

$$\ddot{x}(t) + 3\dot{x}(t) + 2x(t) = 0$$

$$\lambda^2 + 3\lambda + 2 = (\lambda + 1)(\lambda + 2)$$

$$\left(\frac{d}{dt} + 1\right)\left(\frac{d}{dt} + 2\right)x(t) = 0$$

$$\left(\frac{d}{dt} + \frac{d}{dt} + 2\frac{d}{dt} + \frac{d}{dt} + 2\right)x(t) = 0$$

$$\frac{d^2}{dt^2}x(t) + 3\frac{d}{dt}x(t) + 2x(t) = 0$$

hyperabel

$$y(t) = \left(\frac{d}{dt} + 2\right)x(t) = \dot{x}(t) + 2x(t)$$

$$\left(\frac{d}{dt} + 1\right)y(t) = 0$$

$$\dot{y}(t) + y(t) = 0$$

$$y(t) = c_1 e^{-t}$$

$$\dot{x}(t) + 2x(t) = c_1 e^{-t}$$

$$\dot{x} = -2x + c_1 e^{-t} \quad | \circ$$

$$x = c_2 e^{-2t} + c_1 \int_0^t e^{2(s-t)} e^{-s} ds \quad \geq \leq$$

$$= c_2 e^{-2t} + c_1 \int_0^t e^{s-2t} ds$$

$$= c_2 e^{-2t} - c_1 e^{-t}$$

$$x = c_2 e^{-2t} - c_1 e^{-t}$$

$$a_2 \ddot{x}(t) + a_1 \dot{x}(t) + a_0 x(t) = 0$$

$$x(t) = c_1 e^{\lambda_1 t} + c_2 e^{\lambda_2 t}$$

Grunde

$$\dot{x}(t) + \lambda x(t) = f(t) \quad x(0) = x_0$$

Ansatz $x(t) = c(t)e^{-\lambda t}$

$$(c(t)e^{-\lambda t})' = c'(t)e^{-\lambda t} - \lambda c(t)e^{-\lambda t}$$

$$\underbrace{c'(t)e^{-\lambda t} - \lambda c(t)e^{-\lambda t}}_{\dot{x}(t)} + \underbrace{\lambda c(t)e^{-\lambda t}}_{x(t)} = f(t)$$

