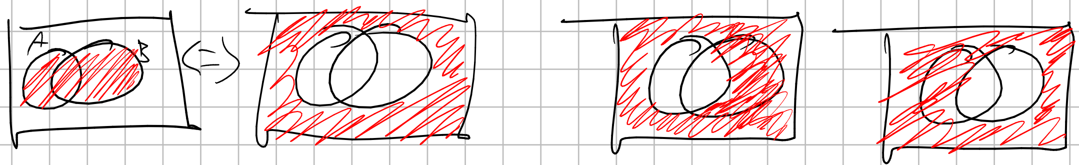
Skriftlig innlevering 1

z_1, z_2, z er alle Z BTW

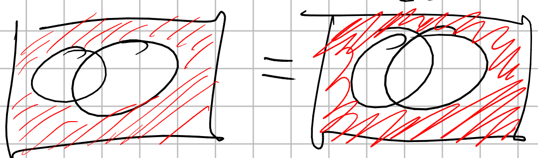
Oppg 1)

$P((A \cup B)') = P(A' \cap B')$?

$P(A \cup B)$ $\quad$ $P((A \cup B)')$ $\quad$ $P(A')$ $\quad$ $P(B')$

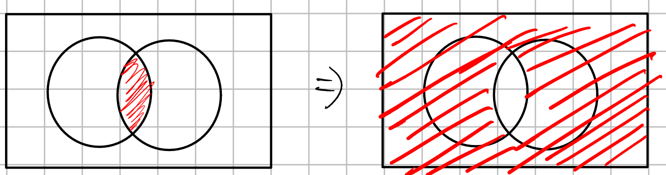


$P(A' \cap B')$ $\quad$ $P((A \cup B)')$

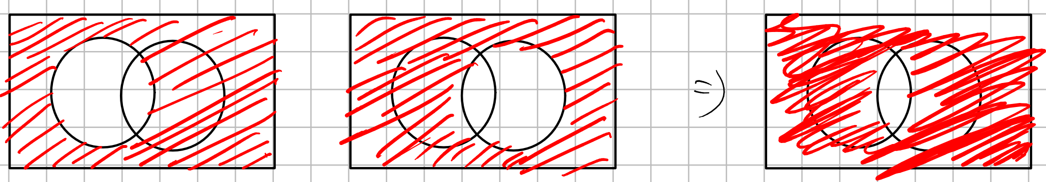


$P((A \cap B)') = P(A' \cup B')$

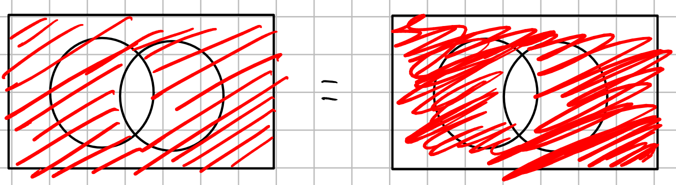
$P(A \cap B)$ $\quad$ $P((A \cap B)')$



$P(A')$ $\quad$ $P(B')$ $\quad$ $P(A' \cup B')$



$P((A \cap B)')$ $\quad$ $P(A' \cup B')$



Oppg 2)

34 kuler, 7 røde, 27 blå, Trekker sju uten tilbakelegging

A: Alle 7 kuler er røde

B: Nøyaktig 4 kuler er røde

C: Trekker 7 med nøyaktig 6 røde, så trekker 1 til hvorav den er rød

Antar uniform sannsynlighetsmodell $\Rightarrow$ $P = \frac{g}{m}$

$P(A) = \frac{g}{m}$

$$\frac{\frac{x}{y}}{\frac{y}{x}} = \frac{x^2}{\frac{y}{y}} = \frac{x^2}{3^2}$$

$m = \binom{34}{7}$

$g = 1$ fordi det er bare 1 kombinasjon der alle er røde

$$P(A) = \frac{1}{\binom{34}{7}} = \underline{\underline{1{,}86 \cdot 10^{-7}}}$$

$P(B) = \frac{g}{m}$

$g = \binom{7}{4}\binom{27}{3} =$

$$P(B) = \frac{\binom{7}{4}\binom{27}{3}}{\binom{34}{7}} = \underline{\underline{0{,}019}}$$

$P(C) = P(6 \text{ røde}) \cdot P(1 \text{ rød} \mid 6 \text{ røde})$

$$P(C) = \frac{\binom{7}{6} \cdot \binom{27}{1}}{\binom{34}{7}} \cdot \frac{\binom{1}{1}\binom{26}{0}}{\binom{27}{1}} = \frac{\binom{7}{6}\binom{27}{1}}{\binom{34}{7}} \cdot \frac{1}{27}$$

$$= \underline{\underline{1{,}30 \cdot 10^{-6}}}$$

Oppgave 3)

