Diffligninger 6

1) $\dot{x}(t) + x(t) = \cos(t) \qquad x(0) = 0$

$$\dot{x} + x = 0 \quad | \cdot e^t$$

$$\dot{x} e^t + x e^t = 0 \qquad \lambda = 1$$

$$\int \frac{d}{dt}(x e^t) = \int 0 \, dt$$

$$x e^t = C$$

$$x = C e^{-t}$$

$$x(0) = 0$$
$$C = 1$$

$$x_h = e^{-t}$$

$$\cos(t) = \frac{1}{2}(e^{it} + e^{-it})$$

$$x = x_h + \int_0^t f(s) e^{s-t} \, ds \quad \underline{\text{brabs}}$$

$$= e^{-t} + \int_0^t \frac{1}{2}(e^{is} + e^{-is}) e^{s-t} \, ds$$

$$= e^{-t} + \frac{1}{2} \int_0^t e^{is+s-t} + e^{-is+s-t} \, ds$$

$$= e^{-t} + \frac{1}{2} \left[\frac{1}{i+1} e^{is+s-t} + \frac{1}{-i+1} e^{-is+s-t} \right]_0^t$$

$$= e^{-t} + \frac{1}{2}\left(\frac{1}{i+1} e^{it+t-t} + \frac{1}{-i+1} e^{-it+t-t} \right) - \frac{1}{2}\left(\frac{1}{i+1} e^{-t} + \frac{1}{-i+1} e^{-t} \right)$$

$$= e^{-t} + \frac{1}{2}\left(\frac{1}{i+1}(e^{it} + e^{-t}) + \frac{1}{-i+1}(e^{-it} + e^{-t}) \right)$$

$$\dot{x}(t) + x(t) = \cos(t) \qquad x(0) = 0$$

$$\dot{x} e^{t} + x\, e^{t} = e^{t} \tfrac{1}{2}\left(e^{it} + e^{-it}\right)$$

$$\frac{d}{dt}\left(x e^{t}\right) = \tfrac{1}{2}\left(e^{(i+1)t} + e^{(-i+1)t}\right) \quad \Big| \int dt$$

$$x e^{t} = \tfrac{1}{2}\left(\frac{1}{i+1} e^{(i+1)t} + \frac{1}{-i+1} e^{(-i+1)t}\right) + C \quad \Big| \cdot e^{-t}$$

$$x(t) = c e^{-t} + \frac{1}{2i+2} e^{(i+1)t} + \frac{1}{-2i+2} e^{(-i+1)t}$$

$$x(0) = 0$$

$$c + \frac{1}{2i+2} + \frac{1}{-2i+2} = 0$$

$$c = -\frac{1}{2i+2} - \frac{1}{-2i+2}$$

$$= -\left(\frac{2i+2-2i+2}{(2i+2)(-2i+2)}\right)$$

$$= -\left(\frac{4}{4+4}\right) = \underline{-\tfrac{1}{2}}$$

2) $\ddot{x} + \dot{x} + x = 1 \qquad x(0) = \dot{x}(0) = 0$

$\lambda^2 + \lambda + 1 = 0$

$\lambda = \dfrac{-1 \pm \sqrt{1-4}}{2}$

$\lambda = -\dfrac{1}{2} \pm \dfrac{\sqrt{3}}{2} i$

$x_h = e^{-\frac{1}{2}t}\left(c_1 \cos\left(\frac{\sqrt{3}}{2}t\right) + c_2 \sin\left(\frac{\sqrt{3}}{2}t\right)\right)$

$x_p = 1$

$x(t) = e^{-\frac{1}{2}t}\left(c_1 \cos\left(\frac{\sqrt{3}}{2}t\right) + c_2 \sin\left(\frac{\sqrt{3}}{2}t\right)\right) + 1$

$x(0) = 0$

$c_1 + 1 = 0$

$c_1 = -1$

$x(t) = e^{-\frac{1}{2}t}\left(-\cos\left(\frac{\sqrt{3}}{2}t\right) + c_2 \sin\left(\frac{\sqrt{3}}{2}t\right)\right) + 1$

$\dot{x}(t) = -\frac{1}{2} e^{-\frac{1}{2}t}\left(-\cos\left(\frac{\sqrt{3}}{2}t\right) + c_2 \sin\left(\frac{\sqrt{3}}{2}t\right)\right)$
$\qquad + e^{-\frac{1}{2}t}\left(\frac{\sqrt{3}}{2} \sin\left(\frac{\sqrt{3}}{2}t\right) + \frac{\sqrt{3}}{2} c_2 \cos\left(\frac{\sqrt{3}}{2}t\right)\right)$

