Skriftlig 2

1.1)

$$P(X \le 2) = 0{,}05 + 0{,}10 + 0{,}25$$

$$= \underline{\underline{0{,}40}}$$

1.2)

```python
# Antall realisasjoner man skal bruke
n = 1000
# Simuler realisasjoner av X ved å kalle på simX-funksjonen i cellen over
simulerte_X = simX(n)
# Approksimer sannsynligheten
count = 0
for i in range(n):
    if simulerte_X[i] == 0 or simulerte_X[i] == 1 or simulerte_X[i] == 2:
        count += 1

P_X_le_2 = count/n
# Skriv ut resultatet
print("Approksimert sannsynlighet: ",P_X_le_2)
```

✓ 0.0s

```
Approksimert sannsynlighet:  0.402
```

2.1)

$$\mu = E[X] = \sum_x x\, f(x) = \underline{\underline{2{,}650}}$$

2.2)

$$Var[X] = \sum_x (x-\mu)^2 f(x) = \underline{\underline{1{,}3275}}$$

$$SD[X] = \sqrt{Var[X]} = \underline{\underline{1{,}15217}}$$

2.3)

```python
def approx_P_X(x):
    count = 0
    for i in range(n):
        if simulerte_X[i] == x:
            count += 1

    return count / n

E_x = 0
for i in range(len(f_x)):
    E_x += approx_P_X(i) * f_x[i]

print("E_X: ", E_x)

Var_x = 0

for i in range(len(f_x)):
    Var_x += (approx_P_X(i) - E_x)**2 * f_x[i]

SD_x = np.sqrt(Var_x)
print("Var_x: ", SD_x)
```

✓ 0.0s

```
E_X:  0.26405
Var_x:  0.1374508912302863
```

3 a)

$$F_X(x) = 1 - e^{-\frac{x^2}{\alpha}} \quad , \quad x \geq 0$$

$$f_X(x) = \left(1 - e^{-\frac{x^2}{\alpha}}\right)' = -\left(-\frac{2x}{\alpha}\right) e^{-\frac{x^2}{\alpha}} = \underline{\underline{\frac{2x}{\alpha} e^{-\frac{x^2}{\alpha}}}}$$

3 b)

$$F_X(x) = u$$

$$1 - e^{-\frac{x^2}{\alpha}} = u$$

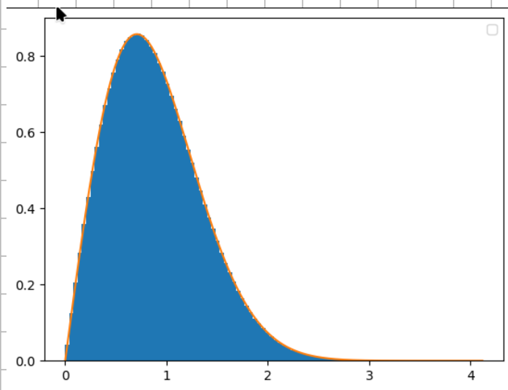
$$e^{-\frac{x^2}{\alpha}} = 1 - u$$

$$-\frac{x^2}{\alpha} = \ln(1-u)$$

$$-x^2 = \alpha \ln(1-u)$$

$$x = \sqrt{-\alpha \ln(1-u)}$$

```python
import numpy as np
import matplotlib.pyplot as plt
def generateX(n,alpha):
    u = np.random.uniform(size=n) #array med n elementer.
    x = np.sqrt(-alpha * np.log(1 - u))
    # fyll inn formelen du fant for x
    return x


# Sett antall realisasjoner og verdien til alpha
n = 10000000
alpha = 1

# simuler realisasjoner av X
simulerte_X = generateX(n, alpha)

# Lag sannsynlighetshistogram for de simulerte verdiene,
# vi spesifiserer antall intervaller ved å sette "bins=100"
plt.hist(simulerte_X, density=True,bins=100) #density=TrueA
#gjør at vi får et sannsynlighetshistogram
# Angi navn på aksene
plt.xlabel("")
plt.ylabel("")

# Regn ut og plott sannsynlighetstettheten til X på samme plott
x = np.linspace(0, np.max(simulerte_X), len(simulerte_X) + 1)
fx = ((2 * x) / alpha) * np.exp(-x**2 / alpha)

plt.plot(x, fx)

# Avslutt med å generere alle elementene du har plottet
plt.legend()
plt.show()
```



