Kapittel 3

Eks 4. Modellering for cruise-control

Eng: control = No: regulering ≠ No: kontroll
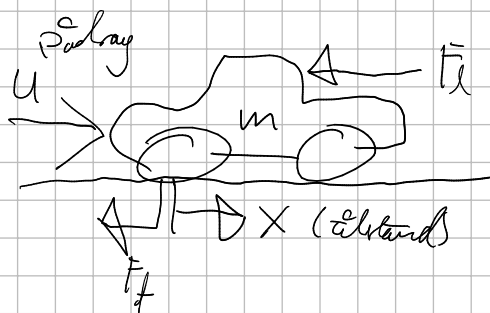
$\sum F = m\alpha$

$U - F_l - F_f = m\dot{v}$

$m\dot{v} = U - F_f = u - k\,v$

$\dot{v} = \frac{1}{m}(u - k\,v)$

$\dot{v} = -\frac{k}{m}v + \frac{1}{m}u$

Utgangspunkt N2. lov

$\sum F = m\alpha$

$\alpha = \dot{v}$

$F_l = k_l v^2$

$F_f = k\,v$

Ikke lineær
ser bort ifra nå

Eks 5 Modellering for nivåregulering
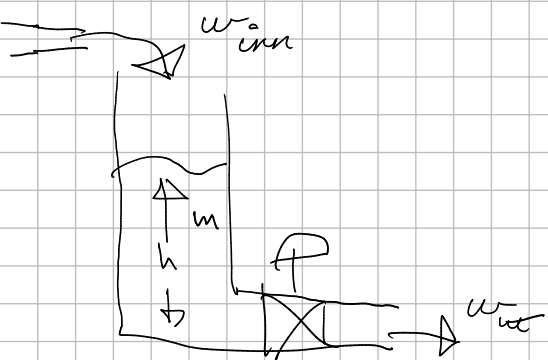
Balanselov: Massebalanse

Endring av masse per tid
= massestrøm inn
− massestrøm ut

$\dot{m} = w_{inn} - w_{ut}$ $m = \rho V = \rho A h$

høyde
Areal
Tetthet

$\dot{m} = \frac{d}{dt}(\rho A h) = w_{inn} - w_{ut}$
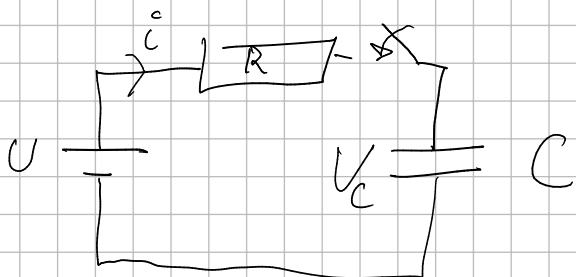
$\rho A \dot{h} = w_{inn} - w_{ut}$

$\dot{h} = \frac{1}{\rho A}(w_{inn} - w_{ut})$

$$w_w = kh$$

$$\dot{n} = \frac{1}{34}(u - kh)$$

$$\dot{n} = \frac{1}{34}u - \frac{k}{34}h$$

Eks 8 RC-krets

$$v_c = \frac{1}{C}\int_0^{\tau} i_c(\tau')\,d\tau' \qquad \text{Merknad, ikke deriver } \tau$$

$$\frac{d v_c}{dt} = \frac{1}{C} i_c(\tau)$$
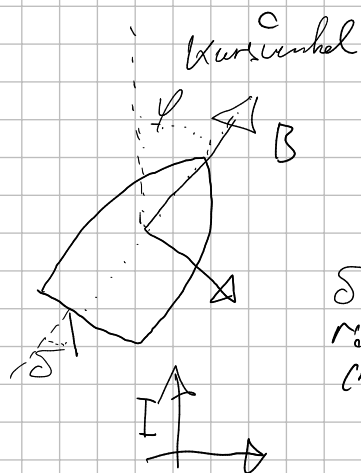
$$\text{Strømbalanse}: \quad i = i_R = i_c$$

$$\frac{u - v_c}{R} = C\frac{d v_c}{dt}$$

$$\dot{v_c} = \frac{1}{C}\left(\frac{u - v_c}{R}\right)$$

$$\dot{v_c} = -\frac{1}{RC}v_c + \frac{1}{RC}u \qquad \underline{\text{Lineær krets}}$$

