# Fixed Income Performance Attribution

EDITOR
Stéphane Daul
RiskMetrics Group, Inc.
stephane.daul@riskmetrics.com

CONTRIBUTORS
Nicholas Sharp
RiskMetrics Group, Inc.
nick.sharp@riskmetrics.com

Lars Qvigstad Sørensen
RiskMetrics Group, Inc.
lars.sorensen@riskmetrics.com

RiskMetrics Group

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**On the Cover:**

Cap volatilities on two dates: 29 May 2009 (grey) and 5 June 2009 (red).

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# Foreword

This document sets the foundation of our fixed income performance attribution application. It builds on our risk analytics and reviews the concepts and mathematical models of fixed income attribution. It should be read as a research review paper and not a product specification. We encourage readers to provide feedback or submit questions to `fia@riskmetrics.com`.

# Contents

# Chapter 1

# Introduction

Performance attribution is a set of techniques used by portfolio managers for the purpose of measuring and explaining portfolio performance relative to a benchmark. The objective is to quantify the impact that active management decisions have on the relative performance (i.e. active return) over a given interval of time.

Each decision taken by the portfolio manager is associated with an attribution effect that quantitatively measures the amount by which the decision affects the relative performance. The attribution effects aggregate together to fully account for the active return. Because attribution effects describe the impact of active management decisions it is important that the attribution model reflects the decision-making process. A long-only government bond portfolio is certainly managed differently from an emerging-market portfolio, or a credit derivatives portfolio.

The standard attribution methodology, introduced by Brinson and Fachler (1985), attributes the active return to asset allocation and security selection decisions. The methodology boils down to grouping positions in a reference portfolio in such a way to make the attribution effects apparent, and hence its current name, the asset-grouping approach. This approach has been widely extended and is particularly well suited for equity portfolios. [1]

However for fixed-income portfolios this approach is not as successful. Fixed-income managers need specialized attribution models that for example incorporate all the effects of yield-curve movements. For instance it is difficult to find any reference portfolio that captures only a butterfly effect of the yield curve. Another route is the factor based approach, where the performance of all securities is first decomposed using systematic factors and then aggregated. We consider a configurable, extendable

[1] This is actually the approach that RiskMetrics has taken for equity attribution, see Soor (2010).

hybrid approach to fixed-income performance attribution where attribution is decomposed into many fixed-income factors as well as simultaneously carrying out an asset-grouping approach to performance attribution where necessary.

To present the content of this review paper, we show what an attribution report could look like in Table 1.1. This fictious example presents a portfolio of fixed income instruments along with their relative attribution to the benchmark. We start by calculating the total return of each securities as shown in Chapter 2, and report the active return (I), i.e. the security return relative to the benchmark. The active return is then decomposed (II) using the fixed income factor model developed in Chapter 3. The factors may not capture all investment decisions, and we apply an asset-grouping approach on the residual (III) as detailed in Chapter 4. Finally we look in more details at how exposures need to be calculated in Chapter 5 and we analyze benchmarks in Chapter 6.

*Table 1.1*
**Illustrative attribution report**

| | (I) | (II) | | | | | | (III) | |
|---|---|---|---|---|---|---|---|---|---|
| | Active Return | Currency | Carry | Yield Curve | Credit | Volatility | Residual | Allocation | Selection |
| Security A | 0.170% | 0.075% | 0.002% | 0.133% | 0.003% | 0.007% | -0.050% | -0.037% | -0.012% |
| Security B | -0.041% | -0.075% | -0.001% | -0.111% | -0.002% | 0.001% | 0.147% | 0.111% | 0.037% |
| Security C | 0.447% | 0.000% | 0.059% | 0.441% | 0.002% | -0.008% | -0.047% | -0.035% | -0.012% |
| Security D | 0.220% | 0.050% | 0.024% | 0.150% | 0.007% | 0.010% | -0.020% | -0.015% | -0.005% |
| Security E | -0.111% | -0.030% | -0.002% | -0.281% | -0.017% | 0.025% | 0.195% | 0.146% | 0.049% |
| Security F | 0.364% | 0.075% | 0.052% | 0.631% | -0.027% | 0.062% | -0.429% | -0.322% | -0.107% |
| Security G | 0.052% | -0.075% | 0.000% | 0.080% | 0.001% | 0.008% | 0.038% | 0.028% | 0.009% |
| Security H | -0.044% | 0.000% | 0.000% | -0.162% | -0.004% | 0.001% | 0.121% | 0.091% | 0.030% |
| Security I | 0.213% | 0.050% | 0.003% | 0.032% | 0.002% | -0.004% | 0.129% | 0.097% | 0.032% |
| Security J | 0.120% | -0.030% | 0.007% | 0.327% | 0.003% | 0.008% | -0.194% | -0.145% | -0.048% |
| Security K | 0.120% | 0.075% | 0.011% | 0.168% | 0.002% | 0.006% | -0.143% | -0.107% | -0.036% |
| Security L | -0.462% | -0.075% | 0.002% | -1.110% | -0.041% | -0.090% | 0.852% | 0.639% | 0.213% |
| Security M | -0.131% | 0.000% | 0.000% | -0.121% | -0.007% | -0.020% | 0.016% | 0.012% | 0.004% |
| Total | 0.916% | 0.041% | 0.154% | 0.177% | -0.078% | 0.005% | 0.617% | 0.463% | 0.154% |

# Chapter 2

# Total Return

The total return is the percentage change in the value of a security resulting from changes in the market value of the security, interest income accrued or received, and reinvestment income over the attribution period. The attribution period is defined as the time that elapses between the start date $t_1$ and end date $t_2$.

The total return $r_{\text{total}}$ of a fixed-income security (in the security's local currency) over the attribution period $[t_1, t_2]$ is defined as

$$r_{\text{total}} = \frac{V(t_2) - V(t_1) + C(t_1, t_2)}{V(t_1)}, \tag{2.1}$$

where $V(t_1)$ is the market value (this is the dirty/invoice price in the case of bonds) of the security at the start of the attribution period, and similarly $V(t_2)$ is the market value of the security at the end of the attribution period. Also, $C(t_1, t_2)$ is the sum of payments received during the attribution period.

## 2.1 Payments

There are two sources of cash that accumulate in a portfolio, these are coupon payments and principal payments, the latter includes both scheduled and unscheduled prepayments.

#### Coupon payments

Interest income in the form of coupon payments is the most common form of cash that accumulates in the portfolio. This form of payment represents the interest income the holder is entitled to based on the total par amount outstanding. The structure and frequency of coupon payments will vary between securities. Examples include U.S. Treasury securities which pay semi-annual coupons and MBS fixed-rate passthrough securities which pay monthly coupons. These are two examples of fixed-coupon securities, there are other coupon types such as floating coupons, step coupons, or inflation-linked coupons where the coupon rate must be multiplied by the appropriate index ratio.

### Principal payments

The three types of principals payments that must also be accounted for when calculating the total return are:

1. Sinking funds – Bonds with sinking fund schedules make partial principal payments prior to the bond's maturity date.
2. Called securities – Bonds with a call provision allow the issuer to buy (call) back the bond at a specified price at specified points in the future. Bonds may be fully or partially called. Being called corresponds to cash at the call price entering the portfolio.
3. Principal prepayments – Principal can be paid early in the form of unscheduled prepayments for securitized products (MBS, ABS and CMBS). The timing of prepayments and the time of cash actually entering the portfolio will vary from security to security.

### Reinvestment of coupon payments

Payments received are typically reinvested at the reinvestment rate $r_e(t_c)$ as

$$C(t_1,t_2) = \sum_{t_c \in [t_1,t_2]} C_{t_c} \cdot [1 + r_e(t_c)]^{(t_2 - t_c)}, \tag{2.2}$$

where payments received, $C_{t_c}$, are coupon payments or principal payments paid during the attribution period $[t_1,t_2]$, and $t_c$ is the time of a payment. It is possible that the payments received, $C_{t_c}$, are reinvested at the reinvestment rate $r_e(t_c)$, e.g. the 1-month LIBID rate (with daily compounding) as of the prior month-end. In this case the number of days between the receipt of the cash flow and the end of the attribution period is $t_2 - t_c$.

It is also permissible that a policy of not reinvesting payments received is followed. On one hand, investors may be able to outperform a benchmark if they reinvest cash but on the other hand, they may also underperform by having to pay transaction costs where a benchmark does not.

## 2.2 Transactions

So far we have assumed that performance measurement has been carried out on a static portfolio basis, i.e. the return is calculated based on holdings snapshots at $t_1$ and $t_2$. A more relevant approach is to take account of all transactions that affect the portfolio. This would allow the contribution of trading activity to the total return to be determined.

To perform transaction-based attribution, we can choose the attribution period to be daily so that the effect of all transactions are applied at the correct time. For a transaction-based approach the total return (2.1) of a security is such that

$$r_{\text{total}} = \frac{V(t+1) - V(t) + C(t,t+1) - P(t) + S(t)}{V(t)}, \tag{2.3}$$

where $P(t)$ and $S(t)$ are purchases made and sales made on day $t$, respectively.[1] For information on the implementation of RiskMetrics transactions-based approach see Soor and Costigan (2010).

It is well known that return calculations based on the buy-and-hold approach are very approximate (see Vann, 1999, for a study supporting this claim) when compared with calculations that take intra-attribution period changes in portfolio holdings into account. Buy-and-hold return calculation is still very important though as the difference between this and the transaction-based return gives a measure of the value added from transactions, and as such is an integral aspect of attribution analysis.

## 2.3 Currency

If a bond is denominated in a currency different from the portfolio's base currency, then part of the return will be due to foreign exchange rate movements. This is treated as a separate component of the return and will be part of our factor model. The total return (in base currency) of a security is defined as the currency return $r_{\text{currency}}$, plus the local total return $r_{\text{total}}^{\text{local}}$ as

$$r_{\text{total}}^{\text{base}} = r_{\text{currency}} + r_{\text{total}}^{\text{local}}. \tag{2.4}$$

The currency return comprises the currency appreciation and its cross product with the local total return. Further details are provided in Section 3.2.

[1] This formulation assumes that external cash flows occur at the end of the day. Assumptions regarding intra-day cash flows are also permissible.

# Chapter 3

# Factor Model

In order to decompose total returns, we need a model that describes how changes in specific factors affect returns. Models provide approximations to a complex real world. Each model has pros and cons, and there are always tradeoffs involved when choosing one particular model over another. In addition, some models may perform well under certain market conditions but poorly under other market conditions. Therefore, a solution that offers the user flexibility is proposed. We want to attribute returns between times $t_1$ and $t_2$ to various decisions taken by the portfolio manager. A common factor model is based on the premise that there exist common factors which affect prices on large classes of securities. A security's price change as a response to a factor is determined by the security's sensitivity to the factor. At a given time, a factor model with $K$ factors for a security $i$ takes the form

$$r_i = \sum_{k=1}^{K} \beta_{i,k} f_k + \varepsilon_i, \tag{3.1}$$

where $r_i$ is the total return on security $i$, $f_k$ is the return of factor $k$, $\beta_{i,k}$ represents the sensitivity of security $i$ to a movement in factor $k$, and $\varepsilon_i$ is the residual return that cannot be explained by factors. We can decompose returns into a systematic and an idiosyncratic component. The systematic component is the part of the return that is attributable to exposure to factors. The residual is the part of the return that remains after the factor return decomposition. It contains the idiosyncratic component and whatever is non-attributable because of data problems. Note that the factor return, $f_k$, is common across all assets and buckets.

## 3.1 Factor duration

There are several ways to measure factor sensitivities and factor realizations. One class of models estimates factor returns by cross-sectional linear regressions, similar to the procedure in Fama and MacBeth (1973). Another class of models constructs factor returns from observable variables and then estimates factor sensitivities by linear regressions. The advantage of this method is that it does not require sophisticated pricing functions. The drawback is that the estimated factors or their sensitivities do not necessarily correspond to concepts portfolio managers can relate to.

Our approach differs from the two above. Our factors are the actual observed changes in the risk factors. We have pricing functions for all fixed income instruments. The sensitivity to a change in a factor is found by repricing the security after applying shifts to the factor. This provides a more intuitive way to assess the sources of returns as the actual factors are used instead of latent variables. The cost, however, is that our approach is more computationally demanding.

To set the stage, assume that the market value of a fixed income instrument is a function of $N$ risk factors

$$V = g(x_1, x_2, \ldots, x_N). \tag{3.2}$$

The effect on market value of small changes in the factors can be approximated with a second order Taylor series expansion

$$\Delta V = \sum_{k=1}^{N} \frac{\partial g}{\partial x_k} \Delta x_k + \frac{1}{2} \sum_{k=1}^{N} \sum_{\ell=1}^{N} \frac{\partial^2 g}{\partial x_\ell \partial x_k} \Delta x_k \Delta x_\ell. \tag{3.3}$$

Typically, cross terms are ignored so that the expression simplifies to

$$\Delta V = \sum_{k=1}^{N} \frac{\partial g}{\partial x_k} \Delta x_k + \frac{1}{2} \sum_{k=1}^{N} \frac{\partial^2 g}{\partial x_k^2} \Delta x_k^2. \tag{3.4}$$

The first order terms are referred to as durations with respect to a specific factor and the second order terms are referred to as convexities. Convexities capture the fact that the relation between a bond's price and the factor change is non-linear. We define the factor duration as the negative of the percent sensitivity of the security's price with respect to a small shift in the factor. The factor duration with respect to factor $i$ is thus

$$D_k = -\frac{1}{V} \frac{\partial V}{\partial x_k}, \tag{3.5}$$

where $\partial V / \partial x_k$ represents the derivative of the security price with respect to the factor, i.e. the factor sensitivity. The return contribution from a specific factor $k$ is thus given by

$$r_{\text{factor k}} = -D_k \times f_k, \tag{3.6}$$

where $f_k = \Delta x_k$ is the ex-post realized return on factor $k$.

