

Schriftlich innderung 1

Opf 1)

$3, 5, 7, 11$ alle \mathbb{Z} BTU

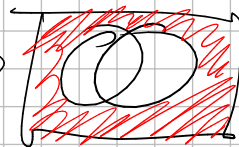
$$P((A \cup B)') = P(A' \cap B') \quad ?$$

$P(A \cup B)$

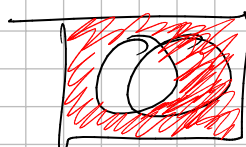


\Leftrightarrow

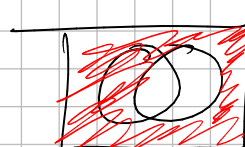
$P(A \cup B')$



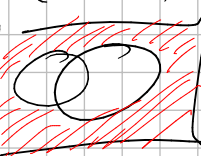
$P(A')$



$P(B')$



$P(A' \cap B')$



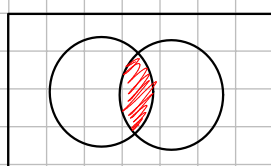
$=$

$P(A \cup B')$



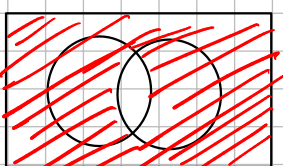
$$P((A \cap B)') = P(A' \cup B')$$

$P(A \cap B)$

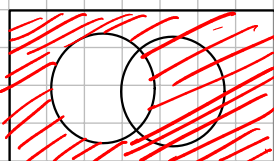


\Rightarrow

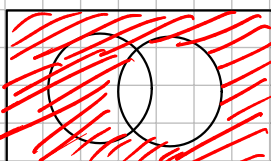
$P((A \cap B)')$



$P(A')$



$P(B')$



\Rightarrow

$P(A' \cup B')$

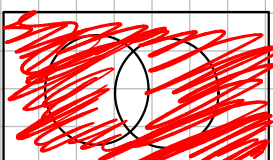


$P((A \cap B)')$



$=$

$P(A' \cup B')$



Oppg 2)

34 kuler, 7 røde, 27 blå, Trekkes en uter i tillegg

A: Alle 7 kuler er røde

B: Nøyaktig 4 kuler er røde

C: Trekkes 7 med nøyaktig 6 røde, så trekkes 1 til hvorav den er rødt

Antas uniform sannsynlighetsmodell $\Rightarrow P = \frac{g}{m}$

$$P(A) = \frac{g}{m}$$

$$m = \binom{34}{7}$$

$$\frac{x}{y} = \frac{x^2}{y} = \frac{x^2}{y^2}$$

$g = 1$ fordi det er bare 1 kombinasjon der alle er røde

$$P(A) = \frac{1}{\binom{34}{7}} = 1,86 \cdot 10^{-7}$$

$$P(B) = \frac{g}{m}$$

$$g = \binom{7}{4} \binom{27}{3} =$$

$$P(B) = \frac{\binom{7}{4} \binom{27}{3}}{\binom{34}{7}} = 0,019$$

$$P(C) = P(6 \text{ røde}) \cdot P(1 \text{ rødt } | 6 \text{ røde})$$

$$P(C) = \frac{\binom{7}{6} \binom{27}{1}}{\binom{34}{7}} \cdot \frac{\binom{1}{1} \binom{26}{0}}{\binom{27}{1}} = \frac{\binom{7}{6} \binom{27}{1}}{\binom{34}{7}} \cdot \frac{1}{27}$$

$$= 1,30 \cdot 10^{-6}$$

Oppgave 3)

300 lodd, 3 er av gevinst A, 3 av B, 294 gir ingen.
Trekket 5 kort

$P(C_1)$ = Vinner nøyaktig 1 lodd av type A = $\frac{3}{n}$

$$n = \binom{300}{5}$$

$$g = \binom{3}{1} \cdot \binom{297}{4}$$

$$P(C_1) = \frac{\binom{3}{1} \binom{297}{4}}{\binom{300}{5}} = \underline{\underline{0,0487}}$$

$P(C_2)$ = minst én av A) = $1 - P(\text{ingen av A})$

$P(\text{ingen av A}) = \frac{g}{n}$

$$n = \binom{300}{5}$$

$$g = \binom{297}{5}$$

$$P(C_2) = 1 - \frac{\binom{297}{5}}{\binom{300}{5}} = \underline{\underline{0,0493}}$$

