Linearalgebra 2

1)

$$\left[\begin{array}{cccc|c} 2 & 1 & 1 & 2 & 4 \\ 1 & 0 & 1 & 1 & 1 \\ 1 & -2 & 1 & -1 & -3 \\ 0 & 1 & 0 & 1 & 0 \end{array}\right]$$

$$\sim \left[\begin{array}{cccc|c} 2 & 0 & 1 & 1 & 4 \\ 1 & 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 & -3 \\ 0 & 1 & 0 & 1 & 0 \end{array}\right]$$

$$\sim \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 3 \\ 1 & 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 & -3 \\ 0 & 1 & 0 & 1 & 0 \end{array}\right]$$

$$\sim \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 3 \\ 0 & 0 & 1 & 1 & -2 \\ 0 & 0 & 0 & 1 & -6 \\ 0 & 1 & 0 & 1 & 0 \end{array}\right]$$

$$\sim \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 3 \\ 0 & 0 & 1 & 0 & 4 \\ 0 & 0 & 0 & 1 & -6 \\ 0 & 1 & 0 & 0 & 6 \end{array}\right]$$

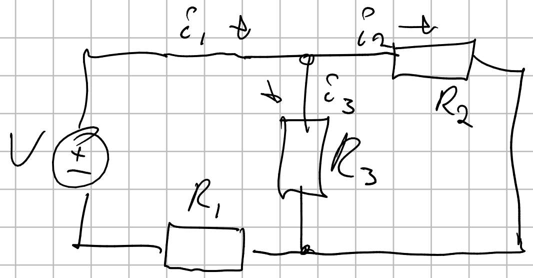
$$\sim \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 3 \\ 0 & 0 & 1 & 0 & 4 \\ 0 & 1 & 0 & 0 & 6 \\ 0 & 0 & 0 & 1 & -6 \end{array}\right]$$

$$\sim \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 3 \\ 0 & 1 & 0 & 0 & 6 \\ 0 & 0 & 1 & 0 & 4 \\ 0 & 0 & 0 & 1 & -6 \end{array}\right]$$

Ikke lineært uavhengig fordi $Ax = b$ har en løsning der $b \neq \vec{0}$

2) $\det(A) = 0$, ikke inverterbar ?

3)



