1 -> Differentialgleichungen

1)

$$\mathring{x}_1 = \alpha_1(x_2 - x_1) \qquad = -\alpha_1 x_1 \quad + \alpha_1 x_2$$

$$\mathring{x}_2 = \alpha_1(x_1 - x_2) + \alpha_2(0 - x_2) \;=\; \alpha_1 x_1 - \alpha_1 x_2 - \alpha_2 x_2 = \alpha_1 x_1 + x_2(-\alpha_1 - \alpha_2)$$

$$\underbrace{\begin{bmatrix} \mathring{x}_1 \\ \mathring{x}_2 \end{bmatrix}}_{\mathring{x}} = \underbrace{\begin{bmatrix} -\alpha_1 & \alpha_1 \\ \alpha_1 & -\alpha_1 - \alpha_2 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}}_{x}$$

2) $$\mathring{x}_1 = (-(R_1 + R_3)x_1 + R_3 x_2)/L_1$$

$$\mathring{x}_2 = (R_3 x_1 - (R_2 + R_3)x_2)/L_2$$

$$\underbrace{\begin{bmatrix} \mathring{x}_1 \\ \mathring{x}_2 \end{bmatrix}}_{\mathring{x}} = \underbrace{\begin{bmatrix} -(R_1 + R_3)/L_1 & R_3/L_1 \\ R_3/L_2 & -(R_2 + R_3)/L_2 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}}_{x}$$

$$\mathring{x} = A x$$

$$x_1(t) = C_1 e^{\lambda t} \qquad x(t) = \vec{C} e^{\lambda t} = \begin{bmatrix} C_1 \\ C_2 \end{bmatrix} e^{\lambda t}$$
$$x_2(t) = C_2 e^{\lambda t}$$

$$\mathring{x} = \lambda e^{\lambda t} \begin{bmatrix} C_1 \\ C_2 \end{bmatrix}$$

$$A\, x(t) = A\, e^{\lambda t} \begin{bmatrix} C_1 \\ C_2 \end{bmatrix}$$

$$\mathring{x} = A x$$

$$\lambda e^{\lambda t} \begin{bmatrix} C_1 \\ C_2 \end{bmatrix} = A e^{\lambda t} \begin{bmatrix} C_1 \\ C_2 \end{bmatrix} \qquad \Big|\cdot e^{-\lambda t}$$

$$\lambda \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} = A \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} \qquad \vec{c} \text{ er egenvektor}$$

$$\lambda \vec{c} = A \vec{c}$$

3) $$A = \begin{bmatrix} -1 & 1 \\ 1 & -2 \end{bmatrix}$$

$$\dot{x} = Ax$$

$$\det(A - \lambda I) = 0$$

$$\begin{vmatrix} -1-\lambda & 1 \\ 1 & -2-\lambda \end{vmatrix} = 0$$

$$(-1-\lambda)(-2-\lambda) - (1)(1) = 0$$

$$(2 + \lambda + 2\lambda + \lambda^2) - 1 = 0$$

$$\lambda^2 + 3\lambda + 1 = 0$$

$$\lambda = \frac{-3 \pm \sqrt{9-4}}{2}$$

$$\lambda = \frac{-3 \pm \sqrt{5}}{2}$$

$$\lambda_1 = -\frac{3}{2} + \frac{\sqrt{5}}{2}$$

$$\lambda_2 = -\frac{3}{2} - \frac{\sqrt{5}}{2}$$

$$A - \lambda_1 I = 0 \qquad \begin{vmatrix} -1-\lambda & 1 \\ 1 & -2-\lambda \end{vmatrix}$$

$$\left[\begin{array}{cc|c} -1 + \frac{3}{2} - \frac{\sqrt{5}}{2} & 1 & 0 \\ 1 & -2 + \frac{3}{2} - \frac{\sqrt{5}}{2} & 0 \end{array}\right]$$

$$\sim \left[\begin{array}{cc|c} \frac{1}{2} - \frac{\sqrt{5}}{2} & 1 & 0 \\ 1 & -\frac{1}{2} - \frac{\sqrt{5}}{2} & 0 \end{array}\right] \qquad \left(-\frac{1}{2} - \frac{\sqrt{5}}{2}\right)\left(\frac{1}{2} - \frac{\sqrt{5}}{2}\right)$$