$P(C_3)$ = Minst én gevinst = $1 - P(\text{ingen gevinst})$

$$= 1 - \frac{\binom{297}{5}}{\binom{300}{5}} = \underline{\underline{0,0967}}$$

Aufgabe 4)

$$A, B, C \in \mathcal{S} \quad P(A) = 0,4 \quad P(B) = 0,3 \quad P(C) = 0,3$$

$$P(A \cup B) = 0,6 \quad P(A \cup C) = 0,5 \quad P(B \cup C) = 0,6$$

$$P(A \cup B \cup C) = 0,7$$

$$P(A \cap B) = P(A) + P(B) - P(A \cup B) = \underline{\underline{0,1}}$$

$$P(A \cap B \cap C) = ?$$

$$P(A \cap C) = P(A) + P(C) - P(A \cup C) = 0,2$$

$$P(B \cap C) = P(B) + P(C) - P(B \cup C) = 0$$

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)$$

$$P(A \cap B \cap C) = P(A \cup B \cup C) + P(A \cap B) + P(A \cap C) + P(B \cap C) - P(A) - P(B) - P(C)$$

$$= 0,7 + 0,1 + 0,2 + 0 - 0,4 - 0,3 - 0,3$$

$$= \underline{\underline{0}}$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{0,1}{0,3} = \underline{\underline{\frac{1}{3}}}$$

A & B unabhängig?

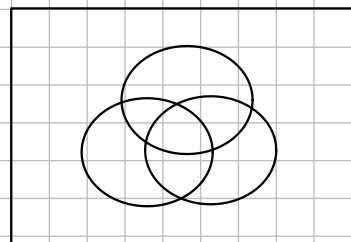
$$P(A|B) \stackrel{?}{=} P(A) :$$

$$\frac{1}{3} \neq 0,4 \Rightarrow \underline{\text{Nicht unabhängig}}$$

A & B disjunkt?

$$P(A \cap B) = P(\emptyset) = 0 ?$$

$$P(A \cap B) = 0,1 \Rightarrow \text{Nicht disjunkt}$$



Aufgabe 5)

$$P(F|M) = 0,08 \quad P(F|K) = 0,003$$

$$P(K|F) = ? \quad P(M) = \frac{1}{3} \quad P(K) = \frac{2}{3}$$

$$P(K|F) = \frac{P(F|K) P(K)}{P(F)}$$

$$P(F) = P(F|M) P(M) + P(F|K) P(K)$$

$$P(K|F) = \frac{P(F|K) P(K)}{P(F|M) P(M) + P(F|K) P(K)}$$

$$= \underline{\underline{0,069}}$$

Oppgave 6)

$$P(X \leq 2) = 0,25 + 0,1 + 0,05 = \underline{\underline{0,40}}$$

$$P(X < 2 | X < 4) = \frac{P(X < 2 \cap X < 4)}{P(X < 4)} = \frac{P(X \leq 2)}{P(X < 4)} = \frac{0,40}{0,40 + 0,40} = \underline{\underline{0,5}}$$

$$P(X \leq 2 | X \geq 1) = \frac{P(X \leq 2 \cap X \geq 1)}{P(X \geq 1)} = \frac{P(X = 2 \cup X = 1)}{1 - P(X < 1)} = \frac{0,10 + 0,25}{1 - 0,05}$$

$$= \underline{\underline{0,3684}}$$

Opgave 7)

a)

$$f'(x) = f(x)$$

$$f(x) = \left(1 - e^{-\frac{x^2}{\alpha}}\right)'$$

$$= -\left(-\frac{x^2}{\alpha}\right)' \cdot e^{-\frac{x^2}{\alpha}}$$

$$= \frac{2x}{\alpha} e^{-\frac{x^2}{\alpha}}$$

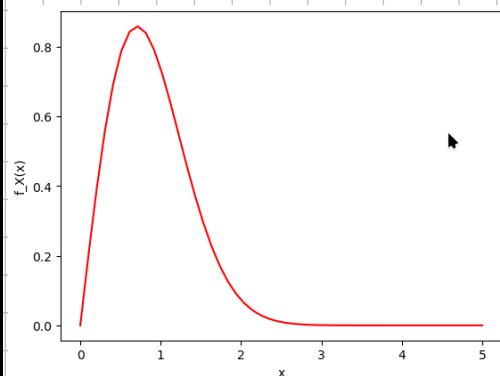
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#Opgave 7a)