$i_1 = i_2 + i_3$ $\quad U = R I$

$V = u_2 = u_3$ $\quad i_2 = \frac{u}{R_2} \Rightarrow$

$R = R_1 + \frac{R_2 R_3}{R_2 + R_3}$

$u_2 = u_3$

$i_2 R_2 = i_3 R_3$

$i_2 R_2 - i_3 R_3 = 0$

$Ax = b$ $\quad x = \begin{matrix} i_1 \\ i_2 \\ i_3 \end{matrix}$

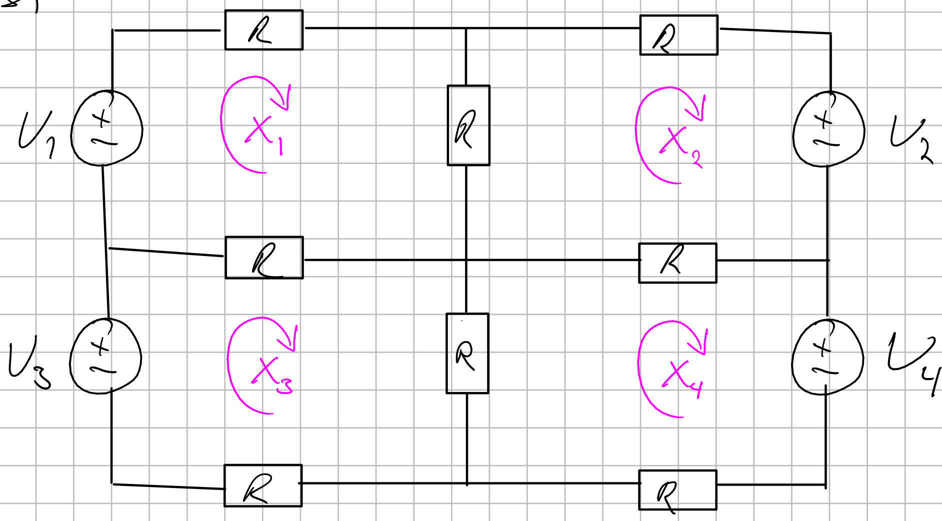
$U = u_1 + u_3$

$i_1 R_1 + i_3 R_3$

$i_1 - i_2 - i_3 = 0$

$$\begin{bmatrix} R_1 & 0 & R_3 \\ 0 & R_2 & -R_3 \\ 1 & -1 & -1 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} U \\ 0 \\ 0 \end{bmatrix}$$

Oppg 4-8)



$$U_1 = R\,x_1 + R(x_1 - x_2) + R(x_1 - x_3)$$

$$= R(x_1 + x_1 - x_2 + x_1 - x_3)$$

$$= R(3x_1 - x_2 - x_3)$$

$$= 3R\,x_1 - R\,x_2 - R\,x_3$$

$$U_2 = R(-x_2) + R(x_1 - x_2) + R(x_4 - x_2)$$

$$= R\,x_1 - 3R\,x_2 + R\,x_4$$

$$U_3 = R(x_3 - x_1) + R(x_3 - x_4) + R(x_3)$$

$$= -R\,x_1 + 3R\,x_3 - R\,x_4$$

$$U_4 = R(x_2 - x_4) + R(x_3 - x_4) + R(-x_4)$$

$$= R\,x_2 + R\,x_3 - 3R\,x_4$$

Opg 4)

$$\begin{bmatrix} 3R & -R & -R & 0 \\ R & -3R & 0 & R \\ -R & 0 & 3R & -R \\ 0 & R & R & -3R \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} v_1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\left[\begin{array}{cccc|c} 3 & -1 & -1 & 0 & 1 \\ 1 & -3 & 0 & 1 & 0 \\ -1 & 0 & 3 & -1 & 0 \\ 0 & 1 & 1 & -3 & 0 \end{array}\right] \sim \begin{array}{cccc|c} 3 & 0 & 0 & -3 & 1 \\ 0 & -3 & 3 & 0 & 0 \\ -1 & 0 & 3 & -1 & 0 \\ 0 & 1 & 1 & -3 & 0 \end{array}$$

$$\sim \begin{array}{cccc|c} 3 & 0 & 0 & -3 & 1 \\ 0 & -1 & 1 & 0 & 0 \\ -1 & 0 & 3 & -1 & 0 \\ 0 & 0 & 2 & -3 & 0 \end{array} \sim \begin{array}{cccc|c} 0 & 0 & 0 & -6 & 1 \\ 0 & -1 & 1 & 0 & 0 \\ -1 & 0 & 3 & -1 & 0 \\ 0 & 0 & 2 & -3 & 0 \end{array}$$

$$\sim \begin{array}{cccc|c} 0 & 0 & 0 & 1 & -1/6 \\ 0 & -1 & 1 & 0 & 0 \\ -1 & 0 & 3 & -1 & 0 \\ 0 & 0 & 2 & -3 & 0 \end{array} \sim \begin{array}{cccc|c} 0 & 0 & 0 & 1 & -1/6 \\ 0 & -1 & 1 & 0 & 0 \\ -1 & 0 & 3 & 0 & -1/6 \\ 0 & 0 & 2 & 0 & -3/6 \end{array}$$

$$\sim \begin{array}{cccc|c} 0 & 0 & 0 & 1 & -1/6 \\ 0 & -2 & 2 & 0 & 0 \\ -2 & 0 & 6 & 0 & -2/6 \\ 0 & 0 & 2 & 0 & -3/6 \end{array} \sim \begin{array}{cccc|c} 0 & 0 & 0 & 1 & -1/6 \\ 0 & -2 & 0 & 0 & 3/6 \\ -2 & 0 & 0 & 0 & 7/6 \\ 0 & 0 & 2 & 0 & -3/6 \end{array}$$