$\dot{x}(0) = 0$

$-\frac{1}{2}(-1) + \left(\frac{\sqrt{3}}{2} c_2\right) = 0$

$\underline{c_2 = -\dfrac{1}{\sqrt{3}}}$

$x(t) = e^{-\frac{1}{2}t}\left(-\cos\left(\frac{\sqrt{3}}{2}t\right) - \frac{1}{\sqrt{3}} \sin\left(\frac{\sqrt{3}}{2}t\right)\right) + 1$

3)

$Ax = b$

$$\left[\begin{array}{cccc|c} 2 & 3 & 4 & 5 & 2 \\ 3 & 4 & 5 & 6 & 2 \\ 4 & 5 & 6 & 7 & 2 \end{array}\right] \sim \begin{array}{cccc|c} 2 & 3 & 4 & 5 & 2 \\ 1 & 1 & 1 & 1 & 0 \\ 2 & 2 & 2 & 2 & 0 \end{array} \sim \begin{array}{cccc|c} 0 & 1 & 2 & 3 & 2 \\ 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}$$

$$\sim \begin{array}{cccc|c} 0 & 1 & 2 & 3 & 2 \\ 1 & 0 & -1 & -2 & -2 \end{array}$$

$x_1 - x_3 - 2x_4 = -2$

$x_2 + 2x_3 + 3x_4 = 2$

$x_3 = s \quad x_4 = t$

$$\underline{x = \begin{bmatrix} -2 \\ 2 \\ 0 \\ 0 \end{bmatrix} + s\begin{bmatrix} -1 \\ 2 \\ 1 \\ 0 \end{bmatrix} + t\begin{bmatrix} -2 \\ 3 \\ 0 \\ 1 \end{bmatrix}}$$

4) $\ddot{x}(t) + \dot{x}(t) + x(t) = u(t-1) \qquad x(0) = \dot{x}(0) = 0$

$$u(t-1) = \begin{cases} 0 & \text{for } t < 1 \\ 1 & \text{for } t \geq 1 \end{cases}$$

$z(t) = x(t - \tau_0),\ \tau_0 = -1 \Rightarrow z(t) = x(t+1)$

$\ddot{z}(t) + \dot{z}(t) + z(t) = u(t - 1 - \tau_0) = u(t) \qquad z(\tau_0) = z(-1) = \dot{z}(-1) = 0$

$\underline{t < 0 \Rightarrow u(t) = 0}$

$\ddot{z}(t) + \dot{z}(t) + z(t) = 0$

$z(t) = e^{-\frac{1}{2}t}\left(c_1 \cos\left(\frac{\sqrt{3}}{2}t\right) + c_2 \sin\left(\frac{\sqrt{3}}{2}t\right)\right) \qquad z(-1) = \dot{z}(-1) = 0$

$z(-1) = 0$

$e^{\frac{1}{2}}\left(c_1 \cos\left(-\frac{\sqrt{3}}{2}\right) + c_2 \sin\left(-\frac{\sqrt{3}}{2}\right)\right) = 0$

$c_1 \cos\left(-\frac{\sqrt{3}}{2}\right) + c_2 \sin\left(-\frac{\sqrt{3}}{2}\right) = 0$

$\dot{z}(t) = -\frac{1}{2}e^{-\frac{1}{2}t}\left(c_1 \cos\left(\frac{\sqrt{3}}{2}t\right) + c_2 \sin\left(\frac{\sqrt{3}}{2}t\right)\right)$
$\quad + e^{-\frac{1}{2}t}\left(-\frac{\sqrt{3}}{2}c_1 \sin\left(\frac{\sqrt{3}}{2}t\right) + \frac{\sqrt{3}}{2}c_2 \cos\left(\frac{\sqrt{3}}{2}t\right)\right)$

$\dot{z}(-1) = 0$

$-\frac{1}{2}e^{\frac{1}{2}}\left(c_1 \cos\left(-\frac{\sqrt{3}}{2}\right) + c_2 \sin\left(-\frac{\sqrt{3}}{2}\right)\right) + e^{\frac{1}{2}}\left(-\frac{\sqrt{3}}{2}c_1 \cos\left(-\frac{\sqrt{3}}{2}\right) + \frac{\sqrt{3}}{2}c_2 \sin\left(-\frac{\sqrt{3}}{2}\right)\right) = 0$

