Skade 3

Oppg 1)

a) $y = 2 \cdot \sqrt{u}$

$0{,}4 = 2 \cdot \sqrt{u}$

$0{,}2 = \sqrt{u}$

$u = 0{,}04$

b) $f(x) = \begin{cases} \frac{32}{x^3} & \text{for } x \geq 4 \\ 0 & \text{for } x < 4 \end{cases}$

$F_X(x) \quad x \geq 4$

$F_X(x) = \int_4^x 32 \frac{1}{u^3} du = 32 \int_4^x u^{-3} du = 32 \left(\frac{1}{-2} u^{-2} \Big|_4^x \right)$

$= 32 \left(\left(\frac{1}{-2} \cdot \frac{1}{x^2} \right) - \left(\frac{1}{-2} \cdot \frac{1}{4^2} \right) \right) = 32 \left(\left(-\frac{1}{2x^2} \right) + \left(\frac{1}{32} \right) \right)$

$= 1 - \frac{32}{2x^2} = 1 - \frac{16}{x^2}$

$F_X = 0{,}95$

$1 - \frac{16}{x^2} = 0{,}95$

$1 - 0{,}95 = \frac{16}{x^2}$

$x = \sqrt{\frac{16}{0{,}05}}$

3 b)

$E[X] = \mu = \sum_x x\, f(x)$

$\sigma^2 = Var[X] = E[(X-\mu)^2] = \sum_x (X-\mu)^2 \cdot f(x)$

4 a) $f(t) = \begin{cases} 1{,}5 e^{-1{,}5t} & , t > 0 \\ 0 & \text{ellers} \end{cases}$

$F(t) = \int_0^t 1{,}5\, e^{-1{,}5s}\, ds = \left(-\frac{1{,}5}{1{,}5} e^{-1{,}5s} \Big|_0^t \right) = 1 - e^{-1{,}5t}$

$E[T] = \int_{-\infty}^{\infty} t \cdot 1{,}5 \cdot e^{-1{,}5t}\, dt = \left(-t e^{-1{,}5t} \Big|_0^\infty \right) + \int_0^\infty e^{-1{,}5t}\, dt$

$= \left(-\frac{1}{1{,}5} e^{-1{,}5t} \Big|_0^\infty \right) = \frac{1}{1{,}5} =$

6a)

$$F(x) = \begin{cases} 0 & x < 0 \\ \dfrac{1 - \cos(2x)}{2}, & 0 \le x \le \frac{\pi}{2} \\ 1 & x > \frac{\pi}{2} \end{cases}$$

$$f(x) = \sin(2x)$$

6c)

$$E[X^2] = \int_0^{\frac{\pi}{2}} x^2 \sin(2x)\,dx = \left(x^2 \cdot \left(-\tfrac{1}{2}\right)\cos(2x)\Big|_0^{\frac{\pi}{2}}\right) + \int_0^{\frac{\pi}{2}} x \cos(2x)\,dx$$

$$= \frac{1}{2}\left(\frac{\pi}{2}\right)^2 + \left(x\,\frac{1}{2}\sin(2x)\Big|_0^{\frac{\pi}{2}}\right) + \int_0^{\frac{\pi}{2}} \frac{1}{2}\sin(2x)\,dx$$

$$= \frac{\pi^2}{8} + \left(\frac{1}{4}\cos(2x)\Big|_0^{\frac{\pi}{2}}\right) = \frac{\pi^2}{8} - \frac{1}{4} - \frac{1}{4} = \frac{\pi^2 - 4}{8}$$

6d)

$$Var(X) = E[(X-\mu)^2] = \int_0^{\frac{\pi}{2}} \left(x - \frac{\pi}{4}\right)^2 \cdot \sin(2x)\,dx$$

$$= \left(\left(x - \frac{\pi}{4}\right)^2 \cdot \left(-\tfrac{1}{2}\right)\cos(2x)\Big|_0^{\frac{\pi}{2}}\right) + \int_0^{\frac{\pi}{2}} \left(x - \frac{\pi}{4}\right)\cos(2x)\,dx$$

$$= \left(-\frac{\pi^2}{16} \cdot \left(-\tfrac{1}{2}\right) \cdot (-1)\right) - \frac{\pi^2}{16} \cdot \left(-\tfrac{1}{2}\right) + \left(\left(x - \frac{\pi}{4}\right)\frac{1}{2}\sin(2x)\Big|_0^{\frac{\pi}{2}}\right) + \int_0^{\frac{\pi}{2}} \frac{1}{2}\sin(2x)\,dx$$

$$= -\frac{\pi^2}{32} + \frac{\pi^2}{16} + \left(-\frac{1}{4}\cos(2x)\Big|_0^{\frac{\pi}{2}}\right) = -\frac{\pi^2}{32} + \frac{1}{4} - \frac{1}{4} = \frac{16 - \pi^2}{32}$$