Kybintro øving 4 - Lars André Roda Jansen

Oppgave 1)

a)

a) $$\frac{d}{dt} m = w_{inn} - w_{ut}$$

$$\frac{d}{dt}(\rho A h) = w_{inn} - w_{ut}$$

$$\frac{d}{dt} h = \frac{1}{\rho A}(w_{inn} - w_{ut})$$

$$w_{ut} = k h$$

$$\frac{d}{dt} h = \frac{1}{\rho A}(w_{inn} - k h)$$

$$\frac{d}{dt} h = -\frac{k}{\rho A} h + \frac{1}{\rho A} w_{inn}$$

$$\frac{d}{dt} h + \frac{k}{\rho A} h = \frac{1}{\rho A} w_{inn}$$

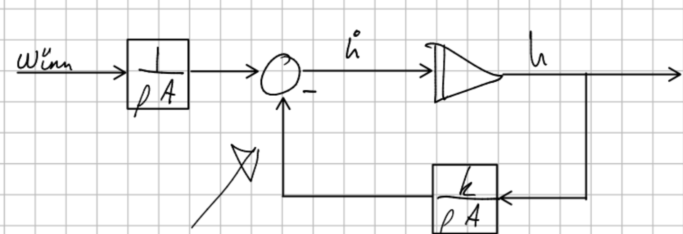
Modell:

$$\frac{d}{dt}h = -\frac{k}{\rho A}h + \frac{1}{\rho A}w_{inn}$$

Vi må anta att massen fra w*inn jevnes ut med engang med en gang den legges til i tanken.*
*Pådragsorganet vil være røret der $w*{inn}$ kommer fra.
Pådraget vil være w_{inn} .

b)

b) $$\frac{d}{dt} h = -\frac{k}{\rho A} h + \frac{1}{\rho A} w_{inn}$$



Naturlig tilbakekobling

c)
Om $w_{inn} = 0$, så vil $t \to \infty$, $h \to 0$. Dette er fordi all massen i tanken vil bare renne ut og bli tom om ingenting annet renner inn.
Dette kan vi se på vår modell. Ettersom $w_{inn} = 0$, så vil ifølge modellen

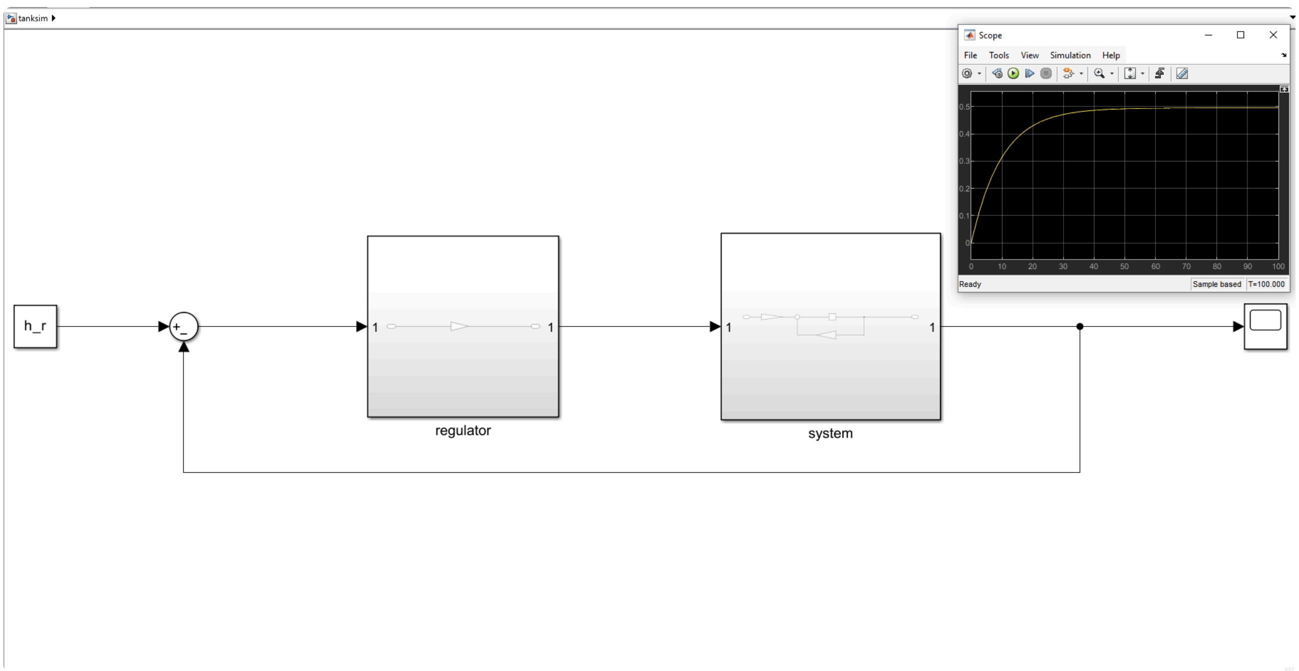
$$\frac{d}{dt}h = -\frac{k}{\rho A}h$$

Dette fører til att endringen $\dot{h}$ vil være negativ helt til $h = 0$, altså att nivået tilsvarer 0.
Modellen vil bli stabil fordi den går mot en bestemt tilstand $h = 0$.

d)
Om vi hadde hatt en positiv tilbakekobling i systemet så hadde dette innebært att $\frac{d}{dt}h$ hadde vært positiv, slik att tanken blir bare fylt opp med væske, i stedet for att væske renner ut.

