Øvinng 9

- 1 a) $y = x^2$, $y = 0$, $x = 1$

$$\int_0^1 x^2\,dx = \left[\tfrac{1}{3}x^3\right]_0^1 = \left(\tfrac{1}{3}\cdot 1\right) - \left(\tfrac{1}{3}\cdot 0\right) = \tfrac{1}{3}$$

- 1 b) $y = x^2$, $y = 1$, $x = 0$

$F(x) = \frac{1}{3}x^3 + C$

$F(0) = 1$

$C = 1$

$F(x) = \frac{1}{3}x^3 + 1$

- 1 c)

$$\int_0^1 \sqrt{x^2 + 4x + 4}\,dx = \int_0^1 \sqrt{(x+2)^2}\,dx = \int_0^1 x + 2\,dx = \underline{\underline{\tfrac{1}{2}x^2 + 2x + C}}$$

- 1 d)

$$\int_0^{\frac{\pi}{2}} \sqrt{1 + \cos x}\,dx$$

$$= \int_0^{\frac{\pi}{2}} (1 + \cos)^{\frac{1}{2}}\,dx$$

$\cos^2 x = \frac{1}{2}(1 - 2x) \Rightarrow \cos^2 \frac{1}{2}x = \frac{1}{2}(1 - \cos x)$

$2\cos^2\left(\frac{1}{2}x\right) = 1 - \cos x$

$$= \int_0^{\frac{\pi}{2}} \sqrt{2\cos^2\left(\tfrac{1}{2}x\right)}\,dx = \int_0^{\frac{\pi}{2}} \sqrt{2}\cos\left(\tfrac{1}{2}x\right)dx = \left[\sqrt{2}\cdot\frac{1}{\frac{1}{2}}\sin\left(\tfrac{1}{2}x\right)\right]_0^{\frac{\pi}{2}} = \left[2\sqrt{2}\sin\left(\tfrac{1}{2}x\right)\right]_0^{\frac{\pi}{2}}$$

$$= \underline{\underline{2\sqrt{2}\sin\left(\tfrac{\pi}{4}\right)}}$$

- 1 3)

$f(x) = \sin^2 x$

$f(x) = 0$

$\sin^2 x = 0$

$(\sin x)(\sin x) = 0$

$x = \frac{\pi}{2} + \pi n \quad n \in N$

$F(x) = \int_0^{\frac{\pi}{2}} \sin^2(x)\, dx \qquad \sin^2(x) = \frac{1}{2}(1 - \cos 2x)$

$= \int_0^{\frac{\pi}{2}} \frac{1}{2}(1 - \cos 2x)\, dx = \frac{1}{2}\int_0^{\frac{\pi}{2}} 1 - \cos 2x\, dx$

$= \frac{1}{2}\left[x - \frac{1}{2}\sin(2x)\right]_0^{\frac{\pi}{2}}$

$= \frac{1}{2}\left(\frac{\pi}{2} - \frac{1}{2}\sin(\pi)\right) \underline{\underline{= \frac{\pi}{4}}}$

- 1 2) $\int \frac{\sin(x)}{1 + \cos(x)}\, dx \qquad u = 1 + \cos(x)$
$\qquad u' = -\sin(x)$

$= \int u' \cdot \left(-\frac{1}{u}\right) dx$

$= \int -\frac{1}{u} \cdot \cancel{\frac{du}{dx}\, dx}$

$= \int -\frac{1}{u}\, du$

$= -\ln|u| + C$

$= -\ln(1 + \cos x) + C$

- 1 1) $\int e^x \sqrt{1 + e^x}\, dx \qquad u = 1 + e^x$

$= \int \sqrt{u}\, du$

$= \int u^{\frac{1}{2}}\, du$

$= \left(\frac{1}{\frac{1}{2}+1}\right) u^{\frac{3}{2}} + C = \frac{1}{\frac{3}{2}} u^{\frac{3}{2}} + C = \frac{3}{2}\sqrt{u^3} + C$

-1 0) $\int x e^{x^2} dx \qquad u = x^2 \qquad u' = 2x \Rightarrow x = \frac{u'}{2}$

$= \int u' \frac{1}{2} e^u \, dx$

$= \frac{1}{2} \int e^u \, du$

$= \frac{1}{2} \cdot \frac{1}{4} e^u + C$

$= \frac{1}{2x^2} e^{x^2} + C$

-1) $\int \frac{1}{\sqrt{x}(x+1)} dx = \int \frac{1}{\sqrt{x}} \cdot \frac{1}{x+1} dx$

