

1 - 5 Lineare Algebra

1) $\mu(x) = ax^2 + bx + c \rightarrow (1,2) (2,3) (3,1)$

$$\mu(1) = 2$$

$$a + b + c = 2$$

$$\mu(2) = 3$$

$$4a + 2b + c = 3$$

$$\mu(3) = 1$$

$$9a + 3b + c = 1$$

$$\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 4 & 2 & 1 & 3 \\ 9 & 3 & 1 & 1 \end{array} \sim \begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 3 & 1 & 0 & 1 \\ 8 & 2 & 0 & -1 \end{array} \sim \begin{array}{ccc|c} -2 & 0 & 1 & 1 \\ 3 & 1 & 0 & 1 \\ 2 & 0 & 0 & -3 \end{array}$$

$$\sim \begin{array}{ccc|c} 0 & 0 & 1 & -2 \\ 3 & 1 & 0 & 1 \\ 2 & 0 & 0 & -3 \end{array} \sim \begin{array}{ccc|c} 0 & 0 & 1 & -2 \\ 6 & 2 & 0 & 2 \\ 2 & 0 & 0 & -3 \end{array} \sim \begin{array}{ccc|c} 0 & 0 & 1 & -2 \\ 0 & 2 & 0 & 2 \\ 1 & 0 & 0 & -3/2 \end{array}$$

$$\underline{\underline{a = -\frac{3}{2} \quad b = \frac{11}{2} \quad c = -2}}$$

4)

$$\begin{array}{ccc|c} 2 & 3 & 4 & 4 \\ 3 & 4 & 5 & 5 \\ 4 & 5 & 7 & 3 \end{array} \sim \begin{array}{ccc|c} 2 & 3 & 4 & 4 \\ 1 & 1 & 1 & 1 \\ 0 & -1 & -1 & -5 \end{array} \sim \begin{array}{ccc|c} 2 & 0 & 1 & -1 \\ 1 & 0 & 0 & -4 \\ 0 & -1 & -1 & -5 \end{array}$$

$$\sim \begin{array}{ccc|c} 0 & 0 & 1 & -3 \\ 1 & 0 & 0 & -4 \\ 0 & -1 & 0 & -2 \end{array}$$

$$\underline{\underline{x = -4, \quad y = 8, \quad z = -3}}$$

$$\begin{array}{l}
 \sim \\
 \begin{array}{ccc|c}
 1 & 3 & -1 & 1 \\
 -1 & 3 & 3 & 0 \\
 0 & -1 & -1 & 0 \\
 \hline
 3 & 0 & 1 & 0 \\
 -3 & 1 & 1 & 0 \\
 3 & 0 & 0 & 0 \\
 \hline
 6 & 7 & -2 & 0 \\
 -2 & 7 & -2 & 0 \\
 -2 & 7 & -2 & 6 \\
 \hline
 6 & -9 & 0 & 0 \\
 0 & 4 & 4 & 0 \\
 0 & -2 & 7 & 0 \\
 0 & 0 & -9 & 6 \\
 \hline
 6 & -9 & 0 & 0 \\
 0 & 0 & 5 & 0 \\
 0 & -2 & 7 & 0 \\
 0 & 0 & -9 & 6 \\
 \hline
 6 & 0 & 0 & 54 \\
 0 & 0 & 0 & 1 \\
 0 & -1 & 0 & -6 \\
 0 & 0 & 0 & -10
 \end{array}
 \end{array}$$

$$\begin{array}{cc|c} 2 & 5 & 10 \\ 4 & 10 & 20 \\ 2 & 4 & 2 \end{array} \sim \begin{array}{cc|c} 2 & 0 & 2 \\ 0 & 8 & 16 \\ 2 & 4 & 2 \end{array}$$

$$\sim \begin{array}{cc|c} 2 & 5 & 10 \\ 0 & 1 & 2 \\ 2 & 4 & 2 \end{array} \sim \begin{array}{cc|c} 2 & 0 & 2 \\ 0 & 1 & 2 \\ 0 & 6 & -6 \end{array}$$

$$\sim \begin{array}{cc|c} 2 & 0 & 2 \\ 0 & 1 & 2 \\ 0 & 0 & -6 \end{array} \sim \begin{array}{cc|c} 0 & 0 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & -6 \end{array}$$

$$2x - y = -6$$

$$y = 2$$

$$x = 1$$

$$2x - y = -6$$

$$y = 6 + 2x$$

$$\rightarrow \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 6 \\ 2 \end{bmatrix}$$