The decomposition of returns can be thought of as a process where returns are attributed to factors successively. For each security in the portfolio, we decompose the local total return into the following components:

1. **Currency**: Each portfolio has a base currency in which market values are computed. If a bond is denominated in a currency different from the base currency, then part of the return in base currency will be due to foreign exchange rate movements. These are separated and treated as a separate factor. Further details are provided in Section 3.2.

2. **Carry**: The carry captures the part of a bond's return that is due simply to the passage of time. It typically consists of two components: the income that accrues to the holder of the bond over time and the change in clean price, provided that the yield curve remains the same. Both these effects are detailed in Section 3.3.

3. **Yield curve**: The yield curve return is the return on the bond that is due to changes in the government yield curve. For regular bonds, this is typically one of the most important factors. It is discussed further in Section 3.4

4. **Credit**: Investors buying bonds that are not backed by the full faith and credit of a government not subject to default risk, will command a premium to take on the risk of not getting back all promised coupons and principal. Widening or tightening of credit spreads will impact the returns on such bonds and is treated in Section 3.5.

5. **Volatility**: The volatility return captures the change in a fixed income security's price because of changes in volatility. This component can be significant for options and instruments that have embedded options. It is discussed more in Section 3.6

6. **Inflation**: Changes in the nominal yield curve can be due to changes in the real interest rate or changes concerning expectations about inflation and associated risk premia. Section 3.7 details how we can decompose the yield curve return into a real rate component and a break-even inflation component.

## 3.2 Currency

In the case of multi-currency portfolios the local currency of a security will often be different from the base currency of the portfolio and there will be a contribution to the total return from movements in the foreign exchange rate during the attribution period. Hence, we separate out the component of return due to currency movements, $r_{\text{currency}}$, as shown in Eq. (2.4).

The total return of a security expressed in terms of the local currency and the exchange rate from local to base currency can be found as follows. First let us consider an investor who buys foreign currency at time $t_1$ and sells the position back in base currency at time $t_2$, the currency appreciation $r_{\text{FX}}$, or the realized return from this investment in foreign currency, is defined as:

$$r_{\text{FX}} = \frac{\chi(t_2) - \chi(t_1)}{\chi(t_1)}, \tag{3.7}$$

where $\chi(t_1)$ and $\chi(t_2)$ are the exchange rates from local currency to base currency at the beginning and the end of the holding period, respectively.

For an investor who buys a security denominated in a foreign currency whose market value in the local currency is $V(t)$ and where the exchange rate from local currency to base currency is $\chi(t)$, the market value in the base currency $V_{\text{base}}(t)$ is given by

$$V_{\text{base}}(t) = V(t)\chi(t). \tag{3.8}$$

Let us assume there is an intermediate cash flow, e.g. a coupon payment, during the holding period, where $C_{t_c}$ is the value of a payment made at time $t_c$ in the local currency during the attribution period $[t_1, t_2]$ . In the presence of intermediate cash flows an assumption about reinvestment must be made to compute the total return. To keep the exposition as simple as possible, we assume that the intra-month payment earns zero interest. Furthermore, we assume that the intermediate payment is kept in local currency until the end of the attribution period, at which point it is converted back into base currency. Relaxing these assumptions is not hard; it is just a matter of bookkeeping. With these assumptions, the local total return can be expressed as

$$r_{\text{local}} = \frac{V(t_2) + C_{t_c}}{V(t_1)} - 1. \tag{3.9}$$

By substituting (3.8) into (2.1) and with these assumptions we can express the total return in base currency as follows

$$\begin{aligned} r_{\text{base}} &= \frac{[V(t_2) + C_{t_c}]}{V(t_1)} \cdot \frac{\chi(t_2)}{\chi(t_1)} - 1 \\ &= (r_{\text{local}} + 1)(r_{\text{FX}} + 1) - 1 \\ &= r_{\text{local}} + r_{\text{FX}} + r_{\text{local}} r_{\text{FX}}. \end{aligned} \tag{3.10}$$

Finally, by rearranging (2.4) and by making the substitution (3.10) for $r_{\text{base}}$ we have an expression for the currency return

$$\begin{aligned} r_{\text{currency}} &= r_{\text{base}} - r_{\text{local}} \\ &= r_{\text{FX}}(1 + r_{\text{local}}). \end{aligned} \tag{3.11}$$

Notice that the currency return contains an interaction component $(r_{\text{FX}} \cdot r_{\text{local}})$ that causes a difference between the return on a currency investment in the cash market and a security's currency return.

**Example**

Assume an investor whose base currency is USD. Assume further that the investor buys a UK gilt denominated in GBP with a dirty price of GBP 95 at an FX rate of 1.5 USD/GBP (here $V(t_1) = 95$ and $\chi(t_1) = 1.5$). His cash outflow at the beginning of the attribution period is thus 142.5 USD. During the month, the investor receives a coupon payment of 5, i.e. $C(t_1, t_2) = 5$, this coupon, by assumption, earns no interest until the end of the attribution period. At the end of the attribution period, the investor sells the bond at a dirty price of GBP 93 (so $V(t_2) = 93$). The FX rate at the end of the period is 1.6 USD/GBP ($\chi(t_2) = 1.6$), at which time the bond is sold and the proceeds from the sale and the received coupon are exchanged back into base currency. The cash inflow is thus $(93 + 5) \times 1.6\,\text{USD/GBP} = 156.8\,\text{USD}$. We can now easily compute the local total return, the total return in base currency, and the currency return. From (3.9) the return in local currency is

$$r_{\text{local}} = \frac{93 + 5}{95} - 1 = 3.16\%. \tag{3.12}$$

From (3.10) the total return in base currency is

$$r_{\text{base}} = \frac{1.6 \times (93 + 5)}{1.5 \times 95} - 1 = 10.04\%. \tag{3.13}$$

Finally, from (3.11) the currency return $r_{\text{currency}}$ is $10.04\% - 3.16\% = 6.88\%$.

### 3.2.1 Forward premium and currency surprise

It is possible to decompose currency appreciation (3.7) into two components (see Ankrim and Hensel, 1994):

1. Forward premium – The expected interest rate differential between two countries.

2. Currency surprise – The unexpected movement of the base currency relative to its forward exchange rate or market predicted rate.

By defining the forward exchange rate of the base currency at time $t_1$ for conversion through a forward contract at time $t_2$ as $F_\chi(t_1,t_2)$, we can rewrite the currency appreciation as follows

$$r_{\text{FX}} = \frac{[\chi(t_2) - F_\chi(t_1,t_2)] + [F_\chi(t_1,t_2) - \chi(t_1)]}{\chi(t_1)}. \tag{3.14}$$

The term in the first set of brackets in the numerator is not known until $t_2$, whereas the term in the second set of brackets is known at $t_1$. Hence, the first term is the currency surprise,

$$r_{\text{surprise}} = \frac{\chi(t_2) - F_\chi(t_1,t_2)}{\chi(t_1)}, \tag{3.15}$$

and the second term is the forward premium,

$$r_{\text{forward}} = \frac{F_\chi(t_1,t_2) - \chi(t_1)}{\chi(t_1)}. \tag{3.16}$$

One of the benefits of splitting the currency return between forward premium and currency surprise is that a portfolio manager can only hedge away the currency exposure at forward rates, he can therefore only expect to neutralize the forward premium element on a fully hedged portfolio.

**Example**

If we take the same market scenario and UK Gilt from our previous example, and we suppose that the 1-month forward exchange rate is $F_\chi(t_1,t_2) = 1.515$. Then from Eq. (3.7), the currency appreciation is

$$r_{\text{FX}} = (1.6 - 1.5)/1.5 = 6.67\%, \tag{3.17}$$

the currency surprise, according to (3.15), is

$$r_{\text{surprise}} = (1.6 - 1.515)/1.5 = 5.67\%, \tag{3.18}$$

and the forward premium, according to (3.16), is

$$r_{\text{forward}} = (1.515 - 1.5)/1.5 = 1\%, \tag{3.19}$$

In this example it is evident that the majority of the currency appreciation is due to the unexpected movement of the base currency (USD) relative to its forward exchange rate.

## 3.3 Carry

The carry return on a bond is deterministic at the outset of the attribution period. It is the holding period return on the bond if bond market conditions do not change, hence the factor is the time. Unlike for equities, even if market conditions stay the same, the return on a coupon bond is not zero. First, the coupon accrues deterministically. If we disregard reinvestment of any intra-period cash flows, the coupon return on a bond between times $t_1$ and $t_2$ is

$$r_{\text{coupon}} = \frac{AI(t_2) - AI(t_1) + C(t_1,t_2)}{P(t_1) + AI(t_1)}, \tag{3.20}$$

where $AI$ is the accrued interest, $P$ is the clean price, and $C(t_1,t_2)$ denotes any payments between times $t_1$ and $t_2$. $C(t_1,t_2)$ typically refers to coupon payments but other deterministic payments between $t_1$ and $t_2$ must also be taken into account (see Section 2.1 for details).

The roll-down return is the return that is due to the change in the clean price of the bond by rolling time forward while keeping the yield curve that prevailed at $t_1$.

$$r_{\text{roll}} = \frac{P(t_2|\text{yield}(t_1)) - P(t_1)}{P(t_1) + AI(t_1)}. \tag{3.21}$$

The carry return is computed as the sum of Eqs. (3.20) and (3.21)

$$r_{\text{carry}} = r_{\text{coupon}} + r_{\text{roll}}. \tag{3.22}$$

### Example

We illustrate the carry computation with an example. The example bond is the U.S. Treasury bond with coupon rate $9\,\frac{1}{8}$ and maturity date May 15, 2018. We compute the carry on this bond between October 30 and November 30, 2009. The dirty price on October 30 was $V(t_1) = 148.38$. The accrued interest was $AI(t_1) = 4.17$ and hence the clean price was $P(t_1) = 144.22$.[1] This bond paid a coupon of $0.5 \times 9.125$ per 100 face value on October 15. The accrued interest $AI(t_2) = 0.38$ is therefore less than $AI(t_1)$. The coupon return is thus given by

$$r_{\text{coupon}} = \frac{0.38 - 4.17 + 0.5 \times 9.125}{148.38} = 0.522\%. \tag{3.23}$$

The computation of the roll-down return is a little more involved than computing the coupon return, and requires repricing the bond. The first step is to calibrate our model to match the observed market

[1] Small numeric differences in this section are due to rounding as the actual computations have been carried out with higher precision than two decimals.

price. This is achieved by adding an option-adjusted spread (OAS) of 20.84 bps to the government spot curve. Second, we need to compute a new 'time to cash flow' vector. This vector is found by setting the settlement date to $t_2$. The third step is to roll along the yield curve in time such that each spot rate $y(\tau_i)$ is replaced with $y(\tau_i - \Delta t)$, where $\Delta t = t_2 - t_1$. We then reprice the bond using the OAS at $t_1$ and find that the hypothetical clean price at $t_2$ is $P(t_2) = 143.98$. The roll-down return can then be computed as the hypothetical change in clean price divided by the dirty price at $t_1$

$$r_{\text{roll}} = \frac{143.98 - 144.22}{148.38} = -0.156\% \tag{3.24}$$

In our example, the carry return on this bond is thus

$$r_{\text{carry}} = 0.522\% - 0.156\% = 0.366\%. \tag{3.25}$$

The roll-down return therefore includes two components: 1) a bond converges to its par value as it approaches maturity. This is the pull-to-par effect. 2) Different rates are used to discount cash flows as we are 'rolling down the yield curve'. A more mathematical treatment of the carry computations and a proof of consistency with the Taylor series expansion framework introduced in Section 3.1 can be found in Sorensen (2010a).

## 3.4 Yield curve

The term yield curve refers to a plot of yield to maturity on either par bonds or zero coupon bonds against time to maturity in a certain market segment, such as Euro government bonds or U.S. corporate AAA bonds. The yield curve reflects market participants' expectations about the future, and as such, summarizes borrowing costs for different maturities and issuers. RiskMetrics Group constructs zero curves for specific maturities for many fixed income markets. Most often, it is the case that there exists market prices for coupon bonds, but zero coupon prices are hard to get. In such cases, we estimate zero coupon bond prices from coupon bond prices using a technique called bootstrapping (see, for example, Malz, 2002 and Shi, 2009 for a description of the construction of curves).

