

Stack 4

1 a)

$$E[X] = 2 \quad SD[X] = 1,5$$

$$E[Y] = 0 \quad SD[Y] = 1$$

$$Z = X + Y$$

$$E[Z] = E[X + Y] = E[X] + E[Y] =$$

$$\mu_z = 2 + 0 = 2.000$$

1 b)

$$Var[Z] = Var[X + Y] = Var[X] + Var[Y] + 2 \underbrace{Cov[X, Y]}_0$$

$$= SD[X]^2 + SD[Y]^2$$

$$= 3,250$$

1 c)

$$g(Z) = 125Z^2 - 250Z$$

$$E[g(Z)] = 125E[Z^2] - 250E[Z]$$

$$Var[Z] = E[Z^2] - E[Z]^2$$

$$E[Z^2] = Var[Z] + E[Z]^2$$

$$E[g(Z)] = 125(Var[Z] + E[Z]^2) - 250E[Z]$$

2 a)

$$P(A) = 0,3 \quad P(B) = 0,8 \quad P(A \cap B) = 0,28$$

$$P(a) = 0,7 \quad P(b) = 0,2$$

$$X = \begin{cases} 0 \\ 1 \end{cases}, A \quad Y = \begin{cases} 0 \\ 1 \end{cases}, B$$

$$E[X] = \sum_x x P_X(x) = 0 \cdot 0,7 + 1 \cdot 0,3 = 0,300$$

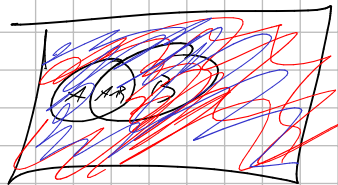
2 b)

$$\text{Var}[X] = E[(X - \mu)^2] = (0 - 0,3)^2 \cdot 0,7 + (1 - 0,3)^2 \cdot 0,3$$

$$= 0,210$$

2 c)

$$E[XY] = \sum_x \sum_y x y P_X(x) P_Y(y)$$



$$P(a) + P(b) - P(a \cap b) + P(A \cap B) = 1$$

$$P(a) + P(b) + P(A \cap B) - 1 = P(a \cap b)$$

$$P(a \cap b) = 0,180$$

$$P(a|b) = \frac{P(a \cap b)}{P(b)} = \frac{0,180}{0,2} = 0,90$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{0,28}{0,8} = 0,350$$

$$E[XY] = a b P(a|b) P(b) + a B(a|B) + A b P(A|b) P(b) + A B P(A|B) P(B)$$

$$= 0 + 0 + 0 + 0,350 \cdot 0,8$$

$$= \underline{0,280}$$

2d)

$$\text{Cov}[X, Y] = E[XY] - E[X] \cdot E[Y]$$

$$E[Y] = 13 \cdot P(B) = 0,8$$

$$\text{Cov}[X, Y] = 0,280 - 0,3 \cdot 0,8 = 0,04$$

$$\text{Corr}[X, Y] = \frac{\text{Cov}[X, Y]}{\sqrt{\text{Var}[X] \cdot \text{Var}[Y]}}$$

$$\text{Var}[Y] = (6 - 0,8)^2 P(A) + (13 - 0,8)^2 P(B)$$

$$= (-0,8)^2 \cdot 0,2 + (0,2)^2 \cdot 0,8$$

$$= 0,160$$

$$\text{Corr}[X, Y] = \frac{0,04}{\sqrt{0,210 \cdot 0,160}}$$

$$= \underline{\underline{0,218}}$$

3)

$$f_{X,Y}(x,y) = \frac{2(2y+x)}{3}$$

a)

$$\begin{aligned} E[X] &= \int_0^1 \int_0^1 x \frac{2}{3}(2y+x) dx dy = \frac{2}{3} \int_0^1 \int_0^1 2xy + x^2 dx dy = \frac{2}{3} \int_0^1 \left(x^2 y + \frac{1}{3} x^3 \Big|_0^1 \right) dy \\ &= \frac{2}{3} \int_0^1 y + \frac{1}{3} dy = \frac{2}{3} \left(\frac{1}{2} y^2 + \frac{1}{3} y \Big|_0^1 \right) = \frac{2}{3} \left(\frac{1}{2} + \frac{1}{3} \right) = \frac{2}{3} \left(\frac{3}{6} + \frac{2}{6} \right) = \underline{\underline{\frac{10}{18}}} \end{aligned}$$