$\frac{1}{x^{\frac{3}{2}} + x^{\frac{1}{2}}}$

$= \int x^{-\frac{1}{2}} \cdot (x+1)^{-1} dx$

$(x^{-\frac{1}{2}})' = -\frac{1}{2} x^{-\frac{3}{2}} \qquad u = x^{-\frac{1}{2}}$

$u' = -\frac{1}{2} x^{-\frac{3}{2}} \qquad u = \frac{1}{x^{\frac{1}{2}}}$

$-2u' = x^{-\frac{3}{2}} \qquad x^{\frac{1}{2}} = \frac{1}{u}$

$-2u' = \frac{1}{x^{\frac{3}{2}}}$

$x^{\frac{3}{2}} = -\frac{1}{2u'}$

$\int \frac{1}{x^{\frac{3}{2}} + x^{\frac{1}{2}}} dx = \int \frac{1}{-\frac{1}{2u'} + \frac{1}{u}} dx = \int \left(\frac{1}{u} - \frac{1}{2u'}\right)^{-1} dx = \int \left(\frac{2u' - u}{2uu'}\right)^{-1} dx$

$= \int \frac{2uu'}{2u' - u} dx = \int \frac{2\frac{du}{dx} u}{u(2\frac{d}{du} - 1)} dx = \int \frac{2\frac{du}{dx}}{2\frac{d}{du} - 1} dx$

nei 😮

$$\int \frac{1}{\sqrt{x}(x+1)}\,dx \qquad u = \sqrt{x} \qquad \frac{du}{dx} = \frac{1}{2\sqrt{x}} = \frac{1}{2u}$$

$$2u\,du = dx$$

$$= \int \frac{1}{u(u^2+1)} \cdot 2u\,du$$

$$= 2\int \frac{1}{u^2+1}\,du$$

$$= 2\tan^{-1}(u) + C$$

$$= \underline{2\tan^{-1}(\sqrt{x}) + C}$$

- 8)

$$f(x) = \frac{x}{(x^2+16)} \qquad y=0,\ x=0,\ x=2$$

$$F(x) = \int \frac{x}{(x^2+16)}\,dx \qquad u = x^2+16$$

$$\frac{du}{dx} = 2x$$

$$\frac{1}{2x}\,du = dx$$

$$= \int \frac{x}{u} \cdot \frac{1}{2x}\,du$$

$$= \int \frac{1}{2u}\,du$$

$$= \ln|2u| + C$$

$$= \ln(x^2+16) + C$$

$$\left[\ln(x^2+16)+C\right]_0^2 = \ln(4+16) = \ln(20) = \ln(2^2\cdot 5) = \underline{2\ln 2 + \ln 5}$$

$-7) \int_a^b f(x)\,g'(x)\,dx = \left[f(x)\,g(x)\right]_a^b - \int_a^b f'(x)\,g(x)\,dx$ Greek

$$\int_0^1 e^x \cos x = \left[e^x \sin x\right]_0^1 - \int_0^1 e^x \sin x \, dx$$

$$= \left[e^x \sin x\right]_0^1 - \int_0^1 (\cos x + i \sin x) \sin x \, dx$$

$$= \left[e^x \sin x\right]_0^1 - \int_0^1 \cos x \sin x + i \sin^2 x \, dx$$

$$= \left[e^x \sin x\right]_0^1 - \int_0^1 \frac{1}{2} \sin 2x + i \sin^2 x$$

$$\int_0^1 e^x \cos x \, dx$$

$$e^{ix} = \cos x + i \sin x$$

$$\cos^x = e^{ix} + i \sin x$$

$$= \int_0^1 \cos(x)\left(\cos\left(\frac{x}{i}\right) + i \sin\left(\frac{x}{i}\right)\right)$$

$$= \int_0^1 \cos(x) \cos\left(\frac{x}{i}\right) + i \cos(x) \sin\left(\frac{x}{i}\right)$$

$$= \int_0^1 \frac{1}{2}\left(\cos\left(\left(1 - \frac{1}{i}\right) x\right) + \cos\left(\left(1 + \frac{1}{i}\right) x\right)\right) + \frac{1}{2}\left(\right.$$

