1 - 1 Differensialligninger

1) $\lambda = 1$

$\dot{x}(\tau) = x(\tau)$

$x(\tau) = e^{\tau}$

2) $\frac{d}{d\tau} c e^{\tau} = \frac{d}{d\tau} e^{\tau} = e^{\tau}$

3) $\dot{x} = x \qquad x(0) = 2$

$x = \lambda e^{\tau}$

$x(0) = 2$

$\lambda e^{0} = 2$

$\lambda = 2$

$x = 2e^{\tau}$

4) $\dot{x} = \lambda x \qquad x(0) = x_0$

$x = e^{\lambda \tau}$

$x(0) = e^{\lambda \tau}$

$x_0 = e^{\lambda 0}$

$x_0 = 1$

$\underline{x(0) = 1}$

5) $x = e^{\lambda \tau} \quad \cancel{x(0) = 1}$ $\qquad \tau$ i timer

$x(\tfrac{1}{3}) = 2x(0)$

$e^{\frac{1}{3}\lambda} = 2$

$\frac{1}{3}\lambda = \ln 2$

$\lambda = \ln 8$

$\underline{\underline{x = e^{\ln 8 \, \tau}}}$

$\dot{x} = \lambda x$

$\int \frac{dx}{d\tau} \cdot \frac{1}{x} \, d\tau = \int \lambda \, d\tau$

$\ln x = \lambda \tau$

$x = e^{\lambda \tau}$

6) $x = e^{\lambda \tau} \quad x(0) = 1 \qquad \tau$ i minutter

$x(20) = 2x(0)$

$e^{20\lambda} = 2$

$20\lambda = \ln 2$

$\lambda = \frac{1}{20}\ln 2$

$$x = e^{\frac{1}{20}\ln 2\,\tau}$$

7) $x = e^{\lambda\tau}$ $x(0) = 1$ τ i sekunder

$x(1200) = 2x(0)$

$e^{1200\lambda} = 2$

$1200\lambda = \ln 2$

$$\lambda = \frac{1}{1200}\ln 2$$

$$x = e^{\frac{1}{1200}\ln 2\,\tau}$$

8) $x = e^{\lambda\tau}$ $x(0) = 1$ τ i 20min intervaller

$x(1) = 2x(0)$

$e^{\lambda} = 2$

$\lambda = \ln 2$

$x = e^{\ln 2\,\tau}$

$$x = 2^{\tau}$$

$\mathring{x}(t) = \lambda x(\tau)$

$\mathring{x}(\tau) - \lambda x(\tau) = 0$ $|\cdot e^{-\lambda\tau}$

$$e^{-\lambda\tau}(\mathring{x}(\tau) - \lambda x(\tau)) = \frac{\partial}{\partial\tau}(e^{-\lambda\tau}x(\tau))$$

9) $\mathring{x}(t) = -\frac{\ln 2}{5700}x(t)$ $x(0) = 1$ $x(\tau_1) = \frac{1}{10}$

$$x(t) = e^{-\frac{\ln 2}{5700}\tau}$$

$x(\tau_1) = \frac{1}{10}$

$$e^{-\frac{\ln 2}{5700}\tau_1} = \frac{1}{10}$$

$$-\frac{\ln 2}{5700}\tau_1 = \ln\frac{1}{10}$$

$$-\ln 2\,\tau_1 = 5700\ln\frac{1}{10}$$

$$\tau_1 = -\frac{5700\ln\frac{1}{10}}{\ln 2}$$

$$\underline{\underline{\tau_1 = 1{,}89 \text{ år}}}$$

1c) $x = e^{\lambda\tau}$ $\qquad x(\frac{51}{12}) = \frac{1}{2}x(0)$

$$e^{\frac{51}{12}\lambda} = \frac{1}{2}$$

$$\frac{51}{12}\lambda = \ln\frac{1}{2}$$

$$\underline{\underline{\lambda = \frac{12\ln\frac{1}{2}}{51}}}$$

11) $\dot{p}(h) = -\frac{mg}{Th}p(h)$

$$p(h) = e^{-\frac{mg}{Th}h}$$

12) $\dot{x} = \lambda x + \beta$

$$\dot{x} = \lambda(x + \frac{\beta}{\lambda})$$

$$\int \frac{\partial x}{\partial \tau} \cdot \frac{1}{(x+\frac{\beta}{\lambda})}\, d\tau = \int \lambda\, d\tau$$

$$\ln(x+\frac{\beta}{\lambda}) = \lambda\tau + C_1$$

$$x + \frac{\beta}{\lambda} = Ce^{\lambda\tau} \qquad C = e^{C_1}$$

$$\underline{\underline{x = Ce^{\lambda\tau} - \frac{\beta}{\lambda}}}$$

13) $\alpha = \lambda$ og $\beta = T_h$

14)

$$\dot{T}(\tau) = \alpha(T_h - T(\tau)) \qquad T(0) = 6^\circ \quad T(2) = 13^\circ \quad T_k = 20^\circ$$

$$\int \frac{dT}{d\tau} \cdot \frac{1}{T_h - T(\tau)} \, d\tau = \int \alpha \, d\tau$$

