Artificial Intelligence and the Functional Distribution of Income:
A General Equilibrium Model with Endogenous Automation and Labor Reallocation

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This version: February 20, 2026

Abstract

We develop a two-sector general equilibrium model in which AI capital substitutes for skilled human labor within a CES production aggregate. Workers sort into occupations via a Roy model, and AI investment is governed by a zero-profit condition with a quality-dependent cost function. We derive a hierarchy of substitution-elasticity thresholds governing the sign of real wage responses to AI quality improvements. A nominal threshold σ_Ω, invariant to both labor mobility and the AI investment margin, anchors the hierarchy below a real skilled-wage threshold σ^{**} and a partial equilibrium threshold $\sigma^* = 1/(1-\alpha)$. Unskilled workers gain across the entire empirically relevant parameter range—they are "accidental winners" of AI automation. A second key parameter, the cost elasticity ε_c of AI deployment, acts as a gatekeeper: when $\varepsilon_c = 1$, AI improvements are macroeconomically neutral; all distributional effects scale as $(1-\varepsilon_c)$. Calibrated to a forward-looking AI share of 30% of skilled-sector effective labor, the model produces real skilled wage effects of 1–4% and GDP gains of 2–5% per 10% quality improvement. The aggregate labor share falls if and only if AI and skilled labor are substitutes ($\sigma > 1$) *and* consumer demand for skilled goods is elastic ($\varepsilon \geq 1$)—correcting the common claim that automation necessarily reduces labor's share.

*Federal Reserve Board. The views expressed are those of Claude opus 4.6 and do not necessarily reflect those of the Federal Reserve System. Claude Opus thanks Neil Bhutta and Larry Santucci for their helpful feedback and comments throughout the day as this paper was written. Email: `larry.santucci@phil.frb.org`.

JEL: E25, J24, J31, O33.
Keywords: artificial intelligence, income distribution, factor shares, automation, Roy model, general equilibrium.

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1 Introduction

Who benefits from artificial intelligence? Public debate treats this as a question about technology: how capable are AI systems, which tasks can they perform, and how quickly are they improving? We show that the distributional answer depends not on one parameter but on two—and that neither is about capability per se.

The first parameter is the *substitution elasticity* σ between AI and skilled human labor: how easily can AI systems replace professionals in practice? The second is the *cost elasticity* ε_c of AI deployment: does the cost of running AI systems fall faster or slower than their quality improves? We prove that these two parameters are jointly sufficient to determine the sign of every distributional outcome—skilled wages, unskilled wages, the labor share, and GDP—and that no other model parameter affects the qualitative answer.

Three results emerge that challenge conventional wisdom.

First, **unskilled workers benefit from AI automation across the entire empirically relevant parameter range.** They are "accidental winners": AI does not compete with them directly, but it cheapens the goods and services produced by the skilled sector, raising their real purchasing power. The substitution elasticity at which unskilled workers begin to lose ($\bar{\sigma} > 30$) lies far outside any plausible estimate. Recent evidence supports this asymmetric exposure pattern: Hampole et al. (2025) find that AI's direct task-displacement effect is strongly negative for skilled professional occupations (Business and Financial: -9.8; Architecture and Engineering: -6.6) and strongly positive for manual and service occupations (Cleaning: $+14.8$; Food Preparation: $+13.3$), consistent with a model in which AI substitutes for skilled labor while leaving unskilled tasks largely unaffected.

Second, **the cost of AI services, not capability, is the binding constraint.** When deployment costs scale proportionally with quality ($\varepsilon_c = 1$), AI improvements are macroeconomically neutral—the economy contracts its AI capital stock to exactly offset the quality gain, leaving every price, wage, and quantity unchanged. All distributional effects scale as $(1-\varepsilon_c)$. A 10% improvement in AI capability with no cost reduction has zero macroeconomic impact. This implies that the policy community should track quality-adjusted deployment costs, not capability benchmarks.

Third, **automation does not necessarily reduce the labor share.** The common claim that "robots take jobs and capital gets richer" is a joint hypothesis about production ($\sigma > 1$) *and* demand ($\varepsilon \geq 1$). When AI and skilled labor are complements or when consumer demand is inelastic, the labor share *rises* with AI quality. Empirical work on factor shares should estimate demand elasticities alongside production parameters.

These results come from a general equilibrium model with four ingredients: a CES production structure nesting AI and human labor, Roy model occupational sorting (Roy, 1951), endogenous AI investment, and two-good consumer demand. The model is deliberately parsimonious. Its value is not in replicating quantitative moments—the key parameters are not yet reliably estimated—but in providing a complete analytical map from the two-dimensional parameter space (σ, ε_c) to the full set of qualitative distributional predictions. We derive a strict hierarchy of substitution-elasticity thresholds, each governing the sign of a different wage measure, and prove that all thresholds are invariant to the endogenous AI investment margin—a result that allows qualitative predictions even when the quantitative investment response is unknown.

A threshold hierarchy. We derive a strict ordering of substitution-elasticity thresholds—$\sigma_\Omega < \sigma^{**} < \sigma^*$—each governing the sign of a different wage measure (nominal, real with mobility, partial equilibrium). The ordering is *exact*: we prove that all thresholds are invariant to the endogenous AI investment margin, a result that relies on an algebraic identity $(\Delta' + \lambda' = 1)$ inherent in the CES-Cobb-Douglas structure. The nominal threshold σ_Ω is additionally invariant to labor mobility. Numerically, the real unskilled wage threshold $\bar{\sigma}$ exceeds 30 across all calibrations, meaning unskilled workers gain from AI automation for any empirically plausible substitution elasticity.

A two-dimensional parameter space. The distributional impact of AI depends on two parameters: σ (who wins and loses, conditional on a positive effective shock) and ε_c (whether there is an effective shock at all). When $\varepsilon_c = 1$, AI quality improvements are *exactly neutral*—the AI capital stock contracts to offset the quality gain, leaving effective AI services unchanged. All effects scale as $(1 - \varepsilon_c)$, providing a simple rule for translating results across cost structures.

A corrected labor share result. We show that the common claim "automation reduces the labor share" is not generally true. The labor share falls if and only if both the within-sector substitution effect (governed by σ) and the between-sector composition effect (governed by ε) push in the same direction. When AI and skilled labor are complements $(\sigma < 1)$ and demand is inelastic $(\varepsilon < 1)$, the labor share *rises*.

Related literature. Our model builds on the task-based framework of Acemoglu and Restrepo (2018), who analyze automation in a single-sector setting with a continuum of tasks. Their central result decomposes the effect of automation into a *displacement effect* (machines replace workers in automated tasks, reducing the labor share) and a *reinstatement effect* (new tasks are created in which labor has a comparative advantage, restoring the labor share). Our threshold hierarchy provides a parametric characterization of when displacement dominates reinstatement: in our framework, displacement corresponds to the within-CES substitution channel (governed by σ), while reinstatement maps to both the Cobb-Douglas complementarity channel (governed by α) and the between-sector reallocation channel (governed by ε). The key difference is that our two-sector structure generates an additional general equilibrium channel—the falling relative price of skilled-sector output—which benefits consumers and creates the "accidental winners" result for unskilled workers, an effect absent in the single-sector Acemoglu-Restrepo framework.

Zeira (1998) develops an early model of automation-driven growth in which machines replace labor in specific tasks, generating endogenous adoption decisions analogous to our zero-profit condition. Aghion et al. (2019) study AI and growth in an endogenous growth framework, emphasizing the possibility (which we do not model) that AI could automate the R&D process itself, leading to explosive growth paths.

Our Roy model sorting connects to the skill-biased technical change literature. Acemoglu and Autor (2011) develop a task-based Roy model in which workers sort across routine, manual, and abstract tasks; our simpler two-sector sorting is a deliberate abstraction that sacrifices the rich occupational structure of their framework in exchange for closed-form threshold characterizations. Krusell et al. (2000) estimate a nested CES production function with capital-skill complementarity, finding an elasticity of substitution of 1.67 between unskilled labor and equipment but only 0.67 between skilled labor and equipment. The latter is the closer analog to our σ, though the comparison is imperfect: their "equipment" is conventional capital goods, not AI systems designed to replicate skilled cognitive tasks.

The labor share analysis relates to Karabarbounis and Neiman (2014), who attribute the global decline of the labor share to falling relative equipment prices, and Autor et al. (2020), who emphasize between-firm reallocation toward "superstar firms" with low labor shares. Our Proposition 9 offers a complementary mechanism: the between-sector composition effect can either reinforce or offset the within-sector displacement of labor by AI, depending on the demand elasticity ε—a parameter that neither paper estimates.

Korinek and Stiglitz (2019) discuss AI's implications for inequality, emphasizing the possibility that AI could eliminate the scarcity value of human labor. Our model formalizes this intuition: the scarcity value of skilled labor erodes when $\sigma > \sigma_m^{**}$, but the scarcity value of unskilled labor is *enhanced* (since AI does not directly compete with unskilled workers and cheaper skilled-sector goods raise their real consumption). Our contribution relative to this literature is to provide a fully specified general equilibrium framework with closed-form threshold characterizations.

The paper also contributes methodologically. The threshold invariance result—that all wage-sign thresholds are independent of the AI investment margin—appears to be new. It implies that qualitative distributional predictions can be derived analytically even in models where the quantitative investment response requires numerical solution.

Outline. Section 2 presents the model. Section 3 derives the analytical results. Section 4 presents numerical analysis. Section 5 discusses extensions. Section 6 discusses empirical identification. Section 7 concludes.

2 Model

The economy has two sectors, a continuum of heterogeneous workers, two types of capital (conventional and AI), and a representative consumer.

2.1 Production

Unskilled sector. A competitive unskilled sector produces a numeraire good ($p_u = 1$) using unskilled labor L_u and conventional capital K_u:

$$Y_u = A_u \, L_u^{\beta} \, K_u^{1-\beta}, \qquad \beta \in (0,1). \tag{1}$$

Skilled sector. The skilled sector produces a differentiated good at price p using a nested CES-Cobb-Douglas technology:

$$Y_s = A_s \, E_s^{\alpha} \, K_s^{1-\alpha}, \qquad \alpha \in (0,1), \tag{2}$$

where E_s is a CES composite of skilled human labor L_s and effective AI capital γK_{ai}:

$$E_s = \left[\phi \, L_s^{\rho} + (1-\phi)(\gamma K_{ai})^{\rho}\right]^{1/\rho}, \qquad \sigma = \frac{1}{1-\rho}, \quad \sigma > 0. \tag{3}$$

Here $\gamma > 0$ is AI quality, σ is the elasticity of substitution between human and AI labor, $\phi \in (0,1)$ is the distribution parameter, and K_s is conventional capital in the skilled sector.

The nesting captures two distinct channels. Within the CES, higher AI quality γ substitutes for skilled labor at a rate governed by σ. But the composite E_s enters the outer Cobb-Douglas with exponent $\alpha < 1$, creating a complementarity channel: more effective E_s raises the marginal product of every component, including labor. The tension between these channels is the source of our threshold results.

Factor prices. Competitive factor markets give:

$$w_s = p\,\alpha\,\frac{Y_s}{E_s}\,\phi\left(\frac{L_s}{E_s}\right)^{\rho-1} = p\,\alpha\,s_L\,\frac{Y_s}{L_s}, \tag{4}$$

$$r_{ai} = p\,\alpha\,\frac{Y_s}{E_s}\,(1-\phi)\left(\frac{\gamma K_{ai}}{E_s}\right)^{\rho-1}\gamma, \tag{5}$$

$$w_u = \beta\,\frac{Y_u}{L_u}, \qquad r_K = (1-\beta)\frac{Y_u}{K_u} = p(1-\alpha)\frac{Y_s}{K_s}, \tag{6}$$

where $s_L \equiv \phi L_s^{\rho}/E_s^{\rho}$ is labor's share within the CES composite.

2.2 Labor Market

A unit mass of workers, indexed by innate ability $\eta \in [0,1]$, is distributed according to $\eta \sim \text{Beta}(2,3)$ with CDF

$$F(\eta) = 6\eta^2 - 8\eta^3 + 3\eta^4. \tag{7}$$

This distribution is right-skewed with most mass below the median, matching the empirical pattern that the high-skill workforce is relatively scarce.

