

Öving 4

Öppgave 1)

$$m = \rho A h \quad w_{ut} = k h$$

$$a) \quad \frac{d}{dt} m = w_{in} - w_{ut}$$

$$\frac{d}{dt} (\rho A h) = w_{in} - w_{ut}$$

$$\frac{d}{dt} h = \frac{1}{\rho A} (w_{in} - w_{ut})$$

$$w_{ut} = k h$$

$$\frac{d}{dt} h = \frac{1}{\rho A} (w_{in} - k h)$$

$$\frac{d}{dt} h = -\frac{k}{\rho A} h + \frac{1}{\rho A} w_{in}$$

$$\frac{d}{dt} h + \frac{k}{\rho A} h = \frac{1}{\rho A} w_{in}$$

Auto w_{inn} er konstant

Auto p er konstant

Auto A er konstant

$$\frac{d}{dt}h + \frac{k}{pA}h = \frac{1}{pA}w_{\text{inn}} \quad | \cdot e^{\frac{k}{pA}\tau}$$

$$\frac{d}{d\tau}h e^{\frac{k}{pA}\tau} + \frac{k}{pA}h e^{\frac{k}{pA}\tau} = \frac{1}{pA}w_{\text{inn}} e^{\frac{k}{pA}\tau}$$

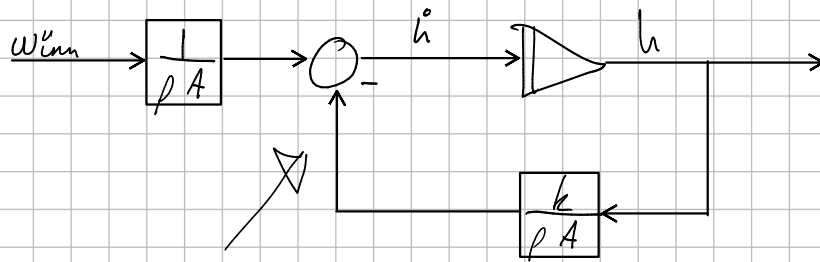
$$\frac{d}{d\tau}(h e^{\frac{k}{pA}\tau}) = \frac{1}{pA}w_{\text{inn}} e^{\frac{k}{pA}\tau} \quad | \int d\tau$$

$$h e^{\frac{k}{pA}\tau} = \frac{pA}{k} \frac{1}{pA} w_{\text{inn}} e^{\frac{k}{pA}\tau} + C \quad | \cdot e^{-\frac{k}{pA}\tau}$$

$$h(\tau) = \frac{1}{k} w_{\text{inn}} + C e^{-\frac{k}{pA}\tau}$$

b)

$$\frac{dh}{dt} = -\frac{k}{\rho A} h + \frac{1}{\rho A} w_{inn}$$



Naturlig tilbakerekobling

c)

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Om $w_{inn} = 0$, så vil $t \rightarrow \infty$, $h \rightarrow 0$. Dette er fordi all massen i tanken vil bare renne ut og bli tom om ingenting annet renner inn.

Dette kan vi se på vår modell. Ettersom $w_{inn} = 0$, så vil ifølge modellen

$$\frac{d}{dt} h = -\frac{k}{\rho A} h$$

Dette fører til at endringen \dot{h} vil være negativ helt til $h = 0$, altså at nivået tilsvarende 0.

Modellen vil bli stabil fordi den går mot en bestemt tilstand $h = 0$.

d)

Oppgave 2)

b) Steady-state: a

$$a = h_r - h(\infty)$$

$$\frac{d}{dt}h = -\frac{k}{\rho A}h + \frac{1}{\rho A}w_{\text{inn}}$$

$$w_{\text{inn}} = K_p(h_r - h)$$

$$\frac{d}{dt}h = -\frac{k}{\rho A}h + \frac{1}{\rho A}K_p(h_r - h)$$

$$\frac{d}{dt}h = -\frac{k}{\rho A}h + \frac{K_p}{\rho A}h_r - \frac{K_p}{\rho A}h \quad -\frac{k}{\rho A} - \frac{K_p}{\rho A} = -\frac{1}{\rho A}(k + K_p)$$

$$\frac{d}{dt}h = \left(-\frac{k + K_p}{\rho A}\right)h + \frac{K_p}{\rho A}h_r$$

$$\frac{d}{dt}h + \frac{k + K_p}{\rho A}h = \frac{K_p}{\rho A}h_r$$

$$\lambda = \frac{k + K_p}{\rho A}$$

$$\beta = \frac{K_p}{\rho A}h_r$$

$$\frac{d}{dt}h + \lambda h = \beta \quad | \cdot e^{\lambda t}$$

$$\frac{d}{dt}h e^{\lambda t} + \lambda h e^{\lambda t} = \beta e^{\lambda t}$$

$$\frac{d}{dt}(h e^{\lambda t}) = \beta e^{\lambda t} \quad | \cdot \int dt$$

$$h e^{\lambda t} = \frac{1}{\lambda} \beta e^{\lambda t} + C \quad | \cdot e^{-\lambda t}$$

$$h(t) = \frac{1}{\lambda} \beta + C e^{-\lambda t}$$

$$\lambda = \frac{k + K_p}{\rho A}$$

$$\beta = \frac{K_p}{\rho A}h_r$$

$$h(t) = \frac{\rho A}{k + K_p} \cdot \frac{K_p}{\rho A}h_r + C e^{-\frac{k + K_p}{\rho A}t}$$

$$h(t) = \frac{K_p}{k + K_p}h_r + C e^{-\frac{k + K_p}{\rho A}t}$$

$$h(0) = h_0$$

$$\frac{k_p}{k + k_p} h_r + C = h_0$$

$$C = h_0 - \frac{k_p}{k + k_p} h_r$$

$$h(t) = \frac{k_p}{k + k_p} h_r + \left(h_0 - \frac{k_p}{k + k_p} h_r \right) e^{-\frac{k + k_p}{\rho A} t}$$

$$a = h_r - h(\infty)$$

$$a = h_r - \frac{k_p}{k + k_p} h_r$$

$$a = h_r \left(1 - \frac{k_p}{k + k_p} \right)$$

$$k_p = 100 \quad k = 1 \quad h_r = 0,5$$

$$a = 0,5 \cdot \left(1 - \frac{100}{1 + 100} \right)$$

$$a = 4,95 \cdot 10^{-5} \approx \underline{\underline{0,05}}$$

4)

$$\frac{d}{dt} x + b x = u, \quad u = k_p(r - x) + k_i \int (r - x) dt$$

$$\frac{d}{dt}$$

