TTK4100 Kybernetikk introduksjon
Eksamen høsten 2016

Løsningsforslag

Oppgave 1. (5%)

1-e 2-a 3-d 4-c 5-b

Oppgave 2. (35%)

a) (1%) Momentbalanse

b) (2%)

$$\begin{aligned} J\dot{\omega} &= M_d - M_l = M_d - k\omega \\ \dot{\omega} &= -\frac{k}{J}\omega + \frac{1}{J}M_d \\ T &= -\frac{1}{a} = -- \frac{J}{k} = \underline{\underline{\frac{J}{k}}} \end{aligned}$$

c) (2%)

$$J_{TOT} = J + J_{LP}$$

Ny tidskonstant:

$$T_{medLP} = \underline{\underline{\frac{J + J_{LP}}{K}}}$$

I og med at $J \gg J_{LP}$ vil $T_{medLP} \approx T$

d) (2%) Likestrømsmotor

e) (5%)

$$
\begin{aligned}
J\ddot{\omega} &= K_m\dot{i}_a - k\dot{\omega} \\
J\ddot{\omega} &= K_m\Big(-\frac{R_a}{L_a}i_a - \frac{K_E}{L_a}\omega + \frac{u}{L_a}\Big) - k\dot{\omega} \\
J\ddot{\omega} &= -k\dot{\omega} - \frac{K_mK_E}{L_a}\omega - \frac{K_mR_a}{L_a}i_a + \frac{K_m}{L_a}u \\
\ddot{\omega} + \frac{k}{J}\dot{\omega} + \frac{K_mK_E}{JL_a}\omega + \frac{K_m}{JL_a}\big(R_ai_a - u\big) &= 0
\end{aligned}
$$

f) (2%)

$$
\omega_r = 33rpm = \frac{33\cdot 2\pi}{60s} = \frac{2\cdot 3\cdot 11\pi}{2\cdot 2\cdot 10s} = \underline{\underline{\frac{11\pi}{10}rad/s}}
$$

g) (6%)

$$
u = R_ai_a + K_p(\omega_r - \omega) + K_d(\dot{\omega}_r - \dot{\omega})
$$

$$
\begin{aligned}
\ddot{\omega} + \frac{k}{J}\dot{\omega} + \frac{K_mK_E}{JL_a}\omega + \frac{K_m}{JL_a}\big(R_ai_a - R_ai_a - K_p(\omega_r - \omega) - K_d(0 - \dot{\omega})\big) &= 0 \\
\ddot{\omega} + \Big(\frac{k}{J} + \frac{K_mK_d}{JL_a}\Big)\dot{\omega} + \frac{K_m(K_E + K_p)}{JL_a}\omega - \frac{K_mK_p}{JL_a}\omega_r &= 0
\end{aligned}
$$

$$
\begin{aligned}
\omega_0^2 &= \underline{\underline{\frac{K_m(K_E + K_p)}{JL_a}}} \\
2\zeta\omega_0 &= \Big(\frac{k}{J} + \frac{K_mK_d}{JL_a}\Big) \\
\zeta &= \underline{\underline{\frac{1}{2}\sqrt{\frac{JL_a}{K_m(K_E + K_p)}}\Big(\frac{k}{J} + \frac{K_mK_d}{JL_a}\Big)}}
\end{aligned}
$$

h) (6%)

$$\begin{aligned}\frac{K_m(K_E+K_p)}{JL_a} > 0 \quad &\Longrightarrow \quad \underline{\underline{K_p > -K_E}} \\ \frac{k}{J}+\frac{K_mK_d}{JL_a} > 0 \quad &\Longrightarrow \quad \underline{\underline{K_d > -\frac{kL_a}{K_m}}}\end{aligned}$$

i) (3%)