300 lodd, 3 er av gevinst A, 3 av B, 294 gir ingen.
Trekker 5 kort

P(C₁ = Vinner nøyaktig 1 lodd av type A) = $\frac{g}{m}$

$m = \binom{300}{5}$

$g = \binom{3}{1} \cdot \binom{297}{4}$

$P(C_1) = \frac{\binom{3}{1}\binom{295}{4}}{\binom{300}{5}} = \underline{0{,}0487}$

P(C₂ = minst én av A) = 1 − P(Ingen er A)

P(Ingen er A) = $\frac{g}{m}$

$m = \binom{300}{5}$

$g = \binom{297}{5}$

$P(C_2) = 1 - \frac{\binom{294}{5}}{\binom{300}{5}} = \underline{\underline{0{,}0493}}$

P(C₃ = Minst én gevinst) = 1 − P(Ingen gevinst)

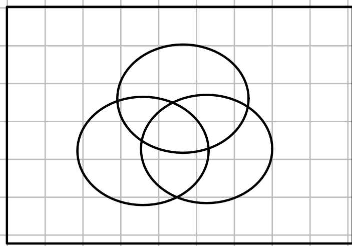
$= 1 - \frac{\binom{294}{5}}{\binom{300}{5}} = \underline{\underline{0{,}0967}}$

Oppgave 4)

$A, B, C \in S \qquad P(A) = 0{,}4 \qquad P(B) = 0{,}3 \qquad P(C) = 0{,}3$

$P(A \cup B) = 0{,}6 \qquad P(A \cup C) = 0{,}5 \qquad P(B \cup C) = 0{,}6$

$P(A \cup B \cup C) = 0{,}7$



$P(A \cap B) = P(A) + P(B) - P(A \cup B) = \underline{\underline{0{,}1}}$

$P(A \cap B \cap C) = ?$

$P(A \cap C) = P(A) + P(C) - P(A \cup C) = 0{,}2$

$P(B \cap C) = P(B) + P(C) - P(B \cup C) = 0$

$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)$

$P(A \cap B \cap C) = P(A \cup B \cup C) + P(A \cap B) + P(A \cap C) + P(B \cap C) - P(A) - P(B) - P(C)$

$= 0{,}7 + 0{,}1 + 0{,}2 + 0 - 0{,}4 - 0{,}3 - 0{,}3$

$= \underline{\underline{0}}$

$P(A \mid B) = \frac{P(A \cap B)}{P(B)} = \frac{0{,}1}{0{,}3} = \underline{\underline{\frac{1}{3}}}$

A & B uavhengige ?

$P(A|B) = P(A)$?

$\frac{1}{3} \neq 0{,}4 \Rightarrow$ <u>Ikke uavhengig</u>

A & B disjunkte ?

$P(A \cap B) = P(\emptyset) = 0$?

$P(A \cap B) = 0{,}1 \Rightarrow$ Ikke disjunkte

Oppgave 5)

$P(F|M) = 0{,}08 \qquad P(F|K) = 0{,}003$

$P(K|F) = ? \qquad P(M) = \frac{1}{3} \quad P(K) = \frac{2}{3}$

$$P(K|F) = \frac{P(F|K)\,P(K)}{P(F)}$$

$$P(F) = P(F|M)\,P(M) + P(F|K)\,P(K)$$

$$P(K|F) = \frac{P(F|K)\,P(K)}{P(F|M)\,P(M) + P(F|K)\,P(K)}$$

$$= \underline{\underline{0{,}069}}$$

Oppgave 6)

$P(X \leq 2) = 0{,}25 + 0{,}1 + 0{,}05 = \underline{\underline{0{,}40}}$

$P(X < 2 | X < 4) = \frac{P(X < 2 \cap X < 4)}{P(X < 4)} = \frac{P(X \leq 2)}{P(X < 4)} = \frac{0{,}40}{0{,}40 + 0{,}40} = \underline{\underline{0{,}5}}$

$P(X \leq 2 | X \geq 1) = \frac{P(X \leq 2 \cap X \geq 1)}{P(X \geq 1)} = \frac{P(X = 2 \cup X = 1)}{1 - P(X < 1)} = \frac{0{,}10 + 0{,}25}{1 - 0{,}05}$

$= \underline{\underline{0{,}3684}}$

Oppgave 7)

a)