Modellen hadde derfor vært ustabil fordi den aldri når en stasjonærverdi, men vil heller forevig øke.

Oppgave 2)
a)



b)

Oppgave 2)

b) Stasjonæravvik: a

$$a = h_r - h(\infty)$$

$$\frac{d}{dt}h = -\frac{k}{\rho A}h + \frac{1}{\rho A}w_{inn}$$

$$w_{inn} = K_p(h_r - h)$$

$$\frac{d}{dt}h = -\frac{k}{\rho A}h + \frac{1}{\rho A}K_p(h_r - h)$$

$$\frac{d}{dt}h = -\frac{k}{\rho A}h + \frac{K_p}{\rho A}h_r - \frac{K_p}{\rho A}h \qquad -\frac{k}{\rho A} - \frac{K_p}{\rho A} = -\frac{1}{\rho A}(k + K_p)$$

$$\frac{d}{dt}h = \left(-\frac{k + K_p}{\rho A}\right)h + \frac{K_p}{\rho A}h_r$$

$$\frac{d}{dt}h + \frac{k + K_p}{\rho A}h = \frac{K_p}{\rho A}h_r \qquad \lambda = \frac{k + K_p}{\rho A} \qquad \beta = \frac{K_p}{\rho A}h_r$$

$$\frac{d}{dt}h + \lambda h = \beta \quad \Big| \cdot e^{\lambda t}$$

$$\frac{d}{dt}h\, e^{\lambda t} + \lambda h e^{\lambda t} = \beta e^{\lambda t}$$

$$\frac{d}{dt}\left(h e^{\lambda t}\right) = \beta e^{\lambda t} \quad \Big| \cdot \int dt$$

$$h e^{\lambda t} = \frac{1}{\lambda}\beta e^{\lambda t} + C \quad \Big| \cdot e^{-\lambda t}$$

$$h(t) = \frac{1}{\lambda}\beta + C e^{-\lambda t} \qquad \lambda = \frac{k + K_p}{\rho A} \qquad \beta = \frac{K_p}{\rho A}h_r$$

$$h(t) = \frac{\rho A}{k + K_p} \cdot \frac{K_p}{\rho A}h_r + C e^{-\frac{k + K_p}{\rho A}t}$$

$$h(t) = \frac{K_p}{k + K_p}h_r + C e^{-\frac{k + K_p}{\rho A}t}$$

$$\overline{k + K_p}$$

$$h(0) = h_0$$

$$\frac{K_p}{k + K_p} h_r + C = h_0$$

$$C = h_0 - \frac{K_p}{k + K_p} h_r$$

$$h(t) = \frac{K_p}{k + K_p} h_r + \left(h_0 - \frac{K_p}{k + K_p} h_r\right) e^{-\frac{k + K_p}{\rho A} t}$$

$$a = h_r - h(\infty)$$

$$a = h_r - \frac{K_p}{k + K_p} h_r$$

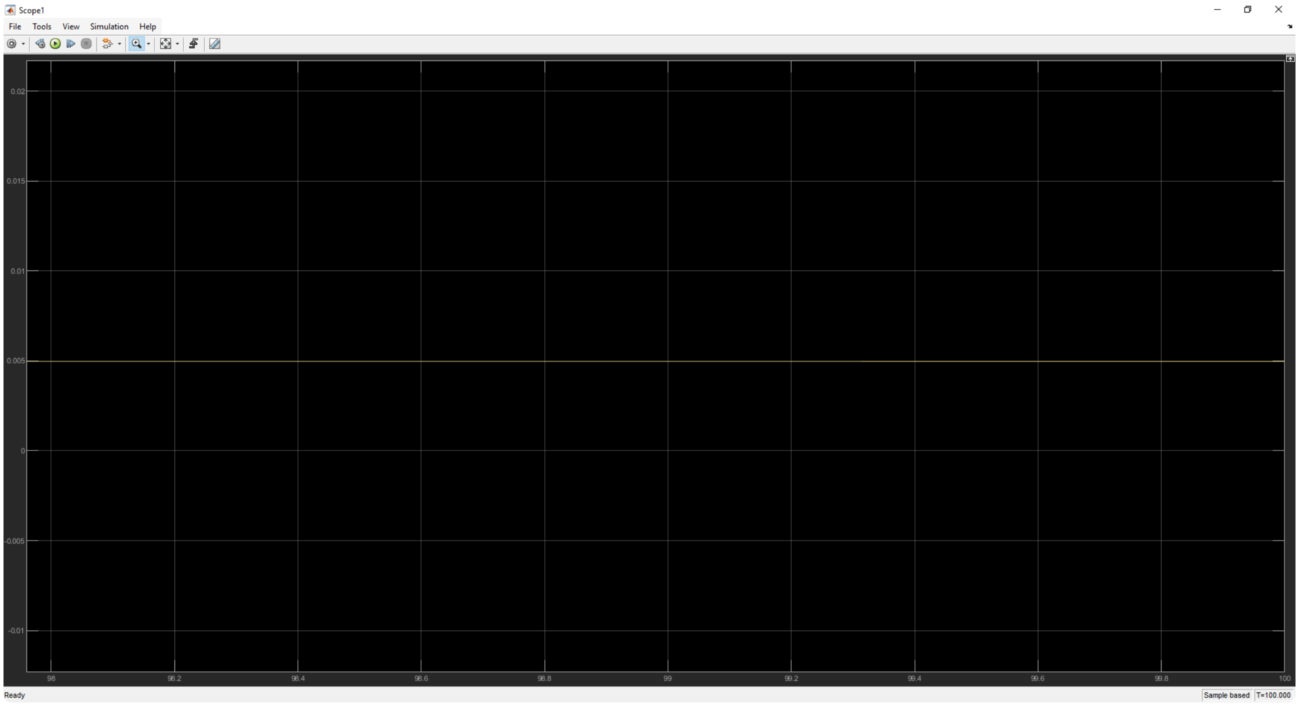
$$a = h_r\left(1 - \frac{K_p}{k - K_p}\right)$$

