1 - 2 - Tall

0) $x(t) = e^{\lambda t}$

$\dot{x}(t) = \lambda e^{\lambda t}$

$\ddot{x}(t) = \lambda^2 e^{\lambda t}$

$\ddot{x}(t) + x(t) = 0$

$\lambda^2 e^{\lambda t} + e^{\lambda t} = 0$

$\lambda^2 + 1 = 0$

$\lambda^2 = -1$

$\lambda = \underline{\pm i}$

2) $\sqrt{-4} = \pm 2i$

3) $x^2 + 2x + 2 = 0$

$$x = \frac{-2 \pm \sqrt{4 - 4(2)(1)}}{2} = \frac{-2 \pm \sqrt{-4}}{2} = \frac{-2 \pm 2i}{2}$$

$x = -1 \pm i$

$p(x) = (x + 1 + i)(x + 1 - i)$

4) $z = 2 + 3i \qquad w = 4 + 5i$

$z + w = 2 + 3i + 4 + 5i$
$= \underline{\underline{6 + 8i}}$

$z - w = 2 + 3i - 4 - 5i$
$= -2 - 2i$

$z \cdot w = (2 + 3i)(4 + 5i)$
$= 8 + 10i + 12i - 15$
$= -7 + 22i$

$$\frac{z}{w} = \frac{(2+3i)}{(4+5i)} = \frac{(2+3i)(4-5i)}{(4+5i)(4-5i)} = \frac{8 - 10i + 12i + 15}{16 + 25}$$

$$= \frac{23 + 2i}{39} = \frac{23}{39} + \frac{2}{39} i$$

4) $z = a + bi \qquad w = c + di$

$z + w = a + bi + c + di$
$= (a + c) + (b + d)i$

$$z - w = a + bi - c - di$$
$$= (a - c) + (b - d)i$$

$$z \cdot w = (a + bi)(c + di) = ac + adi + bci - bd$$
$$= (ac - bd) + (ad + bc)i$$

$$\frac{z}{w} = \frac{a + bi}{c + di} = \frac{(a + bi)(c - di)}{(c + di)(c - di)}$$

$$= \frac{ac - adi + bci + bd}{c^2 + d^2}$$

$$= \frac{ac + bd + (bc - ad)i}{c^2 + d^2}$$

$$= \frac{ac + bd}{c^2 + d^2} + \frac{bc - ad}{c^2 + d^2} i$$

6) $z = a + bi \quad w = c + di$

$$\overline{z + w} = \overline{z} + \overline{w}$$

$$z + w = a + bi + c + di$$
$$= (a + c) + (b + d)i$$

$2+2$

$$\overline{z + w} = (a + c) - (b + d)i$$

$$\overline{z} + \overline{w} = a - bi + c - di$$
$$= (a + c) - (b + d)i$$

$$\underline{\underline{\overline{z + w} = \overline{z} + \overline{w}}}$$

$$\overline{z - w} = \overline{z} - \overline{w} \quad ¿?$$

$$z - w = (a + bi) - (c + di) = a + bi - c - di$$
$$= (a - c) + (b - d)i$$

$$\overline{z - w} = (a - c) - (b - d)i$$

$$\overline{z} - \overline{w} = (a - bi) - (c - di) = a - bi - c + di$$
$$= (a - c) - (b - d)i$$

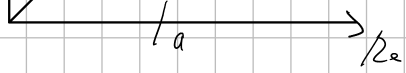
$$\underline{\underline{\overline{z - w} = \overline{z} - \overline{w}}}$$

7) $\frac{1}{z} = \frac{\overline{z}}{|z|^2} \quad ¿?$

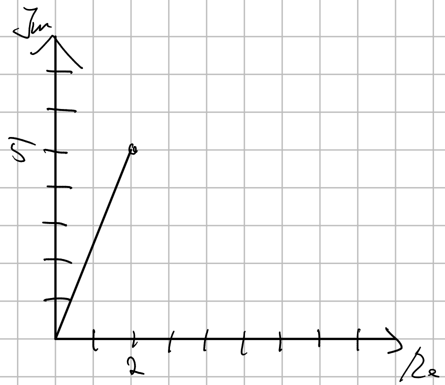
$$\frac{1}{z} = \frac{1}{a + bi} = \frac{a - bi}{a^2 + b^2}$$