$-\frac{1}{2}c_1 \cos\left(-\frac{\sqrt{3}}{2}\right) - \frac{1}{2}c_2 \sin\left(-\frac{\sqrt{3}}{2}\right) - \frac{\sqrt{3}}{2}c_1 \cos\left(-\frac{\sqrt{3}}{2}\right) + \frac{\sqrt{3}}{2}c_2 \sin\left(-\frac{\sqrt{3}}{2}\right) = 0$

$-c_1 \cos\left(-\frac{\sqrt{3}}{2}\right) - c_2 \sin\left(-\frac{\sqrt{3}}{2}\right) - \sqrt{3}c_1 \sin\left(-\frac{\sqrt{3}}{2}\right) + \sqrt{3}c_2 \cos\left(-\frac{\sqrt{3}}{2}\right) = 0$

$c_1\left(-\cos\left(-\frac{\sqrt{3}}{2}\right) - \sqrt{3}\sin\left(-\frac{\sqrt{3}}{2}\right)\right) + c_2\left(-\sin\left(-\frac{\sqrt{3}}{2}\right) + \sqrt{3}\cos\left(-\frac{\sqrt{3}}{2}\right)\right) = 0$

$c_1 = c_2 = 0$ fordi $z(t) = \dot{z}(t) = 0$ når $t < 0$

$t \geq 0$

$\ddot{z}(t) + \dot{z}(t) + z(t) = 1$

$z_p = 1$

$z_h = e^{-\frac{1}{2}t}\left(c_3 \cos\left(\frac{\sqrt{3}}{2}t\right) + c_4 \sin\left(\frac{\sqrt{3}}{2}t\right)\right)$

$z(0) = \dot{z}(0) = 0$

$c_3 = -1 \quad c_4 = -\frac{1}{\sqrt{3}}$

$z(t) = \left(e^{-\frac{1}{2}t}\left(-\cos\left(\frac{\sqrt{3}}{2}t\right) - \frac{1}{\sqrt{3}}\sin\left(\frac{\sqrt{3}}{2}t\right)\right) + 1\right)u(t)$

$z(t) = x(t+1) \Rightarrow x(t) = z(t-1)$

$$\underline{\underline{x(t) = \left(e^{-\frac{1}{2}(t-1)}\left(-\cos\left(\frac{\sqrt{3}}{2}(t-1)\right) - \frac{1}{\sqrt{3}}\sin\left(\frac{\sqrt{3}}{2}(t-1)\right)\right) + 1\right)u(t-1)}}$$

5) $\ddot{x}(t) + \dot{x}(t) + x(t) = u(t-2)$

$z(t) = x(t - t_0)$, $t_0 = -2$ $\quad z(t) = x(t+2)$

$\ddot{z}(t) + \dot{z}(t) + z(t) = u(t)$

$z(t) = \left(e^{-\frac{1}{2}t}\left(-\cos\left(\frac{\sqrt{3}}{2}t\right) - \frac{1}{\sqrt{3}}\sin\left(\frac{\sqrt{3}}{2}t\right)\right) + 1\right)u(t)$

$z(t) = x(t+2) \Rightarrow z(t-2) = x(t)$

$$\underline{\underline{x(t) = \left(e^{-\frac{1}{2}(t-2)}\left(-\cos\left(\frac{\sqrt{3}}{2}(t-2)\right) - \frac{1}{\sqrt{3}}\sin\left(\frac{\sqrt{3}}{2}(t-2)\right)\right)u(t-2)}}$$

6) $\ddot{x}(t) + \dot{x}(t) + x(t) = u(t-1) - u(t-2)$

$x_1 = \dots\dots u(t-1)$

$x_2 = \dots\dots u(t-2)$

$x(t) = x_1(t) - x_2(t)$

7) $\ddot{x}(t) + \dot{x}(t) + x(t) = u(t-1)$ $\quad x(0) = 1 \quad \dot{x}(0) = 0$

$z(t) = x(t - \tau_0) \quad \tau_0 = -1$ $\quad z(-1) = 1 \quad \dot{z}(-1) = 0$

$z(t) = e^{-\frac{1}{2}t}(c_1 \cos(\frac{\sqrt{3}}{2}t) + c_2 \sin(\frac{\sqrt{3}}{2}t)) + 1$

