1 - 8 Lineæralgebra III

1) $\left\{ \begin{pmatrix} 1 \\ 2 \end{pmatrix}, \begin{pmatrix} 2 \\ 4 \end{pmatrix} \right\}$

Er en basis for $\mathbb{R}^2$ fordi vektorene er lineært uavhengige

2) $\left\{ \begin{bmatrix} 0 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 0 \end{bmatrix} \right\}$

3) $\begin{bmatrix} 2 & 1 & 1 & 2 \\ 1 & 0 & 1 & 1 \\ 1 & -2 & 1 & -1 \\ 0 & 1 & 0 & 1 \end{bmatrix}$

$$\left(\begin{array}{cccc|c} 2 & 1 & 1 & 2 & 0 \\ 1 & 0 & 1 & 1 & 0 \\ 1 & -2 & 1 & -1 & 0 \\ 0 & 1 & 0 & 1 & 0 \end{array}\right) \sim \left(\begin{array}{cccc|c} 2 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 & 0 \\ 1 & -1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \end{array}\right)$$

$$\sim \begin{pmatrix} 2 & 0 & 0 & 1 \\ -1 & 0 & 1 & 0 \\ 1 & -1 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{pmatrix} \sim \begin{pmatrix} 2 & 0 & 0 & 1 \\ -1 & 0 & 1 & 0 \\ 0 & -1 & 2 & 0 \\ 0 & 1 & 0 & 1 \end{pmatrix}$$

$$\sim \begin{pmatrix} 0 & 0 & 2 & 1 \\ -1 & 0 & 1 & 0 \\ 0 & -1 & 2 & 0 \\ 0 & 1 & 0 & 1 \end{pmatrix} \sim \begin{pmatrix} 0 & 1 & 0 & 1 \\ -1 & 0 & 1 & 0 \\ 0 & -1 & 2 & 0 \\ 0 & 1 & 0 & 1 \end{pmatrix}$$

$$\sim \begin{pmatrix} 0 & 1 & 0 & 1 \\ -1 & 0 & 1 & 0 \\ 0 & -1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Ikke lineært uavhengig

$$\begin{matrix} 2 & 1 & 1 & 2 \\ 1 & 0 & 1 & 1 \\ 1 & -2 & 1 & -1 \\ 0 & 1 & 0 & 1 \end{matrix} \sim \begin{matrix} 2 & 1 & 1 & 2 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \end{matrix}$$

ikke for $\mathbb{R}^4$

$$\sim \begin{matrix} 0 & 1 & -1 & 0 \\ 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \end{matrix} \sim \begin{matrix} 0 & 1 & -1 & 0 \\ 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 \end{matrix} \sim \begin{matrix} 0 & 1 & -1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \end{matrix}$$

$x_2 = x_3$

$x_1 = 0$

$x_2 = -x_4$

$x_2 = s$

Spenner ikke $\mathbb{R}^4$

$$\left(\begin{matrix} 2 & 1 & 2 \\ 1 & 1 & 1 \\ 1 & 1 & -1 \end{matrix}\middle| \begin{matrix} 0 \\ 0 \\ 0 \end{matrix}\right) \sim \begin{matrix} 1 & 0 & 3 \\ 0 & 0 & 2 \\ 1 & 1 & -2 \end{matrix} \sim \begin{matrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & -2 \end{matrix} \sim \begin{matrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{matrix}$$

Er en basis for $\mathbb{R}^3$

Radene spenner derfor også hele $\mathbb{R}^3$

4)
$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 5 \\ 3 & 4 & 5 & 6 \\ 4 & 5 & 6 & 7 \end{bmatrix}$$

$$\begin{matrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 5 \\ 3 & 4 & 5 & 6 \\ 4 & 5 & 6 & 7 \end{matrix} \sim \begin{matrix} 1 & 2 & 3 & 4 \\ 0 & -1 & -2 & -3 \\ 0 & -2 & -4 & -6 \\ 0 & -3 & -6 & -9 \end{matrix} \sim \begin{matrix} 1 & 2 & 3 & 4 \\ 0 & 1 & 2 & 3 \end{matrix}$$

$$\sim \left(\begin{matrix} 1 & 0 & -1 & -2 \\ 0 & 1 & 2 & 3 \end{matrix}\middle| \begin{matrix} 0 \\ 0 \end{matrix}\right)$$

