

Aufgabe 3

Opptg 1)

a)  $y = 2 \cdot \sqrt{u}$

$$0,4 = 2 \cdot \sqrt{u}$$

$$0,2 = \sqrt{u}$$

$$u = 0,04$$

b)  $f(x) = \begin{cases} \frac{3,2}{x^3} & \text{for } x \geq 4 \\ 0 & \text{for } x < 4 \end{cases}$

$$F_x(x) \quad x \geq 4$$

$$F_x(x) = \int_4^x 3,2 \frac{1}{u^3} du = 3,2 \int_4^x u^{-3} du = 3,2 \left( -\frac{1}{2} u^{-2} \right) \Big|_4^x$$

$$= 3,2 \left( \left( -\frac{1}{2} - \frac{1}{x^2} \right) - \left( -\frac{1}{2} - \frac{1}{4^2} \right) \right) = 3,2 \left( -\frac{1}{2x^2} + \frac{1}{32} \right)$$

$$= 1 - \frac{3,2}{2x^2} = 1 - \frac{1,6}{x^2}$$

$$F_x = 0,95$$

$$1 - \frac{1,6}{x^2} = 0,95$$

$$1 - 0,95 = \frac{1,6}{x^2}$$

$$x = \sqrt{\frac{1,6}{0,05}}$$

3 b)

$$E[X] = \mu = \sum_x x \cdot f(x)$$

$$\sigma^2 = \text{Var}[X] = E[(X - \mu)^2] = \sum_x (X - \mu)^2 \cdot f(x)$$

4 a)  $f(t) = \begin{cases} 1,5 e^{-1,5t} & t > 0 \\ 0 & \text{sonst} \end{cases}$

$$F(t) = \int_0^t 1,5 e^{-1,5s} ds = \left( -\frac{1,5}{1,5} e^{-1,5s} \right) \Big|_0^t = 1 - e^{-1,5t}$$

$$E[X] = \int_{-\infty}^{\infty} t \cdot 1,5 e^{-1,5t} dt = \left( -t e^{-1,5t} \right) \Big|_0^{\infty} + \int_0^{\infty} e^{-1,5t} dt$$

$$= \left( -\frac{1}{1,5} e^{-1,5t} \right) \Big|_0^{\infty} = \frac{1}{1,5} =$$

6a)

$$f(x) = \begin{cases} 0 & x < 0 \\ \frac{1 - \cos(2x)}{2}, & 0 \leq x \leq \frac{\pi}{2} \\ 1 & x > \frac{\pi}{2} \end{cases}$$

$$f(x) = \sin(2x)$$

$$6c) E[X^2] = \int_0^{\frac{\pi}{2}} x^2 \sin(2x) dx = \left( x^2 \cdot \left(-\frac{1}{2}\right) \cos(2x) \right) \Big|_0^{\frac{\pi}{2}} + \int_0^{\frac{\pi}{2}} x \cos(2x) dx$$

$$= \frac{1}{2} \left( \frac{\pi}{2} \right)^2 + \left( x \cdot \frac{1}{2} \sin(2x) \right) \Big|_0^{\frac{\pi}{2}} + \int_0^{\frac{\pi}{2}} \frac{1}{2} \sin(2x) dx$$

$$= \frac{\pi^2}{8} + \left( \frac{1}{4} \cos(2x) \right) \Big|_0^{\frac{\pi}{2}} = \frac{\pi^2}{8} - \frac{1}{4} - \frac{1}{4} = \frac{\pi^2 - 4}{8}$$

6d)

$$\text{Var}(X) = E[(X - \mu)^2] = \int_0^{\frac{\pi}{2}} \left( x - \frac{\pi}{4} \right)^2 \sin(2x) dx$$

$$= \left( \left( x - \frac{\pi}{4} \right)^2 \cdot \left(-\frac{1}{2}\right) \cos(2x) \right) \Big|_0^{\frac{\pi}{2}} + \int_0^{\frac{\pi}{2}} \left( x - \frac{\pi}{4} \right) \cos(2x) dx$$

$$= \left( -\frac{\pi^2}{16} \cdot \left(-\frac{1}{2}\right) \cdot (-1) - \frac{\pi^2}{16} \cdot \left(-\frac{1}{2}\right) + \left( x - \frac{\pi}{4} \right) \frac{1}{2} \sin(2x) \right) \Big|_0^{\frac{\pi}{2}} + \int_0^{\frac{\pi}{2}} \frac{1}{2} \sin(2x) dx$$

$$= -\frac{\pi^2}{32} + \frac{\pi^2}{16} + \left( -\frac{1}{4} \cos(2x) \right) \Big|_0^{\frac{\pi}{2}} = -\frac{\pi^2}{32} + \frac{1}{4} - \frac{1}{4} = \frac{1}{32} - \frac{\pi^2}{32}$$

