Machine Learning, Tutorial 4
University of Bern

Mehdi Noroozi (noroozi@inf.unibe.ch)

17/10/2018

Multivariate Gaussian

1. Prove the following statements regarding the covariance:
   (a) Show that if X and Y are independent then $\text{Cov}[X, Y] = 0$.
   Give an example that shows that the opposite is not true.
   (b) Show that the covariance matrix is always symmetric and positive semidefinite.

2. For a function $f : R^2 \to R$, an isocontour is a set of the form
$$\{x \in R^2 : f(x) = c\}$$
for some $c \in R$.
Derive an analytical form for the isocontours of a multivariate Gaussian.

3. Consider the classifier based on Gaussian Discriminant Analysis, where the distribution of the samples are modelled by
$$p(y) = \phi^y(1-\phi)^{(1-y)}, \tag{1}$$
$$p(x|y=0) = \frac{1}{(2\pi)^{n/2}|\Sigma|^{1/2}} \exp(-\frac{1}{2}(x-\mu_0)^T\Sigma^{-1}(x-\mu_0)), \tag{2}$$
$$p(x|y=1) = \frac{1}{(2\pi)^{n/2}|\Sigma|^{1/2}} \exp(-\frac{1}{2}(x-\mu_1)^T\Sigma^{-1}(x-\mu_1)). \tag{3}$$
Compute $p(y = 1|x)$. How does this relate to Linear Regression?

Naive Bayes

1. Consider a text classification problem using the multinomial naive Bayes classifier. Given the following data we want to classify texts into two classes j(Japanese) and c(Chinese) based on the observed words.

	Doc	Words	Class
Training	1	Chinese Beijing Chinese	c
	2	Chinese Chinese Shanghai	c
	3	Chinese Macao	c
	4	Tokyo Japan Chinese	j
Test	5	Chinese Chinese Chinese Tokyo Japan	?

Give detailed answers to the following questions.

(a) Calculate the probabilities of the two classes $P(c)$ and $P(j)$.
(b) Calculate conditional probabilities $P(word|class)$ by using Laplacian smoothing.
(c) Write the inequality (with explicit number) used to classify the fifth document.