$$\text{Var}[X] = E[X^2] - E[X]^2$$

$$\begin{aligned} E[X^2] &= \int_0^1 \int_0^1 x^2 \frac{2}{3}(2y+x) dx dy = \frac{2}{3} \int_0^1 \int_0^1 2x^2 y + x^3 dx dy = \frac{2}{3} \int_0^1 \left(\frac{2}{3} x^3 y + \frac{1}{4} x^4 \Big|_0^1 \right) dy \\ &= \frac{2}{3} \int_0^1 \frac{2}{3} y + \frac{1}{4} dy = \frac{2}{3} \left(\frac{1}{3} y^2 + \frac{1}{4} y \Big|_0^1 \right) = \frac{2}{3} \left(\frac{1}{3} + \frac{1}{4} \right) = \frac{2}{3} \left(\frac{4+3}{12} \right) = \frac{14}{36} \end{aligned}$$

$$\text{Var}[X] = \frac{14}{36} - \left(\frac{10}{18} \right)^2 = \frac{14}{36} - \frac{100}{324} = \frac{126 - 100}{324} = \frac{26}{324} = \underline{\underline{\frac{13}{162}}}$$

$$\begin{aligned} E[Y] &= \int_0^1 \int_0^1 y \frac{2}{3}(2y+x) dy dx = \frac{2}{3} \int_0^1 \int_0^1 2y^2 + xy dy dx = \frac{2}{3} \int_0^1 \left(\frac{2}{3} y^3 + \frac{1}{2} x y^2 \Big|_0^1 \right) dx \\ &= \frac{2}{3} \int_0^1 \frac{2}{3} + \frac{1}{2} x dx = \frac{2}{3} \left(\frac{2}{3} x + \frac{1}{4} x^2 \Big|_0^1 \right) = \frac{2}{3} \left(\frac{2}{3} + \frac{1}{4} \right) = \frac{2}{3} \left(\frac{8+3}{12} \right) = \frac{22}{36} = \frac{11}{18} \end{aligned}$$

$$\text{Var}[Y] = E[Y^2] - E[Y]^2$$

$$\begin{aligned} E[Y^2] &= \int_0^1 \int_0^1 y^2 \frac{2}{3}(2y+x) dy dx = \frac{2}{3} \int_0^1 \int_0^1 2y^3 + xy^2 dy dx = \frac{2}{3} \int_0^1 \left(\frac{1}{2} y^4 + \frac{1}{3} x y^3 \Big|_0^1 \right) dx \\ &= \frac{2}{3} \int_0^1 \frac{1}{2} + \frac{1}{3} x dx = \frac{2}{3} \left(\frac{1}{2} x + \frac{1}{6} x^2 \Big|_0^1 \right) = \frac{2}{3} \left(\frac{4}{6} \right) = \underline{\underline{\frac{8}{18}}} \end{aligned}$$

$$\text{Var}[Y] = \frac{8}{18} - \left(\frac{11}{18} \right)^2 = \frac{8 - \frac{121}{18}}{18} = \frac{23}{324}$$

$$\begin{aligned}
 E[XY] &= \int_0^1 \int_0^1 xy \cdot \frac{2}{3}(2y+x) dx dy = \frac{2}{3} \int_0^1 \int_0^1 2xy^2 + x^2y dx dy \\
 &= \frac{2}{3} \int_0^1 \left(x^2y^2 + \frac{1}{3}x^3y \Big|_0^1 \right) dy = \frac{2}{3} \int_0^1 y^2 + \frac{1}{3}y dy = \frac{2}{3} \left(\frac{1}{3}y^3 + \frac{1}{6}y \Big|_0^1 \right) \\
 &= \frac{2}{3} \left(\frac{1}{3} + \frac{1}{6} \right) = \frac{2}{3} \left(\frac{1}{2} \right) = \frac{2}{6} = \underline{\underline{\frac{1}{3}}}
 \end{aligned}$$

