

Sluttlig 2

1.1)

$$P(X \leq 2) = 0,05 + 0,10 + 0,25 \\ = \underline{\underline{0,40}}$$

1.2)

```
# Antall realisasjoner man skal bruke
n = 1000
# Simuler realisasjoner av X ved å kalle på simX-funksjonen i cellen over
simulerte_X = simX(n)
# Approksimer sannsynligheten
count = 0
for i in range(n):
    if simulerte_X[i] == 0 or simulerte_X[i] == 1 or simulerte_X[i] == 2:
        count += 1

P_X_le_2 = count/n
# Skriv ut resultatet
print("Approksimert sannsynlighet: ", P_X_le_2)
✓ 0.0s
```

Approksimert sannsynlighet: 0.402

2.1)

$$\mu = E[X] = \sum_x x \cdot f(x) = \underline{\underline{2,650}}$$

2.2)

$$\text{Var}[X] = \sum_x (x - \mu)^2 f(x) = \underline{\underline{1,3275}}$$

$$\text{SD}[X] = \sqrt{\text{Var}[X]} = \underline{\underline{1,15217}}$$

2.3)

```
I
def approx_P_X(x):
    count = 0
    for i in range(n):
        if simulerte_X[i] == x:
            count += 1

    return count / n

E_x = 0
for i in range(len(f_x)):
    E_x += approx_P_X(i) * f_x[i]

print("E_X: ", E_x)

Var_x = 0

for i in range(len(f_x)):
    Var_x += (approx_P_X(i) - E_x)**2 * f_x[i]

SD_x = np.sqrt(Var_x)
print("Var_x: ", SD_x)

✓ 0.0s

E_X: 0.26405
Var_x: 0.1374508912302863
```

3 a)

$$F_X(x) = 1 - e^{-\frac{x^2}{\alpha}}, \quad x \geq 0$$

$$f_X(x) = (1 - e^{-\frac{x^2}{\alpha}})' = -(-\frac{2x}{\alpha}) e^{-\frac{x^2}{\alpha}} = \frac{2x}{\alpha} e^{-\frac{x^2}{\alpha}}$$

3 b)

$$F_X(x) = u$$

$$1 - e^{-\frac{x^2}{\alpha}} = u$$

$$e^{-\frac{x^2}{\alpha}} = 1 - u$$

$$-\frac{x^2}{\alpha} = \ln(1 - u)$$

$$-x^2 = \alpha \ln(1 - u)$$

$$x = \sqrt{-\alpha \ln(1 - u)}$$

```
import numpy as np
import matplotlib.pyplot as plt
def generateX(n, alpha):
    u = np.random.uniform(size=n) #array med n elementer.
    x = np.sqrt(-alpha * np.log(1 - u))
    # fyll inn formelen du fant for x
    return x

# Sett antall realisasjoner og verdien til alpha
n = 10000000
alpha = 1

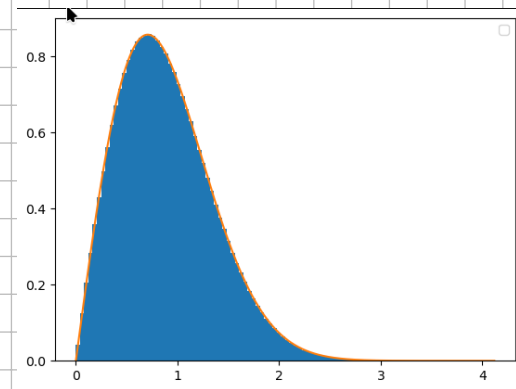
# simuler realisasjoner av X
simulerte_X = generateX(n, alpha)

# Lag sannsynlighetshistogram for de simulerte verdiene,
# vi spesifiserer antall intervaller ved å sette "bins=100"
plt.hist(simulerte_X, density=True, bins=100) #density=TrueA
# gjør at vi får et sannsynlighetshistogram
# Angi navn på aksene
plt.xlabel("")
plt.ylabel("")

# Regn ut og plott sannsynlighetstettheten til X på samme plott
x = np.linspace(0, np.max(simulerte_X), len(simulerte_X) + 1)
fx = ((2 * x) / alpha) * np.exp(-x**2 / alpha)

plt.plot(x, fx)

# Avslutt med å generere alle elementene du har plottet
plt.legend()
plt.show()
```