def f_X(x, alpha):
    return ((2*x)/alpha) * np.exp((-x**2)/alpha)

x = np.linspace(0, 5)
y = np.zeros(len(x))

for i in range(len(x)):
    y[i] = f_X(x[i], alpha)

plt.plot(x, y, color="red")
plt.xlabel("x")
plt.ylabel("f_X(x)")
plt.show()
```



b)

$$1 - F_Z(z) = P(Z \leq z)$$

$$= P(X \leq z \cap Z \leq z) \text{ uafhængig} \Rightarrow \text{Begge må være sandsynlig}$$

$$= P(X \leq z) P(Z \leq z)$$

$$= \left(1 - e^{-\frac{z^2}{\alpha}}\right) \left(1 - e^{-\frac{z^2}{\alpha}}\right)$$

$$= 1 - 2e^{-\frac{z^2}{\alpha}} + e^{-\frac{2}{\alpha} z^2}$$

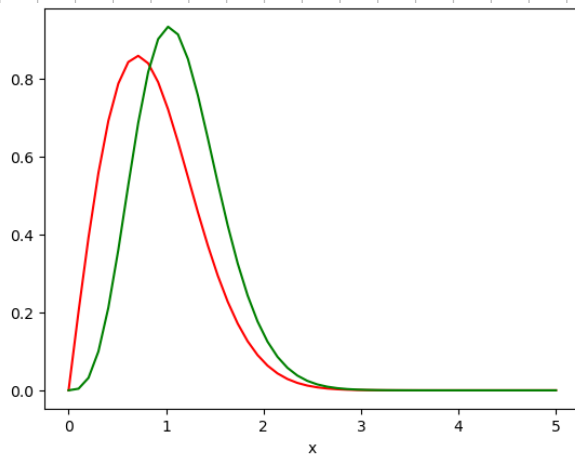
$$f_Z(z) = \frac{2}{\alpha} z e^{-\frac{z^2}{\alpha}} - \frac{4}{\alpha} z e^{-\frac{2}{\alpha} z^2}$$

```
#Oppgave 7b)

def f_Z(z, alpha):
    return (4/alpha)*z*np.exp(-(z**2)/alpha) - (4/alpha)*z*np.exp(-(2*z**2)/alpha)

plt.plot(x, f_X(x, alpha), color="red")
plt.plot(x, f_Z(x, alpha), color="green")
plt.xlabel("x")
plt.ylabel("")
plt.show()

0.0s
```



Kan se att det er en høyere sannsynlighet for instrumentet funker når den består av to komponenter, hvorav bare én trenger å være i stand for att instrumentet skal funke.

Dette ser man fordi den grønne grafen $f_Z(z)$ er forskjøvet mot høyre i forhold til $f_X(x)$.

Opfrage 8)

300 Lodd, trecker 5

3 Lodd grü gewünscht A

3 Lodd grü gewünscht B

294 grü ünger gewünscht

X : Anzahl gewünschter von A

Y : Anzahl gewünschter von B

$$f_X(x) = \frac{1}{5}$$

$$g = \binom{3}{x} \binom{297}{5-x}$$

$$f_X(x) = \frac{\binom{3}{x} \binom{297}{5-x}}{\binom{300}{5}}$$

$$f_{XY}(x, y) = \frac{\binom{3}{x} \binom{3}{y} \binom{294}{5-x-y}}{\binom{300}{5}}$$

$$f_X(x) = \sum_y f_{XY}(x, y) = \sum_y \frac{\binom{3}{x} \binom{3}{y} \binom{294}{5-x-y}}{\binom{300}{5}} = \frac{\binom{3}{x}}{\binom{300}{5}} \sum_y \binom{3}{y} \binom{294}{5-x-y}$$

$$= \frac{\binom{3}{x}}{\binom{300}{5}} \cdot \binom{3+294}{5-x} = \frac{\binom{3}{x} \binom{297}{5-x}}{\binom{300}{5}}$$