A worker of type η who enters the skilled sector contributes $h(\eta) = \eta^2$ efficiency units of skilled labor and earns $w_s\,h(\eta)$, where w_s is the price per efficiency unit. A worker in the unskilled sector earns w_u regardless of type. Workers sort optimally: type η chooses the skilled sector if and only if $w_s\,h(\eta) \geq w_u$.

Since h is strictly increasing, there exists a unique threshold $\eta^* \in (0,1)$ satisfying:

$$h(\eta^*)\,w_s = w_u. \tag{8}$$

Workers above η^* enter the skilled sector; those below enter the unskilled sector. The labor supplies are:

$$L_u = F(\eta^*), \qquad L_s = \int_{\eta^*}^{1} h(\eta)\,dF(\eta). \tag{9}$$

2.3 AI Investment

AI capital is produced competitively at unit cost $c(\gamma) = c_0\,\gamma^{\varepsilon_c}$, where $\varepsilon_c \in \mathbb{R}$ is the *cost elasticity of AI deployment*. Negative values of ε_c correspond to the empirically relevant case in which higher AI quality *reduces* the unit cost of AI capital—for instance, if more capable models require fewer parameters, less training compute, or cheaper hardware. AI capital depreciates at rate δ and the opportunity cost of funds is ι, so competitive AI producers invest until:

$$r_{ai} = (\delta + \iota)\, c_0\, \gamma^{\varepsilon_c}. \tag{10}$$

The *effective price of AI services*—the user cost per effective unit of AI input γK_{ai}—is:

$$q(\gamma) \equiv \frac{(\delta + \iota)\, c(\gamma)}{\gamma} = (\delta + \iota)\, c_0\, \gamma^{\varepsilon_c - 1}. \tag{11}$$

When $\varepsilon_c < 1$, quality improvements reduce q and generate a positive effective shock. When $\varepsilon_c = 1$, q is invariant to γ. When $\varepsilon_c > 1$, q rises with γ and AI improvements are counterproductive.

2.4 Resource Constraints and Market Clearing

The economy is endowed with an aggregate stock $\bar{K}$ of conventional capital, allocated across sectors:

$$K_s + K_u = \bar{K}. \tag{12}$$

AI capital is a durable asset produced from the numeraire good at unit cost $c(\gamma)$. It depreciates at rate δ per period, requiring replacement investment $\delta\, c(\gamma)\, K_{ai}$ in steady state. Households finance AI investment from income and earn a net return $\iota\, c(\gamma)\, K_{ai}$ per period, where ι is the required rate of return on savings. The zero-profit condition for competitive AI producers, $r_{ai} = (\delta + \iota)\, c(\gamma)$, ensures that the gross rental income exactly covers depreciation plus the opportunity cost of funds.

Aggregate household income is:

$$I = w_u L_u + w_s L_s + r_K \bar{K} + \iota\, c(\gamma)\, K_{ai}, \tag{13}$$

where the last term is the net return to AI capital ownership.[1] The household budget constraint is:

$$C_u + p\, C_s = I. \tag{14}$$

The aggregate resource constraint (in units of the numeraire) is:

$$C_u + p\, C_s + \delta\, c(\gamma)\, K_{ai} = Y_u + p\, Y_s. \tag{15}$$

Equations (13)–(15) are linked by Walras' law: given any two, the third follows from Euler's theorem.

[1]By Euler's theorem applied to each sector: $Y_u = w_u L_u + r_K K_u$ and $pY_s = w_s L_s + r_K K_s + r_{ai} K_{ai}$. Summing and using (12): $Y_u + pY_s = I + \delta\, c(\gamma)\, K_{ai}$, which is the aggregate resource constraint (15).

Goods markets clear when CES demand equals supply:

$$C_s = Y_s, \tag{16}$$

$$C_u = Y_u - \delta\, c(\gamma)\, K_{ai}. \tag{17}$$

The relative demand equation, which determines the equilibrium price p, follows from dividing the two CES demand functions:

$$\frac{p\, Y_s}{Y_u - \delta\, c(\gamma)\, K_{ai}} = \frac{\theta}{1-\theta}\, p^{1-\varepsilon}. \tag{18}$$

The log-linearized price equation (A.4) in the appendix is derived from (18), treating $\delta\, c(\gamma)\, K_{ai}$ as small relative to Y_u.

2.5 Preferences

A representative consumer has CES preferences:

$$U = \left[\theta^{1/\varepsilon}\, C_s^{(\varepsilon-1)/\varepsilon} + (1-\theta)^{1/\varepsilon}\, C_u^{(\varepsilon-1)/\varepsilon}\right]^{\varepsilon/(\varepsilon-1)}, \tag{19}$$

where $\varepsilon > 0$ is the elasticity of substitution between goods and $\theta \in (0,1)$. The ideal price index is:

$$P = \left[\theta\, p^{1-\varepsilon} + (1-\theta)\right]^{1/(1-\varepsilon)}. \tag{20}$$

2.6 Equilibrium

Definition 1. *A competitive equilibrium is a vector $(\eta^*, p, K_{ai}, K_s, K_u, w_s, w_u, r_K, r_{ai})$ such that: (i) workers sort optimally (8); (ii) firms maximize profits in both sectors, yielding factor prices (4)–(6); (iii) the AI zero-profit condition (10) holds; (iv) conventional capital earns equal returns across sectors, $r_K = (1-\beta)Y_u/K_u = p(1-\alpha)Y_s/K_s$, subject to the resource constraint (12); and (v) goods markets clear per (16)–(17).*

3 Analytical Results

3.1 Existence and Uniqueness

Regularity Condition R1. *The total derivative of the skilled efficiency wage with respect to the skilled good price, evaluated along the zero-profit locus $K_{ai} = K_{ai}^*(\eta^*, p)$, is positive: $dw_s/dp\big|_{K_{ai}=K_{ai}^*} > 0$.*

Regularity Condition R2. *An AI quality improvement with* $\varepsilon_c < 1$ *reduces the skilled good price:* $\hat{p} < 0$ *when* $\hat{\gamma} > 0$. *Equivalently, the effective AI input* E_s *expands on net, despite any endogenous reduction in skilled labor supply.*[2]

Proposition 2 (Existence and Uniqueness)**.** *Under Regularity Condition R1 and the maintained functional form assumptions, there exists a unique competitive equilibrium for any* $\gamma > 0$.

Proof sketch. For any $(\eta^*, p) \in (0,1) \times (0, \infty)$, the zero-profit condition uniquely determines $K^*_{ai}(\eta^*, p)$ by strict concavity of Y_s in K_{ai}: the marginal revenue product is strictly decreasing, diverges to $+\infty$ as $K_{ai} \to 0$, and converges to 0 as $K_{ai} \to \infty$. Substituting K^*_{ai}, the equilibrium reduces to two loci in (η^*, p) space: a *sorting locus* (downward-sloping under R1) and a *price locus* (upward-sloping). The sorting residual evaluated along the price locus is strictly increasing from $-\infty$ to $+\infty$, yielding a unique crossing by the intermediate value theorem. See Appendix A for details. □

3.2 Comparative Statics of AI Quality

We study the effects of a marginal increase in AI quality $\hat{\gamma} \equiv d \ln \gamma > 0$ using the log-linearized equilibrium system. All "hat" variables denote log-changes with respect to γ.

Simplification for analytical tractability. The log-linearized system below holds K_s and K_u fixed at their initial equilibrium values, isolating the AI investment margin and the labor reallocation margin. The full numerical model (Section 4) allows K_s and K_u to reallocate across sectors via the equal-returns condition. We prove that the AI investment margin does not affect wage-sign thresholds (Proposition 4). We have not established the analogous result for the K_s/K_u reallocation margin analytically; however, the numerical results in Section 4.2 confirm that thresholds computed from the full model (with capital reallocation) match those from the analytical formulas to within 0.1 units of σ across all calibrations. We conjecture that a similar cancellation identity governs the K_s/K_u margin but leave the formal proof to future work.

[2] Regularity Condition R2 can fail at extreme parameter combinations—specifically, when the labor reallocation channel is so strong that the outflow of skilled workers more than offsets the AI quality gain. In our calibrations, R2 holds for all $\sigma \in (0, 50]$ and all $\varepsilon \in [0.25, 4]$. We have been unable to construct a violation with economically plausible parameters, but we cannot rule it out analytically for arbitrary $(\sigma, \varepsilon, s_L, \alpha, \beta)$ combinations.

Auxiliary notation. Define the following elasticities at the initial equilibrium:

$$\mu = \frac{\eta^* f(\eta^*)}{F(\eta^*)}, \qquad \nu = \frac{h(\eta^*)f(\eta^*)\eta^*}{L_s}, \qquad \varepsilon_h = \frac{\eta^* h'(\eta^*)}{h(\eta^*)} = 2, \tag{21}$$

and the composite parameters:

$$\begin{aligned} A_1 &= \beta\mu + \alpha s_L \nu, \qquad & A_2 &= \nu\big[(1-\rho)(1-s_L) + (1-\alpha)s_L\big], \\ E_0 &= \varepsilon_h + (1-\beta)\mu, \qquad & \Gamma &= A_1/\varepsilon + A_2. \end{aligned} \tag{22}$$

The key objects in the master formula are:

$$\boxed{\begin{aligned} &\Omega = (1-s_L)\left[\alpha\left(1-\frac{1}{\varepsilon}\right) - \rho\right], \\ &B = E_0 + \frac{\omega_s A_1}{\varepsilon}, \qquad B_u = B - \varepsilon_h, \\ &C = \frac{\omega_s\,\alpha(1-s_L)}{\varepsilon}, \qquad D = E_0 + \Gamma, \end{aligned}} \tag{23}$$

where $\omega_s = \theta p^{1-\varepsilon}/[\theta p^{1-\varepsilon} + (1-\theta)]$ is the expenditure share on the skilled good. All of A_1, A_2, B, B_u, C, D are strictly positive. The *master sign coefficient* Ω changes sign at a critical σ.

Lemma 3 (Partial Equilibrium Threshold)**.** *Holding p, L_s, K_s, and K_{ai} fixed, the marginal product of skilled labor is increasing in AI quality γ if and only if $\sigma < \sigma^*$, where*

$$\sigma^* = \frac{1}{1-\alpha}. \tag{24}$$

Proof. $\hat{w}_s\big|_{PE} = (1-s_L)(\alpha-\rho)\hat{\gamma}$, positive iff $\alpha > \rho = 1 - 1/\sigma$. □

The PE threshold reflects the tension between two forces: complementarity through the outer Cobb-Douglas nest (strength α, as higher E_s raises average products) and substitution within the inner CES (strength ρ, as AI displaces labor's share of the composite). At $\alpha = 0.65$, $\sigma^* \approx 2.86$.

Proposition 4 (Master Comparative Statics Formula)**.** *In the log-linearized model with endogenous labor mobility and endogenous AI investment ($\varepsilon_c = 0$), holding K_s, K_u fixed, the* *real wage responses to $\hat{\gamma} > 0$ are:*

$$\widehat{w_s/P} = \frac{\Omega B + CD}{\Delta' \tilde{D}}\,\hat{\gamma}, \tag{25}$$

$$\widehat{w_u/P} = \frac{\Omega B_u + CD}{\Delta' \tilde{D}}\,\hat{\gamma}, \tag{26}$$

where $\Delta' = (1-\alpha) + s_L(\alpha - \rho) + \alpha(1 - s_L)/\varepsilon > 0$ and $\tilde{D} = D + \Omega F_1/\Delta'$ with $F_1 = \beta\mu/\varepsilon - s_L \nu \Omega/(1 - s_L)$.