$$\int_0^1 e^x \cos x \, dx$$

$$\int_a^b u v' dx = [uv]_a^b - \int_a^b u' v \, dx$$

$$\int_0^1 v' u \, dx = [uv]_0^1 - \int_0^1 v u' \, dx$$

$u = \cos x \Rightarrow \frac{du}{dx} = -\sin x$

$du = (-\sin x) \, dx$

$$= [e^x \cos x]_0^1 - \int_0^1 e^x (-\sin x) \, dx$$

$\frac{dv}{dx} = e^x \Rightarrow v = e^x$

$dv = e^x dx$

$$= [e^x \cos x]_0^1 + \int_0^1 e^x \sin x \, dx$$

$$= [e^x \cos x]_0^1 + [-e^x \cos x]_0^1 - \int_0^1 e^x (-\cos x) \, dx$$

$$= [e^x \cos x]_0^1 + [-e^x \cos x]_0^1 + \int e^x \cos x \, dx$$

$$\int_0^1 e^x \cos x = J$$

$$= [e^x \cos x]_0^1 - \int_0^1 e^x (-\sin x) \, dx$$

$$= [e^x \cos x]_0^1 + \int_0^1 e^x \sin x \, dx$$

$$\int_0 e^x \sin x = [e^x \sin x]_0^1 - \int_0 e^x \cos x \, dx$$

$$= [e^x \cos x]_0 + [e^x \sin x]_0^1 - \int_0 e^x \cos x \, dx$$

$$= [e^x \cos x]_0^1 + [e^x \sin x]_0^1 - J$$

$$2J = [e^x \cos x]_0^1 + [e^x \sin x]_0^1$$

$$J = \frac{[e^x \cos x]_0^1 + [e^x \sin x]_0^1}{2}$$

$$J = \frac{e \cos 1 + e \sin 1}{2} = \underline{\frac{e(\cos 1 + \sin 1)}{2}}$$

- 6) $\int_1^{\infty} \frac{1}{x} dx = [\ln x]_1^{\infty} = \ln \infty - \ln 1 = \infty$??

- 5) $\int_1^{\infty} \frac{1}{x^2} dx = \int_1^{\infty} x^{-2} dx = [-x^{-1}]_1^{\infty} = [-\frac{1}{x}]_1^{\infty} = -\frac{1}{\infty} + \frac{1}{1} = \underline{\underline{1}}$

- 4) $\int_1^{\infty} \frac{1}{\sqrt{x}} dx = \int_1^{\infty} x^{-\frac{1}{2}} dx = [2x^{\frac{1}{2}}]_1^{\infty}$ = inf

- 3) $\int_0^1 \frac{1}{x} dx = [\ln x]_0^1 = \ln 1 - \ln 0 = 0$ - udefinert

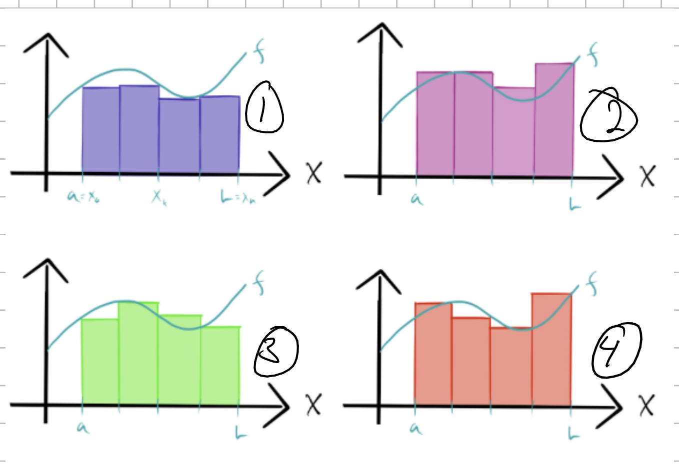
- 2) $\int_0^1 \frac{1}{x^2} dx = [-\frac{1}{x}]_0^1 = -1 -$ inf

- 1) $\int_0^1 \frac{1}{\sqrt{x}} = [2x^{\frac{1}{2}}]_0^1 = 2 - 0 = \underline{\underline{2}}$

1) Nei !