$$\frac{\overline{z}}{|z|^2} = \frac{a - bi}{a^2 + b^2}$$

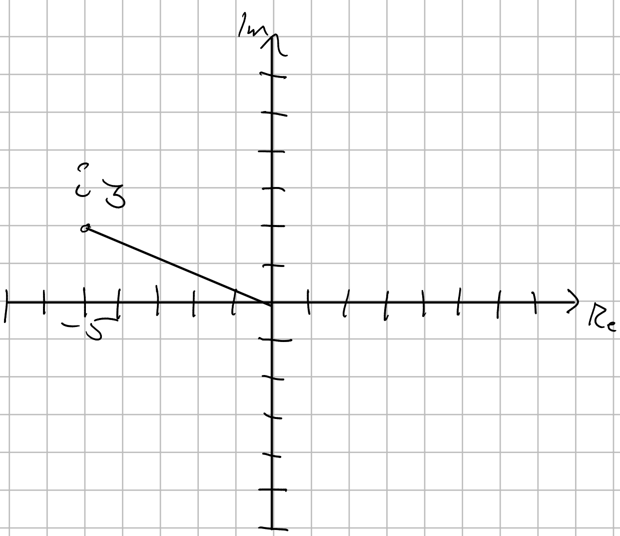
$$\frac{1}{z} = \frac{\overline{z}}{|z|^2}$$



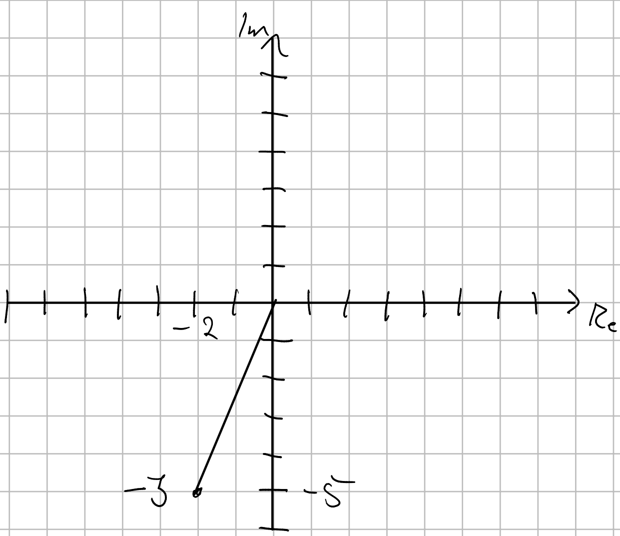
8)



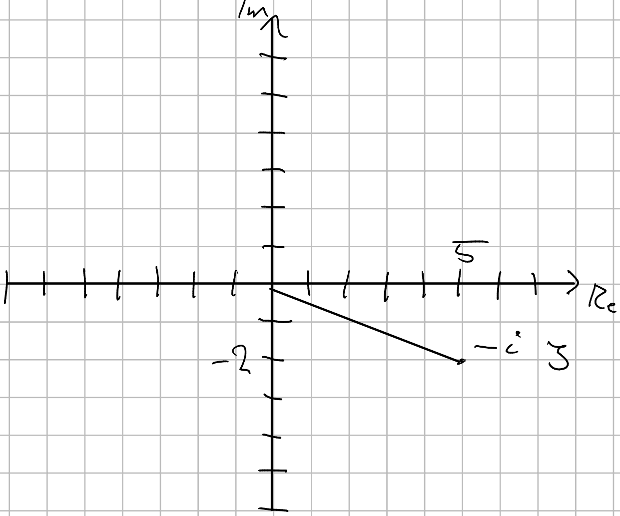
$z = 2 + 5i$



$iz = -5 + 2i$



$-z = -2 - 5i$



$-iz = 5 - 2i$

Rotasjon!

