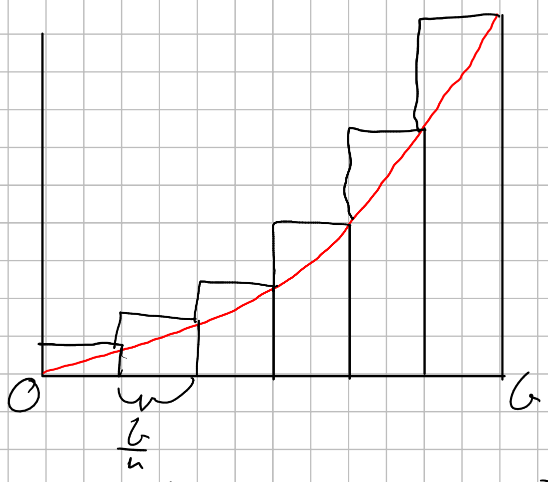
Rekker

1)



$f(x) = x^2$

~~rektangler~~ n

$$S = \left(\frac{b}{n}\right)^2 \cdot \left(\frac{b}{n}\right) + \left(\frac{2b}{n}\right)^2 \cdot \left(\frac{b}{n}\right) + \dots + \left(\frac{nb}{n}\right)^2 \cdot \left(\frac{b}{n}\right)$$

$$= \left(\frac{b}{n}\right)^3 + \left(1^2 + 2^2 + 3^2 + \dots + n^2\right)$$

$$= \left(\frac{b}{n}\right)^3 \sum_{k=1}^{n} k^2$$

2) $$100 = \sum_{k=1}^{n} k = (1 + 100) + (2 + 99) + (3 + 98)$$

$$= \sum_{k=1}^{n=100} (k + (n - k)) \quad ??$$

$$50 \cdot 101 = 5050$$

$$n^2 = \sum_{k=1}^{n} k^2 - \sum_{k-1}^{n} (k-1)^2 = \sum_{k=1}^{n} (2k - 1) = -n + 2\sum_{k=1}^{n} k$$

$$-10 + 2(1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10)$$

$$-10 + 2(55)$$

$$-10 + 110$$

$$\underline{100}$$

?

$$k^2 - (k-1)^2$$

$$k^2 \; (k^2 - 2k + 1)$$

$$k^2 - k^2 + 2k - 1$$

$$2k - 1$$

$$(1 + 100)(2 + 99)(3 + 98) = 50 \cdot 101$$

$$n^2 = -n + 2\sum_{k=1}^{n} k$$

$$n = -1 + \frac{2}{n}\sum_{k=1}^{n} k$$

$$\sum_{k=1}^{n} k =$$

$$n^2 = \sum_{k=1}^{n} k^2 - \sum_{k=1}^{n} (k-1)^2 = \sum_{k=1}^{n} (2k-1) = -n + 2\sum_{k=1}^{n} k$$

$$n^2 = -n + 2\sum_{k=1}^{n} k$$

$$2\sum_{k=1}^{n} k = n^2 + n$$

$$\sum_{k=1}^{n} k = \frac{1}{2}(n^2 + n)$$

$$n = 100$$

$$\sum_{k=1}^{100} k = \frac{1}{2}(10000 + 100)$$

$$= 5000 + 50$$

$$= 5050$$

3)

$$\int_0^n x^2 = \frac{b^3}{n^3}\sum_{k=1}^{n} k^2$$

$$3\left(\sum_{k=1}^{n} k^2\right) - 3\left(\sum_{k=1}^{n} k\right) + n = n^3$$

$$3\left(\sum_{k=1}^{n} k^2\right) = n^3 - n + 3\left(\sum_{k=1}^{n} k\right)$$

$$\sum_{k=1}^{n} k^2 = \frac{1}{3}(n^3 - n) + \sum_{k=1}^{n} k \qquad \sum_{k=1}^{n} k = \frac{1}{2}(n^2 + n)$$

