Difflikninger 4 !

1) $\dot{x}_1 = a x_1 \qquad a > 0$

$x_1 = $ Harebestand

$x_2 = $ Gaupebestand

$\downarrow$

$\dot{x}_1 = a x_1 - b x_1 x_2$ Proporsjonal med mengden gauper og harer

$b > 0$

$\dot{x}_2 = -c x_2$ Antar att gaupe kun spiser hare, vil dø ut uten hare

$\downarrow$

$\dot{x}_2 = -x_2 + d x_1 x_2$ Proporsjonal med hare og gaupebestanden

Total likningssystem

$\dot{x}_1 = a x_1 - b x_1 x_2$

$\dot{x}_2 = -c x_2 + d x_1 x_2$

hvar $a, b, c, d > 0$

1) $\dfrac{\dot{x}_1}{\dot{x}_2} = \dfrac{a x_1 - b x_1 x_2}{-c x_2 + d x_1 x_2}$

$\dfrac{\dot{x}_1}{\dot{x}_2} = \dfrac{x_1(a - b x_2)}{x_2(-c + d x_1)}$

$\dot{x}_1 x_2(-c + d x_1) = \dot{x}_2 x_1 (a - b x_2)$

$\dot{x}_1(-c + d x_1) \cdot \dfrac{1}{x_1} = \dot{x}_2 (a - b x_2) \cdot \dfrac{1}{x_2}$

$-c \dot{x}_1 \cdot \dfrac{1}{x_1} + d \dot{x}_1 = a \dot{x}_2 \cdot \dfrac{1}{x_2} - b \dot{x}_2 \quad | \int dt$

$\int -c \dot{x}_1 \cdot \frac{1}{x_1} dt + \int d \dot{x}_1 = \int a \dot{x}_2 \cdot \frac{1}{x_2} dt - \int b \dot{x}_2$

$-c \int \frac{1}{x_1} dx_1 + dx + C_1 = a \int \frac{1}{x_2} dx_2 - b x_2 + C_2$

$-c \ln|x_1| + d x_1 + C_1 = a \ln|x_2| - b x_2 + C_2$

$d x_1 + b x_2 - c \ln|x_1| - a \ln|x_2| = C$

$dx_1 + b x_2 - c \ln|x_1| - a \ln|x_2| = C$

Algebraisk kurve?