$$c'(t)e^{-\lambda t} = f(t) \quad | \cdot e^{\lambda t}$$

$$c'(t) = f(t)e^{\lambda t} \quad | \int$$

$$c(t) = \int_0^t f(s)e^{\lambda s} ds$$

$$x(t) = c(t)e^{-\lambda t} = e^{-\lambda t} \int_0^t f(s)e^{\lambda s} ds$$

$$\dot{x}(t) + \lambda x(t) = f(t) \quad x(0) = 0 \Rightarrow x(0) = x_0$$

$$(x_0 e^{-\lambda t})' = -\lambda x_0 e^{-\lambda t}$$

$$-\lambda x_0 e^{\lambda t} + \lambda x_0 e^{\lambda t} = f(0)$$

$$f(0) = 0$$

$$x(t) = x_0 e^{-\lambda t} + e^{-\lambda t} \int_0^t f(s)e^{\lambda s} ds$$

$$x(t) = x_0 e^{-\lambda t} + \int_0^t f(s)e^{\lambda(t-s)} ds$$

$$x_0 e^{-\lambda t} = x_h \quad \text{homogen Lösung}$$

$$\int_0^t f(s)e^{\lambda(t-s)} ds = x_p \quad \text{partikuläre Lösung}$$

$$x(t) = x_h(t) + x_p(t)$$

$$\ddot{x}(t) + 3\dot{x}(t) + 2x(t) = 0$$

$$\lambda^2 + 3\lambda + 2 = (\lambda + 1)(\lambda + 2)$$

$$\left(\frac{d}{dt} + 1\right)\left(\frac{d}{dt} + 2\right)x(t) = 0$$

$$y(t) = \left(\frac{d}{dt} + 2\right)x(t) = \dot{x}(t) + 2x(t)$$

$$0 = \left(\frac{d}{dt} + 1\right)y(t) = \dot{y}(t) + y(t)$$

$$y(t) = c_1 e^{-t}$$

$$\dot{x}(t) + 2x(t) = c_1 e^{-t}$$

$$\frac{c_1 e^{-t}}{c_1 e^{-s}} = f(t)$$

$$x(0) = c_2 e^{-2 \cdot 0}$$

$$x(t) = c_2 e^{-2t} + c_1 \int_0^t e^{-2(t-s)} e^{-s} ds$$

$$= c_2 e^{-2t} + c_1 \int_0^t e^{2(s-t)} e^{-s} ds$$

$$= c_2 e^{-2t} + c_1 \int_0^t e^{s-2t} ds$$

$$= c_2 e^{-2t} - c_1 e^{-t} ds$$

c_1 & c_2 er vilkårlige konstanter

$$x(t) = c_1 e^{-t} + c_2 e^{-2t}$$

$$a_2 \ddot{x} + a_1 \dot{x} + a_0 x = 0$$

$$x(t) = c_1 e^{\lambda_1 t} + c_2 e^{\lambda_2 t}$$

λ_1 & λ_2 røtter til karakteristisk polynom

$$a_2 \lambda^2 + a_1 \lambda + a_0 = a_2 (\lambda - \lambda_1)(\lambda - \lambda_2)$$

$$\ddot{x} + a_1 \dot{x} + a_0 x = 0$$

$$\delta = -\frac{a_1}{2} \quad \omega_0 = \sqrt{\frac{a_1^2 - 4a_0}{2}}$$

~~2. Fall~~

1. $a_1^2 - 4a_0 > 0 \Rightarrow$ 2 reelle Werten

$$\lambda = \delta \pm \omega_0$$

$$x(\tau) = C_1 e^{(\delta + \omega_0)\tau} + C_2 e^{(\delta - \omega_0)\tau}$$

$$= e^{\delta\tau} (C_1 e^{\omega_0\tau} + C_2 e^{-\omega_0\tau})$$

2. $a_1^2 - 4a_0 = 0 \Rightarrow$ doppelte reelle $\lambda = \delta$

$$x(\tau) = C_1 e^{\delta\tau} + C_2 \tau e^{\delta\tau}$$

3. $a_1^2 - 4a_0 < 0$

$$\lambda = \frac{-a_1 \pm \sqrt{a_1^2 - 4a_0}}{2} = -\frac{a_1}{2} \pm i \frac{\sqrt{4a_0 - a_1^2}}{2} = \delta \pm i\omega_0$$

$$x(\tau) = C_1 e^{(\delta + i\omega_0)\tau} + C_2 e^{(\delta - i\omega_0)\tau}$$

$$= e^{\delta\tau} (C_1 e^{i\omega_0\tau} + C_2 e^{-i\omega_0\tau}) \quad d_1 = C_1 + C_2 \quad d_2 = i(C_1 - C_2)$$

$$= e^{\delta\tau} (d_1 \cos(\omega_0\tau) + d_2 \sin(\omega_0\tau))$$

2) $a_1 = a_0 = 1$

$$\delta = -\frac{a_1}{2} = -\frac{1}{2} \quad \omega_0 = \sqrt{\frac{a_1^2 - 4a_0}{2}} = \sqrt{\frac{1 - 4}{2}} = \frac{\sqrt{3}}{2} \quad ?$$

$$x(\tau) = e^{-\frac{1}{2}\tau} \left(d_1 \cos\left(\frac{\sqrt{3}}{2}\tau\right) + d_2 \sin\left(\frac{\sqrt{3}}{2}\tau\right) \right)$$

$$3) \quad a_1 = 2 \quad a_0 = 1$$

$$\delta = -\frac{2}{2} = -1$$

$$\omega_0 = \left| \frac{\sqrt{2^2 - 4}}{2} \right| = 0$$

$$X(\tau) = C_1 e^{\tau} + C_2 \tau e^{\tau}$$

$$4) \quad a_1 = 0 \quad a_0 = 1$$

$$\delta = -\frac{0}{2} = 0$$

$$\omega = \left| \frac{\sqrt{0^2 - 4a_0}}{2} \right| \Rightarrow i\omega_0 = \left| \frac{\sqrt{4}}{2} \cdot i \right| = 1$$

$$x = d_1 \cos(\tau) + d_2 \sin(\tau)$$

$$5) \quad a_1 = 3, \quad a_0 = 1$$

$$\delta = -\frac{3}{2} \quad \omega_0 = \left| \frac{\sqrt{9 - 4}}{2} \right| = \frac{\sqrt{5}}{2}$$

$$x = e^{-\frac{3}{2}\tau} \left(C_1 e^{\frac{\sqrt{5}}{2}\tau} + C_2 e^{-\frac{\sqrt{5}}{2}\tau} \right)$$

