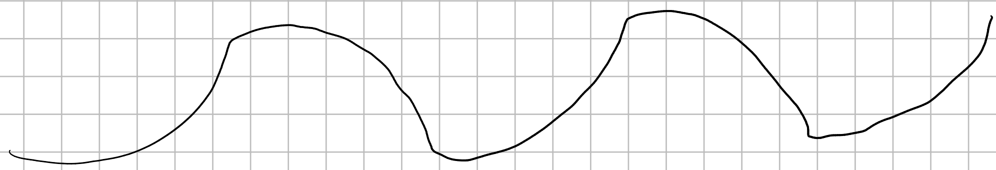
Fourieranalyse 1

0) $\dot{x}(t) + \alpha\, x(t) = f(t) \Rightarrow$



1) $f(t) = i\, R + x(t)$

$$= RC \frac{d}{dt} x(t) + x(t)$$

$$\frac{d}{dt} x(t) + \frac{1}{RC} x(t) = \frac{1}{RC} f(t)$$

3) nei

4) $f = 100\,\text{Hz}$ $\quad f = \frac{1}{T} \quad \omega = \frac{2\pi}{T}$

$$\omega = 2\pi f = \underline{\underline{200\,\pi}}$$

6)

$$\int_{-\frac{T}{2}}^{\frac{T}{2}} e^{in\omega t} e^{-im\omega t}\, dt = \int_{-\frac{T}{2}}^{\frac{T}{2}} e^{(n-m)i\omega t}\, dt = \left(\frac{1}{(n-m)i\omega} e^{(n-m)i\omega t} \Big|_{-\frac{T}{2}}^{\frac{T}{2}} \right)$$

$$= \frac{e^{(n-m)i\omega\frac{T}{2}}}{(n-m)i\omega} - \frac{e^{-(n-m)\frac{T}{2}}}{(n-m)i\omega} = \left(\frac{1}{(n-m)i\omega}\right)\left(e^{(n-m)i\omega\frac{T}{2}} - e^{-(n-m)i\omega\frac{T}{2}}\right)$$

$$= \left(\frac{1}{(n-m)i\omega}\right)\left(\cos((n-m)\omega\tfrac{T}{2}) + i\sin((n-m)\omega\tfrac{T}{2} - \cos((n-m)i\omega\tfrac{T}{2}) + \sin((n-m)i\omega\tfrac{T}{2})\right)$$

$$= \left(\frac{1}{(n-m)i\omega}\right)\left(2i\sin((n-m)\omega\tfrac{T}{2})\right) = \left(\frac{1}{(n-m)i\omega}\right)\left(2i\sin((n-m)\pi)\right)$$

$$\underline{\underline{= 0}}$$

$$X(t) = \sum_{n=-\infty}^{\infty} c_n e^{inwt}$$

$$X(t)\, e^{-imwt} = \sum_{n=-\infty}^{\infty} c_n e^{inwt} e^{-iwmt} \quad \Big|\Big| \int_{-\frac{T}{2}}^{\frac{T}{2}}$$

$$\int_{-\frac{T}{2}}^{\frac{T}{2}} X(t) e^{-imwt} dt = \int_{-\frac{T}{2}}^{\frac{T}{2}} \sum_{n=-\infty}^{\infty} c_n e^{inwt} e^{-iwmt} dt = \sum_{n=-\infty}^{\infty} c_n \int_{-\frac{T}{2}}^{\frac{T}{2}} e^{inwt} e^{-imwt} dt$$

$$= c_m T$$

$$\underline{\underline{C_m = \int_{-\frac{T}{2}}^{\frac{T}{2}} X(t)\, e^{-imwt}\, dt}}$$

