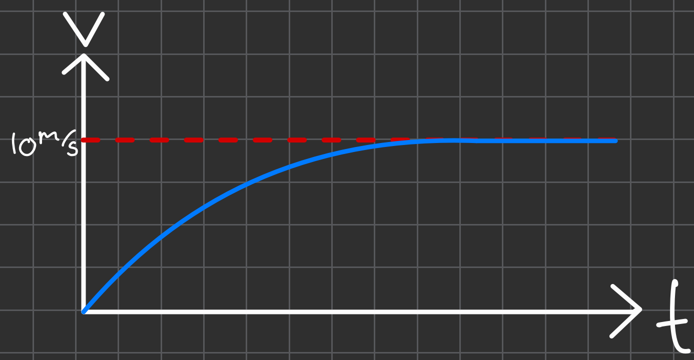
Stasjonær respons



Statisk modell:

$$v = K \cdot U \Rightarrow K = \frac{v}{U} = 0.02$$

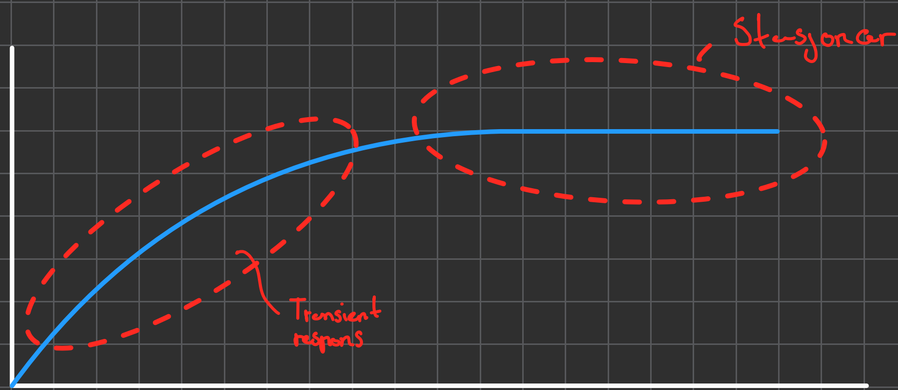
Alternativt:

Har modell: $\dot{v} = -\frac{k}{m} v + \frac{L}{m} U$

Når v er konstant så er $\dot{v} = 0$

$$-\frac{k}{m}\dot{v} + \frac{L}{m} U = 0 \Rightarrow \boxed{v = \frac{1}{k} U}$$

Transient respons



1. ordens system:

$$\dot{x} = ax + bu$$

Når $\dot{x} = 0$ får vi: $ax + bu = 0$

<u>Stasjonær verdi:</u> $\boxed{x_s = -\frac{b}{a}u}$

For $\dot{x} = ax + bu$ så er

$\boxed{T = -\frac{1}{a}}$ Systemets tidskonstant

$\boxed{K = -\frac{b}{a}}$ Systemets forsterkning

$$\dot{x} = ax + bu \Rightarrow \dot{x} - bu = ax$$

$$\frac{1}{a}\dot{x} - \frac{b}{a}u = x \Rightarrow \underbrace{-\frac{1}{a}}_{T}\dot{x} + x = \underbrace{-\frac{b}{a}}_{K}$$

Stasjonært $\dot{x} = 0 \Rightarrow x_s = Ku$

Eks

$$\dot{v} = \underbrace{-\frac{k}{m}}_{a} v + \underbrace{\frac{1}{m}}_{b} u$$

Tidskonstant: $T = -\frac{1}{a} = -\frac{1}{-\frac{k}{m}} = \frac{m}{k}$

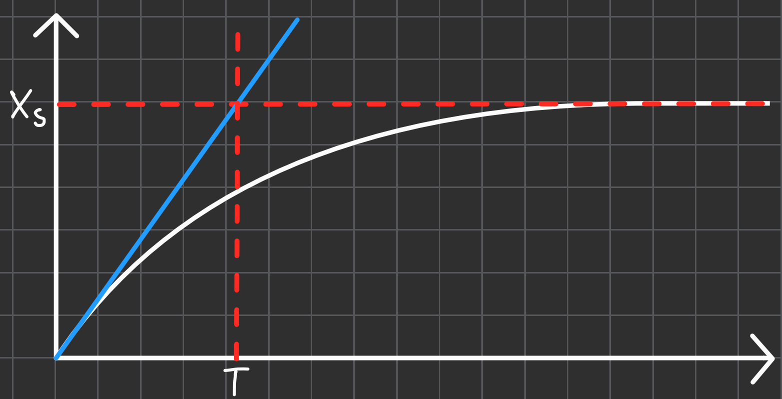
Forsterkning: $K = -\frac{b}{a} = -\frac{1/m}{-k/m} = \frac{1}{k}$

$F = kv \qquad [k] = \frac{[F]}{[v]} = \frac{N}{m/s} = kg/s$

$T = \frac{1000kg}{50\,kg/s} = \underline{\underline{20s}}$ → Tidskonstant må ha enhet sekund

$K = \frac{1}{k} = \frac{1}{50\,kg/s} = \underline{\underline{0.02\ s/kg}}$

Hvordan finne tidskonstant uten matematisk modell?