9) $z = 1 + \sqrt{3}\, i$

$\frac{z}{|z|} = \frac{1+\sqrt{3}\, i}{1-3} = -\frac{1}{2} - \frac{\sqrt{3}}{2} i$

$z^2 = (a + b i)^2$
$= a^2 + 2ab i - b^2$
$= a^2 - b^2 + 2ab i$

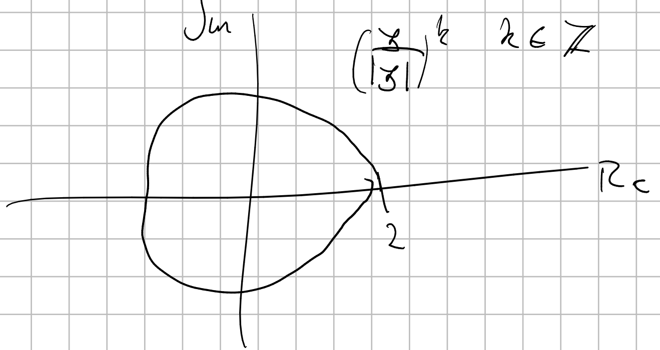
$\left(\frac{z}{|z|}\right)^0 = 1$

$\left(\frac{z}{|z|}\right)^1 = -\frac{1}{2} - \frac{\sqrt{3}}{2} i$

$\left(\frac{z}{|z|}\right)^2 = \left(\frac{1}{4} - \frac{3}{4}\right) + 2 \cdot \left(-\frac{1}{2}\right)\left(-\frac{\sqrt{3}}{2}\right) i$

$= -\frac{2}{4} + \frac{2\sqrt{3}}{2} i$

$\left(\frac{z}{|z|}\right)^k \quad k \in \mathbb{Z}$



10)

$z = \frac{1}{2} + \frac{\sqrt{3}}{2} i \qquad w = 1 + i$

$zw = \left(\frac{1}{2} + \frac{\sqrt{3}}{2} i\right)(1 + i) = \frac{1}{2} + \frac{1}{2} i + \frac{\sqrt{3}}{2} i - \frac{\sqrt{3}}{2}$

$= \frac{1}{2} - \frac{\sqrt{3}}{2} + \frac{1}{2} i + \frac{\sqrt{3}}{2} i$

$= \frac{1 - \sqrt{3}}{2} + \frac{1 - \sqrt{3}}{2} i$

$= \left(\frac{1 - \sqrt{3}}{2}\right)(1 + i)$

$z^k w \quad k \in \mathbb{Z}$ flbe nilber

11) $|z^2| = |z|^2$

$z^2 = (a + b i)^2$
$= a^2 - b^2 + 2ab i$

$|z^2| = \sqrt{(a^2 - b^2)^2 + (2ab)^2}$
$= \sqrt{a^4 - 2a^2b^2 + b^4 + 4a^2b^2}$

$= \sqrt{a^4 + 2a^2b^2 + b^4}$

$= \sqrt{(a^2+b^2)^2}$

$= \underline{a^2+b^2}$

$|z|^2 = (\sqrt{a^2+b^2})^2$

$= \underline{a^2+b^2}$

$\underline{|z^2| = |z|^2}$

$|z|^3 = (\sqrt{a^2+b^2})^3 = (a^2+b^2)^{\frac{3}{2}} = ((a^2+b^2)^3)^{\frac{1}{2}}$

$= ((a^4+2a^2b^2+b^4)(a^2+b^2))^{\frac{1}{2}}$

$= (a^8 + a^4b^2 + 2a^4b^2 + 2a^2b^4 + a^2b^4 + b^8)^{\frac{1}{2}}$

$= (a^8 + 3a^4b^2 + 3a^2b^4 + b^8)^{\frac{1}{2}}$

$|z^3| =$ fakk off

1 2)

$\lambda = i$

$\dot{x} = ix$

$x = ce^{it}$ $\quad \dot{x} = ice^{it}$

$ice^{it} = ice^{it}$

$\dot{y} = iy$

$y(t) = \cos t + i \sin t$

$\dot{y}(t) = -\sin t + i\cos t$

$-\sin t + i\cos t = i(\cos t + i\sin t)$

$-\sin t + i\cos t = i\cos t - \sin t$

$x = ce^{it}$ $\quad y = \cos t + i\sin t$

$\underline{e^{it} = \cos t + i\sin(t)}$

1 3)

$$f(\theta) = \frac{\cos\theta + i\sin\theta}{e^{i\theta}}$$

$$\dot{f}(\theta) = \frac{(\cos\theta + i\sin\theta)'(e^{i\theta}) - (\cos\theta + i\sin\theta)(e^{i\theta})'}{(e^{i\theta})^2}$$