Changes in the yield curve affect a number of market participants. First, movements in the yield curve are one of the main drivers of returns on fixed income securities, and consequently affect bond investors. Second, the short rate is one of the main tools for macroeconomic policy. Third, the yield curve in large part determines the costs of mortgage borrowing. The yield curve is therefore followed both by investors, policymakers, and the general public. As shown by Litterman and Scheinkman (1991), movements in the yield curve can in large part be summarized by the first two principal components. These roughly correspond to a shift in the level of interest rates and a change in the slope of the term structure. A model for returns due to a parallel shift of the yield curve is presented in Section 3.4.1. A model that also incorporates returns due to slope changes is presented in Section 3.4.2.

### 3.4.1 Effective duration and convexity

The yield curve return is modeled as the bond's return sensitivity to changes in the yield curve. The traditional measure for capturing a bullet bond's exposure to changes in interest rates was the Macaulay duration or modified duration. In the case of plain vanilla fixed rate bullet bonds, there exist analytical formulas to compute Macaulay and modified duration. However, for more complex instruments, such as mortgage backed securities or bonds with embedded options, analytical formulas for duration are not available. Instead, we compute the effective duration by taking the central finite difference

$$D = -\frac{1}{V}\frac{V(+\delta y) - V(-\delta y)}{2\delta y}, \tag{3.26}$$

where $D$ is the effective duration, $V$ is the current price and $\mathrm{V}(+\delta y)$ and $V(-\delta y)$ are the prices obtained by shifting the whole yield curve up and down by $\delta y$ basis points respectively. For complex fixed income instruments, duration often loses its interpretation as the weighted average maturity of the instrument. For instance, Interest Only (IO) strips typically have negative duration. In such instances, it is more fruitful to consider effective duration and think of it as a measure of a security's interest rate sensitivity. The benefit of using effective duration is that by applying shifts and repeatedly calling the pricing function, the chain rule is automatically taken into account. For example, if we consider a callable bond, the effective duration will automatically capture the interest rate sensitivity of both the bullet bond and the embedded option. That is, the second term of the effective duration in Eq. (3.26) will be approximately equal to

$$\frac{\partial V_{cb}}{\partial y} = \frac{\partial V_{cb}}{\partial V_{bb}}\frac{\partial V_{bb}}{\partial y} + \frac{\partial V_{cb}}{\partial V_o}\frac{\partial V_o}{\partial y} \tag{3.27}$$

$$= \frac{\partial V_{bb}}{\partial y} - \frac{\partial V_o}{\partial y}. \tag{3.28}$$

where $V_{cb}$ denotes the market value of the callable bond, $V_{bb}$ the market value of the embedded bullet bond, and $V_o$ the market value of the embedded option.

Duration represents the first-order term from a Taylor series expansion of bond prices with respect to yields. As such, it represents a linear approximation of the sensitivity of prices to yield changes. However, the relation between bond prices and interest rates is not linear. Second-order terms are captured by convexity. Effective convexity is computed as

$$C = \frac{1}{V}\frac{V(+\delta y) - 2V + V(-\delta y)}{(\delta y)^2}, \tag{3.29}$$

While the convexity contribution is often small, it can be substantial. Some portfolio strategies, such as the bullet versus barbell, are explicit convexity strategies. We therefore include the convexity effect of a parallel shift in yields in our attribution model. The return contribution due to a parallel shift in yields is thus

$$r_{\text{shift}} = r_{\text{duration}} + r_{\text{convexity}} = -D\overline{\Delta y} + \frac{1}{2}C(\overline{\Delta y})^2. \tag{3.30}$$

The effective duration $D$ and the convexity $C$ in Eq. (3.30) represent sensitivities to the average change in the yield curve. RiskMetrics Group provides spot rates for a large number of maturity nodes. The average change in the yield curve depends on which nodes we choose to compute the average. For consistency with our approach for key rate durations in Section 3.4.2, we compute the average change in yields over a set of $K$ key rates

$$\overline{\Delta y} = \frac{1}{K}\sum_{k=1}^{K}\Delta y_k, \tag{3.31}$$

where $\Delta y_k$ is shorthand notation for the change in the spot rate for constant maturity $\tau_k$ between the start date ($t_1$) and end date ($t_2$) of the attribution period

$$\Delta y_k = y(t_2,\tau_k) - y(t_1,\tau_k). \tag{3.32}$$

While effective duration is a much used and important tool in portfolio analysis and risk management, it is important to understand that it assumes that yield curve changes occur in a parallel way. However, in practice, yield curve changes are often far from parallel. Figure 3.1a shows the yield curves as of month-end October and month-end November, 2009. Visual inspection reveals that the shift was largest in absolute value in the medium term. Figure 3.1b plots the changes in yields for each time to maturity and shows that yields in general decreased between October and November and that the maximum decrease was around the five year point. The figure makes it clear that this was not a parallel shift.

### 3.4.2 Key-rate durations

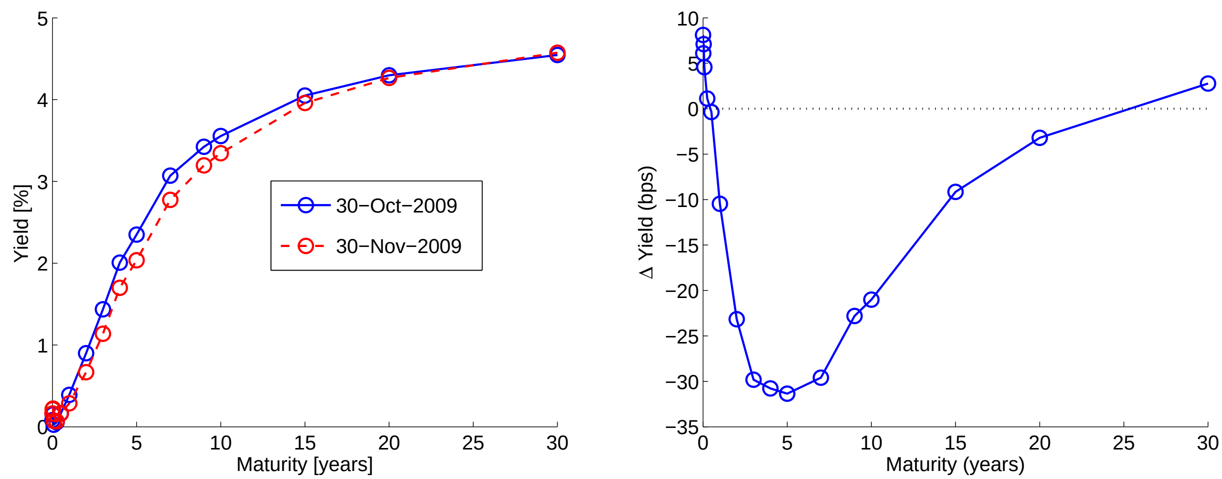
To remedy the shortcomings of the traditional duration measures, Ho (1992) introduced the key rate durations (KRD). The key rate durations are computed by considering a factor model, where the factors are the chosen key rates for the currency in question. Key rate duration is the negative of the percentage price change due to a small shift in that key rate $k$ and is defined as

$$KRD_k = -\frac{1}{V}\frac{V(+\delta y_k) - V(-\delta y_k)}{2\delta y_k}, \tag{3.33}$$

*Figure 3.1*

**Yield Curves and Yield Curve Changes**

*(a)* **Yield curves** *(b)* **Change in yields**




where $\delta y_k$ is the shift applied to the $k$-th key rate. See, for example, Sorensen (2010b) for a detailed description of the implementation of key rate durations. The yield curve component of a bond's return is the part of the return that can be explained by movements in the bond's benchmark curve. Figure 3.2 shows the effective duration and key rate durations for a U.S. Treasury benchmark as of October 30, 2009.

The key rates depends on the market and for example, we use the following key rates for the U.S. Treasury,:

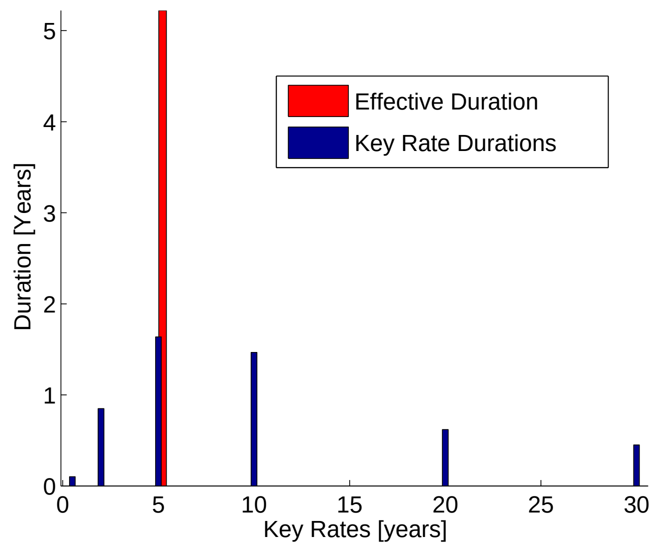
$$\text{Key rates} = \{6m, 2y, 5y, 10y, 20y, 30y\}, \tag{3.34}$$

which have been found representative of the entire yield curve.

We use the key rate durations to capture the contributions to returns that come from a reshaping of the yield curve. It is therefore the effect over and beyond the parallel shift of the yield curve. The additional contribution to returns from key rate durations is labeled the shape return. For each key rate, we compute the difference between that key rate duration and the effective duration divided by the number of key rates. This difference is multiplied with the change in the corresponding key rate and summed over all key rates such that

$$r_{\text{shape}} = -\sum_{k=1}^{K} KRD_k \, \Delta y_k + D\overline{\Delta y}. \tag{3.35}$$

*Figure 3.2*
**Effective duration and key rate durations**




Combining Eqs. (3.30) and (3.35), we get the return contribution from government yield curve changes

$$\begin{aligned} r_{\text{yield curve}} &= r_{\text{shift}} + r_{\text{shape}} \\ &= -\sum_{k=1}^{K} KRD_k \, \Delta y_k + \frac{1}{2} C (\overline{\Delta y})^2. \end{aligned} \tag{3.36}$$

An appealing feature of key rate duration models is that, using six key rates as for US Treasury, we can capture more changes than with the usual twist and butterfly effects. And further unlike for principal components models, we do not require a stationary covariance structure of interest rate changes

## Example

We continue with our example bond from Section 3.3 and consider the return from month-end October to month-end November, 2009 on the 9 $\frac{1}{8}$ U.S. Treasury bond with maturity date May 15, 2018. This U.S. Treasury bond has no embedded option or risk of default. Hence, the two main drivers of returns will be the carry and the yield curve return. The total return on the bond between October and November, 2009 was 1.83%. As we showed in Section 3.3, the carry return was 0.37%. Table 3.1 shows the returns due to key rate duration exposures. The header row shows the term of each key rate. Since this is a U.S. bond, the key rate nodes are those in Eq. (3.34). The first and second rows show the zero

*Table 3.1*
**Key rate yields and durations**

| Time to Maturity [Yrs] | 0.5 | 2 | 5 | 10 | 20 | 30 |
|---|---|---|---|---|---|---|
| Key Rates Start [%] | 0.16 | 0.90 | 2.35 | 3.56 | 4.30 | 4.55 |
| Key Rates End [%] | 0.16 | 0.67 | 2.04 | 3.35 | 4.27 | 4.57 |
| Δ Key Rates [%] | 0.00 | -0.23 | -0.31 | -0.21 | -0.03 | 0.02 |
| KRD | 0.05 | 0.33 | 2.28 | 3.76 | 0.00 | 0.00 |
| KRD Return [%] | 0.00 | 0.08 | 0.71 | 0.79 | 0.00 | 0.00 |

coupon yield for each key rate node at the beginning and end of the attribution period. The third row displays the change in key rate yields and shows that the medium term rates decreased significantly over the period. The fourth row shows the key rate durations. The last row shows the returns due to exposures to the key rate durations and is computed by multiplying the negative of each of the key rate durations with the realized shift in the corresponding key rate. The sum across key rates of the key rate duration returns in Table 3.1 is 1.58%.

Table 3.2 shows the return decomposition when taking the carry, shift, and shape return into account. The second column shows the decomposition of our sample bond. The average change in the key rates of the U.S. government curve was $\overline{\Delta y} = -0.127\%$, which yields $-D\overline{\Delta y} = 0.82\%$. In this example, the positive return due to convexity is only 0.004% and is indistinguishable from zero in the table given our level of precision. The additional return due to a non-parallel shift of the yield curve is thus $r_{\text{shape}} = 1.58\% - 0.82\% = 0.76\%$. The shift and shape returns are of similar size, and illustrate the importance of taking non-parallel movements in they yield curve into account. The decomposition of returns is done security by security. We can aggregate the security level results and decompose the returns of a portfolio. The last column of the table provides the results for a Treasury index. The signs of the return components are the same as for the example bond, but the shift and shape returns are smaller in magnitude. This is consistent with the lower duration of the index. Both return decompositions leave a small but not negligible residual.