$$\sim \begin{array}{cccc|c} 1 & 0 & 0 & 0 & -7/12 \\ 0 & 1 & 0 & 0 & -1/4 \\ 0 & 0 & 1 & 0 & -1/4 \\ 0 & 0 & 0 & 1 & -1/6 \end{array}$$

$$\begin{matrix} v_1 \\ 0 \\ 0 \\ 0 \end{matrix} + \begin{matrix} 0 \\ v_2 \\ 0 \\ 0 \end{matrix} + \begin{matrix} 0 \\ 0 \\ v_3 \\ 0 \end{matrix} + \begin{matrix} 0 \\ 0 \\ 0 \\ v_4 \end{matrix} = \begin{matrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{matrix}$$

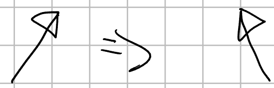
PY:

```python
import numpy as np


A = np.matrix([[3, -1, -1, 0],
               [1, -3, 0, 1],
               [-1, 0, 3, -1],
               [0, 1, 1, -3]])

v_v1 = np.array([1, 0, 0, 0])
v_v2 = np.array([0, 1, 0, 0])
v_v3 = np.array([0, 0, 1, 0])
v_v4 = np.array([0, 0, 0, 1])

print(np.linalg.solve(A, v_v1))
print(np.linalg.solve(A, v_v2))
print(np.linalg.solve(A, v_v3))
print(np.linalg.solve(A, v_v4))
print(np.linalg.solve(A, v_v1) + np.linalg.solve(A, v_v2) + np.linalg.solve(A, v_v3) + np.linalg.solve(A, v_v4))
```

```
┌─larsandre@archlinux ~/Documents/studie/tma4101-ma
└─➤ main ✗  $ /bin/python "/home/larsandre/Document
[0.46666667 0.2        0.2        0.13333333]
[-0.2        -0.46666667 -0.13333333 -0.2        ]
[0.2        0.13333333 0.46666667 0.2        ]
[-0.13333333 -0.2        -0.2        -0.46666667]
[ 0.33333333 -0.33333333  0.33333333 -0.33333333]
```

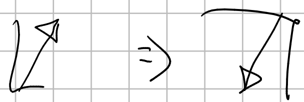
1 0) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ Hmm ?

1 1) $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} x_1 \\ -x_2 \end{bmatrix}$ Speiler langs y-aksen



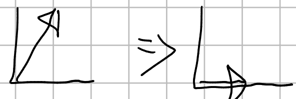
1 2) $\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -x_1 \\ x_2 \end{bmatrix}$ Speiler langs x-aksen

1 3) $\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -x_1 \\ -x_2 \end{bmatrix}$ Bytter direksjon



1 4) $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} x_2 \\ x_1 \end{bmatrix}$ Speiler langs $x = y$

1 5) $\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} x_1 \\ 0 \end{bmatrix}$ Fjerner y-komponent



1 6) $\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ x_2 \end{bmatrix}$ Fjerner x-komponent

1 7) $\begin{bmatrix} a & 0 \\ 0 & a \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} a\,x_1 \\ a\,x_2 \end{bmatrix} = a \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ Skalerer med a

1 8) $\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{matrix} x_1\cos\theta - x_2\sin\theta \\ x_1\sin\theta + x_2\cos\theta \end{matrix}$ Roterer med θ

1 9) ITTNO ?

2 0) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} x_1 \\ x_3 \\ x_2 \end{bmatrix}$ Speiler langs yz-planet

2 1) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} x_1 \\ 0 \\ 0 \end{bmatrix}$ Får lengden til x_1 ?