2) Python

3)



1 nedre

2 øvre

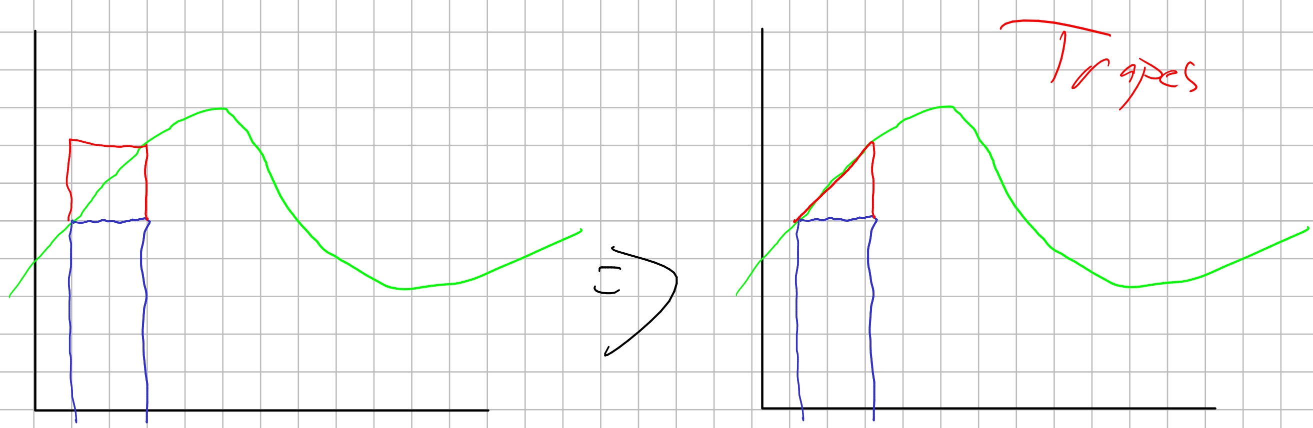
3 venstre

4 høyre

4) $\int (1+x^2)^{\frac{1}{3}}\,dx = \frac{3}{4}(1+x^2)^{\frac{4}{3}} \cdot \frac{1}{2x} + C$

Tydeligvis ikke

7)



9) $p(x) = a x^2 + bx + c$ $(1,2)$ $(2,3)$ $(3,1)$

$p(1) = 2$

$1a + 1b + c = 2$

$p(2) = 3$

$4a + 2b + c = 3$

$p(3) = 1$

$9a + 3b + c = 1$

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 4 & 2 & 1 & 3 \\ 9 & 3 & 1 & 1 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 0 & -2 & -3 & -5 \\ 0 & -6 & -8 & -1 \end{array}\right)$$

$$\sim \left(\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 0 & -2 & -3 & -5 \\ 0 & 0 & 1 & -2 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 0 & -2 & 0 & -11 \\ 0 & 0 & 1 & -2 \end{array}\right)$$

$$\sim \left(\begin{array}{ccc|c} 1 & 0 & 0 & -3/2 \\ 0 & 1 & 0 & 11/2 \\ 0 & 0 & 1 & -2 \end{array}\right)$$

$p(x) = -\frac{3}{2} x^2 + \frac{11}{2} x - 2$

10) $l_k(x) = \prod_{\substack{j=0 \\ j \neq k}}^{n} \frac{(x - x_j)}{(x_k - x_j)}$

$l_k(x_m) = \begin{cases} 1, & m = k \\ 0, & m \neq k \end{cases}$?

$l_k(x) = \prod_{\substack{j=0 \\ j \neq k}}^{n} \frac{(x - x_j)}{(x_k - x_j)} = \frac{(x - x_0)}{(x_k - x_0)} \cdot \frac{(x - x_1)}{(x_k - x_1)} \cdot \frac{(x - x_2)}{(x_k - x_2)} \cdot \ldots \cdot \frac{(x - x_n)}{(x_k - x_n)}$

$n + 1 - j_k$ faktorer $\Rightarrow$ $\underline{\underline{n - \text{faktorer}}}$

$m = k$

$l_k(x_m) = \prod_{\substack{j=0 \\ j \neq k}}^{n} \frac{(x_m - x_j)}{(x_m - x_j)} = 1$ ✓

$m \neq k$

$$l_k(x_m) = \prod_{\substack{j=0 \\ j \neq k}}^{n} \frac{(x_m - x_j)}{(x_k - x_j)} = \frac{(x_m - x_0)}{(x_k - x_0)} \cdot \frac{(x_m - x_1)}{(x_k - x_1)} \cdot \frac{(x_m - x_2)}{(x_k - x_2)} \cdot \ldots \cdot \frac{(x_m - x_n)}{(x_k - x_n)}$$

m er mellom $j=0$ & $j=n$

$$\frac{(x_m - x_m)}{(x_k - x_m)} = \frac{0}{x_k - 0} = 0$$

$\underline{l_k(x_m) = 0}$

11) $p_n(x) = \sum_{k=0}^{n} y_k l_k(x)$ (0,1) (1,0) (2,1) (3,2)