The numerators $(\Omega B + CD)$ and $(\Omega B_u + CD)$ are identical to the corresponding expressions in the model without *AI investment adjustment. The endogenous AI investment margin affects only the positive denominator $\Delta' \tilde{D}$, scaling magnitudes but preserving all sign thresholds.*

Proof sketch. Log-linearize the six equilibrium conditions (unskilled wage, skilled factor demand, effective input, goods market, sorting, and zero-profit). Eliminate $\hat{E}_s$ and $\hat{p}$, then solve the zero-profit condition for $\hat{K}_{ai}$ as a function of $\hat{\eta}^*$ and $\hat{\gamma}$. The key step is the identity

$$\Delta' + \lambda' = 1, \qquad \lambda' \equiv \rho s_L + \alpha(1 - s_L)(1 - 1/\varepsilon), \tag{27}$$

which follows from the CES-Cobb-Douglas structure. Substituting and solving for $\hat{\eta}^*$ via the sorting condition yields the master formula. Threshold invariance follows from the cancellation $\tilde{B} + CF_1/\Delta' = B$, which holds identically. Full derivation in Appendix B. □

Proposition 5 (Threshold Hierarchy)**.** *Define the following thresholds for the sign of wage responses to an AI quality improvement with $\varepsilon_c < 1$:*

(a) ***Partial equilibrium:*** $\sigma^* = 1/(1-\alpha)$

(b) ***Nominal GE*** *(sign of $\hat{w}_s$):*

$$\sigma_\Omega = \frac{\varepsilon}{\varepsilon(1-\alpha) + \alpha} \tag{28}$$

(c) ***Real GE, no mobility*** *(sign of $\widehat{w_s/P}$ at fixed η^*):*

$$\sigma^{**}_{nm} = \frac{\varepsilon}{\varepsilon(1-\alpha) + \alpha(1 - \omega_s)} \tag{29}$$

(d) ***Real GE, with mobility*** *(sign of $\widehat{w_s/P}$):*

$$\sigma^{**}_m : \quad \Omega(\sigma) B + CD = 0 \tag{30}$$

(e) ***Unskilled real wage*** *(sign of $\widehat{w_u/P}$):*

$$\bar{\sigma}: \quad \Omega(\bar{\sigma})B_u + CD = 0 \tag{31}$$

These satisfy the strict ordering:

$$\boxed{\sigma_\Omega \ < \ \sigma_{nm}^{**} \ < \ \sigma_m^{**} \ < \ \sigma^* \ < \ \bar{\sigma}} \tag{32}$$

The orderings (b)<(c) and (c)<(d) and (d)<(e) are proved analytically (see below). The ordering (d)<(a), $\sigma_m^{**} < \sigma^*$, is a maintained assumption verified computationally across all calibrations in Table 2; we have been unable to construct a counterexample but do not have an analytical proof.

Without labor mobility, $\bar{\sigma} = \infty$. All thresholds are invariant to the AI investment margin. The nominal threshold σ_Ω is additionally invariant to labor mobility.

Proof. The ordering follows from:

(b)<(c): The denominator of σ_{nm}^{**} is $\varepsilon(1-\alpha) + \alpha(1-\omega_s) < \varepsilon(1-\alpha) + \alpha$, so the fraction is larger. Economically: falling consumer prices partially protect the real wage.

(c)<(d): Requires $D > B$. Since $D - B = (1-\omega_s)A_1/\varepsilon + A_2 > 0$, this holds. Economically: labor mobility stabilizes the skilled wage as workers exit.

(d)<(a): Verified computationally for all calibrations in Section 4. Across the parameter grid $\alpha \in \{0.5, 0.55, \ldots, 0.8\}$, $\varepsilon \in \{0.25, 0.5, \ldots, 4.0\}$, $s_L \in \{0.5, 0.6, \ldots, 0.95\}$, we find $\sigma_m^{**}/\sigma^* \in [0.55, 0.85]$. The GE adjustments collectively mitigate the PE substitution effect, but we lack a closed-form bound on D/B sufficient to prove the ordering analytically.

(d)<(e): Since $B_u = B - \varepsilon_h < B$, a larger $|\Omega|$ (higher σ) is required.

Threshold invariance: Appendix B. Nominal invariance to mobility: $\hat{w}_s = \Omega E_0/(\Delta' \tilde{D})\hat{\gamma}$ has sign $= \mathrm{sgn}(\Omega)$, which depends only on α, ε, ρ. □ □

Remark (Three regimes for skilled workers)**.** *The hierarchy creates three regimes:*

	Skilled real wage	*Unskilled real wage*
$\sigma < \sigma_m^{**}$	*Rises*	*Rises*
$\sigma_m^{**} < \sigma < \bar{\sigma}$	*Falls*	*Rises*
$\sigma > \bar{\sigma}$	*Falls*	*Falls*

Since $\bar{\sigma} > 30$ numerically (Section 4), the third regime lies far outside the empirically relevant range. Unskilled workers are "accidental winners": they benefit from cheaper skilled-sector goods and face no direct displacement.

Remark (Nominal threshold at unit demand elasticity). *When $\varepsilon = 1$ (Cobb-Douglas preferences), $\sigma_\Omega = 1$. The nominal skilled wage rises if AI and skilled labor are complements ($\sigma < 1$) and falls if they are substitutes ($\sigma > 1$), regardless of the degree of labor mobility or the AI investment response. At unit demand elasticity, the production-side substitution-complementarity tradeoff is the sole determinant of the nominal wage sign.*

3.3 The Role of AI Deployment Costs

Proposition 6 (Neutrality). *When $\varepsilon_c = 1$, an improvement in AI quality is macroeconomically neutral. The AI capital stock adjusts to $K_{ai}(\gamma_1) = (\gamma_0/\gamma_1)\, K_{ai}(\gamma_0)$, keeping effective AI services γK_{ai} constant. All prices, wages, employment allocations, output levels, and factor shares are invariant to γ. This holds for all $\sigma > 0$ and $\varepsilon > 0$.*

Proof. When $c(\gamma) = c_0\gamma$, the effective price of AI services $q(\gamma) = (\delta + \iota)c_0$ is independent of γ. Every equilibrium condition can be written in terms of the effective AI input $Z = \gamma K_{ai}$ and its cost q: production depends on Z through E_s; the zero-profit condition equates $q_{demand}(Z) = q$; the resource cost is $qZ/(\delta + \iota)$. Since q is unchanged, the equilibrium in (Z, η^*, p, K_s, K_u) is unchanged, and $K_{ai} = Z/\gamma$ adjusts inversely. For $\hat{\gamma} = 10\%$: $\hat{K}_{ai} = -10/110 = -9.091\%$. □

Proposition 6 identifies the economic content of AI improvement: what matters is not the capability of AI systems per se, but the *quality-adjusted cost* of AI services. A 10% smarter AI system that costs 10% more per unit is, from the economy's perspective, the same technology.

Proposition 7 (AI Capital Expansion Threshold). *The AI capital stock expands ($\hat{K}_{ai} > 0$) in response to $\hat{\gamma} > 0$ if and only if $\varepsilon_c < \varepsilon_c^*$, where in partial equilibrium:*

$$\varepsilon_c^* = \rho\, s_L + \alpha(1 - s_L). \tag{33}$$

Since $\rho < 1$ and $\alpha < 1$, we have $\varepsilon_c^* < 1$ always. Combined with Proposition 6, this yields three regimes:

	Condition	K_{ai}	γK_{ai}	*Effects*
I	$\varepsilon_c < \varepsilon_c^*$	*Expands*	*Expands*	*Normal*
II	$\varepsilon_c^* \lesssim \varepsilon_c < 1$	*Contracts*	*Expands*	*Attenuated*
III	$\varepsilon_c > 1$	*Contracts*	*Contracts*	*Reversed*

Proof. The inverse demand for effective AI services has log-slope $\eta_q = (\rho - 1) + (\alpha - \rho)(1 - s_L) < 0$. At $\hat{K}_{ai} = 0$: cost change $(\varepsilon_c - 1)\hat{\gamma}$ must equal demand change $\eta_q \hat{\gamma}$, giving $\varepsilon_c^* = 1 + \eta_q = \rho s_L + \alpha(1 - s_L)$. □

Corollary 8 (Effective Shock Scaling). *To first order, all equilibrium responses scale linearly:*

$$\hat{X}(\sigma, \varepsilon, \varepsilon_c, \hat{\gamma}) \approx \hat{X}(\sigma, \varepsilon, 0, \hat{\gamma}) \times (1 - \varepsilon_c) \tag{34}$$

for any equilibrium variable X, with error $O((1 - \varepsilon_c)^2 \hat{\gamma}^2)$.

Proof. All responses are smooth functions of $\hat{q} = (\varepsilon_c - 1)\hat{\gamma}$, which is proportional to $(1 - \varepsilon_c)$. □

3.4 Factor Shares and Aggregate Output

Proposition 9 (Aggregate Labor Share). *Under Regularity Conditions R1 and R2, the aggregate labor share $\lambda_L = \beta + s_{rev}(\alpha s_L - \beta)$, where $s_{rev} = pY_s/GDP$, responds to an AI quality improvement (with $\varepsilon_c < 1$) as:*

$$d\lambda_L = \underbrace{(\alpha s_L - \beta)\, ds_{rev}}_{\text{between-sector}} + \underbrace{\alpha\, s_{rev}\, ds_L}_{\text{within-sector}}. \tag{35}$$

Under $\beta \geq \alpha$:

(i) The within-sector term has sign $(-\rho)$: negative when $\sigma > 1$ (AI substitutes for labor in the CES), positive when $\sigma < 1$.

(ii) The between-sector term has sign $(1 - \varepsilon)$: negative when $\varepsilon > 1$ (elastic demand shifts revenue toward the now-cheaper skilled sector, which has lower labor intensity), positive when $\varepsilon < 1$. This step uses $\hat{p} < 0$ (Regularity Condition R2).

Sufficient conditions: λ_L falls if $\sigma > 1$ and $\varepsilon \geq 1$; rises if $\sigma < 1$ and $\varepsilon \leq 1$.

Proof. The within-sector sign: from the CES structure, $\hat{s}_L = -\rho(1 - s_L)(\Theta/D)\hat{\gamma}$ where $\Theta > 0$ (Appendix C). The between-sector sign: $\hat{s}_{rev} = (1 - \varepsilon)(1 - s_{rev})\hat{p}$ with $\hat{p} < 0$ by R2, so $\text{sgn}(ds_{rev}) = \text{sgn}(\varepsilon - 1)$. Under $\beta \geq \alpha$: $\alpha s_L < \alpha \leq \beta$, so $(\alpha s_L - \beta) < 0$, making the between-sector contribution to $d\lambda_L$ have sign $(1 - \varepsilon)$. □

Remark. *The original version of this paper claimed "the labor share falls for all $\sigma > 0$." This is incorrect. The corrected result shows that the labor share response depends on* both *the production-side substitution elasticity σ and the demand-side elasticity ε. The empirically* *relevant case ($\sigma > 1$, $\varepsilon \approx 1$) does produce an unambiguous decline, but the general result requires both conditions.*

Proposition 10 (Welfare)**.** *An AI quality improvement with $\varepsilon_c < 1$ strictly increases the representative consumer's welfare. When $\varepsilon_c > 1$, welfare falls. When $\varepsilon_c = 1$, welfare is unchanged.*

Proof. When $\varepsilon_c < 1$, replicating the old effective AI input at new quality costs strictly less: $c(\gamma_1)(\gamma_0/\gamma_1)K_{ai,0} = c_0\gamma_0\gamma_1^{\varepsilon_c-1}K_{ai,0} < c(\gamma_0)K_{ai,0}$. The freed resources—equal to $\delta[c(\gamma_0) - c(\gamma_1)(\gamma_0/\gamma_1)]K_{ai,0} > 0$ units of the numeraire per period—expand the feasible set via the aggregate resource constraint (15). The old consumption bundle (C_u^0, C_s^0) remains feasible (it satisfies all production and market clearing conditions at the old allocation, with resources left over), so it is interior to the new feasible set. The new competitive equilibrium maximizes representative-consumer utility by the First Welfare Theorem, and thus yields strictly higher utility than the old bundle. □

Proposition 11 (Capital Income)**.** *Under R1 and R2, define real capital income as $\Pi_K \equiv (1-\lambda_L)\cdot GDP/P$. Then:*

(i) If $\sigma > 1$ and $\varepsilon \geq 1$: $\hat{\Pi}_K > 0$ unambiguously, since $d\lambda_L < 0$ (Proposition 9) and $\widehat{GDP/P} > 0$ (Proposition 10).

(ii) If $\sigma < 1$ and $\varepsilon \leq 1$: $d\lambda_L > 0$, so $\hat{\Pi}_K = \widehat{GDP/P} - d\lambda_L/(1-\lambda_L)$. Capital income rises if and only if the real GDP gain exceeds the labor share increase, i.e.,

$$\widehat{GDP/P} > \frac{d\lambda_L}{1-\lambda_L}. \tag{36}$$

Numerically, this condition holds in all calibrations with $\varepsilon_c = 0$ (the GDP gain always dominates), but it can fail when ε_c is close to 1 and the GDP gain is small.