2 2) Fjerner z-komponent

23) Roterer langs xy-planet med θ

24) Roterer langs xz-planet med θ

25) Roterer langs yz-planet med θ

26)
$$A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 6 & 2 \\ 2 & 2 & 6 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 2 \\ 2 & 6 & 2 \\ 2 & 2 & 6 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}$$

27)
$$\begin{bmatrix} 1 & 2 & 2 \\ 2 & 6 & 2 \\ 2 & 2 & 6 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix} = \begin{bmatrix} 1+4+4 \\ 2+12+4 \\ 2+4+12 \end{bmatrix} = \begin{bmatrix} 9 \\ 18 \\ 18 \end{bmatrix} = 9 \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}$$

28)
$$\begin{bmatrix} 1 & 2 & 2 \\ 2 & 6 & 2 \\ 2 & 2 & 6 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 5 \\ 10 \\ 10 \end{bmatrix}$$

29)
$$\begin{bmatrix} 1 & 2 & 2 \\ 2 & 6 & 2 \\ 2 & 2 & 6 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix} = \begin{bmatrix} 2-2 \\ 6-2 \\ 2-6 \end{bmatrix} = \begin{bmatrix} 0 \\ 4 \\ -4 \end{bmatrix}$$

30)
$$\begin{bmatrix} 1 & 2 & 2 \\ 2 & 6 & 2 \\ 2 & 2 & 6 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix} = \begin{bmatrix} 1-4+6 \\ 2-12+6 \\ 2-4+18 \end{bmatrix} = \begin{bmatrix} 3 \\ -4 \\ 16 \end{bmatrix}$$

31)
$$\begin{bmatrix} 1 & 2 & 2 \\ 2 & 6 & 2 \\ 2 & 2 & 6 \end{bmatrix} \cdot \begin{bmatrix} 4 \\ -1 \\ -1 \end{bmatrix} = \begin{bmatrix} 4-2-2 \\ 8-6-2 \\ 8-2-6 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

32)

$$A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 6 & 2 \\ 2 & 2 & 6 \end{bmatrix}$$

$$\det(A - \lambda I) = 0$$

$$\begin{vmatrix} 1-\lambda & 2 & 2 \\ 2 & 6-\lambda & 2 \\ 2 & 2 & 6-\lambda \end{vmatrix} = 0$$

$$(1-\lambda)(6-\lambda)(6-\lambda) + (2)(2)(2) + (2)(2)(2) - (2)(6-\lambda)(2) - (2)(2)(1-\lambda) - (6-\lambda)(2)(2) = 0$$

$$(1-\lambda)(36 - 12\lambda + \lambda^2) + 8 + 8 - 4(6-\lambda) - 4(1-\lambda) - 4(6-\lambda) = 0$$

$$(36 - 12\lambda + \lambda^2 - 36\lambda + 12\lambda^2 - \lambda^3) + 16 - 24 - + 4\lambda - 4 + 4\lambda - 24 + 4\lambda = 0$$

$$-\lambda^3 + 13\lambda^2 - 48\lambda + 36 + 16 - 48 - 4 + 12\lambda = 0$$

$$-\lambda^3 + 13\lambda^2 - 36\lambda = 0$$

$$\lambda(-\lambda^2 + 13\lambda - 36) = 0 \qquad \lambda_1 = 0$$

$$\lambda = \frac{-13 \pm \sqrt{13^2 - 4(-1)(-36)}}{-2}$$

$$\lambda = \frac{-13 \pm \sqrt{169 - 144}}{-2}$$

$$\lambda = \frac{-13 \pm 5}{-2}$$

$$\lambda_1 = 4 \quad \lambda_2 = 9$$

$$\lambda = 0 \wedge 4 \wedge 9$$

33) $(A - \lambda I) = 0$

$$\left[\begin{array}{ccc|c} 1-\lambda & 2 & 2 & 0 \\ 2 & 6-\lambda & 2 & 0 \\ 2 & 2 & 6-\lambda & 0 \end{array}\right]$$