3 c)

$$P(Y \geq 1 \mid Y \geq 0{,}75)$$

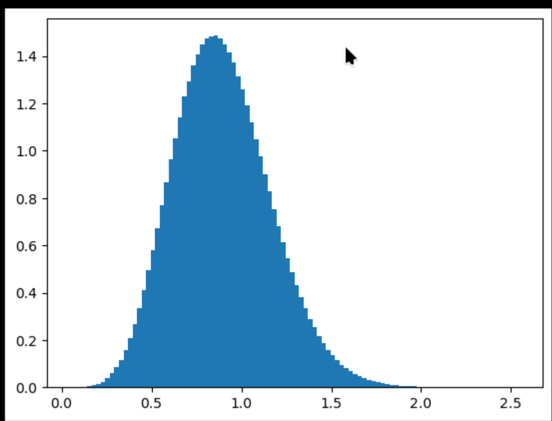
$$= \frac{P(Y \geq 1 \cap Y \geq 0{,}75)}{P(Y \geq 0{,}75)}$$

$$= \frac{P(Y \geq 1)}{P(Y \geq 0{,}75)}$$

```python
import numpy as np
import matplotlib.pyplot as plt

def generateY(n,alpha):
    u = np.random.uniform(size=n) #array med n elementer.
    y = np.sqrt(-alpha * np.log(1 - u))
    # fyll inn formelen du fant for x
    return y

n = 10000000

#Alpha verdiene for de ulike komponentene
alpha = np.array([1, 1, 1.2, 1.2, 1.2])

#Matrise med kjøringer for hvert komponent
simulerte_Y = np.empty((len(alpha), n))

for i in range(len(alpha)):
    simulerte_Y[i] = generateY(n, alpha[i])

# Medianen av matrisen av komponentene, medianen vil bli levetiden
# ettersom det trengs 3 elementer for at instrumentet skal funke
levetid_Y = np.median(simulerte_Y, axis = 0)


#P(Y >= 1) = g / m
pyge1 = np.sum(levetid_Y >= 1) / n
print(pyge1)

#P(Y >= 1 | Y >= 0.75) = P(Y >= 1) / P(Y >= 0.75)
pyge1gyge075 = np.sum(levetid_Y >= 1) / np.sum(levetid_Y >= 0.75)
print(pyge1gyge075)

plt.hist(levetid_Y, density=True,bins=100) #density=TrueA
plt.show()
```

✓ 1.8s

```
0.3310724
0.48074060263266266
```



4)

$f_X(x)$ gyldig hvis $\int_{-\infty}^{\infty} f_X(x)\,dx = 1$

$$\int_{-\infty}^{\infty} e^{-x^4}\,dx = 2\Gamma\left(\frac{5}{4}\right)$$

$$\int_{-\infty}^{\infty} f_X(x)\,dx = \int_{-\infty}^{\infty} \frac{a}{2\Gamma\left(\frac{5}{4}\right)} e^{-(x-1)^4} + \frac{b}{2\Gamma\left(\frac{5}{4}\right)} e^{-(x-4)^4}\,dx = 1$$

$$\frac{a}{2\Gamma\left(\frac{5}{4}\right)} \int_{-\infty}^{\infty} e^{-(x-1)^4}\,dx + \frac{b}{2\Gamma\left(\frac{5}{4}\right)} \int_{-\infty}^{\infty} e^{-(x-4)^4}\,dx = 1$$

$x - 1 = u \qquad x - 4 = v$

$$\frac{a}{2\Gamma\left(\frac{5}{4}\right)} \int_{-\infty}^{\infty} e^{-u^4}\,du + \frac{b}{2\Gamma\left(\frac{5}{4}\right)} \int_{-\infty}^{\infty} e^{-v^4}\,dv = 1$$

$dx \to du$ og $dx \to dv$, fordi vi går fra $-\infty$ til ∞

$$\frac{a}{2\Gamma\left(\frac{5}{4}\right)} \cdot 2\Gamma\left(\frac{5}{4}\right) + \frac{b}{2\Gamma\left(\frac{5}{4}\right)} \cdot 2\Gamma\left(\frac{5}{4}\right) = 1$$

$$\underline{\underline{a + b = 1}}$$

$a + b = 1$ for at $f_X(x)$ skal være en gyldig sannsynlighetstetthet

$$E(X) = \int_{-\infty}^{\infty} x f_X(x)\,dx = \int_{-\infty}^{\infty} x\left(\frac{a}{2\Gamma(\frac{5}{4})} e^{-(x-1)^4} + \frac{b}{2\Gamma(\frac{5}{4})} e^{-(x-4)^4}\right) dx$$

$= \Big(\frac{1}{2}x^2 +$ Hva i alle dager skal jeg gjøre her?