$$x(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}$$

2) $x: \mathbb{R} \to \mathbb{R}^2$

$x_1 + x_2 = 1$

$x_1 = 1 - x_2$

$x_2 = e^t$

$x_1 = 1 - e^t$

$x_1 + x_2 = 1$

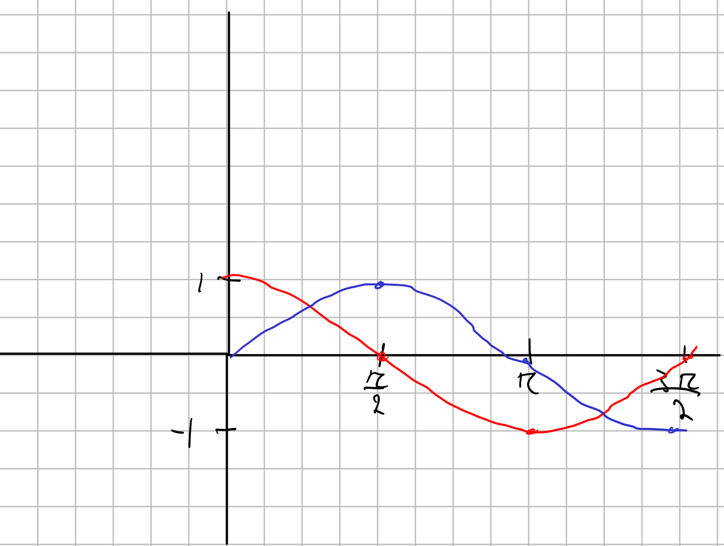
$1 - e^t + e^t = 1$

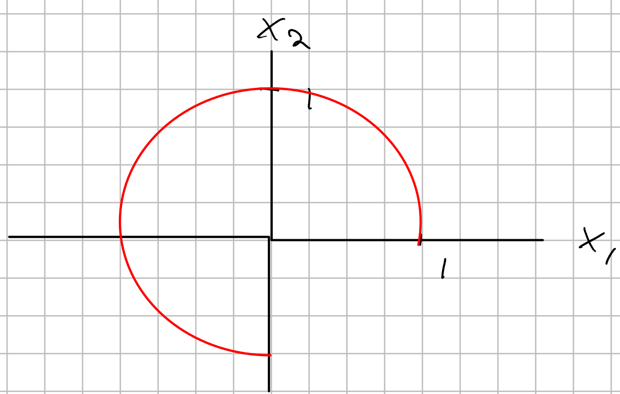
$1 = 1$

$$x(t) = \begin{bmatrix} 1 - e^t \\ e^t \end{bmatrix}$$

3) $x: [0, \frac{3\pi}{2}] \to \mathbb{R}^2$

$$x(t) = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} \cos t \\ \sin t \end{bmatrix}$$

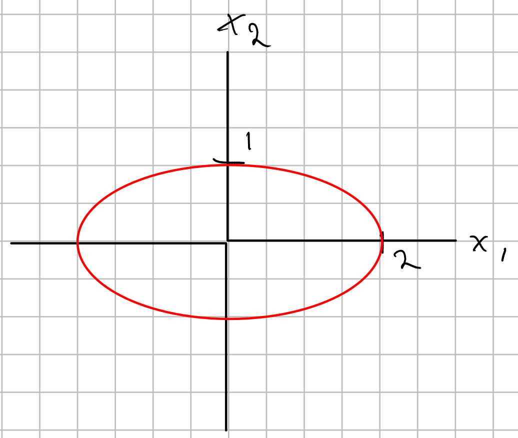




x_1

4) $y:[0, 2\pi] \to \mathbb{R}^2$

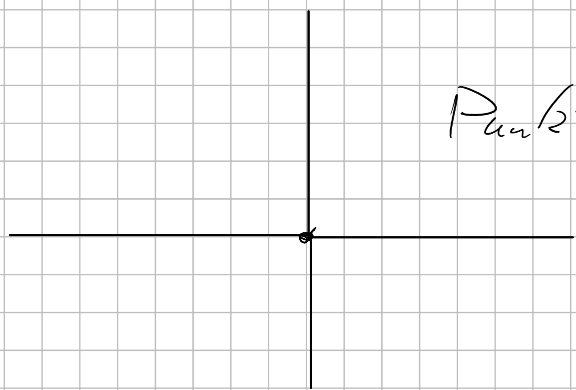
$$y(t) = \begin{bmatrix} 2 \cos t \\ \sin t \end{bmatrix}$$



5) $z:[-1, 1] \to \mathbb{R}^2$

$$z(t) = \begin{bmatrix} t^3 \\ t^2 \end{bmatrix}$$ Tangente i $t = 0$?

$$\dot{z} = \begin{bmatrix} 3t^2 \\ 2t \end{bmatrix} \qquad \dot{z}(0) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$



Punkt 0

6) $x: \mathbb{N} \to \mathbb{N}^3$

$x_1 NO + x_2 O_2 \to x_3 NO_2$

$x_1 NO + x_2 O_2 = x_3 NO_2$

$$x_1 = x_3$$
$$x_1 + 2x_2 = 2x_3$$

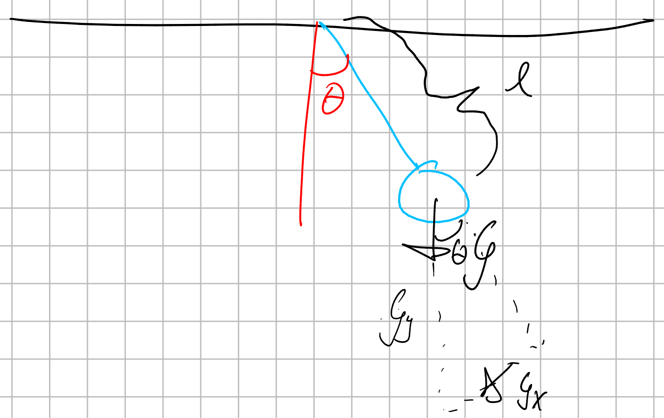
$$\begin{bmatrix} 1 & 0 & 1 \\ 1 & 2 & 2 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 1 \end{bmatrix}$$