5)

$$\begin{array}{ccc|c} 2 & 3 & 4 & 4 \\ 3 & 4 & 5 & 5 \\ 4 & 5 & 6 & 3 \end{array} \sim \begin{array}{ccc|c} 2 & 3 & 4 & 4 \\ 1 & 1 & 1 & 1 \\ 0 & -1 & -2 & -5 \end{array} \sim \begin{array}{ccc|c} 2 & 0 & -2 & -1 \\ 1 & 0 & -1 & -4 \\ 0 & -1 & -2 & -5 \end{array}$$

~ 0 0 0 *Wahrheitswert*

$$\begin{array}{ccc|c} 2 & 3 & 1 & 2 \\ 3 & 4 & 2 & 1 \\ 4 & 5 & 3 & 0 \end{array} \sim \begin{array}{ccc|c} 2 & 3 & 1 & 2 \\ 1 & 1 & 1 & 1 \\ 0 & -1 & -1 & -1 \end{array} \sim \begin{array}{ccc|c} 2 & 0 & -2 & -2 \\ 1 & 0 & 0 & 0 \\ 0 & -1 & -1 & -1 \end{array} \text{ *Wahrheitswert* }$$

7)

$$u_1 x_1 + u_2 x_2 + \dots + u_n x_n = 0 \Rightarrow x_1 = x_2 = \dots = x_n = 0$$

$$c_1 u_1 + c_2 u_2 + \dots + c_n u_n = b$$

$$d_1 u_1 + d_2 u_2 + \dots + d_n u_n = b$$

$$c_1 u_1 + c_2 u_2 + \dots + c_n u_n - (d_1 u_1 + d_2 u_2 + \dots + d_n u_n) = b - b = 0$$

$$(c_1 - d_1)u_1 + (c_2 - d_2)u_2 + \dots + (c_n - d_n)u_n = b - b = 0$$

$$c_1 - d_1 = c_2 - d_2 = \dots = c_n - d_n = 0$$

$$c_1 = d_1, c_2 = d_2, c_3 = d_3$$

8) Alt fra 4, ittro fra 5


$$9) A = \begin{bmatrix} 2 & 3 & 4 \\ 3 & 4 & 5 \\ 4 & 5 & 7 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{bmatrix}$$

$$A \cdot B = \begin{bmatrix} 2 & 3 & 4 \\ 3 & 4 & 5 \\ 4 & 5 & 7 \end{bmatrix} \begin{bmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{bmatrix}$$

$$= \begin{bmatrix} 2 \cdot 1 + 3 \cdot 2 + 4 \cdot 3 & 2 \cdot 4 + 3 \cdot 5 + 4 \cdot 6 & 2 \cdot 7 + 3 \cdot 8 + 4 \cdot 9 \\ 3 \cdot 1 + 4 \cdot 2 + 5 \cdot 3 & 3 \cdot 4 + 4 \cdot 5 + 5 \cdot 6 & 3 \cdot 7 + 4 \cdot 8 + 5 \cdot 9 \\ 4 \cdot 1 + 5 \cdot 2 + 7 \cdot 3 & 4 \cdot 4 + 5 \cdot 5 + 7 \cdot 6 & 4 \cdot 7 + 5 \cdot 8 + 7 \cdot 9 \end{bmatrix}$$

$$= \begin{bmatrix} 2 + 6 + 12 & 8 + 15 + 24 & 14 + 24 + 36 \\ 3 + 8 + 15 & 12 + 20 + 30 & 21 + 32 + 45 \\ 4 + 10 + 21 & 16 + 25 + 42 & 28 + 40 + 63 \end{bmatrix}$$

$$= \begin{bmatrix} 20 & 47 & 74 \\ 26 & 57 & 98 \\ 50 & 83 & 133 \end{bmatrix}$$