$x_1 = x_3 + 2x_4$

$x_2 = -2x_3 - 3x_4$

$x_3 = s, \quad x_4 = t$

Spenner $\mathbb{R}^2$

5)

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 5 \\ 3 & 4 & 5 & 6 \end{bmatrix}$$

$$\begin{matrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 5 \\ 3 & 4 & 5 & 6 \end{matrix} \sim \begin{matrix} 1 & 2 & 3 & 4 \\ 0 & -1 & -2 & -3 \\ 0 & -2 & -4 & -6 \end{matrix}$$

$x_1 = x_3 + 2x_4$

$x_2 = -2x_3 - 3x_4$

$x_3 = s, \quad x_4 = t$

$$v = s\begin{bmatrix} 1 \\ -2 \\ 1 \\ 0 \end{bmatrix} + t\begin{bmatrix} 2 \\ -3 \\ 0 \\ 1 \end{bmatrix}$$

x_3 & x_4 er frie variabler, kan derfor skrives som $\begin{bmatrix} 1 & 2 \\ 2 & 3 \\ 3 & 4 \end{bmatrix}$

Spenner $\mathbb{R}^2$

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 5 \\ 3 & 4 & 5 & 6 \end{bmatrix} \xRightarrow{\text{Trans}} \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 4 & 5 \\ 4 & 5 & 6 \end{bmatrix}$$

$$\left.\begin{matrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 4 & 5 \\ 4 & 5 & 6 \end{matrix}\right| \begin{matrix} 0 \\ 0 \\ 0 \\ 0 \end{matrix} \sim \begin{matrix} 1 & 2 & 3 \\ 0 & -1 & -2 \\ 0 & -2 & -4 \\ 0 & -3 & -6 \end{matrix} \sim \begin{matrix} 1 & 2 & 3 \\ 0 & 1 & 2 \end{matrix}$$

$$\sim \begin{matrix} 1 & 0 & -1 \\ 0 & 1 & 2 \end{matrix}$$

$x_1 = x_3$

$x_2 = -2x_3$

$x_3 = s$

x_3 er fri variabel, kan skrives som $\begin{bmatrix} 1 & 2 \\ 2 & 3 \\ 3 & 4 \\ 4 & 5 \end{bmatrix}$

Spenner derfor $\mathbb{R}^2$ yeah

7)
$$\begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix}$$

$$|A - \lambda I| = 0$$

$$\begin{vmatrix} 2-\lambda & 1 & 1 \\ 1 & 2-\lambda & 1 \\ 1 & 1 & 2-\lambda \end{vmatrix}$$

$$(2-\lambda)((2-\lambda)(2-\lambda)-1) - ((2-\lambda)-1) + (1-(2-\lambda)) = 0$$