$\lambda_1 = 0$

$$\begin{bmatrix} 1 & 2 & 2 \\ 2 & 6 & 2 \\ 2 & 2 & 6 \end{bmatrix} \sim \begin{matrix} 1 & 2 & 2 \\ 0 & 2 & -2 \\ 0 & -2 & 2 \end{matrix} \sim \begin{matrix} 1 & 2 & 2 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{matrix} \sim \left.\begin{matrix} 1 & 4 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{matrix}\right| \begin{matrix} 0 \\ 0 \\ 0 \end{matrix}$$

$x_1 + 4x_2 = 0$

$x_2 - x_3 = 0$

$x_2 = s$

$x_1 = -4s$

$x_3 = s$

$$v_1 = s\begin{bmatrix} -4 \\ 1 \\ 1 \end{bmatrix}$$

$\lambda_2 = 4$

$$\left.\begin{matrix} -3 & 2 & 2 \\ 2 & 2 & 2 \\ 2 & 2 & 2 \end{matrix}\right| \begin{matrix} 0 \\ 0 \\ 0 \end{matrix} \sim \begin{matrix} -3 & 2 & 2 \\ 1 & 1 & 1 \\ 0 & 0 & 0 \end{matrix} \sim \begin{matrix} 0 & 5 & 5 \\ 1 & 1 & 1 \end{matrix} \sim \begin{matrix} 0 & 1 & 1 \\ 1 & 0 & 0 \end{matrix}$$

$x_2 + x_3 = 0$

$x_1 = 0$

$$v_2 = s\begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}$$

$x_2 = s$

$x_3 = -s$

$\lambda_3 = 9$

$$\left.\begin{matrix} -8 & 2 & 2 \\ 2 & -3 & 2 \\ 2 & 2 & -3 \end{matrix}\right| \begin{matrix} 0 \\ 0 \\ 0 \end{matrix} \sim \begin{matrix} 0 & -10 & 10 \\ 2 & -3 & 2 \\ 2 & 2 & -3 \end{matrix} \sim \begin{matrix} 0 & -1 & 1 \\ 2 & -3 & 2 \\ 2 & 2 & -3 \end{matrix} \sim \begin{matrix} 0 & -1 & 1 \\ 2 & 0 & -1 \\ 2 & 0 & -1 \end{matrix}$$

$$\sim \left.\begin{matrix} 0 & -1 & 1 \\ 2 & 0 & -1 \\ 0 & 0 & 0 \end{matrix}\right| \begin{matrix} 0 \\ 0 \\ 0 \end{matrix}$$

$-x_2 + x_3 = 0$

$2x_1 - x_3 = 0$

$x_3 = s$

$x_2 = s$

$x_1 = \frac{1}{2}s$

$$v_3 = s\begin{bmatrix} 1/2 \\ 1 \\ 1 \end{bmatrix}$$

34)

$$A = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

$$\det(A - \lambda I) = 0$$

$$\begin{vmatrix} \cos\theta - \lambda & -\sin\theta \\ \sin\theta & \cos\theta - \lambda \end{vmatrix} = 0$$

$$(\cos\theta - \lambda)(\cos\theta - \lambda) - (-\sin\theta)(\sin\theta) = 0$$

$$\cos^2\theta - 2\lambda\cos\theta - \lambda^2 + \sin^2\theta = 0$$

$$-\lambda^2 - 2\cos\theta\,\lambda + 1 = 0$$

$$\lambda^2 + 2\cos\theta\,\lambda - 1 = 0$$

$$\lambda = \frac{-2\cos\theta \pm \sqrt{4\cos^2\theta - 4(-1)}}{2}$$

$$= \frac{-2\cos\theta \pm \sqrt{4\cos^2\theta + 4}}{2}$$

$$= \frac{-2\cos\theta \pm \sqrt{4(\cos^2\theta + 1)}}{2}$$

$$= -\cos\theta \pm \sqrt{\cos^2\theta + 1}$$