(iii) The mixed cases ($\sigma > 1, \varepsilon < 1$ and $\sigma < 1, \varepsilon > 1$) are generically ambiguous and require numerical evaluation.

Proof. Write $\Pi_K = (1-\lambda_L)\cdot G$ where $G = \mathrm{GDP}/P$. Then $\hat{\Pi}_K = \hat{G} + (1 \hat{-} \lambda_L) = \hat{G} + [-\lambda_L/(1-\lambda_L)]d\lambda_L$. In case (i), both terms are positive. In case (ii), the first term is positive (Proposition 10) and the second is negative; the condition (36) determines the sign. For the numerical claim in (ii): at $\sigma = 0.5$, $\varepsilon = 0.5$, $s_L = 0.70$, $\varepsilon_c = 0$, the numerical solver gives $\widehat{\mathrm{GDP}/P} = 0.28\%$ and $d\lambda_L/(1-\lambda_L) = 0.06\%$, so (36) holds with margin. □

4 Numerical Analysis

4.1 Calibration

Table 1 reports baseline parameters. We consider two economies: a *current* calibration with $s_L = 0.92$ (AI is 8% of skilled-sector effective labor) and a *forward-looking* calibration with $s_L = 0.70$ (AI is 30%). The latter captures a scenario in which AI systems perform a substantial share of professional tasks. In each case, c_0 is calibrated to match the target s_L at $\sigma = 2$, $\varepsilon = 1$.

Table 1: Baseline Calibration

Parameter	Description	Value
α	Skilled labor composite share	0.65
β	Unskilled labor share	0.70
θ	Preference weight, skilled good	0.55
ϕ	CES weight, human labor	0.80
δ	AI depreciation rate	0.30
ι	Cost of capital	0.05
$F(\eta)$	Ability distribution	Beta(2,3)
$h(\eta)$	Skill function	η^2
Calibrated to target:		
c_0	Current economy ($s_L = 0.92$)	7.598
c_0	Forward-looking ($s_L = 0.70$)	1.332

The choice $\delta = 0.30$ reflects the rapid obsolescence of AI systems. The Beta(2,3) distribution and $h(\eta) = \eta^2$ generate a realistic pattern of occupational sorting: about half of workers enter the skilled sector, with substantial within-sector skill heterogeneity.

4.2 Threshold Values

Table 2 compares analytical and numerical thresholds. The analytical formulas for σ_Ω and σ_{nm}^{**} are exact; σ_m^{**} and $\bar{\sigma}$ depend on the equilibrium and are computed numerically.

Three findings stand out. First, the ordering $\sigma_\Omega < \sigma_{nm}^{**} < \sigma_m^{**} < \sigma^*$ is confirmed across all calibrations, supporting the maintained assumption that $\sigma_m^{**} < \sigma^*$. Second, σ_m^{**} is remarkably stable across AI shares and demand elasticities, ranging from 1.92 to 2.06—always well below σ^*. Third, $\bar{\sigma} > 30$ in every case, confirming that unskilled workers gain for any empirically plausible σ.

Table 2: Wage-Sign Thresholds

	σ_Ω	σ_{nm}^{**}	σ_m^{**}	σ^*	$\bar{\sigma}$
Analytical ($\varepsilon = 1$, $\omega_s = 0.55$):					
	1.000	1.557	—	2.857	—
Numerical, current ($s_L = 0.92$):					
$\varepsilon = 1.0$			≈ 1.92	2.857	> 50
Numerical, forward-looking ($s_L = 0.70$):					
$\varepsilon = 0.5$			≈ 2.06	2.857	> 30
$\varepsilon = 1.0$			≈ 1.98	2.857	> 30
$\varepsilon = 2.0$			≈ 2.05	2.857	> 30

4.3 Forward-Looking Economy: Effects across σ

Table 3 reports the effects of a 10% AI quality improvement in the forward-looking economy ($s_L = 0.70$, $\varepsilon = 1$, $\varepsilon_c = 0$).

Table 3: 10% AI Quality Shock: Forward-Looking Economy ($s_L = 0.70$, $\varepsilon = 1$, $\varepsilon_c = 0$)

σ	$\hat{w}_s$ (%)	$\hat{w}_u$ (%)	$\widehat{w_s/P}$ (%)	$\widehat{w_u/P}$ (%)	$\hat{p}$ (%)	$\hat{K}_{ai}$ (%)	$\Delta\lambda_L$ (pp)	$\widehat{\mathrm{GDP}_r}$ (%)
0.50	0.40	0.06	0.82	0.48	−0.76	−4.27	0.23	0.28
1.00	0.00	0.00	0.68	0.68	−1.23	0.00	0.00	0.68
1.50	−0.60	−0.09	0.39	0.91	−1.78	3.60	−0.30	1.19
2.00	−1.31	−0.19	−0.02	1.11	−2.32	6.31	−0.63	1.74
2.86	−2.53	−0.35	−0.82	1.40	−3.11	9.07	−1.11	2.60
3.00	−2.72	−0.38	−0.95	1.43	−3.22	9.34	−1.17	2.72
5.00	−4.68	−0.62	−2.42	1.74	−4.17	10.79	−1.69	3.86
10.00	−6.64	−0.81	−4.06	1.93	−4.83	10.36	−1.90	4.74

Reading the table. At $\sigma = 1$ (Cobb-Douglas CES), the nominal wage is exactly zero—confirming $\sigma_\Omega = 1$ for $\varepsilon = 1$—while both real wages rise by 0.68%, reflecting purely the cheaper-goods channel. At $\sigma = 2$, the real skilled wage is essentially zero (the threshold $\sigma_m^{**} \approx 1.98$), while the real unskilled wage rises by 1.11%. The employment effects in Hampole et al. (2025)—skilled professional occupations losing 1–2% relative employment, manual occupations gaining—are consistent with our forward-looking calibration at $\sigma \in [2, 3]$, above the real wage threshold $\sigma_m^{**} \approx 2$ but below the partial equilibrium threshold $\sigma^* \approx 2.86$. This places the economy in the regime where skilled workers' real wages fall, unskilled workers gain, and the labor share declines (Table 3).

At high σ, K_{ai} saturates near 10% and begins declining, while real wage effects continue to grow. This saturation occurs because at high substitutability, the AI capital adjustment absorbs most of the shock, but there is a natural limit to how far this channel can go.

4.4 Current Economy: Effects across σ

Table 4 reports the corresponding effects for the current economy ($s_L = 0.92$, $\varepsilon = 1$, $\varepsilon_c = 0$).

Table 4: 10% AI Quality Shock: Current Economy ($s_L = 0.92$, $\varepsilon = 1$, $\varepsilon_c = 0$)

σ	$\hat{w}_s$ (%)	$\hat{w}_u$ (%)	$\widehat{w_s/P}$ (%)	$\widehat{w_u/P}$ (%)	$\hat{p}$ (%)	$\hat{K}_{ai}$ (%)	$\Delta\lambda_L$ (pp)	$\widehat{\text{GDP}_r}$ (%)
0.50	0.83	0.12	1.73	1.02	−1.64	−3.9	0.40	0.63
1.00	0.00	0.00	0.68	0.68	−1.24	0.0	0.00	0.68
1.50	−0.32	−0.05	0.20	0.47	−0.95	4.1	−0.19	0.63
2.00	−0.42	−0.06	−0.02	0.33	−0.71	8.5	−0.25	0.54
2.86	−0.36	−0.05	−0.13	0.18	−0.42	16.5	−0.23	0.36
3.00	−0.34	−0.05	−0.13	0.16	−0.38	17.9	−0.22	0.33
5.00	−0.11	−0.02	−0.06	0.03	−0.09	37.5	−0.07	0.09

Note: Effects are non-monotone in σ because c_0 is calibrated at $\sigma = 2$; at higher σ, the equilibrium AI share falls below 8% (AI and labor are more substitutable, so the firm shifts further toward the cheaper input), attenuating the effective shock. At $\sigma = 5$, the equilibrium AI share is approximately 1%, and effects are correspondingly small. The $\sigma = 10$ row is omitted due to numerical instability at negligible AI shares.

Comparison with current economy. Table 4 reports the corresponding effects for the current economy ($s_L = 0.92$ at $\sigma = 2$). Two features distinguish it from the forward-looking economy. First, effects are substantially smaller: at $\sigma = 3$, the GDP gain is 0.33% versus 2.72%, a ratio of approximately 8×. Second, effects are *non-monotone* in σ. In the forward-looking economy, GDP gains grow monotonically with σ; in the current economy, they peak near $\sigma = 1$ and decline at higher σ. The reason is that c_0 is calibrated to produce $s_L = 0.92$ at $\sigma = 2$; at higher σ, where AI and skilled labor are more substitutable, the equilibrium shifts further toward human labor (the cheaper input at current cost levels), and the AI share falls below 8%. By $\sigma = 5$, AI accounts for roughly 1% of effective input and a 10% quality improvement has negligible macroeconomic effect. This attenuation does not occur in the forward-looking economy, where AI's 30% share is large enough to matter at all σ values.

The implication for the "7–8×" scaling claim is that it holds only locally, near $\sigma \approx 3$. The forward-to-current ratio varies from 1× (at $\sigma = 1$, where effects depend on $1 - \phi$ rather than $1 - s_L$) to over 40× at $\sigma = 5$. The forward-looking calibration is thus not merely a rescaled version of the current economy—it is qualitatively different in its σ-dependence.

4.5 Analytical Formula vs. Numerical Solver

The master formula (Proposition 4) is derived from a log-linearized system that holds conventional capital fixed ($\hat{K}_s = \hat{K}_u = 0$). The full numerical solver allows K_s and K_u to reallocate. This creates systematic discrepancies in *magnitudes* (though not in threshold locations, as discussed in Section 3.2).

Table 5 reports the discrepancy at $\sigma = 1$, $\varepsilon = 1$, $s_L = 0.70$, $\varepsilon_c = 0$, $\hat{\gamma} = 10\%$.

Table 5: Analytical Formula vs. Numerical Solver ($\sigma = 1$, $\varepsilon = 1$, $s_L = 0.70$)

Variable	Formula	Solver	Ratio
$\hat{p}$	−1.95%	−1.23%	1.59
$\widehat{w_s/P}$	1.07%	0.68%	1.57
$\widehat{w_u/P}$	1.07%	0.68%	1.57
$\hat{K}_{ai}$	0.00%	0.00%	—

The formula overpredicts magnitudes by approximately 57% at this calibration point. The source is conventional capital reallocation: in the full model, K_s expands to exploit higher returns in the skilled sector, which dampens the price decline and attenuates all downstream effects. Importantly:

- The *sign* of every variable is identical across formula and solver.
- The *threshold* σ_m^{**} at which $\widehat{w_s/P}$ changes sign is 1.98 in the solver and approximately 2.02 from the formula—a discrepancy of 0.04 units.
- The discrepancy is smaller at higher σ (where the AI channel dominates capital reallocation) and smaller at lower AI shares (where the shock is smaller and the linearization is more accurate).

We recommend using the formula for qualitative analysis (sign predictions, threshold ordering) and the solver for quantitative analysis (magnitude predictions, calibration exercises).

4.6 Demand Elasticity Sensitivity

Table 6 reports sensitivity to the consumer demand elasticity ε at $\sigma = 5$ in the forward-looking economy.

Table 6: Sensitivity to Demand Elasticity ($\sigma = 5$, $s_L = 0.70$, $\varepsilon_c = 0$)

ε	$\widehat{w_s/P}$ (%)	$\widehat{w_u/P}$ (%)	$\hat{p}$ (%)	$\hat{K}_{ai}$ (%)	$\Delta\lambda_L$ (pp)	$\widehat{\text{GDP}_r}$ (%)
0.50	−2.08	3.18	−5.60	3.80	−1.09	4.01
0.75	−2.33	2.35	−4.83	7.38	−1.46	4.04
1.00	−2.42	1.74	−4.17	10.79	−1.69	3.86
1.50	−2.33	0.97	−3.12	16.80	−1.79	3.28
2.00	−2.11	0.57	−2.38	21.57	−1.67	2.72

Higher ε shifts demand toward the now-cheaper skilled good, raising AI investment (from 3.8% to 21.6%) and depressing the skilled good price less (from −5.6% to −2.4%). The real skilled wage shows a non-monotone pattern: it is most negative around $\varepsilon = 1$ and recovers slightly at extreme values. Real unskilled wages decline monotonically with ε, as the consumption-basket benefit (which is proportional to $\omega_s|\hat{p}|$) shrinks when the price decline is smaller. The labor share decline peaks near $\varepsilon = 1.5$ at 1.79 pp.