$$= \frac{(-\sin\theta + i\cos\theta)(e^{i\theta}) - (\cos\theta + i\sin\theta)(ie^{i\theta})}{e^{2i\theta}}$$

$$= \frac{(-\sin\theta + i\cos\theta) - (\cos\theta + i\sin\theta)\, i}{e^{i\theta}}$$

$$= \frac{-\sin\theta + i\cos\theta - i\cos\theta + \sin\theta}{e^{i\theta}}$$

$$= \frac{0}{e^{i\theta}} = 0\ ?$$

$$\dot{f}(\theta) = \frac{(\cos\theta + i\sin\theta)'(e^{i\theta}) - (\cos\theta + i\sin\theta)(e^{i\theta})'}{(e^{i\theta})^2}$$

$$= \frac{(-\sin\theta + i\cos\theta)(e^{i\theta}) - (\cos\theta + i\sin\theta)(ie^{i\theta})}{e^{2i\theta}}$$

$$= \frac{(-\sin\theta + i\cos\theta)(e^{i\theta}) - (i\cos\theta - \sin\theta)(e^{i\theta})}{e^{2i\theta}}$$

$$= 0$$

$$\int \dot{f}(\theta) = \frac{\cos\theta + i\sin\theta}{e^{i\theta}} = C$$

$$\cos\theta + i\sin\theta = Ce^{i\theta}$$

1 4) $e^{i(\alpha+\theta)} = e^{i\alpha}e^{i\theta}$?

$$(\cos\alpha + i\sin\alpha)(\cos\theta + i\sin\theta)$$

$$= \cos\alpha\cos\theta + i\cos\alpha\sin\theta + i\cos\theta\sin\alpha - \sin\alpha\sin\theta$$

$$= \underbrace{\cos\alpha\cos\theta - \sin\alpha\sin\theta}_{\text{Trig identitee}} + i\underbrace{(\cos\alpha\sin\theta + \cos\theta\sin\alpha)}_{\text{Trig identitee}}$$

$$= \cos(\alpha+\theta) + i\sin(\alpha+\theta)$$

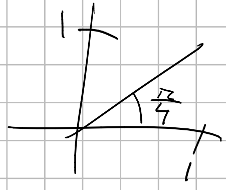
$$\underline{\underline{= e^{i(\alpha+\theta)}}}$$

1 5)

$z = r(\cos\theta + i\sin\theta) = re^{i\theta}$

$z = 1 + i$

$r = \sqrt{2}$ $\quad \theta = \frac{\pi}{4}$



$z = \sqrt{2}(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4})$

$z = \sqrt{2}e^{\frac{\pi}{4}i}$

$w = 1 + \sqrt{3}i$

$r = \sqrt{4} = 2$

$a = r\cos\theta$
$b = r\sin\theta$

$a = 2\cos\theta = 1$
$\cos\theta = \frac{1}{2}$
$\theta = \frac{\pi}{3}$

$w = 2e^{\frac{\pi}{3}i}$

1 6) $z = \sqrt{2}e^{\frac{\pi}{4}i}$ $\quad w = 2e^{\frac{\pi}{3}i}$

$zw = \sqrt{2}e^{\frac{\pi}{4}i}\, 2e^{\frac{\pi}{3}i} = 2\sqrt{2}e^{i\frac{7\pi}{12}}$

$\frac{z}{w} = \frac{\sqrt{2}}{2}e^{i(-\frac{\pi}{12})}$

1 7) $e^{\frac{\pi}{2}i} = \cos\frac{\pi}{2} + i\sin\frac{\pi}{2} = 0 + i = i$

$e^{\pi i} = \cos\pi + i\sin\pi = -1$

$e^{\frac{3\pi}{2}i} = \cos\frac{3\pi}{2} + i\sin\frac{3\pi}{2} = -i$

$e^{2\pi i} = \cos 2\pi + i\sin 2\pi = 1$

1 8) $e^{-i\theta} = \cos\theta - \sin\theta$?