$$\sum_{k=1}^{n} k^2 = \frac{1}{3}(n^3 - n) + \frac{1}{2}(n^2 + n)$$

$$= \frac{1}{3}n^3 + \frac{1}{2}n^2 - \frac{1}{3}n + \frac{1}{2}n$$

$$= \frac{1}{3}n^3 + \frac{1}{2}n^2 + \frac{1}{6}n$$

$$\int_0^b x^2 = \frac{b^3}{n^3}\sum_{k=1}^{n} k^2$$

$$= \frac{b^3}{n^3}\left(\frac{1}{3}n^3 + \frac{1}{2}n^2 + \frac{1}{6}n\right)$$

$$= \frac{b^3}{3} + \frac{1}{2}\cdot\frac{b^3}{n} + \frac{1}{6}\cdot\frac{b^3}{n^2}$$

$$\lim_{n\to\infty} \frac{b^3}{3} + \frac{1}{2}\cdot\frac{b^3}{n} + \frac{1}{6}\cdot\frac{b^3}{n^2} = \underline{\underline{\frac{b^3}{3}}}$$

4) $s_N = \sum_{n=0}^{N} x^n = 1 + x + x^2 + \dots + x^N$

$$(1-x)s_N = (1-x)\sum_{n=0}^{N} x^n$$

$$= (1 + x + x^2 + \dots + x^N) - (x + x^2 + \dots + x^N + x^{N+1})$$

$$= 1 + x^{N+1}$$

$$\underline{\underline{s_N = \sum_{n=0}^{N} x^n = \frac{1 + x^{N+1}}{1-x}}}$$

Dersom $N \to \infty$

$$\sum_{n=0}^{\infty} x^n \begin{cases} \frac{1}{1-x}, & |x| < 1 \\ \infty, & |x| > 1 \\ \infty, & x = 1 \\ ?, & x = -1 \end{cases}$$

5) $\sum_{n=1}^{\infty} z_n = z_1 + z_2 + z_3 + \dots$

$\Leftrightarrow$

$s_N = \sum_{k=}^{n} z_k = z_1 + z_2 + z_3 + \dots + z_N$

$L = 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \dots$

$2L = 2 + 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \dots = 2 + L$

$2L = 2 + L$

$2L - L = 2$

$L = 2$

$$L = 1 + 2 + 4 + 8 + \dots = ?$$

$$2L = 2 + 4 + 8 + \dots = L - 1$$

$$2L = L - 1$$

$$L = -1$$ SUS

4 v 2)

$$\sum_{n=0}^{\infty} (-1)^n = 1 - 1 + 1 - 1 + 1 - 1 + 1$$

$$x = 1 + 1 + 1 + 1 \to \infty$$ Ikke konvergent ?

$$y = -1 - 1 - 1 - 1 \to -\infty$$ Ikke konvergent

7) $$S = \sum_{n=0}^{\infty} \frac{1}{(1+i)^n} = 1 + \frac{1}{1+i} + \frac{1}{2i} + \frac{1}{-2+2i} + \frac{1}{-4} + \frac{1}{-4-4i} + \dots +$$

$$(1 - (1+i))S = (1 - (1+i)) \sum_{n=0}^{N} \frac{1}{(1+i)^n}$$

$$= 1 + \frac{1}{1+i} + \frac{1}{(1+i)^2} + \frac{1}{(1+i)^3} + \dots + \frac{1}{(1+i)^N}$$

$$-(1+i) - 1 - \frac{1}{1+i} - \frac{1}{(1+i)^2} - \dots - \frac{1}{(1+i)^{N-1}}$$

$$(-i)S = -(1+i) + \frac{1}{(1+i)^N}$$

$$S = \frac{1+i}{i} - \frac{1}{i(1+i)^N}$$

$$S = -1 + i + \frac{i}{(1+i)^N}$$

$$S = -1 + i\left(\frac{1}{(1+i)^N}\right)$$? ?

