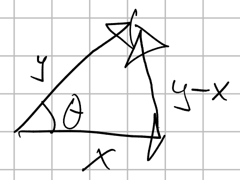
Indre produktrom 1

1)

$x \cdot y = x_1 y_1 + x_2 y_2$

$\|x\| = \sqrt{x \cdot x} = \sqrt{x_1^2 + x_2^2}$

$x_1 y_1 + x_2 y_2 = \|x\|\|y\| \cos\theta$?



$c^2 = a^2 + b^2 - 2ab\cos\theta$

$(y - x)^2 = x^2 + y^2 - 2\|x\|\|y\| \cos\theta$

$x^2 - 2xy + y^2 = x^2 + y^2 - 2\|x\|\|y\|\cos\theta$

$-2xy = -2\|x\|\|y\|\cos\theta$

$x_1 y_1 + x_2 y_2 = \|x\|\|y\|\cos\theta$

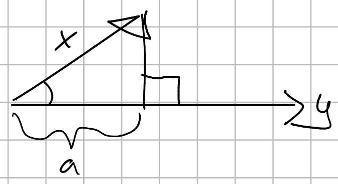
2) $\|x + y\|^2 = (x + y) \cdot (x + y) = \|x\|^2 + 2x \cdot y + \|y\|^2$

$\|x + y\|^2 = \|x\|^2 + \|y\|^2 + 2x \cdot y$

$x \cdot y = 0 \Rightarrow \|x + y\|^2 = \|x\|^2 + \|y\|^2$

x og y er ortogonale dersom $\underline{x \cdot y = 0}$

3)



$a = \|x\| \cos\theta$

$= \frac{\|y\|}{\|y\|} \|x\| \cos\theta$

$= \underline{\underline{\frac{x \cdot y}{\|y\|}}}$

4)

$|a| \leq \|x\|$ $\qquad a = \frac{x \circ y}{\|y\|}$

$\left|\frac{x \circ y}{\|y\|}\right| \leq \|x\|$

$\underline{\underline{|x \circ y| \leq \|x\|\|y\|}}$

5)

$\|x+y\|^2 = \|x\|^2 + 2x \circ y + \|y\|^2 \qquad |x \circ y| \leq \|x\|\|y\|$

$\|x+y\|^2 - \|x\|^2 - \|y\|^2 = 2x \circ y$

$\frac{1}{2}(\|x+y\|^2 - \|x\|^2 - \|y\|^2) \leq \|x\|\|y\|$

$\sqrt{\|x+y\|^2} \leq \sqrt{2\|x\|\|y\| + \|x\|^2 + \|y\|^2}$

$\underline{\underline{\|x+y\| \leq \|x\| + \|y\|}}$

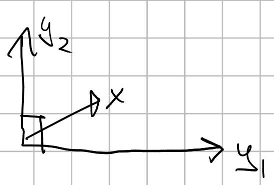
6)

$x = c_1 y_1 + c_2 y_2$

$x \circ y_1 = c_1 y_1 \cdot y_1 + c_2 y_2 \circ y_1$

$x \circ y_1 = c_1 y_1 \cdot y_1$

$c_1 = \frac{x \circ y_1}{y_1 \cdot y_1}$



$x \circ y_2 = c_1 y_1 \circ y_2 + c_2 y_2 \circ y_2$

$x \circ y_2 = c_2 y_2 \cdot y_2$

$c_2 = \frac{x \circ y_2}{y_2 \circ y_2}$

$\underline{x = \frac{x \cdot y_1}{y_1 \cdot y_1} y_1 + \frac{x \circ y_2}{y_2 \circ y_2} y_2}$

7)

$$P = \frac{y y^T}{y^T y} \qquad Px = \frac{y y^T}{y^T y} x \qquad y = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix} \qquad x = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$$

$$\frac{y y^T}{y^T y} x^T = \frac{\begin{bmatrix} y_1 \\ \vdots \\ y_n \end{bmatrix} \cdot \begin{bmatrix} y_1 & \cdots & y_n \end{bmatrix}}{\begin{bmatrix} y_1 & \cdots & y_n \end{bmatrix} \cdot \begin{bmatrix} y_1 \\ \vdots \\ y_n \end{bmatrix}} \begin{bmatrix} x_1 & \cdots & x_n \end{bmatrix} = \frac{y_1^2 + \dots + y_n^2}{y_1^2 + \dots + y_n^2} \cdot \begin{bmatrix} x_1 & \cdots & x_n \end{bmatrix} = x^T$$

$$\cancel{\frac{x^T y}{y^T y} y^T} = x^T$$

8)