$$l_k(x) = \prod_{\substack{j=0 \\ j \neq k}}^{n} \frac{(x - x_j)}{(x_k - x_j)} = \frac{x - x_0}{x_k - x_0} \cdot \frac{x - x_1}{x_k - x_1} \cdot \frac{x - x_2}{x_k - x_2} \cdot \frac{x - x_3}{x_k - x_3}$$

$$= \frac{x - 0}{x_k - 0} \cdot \frac{x - 1}{x_k - 1} \cdot \frac{x - 2}{x_k - 2} \cdot \frac{x - 3}{x_k - 3}$$

$$= \frac{x - 0}{x_k - 0} \cdot \frac{x - 1}{x_k - 1} \cdot \frac{x - 2}{x_k - 2} \cdot \frac{x - 3}{x_k - 3}$$

$$p(x) = \sum_{k=0}^{n} y_k l_k(x) = y_0 l_0(x) + y_1 l_1(x) + y_2 l_2(x) + y_3 l_3(x)$$

$$= 1 \cdot \left(\frac{x-1}{0-1} \cdot \frac{x-2}{0-2} \cdot \frac{x-3}{0-3}\right)$$

$$+ 0 \cdot \left(\frac{x-0}{1-0} \cdot \frac{x-2}{1-2} \cdot \frac{x-3}{1-3}\right)$$

$$+ 1 \cdot \left(\frac{x-0}{2-0} \cdot \frac{x-1}{2-1} \cdot \frac{x-3}{2-3}\right)$$

$$+ 2 \cdot \left(\frac{x-0}{3-0} \cdot \frac{x-1}{3-1} \cdot \frac{x-2}{3-2}\right)$$

$$= 1 \cdot \left(\frac{x-1}{-1} \cdot \frac{x-2}{-2} \cdot \frac{x-3}{-3}\right)$$

$$+ 1 \cdot \left(\frac{x}{2} \cdot (x-1) \cdot \frac{x-3}{-1}\right)$$

$$+ 2 \cdot \left(\frac{x}{3} \cdot \frac{x-1}{2} \cdot (x-2)\right)$$

$$= \frac{(x-1)(x-2)(x-3)}{-6} + \frac{x(x-1)(x-3)}{-2} + \frac{x(x-1)(x-2)}{3}$$

$$= \frac{-(x-1)(x-2)(x-3) - 3x(x-1)(x-3) + 2x(x-1)(x-2)}{6}$$

$$= \frac{1}{6}(x-1)\left(-(x-2)(x-3) - 3x(x-3) + 2x(x-2)\right)$$

$$= \frac{1}{6}(x-1)\left(-x^2 + 5x - 6 - 3x^2 + 9x + 2x^2 - 4x\right)$$

$$= \frac{1}{6}(x-1)\left(-2x^2 + 10x - 6\right)$$

$$= \frac{1}{6}\left(-2x^3 + 10x^2 - 6x + 2x^2 - 10x + 6\right)$$

$$= \frac{1}{6}\left(-2x^3 + 12x^2 - 16x + 6\right)$$

Riktig 8

1 2) $\int_a^b f(x)\,dx$

$a = x_0, \quad b = x_n$

$$\int_a^b f(x)\,dx \approx \int_a^b p(x)\,dx = \int_a^b \sum_{k=0}^{n} f(x_k)\,l_k(x)\,dx = \sum_{k=0}^{n} f(x_k) \int_a^b l_k(x)\,dx$$

$$w_k = \int_a^b l_k(x)\,dx$$

$$\int_a^b f(x)\,dx \approx \sum_{k=0}^{n} w_k f(x_k)$$

$n = 1$

$$\sum_{k=0}^{1} w_k f(x_k) = w_0 f(x_0) + w_1 f(x_1) \qquad l_k(x) = \frac{x - x_0}{x_k - x_0} \cdot \frac{x - x_1}{x_k - x_1}$$

$$= f(x_0)\int_a^b l_0(x)\,dx + f(x_1)\int_a^b l_1(x)\,dx$$

$$= f(x_0)\int_a^b \frac{x - x_1}{x_0 - x_1}\,dx + f(x_1)\int_a^b \frac{x - x_0}{x_1 - x_0}\,dx$$

$$= f(x_0) \frac{1}{x_0 - x_1} \int_a^b x - x_1 \, dx - f(x_1) \frac{1}{x_0 - x_1} \int_a^b x - x_0 \, dx$$