*Table 3.2*
**Treasury return decomposition**

| Return | US T 9 $\frac{1}{8}$ | US T Index |
|---|---:|---:|
| Total | 1.83% | 1.40% |
| Carry | 0.37% | 0.28% |
| Shift (Duration & Convexity) | 0.82% | 0.67% |
| Shape | 0.76% | 0.36% |
| Residual | -0.12% | 0.09% |
| Duration | 6.44 | 5.22 |

## 3.5 Credit

Every bond that is not backed by the full faith and credit of the government is subject to credit risk. Investors command a spread over government yields to accept the possibility of default when investing in corporate bonds. Movements in the spread versus Treasuries therefore influence bond prices and returns. It is generally agreed that there is no risk of default by investing in U.S. Treasuries. However, bond prices for other governments may reflect a probability of default.

RiskMetrics Group provides a large number of curves. The level of detail depends on the market. Figure 3.3 shows the yield curves for US Corp All Industry A, and USD Government on December 31, 2009. As expected, the government zero yields are below the other curve for all maturities.

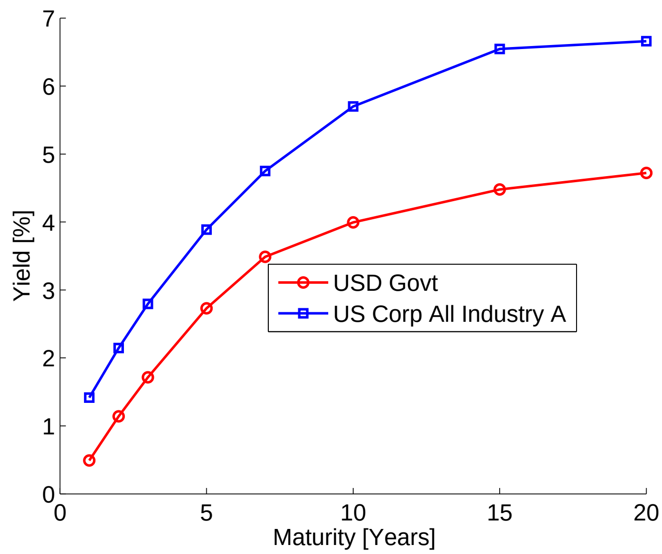
The spread return captures the component of a bond's return that is due to changes in credit markets. Each bond is specified both with a curve for discounting cash flows and a reference government curve. For each bond, we start by computing the total change in spread versus the government curve. Part of this spread will be due to the change in spreads between the sector curve, for example US Corp All Industry A, and the government curve. The part of the total spread change that is not explained by this curve spread change is an issue-specific idiosyncratic spread change. The sensitivity to spread changes is assessed by the spread duration. It is computed as

$$D_{\text{spread}} = -\frac{1}{V}\frac{V(+\delta s) - V(-\delta s)}{2\delta s}, \tag{3.37}$$

where $D_{\text{spread}}$ is the spread duration and $\delta s$ is the shift applied to the spread to compute the central difference.

*Figure 3.3*

**Yield curve for US Corp All Industry A and USD Government on 31 December 2009**




For each bond, we compute the option-adjusted spread, $s$. This is the constant spread that must be added to the government curve such that the bond is priced exactly by the relation

$$P = \sum_{n=1}^{N} CF_n e^{-(y_n+s)\tau_n}, \tag{3.38}$$

where $CF_n$ is the cash flow and $y_n$ the yield for time $\tau_n$. This spread will in general change from $s(t_1)$ to $s(t_2)$ between dates $t_1$ and $t_2$. The spread change, $\Delta s = s(t_2) - s(t_1)$, can be decomposed into the change in the sector spread and the idiosyncratic spread

$$\Delta s = \Delta s_{\text{sector}} + \Delta s_{\text{idiosyncratic}}. \tag{3.39}$$

The return contribution from widening or tightening sector spreads can be modeled as common factor returns. The change in the idiosyncratic spread, which can be due to company-specific news or liquidity changes for a specific issue, constitute the residual return on the bond. This is the part of the bond's total return that common factor changes cannot account for.

The contribution of a change in spreads can be approximated by $-D_{\text{spread}} \times \Delta s$.[2] Using the spread decomposition in Eq. (3.39), we denote the part of the return due to changes in the sector spread the spread return

$$r_{\text{spread}} = -D_{\text{spread}} \times \Delta s_{\text{sector}}. \tag{3.40}$$

[2]Recently, Dor et al. (2007) have suggested that duration times spread be used instead. They show that portfolios with different spreads and spread durations, but similar duration times spread (DTS) exhibit the same return volatility.

The part of the return due to changes in the idiosyncratic spread goes into the residual.

## Example

In this example, we decompose returns from month-end November to month-end December, 2009 on a corporate bond issued by Coca Cola Enterprises. The bond has a coupon rate of 7% and a final maturity date on October 1, 2026. It is mapped to the discount curve US Corp All Industry A. We find that the spread duration of this bond was $D_{\text{spread}} = 10.47$. The spread movements are reported in Panel A of Table 3.3. At the beginning of the attribution period, the bond spread versus the government curve was 130.50 bps. By year-end, the bond spread had decreased to 102.50 bps, a decrease of 28 bps. Next, we consider the spread change in the US Corp All Industry A versus governments. Using the available key rates, we find that this spread decreased by $\Delta s_{\text{sector}} = -16.55$ bps, yielding a positive contribution to the return on the bond by $r_{\text{spread}} = 1.73\%$.

Panel B of Table 3.3 shows the expanded return decomposition for this bond. The total return was negative, $-2.47\%$, despite the positive return contribution from spread returns. This was due to an increase in the Treasury yield curve which is captured by the shift and shape returns, as described in Section 3.4. A visual representation of the return decomposition is given in Fig. 3.4, which clearly shows that the positive contribution from the spread return is not large enough to outweigh the negative contribution from the shift and shape returns. Even after accounting for carry, shift, shape, and spread returns, the remaining residual is not small in this case. This is because Coca Cola spreads decreased more than spreads on the US Corp All Industry A curve between November and December.

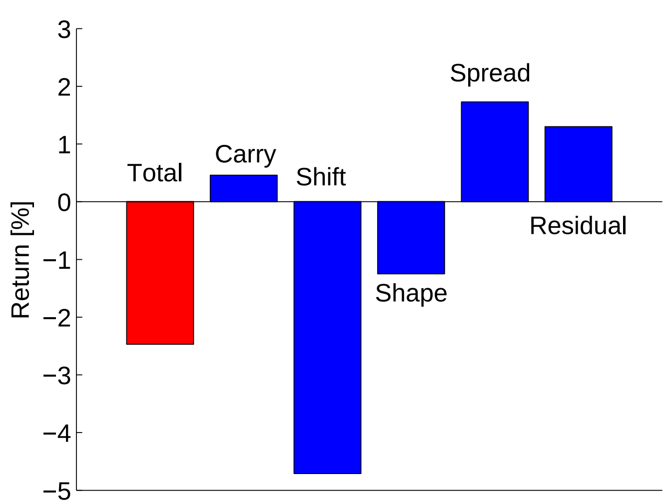
*Table 3.3*

**Spread return and corporate bond return decomposition**

| Statistic | Coca Cola 7 10/01/2026 |
|---|---|
| Panel A: Spread Changes | |
| $D_{\text{spread}}$ | 10.47 |
| $\Delta s$ | -28.00 bps |
| $\Delta s_{\text{sector}}$ | -16.55 bps |
| Panel B: Return Decomposition | |
| Total | -2.47% |
| Carry | 0.46% |
| Shift | -4.71% |
| Shape | -1.25% |
| Spread | 1.73% |
| Residual | 1.30% |

*Figure 3.4*

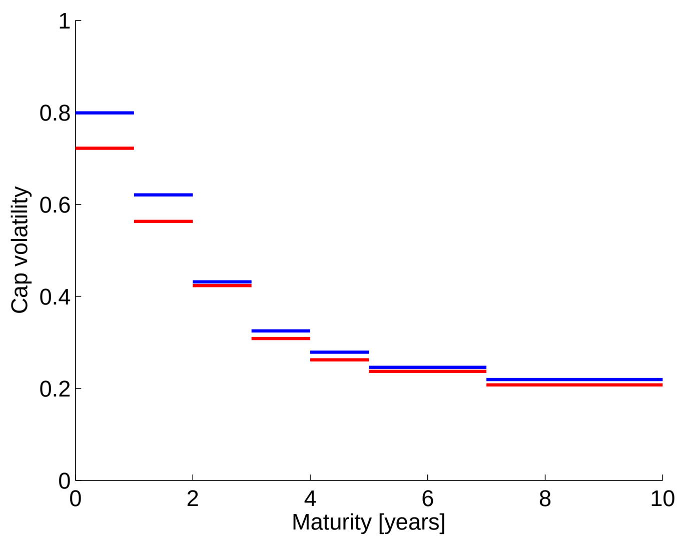
**Corporate bond return decomposition**



## 3.6 Volatility

Several fixed income instruments include options. For such instruments, the market value depends on interest rate volatility. A prominent example is callable bonds, which are standard fixed coupon bonds, but where the issuer retains the right to buy back the bond at par during a specified period before maturity. An investor who is long a callable bond is thus long a regular bond and short a call option on the bond. Hence, part of the return on the callable bond will be driven by the option's price change. The value of the option increases with an increase in interest rate volatility. This implies that the value of the callable bond decreases with an increase in volatility.

*Figure 3.5*
**Cap volatilities on two dates: 29 May 2009 (blue) and 5 June 2009 (red)**




To value embedded options, we resort to models for the interest-rate dynamics, for example the Black-Derman-Toy model (Black et al., 1990). These models typically need a market-implied volatility curve for calibration. Hence any movement in such curve will imply a price change. In Fig. 3.5 we show the volatility curves obtained from cap prices at two different dates.

Our volatility factor is defined as the average shift of the market-implied volatility curve over our $K$ key nodes,

$$\overline{\Delta\sigma} = \frac{1}{K}\sum_{k=1}^{K} \sigma_k(t_2) - \sigma_k(t_1) \tag{3.41}$$

with the key nodes being

$$k \in \{1y, 2y, 3y, 5y, 10y\}. \tag{3.42}$$

The exposure to the volatility factor is given by the bond's vega. To compute vega, the volatility curve is shifted up and down by a small amount $\delta\sigma$ (e.g. 1bp) and obtained as

$$v = \frac{V(+\delta\sigma) - V(-\delta\sigma)}{2\delta\sigma}, \tag{3.43}$$

where $V(+\delta\sigma)$ is the instrument value obtained by increasing all the volatility curve by $+\delta\sigma$ and $V(-\delta\sigma)$ is the instrument value obtained by decreasing all the volatility curve by $-\delta\sigma$, the other factors being constant.

The volatility return is then obtained as

$$r_{\text{volatility}} = \frac{1}{V} v \overline{\Delta\sigma}. \tag{3.44}$$

## Example

*Table 3.4*
**Characteristics of the two bonds**

| | Callable bond | Regular bond |
|---|---:|---:|
| Coupon | 5% | 5% |
| Maturity date | 3 Sep 2020 | 3 Sep 2020 |
| Country | US | US |
| Strike date | 3 Oct 2015 | |
| Strike price | 100 | |

To illustrate the volatility factor we study two similar bonds. The first is a simple US Treasury callable bond and the second is a similar bond, but with no embedded option. Table 3.4 shows the characteristics of the bonds. These bonds have been priced using RiskMetrics models on two consecutive dates, June 1st and 2nd 2009, and we report the various factor sensitivities in Table 3.5.

We first notice that the effective duration of the callable bond is significantly lower than that of the regular bond. Indeed lowering the interest rate will increase the chance the bond is called and hence reduce the value of the bond. Further we see a zero vega as expected for the regular bond and, a negative vega for the callable bond because the higher the volatility the higher the chance to meet the strike price, hence the lower the price.

During the two analysis dates, the cap volatility surface has decreased by

$$\overline{\Delta\sigma} = -1.149\% \tag{3.45}$$

*Table 3.5*

**Duration, convexity and vega of the two bonds**

| | Callable Bond | Regular Bond |
|---|---:|---:|
| Effective duration | 6.95 | 8.27 |
| Effective convexity | 0.731 | 0.838 |
| Vega | -0.163 | 0.000 |

*Table 3.6*

**Return decomposition for the two bonds**

| Return | Callable Bond | Regular Bond |
|---|---:|---:|
| Total | 0.538% | 0.548% |
| Carry | 0.017% | 0.015% |
| Duration | 0.264% | 0.315% |
| Convexity | 0.004% | 0.006% |
| Volatility | 0.187% | 0.000% |
| Residual | 0.066% | 0.212% |

and the US Treasury curve has decreased by

$$\overline{\Delta y} = -0.038\%, \tag{3.46}$$

yielding the results in Table 3.6. We see that the duration effect is not as important for the callable bond as for the regular bond, but this is balanced by the volatility effect. We acknowledge that the residuals are still large and a full key-rate duration decomposition should be applied.

## 3.7 Inflation

In Section 3.4, we assessed the impact of nominal yield curve changes on bond returns. In this section, we decompose returns due to changes in the government curve into those caused by changes in real rates and those caused by changes in break-even inflation.

