

# nimbleHMC: An R package for Hamiltonian Monte Carlo sampling in nimble

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## Summary

Markov chain Monte Carlo (MCMC) algorithms are widely used for fitting hierarchical models to data. MCMC is the predominant tool used in Bayesian analyses to generate samples from the posterior distribution of model parameters conditional on observed data. MCMC is not a single algorithm, but actually a framework that admits assignment of a variety of sampling techniques to unobserved parameters. There exists a vast set of valid samplers to draw upon, which differ in complexity, autocorrelation of samples produced, and applicability.

Hamiltonian Monte Carlo [HMC; Brooks et al. (2011)] sampling is one such technique, applicable to continuous-valued parameters, which uses the gradients to generate large transitions in parameter space. The resulting samples have low autocorrelation, and therefore have high information content, relative for example to an equal-length sequence of highly autocorrelated samples. The No-U-Turn (NUTS) variety of HMC sampling [HMC-NUTS; Hoffman and Gelman (2014)] greatly increases the usability of HMC by self-adapting key sampler tuning parameters upon which the overall performance is highly dependent.

Many software packages offer implementations of MCMC, such as **nimble** (de Valpine et al. 2017), **WinBUGS** (Lunn et al. 2000), **jags** (Plummer 2003), **pyMC** (Fonnesbeck et al. 2015), **NumPyro** (Phan, Pradhan, and Jankowiak 2019), **TensorFlow Probability** (Pang, Nijkamp, and Wu 2020), and **Stan** (Carpenter et al. 2017), among others. These packages differ, however, in their approaches to sampler assignments. As sampling techniques vary in computation and quality of the samples, the effectiveness of the MCMC algorithms will vary depending on the software and model.

Among MCMC software packages, only **nimble** opens the hood of the sampler assignment process. Users may select any valid assignment of one or more samplers to each parameter, selecting from the suite of samplers provided with **nimble**. These include random walk Metropolis-Hastings sampling (Robert and Casella 1999), slice sampling (Neal 2003), elliptical slice sampling (Murray, Adams, and MacKay 2010), automated factor slice sampling (Tibbits et al. 2014), conjugate sampling (George, Makov, and Smith 1993), and others.

The **nimbleHMC** package provides two implementations of HMC-NUTS sampling for use within **nimble**. Specifically, **nimbleHMC** provides an implementation of the original (“classic”) HMC-NUTS algorithm as developed in Hoffman and Gelman (2014), and a modern version of HMC-NUTS sampling matching the HMC sampler available in version 2.32.2 of **Stan** (Stan Development Team 2023). The samplers provided in **nimbleHMC** can be assigned to any continuous-valued parameters, and may be used in combination with other samplers provided with **nimble**.

## Example

The following example demonstrates fitting a hierarchical model to data using **nimbleHMC**. We use the European Dipper *Cinclus cinclus* dataset drawn from ecological capture-recapture (*e.g.*, Lebreton et al. 1992; Turek, de Valpine, and Paciorek 2016). Modelling includes both continuous parameters to undergo HMC

sampling and discrete parameters that cannot be sampled via HMC. As far as we know, this combination is not supported by software other than `nimbleHMC`.

Individual birds are captured, tagged, and potentially recaptured on subsequent sighting occasions. Data is a  $294 \times 7$  binary-valued array of capture histories of 294 uniquely tagged birds over 7 years. Model parameters are detection probability  $p$ , and annual survival rates on non-flood years  $\phi_1$  and flood years  $\phi_2$ . Data is provided in the R package `mra` (McDonald 2018).

```
library(mra)
data(dipper.data)
y <- dipper.data[,1:7]
```

We specify the hierarchical model using uniform priors on the interval  $[0, 1]$  for all parameters. Binary-valued latent states  $x_{i,t}$  represent the true alive (1) or dead (0) state of individual  $i$  on year  $t$ . Doing so allows the survival process to be modelled as  $x_{i,t+1} \sim \text{Bernoulli}(\phi_{f_t} \cdot x_{i,t})$  where  $f_t$  indicates the flood/non-flood history of year  $t$ , and observations are modelled as  $y_{i,t} \sim \text{Bernoulli}(p \cdot x_{i,t})$ .

```
library(nimbleHMC)

code <- nimbleCode({
  phi[1] ~ dunif(0, 1)
  phi[2] ~ dunif(0, 1)
  p ~ dunif(0, 1)
  for(i in 1:N) {
    for(t in (first[i]+1):T) {
      x[i,t] ~ dbern(phi[f[t]] * x[i,t-1])
      y[i,t] ~ dbern(p * x[i,t])
    }
  }
})
```

A `nimble` model object is now built. The argument `buildDerivs = TRUE` allows derivatives of likelihood calculations to be built into the model object to support derivative-based algorithms – here, HMC sampling.

```
Rmodel <- nimbleModel(
  code,
  constants = list(N = nrow(y), T = ncol(y), f = c(1,2,2,1,1,1,1),
    first = apply(y, 1, which.max)),
  data = list(y = y),
  inits = list(phi = c(0.5, 0.5), p = 0.5, x = array(1, dim(y))),
  buildDerivs = TRUE)
```