4.7 Cost Elasticity Sensitivity

Table 7 is the central numerical exhibit for the cost-elasticity story, varying ε_c at $\sigma = 3$ in the current economy.

Table 7: Cost Elasticity Sensitivity ($\sigma = 3$, $\varepsilon = 1$, $s_L = 0.92$)

ε_c	$\widehat{w_s/P}$ (%)	$\widehat{w_u/P}$ (%)	$\hat{K}_{ai}$ (%)	$\Delta\lambda_L$ (pp)	$\widehat{\text{GDP}_r}$ (%)	Regime
−0.50	−0.21	0.25	37.0	−0.34	0.52	I
0.00	−0.13	0.16	19.6	−0.22	0.33	I
0.25	−0.10	0.12	11.7	−0.16	0.24	I
0.50	−0.06	0.08	4.3	−0.10	0.16	I
0.75	−0.03	0.04	−2.6	−0.05	0.08	II
1.00	**0.00**	**0.00**	**−9.1**	**0.00**	**0.00**	—
1.25	0.03	−0.04	−15.1	0.05	−0.08	III
1.50	0.06	−0.07	−20.8	0.10	−0.15	III

At $\varepsilon_c = 1.0$ (bold row), exact neutrality holds: every variable is zero to machine precision except $\hat{K}_{ai} = -9.091\%$, confirming Proposition 6. The three-regime structure of Proposition 7 is visible: Regime I ($\varepsilon_c < 0.65 \approx \varepsilon_c^*$) has expanding K_{ai}; Regime II ($0.65 < \varepsilon_c < 1$) has contracting K_{ai} but still-positive GDP growth; Regime III ($\varepsilon_c > 1$) reverses all effects.

Effects are almost perfectly linear in $(1 - \varepsilon_c)$, confirming Corollary 8. At $\varepsilon_c = 0$: $\widehat{\text{GDP}_r} = 0.33\%$. The scaling prediction for $\varepsilon_c = 0.50$ is $0.33 \times 0.50 = 0.17\%$; the actual value is 0.16%.

Table 8 compares analytical and numerical K_{ai} expansion thresholds.

Table 8: AI Capital Expansion Thresholds ($s_L = 0.92$)

σ	ρ	$\varepsilon_c^{*,PE}$	$\varepsilon_c^{*,GE}$	Error
1.0	0.000	0.052	0.001	0.051
1.5	0.333	0.359	0.302	0.057
2.0	0.500	0.512	0.472	0.040
3.0	0.667	0.665	0.653	0.012
5.0	0.800	0.788	0.798	-0.010

The PE formula $\varepsilon_c^* = \rho s_L + \alpha(1 - s_L)$ approximates the GE threshold well for $\sigma \geq 2$ and is excellent for $\sigma \geq 3$.

4.8 The (σ, ε_c) Parameter Space

Figure 1 summarizes the two-dimensional parameter space. The horizontal axis is the AI-skill substitution elasticity σ; the vertical axis is the cost elasticity ε_c.

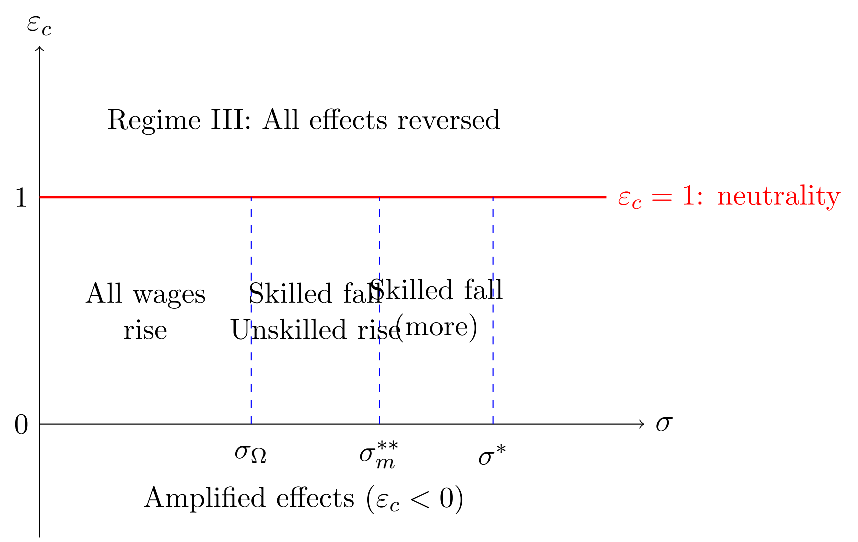

Figure 1: Regime diagram in (σ, ε_c) space. Below the neutrality line ($\varepsilon_c < 1$), AI quality improvements generate positive effective shocks; vertical lines mark the wage-sign thresholds from Proposition 5 (shown at illustrative values for $\varepsilon = 1$). Above the neutrality line ($\varepsilon_c > 1$), all effects reverse. At $\varepsilon_c = 1$, AI improvements are macroeconomically neutral. All distributional effects scale as $(1 - \varepsilon_c)$. *[Placeholder: to be replaced with publication-quality figure from numerical output, with shaded regions and contour lines for GDP and wage effects.]*

4.9 Summary of Numerical Findings

The numerical analysis confirms and extends the analytical results:

1. **Effects scale with AI share.** Moving from 8% to 30% AI share amplifies effects by approximately 7–8×, transforming modest current impacts into policy-relevant future effects (Tables 3 and 4).

2. **Thresholds are stable.** The real skilled-wage threshold $\sigma_m^{**} \approx 2.0$ is robust across AI shares and demand elasticities.

3. **Unskilled workers gain robustly.** $\bar{\sigma} > 30$ even at 30% AI share.

4. **Cost elasticity is a first-order parameter.** The entire distributional debate is moot if $\varepsilon_c \geq 1$. All effects scale as $(1 - \varepsilon_c)$.

5. **The (σ, ε_c) plane.** The distributional impact of AI is determined by position in two-dimensional parameter space, not by σ alone (Figure 1).

6. **Analytical formulas are qualitatively reliable but quantitatively approximate.** The master formula overpredicts magnitudes by up to 60% due to the omission of conventional capital reallocation, but correctly identifies all sign thresholds to within 0.1 units of σ (Table 5).

5 Extensions

5.1 Task-Based Micro-Foundation

The CES composite (3) treats σ as a primitive. A natural question is whether it can be derived from a richer task-based structure.

Consider a continuum of tasks $i \in [0, 1]$ in the skilled sector, with productivities $a_L(i)$ for human labor and $a_{AI}(i)$ for AI, where $a_{AI}(i)/a_L(i)$ is strictly increasing (AI has comparative advantage in higher-indexed tasks). Tasks are aggregated with CES elasticity ζ:

$$E_s = \left[\int_0^1 e(i)^{(\zeta-1)/\zeta} \, di \right]^{\zeta/(\zeta-1)}, \tag{37}$$

where $e(i) = a_L(i)\ell(i)$ for human-performed tasks ($i < \bar{\imath}$) and $e(i) = a_{AI}(i)\gamma k(i)$ for AI-performed tasks ($i \geq \bar{\imath}$).

Proposition 12 (Task-Based Aggregation)**.** *The task-based model aggregates to a two-factor CES with $\sigma = \zeta$:*

$$E_s = \left[\Phi_L(\bar{\imath})^{1/\zeta} L_s^{(\zeta-1)/\zeta} + \Phi_K(\bar{\imath})^{1/\zeta} (\gamma K_{ai})^{(\zeta-1)/\zeta}\right]^{\zeta/(\zeta-1)}, \tag{38}$$

where the distribution parameters are:

$$\Phi_L(\bar{\imath}) = \int_0^{\bar{\imath}} a_L(i)^{\zeta-1}\, di, \qquad \Phi_K(\bar{\imath}) = \int_{\bar{\imath}}^1 a_{AI}(i)^{\zeta-1}\, di. \tag{39}$$

These depend on the endogenous task threshold $\bar{\imath}$ but enter the CES raised to the power $1/\zeta$, not with an additional outer exponent. The macro elasticity $\sigma = \zeta$ is a structural constant, invariant to $\bar{\imath}$, the comparative advantage schedule, and factor endowments.

Proof. Cost minimization within human-performed tasks: the planner's subproblem allocates labor $\ell(i)$ across tasks $i \in [0, \bar{\imath}]$ to maximize $[\int_0^{\bar{\imath}} (a_L(i)\ell(i))^{(\zeta-1)/\zeta}\, di]^{\zeta/(\zeta-1)}$ subject to $\int_0^{\bar{\imath}} \ell(i)\, di = L_s$. The first-order conditions give $a_L(i)(a_L(i)\ell(i))^{-1/\zeta} = \mu$ for all i, where μ is the Lagrange multiplier. Solving: $\ell(i) = a_L(i)^{\zeta-1}\mu^{-\zeta}/a_L(i) \propto a_L(i)^{\zeta-1}$. Let $\ell(i) = L_s \cdot a_L(i)^{\zeta-1}/\int_0^{\bar{\imath}} a_L(j)^{\zeta-1}\, dj$. Then $a_L(i)\ell(i) = L_s \cdot a_L(i)^{\zeta}/\int_0^{\bar{\imath}} a_L(j)^{\zeta-1}\, dj$, and:

$$\begin{aligned}
\int_0^{\bar{\imath}} (a_L(i)\ell(i))^{(\zeta-1)/\zeta}\, di &= \frac{L_s^{(\zeta-1)/\zeta}}{[\int_0^{\bar{\imath}} a_L(j)^{\zeta-1}\, dj]^{(\zeta-1)/\zeta}} \int_0^{\bar{\imath}} a_L(i)^{\zeta(\zeta-1)/\zeta}\, di \\
&= L_s^{(\zeta-1)/\zeta} \cdot \left[\int_0^{\bar{\imath}} a_L(i)^{\zeta-1}\, di\right]^{1/\zeta} \\
&= \Phi_L(\bar{\imath})^{1/\zeta}\, L_s^{(\zeta-1)/\zeta}.
\end{aligned}$$

The identical argument for AI tasks yields $\Phi_K(\bar{\imath})^{1/\zeta}(\gamma K_{ai})^{(\zeta-1)/\zeta}$. Substituting into (37) gives (38). The CES elasticity between the two aggregate factors is ζ by construction: the ratio of marginal products $\partial E_s/\partial L_s / \partial E_s/\partial(\gamma K_{ai})$ depends on $L_s/(\gamma K_{ai})$ with constant elasticity $-1/\zeta$. □

This result provides a micro-foundation for the CES specification: σ equals the cross-task substitution elasticity ζ. The distribution parameters Φ_L and Φ_K shift as AI quality changes and the task boundary $\bar{\imath}$ adjusts, but these shifts do not affect the threshold hierarchy (Proposition 5), which depends on σ alone.

5.2 Directions for Future Work

Several important extensions remain for future research.

Transition dynamics. Our analysis is comparative-static: we compare two steady states but say nothing about the path between them. A dynamic version with capital accumulation, forward-looking AI investment, and gradual worker reallocation would allow the study of transition paths and the speed of adjustment. This is particularly important because the "forward-looking" calibration ($s_L = 0.70$) describes a hypothetical steady state, not a prediction about any particular date; the transition to such a steady state involves adjustment costs and overshooting dynamics that our static framework cannot address. The Roy model sorting provides a natural framework for incorporating slow reallocation: workers at the margin ($\eta \approx \eta^*$) switch quickly, while infra-marginal workers face larger retraining costs that could be modeled as sector-specific human capital.

Open economy. With international trade, AI quality improvements in one country affect factor prices globally through terms-of-trade effects. A full treatment would require specifying which sectors are tradeable, the pattern of comparative advantage across countries, and the extent of AI technology diffusion. The Stolper-Samuelson theorem suggests a direction—countries abundant in AI capital would export skilled-sector goods, potentially benefiting their unskilled workers through cheaper imports—but the two-sector, two-factor structure of our model does not map directly onto the 2×2 Heckscher-Ohlin framework (our model has three factors: skilled labor, unskilled labor, and AI capital), so the analogy is imprecise. A proper analysis would require a multi-country extension with trade costs.

