focus on the more interesting case where additive separability is relaxed in the cost structure of the investment of the incumbent. The First Mover Advantage will let the incumbent observe the investment made by the competitor and assimilate it into her objective function. As before, the competitor take d as given and solves :

$$\max_{x_C} \quad \{(1-d)x_C\pi_H - \frac{\theta_C x_C^2}{2}\}Y_t \tag{A.3}$$

and from the first-order conditions one gest that:

$$x_C = \frac{\pi_H(1-d)}{\theta_C} \tag{A.4}$$

The incumbent now is able to observe and therefore use that information to optimally chose her investments by solving:

$$\max_{x_P,d} \quad \{x_I\pi_H + (1 - x_P - (1-d)^2\frac{\pi_H}{\theta_C})\pi_L - \frac{\theta_P x_P^2 + \sigma d^2}{2}\}Y_t \tag{A.5}$$

Proposition 5.

$$x_P^* = \frac{\Delta(\sigma^2\theta_C + 2\pi_H\pi_L)}{\theta_P^2\pi_H\pi_L} - \frac{\sigma}{\theta_P}; \quad d^* = 1 - \frac{\theta_C\sigma\Delta}{2\theta_P\pi_H\pi_L}; \quad x_C* = \frac{\sigma\Delta}{2\theta_P\pi_L} \tag{A.6}$$

Proof. The F.O.C. of the competitor remains unchanged in this instance as well. From the problem in Eq. (A.6) one derives:

$$d = \frac{2\pi_H\pi_L - \theta_P\theta_C\sigma x_P}{\sigma^2\theta_C + 2\pi_H\pi_L} \quad x_P = \frac{\Delta - \sigma\theta_P d}{\theta_P^2}$$

The solution of the system of three equations in three unknowns gives the optimal quantities. □

Optimal investments are very similar as in Section 3.2.3, and specifically, optimal investment in productive innovation remains invariant for the incumbent.

Assumption 3.

$$\frac{\Delta}{2\pi_H\pi_L + \Delta\theta_C} < \frac{\sigma\theta_P}{2\pi_H\pi_L + \sigma^2\theta_C + \sigma\theta_P\pi_H} \tag{A.7}$$

The probability of a technological advancement is given by:

$$P(\nu_{it} = \nu_H) = x_I^* + (1-d^*)x_C^* = \frac{\Delta(\sigma^2\theta_C + 2\pi_H\pi_L)}{\theta_P^2\pi_H\pi_L} - \frac{\sigma}{\theta_P} + \frac{\theta_C\Delta^2\sigma^2}{2\theta_P^2\pi_H\pi_L^2} \tag{A.8}$$

Proposition 6. *The probability of technological advancement in a given sector is increasing in σ*

Proof.

$$\frac{\delta P}{\delta\sigma} = \frac{-2\theta_P\pi_H\pi_L^2 + 2\sigma\Delta\theta_C\pi_L + \sigma\Delta^2\theta_C}{2\theta_P^2\pi_H\pi_L^2} > 0 \rightarrow 2\sigma\Delta\theta_C\pi_L + \sigma\Delta^2\theta_C < 2\theta_P\pi_H\pi_L^2 \rightarrow (\frac{\pi_H}{\pi_L})^2 > \frac{2\theta_P\pi_H}{\sigma\theta_C} + 1$$

From the fact that $(1-d) < 1$, ensured by Eq. (A.7):

$$\frac{2\theta_P\pi_H}{\sigma\theta_C} > \frac{\pi_H}{\pi_L} - 1$$

which entails that:

$$\frac{\delta P}{\delta \sigma} > 0 \rightarrow (\frac{\pi_H}{\pi_L})^2 > \frac{\pi_H}{\pi_L} \rightarrow \pi_H > \pi_L$$

which is a condition met by assumption. □

Therefore, if the Incumbent has a first Mover Advantage, when the cost of defensive patenting decreases, the probability of a technological advancement declines, as it was the case for the simple case. The result derives by the incumbent being able to observe the competitor's best reply and assimilate it into its objective function and optimally investing accordingly. The result restores the findings of the initial model where the probability of an innovation was decreasing in σ without any need for parameter restrictions. The intuition is that the advantage of the incumbent makes her able to tailor the investment in a more strategic away, therefore leading to a more straightforward mechanism as we have seen in the first model.

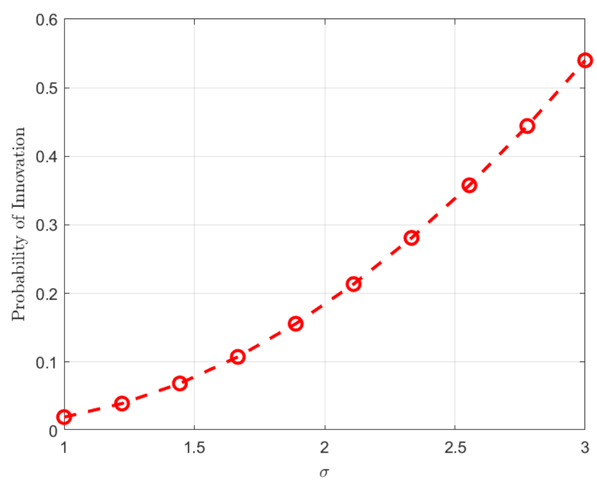

Figure 26: Probability of innovation over values of σ, Sequential Move Model