$e^{-i\theta} = \cos(-\theta) + i\sin(-\theta)$
$= \cos\theta - i\sin\theta$

19) $\cos\theta = \frac{1}{2}(e^{i\theta} + e^{-i\theta})$?

$$\frac{1}{2}(\cos\theta + i\sin\theta + \cos(-\theta) + i\sin(-\theta))$$
$$= \frac{1}{2}(\cos\theta + i\sin\theta + \cos\theta - i\sin\theta)$$
$$= \frac{1}{2}(2\cos\theta)$$
$$= \cos\theta$$

20) $\sin\theta = \frac{1}{2i}(e^{i\theta} - e^{-i\theta})$ $\quad \frac{1}{2i}(e^{-\theta} - e^{\theta}) \Rightarrow e$

$$= \frac{1}{2i}(\cos\theta + i\sin\theta - (\cos\theta - i\sin\theta))$$

$$= \frac{1}{2i}(\cos\theta + i\sin\theta - \cos\theta + i\sin\theta)$$

$$= \frac{1}{2i}(2i\sin\theta)$$
$$= \sin\theta$$

22) $z = re^{i\theta}$ $\quad \bar{z} = re^{i(2\pi - \theta)}$?

$$\bar{z} = \overline{re^{i\theta}} = \overline{r(\cos\theta + i\sin\theta)} = r(\cos\theta - i\sin\theta)$$
$$= r(\cos(-\theta) + i\sin(-\theta)) = re^{-i\theta}$$

23) $\cosh\theta = \frac{e^{\theta} + e^{-\theta}}{2}$ $\quad \sinh\theta = \frac{e^{\theta} - e^{-\theta}}{2}$

$\cosh(i\theta) = \cos\theta$

$\cosh(i\theta) = \frac{e^{i\theta} + e^{-i\theta}}{2} = \cos\theta$ $\quad$ oppg 19

$\sinh(i\theta) = \frac{e^{i\theta} - e^{-i\theta}}{2} = i\sin\theta$ $\quad$ oppg 20

$\cos(a+bi) = \cos a \cosh b - i \sin a \sinh b$?

$$\boxed{\cosh\theta = \frac{e^{\theta}+e^{-\theta}}{2} \qquad \sinh\theta = \frac{e^{\theta}-e^{-\theta}}{2}}$$

$$\cos(a+bi) = \cos(a)\cos(bi) - \sin(a)\sin(bi)$$

$$= \cos(a)\left(\frac{e^{-b}+e^{b}}{2}\right) - \sin(a)\left(\frac{1}{2i}\left(e^{-b}-e^{b}\right)\right)$$

$$= \cos(a)\cosh(b) - \sin(a)\left(\frac{i}{-2}\left(e^{-b}-e^{b}\right)\right)$$

$$\left(i\left(\frac{e^{b}-e^{-b}}{2}\right)\right)$$

$$\underline{\underline{= \cos(a)\cosh(b) - i\sin(a)\sinh(b)}}$$

$\sin(a+bi) = \sin a \cosh b + i \cos a \sinh b$

$$\sin(a+bi) = \sin(a)\cos(bi) + \cos(a)\sin(bi)$$

$$\cos(bi) = \frac{e^{-b}+e^{b}}{2} = \cosh(b)$$

$$\sin bi = \left(i\left(\frac{e^{b}-e^{-b}}{2}\right)\right) = i\sinh b$$

$$\underline{\underline{\sin(a+bi) = \sin(a)\cosh(b) + i\cos(a)\sinh(b)}}$$

2 4) $z^5 = -1$

$$z^5 = e^{i(\pi + 2m\pi)}$$

$$\sqrt[5]{z^5} = -1$$

$$e^{i\frac{1}{5}(\pi+2m\pi)} = (-1)^{\frac{1}{5}}$$

$m = 0, 1, 2, 3, 4, 5$

$e^{i\frac{\pi}{5}} = (-1)^{\frac{1}{5}}$, $e^{i\frac{3\pi}{5}} = (-1)^{\frac{1}{5}}$, osv...

2 5) $f(x) = p x^2 + q x + r$

$f(a+bi) = p(a+bi)^2 + (a+bi)q + r$

$= p(a^2 - b^2 + 2abi) + (a+bi)q + r$

$=$

$f(z) = 0$

$px^2 + qx + r = 0$

$x = \frac{-q \pm \sqrt{q^2 - 4pr}}{2p}$

$x = -\frac{q}{2p} \pm \frac{\sqrt{q^2 - 4pr}}{2p}$

Reell del   Imaginær del ??