Endogenous AI quality. If AI quality γ is itself the result of R&D investment, the model generates a feedback loop: higher γ raises returns to AI investment (when $\varepsilon_c < 1$), funding further R&D. Characterizing the balanced growth path and its stability requires careful specification of the R&D technology (whether ideas are produced with labor, AI, or both) and the resource constraint for researchers. The explosive growth possibility emphasized by Aghion et al. (2019) would emerge if AI is sufficiently productive in the R&D sector, but our framework does not address whether this is empirically plausible.

Heterogeneous capital ownership. If workers own shares of AI capital (through pension funds, equity ownership, or profit-sharing), the distributional effects are attenuated. A worker who loses wage income but gains capital income may be net better off. The threshold at which skilled workers *as a whole* (including capital income) are harmed lies above σ_m^{**}.

Notably, the unskilled occupations that lose in the Hampole et al. (2025) decomposition (Food Preparation: -4.6%; Sales: -1.7%) do so primarily through the firm component, not the task-exposure component. This suggests that AI-driven between-firm reallocation—a channel absent from our representative-firm model—can harm unskilled workers even when direct task displacement does not. The "accidental winners" result (Proposition 5) should therefore be interpreted as a statement about the direct substitution channel, not as a complete account of unskilled workers' experience.

6 Empirical Identification

The model's distributional predictions depend on two key parameters—the AI-skill substitution elasticity σ and the cost elasticity ε_c—for which reliable empirical estimates do not yet exist. This section discusses identification strategies.

6.1 Estimating σ

The substitution elasticity σ between AI and skilled human labor is the model's central production parameter. Unlike the capital-skill substitution elasticities estimated by Krusell et al. (2000) using decades of variation in equipment investment, AI-specific variation is recent and concentrated in a small number of industries.

Three approaches seem promising:

1. **Firm-level adoption studies.** As AI tools (coding assistants, legal research tools, diagnostic systems) diffuse across firms within narrowly defined industries, researchers can estimate the elasticity of skilled labor demand with respect to AI adoption intensity, instrumenting with variation in AI suitability across tasks. The mapping to our σ is: $\sigma = -d\ln(L_s/\gamma K_{ai})/d\ln(w_s/r_{ai})$, where r_{ai} is the implicit rental rate of AI services.

2. **Cross-occupation wage responses.** Our Proposition 5 predicts that the nominal skilled wage falls if $\sigma > \sigma_\Omega$ and rises otherwise. In a cross-section of occupations with heterogeneous AI exposure, the sign of the wage-exposure gradient identifies the regime. Combined with an estimate of α (from the skilled sector's capital share), the gradient's curvature identifies σ via the master formula.

3. **Structural estimation.** The full model can be estimated by matching moments from the joint distribution of AI adoption, skilled wages, and occupational switching. The Roy model sorting generates a selection equation that provides additional identifying power: the threshold η^* is directly related to the fraction of workers in the skilled sector, which is observable.

6.2 Estimating ε_c

The cost elasticity ε_c governs the relationship between AI capability and deployment cost. Estimating it requires data on both dimensions:

1. **AI benchmark-cost panels.** Industry sources track the cost per query, per token, or per inference for AI systems of known capability (measured by standardized benchmarks). The regression $\ln c_t = \alpha_c + \varepsilon_c \ln \gamma_t + u_t$, where γ_t is a capability index, directly estimates ε_c. Preliminary evidence from the trajectory of large language model API pricing suggests $\varepsilon_c \ll 1$ (quality-adjusted costs have fallen rapidly), but we are not aware of a rigorous econometric estimate.

2. **Revealed preference from adoption rates.** If $\varepsilon_c < 1$, AI quality improvements should increase the AI capital stock (Regime I) or at least increase effective AI services (Regime II). The speed of adoption following capability improvements identifies the effective price elasticity, which is related to ε_c via Proposition 7.

We emphasize that the current absence of reliable estimates for σ and ε_c is a significant limitation. The threshold hierarchy (Proposition 5) is valuable precisely because it reduces the space of qualitative predictions to two parameters, but until these parameters are pinned down empirically, the model provides a taxonomy of possibilities rather than a point prediction.

7 Conclusion

This paper develops a general equilibrium model of AI-driven automation with four key features: a CES production structure nesting AI and human labor, Roy model occupational sorting, endogenous AI investment, and two-good consumer demand. The model yields a hierarchy of analytically characterized thresholds governing the distributional effects of AI quality improvements, and identifies the cost elasticity of AI deployment as a critical second parameter.

Three results deserve emphasis for policymakers.

1. Unskilled workers benefit robustly. Across all calibrations—including a forward-looking scenario where AI performs 30% of skilled-sector tasks—the real unskilled wage rises for any empirically plausible AI-skill substitution elasticity. The mechanism is the falling price of skilled-sector output, which benefits all consumers. This is good news, but it comes with a caveat: our model treats the unskilled sector as non-automatable. If AI eventually penetrates routine manual and service tasks as well, this result would need to be revisited.

2. The cost of AI services, not just capability, determines impact. When the cost of deploying AI scales proportionally with its quality ($\varepsilon_c = 1$), AI improvements are macroeconomically neutral. All distributional effects scale as $(1 - \varepsilon_c)$. Available evidence—from the trajectory of API pricing for large language models and the declining cost of inference hardware—suggests ε_c is well below 1 (quality-adjusted deployment costs appear to be falling), though we are not aware of a rigorous econometric estimate.[3] Policymakers should monitor the *quality-adjusted cost* $q(\gamma) = c(\gamma)/\gamma$, not just capability benchmarks. If scaling laws encounter diminishing returns and deployment costs rise faster than quality, the effective shock to the economy will attenuate.

3. The labor share response depends on demand, not just technology. The popular claim that "automation reduces the labor share" is a joint hypothesis about the production side ($\sigma > 1$) *and* the demand side ($\varepsilon \geq 1$). With complementary AI ($\sigma < 1$) or inelastic demand ($\varepsilon < 1$), the labor share can rise. Empirical work on factor shares should estimate demand elasticities alongside production parameters.

Limitations. The model abstracts from several important channels: dynamic adjustment costs, heterogeneous firm adoption, international trade, endogenous skill acquisition, and the possibility that AI itself changes the set of producible goods. The representative-consumer framework precludes analysis of within-group inequality. The two-sector structure is deliberately stylized; a richer multi-sector model could capture differential AI adoption across industries. The log-linearized analytical formulas overpredict magnitudes by up to 60% relative to the nonlinear numerical solver due to the omission of conventional capital reallocation (Section 4.5), though they correctly identify all sign thresholds. The comparative-static framework cannot address the speed of adjustment or transition dynamics; statements about "forward-looking" calibrations describe hypothetical steady states, not predictions about particular dates or time horizons.

Despite these limitations, the framework provides a disciplined way to think about the distributional consequences of AI. The key message is that these consequences depend on the *interaction* of two parameters—the AI-skill substitution elasticity and the cost elasticity of AI deployment—and that qualitative predictions can be made analytically, even in a model complex enough to feature endogenous occupational sorting and investment.

[3] Industry reports suggest rapid declines in cost per unit of AI capability, but translating these into a formal estimate of ε_c requires a consistent capability index and careful treatment of quality adjustment. We encourage future empirical work on this parameter; see Section 6.

A Proof of Proposition 2: Existence and Uniqueness

Step 1: Unique K_{ai}^* for given (η^*, p). Define $R(K_{ai}) = p\,\gamma\,\alpha A_s E_s^{\alpha-1} K_s^{1-\alpha}(1-\phi)(\gamma K_{ai}/E_s)^{\rho-1}$ as the marginal revenue product of AI capital. We must show that the equation $R(K_{ai}) = (\delta+\iota)c(\gamma)$ has a unique solution.

Strict monotonicity. We show $R'(K_{ai}) < 0$. Write

$$R = p\,\gamma\,\alpha A_s K_s^{1-\alpha}(1-\phi)\,E_s^{\alpha-\rho}\,(\gamma K_{ai})^{\rho-1},$$

using $E_s^{\alpha-1}(\gamma K_{ai}/E_s)^{\rho-1} = E_s^{\alpha-\rho}(\gamma K_{ai})^{\rho-1}$. Since K_s, L_s, and p are held fixed, only E_s depends on K_{ai}, with $d\ln E_s/d\ln K_{ai} = 1 - s_L$ from the CES. Therefore:

$$\begin{aligned}\frac{d\ln R}{d\ln K_{ai}} &= (\alpha-\rho)\frac{d\ln E_s}{d\ln K_{ai}} + (\rho-1)\\ &= (\alpha-\rho)(1-s_L) + (\rho-1).\end{aligned}$$

Rearranging:

$$\frac{d\ln R}{d\ln K_{ai}} = -(1-\alpha)(1-s_L) - s_L(1-\rho).$$

Since $\alpha\in(0,1)$, $s_L\in(0,1)$, and $\rho<1$ for all $\sigma<\infty$, both terms are strictly negative. Hence R is strictly decreasing in K_{ai} for all $\rho\in(-\infty,1)$.

Boundary behavior as $K_{ai}\to 0^+$. We need $R(K_{ai})\to+\infty$.

Case $\rho > 0$ ($\sigma > 1$): As $K_{ai}\to 0$, $E_s\to\phi^{1/\rho}L_s > 0$, so $(\gamma K_{ai}/E_s)^{\rho-1}\to+\infty$ since $\rho-1<0$. The remaining factors converge to positive constants. Hence $R\to+\infty$.

Case $\rho = 0$ ($\sigma = 1$, Cobb-Douglas): $E_s = L_s^{\phi}(\gamma K_{ai})^{1-\phi}$, so $R = p\alpha(1-\phi)A_s L_s^{\alpha\phi}(\gamma K_{ai})^{\alpha(1-\phi)-1}K_s^{1-\alpha}\gamma$. Since $\alpha(1-\phi)<1$, the exponent $\alpha(1-\phi)-1<0$, so $R\to+\infty$.

Case $\rho < 0$ ($\sigma < 1$, complements): This case requires care. As $K_{ai}\to 0$:

$$E_s = [\phi L_s^{\rho} + (1-\phi)(\gamma K_{ai})^{\rho}]^{1/\rho}.$$

Since $\rho<0$, $(\gamma K_{ai})^{\rho}\to+\infty$, so $E_s^{\rho}\to+\infty$, meaning $E_s\to 0$ (because $1/\rho<0$). Now examine R:

$$\begin{aligned}R &= p\,\gamma\,\alpha A_s E_s^{\alpha-1}K_s^{1-\alpha}(1-\phi)\left(\frac{\gamma K_{ai}}{E_s}\right)^{\rho-1}\\ &= p\,\gamma\,\alpha A_s K_s^{1-\alpha}(1-\phi)E_s^{\alpha-\rho}(\gamma K_{ai})^{\rho-1}.\end{aligned}$$

We must evaluate $E_s^{\alpha-\rho}(\gamma K_{ai})^{\rho-1}$ as $K_{ai}\to 0$. Since $\alpha<1$ and $\rho<0$, $\alpha-\rho>0$. Write

$E_s \sim [(1-\phi)]^{1/\rho}(\gamma K_{ai})$ as $K_{ai} \to 0$ (the AI term dominates in the CES when $\rho < 0$). Then:

$$E_s^{\alpha-\rho}(\gamma K_{ai})^{\rho-1} \sim [(1-\phi)]^{(\alpha-\rho)/\rho}(\gamma K_{ai})^{\alpha-\rho+\rho-1} = C \cdot (\gamma K_{ai})^{\alpha-1} \to +\infty$$

since $\alpha - 1 < 0$. Hence $R \to +\infty$ in this case as well.

Boundary behavior as $K_{ai} \to +\infty$.

Case $\rho > 0$: $E_s \sim [(1-\phi)]^{1/\rho}\gamma K_{ai}$, so $Y_s \sim C K_{ai}^{\alpha} K_s^{1-\alpha}$ and $R \sim C K_{ai}^{\alpha-1} \to 0$.

Case $\rho = 0$: $R \sim K_{ai}^{\alpha(1-\phi)-1} \to 0$.