$$-\frac{1}{4} + \frac{\sqrt{5}}{4} - \frac{\sqrt{5}}{4} + \frac{5}{4}$$

$$\sim \left[\begin{array}{cc|c} 0 & 0 & 0 \\ 1 & -\frac{1}{2} - \frac{\sqrt{5}}{2} & 0 \end{array}\right] \qquad = 1$$

$$x_1 + \left(-\frac{1}{2} - \frac{\sqrt{5}}{2}\right)x_2 = 0 \qquad v_1 = s \begin{bmatrix} \frac{1}{2} + \frac{\sqrt{5}}{2} \\ 1 \end{bmatrix}$$

$$x_1 = \left(\frac{1}{2} + \frac{\sqrt{5}}{2}\right)x_2$$

$$x_2 = s$$

$$\lambda_2 = -\frac{3}{2} - \frac{\sqrt{5}}{2} \qquad \begin{vmatrix} -1-\lambda & 1 \\ 1 & -2-\lambda \end{vmatrix}$$

$$A - \lambda_2 I = 0$$

$$\left[\begin{array}{cc|c} -1+\frac{3}{2}+\frac{\sqrt{5}}{2} & 1 & 0 \\ 1 & -2+\frac{3}{2}+\frac{\sqrt{5}}{2} & 0 \end{array}\right]$$

$$\sim \begin{array}{cc|c} \frac{1}{2}+\frac{\sqrt{5}}{2} & 1 & 0 \\ 1 & -\frac{1}{2}+\frac{\sqrt{5}}{2} & 0 \end{array} \qquad \left(-\frac{1}{2}+\frac{\sqrt{5}}{2}\right)\left(-\frac{1}{2}-\frac{\sqrt{5}}{2}\right)$$

$$\frac{1}{4} - \frac{5}{4} = -1$$

$$\sim \begin{array}{cc|c} 0 & 1 & 0 \\ 1 & -\frac{1}{2}+\frac{\sqrt{5}}{2} & 0 \end{array}$$

$$x_1 + \left(-\frac{1}{2}+\frac{\sqrt{5}}{2}\right)x_2 = 0$$

$$x_1 = \left(\frac{1}{2}-\frac{\sqrt{5}}{2}\right)x_2$$

$$x_2 = s \qquad v_2 = s\begin{bmatrix} \frac{1}{2}-\frac{\sqrt{5}}{2} \\ 1 \end{bmatrix}$$

$$x(t) = a_1 e^{(3+\sqrt{5})\frac{1}{2}t}\begin{bmatrix} \frac{1}{2}+\frac{\sqrt{5}}{2} \\ 1 \end{bmatrix} + a_2 e^{(3-\sqrt{5})\frac{1}{2}t}\begin{bmatrix} \frac{1}{2}-\frac{\sqrt{5}}{2} \\ 1 \end{bmatrix}$$

4) $$\begin{bmatrix} \dot{y}_1 \\ \dot{y}_2 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$$

$$\begin{bmatrix} 2-\lambda & 1 \\ 1 & 2-\lambda \end{bmatrix}$$

$$(2-\lambda)(2-\lambda) - 1 = 0$$

$$4 - 4\lambda + \lambda^2 - 1 = 0$$

$$\lambda^2 - 4\lambda + 3 = 0$$

$$\lambda = \frac{4 \pm \sqrt{4}}{2} = 0$$

$$\lambda = 2 \pm 1$$

$$\lambda_1 = 3, \quad \lambda_2 = 1$$

$A - \lambda I = 0$

$$\begin{bmatrix} 2-\lambda & 1 \\ 1 & 2-\lambda \end{bmatrix}$$

$$\begin{bmatrix} 2-3 & 1 \\ 1 & 2-3 \end{bmatrix}$$

$$\left[\begin{array}{cc|c} -1 & 1 & 0 \\ 1 & -1 & 0 \end{array}\right] \sim \left[\begin{array}{cc|c} -1 & 1 & 0 \\ 0 & 0 & 0 \end{array}\right]$$

$-x_1 + x_2 = 0$ $\quad v_1 = s\begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$x_1 = x_2$