The consumer price index (CPI) is a measure estimating the average price of consumer goods and services purchased by households. This index can be used to adjust for the effect of inflation on the real value of money, salaries, pensions, and regulated or contracted prices. Many governments have issued inflation-linked bonds. The relevant quantity for pricing these bonds is the break-even inflation, as shown in Couderc (2008). The break-even inflation is not only the expected inflation, it also contains a compensation that investors require to accept inflation risk. Nominal interest rates $y_t(\tau)$ can be expressed as

$$1 + y_t(\tau) = [1 + r_t(\tau)]\,[1 + i_t(\tau)]\,, \tag{3.47}$$

where $r_t(\tau)$ is the real yield curve and $i_t(\tau)$ is the break-even inflation curve. RiskMetrics computes the break-even inflation curve from nominal and inflation-indexed securities as shown for illustration in Fig. 3.6. Here we plot the Euro nominal curve and the French break-even inflation curve on a particular date.

We neglect the cross-product term and rewrite Equation (3.47) as

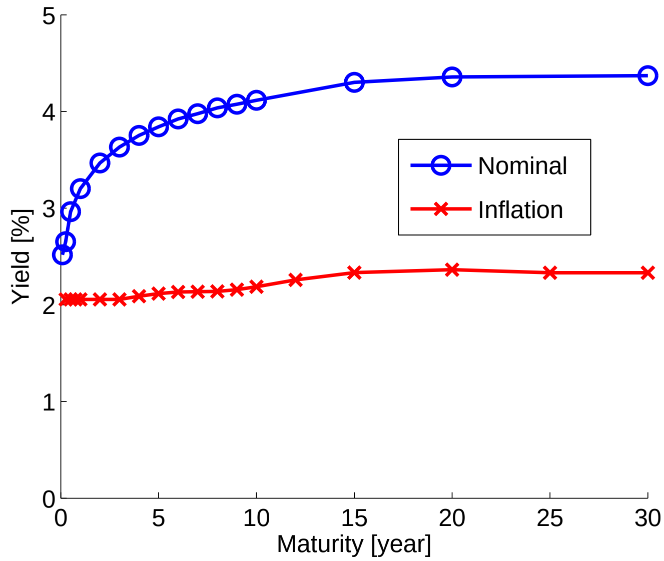
$$y_t(\tau) = r_t(\tau) + i_t(\tau). \tag{3.48}$$

The real yield curve is then obtained by subtracting the break-even inflation from the nominal yield curve.

When the security has no dependency on inflation, its price will change only when the real curve moves. To capture such effects we need to introduce dual durations, namely the real duration

$$D_r = -\frac{1}{V}\frac{\partial V}{\partial r} \tag{3.49}$$

*Figure 3.6*
**EUR nominal yield curve and France break-even inflation curve on 11 May 2006**




and the inflation duration

$$D_i = -\frac{1}{V}\frac{\partial V}{\partial i}. \tag{3.50}$$

As illustration consider a simple zero-coupon inflation-linked bond paying a face value $F$ scaled by inflation at maturity $T$. Consider further a flat real yield curve $r(\tau) = r$ and a flat break-even inflation curve $i(\tau) = i$. The value of the bond is then calculated as

$$V = \frac{F(1+i)^T}{(1+i)^T(1+r)^T}. \tag{3.51}$$

For such bond, we see that $D_i = 0$ and $D_r = T/(1+r)$.

As for the Treasury curve effect we treat parallel shifts in real and break-even inflation rates as factors. Consistent with the computation of the parallel shift for the nominal yield curve in Eq. (3.31), the parallel shift in the real interest rate curve is

$$\overline{\Delta r} = \frac{1}{K}\sum_{k=1}^{K}\Delta r_k, \tag{3.52}$$

where $\Delta r_k = r_{t_2}(\tau_k) - r_{t_1}(\tau_k)$. The same set of $K$ key nodes is used to compute both the nominal and the real shift. The parallel shift in the break-even inflation curve is similarly

$$\overline{\Delta i} = \frac{1}{K}\sum_{k=1}^{K}\Delta i_k, \tag{3.53}$$

where $\Delta i_k = i_{t_2}(\tau_k) - i_{t_1}(\tau_k)$ is the change in the break-even inflation curve between two dates for the constant maturity $\tau_k$.

The effective real duration of the security is calculated as

$$D_{\text{real}} = -\frac{1}{V}\frac{V(+\delta r) - V(-\delta r)}{2\delta r}, \tag{3.54}$$

where $V(+\delta r)$ $(V(-\delta r))$ is the value of the security calculated using the real yield curve shifted up (down) with $\delta r$. The inflation duration is similarly computed as

$$D_{\text{inflation}} = -\frac{1}{V}\frac{V(+\delta i) - V(-\delta i)}{2\delta i}, \tag{3.55}$$

where $V(+\delta i)$ $(V(-\delta i))$ is the value of the security calculated using the inflation curve shifted up (down) with $\delta i$.

The returns due to real yield curve and inflation effects are then

$$r_{\text{real}} = -D_{\text{real}}\overline{\Delta r}, \tag{3.56}$$

and

$$r_{\text{inflation}} = -D_{\text{inflation}}\overline{\Delta i}. \tag{3.57}$$

The sum of the contribution from Eqs. (3.56) and (3.57) is equal to $r_{\text{shift}}$ in Eq. (3.30) less the contribution from convexity for non-inflation linked bonds.

## Example

*Table 3.7*
**Characteristics of the two bonds**

| | Inflation-linked Bond | Regular Bond |
|---|---:|---:|
| Coupon | 2.5% | 2.5% |
| Maturity date | 17.07.2020 | 17.07.2020 |
| Country | US | US |
| CPI | USCPI | – |

To illustrate the inflation factor we study two similar bonds. The first is an inflation-linked bond and the second is a similar bond, but with no inflation protection. Table 3.7 shows the characteristics

*Table 3.8*

**Dual durations of the two bonds**

| Duration | Inflation-linked Bond | Regular Bond |
|---|---|---|
| Nominal | – | 6.36 |
| Inflation | 0.0018 | 6.36 |
| Real | 6.41 | 6.31 |

of the bonds. These bonds have been priced using RiskMetrics models on two consecutive dates, June 1st and 2nd 2009, and we report the various factor sensitivities in Table 3.8.

We first notice that all three durations of the regular bond are equal and that the inflation duration of the inflation-linked bond is almost zero (up to non-linear pricing effects), as expected. During the two analysis dates, the factor realizations are

$$\overline{\Delta i} = -0.038\% \tag{3.58}$$

and

$$\overline{\Delta r} = -0.010\%. \tag{3.59}$$

This yields the results in Table 3.9, where we notice that the inflation-linked bond has zero return attributed to inflation, as expected.

*Table 3.9*

**Return decomposition for the two bonds**

| Return | Inflation-linked Bond | Regular Bond |
|---|---|---|
| Total | 0.098% | 0.531% |
| Carry | 0.013% | 0.013% |
| Inflation | 0.000% | 0.238% |
| Real | 0.066% | 0.065% |
| Residual | 0.019% | 0.214% |

# Chapter 4

# Relative Performance Attribution

## 4.1 Introduction

After having measured the total return of each fixed income security in the portfolio, and further decomposed it into various effects using factors, in this chapter we look at the attribution of the performance of the portfolio relative to the benchmark. We will first describe the method for a single period, which in most cases is daily and then show how to aggregate these daily results to the reporting period, e.g. monthly.

For a single period, the return of the portfolio is obtained as

$$R^P = \sum_i w_i^P r_i^P \tag{4.1}$$

and the return of the benchmark as

$$R^B = \sum_i w_i^B r_i^B, \tag{4.2}$$

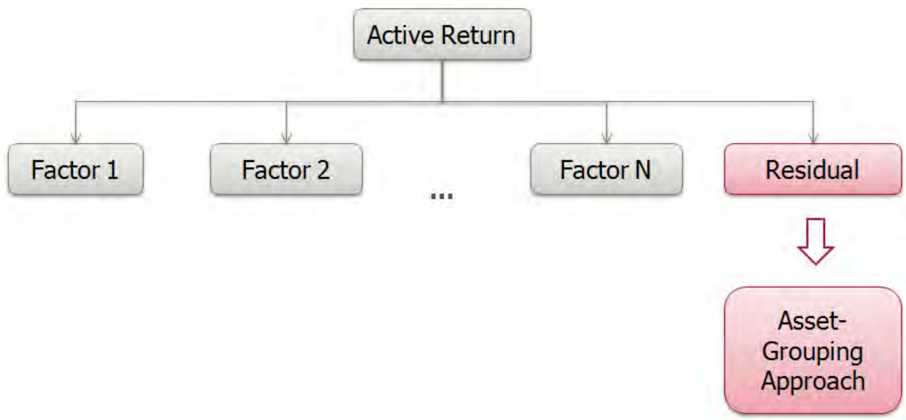
where $w_i^P$ and $w_i^B$ are the weights (exposures) of position $i$ in the portfolio and benchmark, while $r_i^P$ and $r_i^B$ their return over the period. For a given security, these returns can be different for example due to trading, see Section 2.2.

As stated in Chapter 1, the portfolio manager is interested in his active return, i.e. his return relative to the benchmark:

$$R^P - R^B = \sum_i w_i^P r_i^P - \sum_i w_i^B r_i^B. \tag{4.3}$$

The portfolio manager has access to all constituents and characteristics of the benchmark and he

*Figure 4.1*
**Hybrid performance attribution**




takes various bets to try outperform it. Performance attribution is an ex-post analysis that attributes this active return to the various bets the portfolio manager took.

The first method for performance attribution was introduced by Brinson and Fachler (1985) and captures the investment decision by considering reference portfolios or semi-notional funds, which boils down to grouping the assets differently. The asset-grouping approach consists essentially of two main attribution effects, namely asset (or sector) allocation and security selection, and is very efficient for equity portfolios. However this is not not fully satisfactory for fixed income portfolios, where many other effects need to be captured, such as carry or yield curve movements.

Another approach is to resort to a factor model. One first decomposes the return of all securities into systematic factors, and then aggregates all sensitivities as the attribution effect. However the drawback is that some effects cannot be captured by a systematic factor.

We have developed a hybrid attribution method that take the best of both while leaving their disadvantages apart. The hybrid attribution method is schematically shown in Fig. 4.1. We first consider as many factors as needed, which will leave an unexplained part (or residual), on which we apply a standard asset-grouping method. The hybrid method has two interesting limiting cases. The first is when no factors are used, this is then a standard asset-grouping approach. An emerging-market bond manager might like to see the results from this attribution method, if the market is not liquid enough to produce reasonable yield curves. While the other limit is when we use enough factors to produce negligible residuals. This approach could be the choice of a Treasury fund manager.

In the following we first describe the two limiting cases, and then introduce the hybrid approach. We also show to link the single attribution period to a longer reporting period and finally go through the treatment of mispricing.

## 4.2 Pure asset-grouping approach

In addition to the notation introduced in Section 4.1 we suppose the universe of securities is partitioned in sectors (labeled by $J$) and introduce the following notation:

$w_J^B$ is the weight of sector $J$ in the benchmark.

$w_J^P$ is the weight of sector $J$ in the portfolio.

$r_J^B$ is the return of sector $J$ in the benchmark.

$r_J^P$ is the return of sector $J$ in the portfolio.

The sectors typically depend on the investment process the portfolio manager follows, and can be industrial sectors, ratings, or asset classes. Further there are no restrictions to consider also sub-sectors, or more complex hierarchies.

The weights of sector $J$ in the portfolio and benchmark are given by

$$w_J^P = \sum_{i \in J} w_i^P \qquad\qquad w_J^B = \sum_{i \in J} w_i^B \tag{4.4}$$

and the scaled returns of sector $J$ are

$$r_J^P = \frac{1}{w_J^P} \sum_{i \in J} w_i^P r_i^P \qquad\qquad r_J^B = \frac{1}{w_J^B} \sum_{i \in J} w_i^B r_i^B \tag{4.5}$$

Using Equation (4.1) and Equation (4.2), the active return can be written as

$$R^P - R^B = \sum_J w_J^P r_J^P - \sum_J w_J^B r_J^B, \tag{4.6}$$

where the summation is now over all sectors.

A standard way to decompose this active return is to group the terms differently, such as to make apparent a sector allocation attribution term and a security selection term. The allocation and selection attribution term at the sector level are shown in Table 4.1. For each sector $J$, the allocation term is the relative weight of the sector $(w_J^P - w_J^B)$ times the over/underperformance of the sector relative to the total benchmark return $(r_J^B - R^B)$. The selection term for sector $J$ is the portfolio weight of the sector $w_J^P$ times the over/underperformance of the sector return $(r_J^P - r_J^B)$. The sum over all sectors of the allocation and selection returns is equal to the total excess return over the benchmark.

*Table 4.1*
**Asset-grouping performance attribution**

| Sector | Sector Allocation | Security Selection |
|---|---|---|
| $\vdots$ | $\vdots$ | $\vdots$ |
| $J$ | $(w_J^P - w_J^B)(r_J^B - R^B)$ | $w_J^P(r_J^P - r_J^B)$ |
| $\vdots$ | $\vdots$ | $\vdots$ |
| Total | $\sum_J (w_J^P - w_J^B) r_J^B$ | $\sum_J w_J^P (r_J^P - r_J^B)$ |

In this formulation we have chosen a top-down security selection, including also the interaction term. There are various variations of such method as described in Soor (2010).