$F'(X) = f(X)$

$f(X) = \left(1 - e^{-\frac{x^2}{\alpha}}\right)'$

$= -\left(-\frac{x^2}{\alpha}\right)' \cdot e^{-\frac{x^2}{\alpha}}$

$$= \frac{2x}{\alpha} e^{-\frac{x^2}{\alpha}}$$

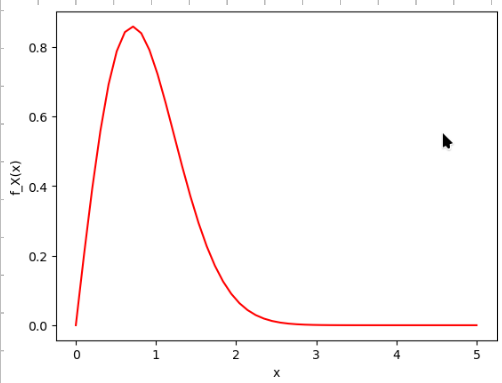
```python
#Oppgave 7a)

def f_X(x, alpha):
    return ((2*x)/alpha) * np.exp((-x**2)/alpha)


x = np.linspace(0, 5)
y = np.zeros(len(x))

for i in range(len(x)):
    y[i] = f_X(x[i], alpha)


plt.plot(x, y, color="red")
plt.xlabel("x")
plt.ylabel("f_X(x)")
plt.show()
```



b) $1 - F_Z(z) = P(Z \le z)$

$= P(X_1 \le z \cap X_2 \le z)$ — Uavhengig ⇒ Begge må skje samtidig

$= P(X_1 \le z)P(X_2 \le z)$

$= \left(1 - e^{-\frac{z^2}{\alpha}}\right)\left(1 - e^{-\frac{z^2}{\alpha}}\right)$

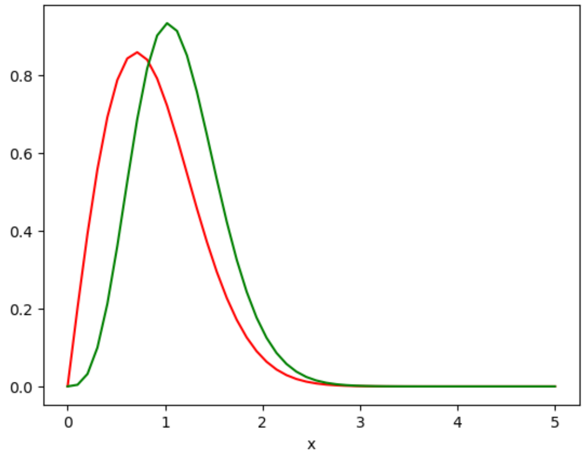
$= 1 - 2e^{-\frac{z^2}{\alpha}} + e^{-\frac{2}{\alpha}z^2}$

$$f_Z(z) = \frac{4}{\alpha} z e^{-\frac{z^2}{\alpha}} - \frac{4}{\alpha} z e^{-\frac{2}{\alpha}z^2}$$

```python
#Oppgave 7b)

def f_Z(z, alpha):
    return (4/alpha)*z*np.exp(-(z**2)/alpha) - (4/alpha)*z*np.exp(-(2*z**2)/alpha)

plt.plot(x, f_X(x, alpha), color="red")
plt.plot(x, f_Z(x, alpha), color="green")
plt.xlabel("x")
plt.ylabel("")
plt.show()
```
✓ 0.0s



Kan se att det er en høyere sannsynlighet for instrumentet funker når den består av to komponenter, hvorav bare én trenger å være i stand for att instrumentet skal funke.
Dette ser man fordi den grønne grafen f_Z(z) er forskjøvet mot høyre i forhold til f_X(x).

Oppgave 8)

300 lodd, trekker 5

3 lodd gir gevinst A

3 lodd gir gevinst B

294 gir ingen gevinst

X: Antall gevinster av A

Y: Antall gevinster av B

$f_X(x) = \frac{g}{n}$ $\qquad g = \binom{3}{x}\binom{297}{5-x}$

$$f_X(x) = \frac{\binom{3}{x}\binom{297}{5-x}}{\binom{300}{5}}$$

$$f_{XY}(x,y) = \frac{\binom{3}{x}\binom{3}{y}\binom{294}{5-x-y}}{\binom{300}{5}}$$

$$f_X(x) = \sum_y f_{XY}(x,y) = \sum_y \frac{\binom{3}{x}\binom{3}{y}\binom{294}{5-x-y}}{\binom{300}{5}} = \frac{\binom{3}{x}}{\binom{300}{5}} \sum_y^{5-x} \binom{3}{y}\binom{294}{5-x-y}$$

$$= \frac{\binom{3}{x}}{\binom{300}{5}} \cdot \binom{3+294}{5-x} = \frac{\binom{3}{x}\binom{297}{5-x}}{\binom{300}{5}}$$