Stack 4

1 a)

$E[X] = 2 \qquad SD[X] = 1{,}5$

$E[Y] = 0 \qquad SD[Y] = 1$

$Z = X + Y$

$E[Z] = E[X + Y] = E[X] + E[Y] =$

$\mu_Z = 2 + 0 = 2{,}000$

1 b)

$Var[Z] = Var[X + Y] = Var[X] + Var[Y] + 2\underbrace{Cov[X, Y]}_{0}$

$= SD[X]^2 + SD[Y]^2$

$= 3{,}250$

1 c)

$g(Z) = 125Z^2 - 250Z$

$E[g(Z)] = 125E[Z^2] - 250E[Z]$

$Var[Z] = E[X^2] - E[X]^2$

$E[X^2] = Var[X] + E[X]^2$

$E[g(Z)] = 125(Var[Z] + E[Z]^2) - 250E[Z]$

2 a)

$P(A) = 0,3 \quad P(B) = 0,8 \quad P(A \cap B) = 0,28$
$P(a) = 0,7 \quad P(b) = 0,2$

$X = \begin{cases} 0, & a \\ 1, & A \end{cases} \qquad Y = \begin{cases} 0, & b \\ 1, & B \end{cases}$

$E[X] = \sum_x x \, p_X(x) = 0 \cdot 0,7 + 1 \cdot 0,3 = 0,300$

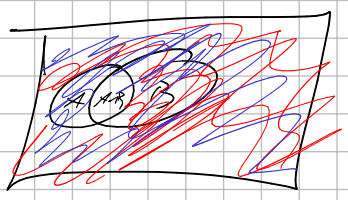
2 b)

$Var[X] = E[(X - \mu)^2] = (0 - 0,3)^2 \cdot 0,7 + (1 - 0,3)^2 \cdot 0,3$

$= 0,210$

2 c)

$E[XY] = \sum_x \sum_y x \, y \, p_X(x) p_Y(y)$



$P(a) + P(b) - P(a \cap b) + P(A \cap B) = 1$

$P(a) + P(b) + P(A \cap B) - 1 = P(a \cap b)$

$P(a \cap B) = 0,180$

$P(a|b) = \frac{P(a \cap b)}{P(b)} = \frac{0,180}{0,2} = 0,90$

$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{0,28}{0,8} = 0,350$

$E[XY] = a\,b\,P(a|b)P(b) + a\,B\,P(a|B) + A\,b\,P(A|b)P(b) + A\,B\,P(A|B)P(B)$

$= 0 + 0 + 0 + 0,350 \cdot 0,8$

$= 0,280$

2 d)

$Cov[X,Y] = E[X \cdot Y] - E[X] \cdot E[Y]$

$E[Y] = B \cdot P(B) = 0,8$

$Cov[X,Y] = 0,280 - 0,3 \cdot 0,8 = 0,04$

$Corr[X,Y] = \frac{Cov[X,Y]}{\sqrt{Var[X] \cdot Var[Y]}}$

$Var[Y] = (b - 0,8)^2 P(b) + (B - 0,8)^2 P(B)$

$= (-0,8)^2 \cdot 0,2 + (0,2)^2 \cdot 0,8$

$= 0,160$

$Corr[X,Y] = \frac{0,04}{\sqrt{0,210 \cdot 0,160}}$

$= \underline{0,218}$

3)

$f_{XY}(x,y) = \frac{2(2y+x)}{3}$

a)

$$E[X] = \int_0^1 \int_0^1 x \tfrac{2}{3}(2y+x)\,dx\,dy = \tfrac{2}{3}\int_0^1 \int_0^1 2xy + x^2\,dx\,dy = \tfrac{2}{3}\int_0^1 \left(x^2y + \tfrac{1}{3}x^3\Big|_0^1\right)dy$$