Hvorfor er det slik?

$$v(t) = \frac{u}{k}\left(1 - e^{-\frac{k}{m}t}\right)$$

$$0 = -\frac{k}{m} v_s + \frac{u}{m} \Rightarrow v_s = \frac{u}{k}$$

$$V(t) = V_s\left(1 - e^{-\frac{1}{T}t}\right)$$

$$\dot{V}(t) = V_s\left(\frac{1}{T}e^{-\frac{1}{T}t}\right)$$

$$\dot{V}(0) = \frac{V_s}{T}e^0 = \frac{V_s}{T}$$

Tangenten går gjennom $(0, 0)$

$$g(t) = \frac{V_s}{T}t$$

$$g(T) = \frac{V_s}{\cancel{T}}\cancel{T} = V_s \quad \left(\begin{array}{l}\text{Tangenten i } t=0 \\ \text{skjærer } V_s \text{ i } t=T\end{array}\right)$$

Hvor langt har $V(t)$ kommer ved $t = T$?

$$V(T) = V_s\left(1 - e^{-\frac{T}{T}}\right) = V_s\left(1 - \frac{1}{e}\right) \approx \underline{\underline{0.63V_s}}$$

$$V(2T) = V_s\left(1 - \frac{1}{e^2}\right) \approx \underline{\underline{0.87V_s}}$$

$$V(3T) = \underline{\underline{0.95V_s}} \qquad V(5T) = \underline{\underline{0.99V_s}}$$

Andre ordens systemer

For systemet

$$\boxed{\ddot{x} + p\dot{x} + qx = 0}$$

har vi størrelsene

ω_0 : Udempet resonans frekvens
ζ : Relativ dempingsfaktor

Eks mfd - system

$$\ddot{x} + \frac{d}{m}\dot{x} + \frac{k}{m}x = 0$$

$$\omega_0 = \sqrt{\frac{k}{m}} \qquad \zeta = \frac{d}{2\sqrt{m}}$$

$$\Rightarrow \quad \ddot{x} + 2\zeta\omega_0\dot{x} + \omega_0 x = 0$$

Har 3 forskjellige tilfeller:

$0 < \zeta < 1$: Underdempet system
$\zeta = 1$: Kritisk dempet system
$\zeta > 1$: Overdempet system

Eks 16 – mfd-system

$m = 1, \quad d = 2, \quad k = 10$

Da er: $\omega_0 = \sqrt{\frac{k}{m}} = \sqrt{10} \approx 3.16$

$$\zeta = \frac{d}{2\sqrt{m}} = \frac{1}{\sqrt{10}} \approx \underline{\underline{0.316}} < 1 \text{ : Underdempet}$$

Vil finne ω_0 fra figuren

Periode: $t = 4.8s - 0.6s$ (2 perioder)

Frekvens: $f = \frac{2}{4.8s} = \underline{\underline{0.48 Hz}}$

Vinkelfrekvens: $\boxed{2\pi f = \omega}$

$$\omega = 2\pi \cdot 0.48 Hz = \underline{\underline{2.88 s^{-1}}}$$

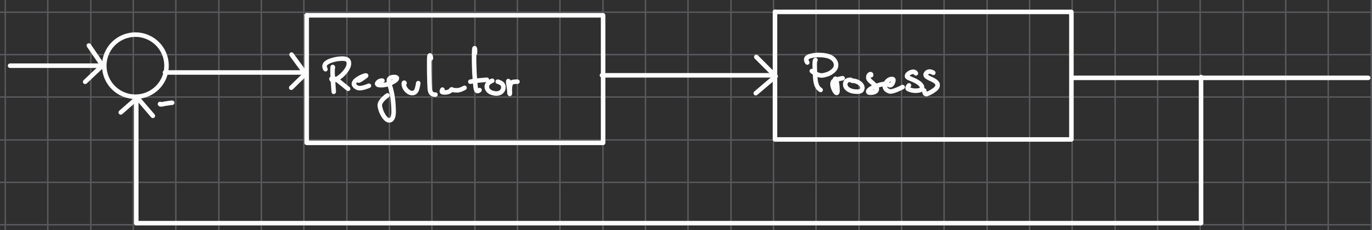
Dempet resonansfrekvens:

$$\boxed{\omega_d = \omega_0 \sqrt{1 - \zeta^2}}$$

Her: $\omega_d = \sqrt{10}\sqrt{1 - \left(\frac{1}{\sqrt{10}}\right)^2} = \sqrt{10}\sqrt{1 - \frac{1}{10}} = \sqrt{10}\,\frac{3}{\sqrt{10}}$