7) $$x(t) = \begin{cases} 0 & -\frac{T}{2} \le t \le 0 \\ 1 & 0 \le t \le \frac{T}{2} \end{cases}$$

$$x(t) = \sum_{n=-\infty}^{\infty} c_n e^{inwt} \qquad c_n = \frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} x(t) e^{-inwt}\, dt$$

$$x(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos(nwt) + b_n \sin(nwt)$$

$$a_n = \frac{2}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} x(t) \cos(nwt)\, dt \qquad b_n = \frac{2}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} x(t) \sin(nwt)\, dt \qquad \omega = \frac{2\pi}{T}$$

$$a_n = \frac{2}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} x(t) \cos(nwt)\, dt = \frac{2}{T}\int_0^{\frac{T}{2}} \cos(nwt)\, dt = \frac{2}{T}\left(\frac{1}{nw} \sin(nwt)\Big|_0^{\frac{T}{2}}\right)$$

$$= \frac{1}{\pi} \sin(\pi n) - \frac{1}{\pi}\sin(0) = 0, \quad n \ge 1$$

$$a_0 = \frac{2}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} x(t) \cos(0wt)\, dt = \frac{2}{T}\int_0^{\frac{T}{2}} x(t)\, dt = \frac{2}{T}\int_0^{\frac{T}{2}} 1\, dt = \underline{1}$$

$$b_n = \frac{2}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} x(t) \sin(nwt)\, dt = \frac{2}{T}\int_0^{\frac{T}{2}} \sin(nwt)\, dt = \frac{2}{T}\left(\frac{1}{nw}(-\cos(nwt))\Big|_0^{\frac{T}{2}}\right)$$

$$= \frac{2}{T}\left(\frac{1}{nw}(-\cos(\pi n)) - \frac{1}{nw}(-\cos(0))\right) = \frac{2}{T}\left(\frac{1}{nw} - \frac{1}{nw}\cos(\pi n)\right)$$

$$\frac{2}{T}\frac{1}{n\omega}(1-\cos(\pi n)) = \frac{1}{n\pi}(1-\cos(\pi n)) = \begin{cases} \frac{2}{\pi n} & n \text{ er odde} \\ 0 & n \text{ er partall} \end{cases}$$

$$x(t) \simeq \frac{a_0}{n} + \sum_{n=1}^{\infty} a_n \cos(n\omega t) + b_n \sin(n\omega t)$$

$$x(t) \simeq \frac{1}{2} + \sum_{n=1}^{\infty} \frac{1}{n\pi}(1-\cos(\pi n))\sin(n\omega t)$$

$$\simeq \frac{1}{2} + \sum_{n=1}^{\infty} \frac{2}{(2n+1)\pi} \sin((2n+1)\omega t)$$

$$\underline{\underline{\simeq \frac{1}{2} + \frac{2}{\pi}\sum_{n=1}^{\infty} \frac{1}{2n+1} \sin((2n+1)\omega t)}}$$

Kompleks variant $\quad n = 0$

$$c_n = \frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} x(t) e^{-in\omega t} = \frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} x(t) = \frac{1}{T}\int_{0}^{\frac{T}{2}} 1 \, dt = \frac{1}{T}\left(\frac{T}{2}-0\right) = \underline{\underline{\frac{1}{2}}}$$

$n \neq 0$

$$c_n = \frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} x(t) e^{-in\omega t} dt = \frac{1}{T}\int_{0}^{\frac{T}{2}} e^{-in\omega t} dt = \frac{1}{T}\left(-\frac{1}{in\omega} e^{-in\omega t}\Big|_0^{\frac{T}{2}}\right)$$

$$= \frac{1}{T}\left(-\frac{1}{in\omega} e^{-in\pi} + \frac{1}{in\omega}\right) = \frac{1}{T}\frac{1}{in\omega}\left(1 - e^{-in\pi}\right) = \frac{1}{2i\pi n}\left(1 - e^{-in\pi}\right)$$

$$= \frac{1}{2\pi i n}(1 - \cos(\pi n) + i\sin(n\pi))$$

$$= \frac{1}{2i\pi n}(1-\cos(\pi n)) = \begin{cases} 0, & n = 2k \\ \frac{1}{i\pi n}, & n = 2k+1 \end{cases}$$

$$x(t) \simeq \sum_{n=-\infty}^{\infty} \frac{1}{2i\pi n}(1-\cos(\pi n)) e^{in\omega t} = \sum_{-\infty}^{\infty} \frac{1}{i\pi(2n+1)} e^{i(2n+1)\omega t}$$

$$\underline{\underline{= \frac{1}{i\pi}\sum_{-\infty}^{\infty} \frac{1}{2n+1} e^{(2n+1)i\omega t}}}$$