$$\ddot{x} + \frac{k}{m}x = 0$$

$$x(t) = c_1 \cos\left(\sqrt{\frac{k}{m}}t\right) + c_2 \sin\left(\sqrt{\frac{k}{m}}t\right)$$

$$b) \quad x(0) = 1, \quad \dot{x}(0) = 0$$

$$\dot{x}(t) = -\sqrt{\frac{k}{m}}c_1 \sin\left(\sqrt{\frac{k}{m}}t\right) + \sqrt{\frac{k}{m}}c_2 \cos\left(\sqrt{\frac{k}{m}}t\right)$$

$$\dot{x}(0) = c_2 \sqrt{\frac{k}{m}} = 0$$

$$c_2 = 0$$

$$x(0) = 1$$

$$c_1 = 1$$

$$\underline{x(t) = \cos\left(\sqrt{\frac{k}{m}}t\right)}$$

$$\dot{x}(t) + \lambda x = f(t)$$

$$x(t) = c(t)e^{-\lambda t}$$

$$\dot{x}(t) = \dot{c}(t)e^{-\lambda t} - c(t) \cdot \lambda e^{-\lambda t}$$

$$\dot{c}(t)e^{-\lambda t} - \lambda c(t)e^{-\lambda t} + \lambda c(t)e^{-\lambda t} = f(t)$$

$$\dot{c}(t)e^{-\lambda t} = f(t) \quad | \cdot e^{\lambda t}$$

$$\dot{c}(t) = f(t)e^{\lambda t}$$

$$(f(t) \cdot g(t))' = f'(t) \cdot g(t) + f(t) \cdot g'(t)$$

$$x(t) = c(t)e^{-\lambda t}$$

$$= c_1 e^{-\lambda t} \int_0^t f(s) e^{\lambda s} ds$$

$$= c_1 \int_0^t f(s) e^{\lambda(s-t)} ds$$



= partikular

$$= x_h + x_p$$

$$= c_1 e^{\lambda_1 t} + c_2 e^{\lambda_2 t}$$

$$\ddot{x}(t) + 2x(t) = c_1 e^{-t} \quad c_1 e^{-t} = f(t)$$

$$\ddot{x}(t) + 2x(t) = f(t)$$

$$x(t) = c_1 \int_0^t f(s) e^{-s} ds$$

$$\ddot{x} + \frac{k}{m}x = 0$$

$$x(t) = c_1 \cos\left(\sqrt{\frac{k}{m}}t\right) + c_2 \sin\left(\sqrt{\frac{k}{m}}t\right)$$

$$\dot{x}(t) = -c_1 \sqrt{\frac{k}{m}} \sin\left(\sqrt{\frac{k}{m}}t\right) + c_2 \sqrt{\frac{k}{m}} \cos\left(\sqrt{\frac{k}{m}}t\right)$$

7) $x(0) = 0, \quad \dot{x}(0) = 1$

$$x(0) = 0$$

$$c_1 = 0$$

$$\dot{x}(0) = 1$$

$$c_2 \sqrt{\frac{k}{m}} = 1$$

$$c_2 = \frac{1}{\sqrt{\frac{k}{m}}}$$

$$x(t) = \frac{1}{\sqrt{\frac{k}{m}}} \sin\left(\sqrt{\frac{k}{m}}t\right)$$

$$\ddot{x}(t) + \gamma \dot{x}(t) = C_1 e^{-t} \\ = 0$$

$$f(t) = C_1 e^{-t} \\ f(s) = C_1 e^{-s}$$

Formel

$$x_h(t) = \int_0^t \underbrace{f(s)} e^{-\lambda(t-s)} ds$$

$$x(t) = \int_0^t C_1 e^{-s} \cdot e^{-2(t-s)} ds$$

$$= \int_0^t C_1 e^{-s} \cdot e^{-2t+2s} ds = \int_0^t C_1 e^{s-2t} ds$$

$$8) \quad \ddot{x} + \frac{k}{m} x = 0$$

$$x(t) = C_1 \cos\left(\sqrt{\frac{k}{m}} t\right) + C_2 \sin\left(\sqrt{\frac{k}{m}} t\right)$$

$$\dot{x}(t) = -C_1 \sqrt{\frac{k}{m}} \sin\left(\sqrt{\frac{k}{m}} t\right) + C_2 \sqrt{\frac{k}{m}} \cos\left(\sqrt{\frac{k}{m}} t\right)$$