3 c)

$$P(X \geq 1 | Y \geq 0, 75)$$

$$= \frac{P(X \geq 1, Y \geq 0, 75)}{P(Y \geq 0, 75)}$$

$$= \frac{P(X \geq 1)}{P(Y \geq 0, 75)}$$

```
import numpy as np
import matplotlib.pyplot as plt

def generateY(n, alpha):
    u = np.random.uniform(size=n) #array med n elementer.
    y = np.sqrt(-alpha * np.log(1 - u))
    # fyll inn formelen du fant for x
    return y

n = 10000000

#Alpha verdiene for de ulike komponentene
alpha = np.array([1, 1, 1.2, 1.2, 1.2])

#Matrise med kjøringer for hvert komponent
simulerte_Y = np.empty((len(alpha), n))

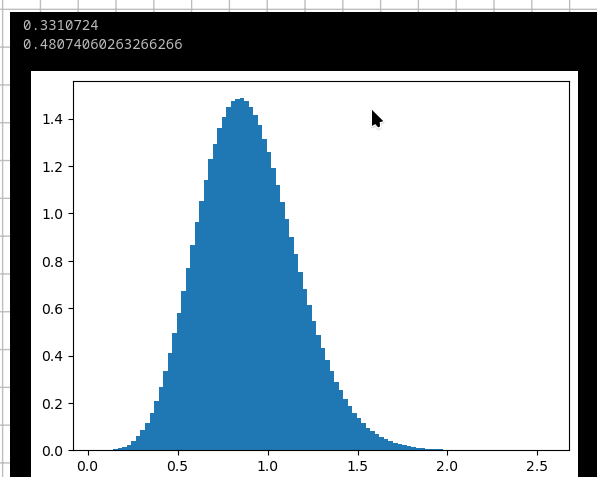
for i in range(len(alpha)):
    simulerte_Y[i] = generateY(n, alpha[i])

# Medianen av matrisen av komponentene, medianen vil bli levetiden
# ettersom det trengs 3 elementer for at instrumentet skal funke
levetid_Y = np.median(simulerte_Y, axis = 0)

#P(Y >= 1) = 0 / m
pygel = np.sum(levetid_Y >= 1) / n
print(pygel)

#P(Y >= 1 | Y >= 0.75) = P(Y >= 1) / P(Y >= 0.75)
pygelgyge075 = np.sum(levetid_Y >= 1) / np.sum(levetid_Y >= 0.75)
print(pygelgyge075)

plt.hist(levetid_Y, density=True, bins=100) #density=TrueA
plt.show()
```



4)

$$f_X(x) \text{ gyldig hvis } \int_{-\infty}^{\infty} f_X(x) dx = 1$$

$$\int_{-\infty}^{\infty} e^{-x^4} dx = 2 \Gamma\left(\frac{5}{4}\right)$$

$$\int_{-\infty}^{\infty} f_X(x) dx = \int_{-\infty}^{\infty} \frac{a}{2 \Gamma\left(\frac{5}{4}\right)} e^{-(x-1)^4} + \frac{b}{2 \Gamma\left(\frac{5}{4}\right)} e^{-(x-4)^4} dx = 1$$

$$\frac{a}{2 \Gamma\left(\frac{5}{4}\right)} \int_{-\infty}^{\infty} e^{-(x-1)^4} dx + \frac{b}{2 \Gamma\left(\frac{5}{4}\right)} \int_{-\infty}^{\infty} e^{-(x-4)^4} dx = 1$$

$$x-1 = u \quad x-4 = v$$

$$\frac{a}{2 \Gamma\left(\frac{5}{4}\right)} \int_{-\infty}^{\infty} e^{-u^4} du + \frac{b}{2 \Gamma\left(\frac{5}{4}\right)} \int_{-\infty}^{\infty} e^{-v^4} dv = 1$$

$dx \rightarrow du$ og $dx \rightarrow dv$, fordi vi går fra $-\infty$ til ∞

$$\frac{a}{2 \Gamma\left(\frac{5}{4}\right)} \cdot 2 \Gamma\left(\frac{5}{4}\right) + \frac{b}{2 \Gamma\left(\frac{5}{4}\right)} \cdot 2 \Gamma\left(\frac{5}{4}\right) = 1$$

$$\underline{a + b = 1}$$

$a + b = 1$ for at $f_X(x)$ skal være en gyldig sandsynlighedstæthed

$$E(X) = \int_{-\infty}^{\infty} x f_X(x) dx = \int_{-\infty}^{\infty} x \left(\frac{a}{2 \Gamma(\frac{5}{4})} e^{-(x-1)^4} + \frac{b}{2 \Gamma(\frac{5}{4})} e^{-(x-4)^4} \right) dx$$