Graph 

10) Gjør ikke mening fordi rader i A & kolonner i B

11)

$$A = \begin{bmatrix} 2 & 3 & 4 \\ 3 & 4 & 5 \\ 4 & 5 & 7 \end{bmatrix} \quad I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad B = \begin{bmatrix} -3 & 1 & 1 \\ 1 & 2 & -2 \\ 1 & -2 & 1 \end{bmatrix}$$

$$A \cdot I = A$$

$$I \cdot A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & 3 & 4 \\ 3 & 4 & 5 \\ 4 & 5 & 7 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 3 & 4 \\ 3 & 4 & 5 \\ 4 & 5 & 7 \end{bmatrix} \quad I \cdot A = A$$

$$AB = \begin{bmatrix} 2 & 3 & 4 \\ 3 & 4 & 5 \\ 4 & 5 & 7 \end{bmatrix} \begin{bmatrix} -3 & 1 & 1 \\ 1 & 2 & -2 \\ 1 & -2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -6+3+4 & 2+6-8 & 2-6+4 \\ -9+4+5 & 3+8-10 & 3-8+5 \\ -12+5+7 & 4+10-14 & 4-10+7 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$BA = \begin{bmatrix} -3 & 1 & 1 \\ 1 & 2 & -2 \\ 1 & -2 & 1 \end{bmatrix} \begin{bmatrix} 2 & 3 & 4 \\ 3 & 4 & 5 \\ 4 & 5 & 7 \end{bmatrix}$$

$$= \begin{bmatrix} -6+3+4 & -9+4+5 & -12+5+7 \\ 2+6-8 & 3+8-10 & 4+10-14 \\ 2-6+4 & 3-8+5 & 4-10+7 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Bruger følelse her og blender

$$B \cdot \begin{bmatrix} 4 \\ 5 \\ 3 \end{bmatrix} = \begin{bmatrix} -3 & 1 & 1 \\ 1 & 2 & -2 \\ 1 & -2 & 1 \end{bmatrix} \begin{bmatrix} 4 \\ 5 \\ 3 \end{bmatrix} = \begin{bmatrix} -12+5+3 \\ 4+10-6 \\ 4-10+3 \end{bmatrix} = \begin{bmatrix} -4 \\ 8 \\ -3 \end{bmatrix}$$

Oppg 4 nye 2

$$\begin{bmatrix} 2 & 3 & 4 & 4 \\ 3 & 4 & 5 & 5 \\ 4 & 5 & 7 & 3 \end{bmatrix} \sim \begin{bmatrix} 2 & 3 & 4 & 4 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 2 & -2 \end{bmatrix} \sim \begin{bmatrix} 2 & 3 & 4 & 4 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & -3 \end{bmatrix}$$

$$\sim \begin{bmatrix} 0 & 1 & 2 & 2 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & -3 \end{bmatrix} \sim \begin{bmatrix} 0 & 1 & 0 & 8 \\ 1 & 1 & 0 & 4 \\ 0 & 0 & 1 & -3 \end{bmatrix} \sim \begin{bmatrix} 0 & 1 & 0 & 8 \\ 1 & 0 & 0 & -4 \\ 0 & 0 & 1 & -3 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 0 & 0 & -4 \\ 0 & 1 & 0 & 8 \\ 0 & 0 & 1 & -3 \end{bmatrix}$$

12)

$$A = \begin{bmatrix} 2 & 3 & 4 \\ 3 & 4 & 5 \\ 4 & 5 & 7 \end{bmatrix}$$

$$A^T = A \quad | \quad I = \begin{bmatrix} 2 & 3 & 4 & 1 & 0 & 0 \\ 3 & 4 & 5 & 0 & 1 & 0 \\ 4 & 5 & 7 & 0 & 0 & 1 \end{bmatrix}$$

$$\sim \begin{array}{ccc|ccc} 2 & 3 & 4 & 1 & 0 & 0 \\ 1 & 1 & 1 & -1 & 1 & 0 \\ 1 & 1 & 2 & 0 & -1 & 1 \end{array}$$

$$\sim \begin{array}{ccc|ccc} 0 & 1 & 2 & 3 & -2 & 0 \\ 1 & 1 & 1 & -1 & 1 & 0 \\ 0 & 0 & 1 & 1 & -2 & 1 \end{array}$$

$$\sim \begin{array}{ccc|ccc} 0 & 1 & 0 & 1 & 2 & -2 \\ 1 & 1 & 0 & -2 & 3 & -1 \\ 0 & 0 & 1 & 1 & -2 & 1 \end{array}$$

$$\sim \begin{array}{ccc|ccc} 0 & 1 & 0 & 1 & 2 & -2 \\ 1 & 0 & 0 & -3 & 1 & 1 \\ 0 & 0 & 1 & 1 & -2 & 1 \end{array}$$

$$\sim \begin{array}{ccc|ccc} 1 & 0 & 0 & -3 & 1 & 1 \\ 0 & 1 & 0 & 1 & 2 & -2 \\ 0 & 0 & 1 & 1 & -2 & 1 \end{array}$$

NICE!