$$K_p = 100 \quad k = 1 \quad h_r = 0{,}5$$

$$a = 0{,}5 \cdot \left(1 - \frac{100}{1 + 100}\right)$$

$$a = 4{,}95 \cdot 10^{3} \approx \underline{\underline{005}}$$

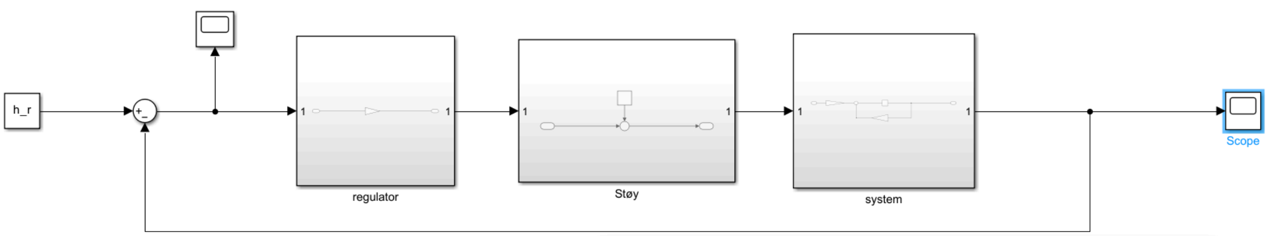
c)

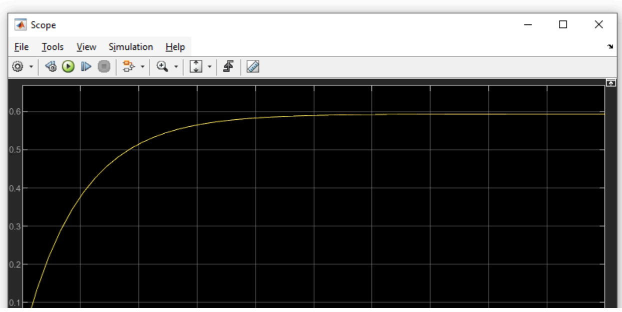


Den er lik og tilsvarer 0.05

d)
Man kan unngå problemer med standardavvik med å legge til ett I-ledd i regulatoren.

e)





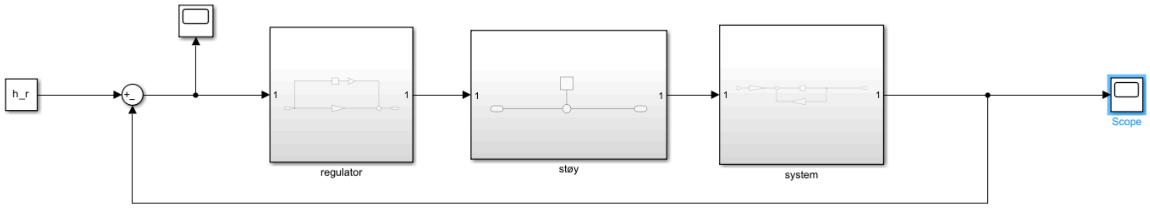
Stasjonæravviket vårt nå blir på ca -0.95

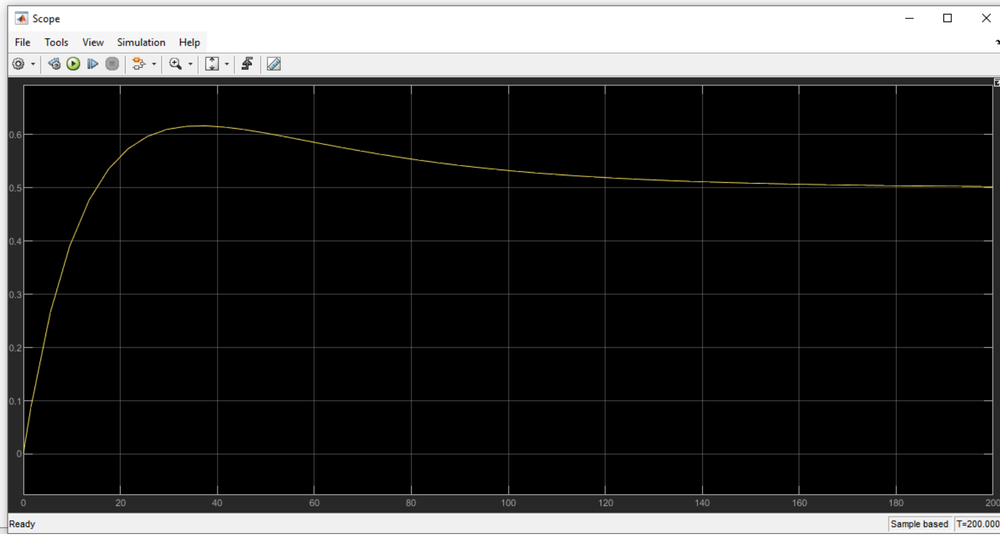
f)