Polynom Reelle koeffisienter $p(z)$ & $p(\bar{z}) = 0$ $\overline{z+w} = \bar{z} + \bar{w}$

$p(z) = a_0 z^0 + a_1 z^1 + a_2 z^2 + \dots + a_n z^n$

$a \in \mathbb{R}$ for hver j, $z \in \mathbb{C}$, slik at

$p(z) = 0 \Rightarrow p(\bar{z}) = 0$

$z, w \in \mathbb{C}$, $\overline{z+w} = \bar{z} + \bar{w}$

$\overline{zw} = \bar{z} \cdot \bar{w}$

$z = \bar{z}$ sier at $z \in \mathbb{R}$

$a + bi = a - bi$

$0 = \bar{0} = \overline{p(z)} = \overline{a_0 z^0 + a_1 z^1 + a_2 z^2 + \dots a_n z^n}$

$= \overline{a_0 z^0} + \overline{a_1 z^1} + \overline{a_2 z^2} + \dots + \overline{a_n z^n}$

$= \bar{a}_0 \overline{z^0} + \bar{a}_1 \overline{z^1} + \bar{a}_2 \overline{z^2} + \dots + \bar{a}_n \overline{z^n}$

$= \bar{a}_0 \bar{z}^0 + \bar{a}_1 \bar{z}^1 + \bar{a}_2 \bar{z}^2 + \dots + \bar{a}_n \bar{z}^n$

$= a_0 + a_1 \bar{z}^1 + a_2 \bar{z}^2 + \dots + a_n \bar{z}^n$

$= p(\bar{z})$

$p(\overline{z}) = 0$

26)

$p(x) = x^3 + x^2 + x - 3 \qquad z = a + bi$

$p(x) = 0$

$p(z) = z^3 + z^2 + z - 3 = 0$

$(a + bi)^3 + (a + bi)^2 + (a + bi) - 3 = 0$

$(a + bi)^3 + (a + bi)^2 + (a + bi) = 3$

$(a + bi)(a^2 - b^2 + 2abi) + (a^2 - b^2 + 2abi) + (a + bi) = 3$

$a^3 - ab^2 + 2a^2bi + a^2bi - b^3i - 2ab^2 + a^2 - b^2 + 2abi + a + bi = 3$

$(a^3 - ab^2 - 2ab^2 + a^2 - b^2 + a) + i(2a^2b + a^2b - b^3 + 2ab + b) = 3$

$(a^3 - 3ab^2 + a^2 - b^2 + a) + i(3a^2b + 2ab + b - b^3) = 3 + 0i$

$a^3 - 3ab^2 + a^2 - b^2 + a = 3$

$3a^2b + 2ab + b - b^3 = 0$

$b(3a^2 + 2a + 1 - b^2) = 0$

$b = 0$

$a^3 - 0 + a^2 - 0 + a = 3$

$p(x) = x^3 + x^2 + x - 3$

$p(x) = 0$

$p(x) = (x - z)(x - \overline{z})(x - 1)$

$\Rightarrow (x^2 - \overline{z}x - zx + z\overline{z})(x - 1)$

$\Rightarrow (x^3 - \overline{z}x^2 - zx^2 + z\overline{z}x - x^2 + \overline{z}x + zx - z\overline{z}) \qquad z = a + bi$

$\Rightarrow (x^3 + x^2(-\overline{z} - z - 1) + x(z\overline{z} + \overline{z} + z) + (-z\overline{z})$

$\Rightarrow (x^3 + x^2(-a + bi - a - bi - 1) + x(a^2 + b^2 + a - bi + a + bi) + (-a^2 - b^2))$

$\Rightarrow x^3 + x^2(-2a - 1) + x(a^2 + b^2 + 2a) + (-a^2 - b^2)$

$-2a - 1 = 1 \qquad a^2 + b^2 + 2a = 1 \qquad -a^2 - b^2 = -3$

$a^2 + b^2 = 3$

$-2a-1=1$
$-2a=2$
$a=\frac{2}{-2}$

$a=-1$

$(-1)^2+b^2+2(-1)=1$
$1+b^2-2=1$
$b^2=2$

$b=\pm\sqrt{2}$

$a^2+b^2=3$

$1+2=3$

$$\underline{\underline{p(x)=(x+1+i\sqrt{2})(x+1-i\sqrt{2})(x+1)}}$$