Eks 9 Modell for autopilotdesign

Kursvinkel

B

δ: rorvinkel
r: svinghastighet
$\left(\frac{rad}{s}\right)$

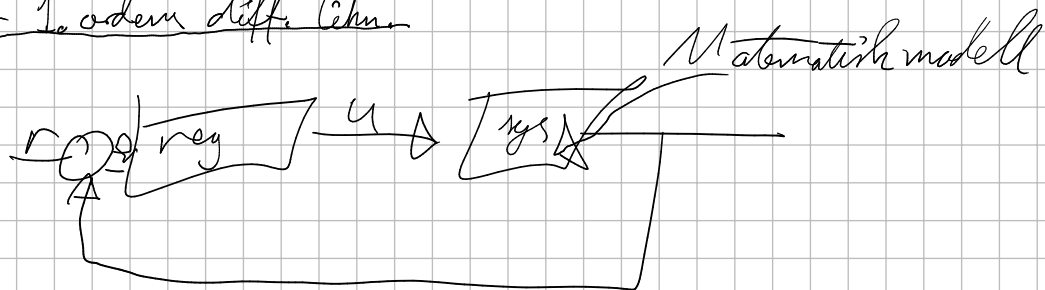$$\boxed{\dot{r} = -\frac{1}{T} + \frac{K}{T}\delta}$$

Nomoto - modell

$$a = -\frac{1}{T} \qquad b = \frac{K}{T} \qquad T = \text{Tidskonstant}$$

$$\dot{\psi} = r$$

Løsning av 1. ordens diff. likn.

Matematisk modell

$\dot{x} = ax + b$ gjemmer $u(\tau)$ for å la reg bare styre nå

$$\frac{dx}{\tau} = ax + b$$

$$\int \frac{1}{ax+b} \frac{dx}{d\tau} \, d\tau = \int 1 \, d\tau$$

$$\frac{1}{a} \ln|ax+b| = \tau + C_1$$

$$|ax+b| = e^{a(\tau + C_1)} > 0$$

$$ax + b = e^{a\tau} \underbrace{e^{aC_1}}_{C_2}$$

$$x(\tau) = Ce^{a\tau} - \frac{b}{a} \qquad C = \frac{C_2}{a} \qquad C = \frac{1}{a}e^{aC_1} \text{ er vilkårlig}$$

$$x(0) = x_0$$

$$x(0) = Ce^{0\tau} - \frac{b}{a} = x_0$$

$$C = x_0 + \frac{b}{a}$$

$$x(\tau) = \left(x_0 + \frac{b}{a}\right)e^{a\tau} - \frac{b}{a}$$

$$= x_0 e^{a\tau} + \frac{b}{a}e^{a\tau} - \frac{b}{a}$$

$$x(\tau) = x_0 e^{a\tau} + \frac{b}{a}\left(e^{a\tau} - 1\right)$$

Kochplatte:

$$\overset{o}{T} = -\frac{h}{C}T + \frac{1}{C}(P + h\,T_{rom})$$

Lösung:

$$T(t) = T_0 e^{-\frac{h}{C}t} - \frac{P + h\,T_{rom}}{h}\left(e^{-\frac{h}{C}t} - 1\right)$$

$T(0) = 20°C$

$P = 500\ W$

$h = 2\ W/°C$

$T_{rom} = 20°$

$C = 400\ J/°C$

Cruise Control

$$\overset{o}{v} = -\frac{h}{m}v + \frac{1}{m}u \;\underline{\left[- h_\ell v^2\right]}$$

$$v(t) = \frac{u}{h}\left(1 - e^{-\frac{h}{m}t}\right), \qquad v(0) = v_0 = 0$$