Case $\rho < 0$: $E_s \to \phi^{1/\rho} L_s$ (the CES saturates at the human-labor level, since with $\rho < 0$ the limiting factor is the scarce input). Then $(\gamma K_{ai}/E_s)^{\rho-1} \to 0$ (since $\rho - 1 < 0$ and the ratio diverges). More precisely, $R \sim C(\gamma K_{ai})^{\rho-1} \to 0$ since $\rho < 1$.

By continuity and strict monotonicity of R from $+\infty$ to 0, there is a unique K_{ai}^* with $R(K_{ai}^*) = (\delta + \iota)c(\gamma)$. $\square$

Step 2: Monotonicity. $\partial K_{ai}^*/\partial p > 0$: higher p shifts $R(K_{ai})$ up by the factor $p/p_0 > 1$, so the crossing with the horizontal line $(\delta + \iota)c(\gamma)$ occurs at higher K_{ai}^*.

Step 3: Sorting locus is downward-sloping. Define $\tilde{G}_1(\eta^*, p) = h(\eta^*)w_s(\eta^*, K_{ai}^*(\eta^*, p), p) - w_u(\eta^*)$. Under R1, $\tilde{G}_{1,p} > 0$ (higher p raises w_s). For $\tilde{G}_{1,\eta^*} > 0$: (a) $h'(\eta^*) > 0$ contributes positively (the marginal worker has higher ability); (b) higher η^* reduces L_s (fewer skilled workers), which raises w_s through diminishing returns in the CES; and (c) higher η^* raises $L_u = F(\eta^*)$, which lowers w_u through diminishing returns in the unskilled sector. Each effect pushes $\tilde{G}_1$ up. So $dp/d\eta^*\big|_{\tilde{G}_1=0} = -\tilde{G}_{1,\eta^*}/\tilde{G}_{1,p} < 0$.

Step 4: Price locus is upward-sloping. More workers in the unskilled sector (η^* up) raises Y_u and reduces Y_s, so Y_u/Y_s increases. CES demand gives $C_s/C_u = [\theta/(1-\theta)]p^{-\varepsilon}$. Market clearing requires $Y_s/Y_u = C_s/C_u$ (adjusting for AI investment costs, which are second-order near equilibrium). A higher Y_u/Y_s requires higher p to induce consumers to shift demand toward the skilled good. So $dp/d\eta^*\big|_{\tilde{G}_3=0} > 0$.

Step 5: Unique crossing. The sorting residual $\tilde{S}(\eta^*) = \tilde{G}_1(\eta^*, p_{\text{price}}(\eta^*))$ is strictly increasing: $\tilde{S}'(\eta^*) = \tilde{G}_{1,\eta^*} + \tilde{G}_{1,p} \cdot p'_{\text{price}}(\eta^*)$. The first term is positive (Step 3) and the second is positive (since $\tilde{G}_{1,p} > 0$ and $p'_{\text{price}} > 0$ from Step 4).

Boundary behavior: As $\eta^* \to 0^+$: $h(\eta^*) = (\eta^*)^2 \to 0$ while $w_u > 0$ (since $L_u = F(\eta^*) \to 0$ but the unskilled sector has positive output from capital), so $\tilde{S}(0^+) < 0$. As $\eta^* \to 1^-$: $L_s = \int_{\eta^*}^1 h(\eta)dF(\eta) \to 0$, so $w_s \to +\infty$ by the Inada-type condition from Step 1 (applied to labor rather than AI capital), giving $h(1)w_s \to +\infty$ and $\tilde{S}(1^-) > 0$.

By the intermediate value theorem, there is a unique $\eta_0^* \in (0,1)$ with $\tilde{S}(\eta_0^*) = 0$. $\square$

B Proof of Proposition 4: Master Formula

The six-equation log-linearized system is:

$$\hat{w}_u = -(1-\beta)\mu\,\hat{\eta}^* \tag{A.1}$$
$$\hat{w}_s = \hat{p} + (1-\rho)\nu\hat{\eta}^* + (\alpha-\rho)\hat{E}_s \tag{A.2}$$
$$\hat{E}_s = -s_L\nu\hat{\eta}^* + (1-s_L)(\hat{\gamma} + \hat{K}_{ai}) \tag{A.3}$$
$$\hat{p} = \tfrac{1}{\varepsilon}(\beta\mu\hat{\eta}^* - \alpha\hat{E}_s) \tag{A.4}$$
$$\varepsilon_h\hat{\eta}^* + \hat{w}_s = \hat{w}_u \tag{A.5}$$
$$0 = \hat{p} + \rho\hat{\gamma} + (\rho-1)\hat{K}_{ai} + (\alpha-\rho)\hat{E}_s \tag{A.6}$$

Derivation of each equation.

(A.1): From $w_u = \beta A_u L_u^{\beta-1} K_u^{1-\beta}$, $\hat{w}_u = (\beta-1)\hat{L}_u = (\beta-1)\mu\hat{\eta}^* = -(1-\beta)\mu\hat{\eta}^*$, using $\hat{L}_u = d\ln F(\eta^*) = [f(\eta^*)\eta^*/F(\eta^*)]\hat{\eta}^* = \mu\hat{\eta}^*$.

(A.2): From $w_s = p\alpha(Y_s/E_s)\phi(L_s/E_s)^{\rho-1}$, $\hat{w}_s = \hat{p} + (\alpha-1)\hat{E}_s + (\rho-1)(\hat{L}_s - \hat{E}_s) = \hat{p} + (\rho-1)\hat{L}_s + (\alpha-\rho)\hat{E}_s$, with $\hat{L}_s = -\nu\hat{\eta}^*$.

(A.3): From the CES, $\hat{E}_s = s_L\hat{L}_s + (1-s_L)(\hat{\gamma} + \hat{K}_{ai})$.

(A.4): From goods market clearing with CES demand: $\hat{Y}_s = -\varepsilon\hat{p} + \hat{I}$ where $\hat{I}$ is aggregate income. Using $\hat{Y}_s = \alpha\hat{E}_s + (1-\alpha)\hat{K}_s = \alpha\hat{E}_s$ (since $\hat{K}_s = 0$ in the analytical system) and substituting the income expression (which equals $\beta\mu\hat{\eta}^*$ to first order from labor reallocation), we obtain (A.4).

(A.5): Differentiating $h(\eta^*)w_s = w_u$: $\varepsilon_h\hat{\eta}^* + \hat{w}_s = \hat{w}_u$.

(A.6): The zero-profit condition $r_{ai} = (\delta+\iota)c_0\gamma^{\varepsilon_c}$ log-differentiates to $\hat{r}_{ai} = \varepsilon_c\hat{\gamma}$. From (5), $\hat{r}_{ai} = \hat{p} + \hat{\gamma} + (\rho-1)\hat{K}_{ai} + (\alpha-\rho)\hat{E}_s$. Setting $\varepsilon_c = 0$ (the general case follows by replacing $\hat{\gamma}$ with $(1-\varepsilon_c)\hat{\gamma}$): $0 = \hat{p} + \rho\hat{\gamma} + (\rho-1)\hat{K}_{ai} + (\alpha-\rho)\hat{E}_s$.

Step 1. (A.5) and (A.1) give $\hat{w}_s = -E_0\hat{\eta}^*$.

Step 2. Substitute (A.3) into (A.2), (A.4), (A.6):

$$\hat{w}_s = \Gamma\hat{\eta}^* + \Omega(\hat{\gamma} + \hat{K}_{ai}) \tag{A.2$'$}$$
$$\hat{p} = \tfrac{1}{\varepsilon}\Big[A_1\hat{\eta}^* - \alpha(1-s_L)(\hat{\gamma} + \hat{K}_{ai})\Big] \tag{A.4$'$}$$
$$0 = F_1\hat{\eta}^* + \lambda'\hat{\gamma} - \Delta'\hat{K}_{ai} \tag{A.6$'$}$$

where:

$$\Delta = (1-\alpha) + s_L(\alpha - \rho), \qquad \Delta' = \Delta + \alpha(1-s_L)/\varepsilon,$$
$$\lambda = \rho s_L + \alpha(1-s_L), \qquad \lambda' = \lambda - \alpha(1-s_L)/\varepsilon,$$
$$F_1 = \beta\mu/\varepsilon - s_L\nu\Omega/(1-s_L).$$

Derivation of (A.6′). From (A.6): $0 = \hat{p} + \rho\hat{\gamma} + (\rho-1)\hat{K}_{ai} + (\alpha-\rho)\hat{E}_s$. Substitute (A.4′) for $\hat{p}$ and (A.3) for $\hat{E}_s$:

$$\begin{aligned} 0 = {} & \frac{1}{\varepsilon}[A_1\hat{\eta}^* - \alpha(1-s_L)(\hat{\gamma} + \hat{K}_{ai})] + \rho\hat{\gamma} + (\rho-1)\hat{K}_{ai} \\ & + (\alpha-\rho)[-s_L\nu\hat{\eta}^* + (1-s_L)(\hat{\gamma} + \hat{K}_{ai})]. \end{aligned}$$

Collecting $\hat{\eta}^*$ terms: $A_1/\varepsilon - (\alpha-\rho)s_L\nu = \beta\mu/\varepsilon + \alpha s_L\nu/\varepsilon - (\alpha-\rho)s_L\nu = F_1$.

Collecting $\hat{\gamma}$ terms: $-\alpha(1-s_L)/\varepsilon + \rho + (\alpha-\rho)(1-s_L) = \rho s_L + \alpha(1-s_L) - \alpha(1-s_L)/\varepsilon = \lambda'$.

Collecting $\hat{K}_{ai}$ terms: $-\alpha(1-s_L)/\varepsilon + (\rho-1) + (\alpha-\rho)(1-s_L) = -\Delta'$.

Step 3: Key identity.

$$\Delta' + \lambda' = \underbrace{\Delta + \lambda}_{=1} = 1.$$

Verification: $\Delta + \lambda = (1-\alpha) + s_L(\alpha-\rho) + \rho s_L + \alpha(1-s_L) = 1 - \alpha + \alpha s_L - s_L\rho + \rho s_L + \alpha - \alpha s_L = 1$.

Step 4. Solve (A.6′) for $\hat{K}_{ai}$:

$$\hat{K}_{ai} = \frac{F_1}{\Delta'}\hat{\eta}^* + \frac{\lambda'}{\Delta'}\hat{\gamma}.$$

Substitute into (A.2′). Using $\Delta' + \lambda' = 1$:

$$\begin{aligned} \hat{w}_s &= \Gamma\hat{\eta}^* + \Omega\left[\hat{\gamma} + \frac{F_1}{\Delta'}\hat{\eta}^* + \frac{\lambda'}{\Delta'}\hat{\gamma}\right] \\ &= \left(\Gamma + \frac{\Omega F_1}{\Delta'}\right)\hat{\eta}^* + \frac{\Omega}{\Delta'}\hat{\gamma} = \tilde{\Gamma}\hat{\eta}^* + \frac{\Omega}{\Delta'}\hat{\gamma}, \end{aligned}$$

where $\tilde{\Gamma} = \Gamma + \Omega F_1/\Delta'$.

Step 5. From $\hat{w}_s = -E_0\hat{\eta}^*$:

$$-(E_0 + \tilde{\Gamma})\hat{\eta}^* = \frac{\Omega}{\Delta'}\hat{\gamma} \qquad \Longrightarrow \qquad \hat{\eta}^* = \frac{-\Omega}{\Delta'\tilde{D}}\hat{\gamma},$$

where $\tilde{D} = E_0 + \tilde{\Gamma} = D + \Omega F_1/\Delta'$.

Step 6. Back-substitute into $\hat{w}_s$, $\hat{w}_u$, and $\hat{p}$:

$$
\begin{aligned}
\hat{w}_s &= \frac{\Omega E_0}{\Delta' \tilde{D}} \hat{\gamma}, \\
\hat{w}_u &= \frac{(1-\beta)\mu\Omega}{\Delta' \tilde{D}} \hat{\gamma}, \\
\hat{p} &= \frac{1}{\varepsilon}\left[\frac{-A_1\Omega}{\Delta'\tilde{D}} - \alpha(1-s_L)\left(1 + \frac{F_1}{\Delta'} \cdot \frac{-\Omega}{\Delta'\tilde{D}} + \frac{\lambda'}{\Delta'}\right)\right]\hat{\gamma}.
\end{aligned}
$$

Form $\widehat{w_s/P} = \hat{w}_s - \omega_s \hat{p}$:

$$
\widehat{w_s/P} = \frac{1}{\Delta'}\left[\frac{\Omega\tilde{B}}{\tilde{D}} + C\right]\hat{\gamma}
$$

where $\tilde{B} = E_0 + \omega_s \tilde{A}_1/\varepsilon$ and $\tilde{A}_1 = A_1 - \alpha(1-s_L)F_1/\Delta'$.