$$u = K_p(r - x) + K_i \int r - x \, dt$$

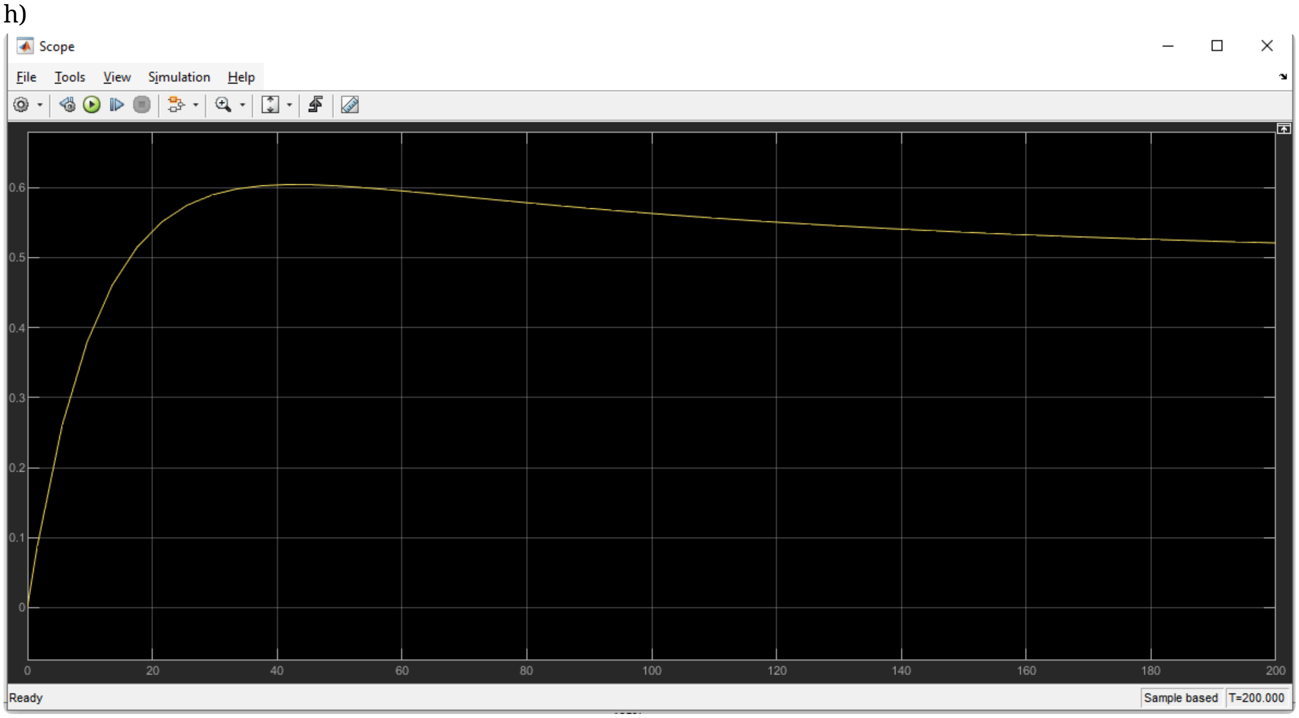
g)

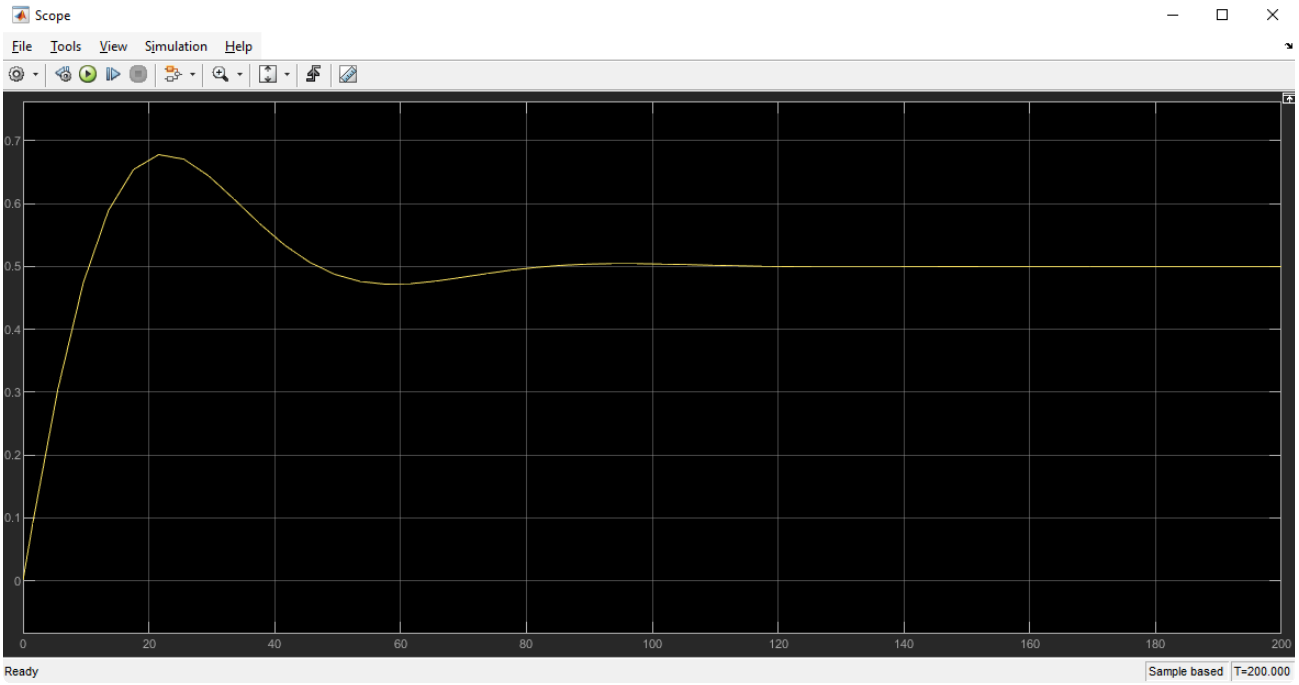
```
%% INIT
h_max = 1;
A = 1;
k = 1;
rho = 1000;

h_0 = 0;
h_r = 0.5;

K_p = 100;
K_i = 2;

w_f = 10;

T = 200;
```