## 4.3 Pure factor model approach

When the factor model introduced in Chapter 3 yields a negligible residual we can use an attribution based on factors only. Assuming a zero residual in Eq. (3.1) we decompose the return of all securities in the portfolio and benchmark as

$$r_i^P = \sum_k \beta_{i,k}^P f_k \qquad r_i^B = \sum_k \beta_{i,k}^B f_k. \tag{4.7}$$

The beta-sensitivities to factor $k$ of the portfolio and benchmark are

$$\beta_k^P = \sum_i w_i^P \beta_{i,k}^P \qquad \beta_k^B = \sum_i w_i^B \beta_{i,k}^B. \tag{4.8}$$

The factor-based attribution terms are shown in Table 4.2. For each factor $k$, the allocation term is the relative beta $(\beta_k^P - \beta_k^B)$ times the realization of the factor $f_k$. The sum over all factor returns is equal to the total excess return over the benchmark.

In case a factor is only uniquely defined for a sector, for example credit spreads for a rating sector, we extend the attribution as followings. We label the sector-specific factor as $f_{J,k}$ and introduce the

*Table 4.2*
**Factor-based performance attribution**

| Factor | Attribution |
|---|---|
| $\vdots$ | $\vdots$ |
| $k$ | $(\beta_k^P - \beta_k^B) f_k$ |
| $\vdots$ | $\vdots$ |
| Total | $\sum_k (\beta_k^P - \beta_k^B) f_k$ |

sector-specific sensitivity for the portfolio and benchmark as

$$\beta_{J,k}^P = \sum_{i \in J} w_i^P \beta_{i,k}^P \qquad \beta_{J,k}^B = \sum_{i \in J} w_i^B \beta_{i,k}^B. \tag{4.9}$$

Notice that here the sector-specific sensitivities are not rescaled by the sector weight as for the sector returns (4.5). The attribution to factor $k$ in sector $J$ is then

$$\left(\beta_{J,k}^P - \beta_{J,k}^B\right) f_{J,k} \tag{4.10}$$

where the factor realization $f_{J,k}$ is the same for all securities in the sector.

Finally, in case the residuals are not negligible we can aggregate them as an additive term:

$$\sum_i (\varepsilon_i^P - \varepsilon_i^B). \tag{4.11}$$

## 4.4 Hybrid attribution model

In some cases it is difficult to construct a consistent factor for a given effect, or the investment process does not consider a common factor for all securities. We then use a mixed approach. In that approach, first the return is decomposed using factors, with an idiosyncratic or residual term remaining. Second, this idiosyncratic term is explained using the asset-grouping approach. For example in the case of corporate bonds, the factors could be carry (Section 3.3) and yield curve (Section 3.4), and the residual include the return due to credit spread movements and all other specifics.

We use the notation introduced in Sections 4.2 and 4.3 to write the return of security $i$ in the portfolio and benchmark as

$$r_i^P = \sum_k \beta_{i,k}^P f_k + \varepsilon_i^P \qquad r_i^B = \sum_k \beta_{i,k}^B f_k + \varepsilon_i^B \tag{4.12}$$

where we introduce the residual return as $\varepsilon_i^P$ for the portfolio and $\varepsilon_i^B$ for the benchmark.

Using the portfolio and benchmark beta defined in Eq. (4.8), the active return is then

$$R^P - R^B = \sum_k (\beta_k^P - \beta_k^B) f_k + \sum_i \left( w_i^P \varepsilon_i^P - w_i^B \varepsilon_i^B \right). \tag{4.13}$$

We introduce the sector residuals

$$\varepsilon_J^P = \sum_{i \in J} \varepsilon_i^P \qquad\qquad \varepsilon_J^B = \sum_{i \in J} \varepsilon_i^B, \tag{4.14}$$

to obtain the hybrid attribution terms by treating factors as in Section 4.3 and residuals as in Section 4.2. Attribution results are shown in Table 4.3.

*Table 4.3*
**Hybrid performance attribution**

| Factor | Sector | Factor Attribution | Sector Allocation | Security Selection |
|---|---|---|---|---|
| $\vdots$ | $\vdots$ | $\vdots$ | $\vdots$ | $\vdots$ |
| $k$ | $J$ | $(\beta_{J,k}^P - \beta_{J,k}^B) f_{J,k}$ | – | – |
| $\vdots$ | $\vdots$ | $\vdots$ | $\vdots$ | $\vdots$ |
| Residuals | $J$ | – | $(w_J^P - w_J^B)\varepsilon_J^B$ | $w_J^P(\varepsilon_J^P - \varepsilon_J^B)$ |
| Total | | $\sum_{k,J}(\beta_{J,k}^P - \beta_{J,k}^B) f_{J,k}$ | $\sum_J (w_J^P - w_J^B)\varepsilon_J^B$ | $\sum_J w_J^P(\varepsilon_J^P - \varepsilon_J^B)$ |

## Example

To illustrate the hybrid attribution method we analyze a small corporate bond portfolio. As shown in Fig. 4.2 we consider two factor effects, carry and duration, and apply the asset-grouping approach to the residual, which will contain the return due to the sector credit spread movements.

The portfolio and benchmark positions are listed in Table 4.4. The holding period is one week, from 29 May 2009 to 5 June 2009 and we show the resulting sector allocation and duration on 29 May 2009 in Table 4.5. The active return is 29.32 bps.

The first step consists of applying our factor model yielding the decomposition shown in Table 4.6. The shift of the yield curve was + 0.26%, resulting in negative returns for all bonds, larger than the carry return in absolute terms. However, not all bonds have negative total returns, due to different

*Figure 4.2*

**Corporate bond attribution**

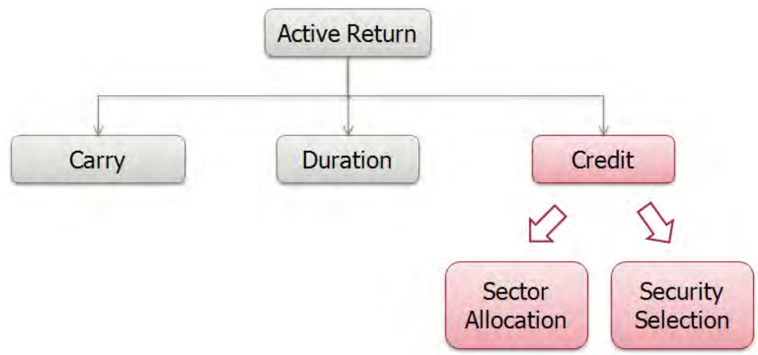



*Table 4.4*

**Universe of corporate bonds**

| Security | Sector | Portfolio Weight | Benchmark Weight |
|---|---|---|---|
| CAM 6.375 07/15/2018 | Energy | 10.1% | 2.8% |
| CSGN 6.125 11/15/2011 | Financials | 0.0% | 23.5% |
| CSGN 6.5 01/15/2012 | Financials | 71.3% | 19.8% |
| CSGN 6 02/15/2018 | Financials | 0.0% | 12.5% |
| NRC 4.375 10/01/2010 | Financials | 0.0% | 3.4% |
| NRC 7.25 03/01/2012 | Financials | 0.0% | 11.0% |
| VZ 4.35 02/15/2013 | Telecom | 18.6% | 5.2% |
| VZ 5.5 02/15/2018 | Telecom | 0.0% | 10.2% |
| VZ 6.4 02/15/2038 | Telecom | 0.0% | 11.6% |

*Table 4.5*

**Sector allocation and duration**

| | | Portfolio | Benchmark |
|---|---|---|---|
| Sector weights | Energy | 10.1% | 2.8% |
| | Financials | 71.3% | 70.2% |
| | Telecom | 18.6% | 27.0% |
| Duration | | 3.0 | 4.5 |

*Table 4.6*

**Return decomposition of all bonds**

| Security | Sector | Total Return | Carry Return | Duration Return | Residual |
|---|---|---|---|---|---|
| CAM 6.375 07/15/2018 | Energy | 0.21% | 0.16% | -1.69% | 1.74% |
| CSGN 6.125 11/15/2011 | Financials | -0.83% | 0.14% | -0.60% | -0.37% |
| CSGN 6.5 01/15/2012 | Financials | -0.82% | 0.05% | -0.62% | -0.25% |
| CSGN 6 02/15/2018 | Financials | -0.88% | 0.08% | -1.68% | 0.72% |
| NRC 4.375 10/01/2010 | Financials | -0.48% | 0.08% | -0.34% | -0.23% |
| NRC 7.25 03/01/2012 | Financials | -1.34% | 0.14% | -0.64% | -0.84% |
| VZ 4.35 02/15/2013 | Telecom | -1.01% | 0.08% | -0.88% | -0.21% |
| VZ 5.5 02/15/2018 | Telecom | -1.26% | 0.11% | -1.76% | 0.39% |
| VZ 6.4 02/15/2038 | Telecom | -2.08% | 0.13% | -3.07% | 0.86% |

*Table 4.7*

**Hybrid performance attribution [in bps]**

| Sector | Active Return | Carry | Duration | Sector Allocation | Security Selection |
|---|---|---|---|---|---|
| Financials | 4.54 | -3.41 | 11.38 | -0.22 | -3.20 |
| Telecom | 23.27 | -1.57 | 41.57 | -3.99 | -12.75 |
| Energy | 1.51 | 1.13 | -12.26 | 12.64 | 0.00 |
| Total | 29.32 | -3.85 | 40.69 | 8.43 | -15.95 |

credit spread shifts included in the residuals. The next step is to apply the asset-grouping approach on the residuals yielding the final attribution of the active return in Table 4.7.

These results are in agreement with the insights we have analyzing the characteristics of the portfolio in Table 4.5. There is a large negative duration bet and since the Treasury curve increased during the analysis period, we get a large positive attribution. We also notice that the sector allocation bets are not large in comparison to stock picking bets, understandable as the portfolio only has one single bond in each sector. This example also shows the importance of using factor for fixed income performance attribution, as highlighted by the fact that duration attribution is much more important than the other effects.

## 4.5 Multi-period attribution

So far we have only considered a single attribution period, which is thought to be daily. This is the basic building block for longer attribution period, such as monthly or quarterly. As we get holdings of the portfolio including transactions and the constituents of the benchmarks on a daily frequency. To report the performance attribution over a longer period, we need to aggregate the daily calculations. For example, to get the performance attribution of an European manager in February 2010, we need to aggregate the 20 daily total returns and attribution.

The returns that we considered so far were discrete relative returns from $t$ to $t+1$ day, defined as

$$r_{i,t,t+1} = \frac{V_i(t+1)}{V_i(t)} - 1, \tag{4.15}$$

where $V_i(t)$ is the market value of the instrument $i$ at time $t$ (measured in days). This formulation is very useful because the return of a portfolio, from time $t$ to $t+1$, can be calculated as the weighted sum of these discrete relative returns,

$$R_{t,t+1} = \sum_i w_i r_{i,t,t+1} \tag{4.16}$$

where $w_i$ is the portfolio weight of instrument $i$ at time $t$. This weight should represent the exposure of the position and is not always simply its market value (see Chapter 5). Further this is consistent with the arithmetic attribution that we have introduced in Section 4.4. In other words the active return can be decomposed in a sum of attribution terms $A^k_{t,t+1}$, as

$$R^P_{t,t+1} - R^B_{t,t+1} = \sum_k A^k_{t,t+1} \tag{4.17}$$

We consider now a reporting period, from $t_1 = 0$ to $t_2 = T$, consisting of $T$ daily steps. The return of security $i$ over that reporting period is

$$r_{i,0,T} = \frac{V_i(T)}{V_i(0)} - 1 \tag{4.18}$$

and the portfolio total return over that period is

$$R_{0,T} = \sum_i w_i r_{i,0,T} \tag{4.19}$$

under a static portfolio assumption. The problem is to account for daily transactions and that factor sensitivities evolve during the reporting period, yielding the question of what value to take for the attribution, i.e. start value or average?

The solution is to perform daily attribution, accounting for any transactions and then aggregate the attributions to the reporting period. The problem is that discrete relative return and arithmetic attribution do not add up over time. A number of methodologies (known as smoothing algorithms) have been developed to achieve this (see Bacon (2008)) and a comparison of these methodologies can be found in Menchero (2004). Among them is a simple yet effective methodology proposed by Carino (1999) for the asset-grouping approach.

The idea is to resort to logarithmic returns defined as

$$\tilde{r}_{i,t,t+1} = \log \frac{V_i(t+1)}{V_i(t)} \tag{4.20}$$

with the property that the full period return is obtained by summing up the daily returns,

$$\tilde{r}_{i,0,T} = \log \frac{V_i(T)}{V_i(0)} = \sum_{t=0}^{T-1} \tilde{r}_{i,t,t+1}. \tag{4.21}$$

In the following $(\tilde{\ })$ stands for log-returns.