$$\ln(T_h - T(\tau)) = \alpha\tau + C_1$$

$$T_k - T(\tau) = Ce^{\alpha\tau} \qquad C = e^{C_1}$$

$$T(\tau) = T_h - Ce^{\alpha\tau}$$

$$= 20 - Ce^{\alpha\tau}$$

$$T(0) = 6$$

$$20 - Ce^{\alpha \cdot 0} = 6$$

$$20 - C = 6$$

$$C = 14$$

$$T(\tau) = 14e^{\alpha\tau} + 20$$

$$T(2) = 13$$

$$14e^{2\alpha} + 20 = 13$$

$$14e^{2\alpha} = -7$$

$$e^{2\alpha} = -\frac{1}{2}$$

$$2\alpha = \ln(-\tfrac{1}{2})$$

Utregningsfeil

$$\alpha = \frac{\ln(-\frac{1}{2})}{2} = \ln(\tfrac{1}{4})$$

$$T(\tau) = 14e^{\ln(\frac{1}{4})\tau} + 20$$

$$= \frac{7}{2}e^{\tau} + 20$$

15) $T(0) = 15$, $T(1) = 12$ $\quad T_h = ?$ $\quad \alpha = \ln(\tfrac{1}{4})$

$$\dot{T} = \alpha(T_h - T)$$

$$\ln(T_h - T) = \alpha\tau + C_1$$

$$T_h - T = Ce^{\alpha\tau}$$

$$T = T_h - Ce^{\alpha\tau}$$

$T(0) = 15$

$T_h - C = 15$

$T_h = 15 + C$

$T = 15 + C - Ce^{\alpha\tau}$

$T(1) = 12$

$15 + C - Ce^{\alpha} = 12$

$15 + C - C \cdot \frac{1}{4} = 12$

$C - \frac{1}{4}C = -3 \quad | \cdot 4$

$4C - C = -12$

$3C = -12$

$C = -4$

$T_h = 15 + C$

$\underline{\underline{T_h = 9}}$

17) $L i'(\tau) + R i(\tau) = 9$

$2 i' = 9 - R i$

$= R(\frac{9}{R} - i)$

$\int L \frac{di}{d\tau} \cdot \frac{1}{\frac{9}{R} - i} d\tau = \int R \, d\tau$

$L \ln(\frac{9}{R} - i) = R\tau + C_1$

$(\frac{9}{R} - i)e^{L} = Ce^{R\tau}$

$\frac{9}{R} - i = Ce^{R\tau - L}$

$i(\tau) = 9\frac{1}{R} - Ce^{R\tau - L}$

18) $RC v(\tau) + v(\tau) = 9V \qquad v(0) = 0$

$RC \dot{v} = 9 - v$

$\dot{v} = \frac{1}{RC}(9 - v)$

$\int \frac{dv}{d\tau} \cdot \frac{1}{9 - v} d\tau = \int \frac{1}{RC} d\tau$

$$\ln(q - v) = \frac{1}{RC}\tau + C_1$$

$$q - v = C_2 e^{\frac{1}{RC}\tau} \qquad C_2 = e^{C_1}$$

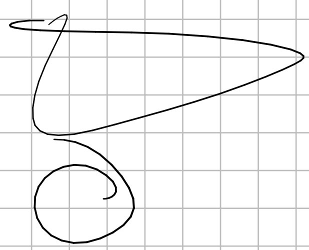
$$v = C e^{\frac{1}{RC}\tau} + q$$

$$v(0) = 0$$

$$C + q = 0$$

$$C = -q$$

$$\underline{\underline{v(\tau) = -q e^{\frac{1}{RC}\tau} + q}}$$



20)

$$\dot{x}(\tau) = \lambda x(\tau) + \beta \qquad x(0) = x_0$$

$$x = \left(x_0 + \frac{\beta}{\lambda}\right) e^{\lambda\tau} - \frac{\beta}{\lambda}$$

$$RC\,\dot{v}(\tau) + v(\tau) = \sin(\omega_0 \tau)$$

$$\dot{x}(\tau) + \lambda x(\tau) = f(\tau) \qquad x(\tau) = c(\tau) e^{-\lambda\tau}$$

$$\dot{x}(\tau) = \dot{c}(\tau) e^{-\lambda\tau} - \lambda c(\tau) e^{-\lambda\tau}$$

$$\dot{c} e^{-\lambda\tau} - \lambda c(\tau) e^{-\lambda\tau} + \lambda c(\tau) e^{-\lambda\tau} = f(\tau)$$

$$\dot{c} e^{-\lambda\tau} = f(\tau)$$

$$\dot{c}(\tau) = f(\tau) e^{\lambda\tau}$$

$$c(\tau) = \int_0^\tau f(s) e^{\lambda s}\, ds \qquad \Big|\cdot e^{-\lambda\tau}$$

$$c(\tau) e^{-\lambda\tau} = e^{-\lambda\tau} \int_0^\tau f(s) e^{\lambda s}\, ds$$

$$x(\tau) = c(\tau) e^{-\lambda\tau} = e^{-\lambda\tau} \int_0^\tau f(s)\, e^{\lambda s}\, ds$$

$$x(0) = x_0$$

$$x(\tau) = x_0 e^{-\lambda\tau} + e^{-\lambda\tau} \int_0^\tau f(s) e^{\lambda s}\, ds$$

$$= x_0 e^{-\lambda\tau} + \int_0^\tau f(s) e^{\lambda(s-\tau)}\, ds$$

$$= x_h(\tau) + x_p(\tau) \qquad x_h = \text{homogen løsning} \qquad x_p = \text{partikulær}$$

2 2) $RC\,\dot{v}(t) + v(t) = \sin(\omega_0 t) \qquad v(0) = v_0$

$$\dot{v}(t) + \frac{1}{RC} v(t) = \frac{1}{RC} \sin(\omega_0 t)$$

$$\underline{\underline{v(t) = v_0 e^{-\frac{1}{RC}t} + \int_0^t \frac{1}{RC} \sin(\omega_0 t)\, e^{\lambda(s-t)}\, ds}}$$