$x_1 = s$

$$\left[\begin{array}{cc|c} 2-1 & 1 & 0 \\ 1 & 2-1 & 0 \end{array}\right]$$

$$\left[\begin{array}{cc|c} 1 & 1 & 0 \\ 1 & 1 & 0 \end{array}\right]$$

$x_1 + x_2 = 0$

$x_1 = -x_2$ $\quad v_2 = s\begin{bmatrix} -1 \\ 1 \end{bmatrix}$

$x_2 = s$

$$\underline{\underline{y(t) = a_1 e^{3t}\begin{bmatrix} 1 \\ 1 \end{bmatrix} + a_2 e^{t}\begin{bmatrix} -1 \\ 1 \end{bmatrix}}}$$

$$\begin{bmatrix} \dot{y}_1 \\ \dot{y}_2 \\ \dot{y}_3 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 6 & 2 \\ 2 & 2 & 6 \end{bmatrix}\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}$$

$|A - \lambda I| = 0$

$$\begin{vmatrix} 1-\lambda & 2 & 2 \\ 2 & 6-\lambda & 2 \\ 2 & 2 & 6-\lambda \end{vmatrix} = 0$$

$(1-\lambda)(6-\lambda)(6-\lambda) + (2)(2)(2) + (2)(2)(2) - (1-\lambda)(2)(2) - (2)(2)(6-\lambda) - (6-\lambda)(2)(2) = 0$

$(1-\lambda)(36 - 12\lambda + \lambda^2) + 16 - 4(1-\lambda) - 8(6-\lambda) = 0$

$(36 - 12\lambda + \lambda^2 - 36\lambda + 12\lambda^2 - \lambda^3) + 16 - 4 + 4\lambda - 48 + 8\lambda = 0$

$-\lambda^3 + 13\lambda^2 - 48\lambda + 12\lambda + 36 + 16 - 4 - 48 = 0$

$-\lambda^3 + 13\lambda^2 - 36\lambda = 0$

$\lambda(-\lambda^2 + 13\lambda - 36) = 0, \qquad \lambda_1 = 0$

$$\lambda = \frac{-13 \pm \sqrt{169 - 144}}{-2}$$

$$\lambda = \frac{-13 \pm 5}{-2}$$

$\lambda_2 = 4 \qquad \lambda_3 = 9$

$\lambda_1 = 0$

$$\begin{vmatrix} 1-\lambda & 2 & 2 \\ 2 & 6-\lambda & 2 \\ 2 & 2 & 6-\lambda \end{vmatrix}$$

$A - \lambda_1 I$

$$\left[\begin{array}{ccc|c} 1 & 2 & 2 & 0 \\ 2 & 6 & 2 & 0 \\ 2 & 2 & 6 & 0 \end{array}\right]$$

$$\sim \left.\begin{array}{ccc|c} 1 & 2 & 2 & 0 \\ 0 & 2 & -2 & 0 \\ 0 & -2 & 2 & 0 \end{array}\right. \sim \left.\begin{array}{ccc|c} 1 & 4 & 0 & 0 \\ 0 & 2 & -2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right. \sim \left.\begin{array}{ccc|c} 1 & 4 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right.$$

$x_1 + 4x_2 = 0$

$x_2 - x_3 = 0 \qquad v_1 = s\begin{bmatrix} -4 \\ 1 \\ 1 \end{bmatrix}$

$x_2 = x_3$

$x_2 = s$

$x_3 = s$

$x_1 = -4s$

$\lambda_2 = 4$

$$\begin{vmatrix} 1-\lambda & 2 & 2 \\ 2 & 6-\lambda & 2 \\ 2 & 2 & 6-\lambda \end{vmatrix}$$

$$\left[\begin{array}{ccc|c} -3 & 2 & 2 & 0 \\ 2 & 2 & 2 & 0 \\ 2 & 2 & 2 & 0 \end{array}\right]$$

$$\sim \left.\begin{array}{ccc|c} -3 & 2 & 2 & 0 \\ 5 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right. \sim \left.\begin{array}{ccc|c} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right.$$

$x_2 = -x_3$

$x_1 = 0 \qquad v_2 = s\begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix}$

$x_3 = s$

$$\begin{vmatrix} 1-\lambda & 2 & 2 \\ 2 & 6-\lambda & 2 \\ 2 & 2 & 6-\lambda \end{vmatrix}$$