$\dot{z}(t) = -\frac{1}{2} e^{-\frac{1}{2}t}(c_1 \cos(\frac{\sqrt{3}}{2}t) + c_2 \sin(\frac{\sqrt{3}}{2}t))$
$\quad + e^{-\frac{1}{2}t}(-\frac{\sqrt{3}}{2} c_1 \sin(\frac{\sqrt{3}}{2}t) + \frac{\sqrt{3}}{2} c_2 \cos(\frac{\sqrt{3}}{2}t))$

$z(-1) = c_1$

Takk deg Name

10)

$$\ddot{x} + x = \cos(\omega t)$$

$$\ddot{x} + x = \frac{1}{2}\left(e^{i\omega t} + e^{-i\omega t}\right)$$

$$\ddot{x} + x = 0$$

$$\lambda^2 + 1 = 0$$

$$\lambda = \pm i$$

$$x_h = C_1 e^{it} + C_2 e^{-it}$$

$$x = A e^{i\omega t} + B e^{-i\omega t}$$

$$\dot{x} = i\omega A e^{i\omega t} - i\omega B e^{-i\omega t}$$

$$\ddot{x} = -\omega^2 A e^{i\omega t} + \omega^2 B e^{-i\omega t}$$

$$-\omega^2 A e^{i\omega t} + \omega^2 B e^{-i\omega t} + A e^{i\omega t} + B e^{-i\omega t} = \frac{1}{2} e^{i\omega t} + \frac{1}{2} e^{-i\omega t}$$

$$-\omega^2 A + A = \frac{1}{2} \qquad \omega^2 B + B = \frac{1}{2}$$

$$A(1 - \omega^2) = \frac{1}{2} \qquad B(1 + \omega^2) = \frac{1}{2}$$

$$A = \frac{1}{2 - 2\omega^2} \qquad B = \frac{1}{2 + 2\omega^2}$$

$$x_p = \frac{1}{2 - 2\omega^2} e^{i\omega t} + \frac{1}{2 + 2\omega^2} e^{-i\omega t}$$

$$x(t) = C_1 e^{it} + C_2 e^{-it} + \frac{1}{2 - 2\omega^2} e^{i\omega t} + \frac{1}{2 + 2\omega^2} e^{-i\omega t}$$

$$\omega = 1$$

$$= C_1 e^{it} + C_2 e^{-it} + \frac{1}{0} e^{it} + \frac{1}{4} e^{-it}$$

Udefinert

1 1)

$$\mathcal{L}(x) = X(s) = \int_0^\infty x(t)\, e^{-st}\, dt$$

$$x(t) = e^{at}$$

$$\mathcal{L}(x) = X(s) = \int_0^\infty e^{at}\, e^{-st}\, dt = \int_0^\infty e^{t(a-s)}\, dt \qquad Re(s) \geq a$$

$$= \left[\frac{1}{a-s} e^{t(a-s)}\right]_0^\infty = \frac{1}{a-s} e^{\infty(a-s)} - \frac{1}{a-s} e^{0(a-s)}$$

$$= \frac{1}{s-a}$$

1 2) $\mathcal{L}(\dot{x}) = s\,\mathcal{L}(x) - x(0)$? $\quad Re(s) > 0$

$$\mathcal{L}(\dot{x}) = \int_0^\infty \dot{x}(t) e^{-st}\, dt = \left[x(t)\, e^{-st}\right]_0^\infty - \int_0^\infty x(t)(-s)\, e^{-st}\, dt$$

$$= -x(0) + s \int_0^\infty x(t)\, e^{-st}\, dt$$

$$= s\,\mathcal{L}(x) - x(0)$$

1 3) $\dot{x} + a x = 0 \qquad x(0) = 1 \qquad x(t) = e^{-at}$

$$s\,\mathcal{L}(x) - x(0) + a\,\mathcal{L}(x) = 0$$

$$s\,\mathcal{L}(x) - 1 + a\,\mathcal{L}(x) = 0$$

$$\mathcal{L}(x)(s+a) = 1$$

$$\mathcal{L}(x) = X(s) = \frac{1}{s+a}$$

$$x(t) = e^{-t}$$

14) $\mathcal{L}(\ddot{x}) = s^2 \mathcal{L}(x) - s\,x(0) - \dot{x}(0)$?

