Øving 4

Oppgave 1)

$m = \rho A h \qquad w_{ut} = k h$

a) $$\frac{d}{dt} m = w_{inn} - w_{ut}$$

$$\frac{d}{dt}(\rho A h) = w_{inn} - w_{ut}$$

$$\frac{d}{dt} h = \frac{1}{\rho A}(w_{inn} - w_{ut})$$

$$w_{ut} = k h$$

$$\frac{d}{dt} h = \frac{1}{\rho A}(w_{inn} - k h)$$

$$\frac{d}{dt} h = -\frac{k}{\rho A} h + \frac{1}{\rho A} w_{inn}$$

$$\frac{d}{dt} h + \frac{k}{\rho A} h = \frac{1}{\rho A} w_{inn}$$

Anta w_{inn} er konstant

Anta ρ er konstant

Anta A er konstant

$$\frac{d}{dt}h + \frac{k}{\rho A}h = \frac{1}{\rho A}w_{inn} \quad \Big| \cdot e^{\frac{k}{\rho A}t}$$

$$\frac{d}{dt}h e^{\frac{k}{\rho A}} + \frac{k}{\rho A} h e^{\frac{k}{\rho A}t} = \frac{1}{\rho A} w_{inn} e^{\frac{k}{\rho A}t}$$

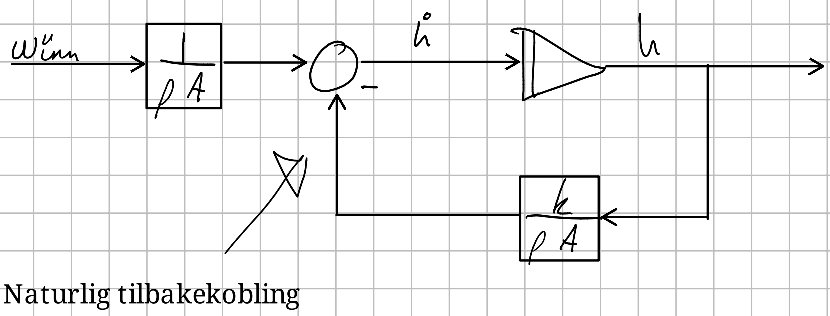
$$\frac{d}{dt}\left(h e^{\frac{k}{\rho A}t}\right) = \frac{1}{\rho A} w_{inn} e^{\frac{k}{\rho A}t} \quad \Big| \cdot \int dt$$

$$h e^{\frac{k}{\rho A}t} = \frac{\rho A}{k}\frac{1}{\rho A} w_{inn} e^{\frac{k}{\rho A}t} + C \quad \Big| \cdot e^{-\frac{k}{\rho A}t}$$

$$h(t) = \frac{1}{k} w_{inn} + C e^{-\frac{k}{\rho A}t}$$

b)

$$\frac{d}{dt}h = -\frac{k}{\rho A}h + \frac{1}{\rho A}w_{inn}$$



c)

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Om $w_{inn} = 0$, så vil $t \to \infty$, $h \to 0$. Dette er fordi all massen i tanken vil bare renne ut og bli tom om ingenting annet renner inn.

Dette kan vi se på vår modell. Ettersom $w_{inn} = 0$, så vil ifølge modellen

$$\frac{d}{dt}h = -\frac{k}{\rho A}h$$

Dette fører til att endringen $\dot{h}$ vil være negativ helt til $h = 0$, altså att nivået tilsvarer 0.

Modellen vil bli stabil fordi den går mot en bestemt tilstand $h = 0$.

d)

Oppgave 2)

b) Stasjonæravvik: a

$$a = h_r - h(\infty)$$

$$\frac{d}{dt}h = -\frac{k}{\rho A}h + \frac{1}{\rho A}w_{inn}$$

$$w_{inn} = K_p(h_r - h)$$

$$\frac{d}{dt}h = -\frac{k}{\rho A}h + \frac{1}{\rho A}K_p(h_r - h)$$

$$\frac{d}{dt}h = -\frac{k}{\rho A}h + \frac{K_p}{\rho A}h_r - \frac{K_p}{\rho A}h \qquad -\frac{k}{\rho A} - \frac{K_p}{\rho A} = -\frac{1}{\rho A}(k + K_p)$$

$$\frac{d}{dt}h = \left(-\frac{k + K_p}{\rho A}\right)h + \frac{K_p}{\rho A}h_r$$

$$\frac{d}{dt}h + \frac{k + K_p}{\rho A}h = \frac{K_p}{\rho A}h_r \qquad \lambda = \frac{k + K_p}{\rho A} \qquad \beta = \frac{K_p}{\rho A}h_r$$

$$\frac{d}{dt}h + \lambda h = \beta \quad \Big| \cdot e^{\lambda t}$$

$$\frac{d}{dt}h\, e^{\lambda t} + \lambda h e^{\lambda t} = \beta e^{\lambda t}$$

$$\frac{d}{dt}\left(h e^{\lambda t}\right) = \beta e^{\lambda t} \quad \Big| \cdot \int dt$$

$$h e^{\lambda t} = \frac{1}{\lambda}\beta e^{\lambda t} + C \quad \Big| \cdot e^{-\lambda t}$$

$$h(t) = \frac{1}{\lambda}\beta + C e^{-\lambda t} \qquad \lambda = \frac{k + K_p}{\rho A} \qquad \beta = \frac{K_p}{\rho A}h_r$$

$$h(t) = \frac{\rho A}{k + K_p} \cdot \frac{K_p}{\rho A}h_r + C e^{-\frac{k + K_p}{\rho A}t}$$

$$h(t) = \frac{K_p}{k + K_p}h_r + C e^{-\frac{k + K_p}{\rho A}t}$$

$h(0) = h_0$

$\frac{K_p}{k + K_p} h_r + C = h_0$

$C = h_0 - \frac{K_p}{k + K_p} h_r$

$h(t) = \frac{K_p}{k + K_p} h_r + \left(h_0 - \frac{K_p}{k + K_p} h_r\right) e^{-\frac{k + K_p}{\rho A} t}$

$a = h_r - h(\infty)$

$a = h_r - \frac{K_p}{k + K_p} h_r$

$a = h_r\left(1 - \frac{K_p}{k - K_p}\right)$

$K_p = 100 \quad k = 1 \quad h_r = 0{,}5$

$a = 0{,}5 \cdot \left(1 - \frac{100}{1 + 100}\right)$

$a = 4{,}95 \cdot 10^{-3} \approx \underline{\underline{0{,}005}}$

f)

$\frac{d}{dt} x + d x = u, \quad u = K_p(r - x) + K_i \int (r - x)\, dt$

$\frac{d}{dt}$