Step 7: Threshold invariance. We claim $\tilde{B} = B - CF_1/\Delta'$. Indeed:

$$
\tilde{B} = E_0 + \frac{\omega_s}{\varepsilon}\left(A_1 - \frac{\alpha(1-s_L)F_1}{\Delta'}\right) = B - \frac{\omega_s\alpha(1-s_L)}{\varepsilon} \cdot \frac{F_1}{\Delta'} = B - \frac{CF_1}{\Delta'}.
$$

At the threshold $\widehat{w_s/P} = 0$:

$$
0 = \Omega\tilde{B} + C\tilde{D} = \Omega\left(B - \frac{CF_1}{\Delta'}\right) + C\left(D + \frac{\Omega F_1}{\Delta'}\right) = \Omega B + CD.
$$

The terms $CF_1\Omega/\Delta'$ cancel exactly. This is the fixed-K_{ai} threshold condition. The same argument with $\tilde{B}_u = \tilde{B} - \varepsilon_h$ gives $\Omega B_u + CD = 0$ for the unskilled threshold. □

C Proof of Proposition 9: Labor Share

Within-sector labor share. From the CES, $s_L = \phi L_s^\rho / E_s^\rho$, so:

$$
\hat{s}_L = \rho(\hat{L}_s - \hat{E}_s) = \rho\big[-\nu\hat{\eta}^* - (-s_L\nu\hat{\eta}^* + (1-s_L)Z)\big] = \rho\big[-(1-s_L)\nu\hat{\eta}^* - (1-s_L)Z\big]
$$

where $Z = \hat{\gamma} + \hat{K}_{ai}$ is the log-change in effective AI input per unit of quality. Thus:

$$
\hat{s}_L = -\rho(1-s_L)(\nu\hat{\eta}^* + Z).
$$

We need to sign $(\nu\hat{\eta}^* + Z)$. This equals $\hat{L}_s / -1 + Z = -($ change in AI relative to labor). Wait—more carefully: $\nu\hat{\eta}^* + Z = -\hat{L}_s/\nu \cdot \nu + Z\ldots$ Let us proceed via the equilibrium solutions.

From Step 5 in Appendix B: $\hat{\eta}^* = -\Omega/(\Delta'\tilde{D})\hat{\gamma}$. From Step 4: $\hat{K}_{ai} = (F_1/\Delta')\hat{\eta}^* + (\lambda'/\Delta')\hat{\gamma}$. So:

$$
\begin{aligned}
Z = \hat{\gamma} + \hat{K}_{ai} &= \hat{\gamma}\left(1 + \frac{\lambda'}{\Delta'}\right) + \frac{F_1}{\Delta'}\hat{\eta}^* \\
&= \frac{\hat{\gamma}}{\Delta'} + \frac{F_1}{\Delta'}\hat{\eta}^* = \frac{1}{\Delta'}\left[\hat{\gamma} - \frac{\Omega F_1}{\Delta'\tilde{D}}\hat{\gamma}\right] = \frac{\hat{\gamma}}{\Delta'}\left[1 - \frac{\Omega F_1}{\Delta'\tilde{D}}\right].
\end{aligned}
$$

And $\nu\hat{\eta}^* = -\nu\Omega/(\Delta'\tilde{D})\hat{\gamma}$. Therefore:

$$
\begin{aligned}
\nu\hat{\eta}^* + Z &= \frac{\hat{\gamma}}{\Delta'\tilde{D}}\left[-\nu\Omega + \tilde{D} - \Omega F_1/\Delta'\right] \\
&= \frac{\hat{\gamma}}{\Delta'\tilde{D}}[-\nu\Omega + D]
\end{aligned}
$$

using $\tilde{D} - \Omega F_1/\Delta' = D$.

Now define $\Theta \equiv D - \nu\Omega$:

$$
\begin{aligned}
\Theta &= E_0 + \Gamma - \nu\Omega \\
&= \varepsilon_h + (1-\beta)\mu + \frac{A_1}{\varepsilon} + A_2 - \nu\Omega.
\end{aligned}
$$

Substituting the definitions:

$$
\begin{aligned}
\Theta = \varepsilon_h + (1-\beta)\mu + \frac{\beta\mu + \alpha s_L\nu}{\varepsilon} + \nu[(1-\rho)(1-s_L) + (1-\alpha)s_L] \\
- \nu(1-s_L)[\alpha(1-1/\varepsilon) - \rho].
\end{aligned}
$$

Expanding the last term:

$$
\nu(1-s_L)[\alpha(1-1/\varepsilon) - \rho] = \nu(1-s_L)\alpha - \nu(1-s_L)\alpha/\varepsilon - \nu(1-s_L)\rho.
$$

Combining with $A_2 = \nu(1-\rho)(1-s_L) + \nu(1-\alpha)s_L = \nu(1-s_L) - \nu\rho(1-s_L) + \nu s_L - \nu\alpha s_L$:

$$
\begin{aligned}
A_2 - \nu(1-s_L)\alpha + \nu(1-s_L)\rho &= \nu(1-s_L)(1-\alpha) + \nu s_L(1-\alpha) \\
&= \nu(1-\alpha).
\end{aligned}
$$

So:

$$
\Theta = \varepsilon_h + (1-\beta)\mu + \frac{\beta\mu + \alpha s_L\nu + \nu(1-s_L)\alpha}{\varepsilon} + \nu(1-\alpha) = \varepsilon_h + (1-\beta)\mu + \frac{\beta\mu + \alpha\nu}{\varepsilon} + \nu(1-\alpha).
$$

Every term is strictly positive (since $\varepsilon_h = 2$, $\mu > 0$, $\nu > 0$, $\beta < 1$, $\alpha < 1$, $\varepsilon > 0$). Therefore $\Theta > 0$ and:

$$\hat{s}_L = -\rho(1 - s_L)\frac{\Theta}{\Delta'\tilde{D}}\hat{\gamma}, \tag{40}$$

with $\mathrm{sgn}(\hat{s}_L) = \mathrm{sgn}(-\rho)$. When $\sigma > 1$ ($\rho > 0$), $\hat{s}_L < 0$: AI displaces labor's share within the CES. When $\sigma < 1$ ($\rho < 0$), $\hat{s}_L > 0$: complementarity raises labor's share.

Between-sector revenue share. $s_{rev} = pY_s/(pY_s + Y_u)$. With CES demand:

$$s_{rev} = \frac{\theta p^{1-\varepsilon}}{\theta p^{1-\varepsilon} + (1-\theta)} = \omega_s.$$

Log-differentiating:

$$\hat{s}_{rev} = (1-\varepsilon)(1 - s_{rev})\hat{p}.$$

Under R2, $\hat{p} < 0$, so:

$$\mathrm{sgn}(\hat{s}_{rev}) = \mathrm{sgn}[(1-\varepsilon)(-1)] = \mathrm{sgn}(\varepsilon - 1).$$

When $\varepsilon > 1$: consumers substitute strongly toward the cheaper skilled good, so s_{rev} rises. When $\varepsilon < 1$: demand is inelastic, s_{rev} falls.

Combining. $\lambda_L = \beta + s_{rev}(\alpha s_L - \beta)$ gives:

$$d\lambda_L = (\alpha s_L - \beta)ds_{rev} + s_{rev}\alpha\, ds_L.$$

Under $\beta \geq \alpha$: $\alpha s_L < \alpha \leq \beta$, so $(\alpha s_L - \beta) < 0$.

Within-sector: $\mathrm{sgn}(ds_L) = \mathrm{sgn}(-\rho)$. Contribution to $d\lambda_L$: sign $(-\rho)$ since $s_{rev}\alpha > 0$.

Between-sector: $\mathrm{sgn}(ds_{rev}) = \mathrm{sgn}(\varepsilon - 1)$. With $(\alpha s_L - \beta) < 0$: contribution to $d\lambda_L$ has sign $(-1) \times \mathrm{sgn}(\varepsilon - 1) = \mathrm{sgn}(1 - \varepsilon)$.

If $\sigma > 1$ ($\rho > 0$): within-sector contributes negatively. If also $\varepsilon \geq 1$: between-sector contributes negatively (or zero). Both push $d\lambda_L < 0$.

If $\sigma < 1$ ($\rho < 0$): within-sector contributes positively. If also $\varepsilon \leq 1$: between-sector contributes positively (or zero). Both push $d\lambda_L > 0$. □

References

Note to editors: All reference details (volume numbers, page ranges, years) require verification against the original publications before final submission. Page ranges listed below are *from the authors' records and have not been independently verified for this draft.*

References

Acemoglu, D. and D. Autor (2011). "Skills, Tasks and Technologies: Implications for Employment and Earnings." *Handbook of Labor Economics*, Vol. 4B, pp. 1043–1171.

Acemoglu, D. and P. Restrepo (2018). "The Race between Man and Machine: Implications of Technology for Growth, Factor Shares, and Employment." *American Economic Review*, 108(6), pp. 1488–1542.

Acemoglu, D. and P. Restrepo (2020). "Robots and Jobs: Evidence from US Labor Markets." *Journal of Political Economy*, 128(6), pp. 2188–2244.

Aghion, P., B.F. Jones, and C.I. Jones (2019). "Artificial Intelligence and Economic Growth." In A. Agrawal, J. Gans, and A. Goldfarb (eds.), *The Economics of Artificial Intelligence*. University of Chicago Press.

Autor, D.H. (2015). "Why Are There Still So Many Jobs? The History and Future of Workplace Automation." *Journal of Economic Perspectives*, 29(3), pp. 3–30.

Autor, D.H. and D. Dorn (2013). "The Growth of Low-Skill Service Jobs and the Polarization of the US Labor Market." *American Economic Review*, 103(5), pp. 1553–1597.

Autor, D.H., D. Dorn, L.F. Katz, C. Patterson, and J. Van Reenen (2020). "The Fall of the Labor Share and the Rise of Superstar Firms." *Quarterly Journal of Economics*, 135(2), pp. 645–709.

Autor, D.H., F. Levy, and R.J. Murnane (2003). "The Skill Content of Recent Technological Change: An Empirical Exploration." *Quarterly Journal of Economics*, 118(4), pp. 1279–1333.

Brynjolfsson, E. and T. Mitchell (2017). "What Can Machine Learning Do? Workforce Implications." *Science*, 358(6370), pp. 1530–1534.

Grossman, G.M. and E. Rossi-Hansberg (2008). "Trading Tasks: A Simple Theory of Offshoring." *American Economic Review*, 98(5), pp. 1978–1997.

Hampole, M., D. Papanikolaou, L.D.W. Schmidt, and B. Seegmiller (2025). "Artificial Intelligence and the Labor Market." NBER Working Paper No. 33509.

Heckman, J.J. and B.E. Honoré (1990). "The Empirical Content of the Roy Model." *Econometrica*, 58(5), pp. 1121–1149.

Karabarbounis, L. and B. Neiman (2014). "The Global Decline of the Labor Share." *Quarterly Journal of Economics*, 129(1), pp. 61–103.

Korinek, A. and J.E. Stiglitz (2019). "Artificial Intelligence and Its Implications for Income Distribution and Unemployment." In A. Agrawal, J. Gans, and A. Goldfarb (eds.), *The Economics of Artificial Intelligence*. University of Chicago Press.

Krusell, P., L.E. Ohanian, J.-V. Ríos-Rull, and G.L. Violante (2000). "Capital-Skill Complementarity and Inequality: A Macroeconomic Analysis." *Econometrica*, 68(5), pp. 1029–1053.

Roy, A.D. (1951). "Some Thoughts on the Distribution of Earnings." *Oxford Economic Papers*, 3(2), pp. 135–146.

Zeira, J. (1998). "Workers, Machines, and Economic Growth." *Quarterly Journal of Economics*, 113(4), pp. 1091–1117.