attenuation_q

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1 Attenuation as Q-value

Damping in a passive material can be described in multiple ways. Examples are a complex wave number $k_r - jk_i$, complex speed of sound $c_r + jc_i$, attenuation coefficient α, or a Q-value.

This note gives relations between the formulations.

2 Complex wave-number

Note the negative sign conventon for k_i

$$k = k_r - jk_i$$

Complex speed of sound

$$c = \omega/k = \frac{\omega}{k_r - jk_i} = \frac{\omega}{k_r}\frac{1}{1 - jk_i/k_r} = \frac{\omega}{k_r}\frac{1 + jk_i/k_r}{1 + (k_i/k_r)^2} \approx \frac{\omega}{k_r}\left(1 + j\frac{k_i}{k_r}\right) = c_r\left(1 + j\frac{k_i}{k_r}\right) = c_r + jc_i$$

$$c_i = c_r\frac{k_i}{k_r} \quad , \quad \frac{c_i}{c_r} = \frac{k_i}{k_r}$$

Attenuation α

$$p(x,t) = p_0 e^{j(\omega t - kx)} = p_0 e^{j(\omega t - (k_r - jk_i)x)} = p_0 e^{j(\omega t - k_r x)} e^{-k_i x} = p_0 e^{j(\omega t - k_r x)} e^{-\alpha x} \quad , \quad \alpha = k_i$$

3 Intensity

Intensity loss from attenuation α

$$I = \frac{|p|^2}{2\rho c} = I_0 e^{-2\alpha x} \quad , \quad I_0 = \frac{p_0^2}{2\rho c}$$

Energy loss per unit distance

$$\frac{dI}{dx} = -2\alpha I(x) \quad , \quad \frac{-\Delta I}{I} = 2\alpha\Delta x$$

Relative energy loss over one wavelength

$$\Delta x = \lambda \quad \frac{(-\Delta I)_\lambda}{I} = 2\alpha\Delta\lambda = 2k_i\lambda = 2k_i\frac{2\pi}{k_r} = 4\pi\frac{k_i}{k_r}$$

4 Q-value

$$Q = 2\pi \frac{I}{(-\Delta I)_\lambda} = \frac{k_r}{2k_i} = \frac{c_r}{2c_i} \quad c_i = \frac{c_r}{2Q}$$
$$c = c_r + jc_i = c_r \left(1 + \frac{j}{2Q}\right) \quad k = k_r - jk_i \approx \frac{\omega}{c_r}\left(1 - \frac{j}{2Q}\right)$$
$$Q = \frac{k_r}{2\alpha} = \frac{\omega}{2c\alpha} = \frac{\pi f}{c\alpha} \qquad \alpha = \frac{1}{2Q}\frac{\omega}{c} = \frac{1}{Q}\frac{\pi f}{c}$$

4.1 Attenuation coeficient in dB

$$p(x,t) = p_0 e^{j(\omega t - k_r x)} e^{-\alpha x}$$
$$20\lg\left|\frac{p}{p_0}\right| = 20\lg\left(e^{-\alpha x}\right) = -20\lg(e) \times \alpha x = -8.69\alpha x$$
$$\alpha_{dB} = 20\lg(e)\alpha = 8.69\alpha$$

4.2 Example: Q-value for water

- Attenuation coefficient α= 0.0022 dB/(cm MHz)
- Frequency 200 kHz α_{dB}=0.044 dB/m
- Expressed in Np (1/m) $\alpha_{dB} = \frac{\alpha_{dB}}{8.69} = 5.1 \times 10^{-3}$ 1/m
- $Q = \frac{\pi f}{c\alpha} = 8 \times 10^4$

[5]:
```python
import numpy as np
import matplotlib.pyplot as plt
import ipywidgets as widgets

def plot_q(c):
    DB_CONST = 20* np.log10( np.exp(1) )

    alpha_dB_cm_MHz = np.linspace( 0.05, 4, 1000 )  # Range of attenuations,␣
 ↪using conventional medical uotrasound unit [dB/(cm MHz)]
    alpha_dB  = alpha_dB_cm_MHz/1e6*100             # Convert to [dB/(m Hz)]
    Q = np.pi/( c * alpha_dB )                      # Corresponding Q-value

    #--- Plot result in graph ---
    plt.figure( 1, ( 10, 6 ) )
    plt.semilogy( alpha_dB_cm_MHz, Q )
    plt.xlabel( 'Attenuation [dB/(cm MHz)]' )
    plt.ylabel( 'Q' )
    plt.title( f'Speed of sound {c:.0f} m/s' )
    plt.grid( True, which= 'major', axis='x' )
    plt.grid( True, which= 'both',  axis='y' )
    plt.xlim( 0, 4 )
    plt.ylim( 1, 1e3 )
```

[6]:

```python
widgets.interact(plot_q, c=widgets.FloatSlider( min=500, max=8000, step=100,␣
 ↪value=1500, description='c [m/s]', orientation='horizontal',␣
 ↪continuous_update=True, readout_format='.0f' ) );
```

```
interactive(children=(FloatSlider(value=1500.0, description='c [m/s]', max=8000.
 ↪0, min=500.0, readout_format='…
```

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