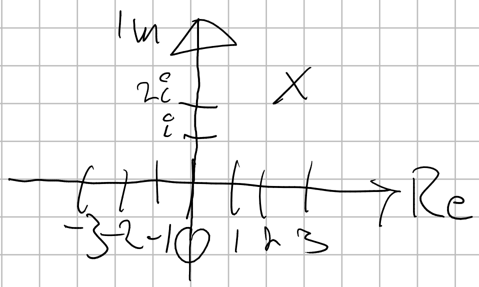
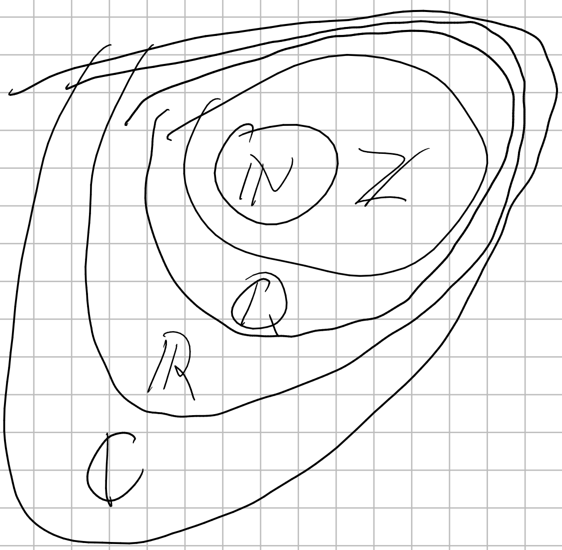
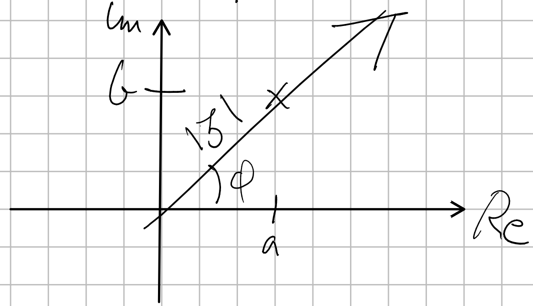
Komplekse tall





$z = a + bi, \quad i = \sqrt{-1}$

$\bar{z} = a - bi$



Andreordens diff likn.

$\ddot{x} + p\dot{x} + qx = 0$

Konstanter

Karakteristisk likning

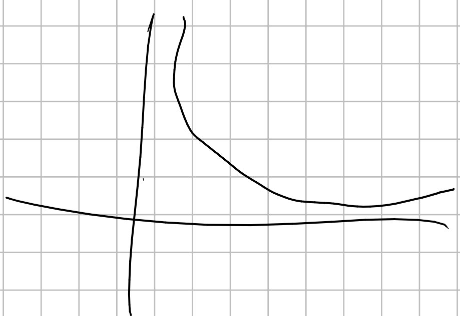
$r^2 + pr + q = 0$

$r_{1,2} = \frac{-p \pm \sqrt{p^2 - 4pq}}{2}$

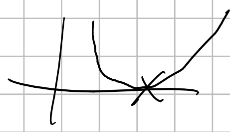
To reelle røtter:

$r_1 \neq r_2 \Rightarrow \boxed{x(t) = Ce^{r_1 t} + De^{r_2 t}}$

I dette tilfelle har vi et overdempet andreordens system



En reell rot: ($p^2 - 4q = 0$)



$$r_1 = r_2 = r \Rightarrow \boxed{x(t) = Ce^{rt} + Dte^{rt}}$$

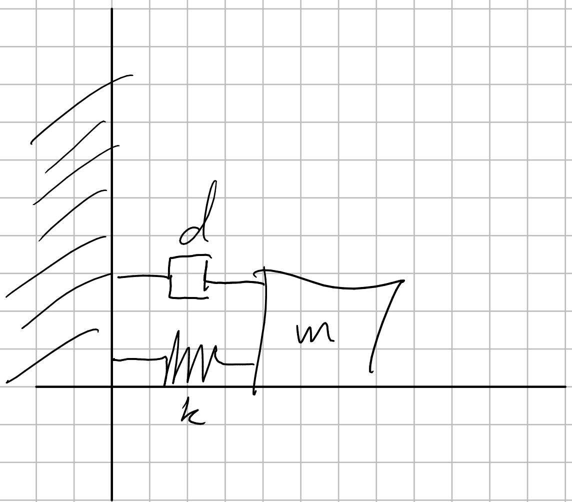
Kritisk dempet system!

To komplekse røtter: ($p^2 - 4q < 0$)

$$\left.\begin{aligned} r_1 &= a + bi \\ r_2 &= a - bi \end{aligned}\right\} \Rightarrow x(t) = e^{at}\left(C\cos(bt) + D\sin(bt)\right)$$

Oscillerende!