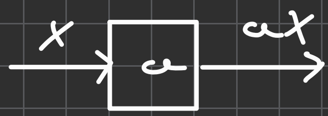
$\underline{\underline{\omega_d = 3}}$

Blokkdiagrammer

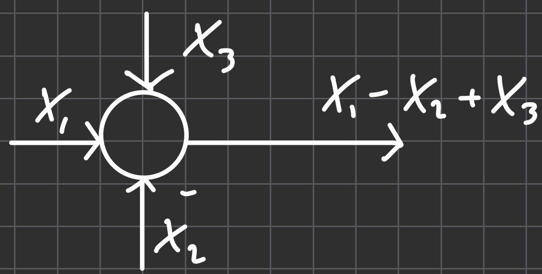


Symboler:

Multiplikasjon med konstant:

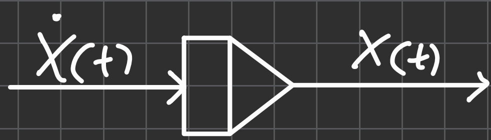


Summasjonspunkt



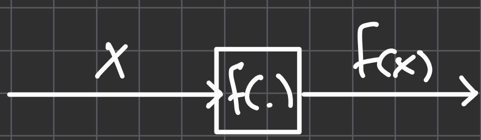
Multiplikator

Integrator



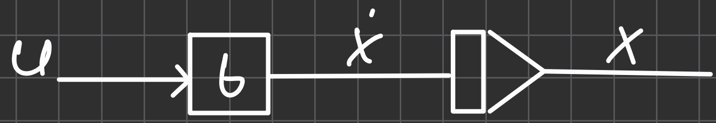
Derivator

Funksjonsgenerator

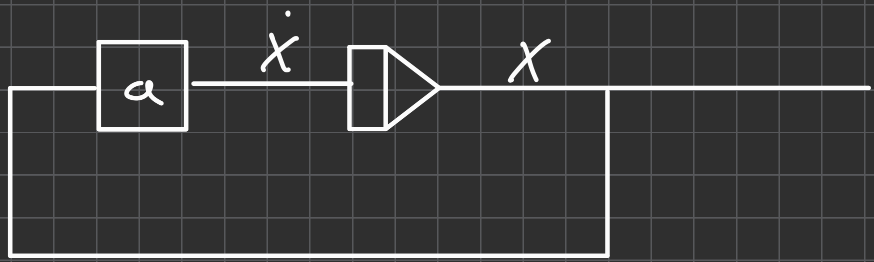


Blokkdiagrammer av diffligninger