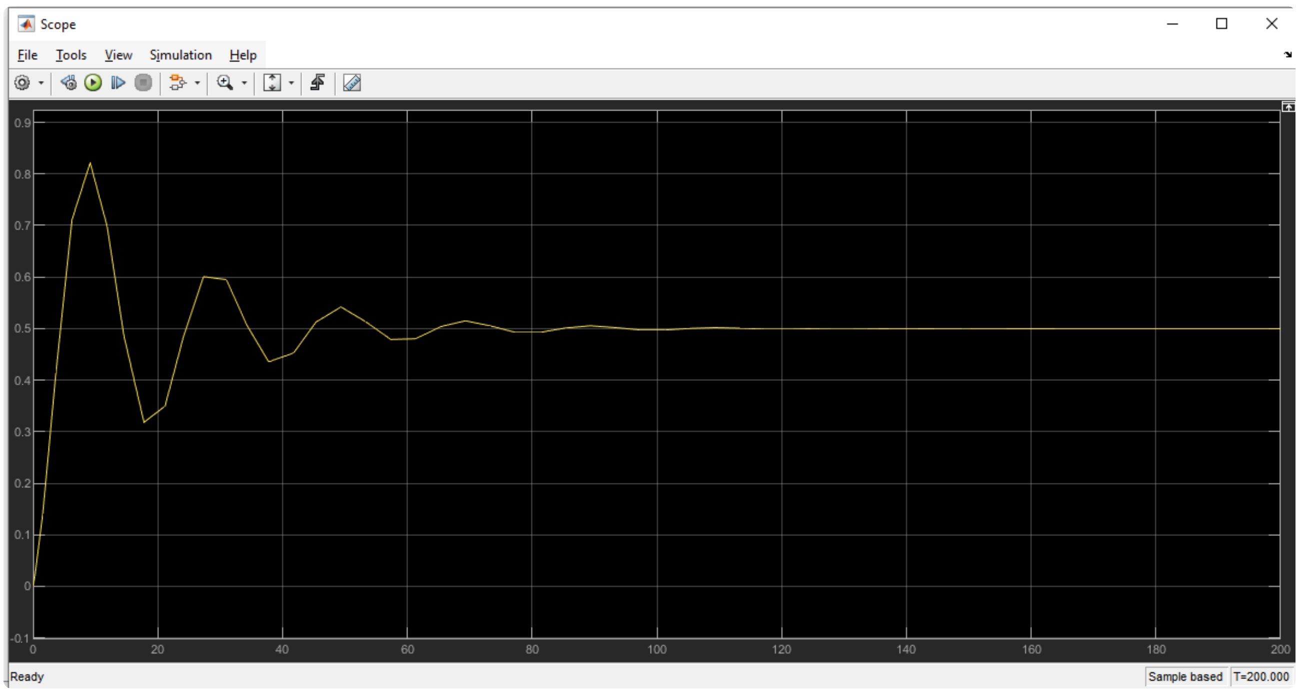

h_r
regulator
støy
system
Scope

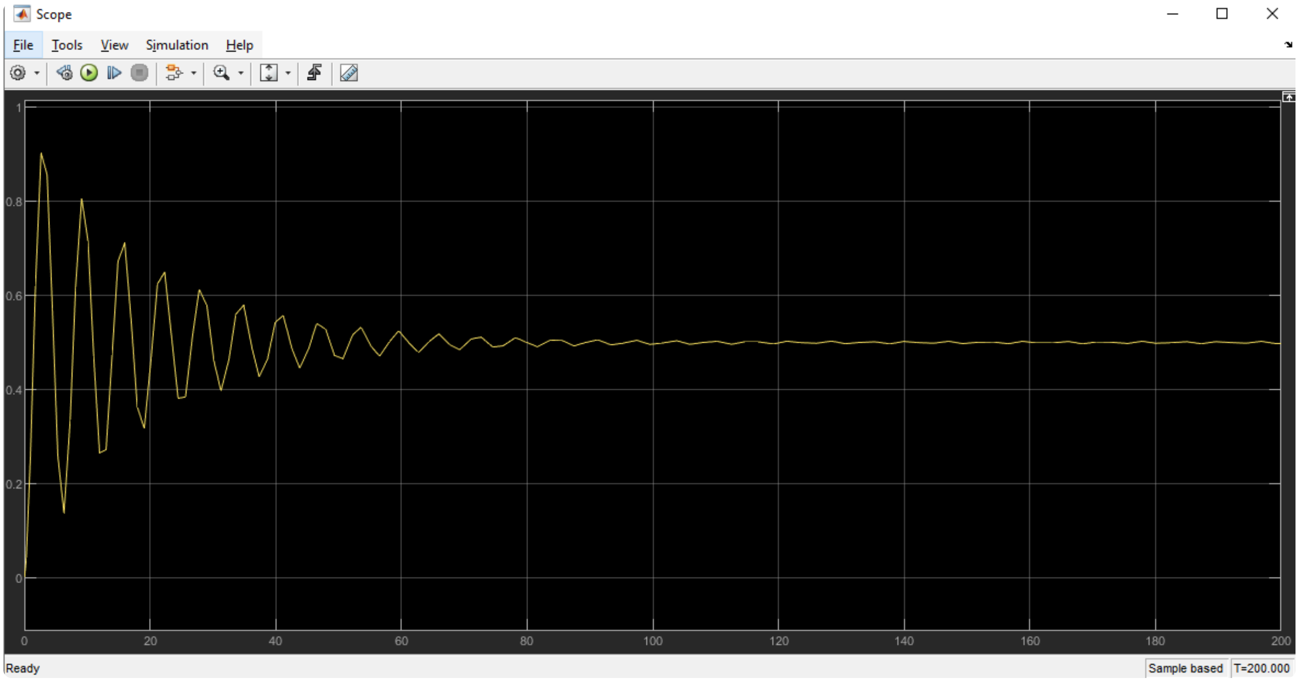

Scope
File Tools View Simulation Help
Ready
Sample based
T=200.000

h)


Scope
File Tools View Simulation Help
Ready
Sample based
T=200.000

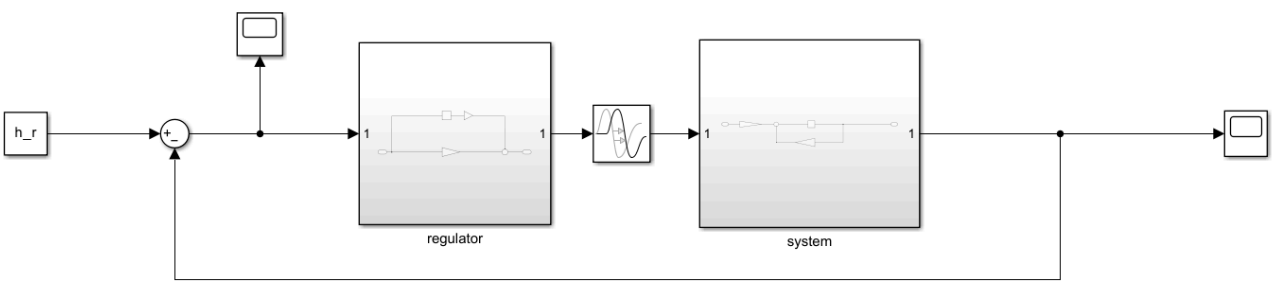

Scope
File Tools View Simulation Help
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0 20 40 60 80 100 120 140 160 180 200
