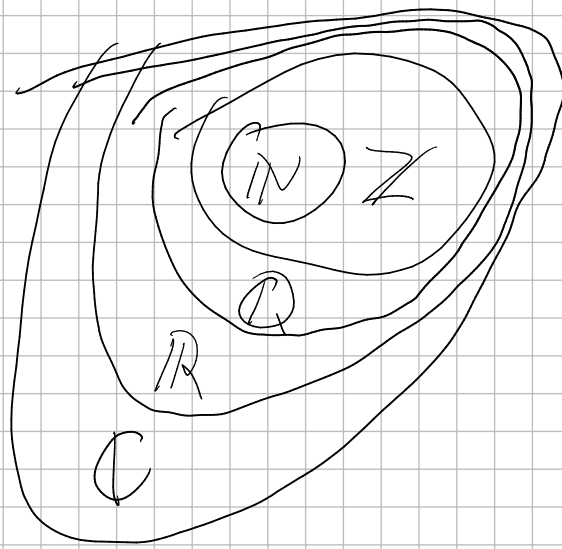
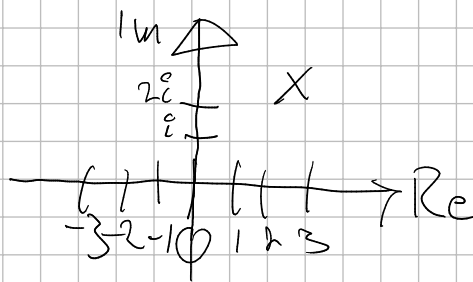
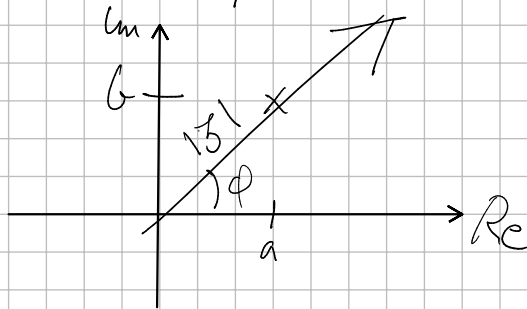


# Komplexe Zahl



$$z = a + b i, \quad i = \sqrt{-1}$$

$$\bar{z} = a - b i$$



## Andere ordnung diff. Gleichung

$$\ddot{x} + \mu \dot{x} + q x = 0$$

$\mu$   $q$  Konstanten

## Karakteristische Gleichung

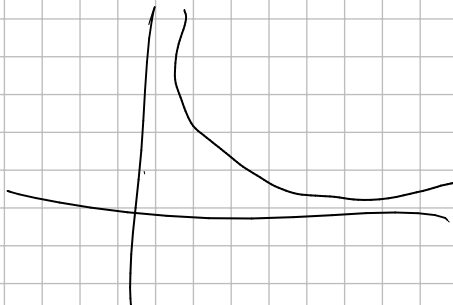
$$r^2 + \mu r + q = 0$$

$$r_{1,2} = \frac{-\mu \pm \sqrt{\mu^2 - 4q}}{2}$$

10 reelle Werten:

$$r_1 \neq r_2 \Rightarrow \boxed{x(t) = C e^{r_1 t} + D e^{r_2 t}}$$

1. Same Werten: hier wir ein überdämpftes and second order system



Ein reelles root: ( $\mu^2 - 4q = 0$ )



$$r_1 = r_2 = r \Rightarrow \boxed{x(t) = C e^{rt} + D t e^{rt}}$$

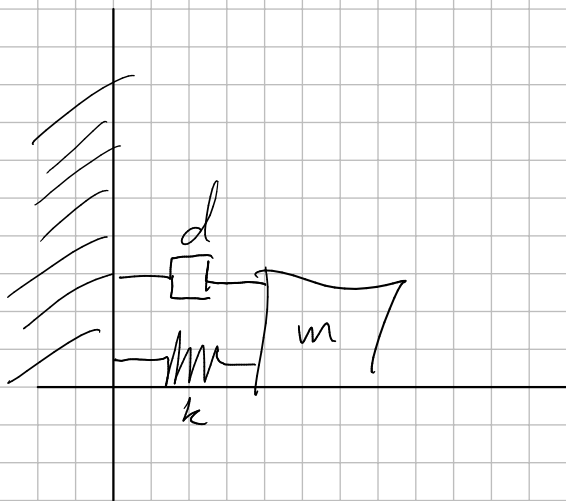
Kritisch gedämpft system?