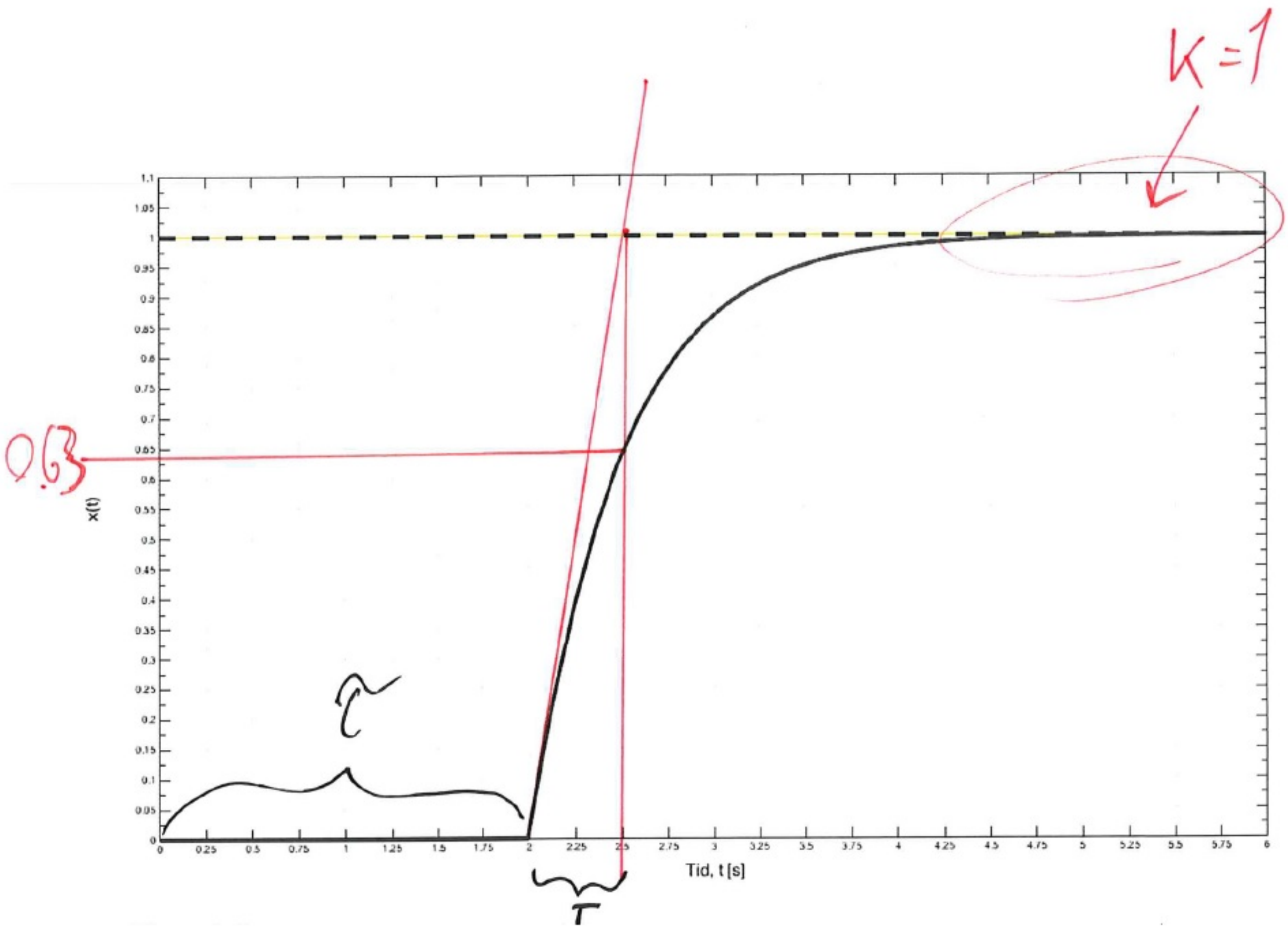
$$\begin{aligned}\frac{K_m(K_E+K_p)}{JL_a}\omega_s &= \frac{K_mK_p}{JL_a}\omega_r \\ \omega_s &= \underline{\underline{\frac{K_p}{K_E+K_p}\omega_r}}\end{aligned}$$

j) (2%) Integraleffekt

k) (4%)

$$\begin{aligned}f_s &\geq 2f_{max} \\ \Longrightarrow f_{max} &= \frac{44.1kHz}{2} = \underline{\underline{22.05kHz}}\end{aligned}$$

Oppgave 3. (6%)



Figur 1: Inngangssignal (stiplet linje) og respons (heltrukken) for et førsteordens system.