$$(2-\lambda)(4-4\lambda+\lambda^2-1) - (1-\lambda) + (-1+\lambda) = 0$$

$$(2-\lambda)(\lambda^2-4\lambda+3) - (1-\lambda) - (1-\lambda) = 0$$

$$\lambda = \frac{4 \pm \sqrt{16-12}}{2} = \frac{4 \pm 2}{2} = 3 \wedge 1$$

$$(2-\lambda)(\lambda-3)(\lambda-1) + 2(\lambda-1) = 0$$

$$(\lambda-1)((2-\lambda)(-3+\lambda)+2)$$

$$(\lambda-1)(-6+2\lambda+3\lambda-\lambda^2+2) = 0$$

$$(\lambda-1)(-\lambda^2+5\lambda-4) = 0 \qquad 1 \cdot -1 \cdot -1$$

$$(1-\lambda)(\lambda^2-5\lambda+4)$$

$$\lambda = \frac{5 \pm \sqrt{25-16}}{2} = \frac{5 \pm 3}{2} = 4 \wedge 1$$

$$(1-\lambda)(\lambda-1)(\lambda-4) = 0 \qquad 1 \cdot -1 \cdot -1$$

$$(1-\lambda)(1-\lambda)(4-\lambda) = 0$$

$$\lambda_1 = 1, \quad \lambda_2 = 4$$

$$\lambda = \lambda_1$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ & & & 0 \\ & & & 0 \end{array}\right]$$

$x_1 = -x_2 - x_3$

$x_2 = s \qquad v_1 = s\begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} + t\begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$

$x_3 = t$

$$E_{\lambda_1} = \begin{bmatrix} -1 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

$\lambda = \lambda_2$

$$\left[\begin{array}{ccc|c} -2 & 1 & 1 & 0 \\ 1 & -2 & 1 & 0 \\ 1 & 1 & -2 & 0 \end{array}\right]$$

$$\sim \begin{array}{ccc} -2 & 1 & 1 \\ 3 & -3 & 0 \\ 3 & 0 & -3 \end{array} \sim \begin{array}{ccc} -2 & 1 & 1 \\ 1 & -1 & 0 \\ 1 & 0 & -1 \end{array} \sim \begin{array}{ccc|c} 0 & 0 & 0 & 0 \\ 1 & -1 & 0 & 0 \\ 1 & 0 & -1 & 0 \end{array}$$

$x_1 = x_2$

$x_1 = x_3$ $\qquad v_2 = s \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

$x_1 = s$

$$E_{\lambda_2} = \begin{bmatrix} 1 & 0 \\ 1 & 0 \\ 1 & 0 \end{bmatrix}$$

8) bruk g

a) $\ddot{x} + \dot{x} + x = 0$

$x_1 = x$

$x_2 = \dot{x}$

$\dot{x}_1 = x_2$

$\dot{x}_2 = -x_1 - x_2$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1 & -1 \end{bmatrix}$$

$|A - \lambda I| = 0$

$$\begin{vmatrix} -\lambda & 1 \\ -1 & -1-\lambda \end{vmatrix} = 0$$

$(-\lambda)(-1-\lambda) - (1)(-1) = 0$

$\lambda + \lambda^2 + 1 = 0$

$$\lambda = \frac{-1 \pm \sqrt{1-4}}{2} = \frac{-1 \pm \sqrt{3}\,i}{2} = -\frac{1}{2} \pm \frac{\sqrt{3}}{2} i$$

$$x = d_1 e^{\left(-\frac{1}{2} + \frac{\sqrt{3}}{2} i\right)t} + d_2 e^{\left(-\frac{1}{2} - \frac{\sqrt{3}}{2} i\right)t}$$

$\ddot{x} + \dot{x} + x = 0$

$$\lambda = \frac{-1 \pm \sqrt{1-4}}{2} = 0$$

$\lambda = -\frac{1}{2} \pm \frac{\sqrt{3}}{2} i$

$$x = e^{-\frac{1}{2}t}\left(c_1 \cos\left(\frac{\sqrt{3}}{2}t\right) + c_2 \sin\left(\frac{\sqrt{3}}{2}t\right)\right)$$

Basis 1 $\left[e^{\left(-\frac{1}{2} + \frac{\sqrt{3}}{2} i\right)t}, e^{\left(-\frac{1}{2} - \frac{\sqrt{3}}{2} i\right)t}\right]$

Basis 2 $\left[e^{-\frac{1}{2}t} \cos \frac{\sqrt{3}}{2}t, e^{-\frac{1}{2}t} \sin \frac{\sqrt{3}}{2}\right]$