$$\mathcal{L}(\ddot{x}) = \int_0^\infty \ddot{x}\, e^{-st} dt = \dot{x}(t) e^{-st}\Big|_0^\infty + s\int_0^\infty \dot{x}\, e^{-st} dt$$

$$= -\dot{x}(0) + s\left(x\, e^{-st}\Big|_0^\infty + s\int_0^\infty x\, e^{-st} dt\right)$$

$$\underline{\underline{= -\dot{x}(0) - s\,x(0) + s^2 \mathcal{L}(x)}}$$

15) $x(t) = \cos t$

$$= \frac{1}{2}\left(e^{it} + e^{-it}\right)$$

$$\mathcal{L}(x) = \frac{1}{2}\int_0^\infty \left(e^{it} + e^{-it}\right) e^{-st}\, dt$$

$$= \frac{1}{2}\int_0^\infty e^{t(i-s)} + e^{t(-i-s)}\, dt \qquad \operatorname{Re}(s) > 0$$

$$= \frac{1}{2}\int_0^\infty e^{(i-s)t} + e^{-(i+s)t}\, dt$$

$$= \frac{1}{2}\left(\frac{1}{i-s} e^{(i-s)t} - \frac{1}{i+s} e^{-(i+s)t}\Big|_0^\infty\right)$$

$$= -\frac{1}{2}\left(\frac{1}{i-s} - \frac{1}{i+s}\right) = -\frac{1}{2}\left(\frac{i-s-i-s}{(i-s)(-i-s)}\right)$$

$$= -\frac{1}{2}\left(\frac{-2s}{s^2+1}\right)$$

$$\underline{\underline{X(s) = \frac{s}{s^2+1}}}$$

1 b) $x(t) = \sin t = \frac{1}{2i}(e^{it} - e^{-it})$

$$X(s) = \frac{1}{2i}\int_0^\infty (e^{it} - e^{-it})e^{-st}\,dt$$

$$= \frac{1}{2i}\int_0^\infty e^{(i-s)t} - e^{-(i+s)t}\,dt$$

$$= \frac{1}{2i}\left(\frac{1}{i-s}e^{(i-s)t} + \frac{1}{i+s}e^{-(i+s)t}\Big|_0^\infty\right)$$

$$= -\frac{1}{2i}\left(\frac{1}{i-s} + \frac{1}{i+s}\right) = -\frac{1}{2i}\left(\frac{i+s+i-s}{-1-s^2}\right)$$

$$= -\frac{1}{2i}\left(\frac{2i}{-s^2-1}\right)$$

$$\underline{\underline{X(s) = \frac{1}{s^2+1}}}$$

18) $\ddot{x}(t) + x(t) = 0 \qquad x(0) = 1 \qquad \dot{x}(0) = 0$

$\mathcal{L}(\ddot{x}) + \mathcal{L}(x) = 0$

$s\,\mathcal{L}(\dot{x}) - \dot{x}(0) + \mathcal{L}(x) = 0$

$s^2\mathcal{L}(x) - s\,x(0) - \dot{x}(0) + \mathcal{L}(x) = 0$

$\mathcal{L}(x)(s^2+1) - s = 0$

$$X(s) = \frac{s}{s^2+1}$$

$$\underline{\underline{x(t) = \cos(t)}}$$

19) $\ddot{x}(t) + x(t) = 0 \qquad x(0) = 0 \qquad \dot{x}(0) = 1$

$\mathcal{L}(\ddot{x}) + \mathcal{L}(x) = 0$

$s\,\mathcal{L}(\dot{x}) - \dot{x}(0) + \mathcal{L}(x) = 0$

$s^2\mathcal{L}(x) - s\,x(0) - \dot{x}(0) + \mathcal{L}(x) = 0$

$X(s)(s^2+1) - 0 - 1 = 0$

$X(s)(s^2+1) = 1$

$$X(s) = \frac{1}{s^2+1}$$

$$x(t) = \text{sen}\,t$$

20) $\ddot{x}(t) + \dot{x}(t) + x(t) = 0 \qquad x(0) = 1 \qquad \dot{x}(0) = 0$

$\mathcal{L}(\ddot{x}) + \mathcal{L}(\dot{x}) + \mathcal{L}(x) = 0$

$s\,\mathcal{L}(\dot{x}) - \dot{x}(0) + s\,\mathcal{L}(x) - x(0) + \mathcal{L}(x) = 0$