8) $x(t) = t \qquad T = 2\pi$

$$x(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos(n\omega t) + b_n \sin(n\omega t)$$

$$a_n = \frac{2}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} x(t)\cos(n\omega t)\,dt \qquad b_n = \frac{2}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} x(t)\sin(n\omega t)\,dt \qquad \omega = \frac{2\pi}{T}$$

$$x(t) = \sum_{-\infty}^{\infty} c_n e^{in\omega t} \qquad c_n = \frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} x(t)\, e^{-in\omega t}\,dt$$

$x(t) = t$

$a_n = 0$ fordi odde funksjon ??

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} t\,dt = \frac{1}{\pi}\left(\frac{1}{2}t^2\Big|_{-\pi}^{\pi}\right) = \frac{1}{\pi}\left(\frac{1}{2}\pi^2 - \frac{1}{2}\pi^2\right) \underline{\underline{= 0}}$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} t\sin(nt)\,dt = \frac{2}{\pi}\int_{0}^{\pi} t\sin(nt)\,dt = \frac{2}{\pi}\left(t\cdot\left(-\frac{1}{n}\cos(nt)\right)\Big|_0^{\pi} + \int_0^{\pi}\frac{1}{n}\cos(nt)\,dt\right)$$

$$= \frac{2}{\pi}\left(-\frac{\pi}{n}\cos(\pi n) + \left(\frac{1}{n^2}\sin(nt)\Big|_0^{\pi}\right)\right) = \frac{2}{\pi}\left(-\frac{\pi}{n}\cos(\pi n)\right) = -\frac{2}{n}\cos(\pi n)$$

$$c_n = \frac{1}{2\pi}\int_{-\pi}^{\pi} t\, e^{-int} = \frac{1}{2\pi}\left(\int_{-\pi}^{\pi} t\cos(nt)\,dt + i\int_{-\pi}^{\pi} t\sin(nt)\right)$$

$$= \frac{1}{2\pi}\left(-\cos(\pi n)\right) = -\frac{1}{2\pi}\cos(\pi n)$$

a)

$$x(t) = \begin{cases} \pi + t, & -\pi \le t < 0 \\ \pi - t, & 0 \le t < \pi \end{cases}$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} x(t)\cos(nt)\,dt = \frac{2}{\pi}\int_0^{\pi} (\pi - t)\cos(nt)\,dt$$

$$= \frac{2}{\pi}\left((\pi - t)\frac{1}{n}\sin(nt)\Big|_0^{\pi} + \frac{1}{n}\int_0^{\pi}\sin(nt)\,dt\right)$$

$$= \frac{2}{\pi}\left(-\frac{1}{n^2}\cos(nt)\Big|_0^{\pi}\right) = \frac{2}{\pi}\left(-\frac{1}{n^2}\cos(\pi n) + \frac{1}{n^2}\right)$$

$$= \frac{2}{\pi n^2}\left(1 - \cos(\pi n)\right) = \begin{cases} 0, & n \text{ er jævn} \\ \frac{4}{n^2} & n \text{ er odde} \end{cases}$$

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} x(t)\,dt = \frac{1}{\pi}\int_0^{\pi} \pi - t\,dt = \frac{1}{\pi}\left(\pi t - \frac{1}{2}t^2\Big|_0^{\pi}\right) = \frac{1}{\pi}\left(\pi^2 - \frac{1}{2}\pi^2\right)$$

$$= \pi - \frac{1}{2}\pi = \frac{1}{2}\pi$$

$$\underline{\underline{x(t) \sim \frac{\pi}{4} + \sum \frac{4}{(2n+1)^2}\cos((2n+1)t)}}$$