Ready
Sample based
T=200.000


Scope
File Tools View Simulation Help
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
-0.1
0 20 40 60 80 100 120 140 160 180 200
Ready
Sample based
T=200.000

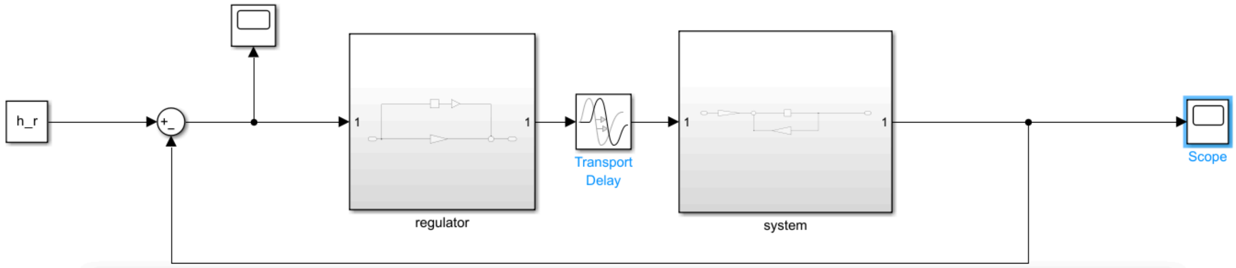


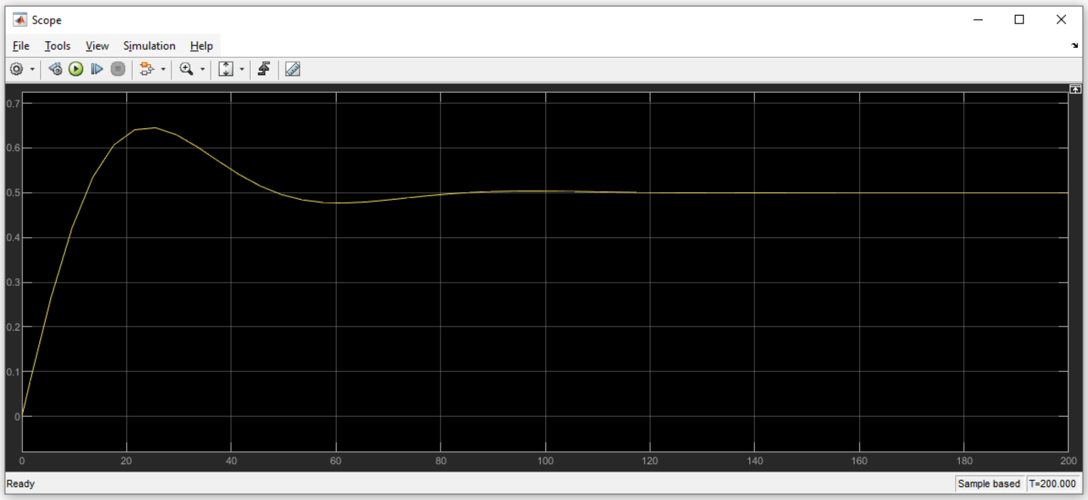
Vi kan se att ved høyere K_i, så vil antall svingninger til systemet før den når stasjonærverdi øke