10) $p(x) = a_2 x^2 + a_1 x + a_0$

$a_2 x^2 + a_1 x + a_0 = 0$

$a_2 = a_1 = a_0 = 0$

$\{x^2, x, 1\}$

Tre dimensjonale Basis

11) $C = a + bi, \ a \in \mathbb{R}, \ b \in \mathbb{R}, \ i^2 = -1$

12) $c_1 \cos 2\tau + c_2 \sin 2\tau + c_3 = 0$

$c_1 = c_2 = c_3 = 0$

$[\cos 2\tau, \sin 2\tau, 1]$ Tre dimensjonale

$c_1 \cos^2\tau + c_2 \sin^2\tau + c_3 = 0$

$c_3 = c_1 \cos^2\tau + c_2 \sin^2\tau$ når $c_1 = c_2 = c_3 = 1$

$[\cos^2\tau, \sin^2\tau]$ To dimensjonale

13) $(3, 1)$ $\left\{\begin{pmatrix}2\\1\end{pmatrix}, \begin{pmatrix}1\\2\end{pmatrix}\right\}$

$\left.\begin{matrix}2 & 1\\1 & 2\end{matrix}\right| \begin{matrix}3\\1\end{matrix} \sim \left.\begin{matrix}2 & 1\\0 & 3/2\end{matrix}\right| \begin{matrix}3\\-1/2\end{matrix} \sim \left.\begin{matrix}2 & 1\\0 & 3\end{matrix}\right| \begin{matrix}3\\-1\end{matrix} \sim \left.\begin{matrix}2 & 0\\0 & 3\end{matrix}\right| \begin{matrix}10/3\\-1\end{matrix}$

$x_1 = 5/3$

$x_2 = -\frac{1}{3}$ $\quad \frac{1}{3}\begin{pmatrix}5\\-1\end{pmatrix}$

Grete

1 4)

$$\ddot{x} + 2\dot{x} + 2x = 0$$

$$\{e^{-\tau}\cos\tau, e^{-\tau}\sin\tau\} \quad \{e^{(-1+i)\tau}, e^{(-1-i)\tau}\}$$

$$x(\tau) = e^{-\tau}(\cos\tau + \sin\tau)$$

$$e^{-\tau}(\cos\tau + \sin\tau) = c_1 e^{-\tau}\cos\tau + c_2 e^{-\tau}\sin\tau$$

$$c = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$e^{-\tau}(\cos\tau + \sin\tau) = c_1 e^{(-1+i)\tau} + c_2 e^{(-1-i)\tau}$$

$$e^{-\tau}(\cos\tau + \sin\tau) = c_1 e^{-\tau+i\tau} + c_2 e^{-\tau-i\tau}$$

$$= e^{-\tau}(c_1 e^{i\tau} + c_2 e^{-i\tau})$$

$$= e^{-\tau}(c_1(\cos\tau + i\sin\tau) + c_2(\cos(-\tau) + i\sin(-\tau)))$$

$$= e^{-\tau}(c_1(\cos\tau + i\sin\tau) + c_2(\cos\tau - i\sin(\tau)))$$

$$= e^{-\tau}(c_1\cos\tau + c_1 i\sin\tau + c_2\cos\tau - c_2 i\sin\tau)$$

$$= e^{-\tau}(\cos(\tau)(c_1 + c_2) + i\sin(\tau)(c_1 - c_2))$$

$$c_1 + c_2 = 1$$

$$i(c_1 - c_2) = 1$$

$$ic_1 - ic_2 = 1$$

$$\left[\begin{array}{cc|c} 1 & 1 & 1 \\ i & -i & 1 \end{array}\right] \sim \left[\begin{array}{cc|c} & 1 & 1 \\ -1 & 1 & i \end{array}\right] \sim \left[\begin{array}{cc|c} 1 & 1 & 1 \\ 0 & 2 & 1+i \end{array}\right] \sim \left[\begin{array}{cc|c} 1 & 0 & \frac{1}{2} - \frac{1}{2}i \\ 0 & 1 & \frac{1}{2} + \frac{1}{2}i \end{array}\right]$$