Eksempel: Masse-fjær-demper-system



Fjærkraft $F_f = kx$, Hookes lov

Dempekraft $F_d = d\dot{x}$

Kraftbalanse

Newtons 2. lov

$$\sum F = ma = m\dot{v} = m\ddot{x}$$

$$-F_f - F_d = m\ddot{x}$$

$$m\ddot{x} + d\dot{x} + kx = 0$$

$$\ddot{x} + \frac{d}{m}\dot{x} + \frac{k}{m}x = 0$$

Karakteristisk ligning

$$r^2 + \frac{d}{m}r + \frac{k}{m} = 0$$

$$r = \frac{-\frac{d}{m} \pm \sqrt{\left(\frac{d}{m}\right)^2 - 4\frac{k}{m}}}{2}$$

$m = 1\,kg \quad d = 2 \quad k = 10$

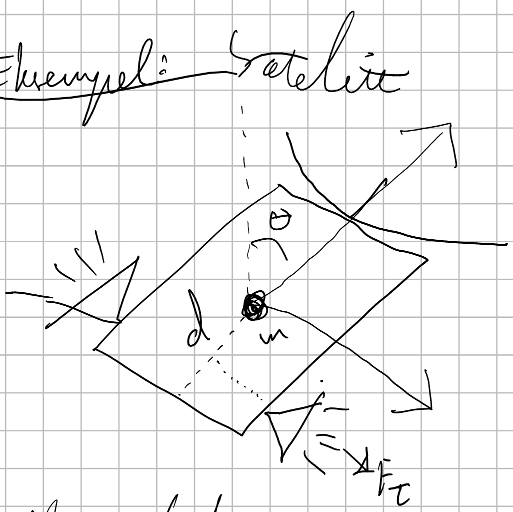
$r_{1,2} = -1 \pm 3i$

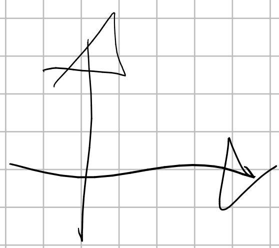
$x(\tau) = e^{a\tau}\left(C\cos(b\tau) + D\sin(b\tau)\right)$

$x(0) = 1$

$\dot{x}(0) = 0$

Eksempel: Satelitt





Momentbalanse

$\Sigma M = J \cdot \ddot{\theta}$

$J\ddot{\theta} = F_\tau \cdot d$

J: Treghetsmoment aka Torque

2. ordens. inhom. likn. med $\mu = q = 0$

$\ddot{\theta} = \frac{F_\tau \cdot d}{J} \Rightarrow \dot{\theta} = \frac{F_\tau d}{J}\tau + C_1$

$\Rightarrow \theta = \frac{F_\tau d}{2J}\tau^2 + C_1\tau + C_2$

2. ordens likn -> 2 1. ordens likn.

$\ddot{x} + \mu\dot{x} + qx = 0$

Innfører to nye tilstander

$x_1 = x$

$x_2 = \dot{x}$

To diff.likn.: $\dot{x}_1 = \dot{x} = x_2 \Rightarrow \dot{x}_1 = x_2$

$\dot{x}_2 = \ddot{x} = -\mu\dot{x} - qx$

$\dot{x}_2 = -\mu x_2 - q x_1$

$$\boxed{\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= -q\,x_1 - \mu x_2 \end{aligned}}$$

Tilstandsrom modell

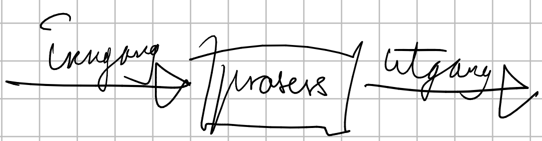
$$\underline{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}, \quad \underline{\dot{x}} = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix}$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -q & -\mu \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$\underline{\dot{x}} = A\underline{x}$$

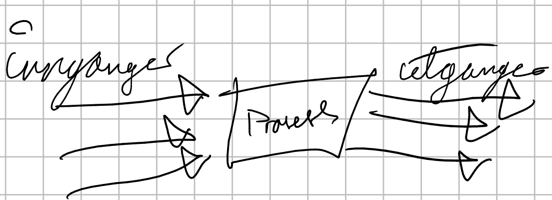
Dynamiske systemer

Monovariabelt system



(eksakt én inngang og utgang)

Multivariable systemer



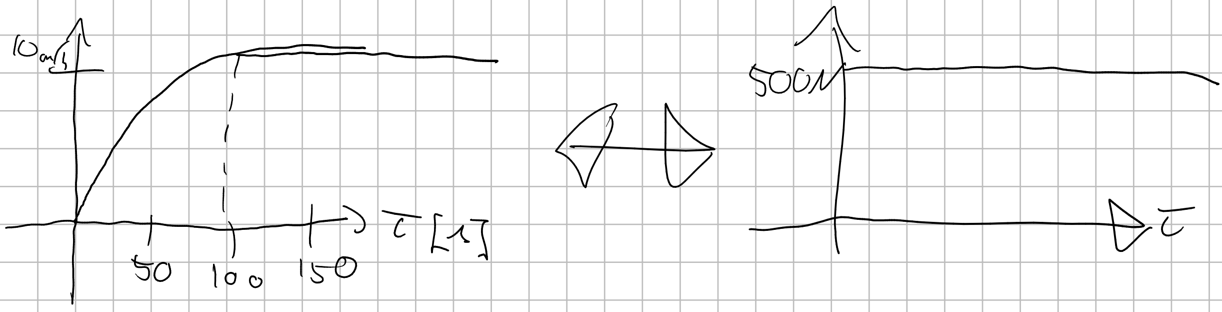
Eks: skip

innganger: ror, thrustere, propeller, ...

utganger: kurs, hastighet, posisjon, ...

Stasjonær respons

Statisk prosess: $y = f(x)$, eks: måleinstrument



Tilnærming $v = 10\,m/s$ etter $100\,s$

konstruerer en statisk modell $v = Ku$, $K = \frac{v}{u} = \frac{10}{500} = 0{,}02$

$v = 0{,}02u$