$x_1 = 1$
$2x_2 = 1$
$x_2 = \frac{1}{2}$

$$\underline{\underline{x = \begin{bmatrix} 2 \\ 1 \end{bmatrix}}}$$

7) $\sum F = ma$ $\qquad$ $E_p = mgl = F_p \cdot l$

$F_p =$



$G_x = G \sin\theta$

$G = mg$

$m = 1$

$\sum F_x = a$ $\qquad$ x langs bue parljón : $\qquad$ $x = l\theta$, $\ddot{x} = l\ddot{\theta}$

$\sum F_x = l\ddot{\theta} = G \sin\theta$

$l\ddot{\theta} = g \sin\theta$

$$\underline{\underline{\ddot{\theta} - \frac{g}{l}\sin\theta = 0}}$$

8) $\ddot{\theta} - \frac{g}{l}\sin\theta = 0 \quad \Big| \cdot m l^2 \dot{\theta}$ $\qquad$ $m = 1$

$\ddot{\theta}\dot{\theta} l^2 - l g \dot{\theta} \sin\theta = 0$

$\int l^2 \ddot{\theta}\dot{\theta} - \int l g \dot{\theta} \sin\theta = 0$

$\frac{1}{2}(l\dot{\theta})^2 + l g \cos\theta = C$

$$\underbrace{\frac{1}{2}m(l\dot{\theta})^2}_{\text{Kinetisk Energi}} + \underbrace{m g l \cos\theta}_{\text{Potentiell Energi}} = C$$

9) $m\ddot{x} + kx = 0 \quad \Big| \cdot \dot{x}$

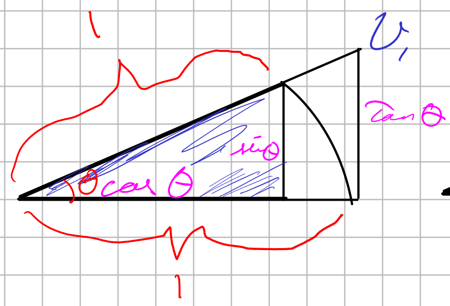
$$m\ddot{x}\dot{x} + kx\dot{x} = 0$$

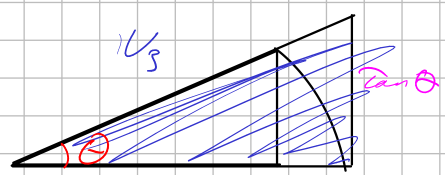
$$\int m\ddot{x}\dot{x}\,dt + \int kx\dot{x}\,dt = 0$$

$$\frac{1}{2}m\dot{x}^2 + \int kx\,dx = 0$$

$$\frac{1}{2}m\dot{x}^2 + \frac{1}{2}kx^2$$

10)





$$\lim_{\theta \to 0} \frac{\sin\theta}{\theta} = 1$$

$$\lim_{\theta \to 0} \frac{\sin\theta}{\theta}$$

$$v_1 = \frac{\cos\theta \sin\theta}{2}$$

$$A = \frac{1}{2}r^2\theta \qquad r = 1$$

$$v_2 = \frac{1}{2}\theta$$

$$A = \pi r^2 \cdot \frac{\theta}{2\pi} = \frac{1}{2}r^2\theta$$

$$v_3 = \tan\theta \cdot 1 \cdot \frac{1}{2} = \frac{1}{2}\tan\theta = \frac{1}{2} \cdot \frac{\sin\theta}{\cos\theta} = \frac{\sin\theta}{2\cos\theta}$$

$$v_1 \le v_2 \le v_3$$

$$\frac{\cos\theta \sin\theta}{2} \le \frac{\theta}{2} \le \frac{\sin\theta}{2\cos\theta}$$

$$\cos\theta \sin\theta \le \theta \le \frac{\sin\theta}{\cos\theta} \qquad \Big| \frac{1}{\sin\theta}$$