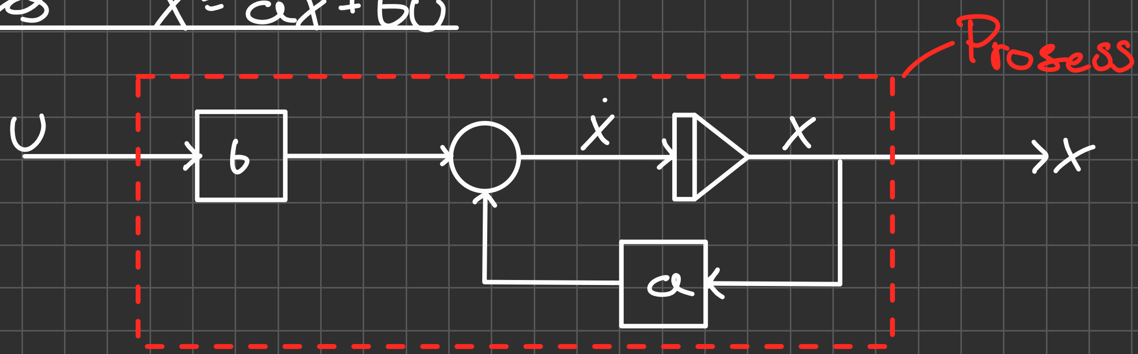
Eks $\dot{x} = bu$



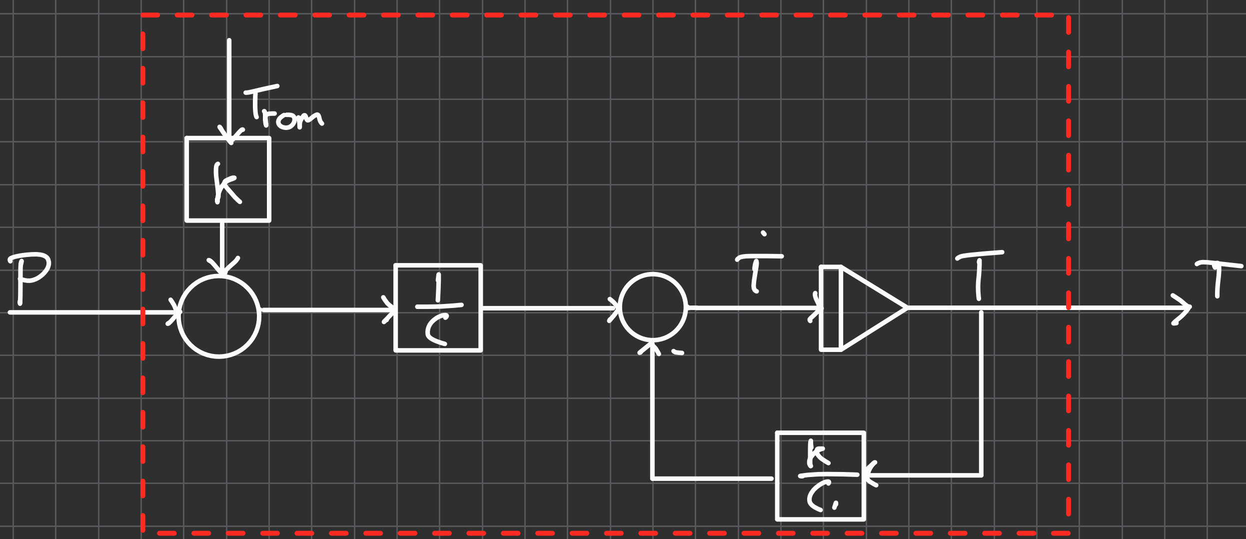
Eks $\dot{x} = ax$



Eks $\dot{x} = ax + bu$

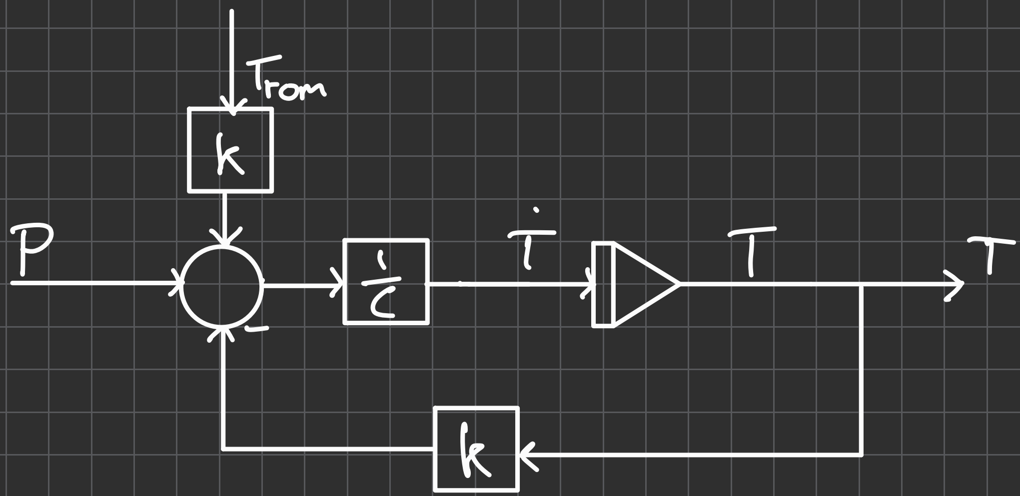


Eks $\dot{T} = -\frac{k}{C}T + \frac{1}{C}(P + kT_{rom})$



Alternativt:

$$\dot{T} = \frac{1}{c}\left(-kT + P + kT_{rom}\right)$$



Alternativt:

$$\dot{X}_1 = X_2$$

$$\dot{X}_2 = -\frac{k}{m}X_1 - \frac{d}{m}X_2$$

$$\dot{X}_2 = \frac{1}{m}\left(-kX_1 - dX_2\right)$$