5)

x	$f(x)$
0	$\frac{1}{18}+\frac{1}{9}+\frac{1}{6}$
1	$\frac{1}{12}+\frac{1}{6}+\frac{1}{4}$
2	$\frac{1}{36}+\frac{1}{18}+\frac{1}{12}$

$\Rightarrow$

x	$f_X(x)$
0	$\frac{1}{3}$
1	$\frac{1}{2}$
2	$\frac{1}{6}$

$$f_{Y|X}(y|x) = \frac{P(Y\cap X)}{P(X)}$$

$x \backslash y$	0	1	2
0	$\frac{1}{6}$	$\frac{1}{3}$	$\frac{1}{2}$
1	$\frac{1}{6}$	$\frac{1}{3}$	$\frac{1}{2}$
2	$\frac{1}{6}$	$\frac{1}{3}$	$\frac{1}{2}$

$$E(X) = \sum_x x f_X(x) = 0\cdot\frac{1}{3} + 1\cdot\frac{1}{2} + 2\cdot\frac{1}{6} = \frac{1}{2}+\frac{1}{3} = \underline{\underline{\frac{5}{6}}}$$

$$E(Y) = \sum_y\sum_x y f_{XY}(x,y) = 0\cdot\left(\frac{1}{18}+\frac{1}{12}+\frac{1}{36}\right) + 1\cdot\left(\frac{1}{9}+\frac{1}{6}+\frac{1}{18}\right) + 2\cdot\left(\frac{1}{6}+\frac{1}{4}+\frac{1}{12}\right)$$

$$= \frac{1}{3} + \frac{12}{12} = \underline{\underline{\frac{4}{3}}}$$

X og Y avhengig? $\Rightarrow Cov(X,Y) \neq 0$

$$Cov(X,Y) = E(XY) - E(X)E(Y)$$

$$E(XY) = \sum_x\sum_y xy\, f_{XY}(x,y) = 0\cdot 0\cdot\left(\frac{1}{18}\right) + 0\cdot 1\cdot \ldots$$
$$+ 1\cdot 1\cdot\frac{1}{6} + 2\left(\frac{1}{4}\right) + 2\left(\frac{1}{18}\right) + 4\left(\frac{1}{12}\right)$$
$$= \frac{1}{6} + \frac{1}{2} + \frac{4}{12} = \frac{1}{6} + \frac{3}{6} + \frac{2}{6} = 1$$

$$Cov(X,Y) = 1 - \frac{4}{3}\cdot\frac{5}{6} = 1 - \frac{20}{18} = -\frac{1}{9}$$

X og Y er <u>ikke</u> uavhengige. $Cov(X,Y) = \underline{\underline{-\frac{1}{9}}}$

6 a)

Forventet bruttovekt = 50 tonn + 200(70 + 20) kg

= $\underline{\underline{68 \text{ tonn}}}$

$SD(X) = \sqrt{Var(X)}$

$Var(X) = 200 \cdot SD_K^2 + 200 \cdot SD_G^2$

$= 200 \cdot 15^2 + 200 \cdot 5^2$

$= 50000$

$SD(X) = \underline{\underline{0{,}224 \text{ tonn}}}$

b)

4 motorer $p = 0{,}01$

P(Minst to svikter) = P(2 svikter) + P(3) + P(4)

= 4choose2*0.01^(2)*(1-0.01)^(4-2) + 4choose3*0.01^(3)*(1-0.01)^(4-3) + 4choose4*0.01^(4)*(1-0.01)^(4-4) Google ♡

= $\underline{\underline{0{,}00059203}}$

c)

P(Funker med 4) > P(Funker med 2)

P(minst 2 virker) < P(minst 1 virker)

$$\binom{4}{2}p^2(1-p)^2+\binom{4}{3}p^3(1-p)+\binom{4}{4}p^4 < \binom{2}{1}p(1-p)+\binom{2}{2}p^2$$

$$6p^2(1-p)^2+4p^3(1-p)+p^4 < 2p(1-p)+p^2 \qquad q = 1-p$$

$$6p^2q^2+4p^3q+p^4 < 2pq+p^2$$

$$6p^2q^2+4p^3q+p^4-2pq-p^2 < 0$$

$$6pq^2+4p^2q+p^3-2q-p < 0$$

$$6p(1-p)^2+4p^2(1-p)+p^3-2+2p-p < 0$$

$$\underline{\underline{p \leq \frac{2}{3}}}$$