$$x(0) = x_0 \quad \dot{x}(0) = v_0$$

$$x(0) = x_0$$

$$C_1 = x_0$$

$$\dot{x}(0) = v_0$$

$$C_2 \sqrt{\frac{k}{m}} = v_0$$

$$C_2 = \frac{v_0}{\sqrt{\frac{k}{m}}}$$

$$x(t) = x_0 \cos\left(\sqrt{\frac{k}{m}} t\right) + \frac{v_0}{\sqrt{\frac{k}{m}}} \sin\left(\sqrt{\frac{k}{m}} t\right)$$

$$\ddot{x}(t) + b \dot{x}(t) = 0 \quad | \cdot e^{dt}$$

$$\dot{x} e^{dt} + b x e^{dt} = 0$$

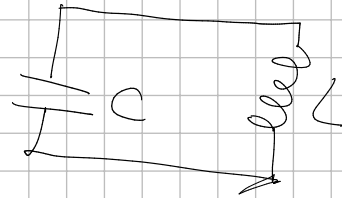
$$\int \frac{d}{dt} (e^{dt} x(t)) dt = 0 \quad dt$$

$$e^{dt} x(t) = C \quad | \cdot e^{-dt}$$

$$x(t) = C e^{-dt}$$

$$L \ddot{i}(t) + \frac{1}{C} \int_0^t i(s) ds = 0$$

$$L \ddot{i}(t) + \frac{1}{C} i(t) = 0$$



$$i(t) = c_1 \cos\left(\frac{1}{\sqrt{LC}} t\right) + c_2 \sin\left(\frac{1}{\sqrt{LC}} t\right)$$

$$\dot{i}(t) = -c_1 \frac{1}{\sqrt{LC}} \sin\left(\frac{1}{\sqrt{LC}} t\right) + c_2 \frac{1}{\sqrt{LC}} \cos\left(\frac{1}{\sqrt{LC}} t\right)$$

$$u(t) = u(0) + \frac{1}{C} \int_0^t i(s) ds$$

$$a) \quad i(0) = 1 \quad \dot{i}(0) = 0$$

$$i(0) = 1$$

$$c_1 = 1$$

$$\dot{i}(0) = 0$$

$$c_2 \frac{1}{\sqrt{LC}} = 0$$

$$c_2 = 0$$

$$\underline{i(t) = -\frac{1}{\sqrt{LC}} \cos\left(\frac{1}{\sqrt{LC}} t\right)}$$

$$10) \quad i(t) = c_1 \cos\left(\frac{1}{\sqrt{2LC}} t\right) + c_2 \sin\left(\frac{1}{\sqrt{2LC}} t\right)$$

$$\dot{i}(t) = -c_1 \frac{1}{\sqrt{2LC}} \sin\left(\frac{1}{\sqrt{2LC}} t\right) + c_2 \frac{1}{\sqrt{2LC}} \cos\left(\frac{1}{\sqrt{2LC}} t\right)$$

$$i(0) = 0$$

$$\dot{i}(0) = 1$$

$$i(0) = 0$$

$$c_2 \frac{1}{\sqrt{2LC}} = 1$$

$$c_1 = 0$$

$$c_2 = \sqrt{2LC}$$

$$\underline{i(t) = \cos\left(\frac{1}{\sqrt{2LC}} t\right)}$$

11)

$$i(0) = i_0$$

$$\dot{i}(0) = \omega_0$$

$$i(0) = i_0$$

$$c_1 = i_0$$

$$\dot{i}(0) = \omega_0$$

$$c_2 \frac{1}{\sqrt{2LC}} = \omega_0$$

$$c_2 = \omega_0 \sqrt{2LC}$$

$$i(t) = i_0 \cos\left(\frac{1}{\sqrt{2LC}} t\right) + \omega_0 \sqrt{2LC} \sin\left(\frac{1}{\sqrt{2LC}} t\right)$$

$$\ddot{x} + a_1 \dot{x}(t) + a_0 x(t) = f(t) \quad x(0) = x_0 \quad \dot{x}(0) = v_0$$