The daily arithmetic attributions are transformed in daily logarithmic attributions, summed up to get the attributions over the reporting period and then transformed back to arithmetic attributions. To that purpose we define the daily smoothing factor

$$\kappa_{t,t+1} = \frac{\tilde{R}^P_{t,t+1} - \tilde{R}^B_{t,t+1}}{R^P_{t,t+1} - R^B_{t,t+1}}. \tag{4.22}$$

In case, $R^P_{t,t+1} = R^B_{t,t+1}$ we set it to $\kappa_{t,t+1} = 1$. While the full period smoothing factor is defined as

$$\kappa_{0,T} = \frac{\tilde{R}^P_{0,T} - \tilde{R}^B_{0,T}}{R^P_{0,T} - R^B_{0,T}}. \tag{4.23}$$

Writing the daily attribution as Eq. (4.17) we get the daily logarithmic active return decomposed as

$$\tilde{R}^P_{t,t+1} - \tilde{R}^B_{t,t+1} = \sum_k \kappa_{t,t+1} A^k_{t,t+1}. \tag{4.24}$$

These attribution terms are summed up over the reporting period and transformed back to relative returns. Using 4.23, the aggregated attribution effects are then

$$A^k_{0,T} = \sum_t \frac{\kappa_{t,t+1}}{\kappa_{0,T}} A^k_{t,t+1} \tag{4.25}$$

providing the following decomposition of the active return of the reporting period:

$$R^P_{0,T} - R^B_{0,T} = \sum_k A^k_{0,T}. \tag{4.26}$$

## 4.6 Impact of pricing difference

It has already been mentioned that security returns in the portfolio may be different from corresponding security returns in the benchmark due to trading, see Section 2.2. Another possible difference in returns can occur due to differences in the benchmark's and portfolio's market prices from which returns are calculated.

It is highly likey that a portfolio manager may take market prices from a different source than the source which is used by a benchmark vendor, which may lead to a different price in the portfolio and benchmark for a given security. Also, a vendor's price may be calculated from an arithmetic mean of two price quotes, which would also lead to a potential difference with the portfolio price (taken from a single quote).[1] For example Iboxx follows this consolidated price or mid-price approach. Mid-prices are also used in FTSE indices, again with prices calculated as the arithmetic mean between two quotes.[2]

It is easy to see that ambiguity may arise as prices are available from multiple sources as well as the use of mid-prices for a given security. As accurate attribution relies on an accurate performance calculation, if there is a difference in prices between the benchmark and the portfolio we must remove this before calculating the active return. Otherwise, the impact of pricing differences could compromise the integrity of the attribution analysis.

[1] Government bonds traditionally have far more price quotes than corporate bonds which will increase the likelihood of pricing differences.

[2] FTSE apply a quality control check to ensure each single price in the index is unbiased and representative of the market so they generally do not include a bond issue if there is only one price available from a single market maker.

To remove the impact of pricing differences on portfolio returns it is necessary to adjust portfolio prices. Let $\pi_i$ be the difference (only due to different price sources) between the portfolio price and the benchmark price

$$\pi_i(t) = V_i^B(t) - V_i^P(t), \tag{4.27}$$

where $V_i^P(t)$ and $V_i^B(t)$ are the market prices of instrument $i$ in the portfolio and benchmark, respectively, at the end of a given day $t$. Then $\tilde{V}_i^P(t)$ is the adjusted portfolio price for security $i$ at time $t$

$$\tilde{V}_i^P(t) = V_i^P(t) + \pi_i(t). \tag{4.28}$$

It is then possible to calculate the adjusted return, $\tilde{r}_i^P$, over the holding period by substitution of (4.28) into the equation for the local total return of an instrument (2.1). Then the adjusted portfolio return, $\tilde{R}^P$, due to adjusted portfolio prices is equal to

$$\tilde{R}^P = \sum_i w_i^P \tilde{r}_i^P. \tag{4.29}$$

The impact (or return), $R^I$, due to the pricing difference, is the difference between the adjusted benchmark return and the benchmark return,

$$R^I = R^P - \tilde{R}^P. \tag{4.30}$$

Finally the active return (4.3) less the impact of any pricing difference $R^I$ is

$$\tilde{R}^P - R^B = \sum_i w_i^P \tilde{r}_i^P - \sum_i w_i^B r_i^B. \tag{4.31}$$

# Chapter 5

# Exposures

For attribution security weights according to market value is straightforward, this is simply the security market value divided by the total portfolio market value. However, when the portfolio contains derivatives calculation of weights becomes more involved. Rather than weight being calculated simply according to market value instead it is more appropriate to calculate weights from the full exposure value of each derivative. The problem with the simple market value approach is that the exposure of the portfolio is vastly understated. We show how full exposure can be used to express derivative securities in terms of their underlying synthetic assets so that their returns contribute towards the portfolio's total return.

## 5.1 Partial and full exposure

The partial exposure of a security is its market value (or present value). The full exposure of a derivative security is captured by decomposing the security according to its underlying synthetic assets. Only then can the appropriate underlying exposure and thus the source of return of the derivative, due to a change in value of its underlying asset(s), be captured.

For a basic instrument partial exposure is equivalent to full exposure, where basic implies the asset is not a derivative instrument, examples include cash holdings and bonds. For a derivative asset the partial exposure represents the value the asset could be sold for, whereas the full exposure quantifies the economic exposure of the derivative due to movements in the underlying asset and as such, weights calculated using the derivatives full exposure is more appropriate.

By restating the portfolio in terms of the full exposure of each security the effective weight of each position can be defined and used for performance measurement and attribution. All derivative positions (futures, options, etc) are replaced by their underlying synthetic assets and notional cash for which performance can be calculated exactly.

## 5.2 Performance legs and notional cash

As full exposure requires derivatives to be broken up into their underlying synthetic assets it is appropriate to introduce the concept of performance legs. A performance leg is either the underlying exposure value or notional cash. Note, all assets expressed in terms of their partial exposure have a single performance leg which is always equal to the market value of the asset. Notional cash is the amount of cash that is necessary to balance the exposure of an underlying asset such that the addition of all performance legs is equal to the market value of the derivative. This adjustment to offset the underlying exposure leg prevents derivative returns from being overstated, the notional cash is a synthetic performance leg.

## 5.3 Portfolio weight

Let us define the weight of each position in a portfolio considered in terms of its partial exposure. For a position $i$, where $i = 1,\ldots,N$ and where $V_i$ is the market value or partial exposure of position $i$, the weight $w_i$ of the position $i$ in the portfolio is

$$w_i = \frac{V_i}{\sum_i V_i}. \tag{5.1}$$

For a portfolio considered in terms of its full exposure, weights are calculated using performance legs, where $\tilde{V}_i$ is the value of each performance leg for a security $i$. For derivatives, $\tilde{V}_i$ is the underlying exposure value or the complementary notional cash and for basic assets, $\tilde{V}_i$ is simply the market value. Hence we define the effective weight $\tilde{w}_i$ as

$$\tilde{w}_i = \frac{\tilde{V}_i}{\sum_i \tilde{V}_i}. \tag{5.2}$$

## Examples

To illustrate the need for full exposure we move to several specific examples. We begin with the trivial case of a bond to introduce the approach.

### Bonds

For bonds full exposure is equivalent to partial exposure as they have a single performance leg, which has value equal to the position market value.

$$\text{Position market value} = \text{Quantity} \times \text{Market price} \times \text{FX rate}. \tag{5.3}$$

A note on currency rates is appropriate. Since the market price of the bond is in the security's local currency the FX rate scales the market price so that exposure value is in the base currency of the portfolio. Quantity is the number of bonds held (which is $> 0$ for long positions and $< 0$ for short positions) multiplied by the notional of the bond (or amount outstanding), i.e. Quantity $=$ Amount $\times$ Notional.

Consider a long position in a single US Treasury bond that trades at 59 cents on a dollar. The notional amount is USD 1,000,000. The portfolio's base currency is EUR and the USD/EUR FX rate is 1.4131. This bond has a single performance leg and its exposure value in the base currency is equal to the position market value, from (5.3) this is as follows

$$\begin{aligned}\text{Exposure value} &= 1,000,000 \times 0.59 \times 1/1.4131 \\ &= 417,521.761. \end{aligned} \tag{5.4}$$

### Bond futures

Futures are the most basic form of derivative. The futures price is set so that no payment is made when the contract is written, such that, at initiation, the futures contract has zero market value. Through marking to market each day the futures contract is effectively rewritten each day at the new futures price, which is determined by the underlying bond price. Hence the value of the futures contract after the daily settlement (of payments towards the eventual purchase of the underlying asset) will always be zero. Purchasing a bond future is equivalent to holding a long bond position and a short cash position. Bond futures have two legs of exposure, one leg is the underlying bond and the other leg is notional cash.

For bond futures, Market Value = 0. The first performance leg is the exposure value of the underlying asset:

$$\text{Underlying asset exposure value} = \text{Quantity} \times \text{Quoted bond price} \times \text{FX rate.} \tag{5.5}$$

Since the bond future's market value is not equal to the underlying bond's exposure value an amount of notional cash is required to offset the difference between the first performance leg and the bond futures market value. The notional cash amount is the second performance leg:

$$\text{Notional cash} = \text{Position market value} - \text{Underlying asset exposure value.} \tag{5.6}$$

Let us illustrate this with a numerical example. Consider a short bond future contract of size 500 on a notional amount of 200,000. The current quoted bond price is 101.4535 and the security currency and base currency are both USD. We first calculate the position market value is zero after marking to market. Thus from (5.5) the first performance leg is

$$\begin{aligned} \text{Underlying asset exposure value} &= -500 \times 200,000 \times 101.4535/100 \times 1 \\ &= -101,453,500, \end{aligned} \tag{5.7}$$

and finally from (5.6) the second performance leg is

$$\begin{aligned} \text{Notional cash} &= 0 - (-101,453,500) \\ &= 101,453,500. \end{aligned} \tag{5.8}$$

From this example it is evident that the position market value/partial exposure of zero is misleading as it does not at all give an intuitive sense of the exposure of the position. This clearly shows that full exposure is needed to decompose the futures contribution to the portfolio's total return.

#### Effect of settlement

If a position has a settlement date which is forward of the trading date, i.e. the position enters the portfolio prior to settlement, the positions market value is not equal to its partial exposure. This produces an extra performance leg in the case of basic assets as an amount of notional cash is required to offset the difference between the position market value and the exposure of the asset.

This is best understood through a simple worked example. If we take the US Treasury bond from the earlier example and say that the bond will settle in three days, and we also assume that the given settlement price has a present value of 0.56, then the positions market value is as follows

$$\begin{aligned}\text{Position market value} &= \text{Quantity} \times (\text{Market price} - \text{Settlement price present value}) \times \text{FX rate} \\ &= 1,000,000 \times (0.59 - 0.56) \times 1/1.4131 \\ &= 21,229.92. \qquad (5.9)\end{aligned}$$

This is very different to the exposure value in (5.4) of 417,521.7607, the resulting notional cash is as follows

$$\text{Notional cash} = 21,229.92 - (417,521.761) = -396,291.841. \qquad (5.10)$$

As before the notional cash offsets the difference between the positions market value and the exposure value of the asset.

## 5.4 Other derivatives

The performance analysis of increasingly complex derivatives using full exposure is possible as long as the derivatives can be split into basic (non-derivative) instruments.

For a pay fixed, receive float swap, the first performance leg is the exposure value of being long the underlying floating rate note, and the second performance leg is the exposure value of being short the corresponding fixed-coupon bond. For a swap there is no notional cash as the swap market value is equal to the addition of the two underlying bond exposures, there is no need to offset anything.

Option contracts like any other asset generate economic exposures. The economic exposure of option contracts is not linear like futures contracts, and will change depending on whether the option is a call or a put and by how much above or below the underlying price is compared to the exercise price. To attribute the performance of an option we express the change in value of the option in terms of the change in value of the underlying asset. To measure the full exposure of an option contract it is appropriate to scale the underlying asset value by the option delta. The delta of an option is the rate of change of the option value with respect to a change in value of the underlying asset. The first performance leg is the delta-adjusted exposure value of the underlying asset. Since the bond option's market value will not be equal to the underlying bond's exposure value an amount of notional cash is required to offset the difference between the first performance leg and the market value of the bond option position. The second performance leg is the notional cash.

Finally consider a credit default swap (CDS), here a protection buyer makes a series of premium payments to the seller in exchange for payment of the notional in the event the underlying bond defaults.

The first performance leg is the exposure value of being long a risky bond. The second performance leg is the exposure value of being long a fixed payer interest-rate swap. The third performance leg is the exposure value of a default free loan. This methodology is also applicable to a CDS index derivative. The approach discussed earlier for options can be used to readily capture the full exposure of swaptions (options on a swap), caps and floors (which are options on bonds). With all derivatives the partial exposure vastly understates the true (full) exposure of the position.

# Chapter 6

# Benchmarks

## 6.1 Introduction

A benchmark is a portfolio that is used as a reference, it consists of a predetermined set of securities against which managers can measure the performance of their portfolio. The set of securities could be a published benchmark or a combination of a set of published benchmarks.

Benchmarks should have the following key characteristics:

- Representative: in terms of span and weight of appropriate markets, instruments and issues that reflect opportunities available to investors;
- Investible and replicable: by including only securities in which an investor can trade at short notice and for which market prices exist;
- Accurate and reliable: so that index return calculations reflect the actual changes in the value of a portfolio consisting of the same securities; *daily and timely*, so managers can measure performance immediately and make adjustments to their investment strategy.