10 komplexes root: ( $\mu^2 - 4q < 0$ )

$$\left. \begin{array}{l} r_1 = a + bi \\ r_2 = a - bi \end{array} \right\} \Rightarrow x(t) = e^{at} (C \cos(bt) + D \sin(bt))$$

Oscillierend?

Beispiel: Masse-feder-dämpfer-system



Federkraft  $\vec{F}_f = kx$ , Hookes law  
Dämpferkraft  $\vec{F}_d = d\dot{x}$

Kraftbalance

Newton's 2. law

$$\sum F = m a = m \ddot{x} = m \ddot{x}$$

$$-F_f - F_d = m \ddot{x}$$

$$m \ddot{x} + d \dot{x} + k x = 0$$

$$\ddot{x} + \frac{d}{m} \dot{x} + \frac{k}{m} x = 0$$

Karakteristische Gleichung

$$r^2 + \frac{d}{m} r + \frac{k}{m} = 0$$

$$r = \frac{-\frac{d}{m} \pm \sqrt{\left(\frac{d}{m}\right)^2 - 4 \frac{k}{m}}}{2}$$

$$m = 1 \text{ kg} \quad d = 2 \quad k = 10$$

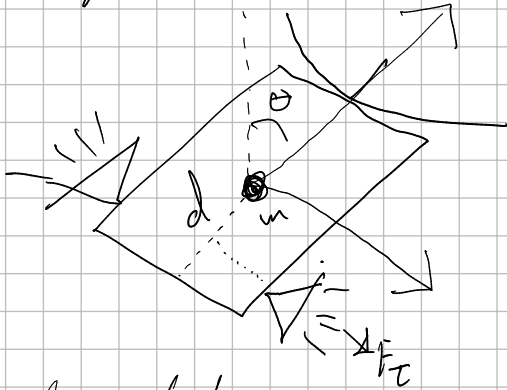
$$p_{1,2} = -1 \pm 3i$$

$$x(\tau) = e^{a\tau} (C \cos(6\tau) + D \sin(6\tau))$$

$$x(0) = 1$$

$$\dot{x}(0) = 0$$

Exempel: Satellit



Momentbalance

$$\sum M = J \ddot{\theta}$$

$$J \ddot{\theta} = F_{\tau} \cdot d$$

$J$ : Tröghetsmoment aka Torque

2. ordens län. län. med  $\mu = \eta = 0$

$$\ddot{\theta} = \frac{F_{\tau} \cdot d}{J} \Rightarrow \ddot{\theta} = \frac{F_{\tau} d}{J} \tau + C_1$$

$$\Rightarrow \theta = \frac{F_{\tau} d}{2J} \tau^2 + C_1 \tau + C_2$$

2. ordens län  $\rightarrow$  2 1. ordens län.

$$\ddot{x} + \mu \dot{x} + q x = 0$$

Omform till nya tillstånd

$$x_1 = x$$

$$x_2 = \dot{x}$$

To differentier  $\dot{x}_1 = \dot{x} = x_2 \Rightarrow \dot{x}_1 = x_2$

$$\dot{x}_2 = \ddot{x} = -\mu \dot{x} - q x$$

$$\dot{x}_2 = -\mu x_2 - q x_1$$

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = -g x_1 - \mu x_2 \end{cases}$$