Begrenzer ansatz $f(t) = e^{i\omega t}$

$$\ddot{x} + b \dot{x} + f(t) \quad x(0) = x_0$$

$$x = x_0 e^{-b\tau} + \int_0^\tau f(s) e^{-\lambda(\tau-s)} ds$$

$$\downarrow$$

$$= x_h(\tau) + x_p(\tau)$$

Homogen Lösung

$$\ddot{x}_h(t) + a_1 \dot{x}_h(t) + a_0 x_h(t) = 0$$

Daher $f(t) = e^{i\omega t}$

$$x_p = H(\omega) e^{i\omega t}$$

$H \rightarrow$ Frequenzgang als Challenge

1 2)

$$\ddot{x} + \dot{x} + x = e^{i\omega t} \quad x(0) = \dot{x}(0) = 0$$

homogen Lösung

$$\delta = -\frac{1}{2} \quad \omega_d = \left| \frac{\sqrt{1^2 - 4}}{2} \right| \Rightarrow i \omega_d = \left| \frac{\sqrt{3}}{2} \right| \cdot i$$

$$x_h = e^{-\frac{1}{2}\tau} \left(d_1 \cos\left(\frac{\sqrt{3}}{2}\tau\right) + d_2 \sin\left(\frac{\sqrt{3}}{2}\tau\right) \right)$$

$$f(t) = e^{i\omega t}$$

$$x_p = H(\omega) e^{i\omega t}$$

$$\dot{x}_p = i \omega H(\omega) e^{i\omega t}$$

$$\ddot{x}_p = -\omega^2 H(\omega) e^{i\omega t}$$

$$-\omega^2 H(\omega) e^{i\omega t} + i \omega H(\omega) e^{i\omega t} + H(\omega) e^{i\omega t} = e^{i\omega t}$$

$$-\omega^2 H(\omega) + i \omega H(\omega) + H(\omega) = 1$$

$$H(\omega)(-\omega^2 + i\omega + 1) = 1$$

$$H(\omega) = \frac{1}{-\omega^2 + i\omega + 1}$$

$$x(\tau) = e^{-\frac{1}{2}\tau} \left(d_1 \cos\left(\frac{\sqrt{3}}{2}\tau\right) + d_2 \sin\left(\frac{\sqrt{3}}{2}\tau\right) \right) + \frac{e^{i\omega\tau}}{(-\omega^2 + i\omega + 1)}$$

$$x(0) = 0$$

$$d_1 + \frac{1}{-\omega^2 + i\omega + 1} = 0$$

$$d_1 = -\frac{1}{-\omega^2 + i\omega + 1} = \frac{1}{\omega^2 - i\omega - 1}$$

$$\begin{aligned} \dot{x}(\tau) = & -\frac{1}{2} e^{-\frac{1}{2}\tau} \left(d_1 \cos\left(\frac{\sqrt{3}}{2}\tau\right) + d_2 \sin\left(\frac{\sqrt{3}}{2}\tau\right) \right) \\ & + \left(-\frac{\sqrt{3}}{2} d_1 \sin\left(\frac{\sqrt{3}}{2}\tau\right) + \frac{\sqrt{3}}{2} d_2 \cos\left(\frac{\sqrt{3}}{2}\tau\right) \right) \\ & + \frac{e^{i\omega\tau}}{-\omega^2 + i\omega + 1} \end{aligned}$$

$$\dot{x}(0) = 0$$

$$d_1 + \frac{\sqrt{3}}{2} d_2 + \frac{1}{-\omega^2 + i\omega + 1} = 0$$

$$\frac{\sqrt{3}}{2} d_2 = 0$$

$$d_2 = 0$$

$$\underline{\underline{x(\tau) = e^{-\frac{1}{2}\tau} \left(\frac{1}{\omega^2 - i\omega - 1} \cos\left(\frac{\sqrt{3}}{2}\tau\right) \right) + \frac{e^{i\omega\tau}}{-\omega^2 - i\omega + 1}}}$$

$$\ddot{x}(t) + \dot{x}(t) + x(t) = \cos(\omega t)$$

Homogen Lösung

$$a_1 = 1 \quad a_0 = 1$$

$$\delta = \frac{-1}{2} \quad \omega = \sqrt{\frac{1-4}{2}} \Rightarrow i \omega = \frac{\sqrt{3}}{2} \cdot i$$

$$x_h(t) = e^{-\frac{1}{2}t} \left(d_1 \cos\left(\frac{\sqrt{3}}{2}t\right) + d_2 \sin\left(\frac{\sqrt{3}}{2}t\right) \right)$$