$$= \tfrac{2}{3}\int_0^1 y + \tfrac{1}{3}\,dy = \tfrac{2}{3}\left(\tfrac{1}{2}y^2 + \tfrac{1}{3}y\Big|_0^1\right) = \tfrac{2}{3}\left(\tfrac{1}{2}+\tfrac{1}{3}\right) = \tfrac{2}{3}\left(\tfrac{3}{6}+\tfrac{2}{6}\right) = \underline{\underline{\tfrac{10}{18}}}$$

$$Var[X] = E[X^2] - E[X]^2$$

$$E[X^2] = \int_0^1 \int_0^1 x^2 \tfrac{2}{3}(2y+x)\,dx\,dy = \tfrac{2}{3}\int_0^1 \int_0^1 2x^2y + x^3\,dx\,dy = \tfrac{2}{3}\int_0^1 \left(\tfrac{2}{3}x^3y + \tfrac{1}{4}x^4\Big|_0^1\right)dy$$

$$= \tfrac{2}{3}\int_0^1 \tfrac{2}{3}y + \tfrac{1}{4}\,dy = \tfrac{2}{3}\left(\tfrac{1}{3}y^2 + \tfrac{1}{4}y\Big|_0^1\right) = \tfrac{2}{3}\left(\tfrac{1}{3}+\tfrac{1}{4}\right) = \tfrac{2}{3}\left(\tfrac{4+3}{12}\right) = \tfrac{14}{36}$$

$$Var[X] = \tfrac{14}{36} - \left(\tfrac{10}{18}\right)^2 = \tfrac{14}{36} - \tfrac{100}{324} = \tfrac{126-100}{324} = \tfrac{26}{324} = \underline{\underline{\tfrac{13}{162}}}$$

$$E[Y] = \int_0^1 \int_0^1 y \tfrac{2}{3}(2y+x)\,dy\,dx = \tfrac{2}{3}\int_0^1 \int_0^1 2y^2 + xy\,dy\,dx = \tfrac{2}{3}\int_0^1 \left(\tfrac{2}{3}y^3 + \tfrac{1}{2}xy^2\Big|_0^1\right)dx$$

$$= \tfrac{2}{3}\int_0^1 \tfrac{2}{3} + \tfrac{1}{2}x\,dx = \tfrac{2}{3}\left(\tfrac{2}{3}x + \tfrac{1}{4}x^2\Big|_0^1\right) = \tfrac{2}{3}\left(\tfrac{2}{3}+\tfrac{1}{4}\right) = \tfrac{2}{3}\left(\tfrac{8+3}{12}\right) = \tfrac{22}{36} = \tfrac{11}{18}$$

$$Var[Y] = E[Y^2] - E[Y]^2$$

$$E[Y^2] = \int_0^1 \int_0^1 y^2 \tfrac{2}{3}(2y+x)\,dy\,dx = \tfrac{2}{3}\int_0^1 \int_0^1 2y^3 + xy^2\,dy\,dx = \tfrac{2}{3}\int_0^1 \left(\tfrac{1}{2}y^4 + \tfrac{1}{3}xy^3\Big|_0^1\right)dx$$

$$= \tfrac{2}{3}\int_0^1 \tfrac{1}{2} + \tfrac{1}{3}x\,dx = \tfrac{2}{3}\left(\tfrac{1}{2}x + \tfrac{1}{6}x^2\Big|_0^1\right) = \tfrac{2}{3}\left(\tfrac{4}{6}\right) = \underline{\underline{\tfrac{8}{18}}}$$

$$Var[Y] = \tfrac{8}{18} - \left(\tfrac{11}{18}\right)^2 = \tfrac{8\cdot 18 - 11^2}{18^2} = \tfrac{23}{324}$$

$$E[XY] = \int_0^1 \int_0^1 xy\,\tfrac{2}{3}(2y+x)\,dx\,dy = \tfrac{2}{3}\int_0^1 \int_0^1 2xy^2 + x^2 y\,dx\,dy$$