A common distinction between benchmarks is by classification of the constituents, for example a dissection of the universe of securities into Treasury, government related (agency or sovereign), corporate, and securitized. A more granular distinction can be made by grouping securities by their style (macro, long-short, relative value, etc), or by their geography (global, Americas, Europe, Asia-Pacific, Africa).

Benchmarks attempt to provide an accurate, comprehensive depiction of the performance and fundamental characteristics of a given market. They are also geared towards a manager's specific investment strategy. For instance J.P. Morgan have two versions of their Emerging Markets Bond Index (EMBI), one is the standard EMBI Global and the other is the EMBI Global Diversified. The EMBI Global uses a traditional market-capitalization approach to determine the weight of each individual bond issue, whereas the EMBI Global Diversified limits the weights of index countries with larger debt stocks by only including a specified portion of these countries' eligible current face amounts of debt outstanding. The former benchmark is geared towards active managers of large portfolios or any portfolio that, regardless of size, faces daily fluctuations in its balance of investable funds. The latter is geared towards managers who face limitations on the amount of portfolio exposure they can take to individual issuers.

This chapter begins by discussing the rules by which benchmark level and composition is determined, we then give examples of the data that is necessary for fixed-income performance attribution. Replication of vendor returns is discussed as this is needed so that validation checks on daily data received can be performed; only then can we ensure the integrity of the data used in performance attribution analysis.

## 6.2 Benchmark rules

The composition and level of a published benchmark is governed by a set of pre-defined rules, these rules are both vendor and benchmark/index specific and can be summarized as follows:

- Selection criteria – How bonds enter and exit the benchmark, for example, by bond type, issuer, time to maturity, or outstanding amount.

- Rebalancing – The details of how the weights of securities are adjusted and at what time interval.

- Total return – The return methodology and how cash flows which arise from the constituent securities are handled in terms of reinvestment.

- Index calculation – How the constituent market prices aggregate to the index price.

### 6.2.1 Selection criteria

It is possible that constituents are removed from the benchmark due to securities that become ineligible for inclusion during the month. This could be due to ratings changes, called bonds, securities falling below a given time-to-maturity. Also, bonds can enter the benchmark due to bond issues that are newly eligible because of ratings changes or newly issued bonds. A very important attribute of any security belonging to a benchmark is that it should be easy to invest in it, as it is essential that the benchmark can be reproduced by a manager. An example of a very different selection criteria exists for J.P. Morgan's EMBI, where a security must belong to a country that has been classified as having a low or middle per capita income by the World Bank for at least two consecutive years.

### 6.2.2 Rebalancing

Rebalancing is performed to ensure that the benchmark accurately reflects the available market supply of investable bonds.

Merrill Lynch and FTSE indices are rebalanced on the last calender day of the month based on information available in the marketplace. For the Iboxx Top 30 Index, rebalancing is performed every quarter, on the last calender day of February, May, August and November. Barclays Capital (formerly Lehman Brothers) benchmark indices belong to either a returns (monthly rebalanced) or statistics universe (no rebalancing) and constituents eligibility for index inclusion is evaluated on a monthly basis to determine index composition, where each index consists of two universes of securities:

1. **Returns universe** – The securities within an index of this type are determined at beginning of each month and held fixed until the beginning of the next month, i.e. securities do not move either into or out of the index. Daily and monthly returns reflect the performance of the returns universe. This monthly rebalancing means the returns universe holds the market cap of the constituents constant throughout the month which means that a fund manager avoids having to hit a moving target.

2. **Statistics universe** – A dynamic set of bonds that changes daily to reflect the latest composition of the market. This universe serves as a projection of the next month's returns universe. Statistics such as market values, sector weightings, and various averages (e.g. coupon, duration, maturity, yield, price) are updated and reported daily. At the end of each month, the latest statistics universe becomes the returns universe for the coming month. The statistics universe allows a manager to monitor changes in the index throughout a month. Active managers can

modify their portfolios as the index changes, while passive managers can be prepared to execute all rebalancing transactions at the end of the month to match the upcoming returns universe.

### 6.2.3 Total return

The standard methodology to calculate total return is based on price changes, income received, and, where applicable, currency value changes. Beyond this each vendor has its own specific rules regarding how cash flows affect returns calculations. For all Lehman Brothers indices intra-month cash flows contribute to monthly returns but the cash flows are not reinvested during the month and do not earn a reinvestment return. For Merrill Lynch intra-month cash flows are reinvested at the beginning-of-month 1-month LIBID (London Interbank Bank Bid Rate), with daily compounding. At the end of each month, cash and reinvestment income are removed from the index and weights are recalculated. For IBoxx indices, cash from received coupons is reinvested in the money market. The interest rate used is the one-month LIBID. LIBID is defined as the US-Dollar one month LIBOR less 1/8 (12.5 basis points) as of the coupon payment date.

### 6.2.4 Index calculation

The calculation of the daily index value requires the following to be known:

1. The list of instruments to be included and their amounts outstanding.
2. The daily total return of each instrument.
3. The weight of each instrument as of the prior business day's close.

An index's total return is the weighted average of the total return of all constituents. Typically the weighting factor is the market value at the start of the period. Where the market value accounts for the market price of an index-eligible security and its accrued interest. One contrasting case is for the EMBI Global Diversified benchmark which applies different proportions of each issuer's current face amount outstanding from countries with larger debt stocks, so that countries with large current face amounts outstanding of index-eligible debt will have instrument allocations and, thus, index capitalization weights reduced.

## 6.3 Data

Daily published data from vendors such as returns, pricing and bond characteristic data should be used where available at both index level and constituent level.

Some typical daily index level data needed for performance attribution is given in Table 6.1. Various statistics such as portfolio duration and the index coupon rate among others are very useful for a portfolio manager to see. Market values and index returns are also required in other currencies if supplied by the vendor.

*Table 6.1*
**Daily index level data**

| | Field name | Example |
|---|---|---|
| 1. | Market value in base currency. | $8384475.78 |
| 2. | Number of securities in the index. | 158 |
| 3. | Index level in base currency. | 6227.12 |
| 4. | Return frequency, e.g. daily, monthly, quarterly, yearly. | DAILY |
| 5. | Hedging – stating if the index returns are hedged or not. | Y |
| 6. | Price return in base currency. | 0.570383% |
| 7. | Total return in base currency. | 0.646883% |
| 8. | Weighted average coupon rate of the index. | 10.743664 |
| 9. | Weighted average remaining maturity of the index. | 11.269746 |
| 10. | Weighted average yield to maturity of the index. | 2.22% |
| 11. | Weighted average yield to worst of the index. | 1.82% |
| 12. | Weighted average effective duration of the index. | 5.27 |
| 13. | Weighted average modified duration of the index. | 5.21 |
| 14. | Weighted average effective convexity of the index. | 0.55 |
| 15. | Weighted average modified convexity of the index. | 0.51 |
| 16. | Weighted average issuer OAS of the index. | 54bps |

Some typical daily constituent level data required for performance attribution is given in Table 6.2. The data needed per security includes security market values so relative weights can be calculated. For returns, prices and cash flows are needed, which in turn requires information on coupon details and conventions, and also includes any non-standard features such as non-uniform first coupons. Again, market values and security returns in other currencies are required if supplied by the vendor.

*Table 6.2*

**Daily constituent level data**

| | Field name | Example |
|---|---|---|
| 1. | Market value in base currency. | $125700 |
| 2. | Outstanding or notional in base currency. | $83800 |
| 3. | Clean quoted price. | 143.473765 |
| 4. | Accrued interest. | 2.445652 |
| 5. | Dirty price. | 145.919417 |
| 6. | Payment made in base currency. | 4137.625 |
| 7. | Coupon type, e.g. fixed, floating, zero, step, hybrid. | FIXED |
| 8. | Annual coupon rate. | 9.875 |
| 9. | Coupon payment frequency per year. | 2 |
| 10. | Day count convention followed e.g. actual/actual. | ACT/ACT |
| 11. | Maturity date. | 09/20/2026 |
| 12. | Price return in base currency. | 0.542782727% |
| 13. | Total return in base currency. | 0.634256839% |
| 14. | Yield to maturity. | 2.22% |
| 15. | Yield to worst. | 1.82% |
| 16. | Effective duration. | 5.23 |
| 17. | Modified duration. | 4.53 |
| 18. | Effective convexity. | 0.55 |
| 19. | Modified convexity. | 0.51 |
| 20. | Issuer OAS. | 54bps |
| 21. | Index ratio, for inflation linked bonds. | 1.606312 |
| 22. | Real price of an inflation linked bonds. | 99.779999 |
| 23. | Real accrued interest of an inflation linked bond. | 1.24448 |

## 6.4 Replicating returns

It is vital to be able to reconcile with published vendor returns. This is necessary to ensure that if vendor data used in performance attribution analysis it is sound. Replication of vendor returns is possible only if each vendor's returns methodology can be replicated.

The main difficulty in replication of fixed-income benchmark returns is the correct calculation of coupon amounts and the timing of the payments. For fixed-coupon bonds, once the inception date, first and last coupon date, maturity date, annual coupon payment and coupon frequency and ex-day convention for each bond is known the coupon my be determined exactly. For bonds paying floating coupons and bonds making unscheduled principal payments (see Section 2.1 for details on all payment types), unless benchmark providers publish their daily security-level rates of returns or at least the payments made, determination of exact payment amounts and timings for returns calculations becomes increasingly difficult.

A further issue with benchmark replication is the treatment of cash. For coupons or principal paid, the vendor may either decide to place the cash received into a holding account or to reinvest the cash in the benchmark's securities.

### Example

To illustrate some of the subtleties involved in replicating benchmark returns we examine the 1-day local return of a bond for a sample of different vendors. The example bond is the Republic of Austria government bond which matures on 01/15/2018, the full terms and conditions are given in Table 6.3. The characteristics of interest needed for replicating returns are the coupon details; the bond pays an annual coupon of 4.65% on January 15.

In Table 6.4 actual pricing data and 1-day returns are given for the 3 sample vendors. The vendors under examination are as follows: Citigroup (shown as Citi), iBoxx Markit (shown as iBoxx), and J.P. Morgan (shown as JPM). The published 1-day local total return (see the column headed *Published*) for 01/15/2009 is compared to the 1-day local total return $r_{\text{total}}$, calculated according to Eq. (2.1) using the clean price $P(t)$, accrued interest $AI(t)$, and coupon payment amount $C_{t_c}$ (reported by each vendor) for 2 consecutive days, the residual is the difference between the published vendor return and the calculated return.

There are some very interesting points to notice about the results.

*Table 6.3*

**Terms and conditions of Republic of Austria Government Bond, 4.65 01/15/2018**

| | |
|---|---|
| ISIN | AT0000385745 |
| Currency | EUR |
| Issue Date | 01/22/03 |
| Maturity Date | 01/15/18 |
| First Coupon Date | 01/15/04 |
| Coupon type | Fixed Annual |
| Coupon | 4.65% |
| Day count | Act/Act |
| Outstanding | 10321379000 |
| Par | 1000 |

*Table 6.4*

**Vendor pricing data and 1-day returns for Republic of Austria Government Bond**

| Date | $P(t)$ | $AI(t)$ | $C_{t_c}$ | Published | $r_{\text{total}}$ | Residual |
|---|---|---|---|---|---|---|
| Citi | | | | | | |
| 01/14/2009 | 107.15362 | 4.6373 | - | | | |
| 01/15/2009 | 107.40437 | 0 | 4.65 | 0.235675 | 0.235663 | 0.000012 |
| iBoxx | | | | | | |
| 01/14/2009 | 107.11179 | 4.6372951 | - | | | |
| 01/15/2009 | 107.40461 | 0 | 4.65 | 0.273403 | 0.273403 | 0 |
| JPM | | | | | | |
| 01/14/2009 | 107.3843 | 0.051 | - | | | |
| 01/15/2009 | 107.6324 | 0.0637 | - | 0.242808 | 0.242751 | 0.000057 |

All value are in percent.

(i) The coupon was deemed to be paid on January 15 for Citi and iBoxx, evident from the accrued interest being zero for these vendors on this day but in the case of JPM, the coupon was deemed to have already been paid by January 15. This leads to a difference in returns between vendors.

(ii) For the same bond the clean price reported by each vendor is different on corresponding days, and thus, so is the 1-day return, which is consistent with Section 4.6, where the impact of pricing differences on returns due to vendors using different data sources is discussed.

(iii) The published vendor return is replicated by $r_{\text{total}}$ to an extremely high degree of accuracy, in the case of iBoxx, the residual is actually zero at this level of precision.

## 6.5 Validation of benchmark data

Daily validation of vendor benchmark data is necessary to ensure the integrity of the data used in all performance attribution analysis. This involves checking data received from a vendor against calculated results and setting allowed tolerances for the difference between the two. If checks exceed specified tolerances then the data feed from the vendor needs to be examined closely for errors. Some examples of data checks are: constituent total return should equal the published constituent return; aggregated constituent returns should equal published index total return; calculated statistics like yield-to-maturity, effective duration, modified duration, etc, should equal published statistics.

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