$s^2\mathcal{L}(x) - s\,x(0) - \dot{x}(0) + s\,\mathcal{L}(x) - x(0) + \mathcal{L}(x) = 0$

$X(s)(s^2+s+1) - s - 1 = 0$

$X(s) = \dfrac{s+1}{s^2+s+1} \qquad \lambda = \alpha + i\,\omega_0 \qquad \bar{\lambda} = \alpha - i\,\omega_0$

$$\frac{s+1}{s^2+s+1} = \frac{A}{s-\lambda} + \frac{B}{s-\bar{\lambda}}$$

$s+1 = A(s-\bar{\lambda}) + B(s-\lambda)$

$A + B = 1$

$A = 1 - B$

$-A\bar{\lambda} - B\lambda = 1$

$$-(1-B)\bar{\lambda} - B\lambda = 1$$

$$-\bar{\lambda} + B\bar{\lambda} - B\lambda = 1$$

$$B(\bar{\lambda} - \lambda) = 1 + \bar{\lambda}$$

$$B = \frac{1+\bar{\lambda}}{\bar{\lambda} - \lambda} = \frac{1+\bar{\lambda}}{\alpha - i\omega_0 - (\alpha + i\omega_0)} = -\frac{1+\bar{\lambda}}{2i\omega_0}$$

$$A = 1 - \frac{1+\bar{\lambda}}{\bar{\lambda}-\lambda} = \frac{\bar{\lambda} - \lambda - 1 - \bar{\lambda}}{\bar{\lambda} - \lambda} = \frac{-\lambda - 1}{-2i\omega_0} = \frac{1+\lambda}{2i\omega_0}$$

$$X(s) = \frac{A}{s-\lambda} + \frac{B}{s-\bar{\lambda}} = \frac{1}{2i\omega_0}\left(\frac{1+\lambda}{s-\lambda} - \frac{1+\bar{\lambda}}{s-\bar{\lambda}}\right)$$

$$x(t) = \frac{1}{2i\omega_0}\left((1+\lambda)e^{\lambda t} - (1+\bar{\lambda})e^{\bar{\lambda}t}\right)$$

$$x(t) = \frac{e^{\alpha t}}{2i\omega_0}\left((1+\lambda)e^{i\omega_0 t} - (1+\bar{\lambda})e^{-i\omega_0 t}\right)$$

$$x(t) = \frac{e^{\alpha t}}{2i\omega_0}\left((1+\lambda)e^{i\omega_0 t} - \overline{(1+\bar{\lambda})e^{-i\omega_0 t}}\right)$$

$$= \frac{e^{\alpha t}}{2i\omega_0}\left((1+\lambda)e^{i\omega_0 t} - (\right.$$

Gilden ikke !

$$x(t) = e^{\alpha t}\left(\cos(\omega_0 t) + \frac{1+\alpha}{\omega_0}\sin(\omega_0 t)\right)$$

2 1) $\ddot{x}(t)+\dot{x}(t)+x(t)=\cos t \qquad x(0)=0 \qquad \dot{x}(0)=0$

$$s\mathcal{L}(\dot{x})-\dot{x}(0)+s\mathcal{L}(x)-x(0)+\mathcal{L}(x)=\frac{s}{s^2+1}$$

$$s^2\mathcal{L}(x)-s\,x(0)-\dot{x}(0)+s(\mathcal{L})-x(0)+\mathcal{L}(x)=\frac{s}{s^2+1}$$

$$X(s)(s^2+s+1)-s\,x(0)-\dot{x}(0)=\frac{s}{s^2+1}$$

$$X(s)=\frac{s}{(s^2+1)}\cdot\frac{1}{s^2+s+1}$$

$$x_h=e^{\alpha t}(A\cos(\omega_0 t)+B\sin(\omega_0 t))$$

$$\ddot{x}+\dot{x}+x=\frac{1}{2}(e^{it}+e^{-it})$$

$$x_p=\frac{1}{2}(H(i)e^{it}+H(-i)e^{-it})$$

$$\dot{x}_p=\frac{1}{2}(H i\,e^{it}-H i\,e^{-it})$$

$$\ddot{x}_p=\frac{1}{2}(-H e^{it}+H e^{-it})$$

$$-He^{it}+He^{-it}+Hi\,e^{it}-Hi\,e^{-it}+He^{it}+He^{-it}=e^{it}+e^{-it}$$

$$e^{it}(-H+Hi+H)+e^{-it}(H-Hi+H)=e^{it}+e^{it}$$

$$e^{it}Hi+e^{-it}Hi=e^{it}+e^{-it}$$

$$H(i)=\frac{1}{i}$$

$$x_p=\frac{1}{2}\left(\frac{1}{i}e^{it}+\frac{1}{-i}e^{-it}\right)=\frac{1}{2i}\left(e^{it}+e^{-it}\right)=\sin t$$