$$f(t) = \cos(\omega t)$$

$$x_p(t) = A t \cos(\omega t) + B t \sin(\omega t)$$

$$\dot{x}_p(t) = -A \omega t \sin(\omega t) + B \omega t \cos(\omega t)$$

$$\ddot{x}_p(t) = -A \omega^2 t \cos(\omega t) - B \omega^2 t \sin(\omega t)$$

$$\begin{aligned} x_p(t) &= -A \omega^2 t \cos(\omega t) - B \omega^2 t \sin(\omega t) \\ &\quad - A \omega t \sin(\omega t) + B \omega t \cos(\omega t) \\ &\quad + A t \cos(\omega t) + B t \sin(\omega t) \\ &= (-A \omega^2 t + B \omega t + A t) \cos(\omega t) \\ &\quad + (-B \omega^2 t - A \omega t + B t) \sin(\omega t) \end{aligned}$$

$$\begin{aligned} x(t) &= e^{-\frac{1}{2}t} \left(d_1 \cos\left(\frac{\sqrt{3}}{2}t\right) + d_2 \sin\left(\frac{\sqrt{3}}{2}t\right) \right) \\ &\quad + (-A \omega^2 + B \omega + A) t \cos(\omega t) \\ &\quad + (-B \omega^2 - A \omega + B) t \sin(\omega t) \end{aligned}$$

$$x(0) = \dot{x}(0) = 0$$

$$x(0) = 0$$

$$d_1 = 0$$

$$\begin{aligned} x(t) &= d_2 e^{-\frac{1}{2}t} \sin\left(\frac{\sqrt{3}}{2}t\right) \\ &\quad + (-A \omega^2 + B \omega + A) t \cos(\omega t) \\ &\quad + (-B \omega^2 - A \omega + B) t \sin(\omega t) \end{aligned}$$

$$\ddot{x}(\tau) = -\frac{1}{2}d_2 e^{-\frac{1}{2}\tau} \sin\left(\frac{\sqrt{3}}{2}\tau\right) + \frac{\sqrt{3}}{2}d_2 e^{-\frac{1}{2}\tau} \cos\left(\frac{\sqrt{3}}{2}\tau\right)$$

$$+(-A\omega^2 + B\omega + A)\cos(\omega\tau) - (A\omega^2 + B\omega + A)\tau \sin(\omega\tau)$$

$$+(-B\omega^2 - A\omega + B)\sin(\omega\tau) + (-B\omega^2 - A\omega + B)\tau \cos(\omega\tau)$$

$$1) 3) m\ddot{x} + \mu\dot{x} + kx = mg$$

$$\ddot{x} + \frac{\mu}{m}\dot{x} + \frac{k}{m}x = g$$

$$f(\tau) = g$$

$$x_h = H(\omega)e^{i\omega\tau} \quad \omega = 0$$

$$x_h = H$$

$$= H(0)$$

$$\dot{x}_h = 0$$

$$= H$$

$$\ddot{x}_h = 0$$

$$0 + 0 + \frac{k}{m}H = g$$

$$H = \frac{mg}{k}$$

$$\underline{x_h = \frac{mg}{k}}$$

$$1) 4)$$

$$\ddot{x} + \mu\dot{x} + x = g$$

$$x(0) = \dot{x}(0) = 0$$

$$x_h =$$

$$\delta = -\frac{\mu}{2} \quad \omega = \sqrt{\frac{4 - \mu^2}{2}}$$

$$x_h = e^{-\frac{\mu}{2}\tau} (C_1 e^{\omega\tau} + C_2 e^{-\omega\tau})$$

$$x_h = H(\omega) \quad \omega = 0$$

$$x_h = g$$

$$\underline{x(\tau) = e^{-\frac{\mu}{2}\tau} (C_1 e^{\omega\tau} + C_2 e^{-\omega\tau})}$$

$$1) 5) \underline{\mu = 2}$$

Übung 1: PID-Regulator

Proportional: $u_p(t) = K_p(r - x)$

Integral: $u_i(t) = K_i \int_0^t r - x$

Derivative: $u_d(t) = K_d(\dot{r} - \dot{x})$

$$u(t) = u_p(t) + u_i(t) + u_d(t)$$

$$r = 1$$

1) $\dot{x} + b x = u_p$

$$\dot{x} + b x = k(1 - x)$$

$$\dot{x} + b x = k - kx$$

$$\dot{x} + b x + k x = k$$

$$\dot{x} + (b+k)x = k \quad f(t) = k \quad d = (b+k)$$

$$x = x_0 e^{-(b+k)t} + \int_0^t k e^{-(b+k)(t-\tau)} d\tau$$

$$= x_0 e^{-(b+k)t} - \frac{k}{b+k}$$