$\lambda_3 = 9$

$$\left[\begin{array}{ccc|c} -8 & 2 & 2 & 0 \\ 2 & -3 & 2 & 0 \\ 2 & 2 & -3 & 0 \end{array}\right] \sim \begin{matrix} -8 & 2 & 2 \\ 10 & -5 & 0 \\ 10 & 0 & -5 \end{matrix}$$

$$\sim \begin{matrix} -4 & 1 & 1 \\ 2 & -1 & 0 \\ 2 & 0 & -1 \end{matrix} \sim \begin{matrix} 0 & 0 & 0 \\ 2 & -1 & 0 \\ 2 & 0 & -1 \end{matrix}$$

$2 \cdot x_1 = x_2$

$2 x_1 = x_3 \qquad v_3 = s \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}$

$x_1 = s$

$$z(t) = a_1 e^{0t} \begin{bmatrix} -4 \\ 1 \\ 1 \end{bmatrix} + a_2 e^{4t} \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix} + a_3 e^{9t} \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}$$

5) $\begin{bmatrix} \dot{\xi}_1 \\ \dot{\xi}_2 \\ \dot{\xi}_3 \end{bmatrix} = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix} \begin{bmatrix} \xi_1 \\ \xi_2 \\ \xi_3 \end{bmatrix}$

$|A - \lambda I| = 0$

$\begin{vmatrix} 2-\lambda & 1 & 1 \\ 1 & 2-\lambda & 1 \\ 1 & 1 & 2-\lambda \end{vmatrix} = 0$

$(2-\lambda)(2-\lambda)(2-\lambda) + (1)(1)(1) + (1)(1)(1) - (2-\lambda)(1)(1) - (1)(2-\lambda)(1) - (1)(1)(2-\lambda) = 0$

$(2-\lambda)(4 - 4\lambda + \lambda^2) + 2 - 2 + \lambda - 2 + \lambda - 2 + \lambda = 0$

$(8 - 8\lambda + 2\lambda^2 - 4\lambda + 4\lambda^2 - \lambda^3) - 4 + 3\lambda = 0$

$-\lambda^3 + 6\lambda^2 - 9\lambda + 4 = 0$

$(2-\lambda)((2-\lambda)^2 - 1) - (2 - \lambda - 1) + (1 - (2-\lambda)) = 0$ $\qquad x = \frac{5 \pm \sqrt{25 - 16}}{2}$

$(2-\lambda)((2-\lambda+1)(2-\lambda-1)) - (1-\lambda) + (-1+\lambda) = 0$ $\qquad x = \frac{5 \pm 3}{2}$

$(2-\lambda)(3-\lambda)(1-\lambda) - (1-\lambda) - (1-\lambda) = 0$

$(1-\lambda)((2-\lambda)(3-\lambda) - 2) = 0$

$(1-\lambda)(6 - 2\lambda - 3\lambda + \lambda^2 - 2) = 0$

$(1-\lambda)(4 - 5\lambda + \lambda^2) = 0$

$(1-\lambda)(\lambda - 4)(\lambda - 1) = 0$ $\qquad |\cdot -1 \quad \cdot -1$

$(1-\lambda)(4-\lambda)(1-\lambda)$

$\lambda_1 = 1, \quad \lambda_2 = 4, \quad \lambda_3 = 1$

$(A - \lambda_2 I)v = 0$

$\left[\begin{array}{ccc|c} -2 & 1 & 1 & 0 \\ 1 & -2 & 1 & 0 \\ 1 & 1 & -2 & 0 \end{array}\right] \sim \begin{matrix} -2 & 1 & 1 \\ 0 & -3/2 & 3/2 \\ 0 & 3/2 & -3/2 \end{matrix} \sim \begin{matrix} -2 & 1 & 1 \\ 0 & -1 & 1 \\ 0 & 0 & 0 \end{matrix} \sim \left.\begin{matrix} -2 & 0 & 2 \\ 0 & -1 & 1 \end{matrix}\right| \begin{matrix} 0 \\ 0 \\ 0 \end{matrix}$

$x_1 = x_3$

$x_2 = x_3 \qquad v_1 = s\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

$x_3 = s$

$(A - 1I)v = 0$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 1 & 1 & 0 \end{array}\right]$$

