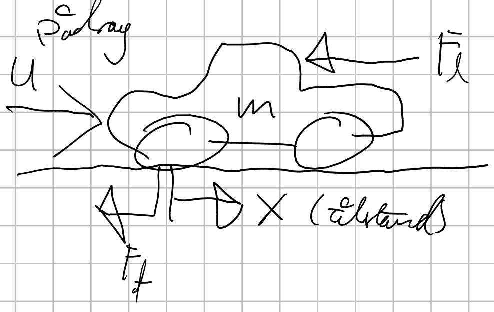
Kapittel 3

Eks 4. Modellering for cruise-control

Eng: control = No: regulering ≠ No: kontroll



Utgangspunkt N's 2. lov

$$\Sigma F = m\alpha$$

$$\alpha = \dot{v}$$

$$F_l = k_l v^2$$

$$F_f = k v$$

Ikke lineær
ser bort ifra nå!

$$\Sigma F = m\alpha$$

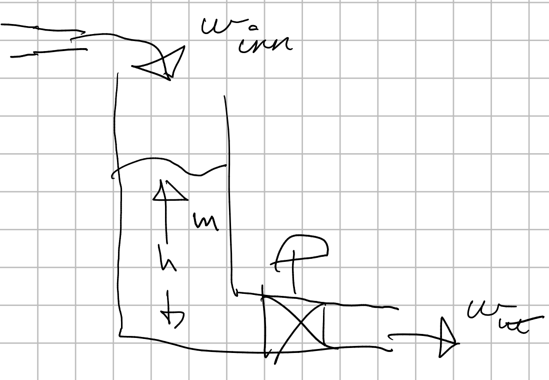
$$u - \cancel{F_l} - F_f = m\dot{v}$$

$$m\dot{v} = u - F_f = u - kv$$

$$\dot{v} = \frac{1}{m}(u - kv)$$

$$\dot{v} = -\frac{k}{m}v + \frac{1}{m}u$$

Eks 5 Modellering for nivåregulering



Balanselov: Massebalanse

Endring av masse per tid
= massestrøm inn
− massestrøm ut

$$\dot{m} = w_{inn} - w_{ut}$$

$$m = \rho V = \rho A h$$

høyde
Areal
Tetthet

$$\dot{m} = \frac{d}{dt}(\rho A h) = w_{inn} - w_{ut}$$

$$\rho A \dot{h} = w_{inn} - w_{ut}$$

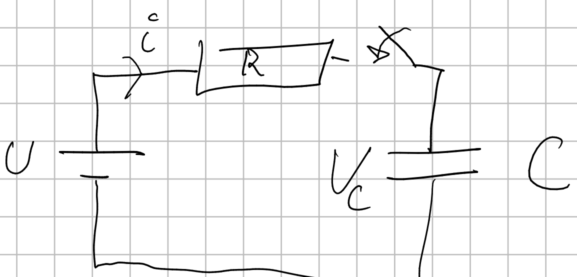
$$\dot{h} = \frac{1}{\rho A}(w_{inn} - w_{ut})$$

$w_{ut} = k h$

$\dot h = \frac{1}{\rho A}(u - k h)$

$\dot h = \frac{1}{\rho A} u - \frac{k}{\rho A} h$

Eks 8 RC-krets



$v_c = \frac{1}{C}\int_0^t i_c(\tau')\,d\tau'$ Merket, ikke derivert τ'

$\frac{dV_c}{dt} = \frac{1}{C} i_c(t)$

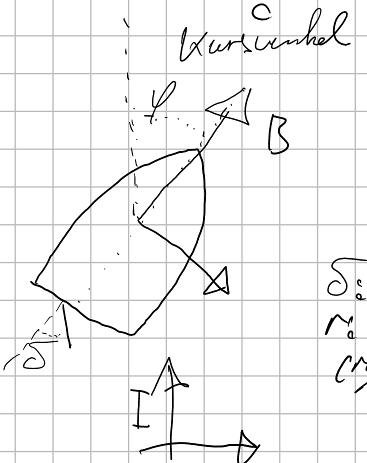
Strømbalanse: $i = i_R = i_C$

$\frac{u - V_c}{R} = C\frac{dV_c}{dt}$

$\dot v_c = \frac{1}{C}\left(\frac{u - v_c}{R}\right)$

$\dot v_c = -\frac{1}{RC} v_c + \frac{1}{RC} u$ Lineær krets

Eks 9 Modell for autopilotdesign



δ: rorvinkel
r: vinkelhastighet (rad/s)