14)

$$A = \begin{bmatrix} 2 & 3 & 4 \\ 3 & 4 & 5 \\ 4 & 5 & 7 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{bmatrix}$$

$$A \cdot B = \begin{bmatrix} 2 & 3 & 4 \\ 3 & 4 & 5 \\ 4 & 5 & 7 \end{bmatrix} \begin{bmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{bmatrix}$$

$$= \begin{bmatrix} 2+6+12 & 8+15+24 & 14+24+36 \\ 3+8+15 & 12+20+30 & 21+32+45 \\ 4+10+21 & 16+25+42 & 20+40+63 \end{bmatrix}$$

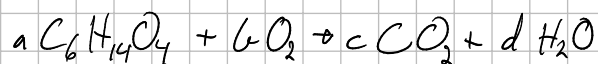
$$= \begin{bmatrix} 20 & 47 & 74 \\ 26 & 62 & 98 \\ 35 & 83 & 131 \end{bmatrix}$$

$$B \cdot A = \begin{bmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{bmatrix} \begin{bmatrix} 2 & 3 & 4 \\ 3 & 4 & 5 \\ 4 & 5 & 7 \end{bmatrix}$$

$$= \begin{bmatrix} 2+12+28 & 3+16+35 & 4+20+49 \\ 4+15+32 & 6+20+40 & 8+25+56 \\ 6+18+36 & 9+24+45 & 12+30+63 \end{bmatrix}$$

$$= \begin{bmatrix} 42 \\ 51 \end{bmatrix}$$

15)



$$6 x_1 = x_3$$

$$14 x_1 = 2 x_4$$

→

$$6 x_1 - x_3 = 0$$

$$14 x_1 - 2 x_4 = 0$$

$$4 x_1 + 2 x_2 = 2 x_3 + x_4$$

$$4 x_1 + 2 x_2 - 2 x_3 - x_4 = 0$$

$$\begin{array}{cccc|c} 6 & 0 & -1 & 0 & 0 \\ 14 & 0 & 0 & -2 & 0 \\ 4 & 2 & -2 & -1 & 0 \end{array}$$

$$\sim \begin{array}{cccc|c} 6 & 0 & -1 & 0 & 0 \\ 7 & 0 & 0 & -1 & 0 \\ 4 & 2 & -2 & -1 & 0 \end{array}$$

$$\sim \begin{array}{cccc|c} 6 & 0 & -1 & 0 & 0 \\ 7 & 0 & 0 & -1 & 0 \\ -3 & 2 & -2 & 0 & 0 \end{array}$$

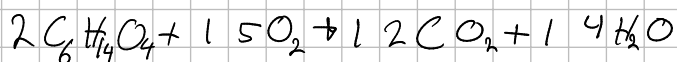
$$\sim \begin{array}{cccc|c} 6 & 0 & -1 & 0 & 0 \\ 7 & 0 & 0 & -1 & 0 \\ -15 & 2 & 0 & 0 & 0 \end{array}$$

$$6 x_1 - x_3 = 0 \quad x_3 = 6$$

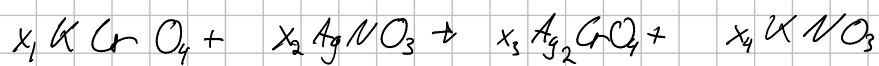
$$7 x_1 - x_4 = 0$$

$$-15 x_1 + 2 x_2 = 0$$

$$x = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} + s \begin{bmatrix} 1 \\ 15/2 \\ 6 \\ 7 \end{bmatrix} = \frac{s}{2} \begin{bmatrix} 2 \\ 15 \\ 12 \\ 14 \end{bmatrix}$$



16)



K	1	0	0	-1
Cr	1	0	-1	0
O	4	3	-4	-3
Ag	0	1	-2	0
N	0	1	0	-1

$$\begin{bmatrix} 1 & 0 & 0 & -1 \\ 1 & 0 & -1 & 0 \\ 4 & 3 & -4 & -3 \\ 0 & 1 & -2 & 0 \\ 0 & 1 & 0 & -1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{matrix} 2 \\ 1 \\ 1 \\ 4 \\ 3 \\ 0 \end{matrix} \begin{matrix} 1 \\ 0 \\ 0 \\ 1 \\ 1 \\ 1 \end{matrix} \begin{matrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{matrix} \begin{matrix} -1 \\ 0 \\ 0 \\ 0 \\ 0 \\ -1 \end{matrix}$$