$x_1 + x_2 + x_3 = 0$

$x_2 = -s$

$x_3 = -t$

$x_1 = s + t$

$$v_2 = s\begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} + t\begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$$

$$x(t) = C_1\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}e^{4t} + \left(C_2\begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} + C_3\begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}\right)e^{t}$$

Oppg 6)

a) $$y(t) = a_1 e^{3t}\begin{bmatrix} 1 \\ 1 \end{bmatrix} + a_2 e^{t}\begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

$$y(0) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$a_1\begin{bmatrix} 1 \\ 1 \end{bmatrix} + a_2\begin{bmatrix} -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$\left[\begin{array}{cc|c} 1 & -1 & 1 \\ 1 & 1 & 0 \end{array}\right] \sim \left[\begin{array}{cc|c} 2 & 0 & 1 \\ -1 & 1 & 0 \end{array}\right] \sim \left[\begin{array}{cc|c} 1 & 0 & 1/2 \\ 0 & 1 & -1/2 \end{array}\right]$$

$$y(t) = \frac{1}{2}e^{3t}\begin{bmatrix} 1 \\ 1 \end{bmatrix} - \frac{1}{2}e^{t}\begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

$$y(t) = a_1 e^{0t}\begin{bmatrix} -4 \\ 1 \\ 1 \end{bmatrix} + a_2 e^{4t}\begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix} + a_3 e^{9t}\begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}$$

$$= a_1\begin{bmatrix} -4 \\ 1 \\ 1 \end{bmatrix} + a_2 e^{4t}\begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix} + a_3 e^{9t}\begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix} \qquad y(0) = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$\left[\begin{array}{ccc|c} -4 & 0 & 1 & 1 \\ 1 & -1 & 2 & 0 \\ 1 & 1 & 2 & 0 \end{array}\right] = \left[\begin{array}{ccc|c} -4 & 0 & 1 & 1 \\ 0 & -2 & 0 & 0 \\ 1 & 1 & 2 & 0 \end{array}\right] \sim \left[\begin{array}{ccc|c} 0 & 0 & 9 & 1 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 2 & 0 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 0 & 0 & 1 & 1/9 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & -2/9 \end{array}\right]$$

$$y(t) = -\frac{2}{9}\begin{bmatrix} -4 \\ 1 \\ 1 \end{bmatrix} + \frac{1}{9}\begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}$$

8)

$$\dot{x} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} x$$

$$|A - \lambda I|$$

$$\begin{bmatrix} 1-\lambda & 1 \\ 0 & 1-\lambda \end{bmatrix}$$

$$(1-\lambda)(1-\lambda) - 0$$

$$\lambda = 1$$

$$A - 1I$$

$$\left[\begin{matrix} 0 & 1 \\ 0 & 0 \end{matrix}\right|\left.\begin{matrix} 0 \\ 0 \end{matrix}\right]$$

$$x_2 = 0$$

$$x_1 = s \qquad v_1 = s\begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$x = c\begin{bmatrix} 1 \\ 0 \end{bmatrix} e^t$$

Defekt matrise?? ?

9) $$x(t) = a_1 e^{(-3-\sqrt{5})\frac{t}{2}} \begin{pmatrix} -2 \\ 1-\sqrt{5} \end{pmatrix} + a_2 e^{(-3+\sqrt{5})\frac{t}{2}} \begin{pmatrix} -2 \\ 1+\sqrt{5} \end{pmatrix}$$

$$x(0) = \begin{bmatrix} 88 \\ 4 \end{bmatrix}$$

$$1^2 + 2\sqrt{5} + 5$$
$$6 + 2\sqrt{5}$$

$$\left[\begin{matrix} -2 & -2 \\ 1-\sqrt{5} & 1+\sqrt{5} \end{matrix}\right|\left.\begin{matrix} 88 \\ 4 \end{matrix}\right] \sim \left.\begin{matrix} 1 & 1 \\ 1-\sqrt{5} & 1+\sqrt{5} \end{matrix}\right|\begin{matrix} -44 \\ 4 \end{matrix} \qquad 1 \cdot 1 + \sqrt{5}$$

$$\sim \left.\begin{matrix} 1 & 1 \\ -4 & 6+2\sqrt{5} \end{matrix}\right|\begin{matrix} -44 \\ 4+4\sqrt{5} \end{matrix} \sim$$

Nei!

10)