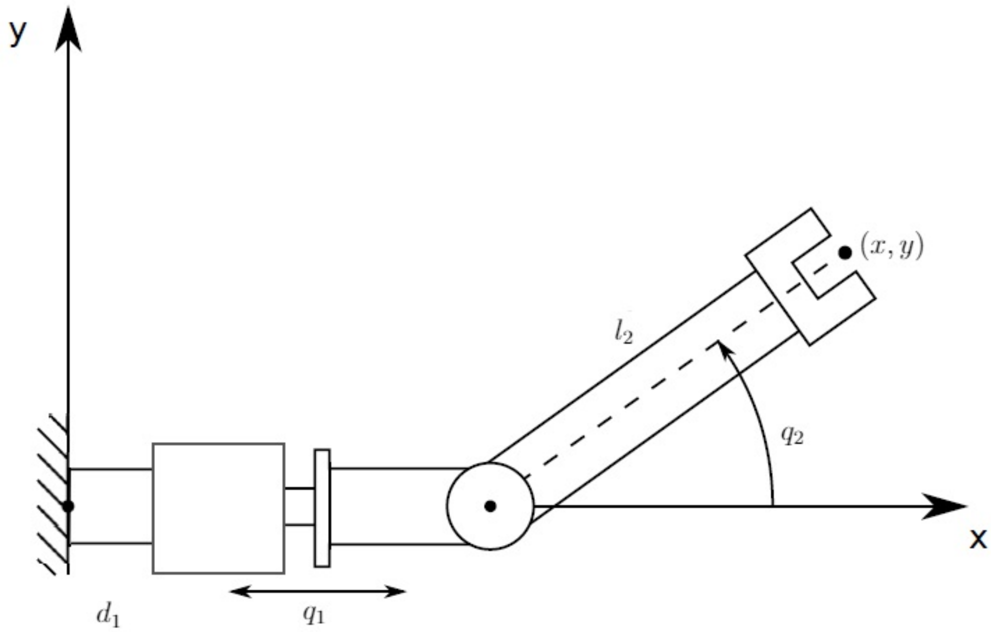
$$T = -\frac{1}{a} \implies \frac{1}{2} = -\frac{1}{a} \implies \underline{\underline{a = -2}}$$

$$K = -\frac{b}{a} \implies 1 = -\frac{b}{-2} \implies \underline{\underline{b = 2}}$$

$$\underline{\underline{\tau = 2}}$$

$$\underline{\underline{\dot{x} = -2x + 2u(t-2)}}$$

Oppgave 4. (10%)



Figur 2: Robot med translasjonsledd og rotasjonsledd.

a) (5%) Forover-kinematikk:

$$\begin{aligned} x &= d_1 + q_1 + l_2 \cos q_2 \\ y &= l_2 \sin q_2 \end{aligned}$$

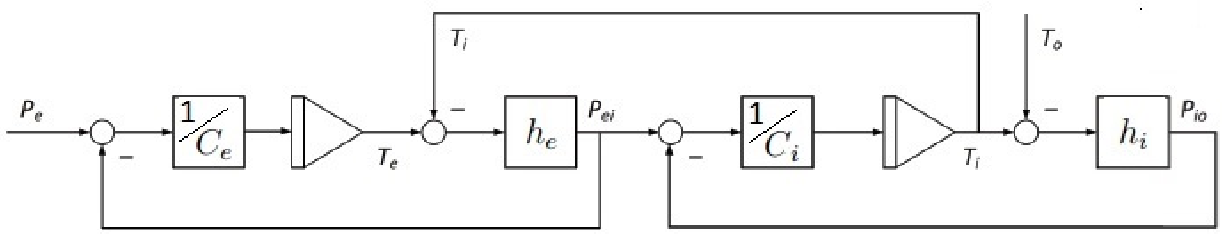
b) (5%) Invers-kinematikk:

$$\begin{aligned} \sin q_2 &= \frac{y}{l_2} \\ \cos q_2 &= \pm\sqrt{1 - \sin^2 q_2} \end{aligned}$$

$$\begin{aligned} q_2 &= \text{atan2}(s_2, c_2) \\ q_1 &= x - d_1 - l_2 \cos q_2 \end{aligned}$$