$$x(t)=e^{\alpha t}(A\cos(\omega_0 t)+B\sin(\omega_0 t))+\sin t \qquad x(0)=\dot{x}(0)=0$$

$$x(0)=0$$

$$A=0$$

$$\dot{x}(t)=\frac{1}{\alpha}e^{\alpha t}(A\cos(\omega_0 t)+B\sin(\omega_0 t))+e^{\alpha t}(-\omega_0 A\sin(\omega_0 t)+\omega_0 B\cos(\omega_0 t))+\cos t$$

$$=\frac{1}{\alpha}e^{\alpha t}B\sin(\omega_0 t)+e^{\alpha t}\omega_0 B\cos(\omega_0 t)+\cos t$$

$$\dot{x}(0)=0$$

$$\omega_0 B+1=0$$

$$B=-\frac{1}{\omega_0}$$

$$x(t)=-\frac{1}{\omega_0}e^{\alpha t}\sin(\omega_0 t)+\sin t$$

22) $\ddot{x} + \dot{x} + x = \sin t \qquad x(0) = \dot{x}(0) = 0$

$s^2 X(s) + s X(s) + X(s) + s\,\dot{x}(0) + x(0) = \frac{s}{s^2+1}$

$X(s) = \frac{s}{s^2+1} \cdot \frac{1}{s^2+s+1}$

24) $\ddot{x} + \dot{x} + x = e^{i\omega t} \qquad x(0) = \dot{x}(0) = 0$

$s^2 X(s) - s\,x(0) - \dot{x}(0) + s X(s) - x(0) + X(s) = \frac{1}{s - i\omega}$

Samme deiten

25) $\ddot{x} + x = \cos(t) \qquad x(0) = \dot{x}(0) = 0$

$s^2 X(s) + s\,x(0) + \dot{x}(0) + X(s) = \frac{s}{s^2+1}$

$X(s) = \frac{s}{(s^2+1)^2}$

$Z = X\,Y$

$z = \int_0^t x(u)\,y(t-u)\,du$ Konvulsjon

26) $\dot{x}(t) + x(t) = e^{-t} \qquad x(0) = 1$

$s X(s) - x(0) + X(s) = \frac{1}{s+1}$

$X(s)(s+1) - 1 = \frac{1}{s+1}$

$X(s) = \frac{1}{(s+1)^2} + \frac{1}{s+1}$

$\frac{d}{ds} \frac{1}{s+1} = -\frac{1}{(s+1)^2}$

$X(s) = -\frac{d}{ds}\left(\frac{1}{s+1}\right) + \frac{1}{s+1}$

$\underline{\underline{x(t) = e^{-t} + t e^{-t}}}$

27) $y(t) = t\,x(t)$ $\qquad Y(s) = -\frac{d}{ds} X(s)$

$$Y(s) = \int_0^\infty t\,x(t)\,e^{-st}\,dt$$

$$= \int_0^\infty \left(-\frac{d}{ds}\,x(t)\,e^{-st}\right)dt$$

$$= -\frac{d}{ds}\int_0^\infty x(t)\,e^{-st}\,dt$$

$$= -\frac{d}{ds} X(s)$$

$\dot{x}_1(t) = -x_1(t) + x_2(t)$ $\qquad x_1(0) = x_2(0) = 1$ $\qquad x_1 = x$

$\dot{x}_2(t) = \qquad -x_2(t)$ $\qquad\qquad x_2 = \dot{x}$

$\ddot{x} = -\dot{x}(t) - x(t)$

$\ddot{x} + \dot{x} + x = 0$

$s^2 X(s) + s\,x(0) + \dot{x}(0) + s\,X(s) + x(0) + X(s) = 0$

$X(s)(s^2 + s + 1) + s + 1 + 1 = 0$

$$X(s) = -\frac{s+2}{s^2+s+1}$$

$Y(s) = -\frac{d}{ds}\frac{1}{s^2+1}$ Fall off

28) $\dot{x}(t) + x(t) = e^{-t}$ $\qquad x(0) = 1$

$s\,X(s) - x(0) + X(s) = \frac{1}{s+1}$