$= \frac{1}{2} x^2 + \text{Hov i alle deger skal jeg gjøre her?}$

5)

$$x \quad f(x)$$

$$0 \quad \frac{1}{18} + \frac{1}{9} + \frac{1}{6}$$

$$1 \quad \frac{1}{12} + \frac{1}{6} + \frac{1}{4} \quad \Rightarrow$$

$$2 \quad \frac{1}{36} + \frac{1}{18} + \frac{1}{12}$$

$$x \quad f_X(x)$$

$$0 \quad \frac{1}{3}$$

$$1 \quad \frac{1}{2}$$

$$2 \quad \frac{1}{6}$$

$$f_X(Y|X) = \frac{P(X,Y)}{P(X)}$$

$$X \quad Y \quad 0 \quad 1 \quad 2$$

$$0 \quad \frac{1}{6} \quad \frac{1}{3} \quad \frac{1}{2}$$

$$1 \quad \frac{1}{6} \quad \frac{1}{3} \quad \frac{1}{2}$$

$$2 \quad \frac{1}{6} \quad \frac{1}{3} \quad \frac{1}{2}$$

$$E(X) = \sum x \cdot f_X(x) = 0 \cdot \frac{1}{3} + 1 \cdot \frac{1}{2} + 2 \cdot \frac{1}{6} = \frac{1}{2} + \frac{1}{3} = \underline{\underline{\frac{5}{6}}}$$

$$E(Y) = \sum_y \sum_x y \cdot f_{XY}(x,y) = 0 \cdot \left(\frac{1}{18} + \frac{1}{12} + \frac{1}{36}\right) + 1 \cdot \left(\frac{1}{9} + \frac{1}{6} + \frac{1}{18}\right) + 2 \cdot \left(\frac{1}{6} + \frac{1}{4} + \frac{1}{12}\right)$$

$$= \frac{1}{3} + \frac{1 \cdot 2}{1 \cdot 2} = \underline{\underline{\frac{4}{3}}}$$

X og Y unabhängig? $\Rightarrow \text{Cov}(X,Y) \neq 0$

$$\text{Cov}(X,Y) = E(XY) - E(X)E(Y)$$

$$E(XY) = \sum_x \sum_y x y \cdot f_{XY}(x,y) = 0 \cdot 0 \cdot \left(\frac{1}{18}\right) + 0 \cdot 1 \cdot \dots$$

$$+ 1 \cdot 1 \cdot \frac{1}{6} + 2 \left(\frac{1}{4}\right) + 2 \left(\frac{1}{18}\right) + 4 \left(\frac{1}{12}\right)$$

$$= \frac{1}{6} + \frac{1}{2} + \frac{4}{18} = \frac{1}{6} + \frac{2}{3} + \frac{2}{9} = 1$$

$$\text{Cov}(X,Y) = 1 - \frac{4}{3} \cdot \frac{5}{6} = 1 - \frac{20}{18} = -\frac{1}{9}$$

X og Y er ikke uafhængige $\text{Cov}(X,Y) = \underline{\underline{-\frac{1}{9}}}$

6 a)

$$\text{Forcutter bruttolast} = 50 \text{ tonn} + 20 (70 + 20) \text{ kg} \\ = \underline{\underline{68 \text{ tonn}}}$$

$$SD(X) = \sqrt{Var(X)}$$

$$Var(X) = 200 \cdot SD_{\mu}^2 + 200 \cdot SD_{\sigma}^2 \\ = 200 \cdot 15^2 + 200 \cdot 5^2 \\ = 50000$$

$$SD(X) = \underline{\underline{0,224 \text{ tonn}}}$$

b)

$$4 \text{ motoren } \mu = 0,01$$

$$P(\text{Miner to werden}) = P(2 \text{ werden}) + P(3) + P(4)$$

$$= 4 \text{ choose } 2 \cdot 0.01^{(2)} \cdot (1-0.01)^{(4-2)} + 4 \text{ choose } 3 \cdot 0.01^{(3)} \cdot (1-0.01)^{(4-3)} + 4 \text{ choose } 4 \cdot 0.01^{(4)} \cdot (1-0.01)^{(4-4)}$$

$$= \underline{\underline{0,00059203}}$$

Google ♥

c)

$$P(\text{Funker mod } 4) > P(\text{Funker mod } 2)$$

$$P(\text{mindest 2 Wähler}) < P(\text{mindest 1 Wähler})$$

$$\binom{4}{2}\mu^2(1-\mu)^2 + \binom{4}{3}\mu^3(1-\mu) + \binom{4}{4}\mu^4 < \binom{2}{1}\mu(1-\mu) + \binom{2}{2}\mu^2$$

$$6\mu^2(1-\mu)^2 + 4\mu^3(1-\mu) + \mu^4 < 2\mu(1-\mu) + \mu^2 \quad q = 1 - \mu$$

$$6\mu^2q^2 + 4\mu^3q + \mu^4 < 2\mu q + \mu^2$$

$$6\mu^2q^2 + 4\mu^3q + \mu^4 - 2\mu q - \mu^2 < 0$$

$$6\mu q^2 + 4\mu^2q + \mu^3 - 2q - \mu < 0$$

$$6\mu(1-\mu)^2 + 4\mu^2(1-\mu) + \mu^3 - 2 + 2\mu - \mu < 0$$

$$\underline{\underline{\mu \leq \frac{2}{3}}}$$