Next we create an MCMC configuration object, which specifies the sampling algorithm to be applied to each parameter. By default, `configureMCMC` uses `nimble`'s default sampler assignments of adaptive random walk Metropolis-Hastings [RW sampler; Robert and Casella (1999)] for each parameter, and a `binary` Gibbs sampler for each  $x_{i,t}$  latent state.

```
conf <- configureMCMC(Rmodel)

## RW sampler (3)
## - phi[] (2 elements)
## - p
## binary sampler (848)
## - x[] (848 elements)
```

Now we customize the MCMC configuration object to use HMC sampling for the model parameters. `replaceSamplers` replaces the samplers operating on  $\phi_1$ ,  $\phi_2$  and  $p$  with the modern HMC-NUTS sampler (called the NUTS sampler) provided in `nimbleHMC`. The classic version of the HMC-NUTS sampler could

be assigned by specifying `type = "NUTS_classic"`.

```
conf$replaceSamplers(target = c("phi", "p"), type = "NUTS")
conf$printSamplers(byType = TRUE)
```

```
## NUTS sampler (1)
##   - phi, p
## binary sampler (848)
##   - x[] (848 elements)
```

Alternatively, the convenience function `configureHMC(Rmodel)` may be used to create an identical MCMC configuration, applying HMC-NUTS sampling to  $\phi_1$ ,  $\phi_2$  and  $p$ , and default binary samplers for discrete parameters.

Now we build and compile the MCMC algorithm.

```
Rmcmc <- buildMCMC(conf)
Cmodel <- compileNimble(Rmodel)
Cmcmc <- compileNimble(Rmcmc, project = Rmodel)
```

We execute the MCMC for 20,000 iterations, and discard the initial 10,000 samples as burn-in.

```
set.seed(0)
samples <- runMCMC(Cmcmc, niter = 20000, nburnin = 10000)
```

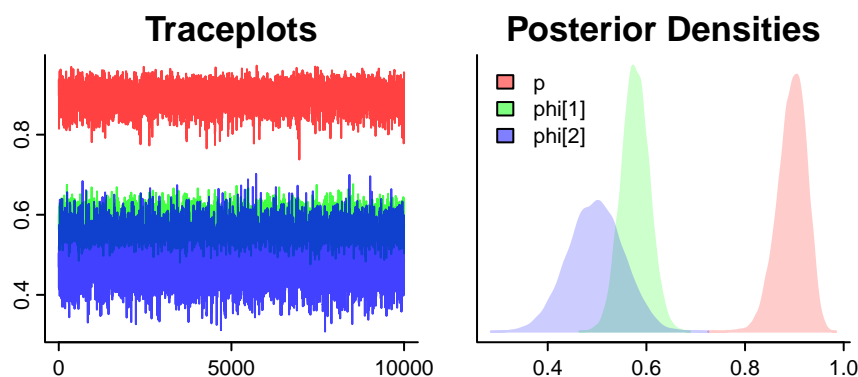
Finally, posterior summary statistics are calculated for the model parameters.

```
samplesSummary(samples, round = 2)
```

```
##      Mean Median St.Dev. 95%CI_low 95%CI_upp
## p      0.89   0.90   0.03     0.83     0.94
## phi[1] 0.58   0.58   0.03     0.52     0.63
## phi[2] 0.50   0.50   0.06     0.39     0.60
```

Traceplots and posterior density plots are generated using the `samplesPlot` function from the `basicMCMCplots` package.

```
basicMCMCplots::samplesPlot(samples, legend.location = "topleft")
```



## Statement of need

HMC is recognized as a state-of-the-art MCMC sampling algorithm. A testimony to this, software packages such as **Stan** exclusively employ HMC sampling. Consequently, such software cannot operate on models containing discrete parameters (upon which HMC cannot operate). Models with discrete parameters arise in a range of statistical motifs including hidden Markov models, finite mixture models, and generally in the presence of unobserved categorical data (Bartolucci, Pandolfi, and Pennoni 2022). In contrast, other

mainstream MCMC packages (`WinBUGS`, `OpenBUGS` and `jags`) can sample discrete parameters, but provide no facilities for HMC sampling. This leaves a gap, as there is no support for HMC sampling of hierarchical models that also contain discrete parameters.

`nimbleHMC` fills this gap, by providing two HMC samplers that operate inside `nimble`’s MCMC engine. The base `nimble` package provides a variety of MCMC sampling algorithms, as well as the ability to customize MCMC sampler assignments. `nimbleHMC` augments the set of sampling algorithms provided in `nimble` with two options for HMC sampling, which can be used alongside any other samplers. The example presented here demonstrates precisely that: HMC sampling operating alongside discrete samplers, which is not possible without the use of `nimbleHMC`.

It’s an open question of what combination of samplers will optimize MCMC efficiency. One metric of comparison is the effective sample size of the samples generated per unit runtime of the algorithm. That is, how quickly an MCMC algorithm generates information about the parameters. This metric is studied in Turek et al. (2017) and Ponisio et al. (2020), but without any conclusive result. For that reason, the ability to mix-and-match samplers from a large pool of candidates is important from both practical and theoretical standpoints. Indeed, packages such as `compareMCMCs` (de Valpine, Paganin, and Turek 2022) exist specifically to compare the relative performance of MCMC algorithms. The addition of HMC sampling provided by `nimbleHMC` supports new practical combinations for applied MCMC, as well as facilitates a deeper study of Bayesian modelling.

## References

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