$$s = \sum_{n=0}^{\infty} \frac{1}{(1+i)^n}$$

$$\frac{1}{(1+i)} \frac{(1-i)}{(1-i)} = \frac{1-i}{1^2-(-1)} = \frac{1}{2} - \frac{1}{2} i$$

$$\left|\frac{1}{2} - \frac{1}{2} i\right| = \sqrt{\left(\frac{1}{2}\right)^2 + \left(-\frac{1}{2}\right)^2} = \sqrt{\frac{1}{4} + \frac{1}{4}} = \sqrt{\frac{1}{2}} = \frac{\sqrt{2}}{2} < 1$$

$$s = \frac{1}{1 - \frac{1}{1+i}} = \frac{1}{\frac{1+i-1}{1+i}} = \frac{1}{\frac{i}{1+i}} = \frac{1+i}{i} = 1 + \frac{1}{i} = 1 + \frac{i}{-1} = \underline{\underline{1-i}}$$

$$\sum_{k=n}^{\infty} x^k = x^n + x^{n+1} + x^{n+2} + \ldots = k^n (1 + x^1 + x^2 + x^3 + \ldots) = k^n \sum_{k=0} x^k = \frac{x^n}{1-x}$$

Retard

$$\sum_{k=n}^{\infty} x^k = x^n + x^{n+1} + x^{n+2} + \ldots = x^n (1 + x + x^2 + x^3) = x^n \cdot \sum_{k=0} x^k = x^n \cdot \frac{1}{1-x}$$

8) $$\sum_{n=1}^{\infty} \frac{1}{(1+i)^n} = \frac{1}{(1+i)} \cdot \sum_{n=0} \frac{1}{(1+i)^n} = \frac{1}{1+i} \cdot \frac{1}{1 - \frac{1}{1+i}}$$

$$= \frac{1}{1+i} \cdot \frac{1}{\frac{i}{1+i}} = \frac{1}{1+i} \cdot \frac{1+i}{i} = \frac{1}{i} = \underline{\underline{-i}}$$

9) $$\sum_{n=10} \frac{1}{(1+i)^n} = \frac{1}{(1+i)^{10}} \sum_{n=0} \frac{1}{(1+i)^n} = \frac{1}{(1+i)^{10}} \cdot \frac{1+i}{i} = \frac{1}{i(1+i)^9} = \frac{1}{-16+16i}$$

$$= \frac{1}{-16} \cdot \frac{1}{1-i} = \frac{1}{-16} \cdot \frac{1+i}{2} = \underline{\underline{-\frac{1}{32}(1+i)}}$$

10) $h + \frac{3}{4}h + \frac{3}{4}h + \left(\frac{3}{4}\right)^2 h + \left(\frac{3}{4}\right)^2 h + \dots$

$= h + 2 \cdot \frac{3}{4}h + 2 \cdot \left(\frac{3}{4}\right)^2 h + \dots$

$= h + 2h\left(\frac{3}{4} + \left(\frac{3}{4}\right)^2 + \left(\frac{3}{4}\right)^3 + \dots\right)$

$= h + 2h \sum_{n=1}^{\infty} \left(\frac{3}{4}\right)^n = h + 2h \cdot \frac{3}{4} \cdot \frac{1}{1-\frac{3}{4}} = h + 2h \cdot \frac{3}{4} \cdot 4 = h + 6h = 7h$

$h = 3$

$s = 7 \cdot 3 = \underline{\underline{21}}$

11) $O_0 = 8a_0$

$2a_1 = \sqrt{a_0^2 + a_0^2}$

$O_1 = 8a_1$

$O_1 = 4\sqrt{a_0^2 + a_0^2}$

$= 4\sqrt{\left(\frac{1}{8}O_0\right)^2 + \left(\frac{1}{8}O_0\right)^2}$

$= 4\sqrt{\frac{O_0^2}{16} + \frac{O_0^2}{16}}$

$= 4\sqrt{\frac{1}{8}O_0^2}$

$= 4 \cdot \frac{O_0}{\sqrt{8}} = 4 \cdot \frac{O_0}{2\sqrt{2}} = 2O_0 \cdot \frac{1}{\sqrt{2}} = \frac{2O_0\sqrt{2}}{2} = O_0\sqrt{2}$

$O = \sum_{n=0}^{\infty} \sqrt{2}^n = \infty$

12) $A_0 = \frac{l_0 \cdot l_0}{2} = \frac{1}{2} l_0^2$

$l_1 = \frac{1}{3} l_0$

$A_1 = \frac{1}{2} l_1^2 = \frac{1}{2}\left(\frac{1}{3} l_0\right)^2 = \frac{1}{2} \frac{l_0^2}{9} = \frac{1}{18} l_0^2$