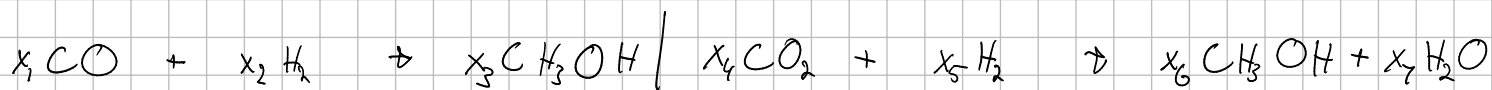
$$\begin{matrix} 2 \\ 1 \\ 1 \\ 0 \\ 0 \\ 0 \end{matrix} \begin{matrix} 1 \\ 0 \\ 0 \\ 1 \\ 1 \\ 1 \end{matrix} \begin{matrix} 0 \\ 0 \\ 1 \\ 4 \\ -2 \\ 0 \end{matrix} \begin{matrix} -1 \\ 0 \\ 0 \\ 0 \\ 0 \\ -1 \end{matrix}$$

$$\begin{matrix} 2 \\ 1 \\ 1 \\ 0 \\ 0 \\ 0 \end{matrix} \begin{matrix} 1 \\ 0 \\ 0 \\ 3 \\ 1 \\ 1 \end{matrix} \begin{matrix} 0 \\ 0 \\ 1 \\ 0 \\ -2 \\ 0 \end{matrix} \begin{matrix} -1 \\ 0 \\ 0 \\ 0 \\ 0 \\ -1 \end{matrix}$$

$$\begin{matrix} 2 \\ 1 \\ 1 \\ 0 \\ 0 \\ 0 \end{matrix} \begin{matrix} 1 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{matrix} \begin{matrix} 0 \\ 0 \\ 1 \\ 0 \\ 1 \\ 0 \end{matrix} \begin{matrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ -1 \end{matrix}$$

$$x = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

18)



C	1	0	-1	1	0	-1	0
O	1	0	-1	2	0	-1	1
H	0	2	-4	0	2	-4	-2

$$\begin{array}{cccccc} 1 & 0 & -1 & 1 & 0 & -1 & 0 \\ 1 & 0 & -1 & 2 & 0 & -1 & 1 \\ 0 & 2 & -4 & 0 & 2 & -4 & -2 \end{array}$$

$$\sim \begin{array}{cccccc} 1 & 0 & -1 & 1 & 0 & -1 & 0 \\ 1 & 0 & -1 & 2 & 0 & -1 & 1 \\ 0 & 1 & -2 & 0 & 1 & -2 & -1 \end{array}$$

$$\sim \begin{array}{cccccc} 1 & 0 & -1 & 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & -2 & 0 & 1 & -2 & -1 \end{array}$$

$$\sim \begin{array}{cccccc} 1 & 0 & -1 & 0 & 0 & -1 & -1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & -2 & 0 & 1 & -2 & -1 \end{array}$$

$$\sim \begin{array}{cccccc} 1 & 0 & -1 & 0 & 0 & -1 & -1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ -2 & 1 & 0 & 0 & 1 & 0 & 1 \end{array}$$

$$x_1 - x_3 - x_6 - x_7 = 0$$

$$x_4 + x_7 = 0$$

$$-2x_1 + x_2 + x_5 + x_7 = 0$$

$$1 - x_3 - u - v = 0$$

$$x_3 = 1 - u - v$$

$$x_4 + v = 0$$

$$x_4 = -v$$

$$-2x_1 + x_2 + v + v = 0$$

$$x_2 = 2x_1 - v - v$$

$$\begin{array}{l} x_1 = s \\ x_5 = z \\ x_6 = u \\ x_7 = v \end{array}$$

$$X = s \begin{bmatrix} 1 \\ 2 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} + z \begin{bmatrix} 0 \\ -1 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} + u \begin{bmatrix} 0 \\ 0 \\ -1 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} + v \begin{bmatrix} 0 \\ -1 \\ -1 \\ -1 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