$$\begin{aligned}
 \text{Cov}[X, Y] &= E[XY] - E[X] \cdot E[Y] \\
 &= \frac{1}{3} - \frac{1}{18} \cdot \frac{1}{18} = \frac{1}{3} - \frac{1 \cdot 1 \cdot 1}{3 \cdot 2 \cdot 4} = \frac{108}{324} - \frac{1}{324} = \frac{107}{324} = \underline{\underline{-\frac{1}{168}}}
 \end{aligned}$$

$$\begin{aligned}
 \text{Var}[Y - X] &= E[(Y - X)^2] - E[Y - X]^2 \\
 &= E[Y^2] - 2E[XY] + E[X^2] - (E[Y] - E[X])^2 \\
 &= E[Y^2] - 2E[XY] + E[X^2] - (E[Y]^2 - 2E[X]E[Y] + E[X]^2) \\
 &= E[Y^2] - 2E[XY] + E[X^2] - E[Y]^2 + 2E[X]E[Y] - E[X]^2 \\
 &= \text{Var}[Y] + \text{Var}[X] - 2E[XY] + 2E[X]E[Y] \\
 &= \frac{13}{162} + \frac{23}{324} - \frac{2}{3} + 2 \cdot \frac{10}{18} \cdot \frac{1}{18} \\
 &= \frac{26 + 23}{324} - \frac{2}{3} + \frac{220}{18^2} \\
 &= \frac{49}{324} - \frac{2}{3} + \frac{220}{324} \\
 &= 0,16
 \end{aligned}$$

$$\begin{aligned}
 \text{Corr}(X, Y) &= \frac{\text{Cov}(X, Y)}{\sqrt{\text{Var}(X) \cdot \text{Var}(Y)}} = \frac{-\frac{1}{168}}{\sqrt{\frac{13}{162} \cdot \frac{23}{324}}} = \\
 &= -0,08
 \end{aligned}$$

4 a)

$$E(X) = 1,800$$

$$\text{Var}(X) = 18 \cdot 0,1(1-0,1) = 18 \cdot 0,1 \cdot 0,9$$

$$P(\bar{n}) = 0,9$$

$$P(\text{Niemand stimmt}) = 1 - P(\text{Jeder stimmt})$$

$$= 1 - P(\bar{n})^{18}$$

$$= \underline{\underline{0,85}}$$

$$b) P(\text{Nur einer stimmt}) = \binom{18}{1} \cdot 0,1 \cdot (1-0,1)^{18-1}$$

$$= \binom{18}{1} \cdot 0,1 \cdot (0,9)^{17}$$

$$= 0,300$$

$$0,9^{17} = 0,167$$

$$P(\text{Nur einer stimmt} | \text{Ein hat stimmt}) = \frac{P(\text{Nur einer stimmt} \cap \text{Ein hat stimmt})}{P(\text{Ein hat stimmt})}$$

5)

700 ~~total~~

150 parallel 1 : A $P(A) = 0,75$

250 parallel 2 : B $P(B) = 0,85$

300 ingen förslutning : C $P(C) = 0,55$

$$a) 150 \cdot 0,75 = 112,50$$

$$b) 150 \cdot 0,75 + 250 \cdot 0,85 + 300 \cdot 0,55 \\ = 490$$

6)

X og Y uafhængige $\mu = 0, 2, \dots, n = 10$

a)

$$P(X=3) = \binom{10}{3} (0,2)^3 \cdot (0,8)^7$$

$$= \underline{\underline{0,201}}$$

$$b) P(X \leq 3) = P(X=1) + P(X=2) + P(X=3) + P(X=0)$$

$$= 0,879$$

$$c) P(X+Y=2) = P(X=0 \cap Y=2) + P(X=1 \cap Y=1) + P(X=2 \cap Y=0)$$

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