$A = \sum_{n=0}^{\infty} \frac{1}{2} l_0^2 \cdot \left(\frac{1}{9}\right)^n = \frac{1}{2} l_0^2 \cdot \frac{9}{8} = \underline{\underline{l_0^2 \cdot \frac{9}{16}}}$

$O_0 = 3 l_0$

$O_1 = 3 \cdot l_1 \cdot 3$

$= 3\left(\frac{1}{3} l_0\right) \cdot 3$

$= 3 l_0$

$O = \sum_{n=0}^{\infty} 3 l_0 = \infty$

H) $\{y\}, \{z\}$, $y_n \leq z$ for alle n

Σy divergerer $\Rightarrow$ Σz divergerer

Σz konvergerer $\Rightarrow$ Σy konvergerer

$\sum_{n=1}^{\infty} \frac{1}{n} = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6} + \frac{1}{7} + \frac{1}{8} + \frac{1}{9} + \frac{1}{10} + \frac{1}{11} + \frac{1}{12}$

$> 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{4} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{16} + \frac{1}{16} \ldots$

$> 1 + \frac{1}{2} + \frac{1}{2} + \quad \frac{1}{2} \rightarrow \infty$

$\underline{\underline{\sum_{n=1}^{\infty} \frac{1}{n} \rightarrow \infty}}$

$$\sum_{n=1}^{\infty} \frac{1}{n^2} = 1 + \frac{1}{4} + \frac{1}{9} + \frac{1}{16} + \frac{1}{25} + \frac{1}{36} + \frac{1}{49} + \frac{1}{64} + \frac{1}{81} + \ldots$$

$$= \frac{\pi^2}{6} ??$$

1 3)

$$\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}} = \frac{1}{1} + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} + \frac{1}{2} + \frac{1}{\sqrt{5}} + \frac{1}{\sqrt{6}} + \frac{1}{\sqrt{7}} + \frac{1}{2\sqrt{2}}$$

$$= 1 + \frac{\sqrt{2}}{2} + \frac{\sqrt{3}}{3} + \frac{2}{4} + \frac{\sqrt{5}}{5} + \frac{\sqrt{6}}{6} + \frac{\sqrt{7}}{7} + \frac{2\sqrt{2}}{8}$$

$$> 1 + \frac{1}{2} + \frac{1}{3} + \frac{2}{4} + \frac{1}{5} + \ldots$$

$$= \infty$$

$\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}} = \infty$ Divergent

1 4)

$\sum_{n=1}^{\infty} \frac{1}{n^3} < \sum_{n=1}^{\infty} \frac{1}{n^2}$ Konvergerer

1 5)

$$\sum_{n=1}^{\infty} \frac{1}{n(n+1)} = \frac{1}{2} + \frac{1}{6} + \frac{1}{11} + \frac{1}{20} + \ldots$$

$$< \sum_{n=1}^{\infty} \frac{1}{n^2} = 1 + \frac{1}{4} + \frac{1}{9} = \frac{\pi^2}{6}$$

$\sum_{n=1}^{\infty} \frac{1}{n(n+1)}$ Konvergerer

1 6)

$$\sum_{n=1}^{\infty} \frac{1}{n\,3^n} = \frac{1}{3} + \frac{1}{18} + \frac{1}{81} + \ldots$$

$$> \sum_{n=1}^{\infty} \frac{1}{n^2} = 1 + \frac{1}{4} + \frac{1}{9} = \frac{\pi^2}{6}$$

$\sum_{n=1}^{\infty} \frac{1}{n\,3^n} \to$ Konvergerer

1 2)

$$\sum_{n=1}^{\infty} \frac{1}{n+\sqrt{n}} = \frac{1}{2} + \frac{1}{2+\sqrt{2}} + \frac{1}{3+\sqrt{3}}$$

$$> \sum_{n=1}^{\infty} \frac{1}{n} = \quad 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} = \infty$$

$$\sum_{n=1}^{\infty} \frac{1}{n+\sqrt{n}} = \infty$$