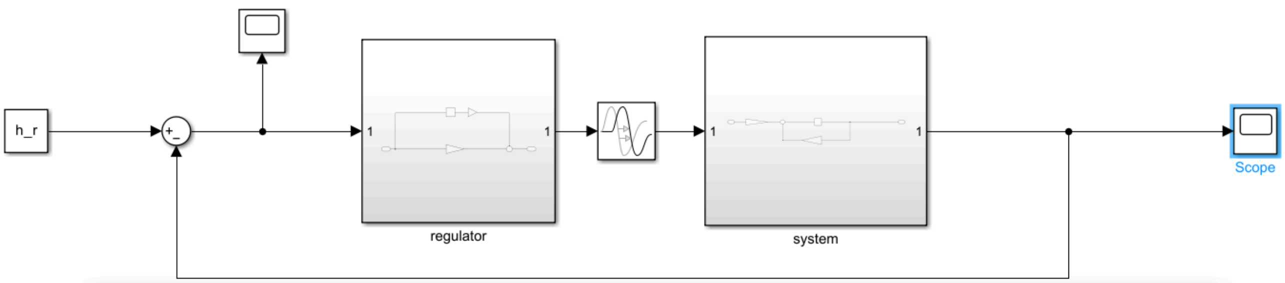
i)

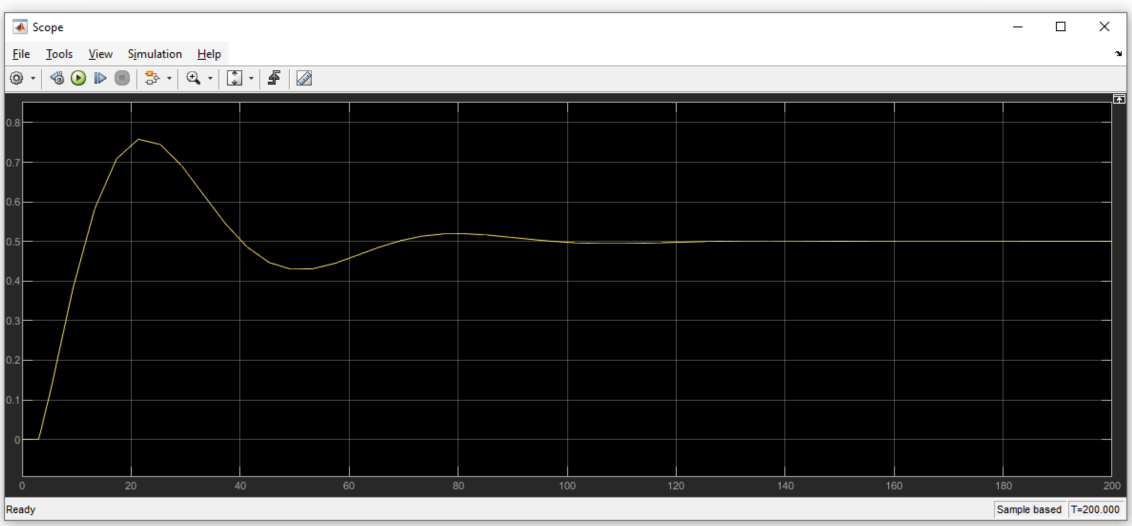


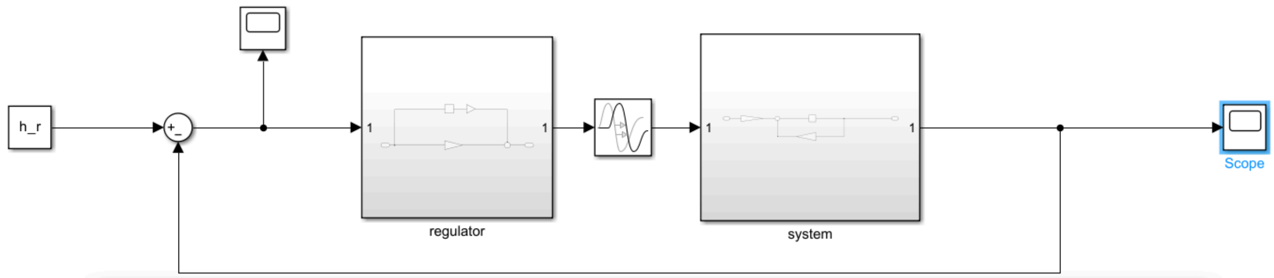
j)