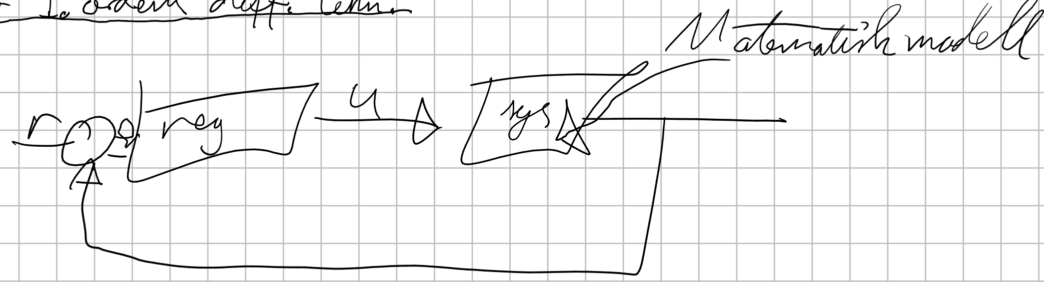
$\boxed{\dot r = -\frac{1}{T} + \frac{K}{T}\delta}$

Nomoto-modell

$a = -\frac{1}{T}$ $b = \frac{K}{T}$ T = tidskonstant

$\dot\psi = r$

Løsning av 1. orden diff. likn.



$$\dot{x} = ax + b$$

gjemmer $u(t)$ for å la reg løse for nå

$$\frac{dx}{dt} = ax + b$$

$$\int \frac{1}{ax+b}\frac{dx}{dt}\,dt = \int 1\,dt$$

$$\frac{1}{a}\ln|ax+b| = t + C_1$$

$$|ax+b| = e^{a(t+C_1)} > 0$$

$$ax+b = e^{at}\underbrace{e^{aC_1}}_{C_2}$$

$$x(t) = Ce^{at} - \frac{b}{a} \qquad C = \frac{C_2}{a} \qquad C = \frac{1}{a}e^{aC_1}$$ er vilkårlig

$$x(0) = x_0$$

$$x(0) = Ce^{0t} - \frac{b}{a} = x_0$$

$$C = x_0 + \frac{b}{a}$$

$$x(t) = \left(x_0 + \frac{b}{a}\right)e^{at} - \frac{b}{a}$$

$$= x_0 e^{at} + \frac{b}{a}e^{at} - \frac{b}{a}$$

$$x(t) = x_0 e^{at} + \frac{b}{a}\left(e^{at} - 1\right)$$

Kokeplate:

$$\dot{T} = -\frac{k}{C}T + \frac{1}{C}(P + k\,T_{rom})$$

Løsning:

$$T(\tau) = T_0 e^{-\frac{k}{C}\tau} - \frac{P + k\,T_{rom}}{k}\left(e^{-\frac{k}{C}\tau} - 1\right)$$

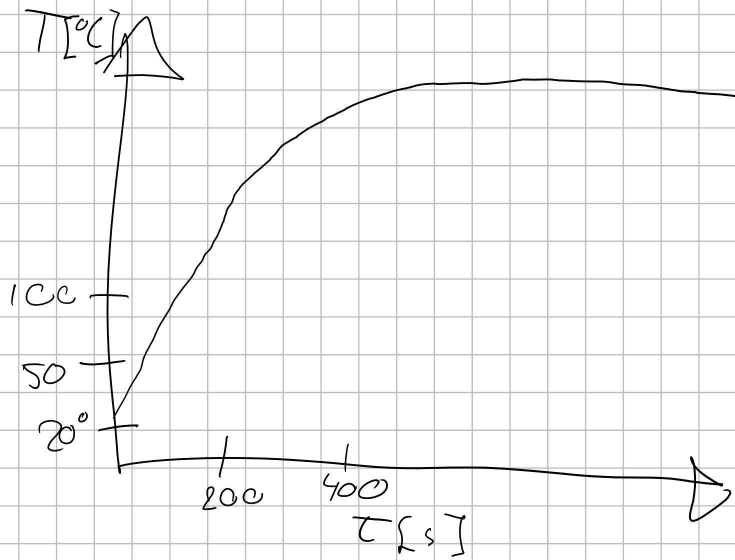
$T(0) = 20°C$

$P = 500\ W$

$k = 2\ W/°C$

$T_{rom} = 20°$

$C = 400\ J/C$



Cruise Control

$$\dot{v} = -\frac{k}{m}v + \frac{1}{m}u - k_l v^2$$

$$v(\tau) = \frac{u}{k}\left(1 - e^{-\frac{k}{m}\tau}\right), \quad v(0) = v_0 = 0$$