$$\begin{matrix} 1 & 1 & 1 & -1 \\ 0 & -\sqrt{2} & \sqrt{2} & 0 \\ -1 & 1 & 1 & -7 \end{matrix} \qquad Ax = b$$

$$A^T A x = A^T b$$

$$\begin{bmatrix} 1 & 0 & -1 \\ 1 & -\sqrt{2} & 1 \\ 1 & \sqrt{2} & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 0 & -\sqrt{2} & \sqrt{2} \\ -1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix}$$

$$(A^T A)^{-1} = \left.\begin{matrix} 2 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{matrix}\right| \begin{matrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{matrix} = \begin{matrix} \frac{1}{2} & & \\ & \frac{1}{4} & \\ & & \frac{1}{4} \end{matrix}$$

$$x = (A^T A)^{-1} A^T b$$

$$= \begin{bmatrix} \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{4} & 0 \\ 0 & 0 & \frac{1}{4} \end{bmatrix} \begin{bmatrix} 1 & 0 & -1 \\ 1 & -\sqrt{2} & 1 \\ 1 & \sqrt{2} & 1 \end{bmatrix} \begin{bmatrix} -1 \\ 0 \\ -7 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{2} & 0 & -\frac{1}{2} \\ \frac{1}{4} & -\frac{\sqrt{2}}{4} & \frac{1}{4} \\ \frac{1}{4} & \frac{\sqrt{2}}{4} & \frac{1}{4} \end{bmatrix} \begin{bmatrix} -1 \\ 0 \\ 7 \end{bmatrix}$$

$$= \begin{bmatrix} -\frac{1}{2} - \frac{7}{2} \\ -\frac{1}{4} + \frac{7}{4} \\ -\frac{1}{4} + \frac{7}{4} \end{bmatrix} = \underline{\underline{\begin{bmatrix} -4 \\ \frac{3}{2} \\ \frac{3}{2} \end{bmatrix}}}$$

10)

$$c_1 x_1^T x_1 + c_2 x_2^T x_2 + \dots + c_n x_n^T x_n = 0$$

$$c_1 x_1^T x_1 + c_2 x_2^T x_2 + \dots = c_n x_n^T x_n \qquad x_n = \begin{bmatrix} 0 \\ \vdots \\ 0 \end{bmatrix}$$, ikke lineært uavhengig!

$$1 \leq k \leq n$$

$$v_k^T v_l = \begin{cases} \|v_l\|^2 & , k = l \\ 0 & , k \neq l \end{cases}$$

$\vec{0} = \sum_{k=1}^{n} c_k v_k$ impliserer $c_k = 0$ for alle k

$$0 = v_l^T \vec{0} = v_l^T \sum_{k=1}^{n} c_k v_k = \sum_{k=1}^{n} c_k v_l^T v_k = \sum_{k=1}^{n} c_k v_l^T v_l = c_l v_l^T v_l = c_l \|v_l\|^2$$

$v_l \neq \vec{0} \Rightarrow \|v_l\|^2 \neq 0 \Rightarrow c_l = 0$

lineært uavhengig!

1 1) $x = \sum_k \frac{x^T v_k}{v_k^T v_k} v_k$?

$$x = \sum_k c_k v_k$$

$$x v_l^T = v_l^T \sum_k c_k v_k = \sum_k c_k v_l^T v_k = c_l v_l^T v_l$$

$$c_l = \frac{x v_l^T}{v_l^T v_l}$$

$$x = \sum_l \frac{x v_l^T}{v_l^T v_l} v_l = \underline{\underline{\sum_l \frac{x^T v_l}{v_l^T v_l} v_l}}$$

1 2)

$$v_1 = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} \quad v_2 = \begin{bmatrix} 1 \\ -\sqrt{2} \\ 1 \end{bmatrix} \quad v_3 = \begin{bmatrix} 1 \\ \sqrt{2} \\ 1 \end{bmatrix} \quad b = \begin{bmatrix} -1 \\ 0 \\ -7 \end{bmatrix}$$

$$x_k = \frac{b^T v_k}{v_k^T v_k}$$

$$Ax = b$$

$$A^T A x = A^T b$$

$$v_k^T v_k x_k = v_k^T b$$

$$x_k = \frac{v_k^T b}{v_k^T v_k} = \underline{\underline{\frac{b^T v_k}{v_k^T v_k}}}$$

13)

$$z' = \frac{v_1^T z}{v_1^T v_1} v_1 + \frac{v_2^T z}{v_2^T v_2} v_2$$

14) <u>ka?</u>