

## GROUP 4

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### Exercise 1

The I-combinator returns the argument that was passed in.  
We can use it to extract encapsulated values.

$\lambda x.x$

### Exercise 2

**1. Which lambda expression is alpha equivalent to  $\lambda x.x$**

option (b) - " $\lambda a.a$ "

**2. Which lambda expression is alpha equivalent to  $\lambda xy.yx$**

option (b) - " $\lambda a(\lambda b.ba)$ "

**3. Which lambda expression is alpha equivalent to  $\lambda xy.xz$**

option (b) - " $\lambda mn.mz$ "

### Exercise 3

**1.**

$(\lambda x.x)y$   
 $= x [x := y]$   
 $= y$

**2.**

$\lambda x.xx$   
 $= x$

**3.**

$(\lambda z.zz)(\lambda y.yy)$   
 $= zz [z := \lambda y.yy]$   
 $= (\lambda y.yy)(\lambda y.yy)$   
 $= yy [y := \lambda y.yy]$   
 $= (\lambda y.yy)(\lambda y.yy)$

divergent

**4.**

$(\lambda x.xx)y$   
 $= xx [x := y]$   
 $= yy$

## Exercise 4

1.

$$\begin{aligned} & (\lambda y.zy)a \\ &= zy [y:=a] \\ &= za \end{aligned}$$

2.

$$\begin{aligned} & (\lambda x.x)(\lambda x.x) \\ &= x [x:=\lambda x.x] \\ &= \lambda x.x \end{aligned}$$

3.

$$\begin{aligned} & (\lambda x.xy)(\lambda x.xx) \\ &= xy [x:=\lambda x.xx] \\ &= (\lambda x.xx)y \\ &= xx[x:=y] \\ &= yy \end{aligned}$$

4.

$$\begin{aligned} & (\lambda z.z)(\lambda a.aa)(\lambda z.zb) \\ &= (z [z:=\lambda a.aa])(\lambda z.zb) \\ &= (\lambda a.aa)(\lambda z.zb) \\ &= aa [a:=\lambda z.zb] \\ &= (\lambda z.zb)(\lambda z.zb) \\ &= zb [z:=\lambda z.zb] \\ &= (\lambda z.zb)b \\ &= zb [z:=b] \\ &= bb \end{aligned}$$

## Exercise 5

1.

$$\lambda x. zx$$
$$= z$$

2.

$$\lambda x. xz$$
$$= z$$

3.

$$(\lambda x. bx)(\lambda y. ay)$$
$$= bx [x := \lambda y. ay]$$
$$= b(\lambda y. ay)$$

4.

$$\lambda xyz. xy(zxy)$$
$$= xy [x := z, y := x, z := y]$$
$$= zx$$

## Exercise 6

1. is a combinator
2. not a combinator
3. is a combinator
4. is a combinator

## Exercise 7

$$Y = \lambda f. (\lambda x. f(xx))(\lambda x. f(xx))$$
$$Y(g)$$
$$= \lambda f. (\lambda x. f(xx))(\lambda x. f(xx)) g$$
$$= (\lambda x. f(xx))(\lambda x. f(xx)) [f := g]$$
$$= (\lambda x. g(xx))(\lambda x. g(xx))$$
$$= g(xx) [x := \lambda x. g(xx)]$$
$$= g((\lambda x. g(xx))(\lambda x. g(xx)))$$
$$Y(g) = (\lambda x. g(xx))(\lambda x. g(xx)) = g((\lambda x. g(xx))(\lambda x. g(xx))) = g(Y(g))$$

## Exercise 8

NOT FALSE

= (\x. IF x FALSE TRUE) (FALSE)  
= IF x FALSE TRUE [x:=FALSE]  
= IF FALSE FALSE TRUE  
= (\btf.btf) FALSE FALSE TRUE  
= btf [b:=FALSE, t:=FALSE, f:=TRUE]  
= FALSE FALSE TRUE  
= (\xy.y)FALSE TRUE  
= y [x:=FALSE, y:= TRUE]  
= TRUE

IF (OR TRUE FALSE)

= (\btf.btf) (OR TRUE FALSE)  
= btf [b:= OR, t:= TRUE, f:= FALSE]  
= OR TRUE FALSE  
= (\xy. IF x TRUE y) (TRUE FALSE)  
= IF x TRUE y [x:=TRUE, y:=FALSE]  
= IF TRUE TRUE FALSE  
= (\btf.btf)(TRUE TRUE FALSE)  
= btf [b:=TRUE, t:=TRUE, f:=FALSE]  
= TRUE TRUE FALSE  
= (\xy.x) TRUE FALSE  
= x [x:=TRUE, y:= FALSE]  
= TRUE

IF (AND TRUE TRUE)

= (\btf.btf)(AND TRUE TRUE)  
= btf [b:=AND, t:=TRUE, f:=TRUE]  
= AND TRUE TRUE  
= (\xy. IF x y FALSE) (TRUE TRUE)  
= IF x y FALSE [x:= TRUE, y:= TRUE]  
= IF TRUE TRUE FALSE  
= (\btf.btf)(TRUE TRUE FALSE)  
= btf [b:=TRUE,t:=TRUE,f:=FALSE]  
= TRUE TRUE FALSE  
= (\xy.x) TRUE FALSE  
= x [x:=TRUE, y:=FALSE]  
= TRUE