$$\cos\theta \le \frac{\theta}{\sin\theta} \le \frac{1}{\cos\theta} \qquad \Big| \cdot x^{-1}$$

$$\frac{1}{\cos\theta} \ge \frac{\sin\theta}{\theta} \ge \cos\theta$$

$$\lim_{\theta \to 0} \cos\theta = 1$$

$$\lim_{\theta \to 0} \frac{\sin\theta}{\theta} = 1$$

1 1) $\ddot{\theta} + \frac{g}{l}\theta = 0$

$\lambda = \frac{0 \pm \sqrt{-4\frac{g}{l}}}{2}$

$\lambda = \pm i \sqrt{\frac{g}{l}}$

$\theta(t) = C_1 \cos\left(\sqrt{\frac{g}{l}}t\right) + C_2 \, i \sin\left(\sqrt{\frac{g}{l}}t\right)$

1 2) Nei

1 3) Bruk

$\dot{x}(t_n) = f(t_n, x_n)$

$x_{n+1} = x_n + h f(t_n, x_n)$

1 4) Faseplottet er feil fordi den er ikke kontinuerlig

1 5) $\dot{x} = x - xy$ Lotka-Volterra

$\dot{y} = -y + xy$

1 6) $\ddot{\theta} + \frac{g}{l}\sin\theta = 0$

$p = \theta \quad q = \dot{\theta}$

$\dot{p} = q$

$\dot{q} = -\frac{g}{l}\sin p$

1 7)

$$\ddot{x} + \mu(x^2 - 1)\dot{x} + x = 0$$

$$x_1 = x$$

$$x_2 = \dot{x}$$

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = -\mu(x_1^2 - 1)x_2 - x_1$$

Heun-metode:

$$x_{n+1} = x_n + \frac{h}{2}\left(f(x_n) + f(x_n + h\, f(x_n))\right)$$

1 8)

Simpsons: $\int_a^b f(x) \approx \frac{b-a}{6}\left(f(a) + 4\, f\left(\frac{b+a}{4}\right) + f(b)\right)$

$$\left\{a, \frac{a+b}{2}, b\right\}$$

$a = x_n \qquad b = x_{n+1}$

Trapes: $x_{n+1} = x_n + \frac{h}{2}\left(f(x_n) + f(x_{n+1})\right)$

$$b = a + \frac{h}{2}\left(f(a) + f(b)\right)$$

$$x_{n+1} = x_n + \frac{h}{2}\left(f(x_n) + f\left(\frac{x_{n+1} + x_n}{2}\right)\right) + \frac{h}{2}\left(f\left(\frac{x_{n+1} + x_n}{2}\right) + f(x_{n+1})\right)$$

$$= x_n + \frac{h}{2}\left(f(x_n) + 2\, f\left(\frac{x_{n+1} + x_n}{2}\right) + f(x_{n+1})\right)$$

$$= x_n + \frac{h}{2}\left(f(x_n) + 2\, f\left(\frac{x_n + h f(x_n) + x_n}{2}\right) + f(x_n + h\, f(x_n))\right)$$

$$= x_n + \frac{h}{2}\left(f(x_n) + 2\, f\left(\frac{2x_n + h\, f(x_n)}{2}\right) + f(x_n + h\, f(x_n))\right)$$

$$x_{n+1} = x_n + \int_{t_n}^{t_{n+1}} f(x(t))\, dt$$

$$x_{n+1} = x_n + \frac{x_{n+1} - x_n}{6}\left(f(x_n) + 4 f\left(\frac{x_{n+1} + x_n}{4}\right) + f(x_{n+1})\right)$$

$$= x_n + \frac{x_n + h f(x_n) - x_n}{6}\left(f(x_n) + 4 f\left(\frac{2x_n + h f(x_n)}{4}\right) + f(x_n + h f(x_n))\right)$$

$$= x_n + \frac{h}{6} f(x_n)\left(f(x_n) + 4 f\left(\frac{2x_n + h f(x_n)}{4}\right) + f(x_n + h f(x_n))\right)$$

$\dot{x} = x \qquad x(0) = 1$

$\underline{\underline{x(t) = e^t}}$