Tilstandsom modell

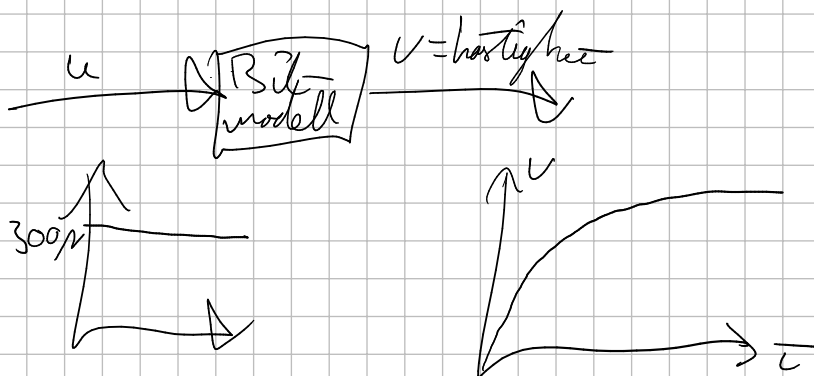
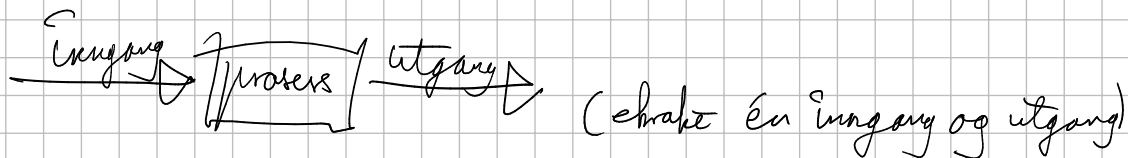
$$\underline{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}, \quad \underline{\dot{x}} = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix}$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -g & -\mu \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

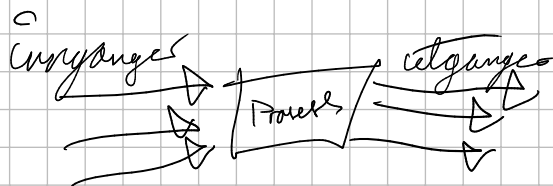
$$\underline{\dot{x}} = A \underline{x}$$

Dynamiske systemer

Monovariable system



Multivariable systemer



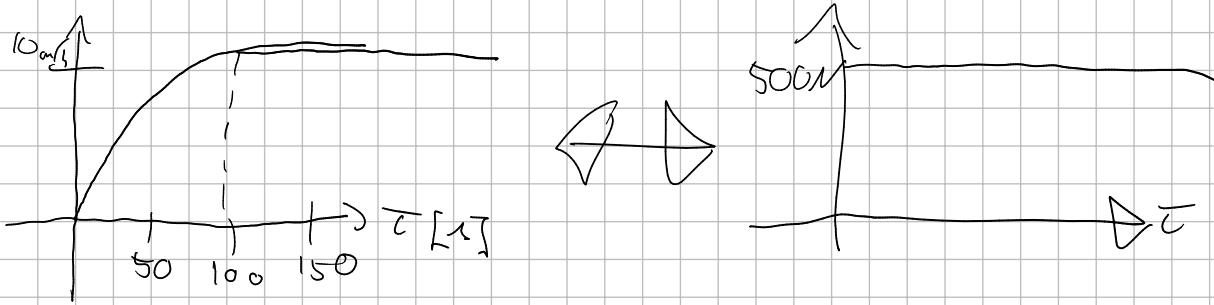
Eksempel:

Indgange: roer, thruster, propeller, ...

Udgange: kurs, hastighed, position, ...

# Støjansvarssystem

statisk proces:  $y = f(x)$ , dvs.: måleinstrument



Tilnærmelse  $v = 10 \text{ m/s}$  efter  $100 \text{ s}$

konstruerer en statisk model  $v = Ku$ ,  $K = \frac{v}{u} = \frac{10}{500} = 0,02$   
 $v = 0,02u$