$$= \tfrac{2}{3}\int_0^1 \left(x^2y^2 + \tfrac{1}{3}x^3 y\Big|_0^1\right) dy = \tfrac{2}{3}\int_0^1 y^2 + \tfrac{1}{3}y\,dy = \tfrac{2}{3}\left(\tfrac{1}{3}y^3 + \tfrac{1}{6}y\Big|_0^1\right)$$

$$= \tfrac{2}{3}\left(\tfrac{1}{3} + \tfrac{1}{6}\right) = \tfrac{2}{3}\left(\tfrac{1}{2}\right) = \tfrac{2}{6} = \underline{\underline{\tfrac{1}{3}}}$$

$$Cov[X,Y] = E[XY] - E[X]\cdot E[Y]$$

$$= \tfrac{1}{3} - \tfrac{10}{18}\cdot\tfrac{11}{18} = \tfrac{1}{3} - \tfrac{110}{324} = \tfrac{108 - 110}{324} = \tfrac{-2}{324} = \underline{\underline{-\tfrac{1}{168}}}$$

$$Var[Y-X] = E[(Y-X)^2] - E[Y-X]^2$$

$$= E[Y^2] - 2E[XY] + E[X^2] - (E[Y] - E[X])^2$$

$$= E(Y^2) - 2E(XY) + E(X^2) - (E(Y)^2 - 2E(X)E(Y) + E(X)^2)$$

$$= E(Y^2) - 2E(XY) + E(X^2) - E(X)^2 + 2E(X)E(Y) - E(X)^2$$

$$= Var(Y) + Var(X) - 2E(XY) + 2E(X)E(Y)$$

$$= \tfrac{13}{162} + \tfrac{23}{324} - \tfrac{2}{3} + 2\cdot\tfrac{10}{18}\cdot\tfrac{11}{18}$$

$$= \tfrac{26+23}{324} - \tfrac{2}{3} + \tfrac{220}{18^2}$$

$$= \tfrac{49}{324} - \tfrac{2}{3} + \tfrac{220}{324}$$

$$= 0{,}16$$

$$Corr(X,Y) = \frac{Cov(X,Y)}{\sqrt{Var(X)\cdot Var(X)^2}} = \frac{-\frac{1}{168}}{\sqrt{\frac{13}{162}\cdot\frac{23}{324}^2}} =$$

$$= -0{,}08$$

4 a)

$E(X) = 1,800$

$Var(X) = 18 \cdot 0,1(1-0,1) = 18 \cdot 0,1 \cdot 0,9$

$P(\bar{N}) = 0,9$

$P(\text{Minst en stemmer}) = 1 - P(\text{Ingen stemmer})$

$= 1 - P(\bar{N})^{18}$

$= \underline{0,85}$

b) $P(\text{Nøyaktig en stemmer}) = \binom{18}{1} \cdot 0,1 \cdot (1-0,1)^{18-1}$

$= \binom{18}{1} \cdot 0,1 \cdot (0,9)^{17}$

$= 0,300$

$0,9^{17} = 0,167$

$P(\text{Nøyaktig en stemte} \mid \text{En har stemt}) = \frac{P(\text{Nøyaktig en stemte} \cap \text{En har stemt})}{P(\text{En har stemt})}$

5)

700 totalt

150 parallell 1 : A $P(A) = 0,75$

250 parallell 2 : B $P(B) = 0,85$

300 ingen forelesning : C $P(C) = 0,55$

a) $150 \cdot 0,75 = 112,50$

b) $150 \cdot 0,75 + 250 \cdot 0,85 + 300 \cdot 0,55$

$= 490$

6)

X og Y uafhængige $p = 0,2$, $n = 10$

a)

$$P(X = 3) = \binom{10}{3} (0,2)^3 \cdot (0,8)^7$$

$$= 0,201$$

b) $P(X \leq 3) = P(X = 1) + P(X = 2) + P(X = 3) + P(X = 0)$

$$= 0,8 > 9$$

c) $P(X + Y = 2) = P(X = 0 \cap Y = 2) + P(X = 1 \cap Y = 1) + P(X = 2 \cap Y = 0)$

$$=$$