$X(s)(s+1) = \frac{1}{s+1} + 1$

$$X(s) = \frac{1}{(s+1)^2} + \frac{1}{s+1}$$

$$X(s) = -\frac{d}{ds}\frac{1}{s+1} + \frac{1}{s+1}$$

$$\underline{\underline{x(t) = t e^{-t} + e^{-t}}}$$

2 a) $\mathcal{L}(u(t-a)) = \int_0^\infty u(t-a)\, e^{-st}\, dt$

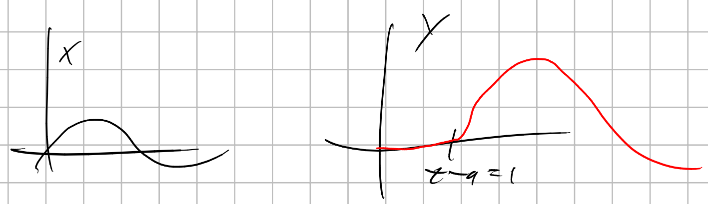
$$= \int_a^\infty e^{-st}\, dt = \frac{1}{-s} e^{-st} \Big|_a^\infty = \underline{\frac{e^{-as}}{s}}$$

$y(t) = x(t-a)\, u(t-a) \Rightarrow Y(s) = e^{-as} X(s)$?

$$Y(s) = \int_0^\infty x(t-a)\, u(t-a)\, e^{-st}\, dt = \int_a^\infty x(t-a)\, e^{-st}\, dt$$

$$= \int_0^\infty x(t)\, e^{-s(t+a)}\, dt = \int_0^\infty x(t)\, e^{-st-sa}\, dt = e^{-as} \int_0^\infty x(t)\, e^{-st}\, dt$$

$$= \underline{e^{-as} X(s)}$$



3 1) $y(t) = e^{at} x(t) \Rightarrow Y(s) = X(s-a)$

$$Y(s) = \int_0^\infty x(t)\, e^{at} e^{-st}\, dt = \int_0^\infty x(t)\, e^{-(s-a)t}\, dt)$$

$$= X(s-a)$$

3 2) $x(t) = e^{\alpha t} \cos \omega t$

$$\mathcal{L}(x) = \int_0^\infty \cos(\omega t)\, e^{\alpha t} e^{-st}\, dt = \int_0^\infty \cos(\omega t) e^{-(s-\alpha)t}$$

$$= X(s-a) \qquad X(s) = \frac{s}{s^2-\omega^2} \qquad \text{~~fehlt delta~~}$$

3 3) $$g_k(s) = \begin{cases} \frac{1}{k}, & 0 < s < k \\ 0 \end{cases}$$

$$\delta(t) = \lim_{k \to 0} g_k(t)$$

$$\int_{-\infty}^{\infty} f(s)\,\delta(x-s)\, ds = f(x)\ ? \qquad \begin{matrix} x - s < k \\ x = s \end{matrix}$$

$$\int_{-\infty}^{\infty} f(s) \lim_{k\to 0} g_k(x-s)\, ds = \lim_{k \to 0} \frac{1}{k} \int_0^k f(s)\, ds = \int_0^0 f(s)\, ds = \underline{\underline{f(x)}}$$

3 4) $$\int_a^b \delta(t-s)\, ds = \begin{cases} 1, & a \le t \le b \\ 0 \end{cases}$$

$$\mathcal{L}(\delta(t-s)) = \int_{-\infty}^{\infty} \delta(t-s) e^{-st}\, dt = \mathcal{L}(f(t)) = X(s) = \underline{\underline{e^{-as}}}$$

3 5) $\dot{x}(t) + x(t) = \delta(t-s) \qquad x(0) = 0$

$$s\,\mathcal{L}(x) - x(0) + \mathcal{L}(x) = \mathcal{L}(\delta(t-s))$$

$$X(s)(s+1) = e^{-s}$$

$$X(s) = \frac{e^{-s}}{s+1}$$

$$x(t) = u(t-1)\, e^{-(t-1)}$$

3 b) $\dot{x}(t) + x(t) = \delta(t-2) \quad x(0) = 0$

$$X(s) = \frac{e^{-2s}}{s+1}$$

$$x(t) = \underline{u(t-1)\, e^{-(t-2)}}$$