Oppgave 5. (24%)

a) (4%)



Figur 3: Blokkdiagram for varmeskap med varmeelement

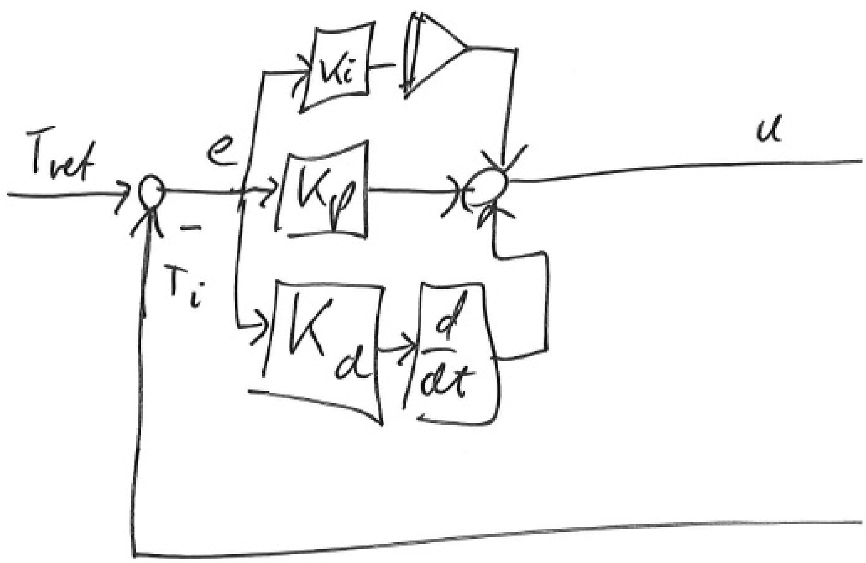
b) (6%)

```
h=0.1;
Tsim=200;

Te(1)=20;
Ti(1)=20;
To=20;

Ce=40;
Ci=400;
he=10;
hi=10;
Pe=2000;

for i=2:Tsim/h+1,
    Te(i)=Te(i-1)+h*(1/Ce*(Pe-he*(Te(i-1))));
    Ti(i)=Ti(i-1)+h*(1/Ci*(he*(Te(i-1)-Ti(i-1))-hi*(Ti(i-1)-To)));
end

t=0:h:Tsim;

figure(1)
plot(t,Te);
hold on
plot(t,Ti);
grid on
xlabel('Tid');
ylabel('Temperatur');
```

c) (6%)

$$\begin{aligned} Pe = u &= K_p e + K_i \int e\,dt + K_d \dot{e} \\ \text{der vi har} \qquad e &= T_{ref} - y = T_{ref} - T_i \end{aligned}$$

Systemet er monovariabelt.



Figur 4: Blokkdiagram med PID-regulator for inngangsreferansen. Legges til på venstre side av blokkdiagrammet i figur 3, da $P_E = u$.

d) (6%)

$$
\begin{aligned}
K_{pk} &= 506, \qquad T_k = 12 \\
K_p &= 0.6 \cdot 506 = \underline{\underline{303.6}} \\
T_i &= 0.5 \cdot T_k = 6, \qquad T_d = 0.125 \cdot T_k = 0.125 \cdot 12 = 1.5 \\
K_i &= \frac{K_p}{T_i} = \frac{303.6}{6} = \underline{\underline{50.6}} \\
K_d &= K_p T_d = 303.6 \cdot 1.5 = \underline{\underline{455.4}}
\end{aligned}
$$

e) (2%) $\underline{\underline{\tau = 1s}}$