$$c_1 = \frac{1}{2}(1 - i)$$

$$c_2 = \frac{1}{2}(1 + i)$$

$$A = \begin{bmatrix} 2 & 3 & 4 \\ 3 & 4 & 5 \\ 4 & 5 & 7 \end{bmatrix} \quad x = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \quad y = \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}$$

1 5)

$$Ax = \begin{bmatrix} 2+6+12 \\ 3+8+15 \\ 4+10+21 \end{bmatrix} = \begin{bmatrix} 20 \\ 26 \\ 35 \end{bmatrix}$$

1 6)

$$Ay = \begin{bmatrix} 8+15+24 \\ 12+20+30 \\ 16+25+42 \end{bmatrix} = \begin{bmatrix} 47 \\ 62 \\ 83 \end{bmatrix}$$

1 7)

$$A(x+y) = A\begin{bmatrix} 5 \\ 7 \\ 9 \end{bmatrix} = \begin{bmatrix} 10+21+36 \\ 15+28+45 \\ 20+35+63 \end{bmatrix} = \begin{bmatrix} 67 \\ 88 \\ 118 \end{bmatrix}$$

1 8)

$$Ax + Ay = \begin{bmatrix} 20+47 \\ 26+62 \\ 35+83 \end{bmatrix} = \begin{bmatrix} 67 \\ 88 \\ 118 \end{bmatrix}$$

$D(x) = \dot{x} \qquad y(t) = t^2 \qquad z(t) = t$

1 5) $\dot{y} = 2t \qquad$ 1 6) $\dot{z} = 0 \qquad$ 1 7) $\frac{d}{dt}(y \times z) = (t^2 + t)' = 2t \qquad$ Greit

2 1) Ja vi trur

2 3) $\dot{v}(t) + \frac{1}{RC} v(t) = 9$

$v(t) = 9(1 - e^{\frac{t}{RC}}) \qquad\qquad RC\,\dot{v}(t) + v(t) = 18$

$v(0) = 0$

Superposisjon $\quad$ då $v(t) = 2v(t) = 18(1 - e^{\frac{1}{RC}})$

2 4) $\quad D(x) = \dot{x}$

$D(x) = \left(\frac{d}{dt}\right) x$

$(e^{at})' = a\,e^{at}$

$\frac{d}{dt} e^{at} = a e^{at}$

$\lambda = a$

2 5)

$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 0 \\ 2 & 3 & 4 & 1 \\ 3 & 4 & 5 & 2 \\ 4 & 5 & 6 & 3 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 2 & 3 & 0 \\ 0 & -1 & -2 & 1 \\ 0 & -2 & -4 & 2 \\ 0 & -3 & -6 & 3 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 2 & 3 & 0 \\ 0 & 1 & 2 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$$\sim \begin{array}{ccc|c} 1 & 0 & -1 & 2 \\ 0 & 1 & 2 & -1 \end{array}$$

$x_1 - x_3 = 2 \Rightarrow x_1 = 2 + x_3$

$x_2 + 2x_3 = -1 \Rightarrow x_2 = -1 - 2x_3$

$x_3 = s$

$$v = \begin{bmatrix} 2 \\ -1 \\ 0 \end{bmatrix} + s \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$$

Homogen   Inhomogen

2 6) ???

2 7) Vektor går fra origo $\Rightarrow x_1 \parallel x_2 \Rightarrow c_1 x_1 = x_2$

Lineært avhengig

2 8) hvis $Ax = y$ har uendelig med løsninger, så $(\mathbb{R}^1)$ eller, nei??

$Ax = 0$

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$