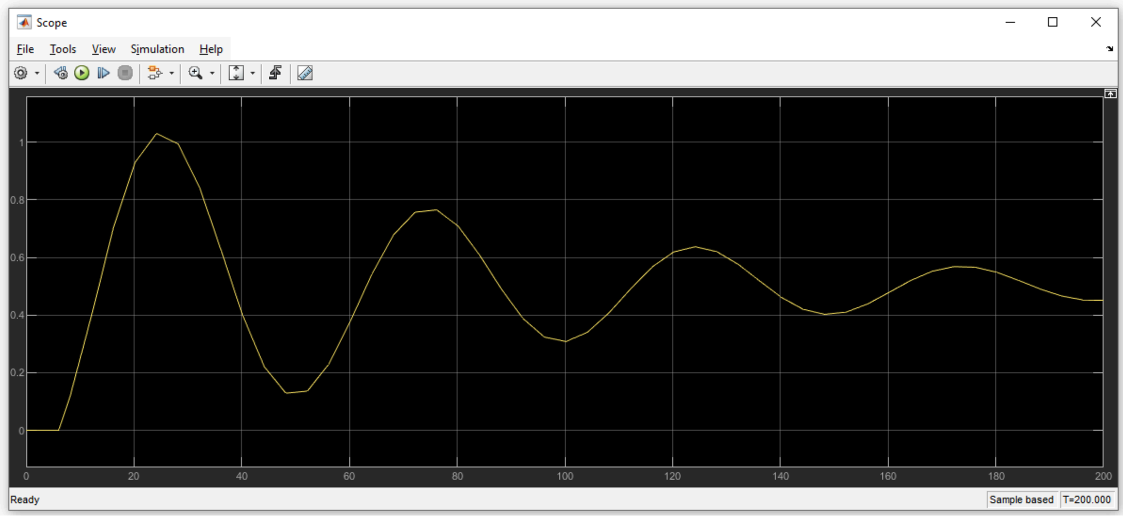

h_r
regulator
Transport Delay
system
Scope

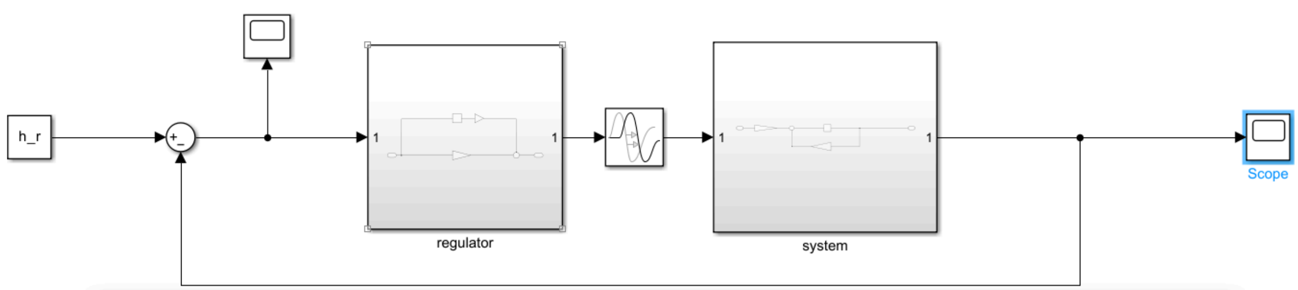

Scope
File Tools View Simulation Help
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0 20 40 60 80 100 120 140 160 180 200
Ready
Sample based
T=200.000

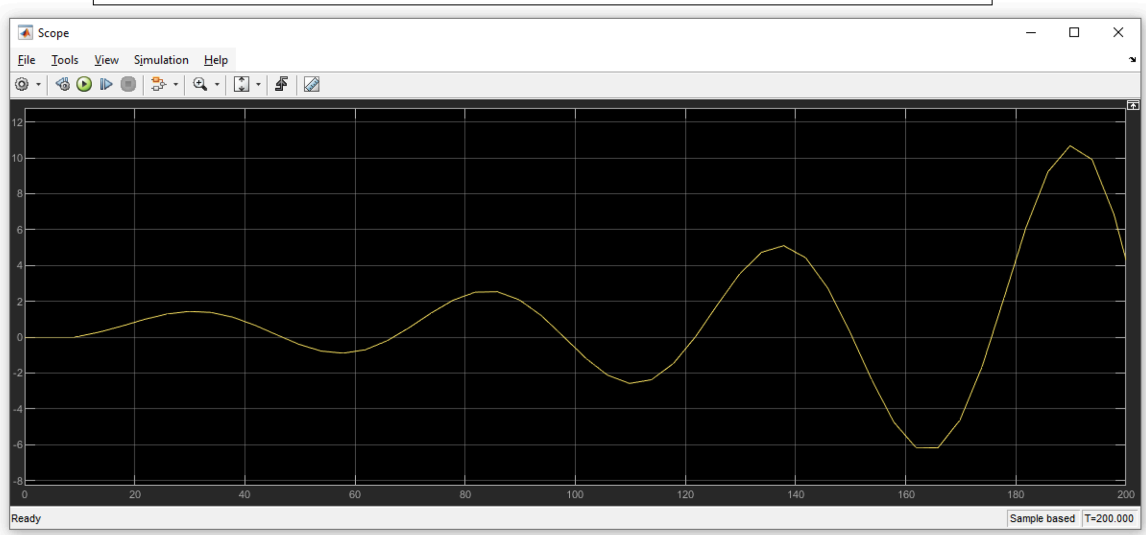

h_r
regulator
system
Scope


Scope
File Tools View Simulation Help
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0 20 40 60 80 100 120 140 160 180 200
Ready
Sample based
T=200.000









Vi kan se att den slutter å bli stabil når $\tau = 9$, for alle verdier før det så er systemet stabilt.