$$= \frac{1}{x_0 - x_1} \left(f(x_0) \cdot \left[\frac{1}{2} x^2 - x_1 x \right]_a^b - f(x_1) \cdot \left[\frac{1}{2} x^2 - x_0 x \right]_a^b \right)$$

$$= \frac{1}{x_0 - x_1} \left(f(x_0) \cdot \left(\left(\frac{1}{2} b^2 - b x_1 \right) - \left(\frac{1}{2} a^2 + x_1 a \right) \right) - f(x_1) \left(\left(\frac{1}{2} b^2 - b x_0 \right) - \left(\frac{1}{2} a^2 - a x_0 \right) \right) \right)$$

$$= \frac{1}{x_0 - x_1} \left(f(x_0) \cdot \left(\frac{1}{2} b^2 - \frac{1}{2} a^2 \right) - f(x_0) \cdot \left(-b x_1 - a x_1 \right) - f(x_1) \left(\frac{1}{2} b^2 - \frac{1}{2} a^2 \right) + f(x_1) \cdot \left(-b x_0 - a x_0 \right) \right)$$

$$= \frac{1}{a - b} \left(f(a) \cdot \left(\frac{1}{2} b^2 - \frac{1}{2} a^2 \right) - f(a) \cdot \left(-b^2 - ab \right) - f(b) \left(\frac{1}{2} b^2 - \frac{1}{2} a^2 \right) + f(b) \cdot \left(-ab - a^2 \right) \right)$$

$$= \frac{1}{a - b} \left(f(a) \cdot \frac{1}{2} \left(\right. \right.$$

$a = x_0 \quad b = x_n$

$$\sum_{k=0}^{1} w_k f(x_k) = w_0 f(x_0) + w_1 f(x_1) \qquad l_k(x) = \frac{x - x_0}{x_k - x_0} \cdot \frac{x - x_1}{x_k - x_1}$$

$$= f(x_0)\int_a^b l_0(x)\,dx + f(x_1)\int_a^b l_1(x)\,dx$$

$$= f(x_0)\int_a^b \frac{x - x_1}{x_0 - x_1}\,dx + f(x_1)\int_a^b \frac{x - x_0}{x_1 - x_0}\,dx$$

$$= f(a)\int_a^b \frac{x - b}{a - b}\,dx + f(b)\int_a^b \frac{x - a}{b - a}\,dx$$

$$= f(a) \cdot \frac{1}{a-b}\int_a^b x - b\,dx + f(b) \cdot \frac{1}{b-a}\int_a^b x - a\,dx$$

$$= f(a) \cdot \frac{1}{a-b}\left[\frac{1}{2}x^2 - bx\right]_a^b + f(b) \cdot \frac{1}{b-a}\left[\frac{1}{2}x^2 - ax\right]_a^b$$

$$= f(a) \cdot \frac{1}{a-b}\left(\frac{1}{2}b^2 - b^2 - \frac{1}{2}a^2 + ba\right) + f(b) \cdot \frac{1}{b-a}\left(\frac{1}{2}b^2 - ab - \frac{1}{2}a^2 + a^2\right)$$

$$= f(a) \cdot \frac{1}{a-b}\left(-\frac{1}{2}b^2 + ba - \frac{1}{2}a^2\right) + f(b) \cdot \frac{1}{b-a}\left(\frac{1}{2}a^2 - ab + \frac{1}{2}b^2\right)$$

$$= f(a) \cdot \frac{1}{a-b}\left(-\frac{1}{2}(b^2 - 2ab + a^2)\right) + f(b) \cdot \frac{1}{b-a}\left(\frac{1}{2}(a^2 - 2ab + \frac{1}{2}b^2\right)$$

$$= f(a) \cdot \frac{1}{a-b}\left(-\frac{1}{2}(a - b)^2\right) + f(b) \cdot \frac{1}{b-a}\left(\frac{1}{2}(b - a)^2\right)$$

$$= f(a) \cdot \left(-\frac{1}{2}(a - b)\right) + f(b) \cdot \frac{1}{2}(b - a)$$

$$= f(a) \cdot \frac{1}{2}(b - a) + f(b) \cdot \frac{1}{2}(b - a)$$

$$= \frac{1}{2}(b - a)\left(f(a) + f(b)\right)$$

$$\underline{= \frac{b - a}{2}\left(f(a) + f(b)\right)}$$

Trapez metode

1 3) Neitoe